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ECONOMIC INSTITUTIONS AND STABILITY: A NETWORK APPROACH

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Economic Institutions and Stability: A Network Approach*

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Abstract

We consider a network economy in which economic agents are connected within a structure of value-generating relationships. Agents are assumed to be able to participate in three types of economic activities: autarkic self-provision; binary matching interactions; and multi-person cooperative collaborations. We introduce two concepts of stability and provide sufficient and necessary conditions on the prevailing network structure for the existence of stable assignments, both in the absence of externalities from cooperation as well as in the presence of size-based externalities. We show that institutional elements such as the emergence of socioeconomic roles and organizations based on hierarchical leadership structures are necessary for establishing stability and as such support and promote stable economic development.

Keywords: Cooperatives, Networks, Clubs, Network economies, Stable matchings

JEL code: C72, D71, D85

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1 Market makers and stability

An age-old question in economics is how complex structures and organizations emerge from choices made by individual decision makers in a decentralized economy. In this paper we investigate the emergence of organizations that are formed through the leadership of endogenously emerging market makers. Our model allows not only for the endogenous selection of such market makers, but also for the endogenous determination of the size of these organizations. Our main insight is that behavioral rules support the emergence of stable network structures or institutions in such economies.

Our approach considers economic agents as being embedded in a given network of potential value-generating economic relationships that they can activate. Within this framework, we focus on the stability of emerging patterns of activated relationships. We use straightforward extensions of standard equilibrium concepts from matching theory (Roth and Sotomayor, 1990) and network formation theory (Jackson and Wolinsky, 1996) to describe such stable assignments. We then identify conditions on the network structure of value-generating activities that guarantee the existence of such stable assignments. These conditions point unquestionably to institutional features of the network as representing the social capital instilled in these networks. (Portes, 1998; Dasgupta, 2005) This allows us to additionally interpret economic institutions as social rules that support and guarantee universal stability in an economy. Instability of such patterns, on the other hand, is manifested in a dysfunctional institutional organization in the economy.

Through the work of Coase (1937), North and Thomas (1973), Williamson (1975), North (1990) and Greif (2006), institutions are usually understood as devices that lower market transaction costs. Lower transaction costs in turn result into increased market efficiency and consequently economic growth and development. Our approach, instead, takes these institutions as fitting specifications of underlying network properties and forces. We consider institutions as functional stabilizers and promoters of economic development and growth (Klaes, 2000). From this viewpoint institutional development is more closely related to Smithian development based on the deepening of the social division of labor, seminally proposed by Smith (1776).

More specifically, our approach is based on a notion of stability as being attributed to the network of socioeconomic relations itself; we identify network properties such that for *every* possible configuration of individuals' productive abilities and preferences, the economy possesses at least one equilibrium state. Individuals deliberately activate certain potential relationships when engaging in economic interaction. We identify stable configurations of activated relationships and determine the structural conditions under which stable configurations arise for arbitrary distributions of skills, productive capabilities and preferences. In particular, we find that certain forms of network acyclicity form a class of crucial properties that allow for equilibrium configurations of matches to emerge. We follow a two-stage development of our theory. First, we discuss stability in bilateral matching markets and, subsequently, in the setting of multi-agent collaborations in network economies.

To show the fundamental principles of our approach, we present some simple configurations and debate concepts that are required to describe the endogenous emergence of stable interaction patterns.

1.1 Matching economies

We first address how economic agents engage in pairwise interactions that are mutually beneficial. Throughout, we assume that economic agents can only potentially engage in economic interaction with a limited set of partners. Thus, the society is endowed with a set of potential economic relationships that can be activated. Such a network of potential relationships is depicted in Figure 1 on a set of five economic agents $N = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}\}$. Hence, the situation depicted in Figure 1 does not allow agents **a** and **c** to engage in mutually beneficial interaction. Similarly for the pairs **a,d** and **c,d**.

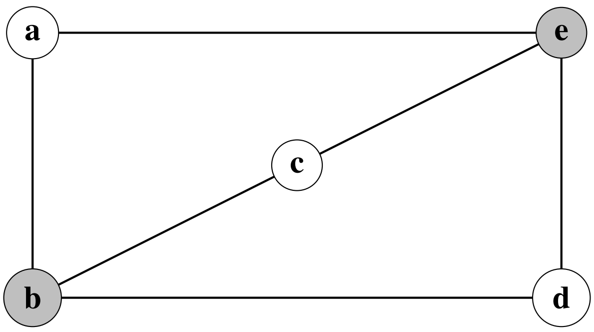

Figure 1: Network structure $\mathcal{A}$

We assume that in the network structure $\mathcal{A}$ depicted in Figure 1, agents interact by exchanging (indivisible) favors (**?**). Thus, two linked individuals i and j in N can mutually decide to activate the relationship ij and exchange favors. Such exchanges result into a hedonic utility values $u_i(ij)$ that accrues to agent i from forming the exchange relationship ij with j. We represent these hedonic utility values in the following matrix.[1]

[1] The matrix is actually the incidence matrix representing network structure $\mathcal{A}$ in which potential payoffs are reported. The number reported in field (i, j) is $u_i(ij)$. Similarly, the field (j, i) reports $u_j(ij)$. If no relationship can be formed, no payoff is reported. Note that we assume that agent $i \in N$ does not exchange favors with herself in autarky, i.e., $u_i(ii) = 0$.

	a	**b**	**c**	**d**	**e**
a	0	2	–	–	1
b	1	0	2	2	–
c	–	1	0	–	2
d	–	1	–	0	2
e	2	–	1	1	0

In a matching economy we investigate a standard notion of stability. We assume that every agent can activate *at most one* (potential) exchange relationship, forming a so-called *exchange pattern*. An exchange pattern is *stable* if (i) there is no agent who prefers to remain in autarky rather than engage with the exchange in the proposed pattern ("individual rationality"); and (ii) there is no pair of agents who prefer to exchange favors rather than exchanging favors with their assigned partners ("pairwise stability"). In the constructed example there emerge two stable exchange patterns: $\pi_1 = \{\mathbf{ae}, \mathbf{bc}, \mathbf{dd}\}$ and $\pi_2 = \{\mathbf{ae,bd,cc}\}$. In fact, our main existence result stated as Theorem 3.5 implies that for any distribution of utilities resulting from favor exchange for network structure $\mathcal{A}$ there exists a stable exchange pattern that satisfies (i) and (ii) above.

Next we modify the structure of potential relationships on N as depicted in Figure 2. It is clear that agent **c** now occupies a centralized position and can interact with any of the four other agents.

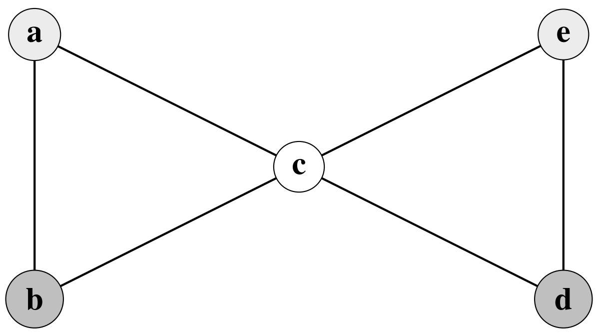

Figure 2: Network structure $\mathcal{B}$

We report the payoffs from favor exchange for structure $\mathcal{B}$ in the matrix below. The underlying favors are similarly distributed as the case considered in structure $\mathcal{A}$.

	a	**b**	**c**	**d**	**e**
a	0	2	1	–	–
b	1	0	2	–	–
c	2	1	0	2	1
d	–	–	1	0	2
e	–	–	2	1	0

We now claim that for the given payoffs there does not exist a stable exchange pattern in B. Indeed, consider pattern $\pi' = \{\mathbf{ab}, \mathbf{cd}, \mathbf{ee}\}$, then both agents **d** and **e** would prefer to exchange favors rather than being engaged with **c** and being autarkic, respectively. Other patterns can be shown to be unstable as well.

What makes exchange structure $\mathcal{B}$ more prone to instability than exchange structure $\mathcal{A}$? We find that the unique feature of a network economy that allows agents to be divided into two distinct economic roles—dark grey nodes versus white ones in Figure 1—such that favor exchange is potentially only carried out between any two agents of distinct colors or "roles". On the other hand, structure $\mathcal{B}$ requires three distinct colors or roles, indicated as dark grey, light grey and white. This feature ensures the stability in a structure like $\mathcal{A}$, and conversely, the impossibility of stability in a structure like $\mathcal{B}$. Thus, stability of exchange is founded on the property that the network structure has an institutional foundation on exactly two socio-economic roles.

1.2 Introducing market makers

Next we introduce the ability of economic agents to engage in multi-agent collaborations or *cooperative economic activities*. In our network setting, a cooperative activity requires the active involvement of a *convener*, who brings together the group of economic agents that forms this value-generating cooperative economic activity. Thus, the convener facilitates the cooperative activity and acts in all respects as a "market maker".[2] In this regard these cooperative activities are relational forms of clubs introduced seminally by Buchanan (1965). We consider the case that a convener can only invite economic agents to participate in a cooperative activity if they have a potential relationship with her. In other words, the convener only collaborates with acquaintances.

Furthermore, the economic values generated through these cooperative activities are expressed as hedonic utilities. The notion of hedonic games in the context of coalition formation was seminally introduced by Drèze and Greenberg (1980) and further studied by Bogomolnaia and Jackson (2004), Banerjee, Konishi, and Sonmez (2001), and Pápai (2004), among others.[3] This is a standard technique from the theory of clubs as well. We refer to the review of Scotchmer (2002) for a discussion of this technique. It allows us to reduce the analysis of the formation of relational cooperative activities to a single dimension, expressed through the hedonic utility functions of the various economic agents.

We thus arrive at a network economy in which economic agents can engage into three types of economic activities: autarkic self-provision, binary matching interactions, and cooperative activities formed through intermediation of a convener. Each of these three types

[2] In this setting a market is now a cooperative economic activity in which the market auctioneer acts as its convener. So, standard markets can be viewed as a special category of cooperative economic activity.

[3] We point out that what distinguishes our work from those studying coalition formation games is that we employ a network approach.

of activities generates different hedonic utility levels for its participants. We explicitly assume that there are no widespread externalities among the various distinct activities; the generated values are solely the outcome of the activities themselves.[4]

Returning to the examples developed above for network patterns A and B, we introduce simple additive multi-agent collaborations as follows. A collaborative is now a star-structured subnetwork of the imposed network pattern. Thus, in structure $\mathcal{A}$ agent **a** can collaborate with **b** and **e**, while agent **b** could principally collaborate with **a, c** and **d**. The convener is now the agent in the center of the star-structured subnetwork.

In the example we assume that there are no spillovers in the payoffs from multi-agent collaboratives. This means that, if an agent $i \in N$ collaborates *as a convener* with agents $j \in G_i \subset N \setminus \{i\}$, then agent i collects a net benefit of

$$u_i(G) = \sum_{j \in G_i} u_i(ij) - \delta, \tag{1}$$

where $G = G_i \cup \{i\}$ is the complete cooperative and $\delta \geqslant 0$ is a common cost of cooperation. Thus, if a convenes $G = \mathbf{abe} = \{\mathbf{a}, \mathbf{b}, \mathbf{e}\}$ in structure $\mathcal{A}$, then he receives $u_a(\mathbf{abe}) = u_a(\mathbf{ab}) + u_a(\mathbf{ae}) - \delta = 3 - \delta$. On the other hand, agent j acting as a regular member of a collaborative around convener i just receives $u_j(ij)$ from her participation in this collaborative. Thus, $u_b(\mathbf{abe}) = u_b(\mathbf{ab}) = 1$.

We devise a standard equilibrium concept in which each agent participates in exactly one permissible economic activity. In equilibrium, no agent has an incentive to join another potentially accessible activity. Such an equilibrium is called a *stable assignment*.

In the case $\delta < 1$, there is no such stable assignment in structure $\mathcal{A}$ as depicted in Figure 1. Indeed, take $\{\mathbf{ab}, \mathbf{ecd}\}$, then agents **a** and **b** engage in pairwise exchange and obtain $u_a(\mathbf{ab}) = 2$ and $u_b(\mathbf{ab}) = 1$, respectively. On the other hand, agent **e** as a convener receives $u_e(\mathbf{ecd}) = 2 - \delta$, while (regular) members **c** and **d** receive $u_c(\mathbf{ecd}) = u_d(\mathbf{ecd}) = 2$. Now, agents **a** and **e** can mutually improve their positions and agent **b** will not suffer by engaging in collaborative **abe**, where **a** acts as its convener. Indeed, $u_a(\mathbf{abe}) = 3 - \delta > 2 = u_a(\mathbf{ab})$, $u_e(\mathbf{abe}) = 2 > 2 - \delta = u_e(\mathbf{ecd})$ and $u_b(\mathbf{abe}) = u_b(\mathbf{ab}) = 1$. Similarly, one can show that in all other exchange patterns there will be a profitable deviation, showing universal instability.

In contrast, in structure $\mathcal{B}$ depicted in Figure 2 there exists a stable assignment or exchange pattern, namely the complete collaboration $\{\mathbf{cabde}\}$ convened around agent **c**. Here, $u_c(\mathbf{cabde}) = 6 - \delta$, $u_a(\mathbf{cabde}) = u_d(\mathbf{cabde}) = 1$ and $u_b(\mathbf{cabde}) = u_e(\mathbf{cabde}) = 2$. Now, agent **a** would rather be exchanging favors with **b**, but agent **b** would not agree due to lowering her payoff. We show in Theorem 4.6 that in fact for any payoff structure without spillovers there exists a stable exchange pattern in structure $\mathcal{B}$.

