

Tilburg University

Effects of a Red Card on Goal-Scoring in World Cup Football Matches

Cervený, Jakub; van Ours, Jan; van Tuijl, Martin

Publication date:
2016

Document Version
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):
Cervený, J., van Ours, J., & van Tuijl, M. (2016). *Effects of a Red Card on Goal-Scoring in World Cup Football Matches*. (CentER Discussion Paper; Vol. 2016-031). Economics.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

No. 2016-031

**EFFECTS OF A RED CARD ON GOAL-SCORING IN WORLD-
CUP FOOTBALL MATCHES**

By

Jakub Červený, Jan C. van Ours, Martin A. van Tuijl

22 August 2016

ISSN 0924-7815
ISSN 2213-9532

Effects of a Red Card on Goal-Scoring in World Cup Football Matches

Jakub Červený*

Jan C. van Ours[†]

Martin A. van Tuijl[‡]

August 22, 2016

Abstract

We examine the effect of the sending-off of a player on the goal-scoring rates in FIFA World Cup matches in tournaments from 1998 to 2014. We use a hazard rate framework in which the effect of a red card is modeled as a shift in the goal-scoring rate. A red card may harm the team that receives a red card and may be beneficial for their opponent. Indeed, we find that the goal-scoring rate of the sanctioned team goes down, while the goal-scoring rate of the non-sanctioned team goes up.

Keywords: Association football, goal-scoring, red card, world cup football matches

JEL-codes: Z29

The authors thank two anonymous reviewers for their helpful comments on a previous version of the paper.

*Department of Economics, CentER, Tilburg University, The Netherlands; j.cerveny@uvt.nl

[†]Corresponding author: Department of Economics, CentER, Tilburg University, P.O. Box 90153, 5000LE Tilburg, The Netherlands; Department of Economics, University of Melbourne, Australia; CEPR (London), CESifo (Munich), IZA (Bonn); vanours@uvt.nl.

[‡]Department of Economics, Tilburg University, The Netherlands; m.a.vtuijl@uvt.nl

1 Introduction

A red card in association football results in the sending-off of a player. A player receives a red card after committing either a serious foul or after misconduct. A player may receive a yellow card as a caution. Receiving a second yellow card also immediately results in a red card. The team receiving the red card faces a significant disadvantage, since the number of players of such a team is reduced from eleven to ten. At the same time, the resulting power-play potentially presents a goal-scoring advantage for the opponents. However, it is also conceivable that the sending-off of a player leads to a psychological “ten do it better” effect resulting in an enhanced spirit and a better defense of the sanctioned team. This would be in line with social impact theory according to which an increase in group size decreases the perceived pressure by members to put in effort. Consequently, the sending-off of a teammate motivates the remaining players to put in more effort.

According to Anderson and Sally (2013), the strength of a team crucially depends on the weakest player. Moreover, weak players are more likely to incur a red card than more talented players. Previous research on the effects of a red card shows mixed results. This research uses various estimation methods, focusing mainly on winning or losing or on goal-scoring. The timing of a red card sanction appears to be important. For example, Ridder, Cramer, and Hopstaken (1994) examine 340 matches from the two Dutch professional divisions from 1991-1992, focusing on the scoring intensity before and after a red card. They find that the goal-scoring intensity of the team playing with 11 players increases by 88 percent, while the goal-scoring intensity of the team with 10 players does not change. Furthermore, using the estimated results, they conclude that a red card occurring early in the match increases the odds of winning substantially, whereas the probability of victory for the sanctioned team decreases even more. They also find that around the 70th minute of a match, it becomes optimal for the defending team to commit a red-card type of foul to an opposing player with a clear way to the goal.

Caliendo and Radic (2006) study FIFA World Cup matches between 1930-2002. Following the modeling approach of Ridder, Cramer, and Hopstaken (1994), they also find that the earlier a sending-off occurs in a match, the higher the odds of winning for the non-sanctioned team are. The turning point of this effect occurs around half-time, with sending-offs in the second half of a match having no effect.

They do not find any supporting evidence for a “ten do it better” effect. Bar-Eli, Tenenbaum, and Geister (2006) study a sample of 743 games from the highest German league from 1963 to 2004. They argue that a red card is an indicator of a psychological performance crisis. Results show that when the first red card was issued, the score was more likely to be in favor of the opposing, unsanctioned team than in favor of the sanctioned team. Furthermore, a player’s sending off weakened the sanctioned team. Vecer, Kopriva, and Ichiba (2009) analyze the 2006 World Cup and 2008 Euro tournaments to estimate goal-scoring intensities from live betting odds. Such odds are updated during the match, reflecting the probability of an event such as one of the teams scoring a next goal. After a red card is given, the odds for scoring a next goal for both teams are updated to reflect the situation. They show that the goal-scoring intensity of the sanctioned team decreased, while the scoring intensity of an opposing team increased slightly. They find that goal-scoring in a match with a red card depends on the strength of a team. A sanction for the stronger team usually leads to fewer goals scored, whereas a sending-off for a weaker team usually leads to an increased number of goals. They also find that the optimal time to commit a foul depends on the score in a match.