It is clear from the discussion above that stability is again determined by the (insti-

[4]This does not, however, exclude various forms of externalities among the members of a cooperative.

tutional) features of the network structure underlying the economy. Our main existence theorems exactly determine these conditions. In its full development, we consider different forms of stability that implement certain features of multi-agent collaboration. We distinguish "open" cooperatives from "closed" cooperatives in that in the latter a convener fully controls the admittance of agents to the cooperative, while in the former this control is limited. Openness is a requirement for stability if there are certain spillover effects among collaborating agents. Closedness can be supported in the absence of such externalities.

The remainder of this paper is organized as follows. Section 2 introduces our network approach to economic interaction. Section 3 discusses stability in matching economies , while in Section 4 we extend this model to include multi-agent cooperative economic activities. In this setting we analyze the emergence of stable interaction patterns if there are no externalities and discuss the implications of the introduction of certain externalities. Section 5 offers some concluding remarks and directions for future research. Proofs are collected in the appendices.

2 A network approach to economic activities

In this section we introduce some basic definitions and fundamental concepts from social network theory[5] and we develop the key concepts in our network approach to describing economic activities. In particular, we use links between economic agents to describe primitive binary economic interaction.

The main postulate of our theoretical construction is that all economic activities are principally relational. More precisely, a multi-agent economic collaboration is assumed to be structured as a collection of binary economic network relationships: Each cooperative economic activity is embedded within the prevailing network of binary economic relationships in the sense of Granovetter (1985). Henceforth, our theory is founded on purely abstract relational activities without explicit reference to other primitive concepts such as commodities or trade and production technologies. Therefore, an *economic activity* is abstractly defined as any economic interaction between a group of linked economic agents that generates a hedonic utility value for each of its participants (Granovetter, 2005).

We emphasize that in our approach the economy solely consists of relational activities. Within this framework a market is viewed as a value-generating cooperative activity. But a market is local rather than global and anonymous; it is only open to its members, where potential membership is determined by the underlying structure of potential trade relationships.[6]

[5] Here we refer to Jackson and Wolinsky (1996), Bala and Goyal (2000), Jackson (2003), and Gilles and Sarangi (2010) for a comprehensive overview of network theory.

[6] We believe that this view conforms with markets in the global economy such as the New York Stock

Relational economic activities

Throughout we work with a finite set of economic agents denoted by $N = \{1, ..., n\}$. These economic agents can engage in three different relational economic activities that generate individual values for the participants. The first and most primitive form of economic activity is that of *economic autarky*. In this case an agent $i \in N$ engages in home production. For an individual economic agent $i \in N$ we denote by ii the agent's possibility to engage autarkically in activities that allow her to attain a subsistence level. Thus we arrive at the *class of all autarkic activities* denoted by

$$\Omega = \{ii \mid i \in N\}. \tag{2}$$

The second level of economic activity is that of an economic *matching* in the sense that two agents i and j engage into some interaction that generates (hedonic utility) values for both of these agents. This form of relational activity is purely binary. These matchings capture any exchange relationship or interaction among intermediate good producers who provide specialized inputs for final good producers.

Formally, consider any $i, j \in N$ with $i \neq j$. Now we denote by the mathematical expression $ij = \{i, j\}$ a binary economic activity involving agents i and j.[7] The binary economic activity ij is called the *matching* of agents i and j if i and j can potentially engage into the relationship and achieve mutual benefits from this relationship. Clearly, not every arbitrary pair of economic agents can potentially form a matching. Formally, we introduce Γ as the class of all (feasible) matchings with

$$\Gamma \subseteq \Gamma_N = \{ij \mid i, j \in N \text{ and } i \neq j\}. \tag{3}$$

Throughout we assume that for every agent $i \in N$ there is some $j \in N$ with $ij \in \Gamma$.

We regard a matching $ij \in \Gamma$ to be purely potential in nature in the sense that the members constituting the activity ij have to consent to participate in this activity before it is realized. Since Γ is a subset of the set of all possible pairings Γ_N, it is designed to capture physical, institutional, or any other constraints that may prohibit the occurrence of economic matching activities between certain agents. In this regard the structure Γ reflects the relational trust that is present among the agents in the economy. Indeed, an agent trusts that engaging in an economically relevant relationship with another distinct agent will indeed result in a beneficial outcome for herself. These restrictions, however, cannot preclude opting out of any engagement in any economic relationship with others in the sense that each $i \in N$ can

exchange (NYSE), NASDAQ, and the Chicago commodity markets. All of these markets are essentially clubs, only accessible to its members.

[7] We remark here that $ij = ji$. Note that if $i = j$, the relational activity ii represents again the economic autarky of agent i.

always decide to initiate her autarkic activity $ii \in \Omega$.

In terms of our framework one can think of the pair (Ω, Γ) as the foundation of a social activity structure in the economy. These foundational autarkic and matching activities are called *simple* activities. Thus, $\Delta^m = \Omega \cup \Gamma$ is now referred to as a *simple interaction structure*.

For any sub-structure $H \subseteq \Delta^m = \Omega \cup \Gamma$ we denote

$$N(H) = \{i \in N \mid \text{There is some } j \neq i \text{ such that } ij \in H\} \tag{4}$$

as the set of economic agents that are engaged within the sub-structure H. It is easy to see that $N(H) = N(H \setminus \Omega)$. Also, for every $H \subseteq \Gamma$, if $H \neq \varnothing$, then $N(H) \neq \varnothing$. Finally, due to the feasibility hypothesis, it holds that $N(\Delta^m) = N(\Gamma) = N$.

Finally, within the setting of the simple interaction structure (Ω, Γ) we introduce the third type of relational economic activity, that of a (relational) *cooperative activity*. Such cooperative activities are assumed to be centered around a market maker or "convener", representing an agent who acts as a hub in the network structure of this activity.

Formally, consider the matching structure Γ on agent set N. Now, a *cooperative* is understood as a combination of matchings in Γ formed around some "convener". Thus, a convener brings together a number of economic agents with whom she already has an established economic relationship in the form of a matching. This is formalized as follows:

Definition 2.1 *Let $\Gamma \subseteq \Gamma_N$ be a matching structure on N. A **cooperative activity**—or simply a "cooperative"—is defined as a sub-structure $G \subseteq \Gamma$ of matchings such that $|N(G)| \geqslant 3$ and there is a unique agent $i \in N(G)$ such that $N_i(G) = N(G) \setminus \{i\}$ and that for all other agents $j \in N(G) \setminus \{i\}$ it holds that $N_j(G) = \{i\}$. The agent i is called the **convener** of the cooperative G and denoted by $N^\star(G) \in N(G)$.*

Our definition of a cooperative imposes that a cooperative has at least three members. Furthermore, a cooperative has an explicit star structure in the matching structure Γ. This implies that the cooperative has a relational center, representing the convener as a "middleman" binding and coordinating all constituting matchings of the cooperative.

Using this definition of cooperative activities, we can introduce some auxiliary concepts and notation.

Definition 2.2 *Let $\Gamma \subseteq \Gamma_N$ be some matching structure.*

(a) *The collection of all (potential) cooperative activities is now given by*

$$\Sigma(\Gamma) = \{G \mid G \subset \Gamma \text{ is a cooperative activity}\} \tag{5}$$

*$\Sigma(\Gamma)$ is denoted as the **permissible cooperative structure** on Γ.*
*The triple $(\Omega, \Gamma, \Sigma(\Gamma))$ is referred to as the **permissible activity structure** on N consisting of all autarkies $G_1 \in \Omega$, all matchings $G_2 \in \Gamma$, and all feasible cooperatives*

$G_3 \in \Sigma(\Gamma)$. *The union of a permissible activity structure,* $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$, *serves as its alternative description.*

(b) *The* ***set of conveners*** *in* Γ *is defined as the collective of conveners of cooperative activities in* $\Sigma(\Gamma)$*:*

$$N^{\star}(\Gamma) = \{i \in N \mid i = N^{\star}(G) \text{ for some } G \in \Sigma(\Gamma)\}. \tag{6}$$

We conclude our discussion with introducing some auxiliary network theoretic concepts that describe features of any sub-structure of matching activities $H \subseteq \Gamma$.

We define a *path* between any two distinct agents $i \in N$ and $j \in N$ in $H \subseteq \Gamma$ as a sequence of distinct agents $P_{ij}(H) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = j$, $i_k \in N$ and $i_k i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. We define a *cycle* in H to be a path of an agent from herself to herself which contains at least two other distinct agents, *i.e.*, a cycle in H from i to herself is defined as a path $C = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = i$, $m \geqslant 4$, $i_k \in N$, and $i_k i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. The length of the cycle C is denoted by $\ell(C) = m - 1 \geqslant 3$. A sub-structure $H \subseteq \Gamma$ is called *acyclic* if H does not contain any cycles.

Furthermore, there may be agents in N between whom there is no path in Γ; such agents are located in different components of the structure Γ. These components are exactly the maximally connected sub-structures within Γ. Formally, a sub-structure $H \subseteq \Gamma$ is a *component* of Γ if

(i) for all $i \in N(H)$ and $j \in N(H)$ there is a path $P_{ij}(H)$ connecting agents i and j in H;

(ii) for all $i \in N(H)$ and $j \in N(H)$, $ij \in \Gamma$ implies that $ij \in H$;

(iii) and $i \in N(H)$ and $ij \in \Gamma$ imply that $j \in N(H)$.

The *set of all components* in a network Γ is denoted by $\mathsf{c}(\Gamma) = \{H \mid H \subseteq \Gamma \text{ is a component }\}$. Note that as a consequence of feasibility property of the matching structure Γ we have that $\Gamma = \cup_{H \in \mathsf{c}(\Gamma)} H$ and $N = \cup_{H \in \mathsf{c}(\Gamma)} N(H)$.

The location of an agent within a network is an important characteristic. Let Γ be some matching structure and let $H \subseteq \Delta^m = \Omega \cup \Gamma$ be some structure of simple activities. We define agent i's *neighborhood* in H as $N_i(H) = \{j \in N \mid ij \in H\}$. Note here that if $i \in N_i(H)$, agent i's autarkic activity ii is listed in H, i.e., $ii \in H$. Also, by the definition of a matching structure, it holds that $N_i(\Gamma) \neq \varnothing$ for any $i \in N$. We can also express the neighborhood of an agent within an arbitrary structure $H \subset \Delta^m$ in terms of its link based analogue, *i.e.*, $L_i(H) = \{ij \in H \mid j \in N_i(H)\} \subset H$. For example, $L_i(\Delta^m) = \{ii\} \cup L_i(\Gamma)$ is the set of feasible simple activities that agent i can potentially participate in.

3 Stability in bilateral matching economies

We first address stable interaction in an economy with autarkic and matching activities only. Based on these "simple" economic activities Gilles, Lazarova, and Ruys (2007) introduced the notion of a "matching economy". Here, we build upon this discussion.

Throughout we assume that every individual $i \in N$ has complete and transitive preferences over the permissible simple activities $L_i(\Delta^m) \subset \Delta^m = \Omega \cup \Gamma$ in which she can engage. Thus, by finiteness of Δ^m, these preferences can be represented by a *hedonic utility function* given by $u_i^m : L_i(\Delta^m) \to \mathbb{R}$. Let $u^m = (u_1^m, \dots, u_n^m)$ now denote a *hedonic utility profile*.

Here we remark that our formal theory follows, in a way, a Lancasterian approach—separating a commodity from its properties and explaining the value of a commodity from the utility of its properties (Lancaster, 1966)—to the performance of relational activities.[8] We separate the concrete content of an economic activity from the basic abstract network framework in which such an interaction is embedded and supported. This is represented in the hedonic utilities introduced above.

Definition 3.1 *A* ***matching economy*** *is defined as a triple $\mathbb{E}^m = (N, \Delta^m, u^m)$ in which N is a finite set of individuals, $\Delta^m = \Omega \cup \Gamma$ is a simple activity structure on N, and $u_i^m : L_i(\Delta^m) \to \mathbb{R}$, $i \in N$, is a hedonic utility profile on Δ^m.*

Within the context of a matching economy we investigate stability of an allocation of activities. The main hypothesis in the definition of stability is that in a matching economy $\mathbb{E}^m$ each individual $i \in N$ activates *exactly one* of her matchings in $L_i(\Delta^m)$. This hypothesis is founded on the fact that a matching economy is not endowed with advanced economic or social institutions. In such a primitive setting—in which one problem is addressed at a time—it seems natural to assume that individuals only interact with a single other individual at a time.

Definition 3.2 *An* ***assignment*** *in the matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ is a mapping $\pi : N \to \Delta^m$ such that*

(i) *$\pi(i) \in L_i(\Delta^m)$ for all $i \in N$ and*

(ii) *$\pi(i) = ij$ implies that $\pi(j) = ij$ for all $i, j \in N$.*

An assignment π is equivalently represented by the induced sub-structure $\Pi \subset \Delta^m$ with

$$\Pi = \pi(N) = \{\pi(i) \mid i \in N\}. \tag{7}$$

The set of all assignments in E^m is denoted by Π^m.

[8] Ruys (2011) follows a similar strategy in developing what he calls a relational capacity approach to the social enterprise.

We remark that by the applied hypotheses and definitions, the set of assignments is non-empty. In particular, $\Omega \in \Pi^m \neq \varnothing$. Moreover, according to the feasibility assumption on Γ, there exist other assignments in which agent $i \in N$ is engaged with some other agent $j \neq i$; indeed, there is some $\pi \in \Pi^m$ with $\pi(i) = ij$ for any $ij \in \Gamma$.

With some slight abuse of notation, we indicate by $u_i^m(\pi)$ the payoff or utility that agent $i \in N$ receives under assignment $\pi \in \Pi^m$, i.e., $u_i^m(\pi) = u_i^m(\pi(i))$.

We introduce stability on an assignment founded on the standard assumptions of individual rationality and a no-blocking condition from matching theory, denoted here as "pairwise stability". (Roth and Sotomayor, 1990; Jackson and Wolinsky, 1996).

Definition 3.3 *An assignment $\pi \in \Pi^m$ is* ***stable*** *in the matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ if all matchings generated by π satisfy*

Individual Rationality (IR)**:** $u_i^m(\pi) \geqslant u_i^m(ii)$ *for all $i \in N$, and;*

Pairwise stability (PS)**:** *There is no blocking matching with regard to π, in the sense that for all $i, j \in N$, $i \neq j$, with $\pi(i) \neq ij$ it holds that*

$$u_i^m(ij) > u_i^m(\pi) \text{ implies that } u_j^m(ij) \leqslant u_j^m(\pi). \tag{8}$$

For an economy to have persistent access to gains from organization, the social structure of the economy has to *universally* admit stable matchings. Hence, whatever capabilities and desires of the individuals—represented by their (hedonic) utility functions and (possibly) other individualistic features—a stable assignment has to exist in the matching economy.

Definition 3.4 *A matching structure Γ on N is* ***universally stable*** *if for every hedonic utility profile u^m on $\Delta^m = \Omega \cup \Gamma$ there exists at least one stable assignment in the corresponding matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$.*

Clearly, a universally stable feasible structure thus reflects that the institutional organization of the economy supports stability regardless of the exact individual preferences. In this regard it reflects a network structure of the economy, which promotes and enhances the economic activities selected by the economic agents.

The next result identifies the necessary and sufficient conditions for universal stability. Similar insights have already been established in the literature on matching markets.

Theorem 3.5 *A matching structure Γ on N is universally stable if and only if the matching structure Γ is bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that*

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}\,. \tag{9}$$

For a proof of this result we refer to Appendix A of the paper.

The generic existence result in Theorem 3.5 has a clear interpretation. Any universally stable matching structure has to be based on two socioeconomic roles such that economic matching activities can only occur between pairs of agents of distinct roles.

4 Stability in network economies

Next we extend the scope of the stability concept to include cooperation among multiple economic agents.

Let $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ be a permissible economic activity structure on the agent set N. An agent $i \in N$ has complete and transitive preferences over the activities in which he or she potentially can participate. We assume that these preferences can be represented by a hedonic utility function. Such a hedonic utility function is essentially *indirect* in that it captures the utility of the value generating activities.

For $i \in N$ we introduce $\mathcal{A}_i(\Delta)$ as the set of all permissible activities in which agent i participates. Formally,

$$\mathcal{A}_i(\Delta) = \{ii\} \cup \{ij \mid ij \in \Gamma\} \cup \{G \mid G \in \Sigma(\Gamma) \text{ and } i \in N(G)\}. \tag{10}$$

We denote by $\mathcal{A}(\Delta) = \cup_{i \in N} \mathcal{A}_i(\Delta)$ the collection of all activities available to the agents in the economy.

For every economic agent $i \in N$, preferences are now introduced through the hedonic utility function $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$. Let $u = (u_1, \ldots, u_n)$ be a profile of hedonic utility functions for all agents in N. Let $\mathcal{U}$ be the set of all profiles of hedonic utility functions on Γ.

Now a *network economy* is defined to be the set of permissible relational activities (autarky, matchings, and cooperatives) and a hedonic utility function that assigns a value to every agent in each of these permissible activities. This is formalized as follows.

Definition 4.1 *A* ***network economy*** *is introduced as a triple $\mathbb{E} = (N, \Delta, u)$ in which N is a finite set of economic agents $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ is a permissible economic activity structure based on some matching structure Γ, and $u \in \mathcal{U}$ is a profile of hedonic utility functions such that $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$ for every $i \in N$.*

Finally we discuss two notions of equilibrium for such network economies. To analyze stability we again adapt the concept of pairwise stability in the same fashion as formalized for matching economies in Definition 3.3.

Definition 4.2 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy.*

(a) *An* ***assignment*** *in $\mathbb{E}$ is a mapping $\lambda \colon N \to \mathcal{A}(\Delta)$ such that*

(i) $\lambda(i) \in \mathcal{A}_i(\Delta)$, *and*

(ii) $\lambda(i) = G \in \Delta$ *implies that* $\lambda(j) = G$ *for all* $j \in N(G)$.

An assignment λ *generates a corresponding partitioning given as its image* $\Lambda = (G_1, \ldots, G_m) \equiv \lambda(N) \subset \Delta$.

(c) *An assignment* $\lambda^\star : N \to \mathcal{A}(\Delta)$ *generating* $\Lambda^\star = (G_1^\star, \ldots, G_m^\star)$ *is* ***stable*** *in the network economy* $\mathbb{E}$ *if for every* $p \in \{1, \ldots, m\}$ *the activity* $G_p^\star \in \Lambda^\star$ *satisfies the* individual rationality *[IR] and two* pairwise stability *conditions [PS] and [PS*] as specified below:*

IR *for all* $i \in N(G_p^\star)$ *it holds that* $u_i(G_p^\star) \geqslant u_i(ii)$;

PS *for all distinct agents* $i \in N(G_p^\star)$ *and* $j \in N(G_q^\star)$ *with* $q \in \{1, \ldots, m\}$ *and* $ij \in \Gamma$, $ij \notin G_p^\star \cap G_q^\star$:

$$u_i(ij) > u_i(G_p^\star) \quad \text{implies} \quad u_j(ij) \leqslant u_j(G_q^\star); \tag{11}$$

PS* *and for all distinct agents* $i \in N(G_p^\star)$ *and* $j \in N(G_q^\star)$ *with* $q \in \{1, \ldots, m\}$ *with* $ij \in \Gamma$, $ij \notin G_p^\star \cap G_q^\star$ *and either* $j = N^\star(G_q^\star)$ *or* $G_q^\star \in \Gamma$:

$$u_i(G_q^\star \cup \{ij\}) > u_i(G_p^\star) \quad \text{implies} \quad u_j(G_q^\star \cup \{ij\}) \leqslant u_j(G_p^\star). \tag{12}$$

(d) *An assignment* $\lambda^\star : N \to \mathcal{A}(\Delta)$ *generating* $\Lambda^\star = (G_1^\star, \ldots, G_m^\star)$ *is* ***strongly stable*** *in the network economy* $\mathbb{E}$ *if* $\lambda^\star$ *is stable in* $\mathbb{E}$ *and for every* $p \in \{1, \ldots, m\}$ *the activity* $G_p^\star \in \Lambda^\star$ *satisfies additionally* Reduction Proofness *[RP]:*

RP *If* $G_p^\star$ *is a cooperative economic activity, i.e.,* $G_p^\star \in \Sigma(\Gamma) \cap \Lambda^\star$, *it holds that for every sub-structure* $G \subset G_p^\star$

$$u_i(G) \leqslant u_i(G_p^\star) \tag{13}$$

where $i = N^\star(G_p^\star) = N^\star(G)$ *is the convener of both cooperative economic activities.*

As in the case with matching economies, here it is again assumed that agents participate in a single activity. An assignment is defined to be stable if it satisfies certain standard stability conditions from game theory, in particular matching theory (Roth and Sotomayor, 1990), network formation theory (Jackson and Wolinsky, 1996), and core theory for Tiebout and club economies (Gilles and Scotchmer, 1997).

Condition IR is a standard individual rationality condition that allows an individual to opt out of an economic activity if she is better off being autarkic. The first pairwise stability condition PS rules out blocking opportunities for pairs of agents who are not connected to each other in the same cooperative. It requires that there are no pairs of such agents who prefer to be linked to each other rather than to the agents with whom they are linked in the present assignment. Condition PS has already been applied in Definition 3.3 of a stable assignment for matching economies.

The second pairwise stability condition PS* rules out blocking opportunities for pairs of agents at least one of whom can add a link without severing his existing links in the present assignment. Hence, such an agent is either a convener in the present assignment, or she is linked in a matching with another distinct agent and not member of a cooperative. This condition requires that there are no two distinct agents who want to be linked to each other in a cooperative in which one of them is a convener.[9]

Both PS and PS* are concerned with the re-structuring of the prevailing assignment. These conditions still do not allow the convener of a cooperative to block access to this cooperative by an existing member, if it is to their gain. Hence, stability of an assignment defines a notion of cooperatives that are principally "open" in the sense that once an agent has fulfilled certain entry requirements she cannot be stropped off her membership. There are numerous cooperative activities that satisfy the principle of openness such as trading posts (stores) and markets, open source communities, and many economic service provision cooperatives (clubs). In most of these cases, if entrants follow the house rules of the cooperative in question, they will not be excluded from participation.

The stronger notion of strong stability excludes the possibility of open cooperative activities. Condition RP explicitly "closes" a cooperative in the sense that the convener is allowed to discontinue participation of existing members based on her own preferences. In economic practice we encounter many such closed cooperatives as well. We mention as examples team production situations (e.g., health care provision), particular upstream-downstream relationships in which a primary input producer may discontinue supply to a final good producer and vice versa, and exclusive clubs (guilds and unions).

Under (regular) stability, a convener is merely a coordinator of a cooperative economic activity who is open to participation, while under strong stability a convener is considered to be a manager of the cooperative activity under consideration. We emphasize that strong stability implies stability, i.e., management implies coordination, but that the reverse is not true. We refer to the example in Section 1.1 for an explicit comparison of stability and strong stability.

[9]Note that a convener and an agent linked in a matching with another distinct agent have multiple blocking opportunities available: such agents can add a link with or without severing their current links. Such agents are subject to both (no blocking) conditions PS and PS*.

4.1 Separability: The absence of externalities

After having established a model of relational economic activities, we investigate the existence of stable assignments. We have to distinguish two types of network economies: economies with relational externalities affecting the performance of cooperatives and economies without such network externalities. We first investigate economies without network externalities.

Definition 4.3 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy.*

(i) *The hedonic utility function $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$* ***exhibits no externalities*** *if for all $G_i \in \mathcal{A}_i(\Delta)$ and $H_i \in \mathcal{A}_i(\Delta)$ with $N_i(G_i) = N_i(H_i)$, it holds that $u_i(G_i) = u_i(H_i)$. The collection of all utility profiles exhibiting no externalities is denoted by $\mathcal{U}_n \subset \mathcal{U}$.*

(ii) *The network economy $\mathbb{E} = (N, \Delta, u)$ is* ***separable*** *if $u_i \in \mathcal{U}_n$ for every agent $i \in N$.*

The non-externality property on a hedonic utility function imposes that an agent derives value only from matchings with agents with whom she is linked *directly*. Thus, changes in cooperatives regarding third parties do not affect the hedonic utility value of a member of that cooperative. Although this seems to be a very severe condition, it is a common assumption in traditional public economics, where the public good itself acts as a convener in our terms.[10]

In addition to separability, we introduce a second property of the hedonic utility functions and that is superadditivity. This superadditivity property reflects synergies which are assumed to be attributed to the convener who acts as a coordinator in the value generation process.

Definition 4.4 *Let Γ be a permissible activity structure on N. For agent $i \in N$, the hedonic utility function $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$ is* ***superadditive*** *if for any $G_i \in \mathcal{A}_i(\Delta)$ and $H_i \in \mathcal{A}_i(\Delta)$ with $G_i \cup H_i \in \mathcal{A}_i(\Delta)$ and $G_i \cap H_i = \varnothing$ it holds that $u_i(G_i \cup H_i) \geqslant u_i(G_i) + u_i(H_i)$.*
Furthermore, we say that a utility profile $u \in \mathcal{U}$ on Γ is superadditive if the hedonic utility function u_i is superadditive for every agent $i \in N$. The collection of all superadditive utility profiles is denoted by $\mathcal{U}_s \subset \mathcal{U}$.

Within the context of network economies we address the existence of stable assignments for arbitrary separable and superadditive hedonic utility profiles. Formally, we introduce:

Definition 4.5 *Let Γ be a matching structure and let $\mathcal{U}^\star \subseteq \mathcal{U}$ be some given class of permissible utility profiles on the matching structure Γ. The matching structure Γ is* ***universally (strongly) stable*** *on the class $\mathcal{U}^\star$ if for every utility profile $u \in \mathcal{U}^\star$ there exists a (strongly) stable assignment $\lambda^\star$ in the network economy $\mathbb{E} = (N, \Delta, u)$.*

[10] In this regard if all cooperatives exhibit such non-externalities towards its members, the activities represented through these cooperatives are separable and, thus, can in principle be evaluated objectively. This is the principle of pricing membership of clubs in a club economy (Gilles and Scotchmer, 1997), or the Samuelson conditions in the efficient provision of a pure public good in the sense of Samuelson (1954).

We denote by $\overline{\mathcal{U}} = \mathcal{U}_s \cap \mathcal{U}_n$ the class of all hedonic utility profiles that satisfy the superadditivity as well as the non-externality properties.

Theorem 4.6 *The matching structure* Γ *is universally strongly stable on the class* $\overline{\mathcal{U}}$ *of superadditive hedonic utility profiles exhibiting no externalities if and only if* Γ *satisfies the property that, if* Γ *contains a cycle* $C \subset \Gamma$*, then* C *has length* $\ell(C) = 3k$ *where* $k \in \mathbb{N}$.

The proof of Theorem 4.6 is given in Appendix B.

Theorem 4.6 states that under some regularity conditions, a permissible activity structure is universally strongly stable for hedonic utility profiles without externalities if and only if the network structure exhibits a certain acyclicity property. Unfortunately, the partial acyclicity condition on the permissible activity structure stated in the assertion is more difficult to interpret than the condition stated in Theorem 3.5.[11]

However, from Theorem 4.6 we may derive some more directly interpretable conclusions. In particular, if the network structure is acyclic, then the permissible activity structure is universally stable for utility profiles exhibiting no externalities.

Corollary 4.7 *If the matching structure* Γ *is acyclic, then* Γ *is universally strongly stable on the class* $\overline{\mathcal{U}}$ *of superadditive hedonic utility profiles exhibiting no externalities.*

One particular interesting class of acyclic matching structures is that of the *hierarchical* structures. Within a hierarchical structure, multiple levels can be distinguished in which agents in a certain level can only communicate with agents in lower and higher levels. It is well-accepted that hierarchical structures are common institutional features of any contemporary society. In particular, social roles are usually assigned to correspond to the various levels within the hierarchical power structure in the economic and political sphere of a society.

The main conclusion from the assertion stated in the corollary is that if a society is hierarchically structured, it is universally strongly stable. In this regard a hierarchical organization structure is a "mode of governance" and as such the corresponding social role and authority structure steer the society towards a (strongly) stable state. As such, a hierarchical organization of a society supports and promotes economic development and stability.

4.2 Introducing size-based externalities

Next we consider certain conditions under which stable assignments emerge in the presence of externalities. We investigate a simple size-based formulation of externalities. The more

[11]The condition formulated in Theorem 4.6 can partially be interpreted. We note that if all agents in N assume one of three socioeconomic roles such that in Γ: (i) agents of two of the three types link with at most one of any other type; and (ii) agents of one particular type can link with multiple other types. The third type clearly refers to a class of market makers or middlemen. This illustrated in the example of an island economy discussed in Section 1.1.

members a cooperative has, the more it affects the resulting value for its members. Such size-based externalities are very common as every bounded facility is subject to crowding. In the literature on Tiebout and club economies such crowding externalities have been investigated extensively. We refer here to the seminal paper by Conley and Wooders (1997) and the subsequent work by Conley and Konishi (2002). For the case of size-based externalities we are able to state a rather general result concerning existence of stable assignments.

For utility profiles with size-based externalities, the number of agents in a cooperative is determining the size of the externality. The identity of the convener of the cooperative determines whether the externality is positive or negative, but the identity of the remainder of the cooperative membership is irrelevant for the amount of externality generated.

Definition 4.8 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy. Then the utility function u* ***exhibits a (linear) size-based externality*** *if for every cooperative $G \in \Sigma(\Gamma)$:*

$$u_i(G) = \sum_{j \in N_i(G)} u_i(ij) + \alpha_c \cdot [\,\#N(G) - 2\,] \tag{14}$$

for all $i \in N(G)$, where $c = N^\star(G)$ and $\alpha_c \in \mathbb{R}$.

If a convener c has an externality parameter $\alpha_c > 0$, she brings about a positive externality in the cooperative. This refers to "economies to club size" based on the total size of the cooperative gathered around this convener. If, on the other hand, this convener has an externality parameter $\alpha_c < 0$, she causes a negative externality in the cooperative. This can be referred to as "crowding" (Conley and Wooders, 1997).

First we report that there exist network economies exhibiting size-based externalities in which there is no stable assignment. An example is presented below.

Example 4.9 Let $N = \{1, 2, 3, 4\}$ and $\Gamma = \{12, 23, 34\}$. Let $\alpha_2 = 200$ and $\alpha_3 = -50$. Let the utility function be such that $u_1(12) = u_2(22) = u_3(33) = -100$, $u_1(11) = u_2(12) = 0$, $u_2(23) = u_4(34) = 100$, $u_4(44) = 90$, $u_3(23) = 60$, and $u_3(34) = 300$. Using the linear size-based externality function, we can compute the utility levels in the two possible cooperatives[12] 213 and 314 in a straightforward manner: $u_1(213) = 100$, $u_2(213) = 300$, $u_3(213) = 260$, $u_2(324) = u_4(324) = 50$, and $u_3(324) = 310$.
We now claim that in this example there is no stable assignment. First, consider the assignment generating $(12, 34)$. It is not stable because [PS*] is not satisfied: $50 = u_2(324) > u_2(12) = 0$ and $310 = u_3(324) > u_3(34) = 300$. Also, since $-100 = u_2(22) < u_2(324) = 50$, the [PS*] condition is not satisfied and the assignment generating $(11, 22, 34)$ is not stable either. Next, consider $(1, 324)$, which is not stable since [IR] for agent 4 is not satisfied:

[12] Here we introduce the following shorthand notation in the form of triples ijk to denote a permissible cooperative consisting of the three agents i, j, and k where i acts as a convener. Similarly we use the quadruplet $ijkl$ to describe a four-agent cooperative with convener i. This notation is adopted in the rest of the paper.

$50 = u_4(324) < u_4(44) = 90$. Moving on, assignment $(11, 23, 44)$ is not stable due to a violation of [PS*]: $0 = u_1(11) < u_1(213) = 100$ and $100 = u_2(23) < u_2(213) = 300$. Finally, $(213, 44)$ is not stable due to a violation of [PS]: $260 = u_3(213) < u_3(34) = 300$ and $90 = u_4(44) < u_4(34) = 100$. Using the same reasoning, we find that $(12, 33, 44)$ and $(11, 22, 33, 44)$ are not stable either. ♦

Second, stable assignments may not exist even when we impose *uniform* linear size-based externalities on all conveners. The following two examples illustrate this point. The first example imposes uniform, but negative, size-based externalities.

Example 4.10 Let $N = \{1, 2, 3\}$ and let $\Gamma = \{12, 23\}$. Now consider $\alpha_2 = -2$. Let the utility function be such that $u_i(ii) = 0$ for all $i = 1, 2, 3$ and $u_1(12) = u_2(12) = 3$, $u_2(23) = 4$, and $u_3(23) = 1$. Using the linear size-based externality function, we can now compute the utility levels in the cooperative 213 in a straightforward manner: $u_1(213) = 1$, $u_3(213) = -1$, and $u_2(213) = 5$. We now claim that there is no stable assignment in this economy.
To show this, first, consider $(12, 33)$. This assignment is not stable due to a violation of [PS]: $3 = u_2(12) < u_2(23) = 4$ and $0 = u_3(33) < u_3(23) = 1$. Similarly $(11, 22, 33)$ is not stable. Next, $(11, 23)$ is not stable due to a violation of [PS*]: $0 = u_1(11) < u_1(213) = 1$ and $4 = u_2(23) < u_2(213) = 5$. Finally, (213) is not stable due to a violation of [IR] for agent 3: $-1 = u_3(213) < u_3(33) = 0$. ♦

Finally, we consider a 5-agent circular matching structure. Here, uniformity of the the size-based externality for conveners is positive. However, the emergence of a Condorcet-like cycle in the economy prevents the desired stability.

Example 4.11 Let $N = \{1, 2, 3, 4, 5\}$ and let $\Gamma = \{12, 15, 23, 34, 45\}$. Furthermore, let $\alpha_c = \alpha = 2$ for all potential conveners $c \in N^\star(\Sigma(\Gamma)) = N$. The utility levels for each matching is given as follows: $u_i(ii) = 0$ for all $i \in N$, $u_1(12) = u_2(23) = u_3(34) = u_4(45) = 2$, $u_1(15) = u_2(12) = u_3(23) = u_4(34) = u_5(45) = 10$ and $u_5(15) = -1$. The utility levels in all possible cooperatives are computed in a straightforward manner from the linear size-based externality function: $u_5(125) = 1$, $u_1(213) = u_2(324) = u_3(435) = u_4(514) = 4$, $u_5(514) = 11$, $u_1(514) = u_2(125) = u_3(213) = u_4(324) = u_5(435) = 12$, and $u_1(125) = u_2(213) = u_3(324) = u_4(435) = 14$. We now claim that also in this example there is no stable activity structure. ♦

We conclude from these three examples that size-based externalities prevent the emergence of a stable assignment if there are non-uniform externalities, there are negative size-based externalities, or there are cycles in Γ. However, if these three conditions are excluded, stability can still be established.

Theorem 4.12 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy where u exhibits size-based externalities such that $\alpha_c = \alpha > 0$ for all potential conveners $c \in N^\star(\Gamma)$. If Γ is acyclic, then $\mathbb{E}$ admits a stable assignment.*

A proof of this existence result is available upon request from the authors.

This assertion cannot be strengthened to cover strong stability rather than regular stability. The next example devises a simple case satisfying the conditions of Theorem 4.12 in which no strongly stable assignment can be constructed.

Example 4.13 Let $N = \{1, 2, 3\}$. Consider the matching structure $\Gamma = \{12, 23\}$ and the resulting permissible cooperative structure $\Sigma(\Gamma) = \{213\}$. We consider the hedonic utility profile with size-based externalities generated by $\alpha = 2$ and $u_1(11) = u_3(33) = 0$, $u_2(22) = -4$, $u_1(12) = -1$, $u_2(23) = -3$, and $u_2(12) = u_3(23) = 1$. Now we derive that $u_1(213) = -1 + 2 = 1$, $u_2(213) = 1 - 3 + 2 = 0$, and $u_3(213) = 1 + 2 = 3$.
We now check that in this economy there is no strongly stable assignment: $\{11, 23\}$ is not stable since agent 1 wants to join agent 2 in the cooperative 213 and its convener, agent 2, agrees; $\{12, 33\}$ is not stable since [IR] is not satisfied for agent 1; $\{213\}$ is not strongly stable since its convener, agent 2, prefers 12 over 213 and thus severs the participation of agent 3; and $\{11, 22, 33\}$ is not stable since agents 2 and 3 prefer the matching 23 over being autarkic.
Although there is no strongly stable assignment in this network economy, cooperative $\{213\}$ forms a stable assignment. ♦

Example 4.13 confirms that the presence of simple size-based externalities prevents the emergence of strongly stable assignments. Thus, in the presence of these externalities only economies with "open" cooperative economic activities can achieve stability.

5 Some concluding remarks

In this paper we address the question how economic stability is founded on certain identified properties of the structure of relationships among the constituents of an economy. We interpret these structural properties of such economic networks as "institutional", in particular through the assumption of socioeconomic roles by individuals in these networks. The adoption of such roles induces acyclicity of these networks, implying economic stability. The main conclusion from our theory is that institutional frameworks increase the stability in an economy.

There is a significant link of our theory with the work of Burt (1992) on structural holes and the developed network framework of economic activities. According to Burt, optimizing the number of nonredundant contacts is a way to increase the efficiency of a social network: while the presence of cycles allows for at least two distinct paths between two distinct individuals, in the absence of cycles, there is at most one path between any two distinct individuals. Thus, in an acyclic structure one does not support links that provide the same accessibility. Given that the generation and maintenance of links is costly, a structure without cycles is more efficient than such in which cycles are present. Last, we should mention some limitations of our general framework. In particular, in our work we focus on very special class of assignments, which consists of star structures. For complex production processes, such as hierarchies of several levels, predominant in today's economic world, these tools are inadequate. A clear goal for future work is the development of a framework where more complex patterns can be analyzed.

Future research may consider extending the proposed theory to include several closely related concepts. First, the multi-person cooperative economic activities considered in this paper can easily incorporate the notion of organizational "scope". The size of a cooperative can be interpreted as its scope and include the possibility to model market size and even such ideas as globalization. In the current paper we did not address the effects of changes in scope of a cooperative; these could form the basis for a more dynamic theory of market formation and economic globalization.

Second, the model presented here can be extended to include the notion of supply chains. Our model considers the conveners of cooperatives to be fully independent from each other in their operations and economic activities. However, one could extend the model by linking conveners with each other in a reduced network linking only fully developed cooperatives. Such linked cooperatives could form chains and stand for production processes that include the use of intermediate products, representing the traditional idea of a supply chain. The employment of supply chains into the theory would significantly alter the model, in particular the generation of economic values. These could no longer be abstract, but should be based explicitly on the trade of commodities, including final and intermediate products.

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Appendices

A Proof of Theorem 3.5

In this section we show the existence of a universally stable matching structure. In Lemma 1, we establish some parallels with existing notions in the one-to-one matching literature.

Lemma 1 *Consider a matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$. Let the matching structure Γ be bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that*

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}.$$

Then there exists a corresponding marriage problem (cf. Gale and Shapley (1962)) such that a stable matching in the marriage problem corresponds to a stable assignment in the matching economy $\mathbb{E}^m$.

Proof. A marriage problem as introduced by Gale and Shapley (1962) consists of two finite and disjoint sets of players M and W. Each agent $m \in M$ has complete and transitive preferences, $\succeq^M_m$, over $W \cup \{m\}$ and each agent $w \in W$ has complete and transitive preferences, $\succeq^W_w$, over $M \cup \{w\}$. A matching is a function $\mu : M \cup W \to M \cup W$ of order two, i.e., $\mu(\mu(i)) = i$, $\mu(m) \in W \cup \{m\}$ and $\mu(w) \in M \cup \{w\}$. A matching μ is stable if there is no (a) player $m \in M$ or $w \in W$ who prefers to be matched to herself than to her partner in μ, or (b) pair of distinct players (m, w) who are not matched by μ and $w \succeq^M_m \mu(m)$ and $m \succeq^W_w \mu(w)$. Notice that conditions (a) and (b) correspond to conditions [IR] and [PS] of Definition 3.3, respectively.
Consider a matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ with a bipartite matching structure Γ such that there exists a partitioning $\{N_1, N_2\}$ of N with

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}.$$

Let $\widetilde{\Gamma} = N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}$. Next consider utility profile $\tilde{u}^m : \widetilde{\Gamma} \cup \Omega \to \mathbb{R}$ such that for all agents $i \in N$ and all matchings ij that satisfy the bipartite property but are *not* feasible, i.e., $ij \in \widetilde{\Gamma} \setminus \Gamma$, we set $\tilde{u}_i{}^m(ij) < u^m_i(ii)$, and for all matchings $ij \in \Delta^m$, we set $\tilde{u}^m = u^m$. Clearly, $\tilde{u}^m$ represents complete and transitive preferences on $\widetilde{\Gamma} \cup \Omega$. .
Let $M = N_1$, $W = N_2$, and let preference profiles $\succeq^M$ and $\succeq^W$ be represented by hedonic utility functions $\phi^M_i : W \cup \{m\} \to \mathbb{R}$ with $\phi^M_i(N_i(ij)) = \tilde{u}_i(ij)$ for all $i \in M$ and all $ij \in \widetilde{\Gamma} \cup \Omega$ and $\phi^W_k(N_k(kl)) = \tilde{u}_k(kl)$ for all $k \in W$ and all $kl \in \widetilde{\Gamma} \cup \Omega$. The tuple $(M, W, \succeq^M, \succeq^W)$ defines a marriage problem.
Suppose μ^* is a stable matching in the marriage problem $(M, W, \succeq^M, \succeq^W)$. Consider, an assignment π^* in economy $\mathbb{E}$ such that $N_i(\pi^*(i)) = \mu^*(i)$ for all $i \in N$. Notice that $\pi^* \in \Delta^m$ follows from the stability of μ^*, which implies that for all $i \in M \cup W$, $\mu^*(i) \in N_i(\Delta^m)$, otherwise there is a contradiction to the stability of μ^* as there are two distinct players $k \in M$ and $l \in W$ with $\mu^*(k) = l$ and $kl \notin \Gamma$ such that k and l each prefer to be matched to themselves than to each other, i.e. $k \succeq^M_k l$ and $l \succeq^W_l k$ given by the construction of $\tilde{u}$, ϕ^M, and ϕ^W.
Lastly, we show that the stability of the matching function μ^* in the marriage problem implies the stability of the assignment π^* in the matching economy (N, Δ^m, u^m). The proof follows by contradiction. Suppose the matching μ^* is stable and the assignment π^* is not stable. Therefore either [IR] or [PS] of Definition 3.3 must be violated.
Suppose, first, that [IR] does not hold and that there is an agent $i \in N$ such that $u_i(\pi^*) < u_i(ii)$.

By construction, this implies that there is a player $i \in M$[13] such that $i \succeq_i^M \mu(i)$, which establishes a contradiction to the stability of μ^*.
Next, suppose that [PS] does not hold and that there are two distinct agents $i \in N_1$ and $j \in N_2$ with $ij \in \Gamma$ such that $u_i(ij) > u_i(\pi^*)$ and $u_j(ij) > u_j(\pi^*)$. By construction this implies that there are two distinct agents $i \in M$ and $j \in W$ with $\mu^*(i) \neq j$ such that $j \succeq_i^M \mu^*(i)$ and $i \succeq_j^W \mu^*(j)$ which contradicts to the stability of μ^*. ■

Proof of Theorem 3.5

If: Consider a matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$. Let the matching structure Γ be bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that

$$\Gamma \subseteq N_1 \otimes N_2 = \{ \, ij \mid i \in N_1 \text{ and } j \in N_2 \, \} \, .$$

For any preference profile u^m, we can obtain a corresponding marriage problem as shown in Lemma 1. The existence of a stable matching in any marriage problem is shown by means of the constructive proof of Gale and Shapley (1962) and by means of the non-constructive proof in Sotomayor (1996). By analogy, this proves the existence of a stable assignment in matching economy $\mathbb{E}^m$ for any preference profiles u^m, given matching structure Γ.

Only If: We show that if the matching structure is not bipartite, there exists a preference profile for which there is no stable assignment in a matching economy.
Consider a matching economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ with $N = \{i, j, k\}$, and matching structure $\Gamma = \{ij, ik, jk\}$. Consider the following preference profile: $u_i(ij) = u_j(jk) = u_k(ik) = 2$, $u_i(ik) = u_j(ij) = u_k(jk) = 1$, and $u_l(ll) = 0$ for all $l \in \{i, j, k\}$. It is easy to see that there is no stable assignment in this matching economy. For example, consider the assignment $\pi(i) = \pi(j) = ij$ and $\pi(k) = kk$. It is not stable because pairwise stability is not satisfied: $u_k(jk) > u_k(kk)$ and $u_j(jk) > u_j(ij)$. Similarly, one can show that no other assignment is stable.
This completes the proof of Theorem 3.5

B Proof of Theorem 4.6

The following Lemma states an intermediate result that is required for the proof of existence of a strongly stable assignment in a network economy without any externalities.
Throughout we let $\mathbb{E} = (N, \Gamma, u)$ be some network economy. As before let $\Delta^m = \Omega \cup \Gamma$ be a structure of feasible simple activities on N and let $u \in \mathcal{U}$ be an arbitrary profile of utility functions, we denote by

$$B_i(\Delta^m, u) = \{ \, j \in N \mid ij \in \Delta^m \text{ and } u_i(ij) \geqslant u_i(ik) \text{ for all } k \in N \text{ with } ik \in \Delta^m \, \} \tag{15}$$

the *set of most preferred partners* of agent i for all $i \in N$.[14]

[13]Here we assume, without loss of generality, that $i \in M$. If we were to assume, instead, that $i \in W$ the argument follows analogously.
[14]Here $i \in B_i(\Delta^m, u)$ refers to agent i preferring to remain in autarky over being member of any matching with another agent.

Lemma 2 *Let the matching structure* Γ *be acyclic. Then there is an agent* $i \in N$ *such that* $i \in B_i(\Delta^m, u)$ *and/or there is a pair of agents* $i, j \in N$ *with* $i \neq j$ *such that* $j \in B_i(\Delta^m, u)$ *and* $i \in B_j(\Delta^m, u)$.

Proof. If there is some agent $i \in N$ with $i \in B_i(\Delta^m, u)$ the assertion is obviously valid. Next assume that for every agent $i \in N$ it holds that $i \notin B_i(\Delta^m, u)$ and the second part of the assertion is not true. Then for all agents $i, j \in N$ with $i \neq j$ such that $j \in B_i(\Delta^m, u)$ it holds that $i \notin B_j(\Delta^m, u)$. Consider agent $i \in N$ and without loss of generality we may assume that the set of most preferred agents is a singleton, i.e., $B_i(\Delta^m, u) = \{j\}$. So, it must hold that $j \neq i$. Next, consider the set of most preferred partners of agent j. Without loss of generality we again may assume that B_j is a singleton, say $B_j(\Delta^m, u) = \{k\}$. It must again hold that $k \notin \{i, j\}$. Subsequently, consider the set of most preferred partners of agent k. Without loss of generality we again assume uniqueness, say $B_k(\Delta^m, u) = \{l\}$. It must be that $l \notin \{j, k\}$, moreover $l \neq i$ otherwise Γ contains a cycle. Hence, $l \notin \{i, j, k\}$. By continuing this process in a similar fashion, given that the player set N is finite, we construct a cycle. Therefore, we have established a contradiction. ■

Proof of Theorem 4.6

If: Consider a separable network economy $\mathbb{E} = (N, \Gamma, u)$ such that $u \in \overline{\mathcal{U}}$ exhibits no externalities and is superadditive. We consider two separate cases: (I) when Γ does not contain any cycle and (II) when Γ contains a cycle with a number of connected agents that is a multiple of 3. Let $M \subseteq N$ be some subset of economic agents. Then we denote by

$$\Gamma(M) = \Delta^m \cap \{ij \mid i, j \in M\}$$

the structure of economic matching activities and autarkic positions restricted to the subset M. Using this auxiliary notation we proceed with the proof of the two cases.

Case I: Assume that Γ is acyclic. We now device an algorithm to construct a stable assignment in the economy $\mathbb{E}$ introduced above. This construction consists of several steps and collects agents in various cooperatives such that the resulting pattern is stable.
We define $\Gamma_1 = \Delta^m$, $N_1 = N$, and $\Lambda_1 = \varnothing$. We now proceed by constructing the desired strongly stable assignment in a step-wise fashion:

Let Γ_k, N_k, and Λ_k be given for k, emphasizing that $\Gamma_k \subseteq \Gamma(N_k)$ and that $\Lambda_k \subseteq \Gamma$ is some partial assignment. We now proceed by constructing these elements for step $k+1$. With application of Lemma 1 to Γ_k, there might be an agent $i \in N_k$ such that $i \in B_i(\Gamma_k, u)$. If that is the case, we define

$$N_{k+1} = N_k \setminus \{i\} ;$$
$$\Gamma_{k+1} = \Gamma(N_{k+1}) ;$$
$$\Lambda_{k+1} = \Lambda_k \cup \{ii\}.$$

Subsequently we proceed to step $k+1$ in our construction process.

If that is not the case, then for every $i \in N$ it holds that $i \notin B_i(\Gamma_k, u)$, but according to Lemma 1 there exist at least two agents $i, j \in N_k$ with $i \neq j$ and $i \in B_j(\Gamma_k, u)$ as well as $j \in B_i(\Gamma_k, u)$. Take two agents $i, j \in N_k$ as indicated and define $M = \varnothing$ as well as $G = \{ij\} \in \Gamma$. We now check whether the activity $G = \{ij\}$ can be enhanced into a cooperative. This is done as

follows.
We first introduce some auxiliary notation. Let $\Gamma_k^{-pq} = \Gamma(N_k \backslash \{pq\})$ for any matching $pq \in \Gamma_k$.

If for every agent $h \in N_k \setminus \{i, j\}$ it holds that $i \notin B_h(\Gamma_k^{-jh}, u)$ or $u_i(ih) > -\alpha_i$, and $j \notin B_h(\Gamma_k^{-ih}, u)$ or $u_j(jh) > -\alpha_j$, then we proceed by defining[15]

$$
\begin{aligned}
N_{k+1} &= N_k \setminus \{i, j\} \ ; \\
\Gamma_{k+1} &= \Gamma(N_{k+1}) \ ; \\
\Lambda_{k+1} &= \Lambda_k \cup \{ij\}.
\end{aligned}
$$

Subsequently we proceed to step $k + 1$ in our construction process.
If not, then without loss of generality we may suppose there is some agent $h \in N_k \setminus \{i, j\}$ such that $i \in B_h(\Gamma_k^{-jh}, u)$ and $u_i(ih) > -\alpha_i$. Then ih is an optimal matching for agent h in Γ_k knowing that agent j is engaged with agent i as well. In that case we make agent i a convener and we add agent h to that cooperative. Thus, we redefine $G = \{ij, ih\}$ and we let $M = \{j, h\}$.

We now follow the subsequent iterative procedure:

(♣) We first introduce $\Gamma'_k = \Gamma(N_k \setminus M) \subset \Gamma_k$. We proceed as before and check whether there is some agent $h' \in N_k \setminus (M \cup \{i\})$ such that $i \in B_{h'}(\Gamma'_k, u)$. If that is not the case, then we proceed to (♦). Otherwise, we proceed to (♠).

(♠) Suppose that an agent h' can be selected as identified in (♣), then we proceed by redefining $G = G \cup \{ih'\}$ and $M = M \cup \{h'\}$. In this case the identified agent h' is added to the cooperative under construction G and removed from consideration. We then return to (♣) to repeat the process described there for the redefined G and M.

(♦) Suppose there is no agent h' that has an optimal matching with the identified convener i of cooperative G as described in (♣). Then we proceed to the next step by defining

$$
\begin{aligned}
N_{k+1} &= N_k \setminus (M \cup \{i\}) \ ; \\
\Gamma_{k+1} &= \Gamma(N_{k+1}) \ ; \\
\Lambda_{k+1} &= \Lambda_k \cup \{G\}.
\end{aligned}
$$

Subsequently we proceed to step $k + 1$ in our construction process.

We proceed through the procedure until for some $k = \bar{k}$ we arrive at the situation that $N_{\bar{k}} = \varnothing$. (Note that such a $\bar{k} \leqslant n$ always exists.) Now consider $\Lambda^\star = \Lambda_{\bar{k}}$. First, since the procedure devised above assigns every agent to either an autarkic activity, a matching activity, or a cooperative activity, $\Lambda^\star$ is an assignment. Furthermore, each constructed activity in $\Lambda^\star$ is based on either the optimality of an autarkic activity or the optimality of a matching activity. In the latter case, the non-externality and superadditivity properties of the hedonic utilities imply that the utilities generated in the constructed cooperatives in $\Lambda^\star$ are maximal under the imposed restrictions as well. Finally, this also guarantees that the convener of cooperative $G \in \Sigma(\Gamma) \cap \Lambda^\star$ does not have any incentives to break any relationships with members $i \in N(G)$. This implies, therefore, that the constructed assignment $\Lambda^\star$ is indeed strongly stable as required.
This concludes the proof of Case I.

[15]In this case there is no agent who has an optimal matching with agent i or j. In that case the matching ij is assigned to the assignment under construction.

Case II: The proof of Case II is based on the constructed proof for Case I above. Let the set of matchings Γ contain a cycle $C = (i_1, \ldots, i_m)$ with $i_{k-1}i_k \in \Gamma$ and $m \geqslant 4$ with $m - 1 = 3s$ with $s \in \{1, 2, \ldots\}$. Depending on the utility profile, we will distinguish two sub-cases.

Case II.1 First, consider a utility functions $u_i \in \overline{\mathcal{U}}$ which satisfies superadditivity and the non-externality property, such that either (a) there exists an agent i_k with $k = 1, \ldots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$; or (b) there are two consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$; or (c) there is a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 2, \ldots, m-1$ and $j \notin C$ such that $j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, we can use the algorithm described in Case I to construct a stable assignment since the utility profile ensures that in any of the three cases described above, we can identify agents that fit the requirements stated in Lemma 1.

Case II.2 Last, consider a profile of utility functions $u_i \in \overline{\mathcal{U}}$ such that there is no agent i_k with $k = 1, \ldots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$, or there are no consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$, nor is there a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 1, \ldots, m-1$ and $j \notin C$ such that $i_j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, without loss of generality, we may assume that $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1}i_k) < u_{i_k}(i_k, i_{k+1})$ or $u_{i_k}(i_{k-1}i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$.[16] Suppose, the profile of utility function is $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1}i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial assignment $\Lambda^\star$ can be introduced that consists of exactly s cooperatives of the type

$$\{\{i_2 i_1 i_3\}, \{i_5 i_4 i_6\}, \ldots, \{i_{m-2} i_{m-3} i_{m-1}\}\} \subseteq \Lambda^\star.$$

Next, all other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) assignment $\Lambda^\star$, which furthermore is stable: all agents who are not linked to their most preferred partner have their most preferred partner linked to her own most preferred partner. This implies that they have no incentive to sever their links; moreover, these agents are not in a matching activity and, therefore, they cannot add a link without severing an existing link.

Last, suppose, the profile of utility function is $u_{i_k}(i_{k-1}i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial assignment $\Lambda^\star$ can be introduced that consists of exactly $m-1$ autarkic agents

$$\{\{i_1 i_1\}, \{i_2 i_2\}, \ldots, \{i_{m-1} i_{m-1}\}\} \subseteq \Lambda^\star.$$

All other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) assignment $\Lambda^\star$, which furthermore is strongly stable: all along the cycle are autarkic as the only partner whom they prefer to being autarkic prefers to be autarkic himself than to be matched with them.

This completes the proof of Case II.

Only if: Let $\Gamma = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ be a feasible activity structure and let $\overline{\mathcal{U}}$ be the collection of all superadditive and non-externality hedonic utility profiles. We show by contradiction the necessity of the condition that Γ contains no cycles or if it contains a cycle it is a cycle with

[16] Alternatively, the profile of utility functions $u_i \in \mathcal{U}_s$ must be such that $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_k i_{k+1}) < u_{i_k}(i_{k-1}i_k)$ or $u_{i_k}(i_k i_{k+1}) < u_{i_k}(i_k i_k) < u_{i_k}(i_{k-1}i_k)$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$.

a number of connected agents equal $m \geqslant 4$ with $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$.
Let there be a stable assignment in the network economy (N, Γ, u) for all $u \in \overline{\mathcal{U}}$. Let the set of matchings Γ contain a cycle $C = (i_1, i_2, \ldots, i_m\}$ with $i_k, i_{k+1} \in \Gamma$ for all $k = 1, \ldots, m-1$ and $m \geqslant 4$ and $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$.
Now, consider a utility profile $u \in \overline{\mathcal{U}}$ such that $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k, i_{k+1}) < u_{i_k}(i_k i_{k-1} i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(\Gamma) \backslash \{i_{k-1}, i_{k+1}\}$.
Let $\Lambda^\star$ be a stable assignment in this network economy. Note that in the stable assignment $\Lambda^\star$ the largest number of agents that can form a cooperative that satisfies the [IR] condition is three. We again consider two sub-cases.

Case A. First, suppose that $i_k i_k \in \Lambda^\star$ for some $k = 1, \ldots, m-1$. Since $\Lambda^\star$ is a stable assignment, the individual rationality condition is satisfied for all agents in N. Hence, agent i_{k-1} is in a state of autarky or connected to agent i_{k-2} either in the matching $g' = \{i_{k-1} i_{k-2}\}$, or in the cooperative $g'' = \{i_{k-2} i_{k-1} i_{k-3}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, and $i_{-2} = i_{m-3}$. In all three cases the [PS] condition is violated: $u_{i_k}(i_{k-1} i_k) > u_{i_k}(i_k i_k)$ and $u_{i_{k-1}}(i_{k-1} i_k) > u_{i_{k-1}}(g'') = u_{i_{k-1}}(g') > u_{i_{k-1}}(i_{k-1} i_{k-1})$. Since $\Lambda^\star$ is stable, then it cannot be that $\{i_k i_k\} \in \Lambda^\star$ for some $i_k \in C$.

Case B. Next, suppose that there is no agent along the cycle such that $i_k i_k \in \Lambda^\star$. Since $\Lambda^\star$ is a stable assignment, the [IR] condition is satisfied for all agents in N. Since $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$, $m-1 \geqslant 4$ and there is no agent i_k along the cycle such that $i_k i_k \in \Lambda^\star$, there must be at least two distinct agents along the cycle, i_{k-1} and i_k for some $k = 1, \ldots, m-1$ and $k_0 = m-1$, such that the matching $\{i_{k-1}, i_k\} \in \Lambda^\star$. Then, agent i_{k-2} is connected to agent i_{k-3} either in the matching $g' = \{i_{k-2} i_{k-3}\}$, or in the cooperative $g'' = \{i_{k-3} i_{k-2} i_{k-4}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, $i_{-2} = i_{m-3}$, and $i_{-3} = i_{m-4}$. In all cases the no blocking condition [PS*] is violated: $u_{i_{k-2}}(i_{k-1} i_{k-2} i_k) > u_{i_{k-2}}(g') = u_{i_{k-2}}(g'')$ as the the matching $i_k i_{k-2} \notin \Gamma$ and $u_{i_{k-1}}(\{i_{k-1} i_{k-2} i_k\}) \geqslant u_{i_{k-1}}(i_{k-1} i_k)$ with $k_{-1} = m-2$ due to superadditivity.
Hence, when Γ contains a cycle with a number of connected agents not a multiple of three, there are such utility profiles that satisfy superadditivity and non-externality properties, for which there is no stable assignment in the network economy.

This completes the proof of Theorem 4.6.

<u>**Not for Publication**</u>

C Proof of Theorem 4.12

Before we present the proof we will introduce additional shorthand notation and some auxiliary results.

First, we introduce some new terms. Let $\mathbb{E} = (N, \Gamma, u)$ be a network economy. Let Λ be an assignment. The *neighborhood of agent* $i \in N$ *in assignment* Λ is denoted by $N_i(\Lambda)$. The *utility of agent* i *in assignment* Λ is denoted by $u_i(\Lambda)$. Furthermore, we say that agents $i \in N$ and $j \in N$ form a *blocking pair* if one of the conditions in Definition 4.2 is not satisfied with respect to these agents. Last, we introduce several relationships between assignments. We will say that a *blocking pair in assignment* Λ *is satisfied in assignment* Λ' if assignment Λ' is formed by satisfying the condition in Definition 4.2 that is violated in assignment Λ for a given blocking pair of agents i and j.

Let the assignment Λ' be formed by severing all links of agent i in assignment Λ and forming the autarky ii. Then the relationship between assignments Λ and Λ' will be denoted as $\Lambda' = \Lambda \cup \{ii\}$.

Let the assignment Λ' be formed by severing all links of two distinct agents i and j in assignment Λ with $j \notin N_i(\Lambda)$ and forming the matching ij. Then the relationship between assignments Λ and Λ' will be denoted as $\Lambda' = \Lambda \cup \{ij\}$. Notice that for all agents $k \in N_i(\Lambda)$ such that $N_k(\Lambda) = \{i\}$, it will hold that $\{kk\} \subseteq \Lambda'$. Similarly, for all agents $l \in N_j(\Lambda)$ such that $N_l(\Lambda) = \{j\}$, it will hold that $\{ll\} \subseteq \Lambda'$.

Last, let the assignment Λ' be formed by severing all links of agent i in assignment Λ and forming the link between agents i and j with $j \notin N_i(\Lambda)$ such that agent j keeps all his links present in assignment Λ. Then the relationship between assignments Λ and Λ' will be denoted as $\Lambda' = \Lambda \oplus^j \{ij\}$ where $\oplus^j$ indicates that agent j acts as a convener and keeps all his links. Notice that for all agents $k \in N_i(\Lambda)$ such that $N_k(\Lambda) = \{i\}$, it will hold that $\{kk\} \subseteq \Lambda'$.

Below we present some preliminary results.

Lemma 3 *Let (N, Γ, u) be a network economy such that the utility function u exhibits multiplicative size-based externalities with $\alpha_c > 0$ for all feasible conveners $c \in N$ in $\Sigma(\Gamma) = \Sigma(\Gamma)$. Then for any agent $i \in N$ and any two cooperatives G and $H \in \Sigma(\Gamma)$ with $i \in N(G)$ and $i \in N(H)$ and $N^*(G) = N^*(H)$ and $N^*(G) \neq \{i\}$, it holds that*

(i) *$u_i(G) = u_i(H)$ if and only if $\#N(G) = \#N(H)$*

(ii) *$u_i(G) < u_i(H)$ if and only if $\#N(G) < \#N(H)$.*

The proof of Lemma 3 follows directly from the definitions and is therefore omitted.

Lemma 4 *Let (N, Γ, u) be a network economy. Let Γ be acyclic. Then there is at most one path between any two distinct agents in N.*

The proof of Lemma 4 follows immediately from the fact that Γ is acyclic. As a corollary of Lemma 4, we know that for any agent $i \in N$ and any two distinct agents $j, k \in N_i(\Gamma)$, it holds that $jk \notin \Gamma$.

Lemma 5 *Let (N, Γ, u) be a network economy and Γ be acyclic. Let Λ and Λ' be two assignments in this network economy such that Λ' is formed by satisfying a blocking pair between two agents $s, t \in N$. Consider an agent $j \in N \setminus \{s, t\}$ such that $p_{js} = (i_1, \ldots, i_m)$ with $i_1 = j$ and $i_m = s$ and $t \notin p_{js}$ who does not form a blocking pair in Λ. Then:*

(i) If $j \in N_s(\Lambda)$ and $\Lambda' = \Lambda \oplus^s \{st\}$, j cannot form a blocking pair in Λ';

(ii) If $m > 4$, then agent j cannot form a blocking pair in Λ';

(iii) If $m \geqslant 3$ and $i_{m-1}s \notin \Lambda$, then agent j cannot form a blocking pair in Λ';

(iv) If $m = 2$ and $js \notin \Lambda$, then the only blocking pair agent j may form in Λ' is with agent s in which PS condition of Definition 4.2 is not satisfied and agent s acts as a convener;*

(v) If $m = 4$, then agent j may only form a blocking pair in Λ' with agent i_2 and only if $N_{i_2}(\Lambda) = N_s(\Lambda)$;

Proof. Consider a network economy (N, Γ, u) with Γ be acyclic. Let Λ and Λ' be two assignments such that Λ' is formed by satisfying a blocking pair between two agents $s, t \in N$. Consider an agent $j \in N \setminus \{s, t\}$ such that $p_{js} = (i_1, \dots, i_m)$ with $i_1 = j$ and $i_m = s$ and $t \notin p_{js}$ who does not form a blocking pair in Λ.

(i) Let $j \in N_s(\Lambda)$ and $\Lambda' = \Lambda \oplus^s \{js\}$. By Lemma 3, $u_j(\Lambda) < u_j(\Lambda')$ and by Lemma 4 for all $h \in N_j(\Gamma)$ with $j \neq s$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Hence if agent j could form a blocking pair in Λ', he could form the same blocking pair in Λ.

(ii) Let $m > 4$. By Lemma 4, $m > 4$, and $t \notin p_{js}$ it follows that $N_j(\Gamma) \cap N_s(\Gamma) = \emptyset$ and $N_j(\Gamma) \cap N_t(\Gamma) = \emptyset$. Hence, for agent j it holds that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$. Moreover, since $m > 4$ for all agents $h \in N_j(\Gamma)$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Since agent j can only form a blocking pair with an agent $h \in N_j(\Gamma)$, it follows that if j does not form a blocking pair in Λ, j cannot form a blocking pair in Λ' either.

(iii) If $m > 4$, the proof follows the proof of case (ii) above. Let $m = 3$ or $m = 4$ and $i_{m-1}s \notin \Lambda$. By $i_{m-1}s \notin \Lambda$ and using Lemma 4, it follows that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$ and that for all agents $h \in N_j(\Gamma)$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Since agent j can only form a blocking pair with an agent $h \in N_j(\Gamma)$, it follows that if j does not form a blocking pair in Λ, j cannot form a blocking pair in Λ' either.

(iv) Let $m = 2$ and $js \notin \Lambda$. First suppose that agent j can form a blocking pair in Λ' with an agent $h \in N_j(\Gamma)$ with $h \neq s$. This is not possible due to case (iii) above.

Next, suppose that agents j and s form a blocking pair in Λ' because the PS condition of Definition 4.2 is not satisfied. Hence, it must be that $u_s(\Lambda') < u_s(js)$ and $u_j(\Lambda') < u_j(js)$. Since $u_j(\Lambda) = u_j(\Lambda')$ and $u_s(\Lambda) < u_s(\Lambda')$, agents j and s could form a blocking pair in Λ, which establishes a contradiction to the fact that agents s and t form the only blocking pair in Λ.

Last, suppose that agents j and s form a blocking pair in Λ' becasue the PS* condition of Definition 4.2 is not satisfied and agent j acts as a convener. Hence it must be that $u_s(\Lambda') < u_s(js) + \alpha_j \# N_s(\Lambda')$ and $u_j(js) > -\alpha_j$. Since $u_s(\Lambda') > u_s(\Lambda)$ it follows that agents j and s could form a blocking pair in Λ, which establishes a contradiction to the fact that agents s and t form the only blocking pair in Λ.

Hence the only blocking pair agents j and s can form in Λ' is if the PS* condition of Definition 4.2 is not satisfied with agent s acting as a convener.

(v) Let $m = 4$. Lemma 4, $m = 4$, and $t \notin p_{js}$ imply that $N_j(\Gamma) \cap N_s(\Gamma) = \emptyset$ and $N_j(\Gamma) \cap N_t(\Gamma) = \emptyset$. Hence, for agent j it holds that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$. Moreover, since $m = 4$ there is only one agent $k \in N_j(\Gamma)$ for whom it may hold that $u_k(\Lambda) > u_k(\Lambda')$ and it can only hold if $N_k(\Lambda) = N_s(\Lambda)$: for all other agents $h \in N_j(\Gamma) \setminus \{k\}$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Therefore, if j does not form a blocking pair in Λ, the only blocking pair he can form in Λ' is with agent k.

This completes the proof of Lemma 5. ■

Proof of Theorem 4.12

Let $\mathbb{E} = (N, \Gamma, u)$ be a network economy such that u exhibits multiplicative size-based externalities such that $\alpha_c > 0$ for all potential conveners $c \in N^*(\Sigma(\Gamma))$. Suppose Γ is acyclic.
Suppose, that $\mathbb{E}$ does not admit a stable assignment. Therefore there exists a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \dots, \Lambda_r)$ with Λ_{k+1} constructed by satisfying a blocking pair in Λ_k for $k = 1, \dots, r-1$ such that $\Lambda_r = \Lambda_1$. If not, due to the finite number of assignments, we can construct a stable assignment by satisfying blocking pairs sequentially.
Furthermore, all assignments have a blocking pair. Hence, starting from any sequence of assignments $\mathbf{\Lambda}' = (\Lambda'_1, \dots, \Lambda'_r)$ with $r \geqslant 4$ such that any assignment $\Lambda'_f \subseteq \mathbf{\Lambda}'$ is formed by satisfying a blocking pair in the preceding assignment Λ'_{f-1} for $f = 1, \dots, r-1$ contains an assignment $\Lambda'_k \subseteq \mathbf{\Lambda}$ such that $\Lambda'_r = \Lambda'_k$. Otherwise, due to the finite number of possible assignments, we can construct stable assignment by satisfying blocking pairs sequentially.
Without loss of generality, suppose that there is exactly one such sequence $\mathbf{\Lambda} = (\Lambda_1, \dots, \Lambda_r)$ with $r \geqslant 4$ such that any assignment $\Lambda_k \subseteq \mathbf{\Lambda}$ is formed by satisfying a blocking pair in the preceding assignment Λ_{k-1} for $k = 1, \dots, r-1$ with $\Lambda_r = \Lambda_1$. Hence starting from any assignment Λ by satisfying blocking pairs we reach some assignment $\Lambda_k \subseteq \mathbf{\Lambda}$. Moreover, each assignment $\Lambda_1, \dots, \Lambda_r \subseteq \mathbf{\Lambda}$ has exactly one blocking pair, otherwise, there are other sequences of assignments $(\Lambda'_1, \dots, \Lambda'_r)$ with $\Lambda'_r = \Lambda'_1$.
We will discuss all possible types of blocking pairs in Λ_1 and show that it cannot be that $\Lambda_r = \Lambda_1$.

Case I: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ with $\{ii\} \subseteq \Lambda_1$ and $\{jj\} \subseteq \Lambda_1$ such that agents i and j form a blocking pair. Hence $u_i(ii) < u_i(ij)$ and $u_j(jj) < u_j(ij)$. Since $\Lambda_r = \Lambda_1$, there must be an assignment $\Lambda_q \subseteq \mathbf{\Lambda}$ with $1 < q < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.
Without loss of generality, suppose agent i deletes the link with agent j. For agent i to delete this link there must be an agent $t \in N_i(\Gamma)$ with $t \neq j$ such that $u_t(\Lambda_1) \neq u_t(\Lambda_q)$, so that agents t and i form a blocking pair in Λ_q but not in Λ_1. For $u_t(\Lambda_q) \neq u_t(\Lambda_1)$ it must be that agent t forms a blocking pair in some assignment Λ_k with $1 < k < q$. By Lemmas 4 and 5 cases (ii) and (iii) it follows that no agent $h \notin N_i(\Gamma) \cup N_j(\Gamma)$ may form a blocking pair before forming a blocking pair with agent i or j. Hence, agent t must form a blocking pair with agent i in Λ_k and by Lemma 5 case (iii), it follows that the agents i acts as convener in that blocking pair. Hence it must be that $u_i(it) > -\alpha_i$ and $u_t(\Lambda_1) < u_t(it) + \alpha_i \#N_i(\Lambda_k)$. By Lemmas 4 and 5 cases (i) and (iii), it follows that agent j will thus not form a blocking pair that requires him to delete the link with agent i in any assignment $\Lambda_{k+1}, \dots, \Lambda_q$.
Since agents i and t form a blocking pair in Λ_q, they are not linked in Λ_q and since agent i cannot delete a link with agent t without deleting a link with agent j, there must be another assignment Λ_m with $k < m < q$ in which agent t forms a blocking pair that requires him to

sever his link with agent i.
Because agent t forms a blocking pair in Λ_m by deleting the link with agent i, by Lemma 5 cases (i) and (iii), it must be that $\#N_i(\Lambda_q) = \#N_i(\Lambda_k)$. Since there is only one blocking pair in Λ_q and it requires agent i to delete its links, it must be that $u_t(\Lambda_q) > u_t(\Lambda_m) = u_t(it) + \alpha_i \#N_i(\Lambda_k) > u_t(it)$, otherwise agents i and t could form a blocking pair when i acts as a convener. Therefore, it cannot be that agents i and t form a blocking pair because the PS condition of Definition 4.2 is not satisfied. So it must be that agents i and t form a blocking pair in Λ_q because the PS* condition of Definition 4.2 is not satisfied and agent t acts as a convener. Hence $u_t(it) > -\alpha_t$. If agents i and t did not form a blocking pair in Λ_1 it must be that either agent t could not act as a convener in Λ_1, or $\#N_t(\Lambda_1) < \#N_t(\Lambda_q)$.

First suppose agents i and t cannot form a blocking pair in Λ_1 because agent t cannot act as a convener.

1. Suppose $\{tt\} \in \Lambda_1$. By Lemmas 4 and 5, we know that $N_l(\Lambda_1) = N_l(\Lambda_m)$ or all $h \in N_t(\Gamma) \setminus \{i\}$, thus, $u_h(\Lambda_1) = u_h(\Lambda_m)$. If agent t forms a blocking pair in Λ_m, such that he deletes the link with i, t could have formed a blocking pair in Λ_1 because $u_t(\Lambda_1) < u_t(\Lambda_m)$ by Lemma 3 and the fact that agent i cannot delete a link without deleting all its links. Thus establishing a contradiction that there is only one blocking pair in Λ_1 and it involves agents $i \neq t$ and $j \neq t$.

2. Suppose $st \in (\Lambda_1)$ with $s \in N^*(\Lambda_1)$. If agents i and t form a blocking pair in Λ_k such that agent i acts as a convener, it must be that $u_t(\Lambda_k) < u_t(it) + \alpha_i \#N_i(\Lambda_k)$. By Lemmas 4 and 5 if agent t forms a blocking pair in Λ_m that requires him to delete the link with agent i, it must be to form a blocking pair with agent s as for all $h \in N_t(\Gamma)$ with $h \neq s$ and $h \neq i$, $N_h(\Lambda_1) = N_h(\Lambda_k) = N_h(\Lambda_m)$. If agents s and t form a blocking pair in Λ_m, it must be that $\#N_s(\Lambda_m) > \#N_s(\Lambda_1)$ and agent s acts as a convener with agent t.
Hence agent t cannot form a blocking pair with agent i as t cannot act as a convener. Moreover, by Lemmas 4 and 5 and the fact that there is only one blocking pair in each assignment, then agent t could only form a blocking pair if agent s deletes all the links agent t will be autarkic and since $u_t(tt) < u_t(it) + \alpha_i \#N_i(\Lambda_k)$ the only blocking pair he could form is with agent i acting as a convener.

Second, suppose that agents i and t did not form a blocking pair in Λ_1 because $\#N_t(\Lambda_1) < \#N_t(\Lambda_q)$ and agent t could form a blocking pair when acting as a convener in Λ_1. Since there is no assignment $\Lambda' \subseteq \mathbf{\Lambda}$ such that t forms a blocking pair when acting as a convener with an agent $p \notin N_t(\Lambda_1)$ unless $\#N_t(\Lambda') > \#N_t(\Lambda_1)$, otherwise agent t could form this blocking pair in Λ_k, there is no assignment Λ_q in which agent i deletes the link with j to join agent t as a convener.

Thus we have shown that agents i and j will not delete their link, hence $\Lambda_r \neq \Lambda_1$. Therefore, a blocking pair when the PS condition of Definition 4.2 is not satisfied for two autarkic agents cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case II: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{jj\} \in \Lambda_1$ and $\{ih\} \in \Lambda_1$ with $i \neq h$ such that agents i and j form a blocking pair because the PS* condition in Definition 4.2 is not satisfied. Hence $u_j(jj) < u_i(ij) + \alpha_i$ and $u_i(ij) > -\alpha_i$. Consider assignment $\Lambda_2 = \Lambda_1 \oplus^i \{ij\}$. Since $\Lambda_r = \Lambda_1$, there must be an assignment $\Lambda_m \subseteq \mathbf{\Lambda}$ with $1 < m < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.
Since there is no stable assignment, there must be a blocking pair in assignment Λ_2. By Lemma 5 cases (i), (ii), and (iii) the blocking pair must involve agent i or j.

Suppose the blocking pair involves agent i. By Lemmas 4 and 5 and the fact that there is only one blocking pair in Λ_1 any assignment formed by satisfying a blocking pair that involves an agent l with $i \in p_{jl}$ agent j or agent h will not form a blocking pair, unless satisfying the blocking pair does not require for agent i to delete simultaneously his links with agents j and h. So, for $\Lambda_1 = \Lambda_r$, there must be an assignment in which agent i deletes his links with agents j and h. To find a contradiction we can follow the reasoning in Case I.
Suppose instead the blocking pair involves agent j. It must be that it requires from agent j to sever his link with agent i. By Lemma 4 and Lemma 5 there is no such agent with whom j can form a blocking pair, otherwise j could form an alternative blocking pair in Λ_1.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for an autarkic agent and an agent in a matching cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case III: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{jj\} \in \Lambda_1$ and $i \in N^*(\Lambda_k)$ such that agents i and j form a blocking pair because the PS* condition in Definition 4.2 is not satisfied. Hence $u_j(jj) < u_i(ij) + \alpha_i \# N_i(\Lambda_1)$ and $u_i(ij) > -\alpha_i$. Consider assignment $\Lambda_2 = \Lambda_1 \oplus^i \{ij\}$. Since $\Lambda_r = \Lambda_1$, there must be an assignment $\Lambda_m \subseteq \mathbf{\Lambda}$ with $1 < m < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.
Following the method and reasoning of Cases I and II, we can show a contradiction.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for an autarkic agent and a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case IV: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$. Let agent i form a blocking pair because the IR condition of Definition 4.2 is not satisfied. Hence $u_i(ii) > u_i(ij)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ii\}$. Since there is no stable assignment, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$ and $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i or j and an agent $h \in N_i(\Gamma) \cup N_j(\Gamma)$. By Lemma 5 case (iv) this is not possible as neither agent i nor j can act as a convener in a blocking pair.
Therefore, a blocking pair in which the IR condition of Definition 4.2 is not satisfied for an agent in a matching cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case V: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $N_i(\Lambda_1) = \{j\}$ and $j \in N^*(\Lambda_1)$. Let agent i form a blocking pair because the IR condition of Definition 4.2 is not satisfied. Hence $u_i(ii) > u_i(ij) + \alpha_j[\# N_j(\Lambda_1) - 1]$.
Consider assignment $\Lambda_2 = \Lambda_k \cup \{ii\}$. Since there is no stable assignment, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and for all agents $l \in N \setminus \{i, j, N_j(\Lambda_2)\}$, $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i or j or an agent $h \in N_j(\Lambda_2)$.
Following the analysis in Case IV, we can show that agent i does not form a blocking pair before he forms a blocking pair with agent j. Moreover, if there is an assignment such that $\# N_j(\Lambda_m) \geq \# N_j(\Lambda_1)$, then it must be that $\{ii\} \in \Lambda_m$ and agents i and j form blocking pair in which agent j acts as a convener and agent i is autarkic. By Case III we know that a blocking pair between an autarkic agent and an agent who is acting as a convener cannot be part of a sequence of assignments such that $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_1 = \Lambda_r$, and hence it cannot be that i and j form a blocking pair. Hence $\Lambda_r \neq \Lambda_1$.
Therefore, a blocking pair in which the IR condition of Definition 4.2 is not satisfied for an agent in a cooperative who is not the convener of the cooperative cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case VI: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $\{js\} \in \Lambda_1$ with $j \neq s$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > u_j(js)$. Since this is the only blocking pair and $\alpha_j > 0$, it must also be that $u_j(ij) < -\alpha_j$, otherwise agents i and j could form a blocking pair because the PS* condition of Definition 4.2 is not satisfied. We will show that there cannot be an assignment $\Lambda_q \subseteq \mathbf{\Lambda}$ with $\{js\} \in \Lambda_q$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ij\}$. Since there is no stable assignment, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j, s\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and for all agents $l \in N \setminus \{i, j, s\}$ it must be that $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i, j or s.
Suppose the blocking pair in Λ_2 involves agent i, then using the analysis for agent i in Case I, it can be shown that agent i will not delete the link with agent j. And by Lemmas 4 and 5, if agents j and s do not form a blocking pair in Λ_2, they will not form a blocking pair.
Next, suppose that the blocking pair in Λ_2 involves agent j. Since $u_j(js) < u_j(ij)$ and $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with agent s when acting as a convener. Since $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with an agent $h \in N_j(\Gamma)$ with $h \notin N_j(\Lambda_1)$ otherwise agent j could form another blocking pair in Λ_1.
It must be that agent s forms a blocking pair in Λ_2. Hence, by Lemmas 4 and 5 and the fact that there is only one blocking pair in each assignment in $\mathbf{\Lambda}$ the first blocking pair agent j is with agent s. Suppose agent j and s form a blocking pair in some assignment $\Lambda_k \subseteq \mathbf{\Lambda}$ with $2 < k < q$. By the above discussion, it follows that j and s form a blocking pair because the PS* condition of Definition 4.2 is not satisfied and agent s acts as a convener. Hence agent i is autarkic in Λ_{k+1} and does not form a blocking pair unless it is with agent j. In addition, there is at least one agent $h \in N_s(\Gamma)$ with $s \neq j$ such that $h \in N_s(\Lambda_k)$. Note that by Lemmas 4 and 5 and the fact that there is only one blocking pair, h does not form a blocking pair until agent s does not delete the link and agent s cannot delete the link with agent h without deleting the link with agent j as well. Hence, $\{js\}$ cannot be an element of an assignment unless agent s deletes all his links as a convener.
Suppose there is an assignment Λ_m with $k < m < r$ such that agent s deletes all his links as a convener and thus agent j is autarkic. Since $u_j(jj) < u_j(ij)$ (otherwise agent j could form a different blocking pair in Λ_1), it must be that the only blocking pair in Λ_m must be by agent j and i because the PS condition of Definition 4.2 is not satisfied. Since the blocking pair of agents i and j entails two autarkic agents who form a blocking pair and by Case I, we know that such blocking pair cannot be part of a sequence of assignments $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$, thus, we have established a contradiction.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for an autarkic agent and an agent in a matching cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case VII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $\{j\} \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > \sum_{h \in N_j(\Lambda_1)} u_j(jh) + \alpha_j[\#N_j(\Lambda_1) - 1]$. Since this is the only blocking pair and $\alpha_j > 0$, it must also be that $u_j(ij) < -\alpha_j$, otherwise agents i and j can form a blocking pair because the PS* condition of Definition 4.2 is not satisfied. We will show that there cannot be an assignment $\Lambda_q \subseteq \mathbf{\Lambda}$ with $N_j(\Lambda_1) = N_j(\Lambda_q)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ij\}$. Since there is no stable assignment, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j, N_j(\Lambda_1)\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i, j or an agent $s \in N_j(\Lambda_1)$.

Suppose the blocking pair in Λ_2 involves agent i, then using the analysis for agent i in Case I, it can be shown that agent i will not delete the link with agent j. And by Lemmas 4 and 5, if agents j and any agent $s \in N_j(\Lambda_1)$ do not form a blocking pair in Λ_2, they will not form a blocking pair unless agent i deletes his link with j.
Next, suppose the blocking pair in Λ_2 involves agent j and no agent $s \in N_j(\Lambda_1)$. Since $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with an agent $h \in N_j(\Gamma)$ with $h \notin N_j(\Lambda_1)$ otherwise agent j could form another blocking pair in Λ_1.
Suppose agent j forms a blocking pair in Λ_2 with an agent $s \in N_j(\Lambda_1)$ with $\{ss\} \in \Lambda_2$. This, however, contradicts either Case II or Case VI.
Lastly, suppose that an agent $s \in N_j(\Lambda_1)$ forms a blocking pair in Λ_2 with an agent $f \in N_s(\Gamma)$ with $f \neq j$. Hence if there is an assignment Λ_q with $N_j(\Lambda_q) = N_j(\Lambda_k)$ agent j must form a blocking pair and the first blocking pair j can make by Lemmas 4 and 5 is with agent s. Let the assignment in which agents j and s form a blocking pair is Λ_k. It must be that j and s form a blocking pair because the PS* condition of Definition 4.2 such that agent s acts as a convener is not satisfied otherwise j and s could form a blocking pair in Λ_2. Hence by Lemmas 4 and 5 and the fact that there is only one blocking pair in each assignment agent j cannot form a blocking pair with an agent $h \in N_j(\Lambda_1)$ with $h \neq s$, unless agent s deletes all his links. If agent s deletes all his links, agent j will be autarkic, and hence, must form a blocking pair with agent i, otherwise agents i and j could not form a blocking pair in Λ_1. Since j is autarkic and i is autarkic when making a blocking pair with i, we know by Case I that this blocking pair cannot be part of a sequence of assignments $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$ and thus we have established a contradiction.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for an autarkic agent and a convener of a cooperative cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case VIII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $N_j(\Lambda_1) = \{s\}$ and $s \in N^*(\Lambda_k)$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1]$. We will show that there cannot be an assignment $\Lambda_q \subseteq \mathbf{\Lambda}$ with $N_s(\Lambda_q) = N_s(\Lambda_1)$.
For $N_s(\Lambda_q) = N_s(\Lambda_k)$, agents j and s must form a blocking pair. Agents j and s can form a blocking pair if and only if one of them acts as a convener.
Suppose agents j and s form a blocking pair in $\Lambda_k \subseteq \mathbf{\Lambda}$ and agent j acts as a convener. For $N_s(\Lambda_q) = N_s(\Lambda_1)$ agent s must be able to form blocking pairs as a convener, hence, there must be an assignment $\Lambda_m \subseteq \mathbf{\Lambda}$ with $k < m < q$ such that either $\{js\} \in \Lambda_m$ or s severs his link with j. By Lemma 5 case (i) if agent j acts as a convener, agent i will not sever his link with him, otherwise there could be another blocking pair in Λ_1. Hence, it must be that s severs his link with j, which implies that j must join agent s as a convener, and the analysis below will hold.
Suppose agents j and s form a blocking pair in $\Lambda_k \subseteq \mathbf{\Lambda}$ and agent s acts as a convener. For agent j to sever his link with i to join s as a convener, it must be that $N_s(\Lambda_k) > N_s(\Lambda_1) - 1$. Hence, there is an agent $h \in N_s(\Gamma)$ with $h \in N_s(\Lambda_k)$ and $h \notin N_s(\Lambda_1)$. So, $\Lambda_{k+1} = \Lambda_k \oplus^s \{js\}$. Hence by Lemma 5 no agent $h \in N_s(\Lambda_k)$ will form a blocking pair unless agent s severs all his links. Suppose there is assignment $\Lambda_m \subseteq \mathbf{\Lambda}$ with $k+1 < m < q$ agent s severs all his links, then agent j will be autarkic and must form a blocking pair with agent i as another autarkic agent or as i acting as a convener, otherwise agents i and j could not form a blocking pair in Λ_1 and by Cases I, II, and III such blocking pair cannot be part of a sequence of assignments $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for an autarkic agent and an agent linked in a cooperative but not acting as the convener of the cooperative cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case IX: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$. Let agent i form a blocking pair because the IR condition of Definition 4.2 is not satisfied. Hence $\Lambda_2 = \Lambda_1 \cup \{ii\}$. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1 - 1)] < u_i(ii)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ii\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and all agents $h \in N_i(\Lambda_1))$ form blocking pairs in some assignments. Note that $\{ii\} \in \Lambda_2$ and $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus agent i and each agent $h \in N_i(\Lambda_1)$ must form a blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair when the IR condition of Definition 4.2 is not satisfied for a convener of a cooperative cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case X: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{st\} \in \Lambda_1$ with $s \in N_j(\Gamma)$ and $s \neq i$. Let agents j and s form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_j(ij) < u_j(js)$ and $u_s(st) < u_s(js)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and t form blocking pairs in some assignments. Note that $\{ii\} \in \Lambda_2$ and $\{tt\} \in \Lambda_2$ and thus i and t form blocking pairs as autarkic agents. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for two agents linked in matchings cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XI: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{st\} \in \Lambda_1$ with $s \in N_j(\Gamma)$ and $s \neq i$. Let agents j and s form a blocking pair because the PS* condition of Definition 4.2 is not satisfied. Hence $u_j(ij) < u_j(js) + \alpha_S$ and $u_s(js) > -\alpha_s$.
Consider assignment $\Lambda_2 = \Lambda_1 \oplus^s \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agent i and j form blocking pairs in some assignment. Note that $\{ii\} \in \Lambda_2$ and thus i must form a blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for two agents linked in matchings cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_k)$. Let agents j and s form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_j(ij) < u_j(js)$ and $\sum_{h \in N_s(\Lambda_1)} u_s(hs) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_s(js)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and $h \in N_s(\Lambda_1)$ form blocking pairs in some assignments. Note that $\{ii\} \in \Lambda_2$ and $\{hh\} \in \Lambda_2$ for all $h \in N_s(\Lambda_1)$ and thus i and each $h \in N_s(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for an agent linked in a matchings and a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XIII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 4.2 is not satisfied and agent s acts as a convener. Hence $u_s(js) > -\alpha_s$ and $u_j(ij) < u_j(js) + \alpha_s \# N_s(\Lambda_1)$.
Consider assignment $\Lambda_2 = \Lambda_1 \oplus^s \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and j form blocking pairs in some assignments. Note that $\{ii\} \in \Lambda_2$ and thus agent i must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for an agent linked in a matchings and a convener such that the agent in the matching acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XIV: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 4.2 is not satisfied and agent j acts as a convener. Hence $\sum_{h \in N_s \Lambda_1} u_s(hs) + \alpha_s[\# N_s(\Lambda_1) - 1] < u_s(js) + \alpha_j$ and $u_j(js) > -\alpha_j$.
Consider assignment $\Lambda_2 = \Lambda_1 \oplus^j \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_s(\Lambda_1)$ and s form blocking pairs in some assignments. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_s(\Lambda_1)$ and thus each $h \in N_s(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for an agent linked in a matchings and a convener such that the convener of the cooperative matching acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XV: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $j \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\# N_i(\Lambda_1) - 1] < u_i(ij)$ and $\sum_{f \in N_j(\Lambda_1)} u_j(jf) + \alpha_j[\# N_j(\Lambda_1) - 1] < u_j(ij)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_i(\Lambda_1)$ form blocking pairs with agent i and agents $f \in N_j(\Lambda_1)$ form blocking pairs with agent j in some assignments. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and $\{ff\} \in \Lambda_2$ for all $f \in N_j(\Lambda_1)$ and each $h \in N_i(\Lambda_1)$ and each $f \in N_j(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases II, III, IV, VII, VIII, and XI, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for two conveners cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XVI: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $j \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS* condition of Definition 4.2 is not satisfied. Without loss of generality let $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\# N_i(\Lambda_1) - 1] < u_i(ij) + \alpha_j \# N_j(\Lambda_1)$ and $u_j(ij) > -\alpha_j$.
Consider assignment $\Lambda_2 = \Lambda_1 \oplus^j \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_i(\Lambda_1)$ form blocking pairs with agent i in some assignments. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus each $h \in N_i(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for two conveners cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XVII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\}$ and $N_s(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1$. Let agents j and s form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_j(ij) < u_j(js)$ and $u_s(st) + \alpha_t[\#N_t(\Lambda_1) - 1] < u_s(js)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and j form a blocking pair in some assignments. Note that $\{ii\} \in \Lambda_2$ and thus agent i form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for an agent in a matching and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XVIII: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\}$ and $N_s(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 4.2 is not satisfied and j acts as a convener. Hence $u_j(js) > -\alpha_j$ and $u_s(st) + \alpha_t[\#N_t(\Lambda_1) - 1] < u_s(js) + \alpha_j$. We will show that there is no activity patter $\Lambda_q \subseteq \mathbf{\Lambda}$ such that $N_t(\Lambda_q) = N_t(\Lambda_1)$. A contradiction can be established following the same analysis as in Case VIII.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for an agent in a matching and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XIX: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{s\}$ with $s \in N^*(\Lambda_1)$ and $i \neq s$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1) - 1] < u_i(ij)$ and $u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_j(ij)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agent i and an agent $h \in N_i(\Lambda_1)$ form a blocking pair in some assignments. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus agent each agent $h \in N_i(\Lambda_1)$ forms at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, assignments in which autarkic agents form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for a convener and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XX: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{s\}$ with $s \in N^*(\Lambda_1)$ and $i \neq s$. Let agents i and j form a blocking pair because the PS* condition of Definition 4.2 is not satisfied. Hence $u_i(ij) > -\alpha_i$ and $u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_j(ij) + \alpha_i \#N_i(\Lambda_1)$. We will shows that there cannot be an assignment $\Lambda_q \subseteq \mathbf{\Lambda}$ such that $N_s(\Lambda_q) = N_s(\Lambda_1)$.
A contradiction can be established following the same analysis as in Case VIII.
Therefore, a blocking pair in which the PS* condition of Definition 4.2 is not satisfied for a convener and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

Case XXI: Consider assignment $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $N_i(\Lambda_1) = s$ with $s \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1)$ and $i \neq j$. Let agents i and j form a blocking pair because the PS condition of Definition 4.2 is not satisfied. Hence $u_i(is) + \alpha_s[N_s(\Lambda_1) - 1] < u_i(ij)$ and $u_j(jt) + \alpha_t[\#N_t(\Lambda_1) - 1] < u_j(ij)$.
Consider assignment $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and s form

a blocking pair in some assignments and agents j and t forma a blocking pair in some assignment. Note that $\{ij\} \in \Lambda_2$ and thus at least one of agents i and j forms at least one blocking pair as an agent in a matching. As proven in Cases II, IV, VI, X, XI, XII, XIII, XIV, XVII, and XVIII assignments in which agents in a matching form blocking pairs cannot be part of a sequence of assignments $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.
Therefore, a blocking pair in which the PS condition of Definition 4.2 is not satisfied for two agents linked in a cooperatives none of whom acts as a convener cannot be part of the sequence of assignments $\mathbf{\Lambda}$.

This completes the proof of Theorem 4.12.