Mechtel, Bäker, Brändle, and Vetter (2011) analyzing matches from the highest German league over the period 1999-2009 find that a red card given to the home team worsens their final score and result of the match. Contrary to this, the authors do not find such an effect for an away team. A sending-off occurring after the 70th minute leads to a better performance of the penalized guest team. Titman, Costain, Ridall, and Gregory (2015) study goals and bookings – both yellow cards and red cards – in two seasons (2009-2011) of English Premier League and Championship games. They distinguish between bookings and goal scoring for the home team and the away team. Except for home red cards that only have a modest effect on home team’s scoring rates for the three other combinations, home team red cards – away team scoring and away team red card on both home team and away team scoring, the red card has significant and substantial effects. Yellow cards do not affect goal scoring rates.

Not only the effects of issuing a red card have been studied. Also, the determinants of a red card sanction are analyzed. Some studies examine whether there is a referee bias. A comprehensive analysis of yellow and red card sanctions

in the 1996-2003 English Premier League seasons by Dawson, Dobson, Goddard, and Wilson (2007) shows that weaker teams tend to be more sanctioned than favorites. They also find that the incidence of sanctions is higher in matches with evenly balanced teams and they find evidence for a home team bias. After controlling for team strength, the authors find that away teams collect more sanctions than home teams. Similar effects are found by Buraimo, Forrest, and Simmons (2010) in their analysis of matches over the period 2000-2006 from the highest German and English Leagues. Using betting data to distinguish between “underdogs” and favorites, they also find evidence for home-team favoritism in both leagues. Pope and Pope (2015) find evidence for a player-referee own-nationality bias. They focus on the difference between fouls incurred and fouls committed by a player. A larger foul difference hence implies a larger number of beneficial foul calls. Using player-match data from the 2001-2013 UEFA Champions League seasons, they find that players officiated by a referee from the same country have a 10 percent increase in beneficial foul calls. This effect is even stronger for national team players as well as in later stages of the tournament. Furthermore, they find that experienced referees exhibit the same or even stronger nationality-bias as less experienced referees.

In our paper, we build on previous research analyzing both the determinants of the issuing of a red card and the consequences of a red card on goal scoring. We study World Cup matches, as these are characterized by the presence of top players and top referees. Furthermore, these matches are closely monitored (across the world). In these matches home advantage usually does not play a role as the participating teams (except for one) never play a home match. Therefore, World Cup matches are well-suited to study the effects of a red card on team performance. Our contribution to the literature is threefold: First, we investigate the determinants of the red card sanction rate focusing on the effect of the score during the match and the possible referee bias. Second, we use hazard rate models to determine the effects of a red card on the goal-scoring rate. Our model allows us to precisely determine the sequence and timing of events within a match, including also matches where no goals were scored. This is in contrast compared to Ridder, Cramer, and Hopstaken (1994) and Caliendo and Radic (2006) whose modeling approach requires the exclusion of such matches and assume that goal-scoring intensity increases linearly during the match. We allow for a more flexible

specification. Furthermore, our approach reflects the goal-scoring intensity estimates depending on actual goals scored without any further assumptions as in e.g. Vecer, Kopriva, and Ichiba (2009), where estimated intensities rather reflect the evolution of betting odds during the match. Third, our data allow us to study whether there is any heterogeneity in the effects of a red card, for example whether it matters which type of player is sent off. We find that teams who are behind are more likely to receive a red card. We also find that a red card has a significantly positive effect on the team playing with more players. Contrary to this, a sending-off negatively affects the goal-scoring rate of the sanctioned team. Finally, we find that the only relevant heterogeneity in the effects of a red card is the time at which a red card was issued.

The remainder of the paper is organized as follows. Section 2 presents an overview of association football rules and the dataset used in our analysis. Section 3 describes the empirical model. Section 4 presents and discusses the parameter estimates. Section 5 concludes.

2 Rules, theory and data

2.1 The rules and history

In the quarter finals of the 1966 FIFA World Cup, England, the hosts, faced Argentina, one of the other favorites, in London, at Wembley. Ten minutes before half-time, Rudolf Kreitlein, the West-German referee, cautioned Antonio Rattin, the Argentinean captain, for the second time. This second caution inevitably meant an expulsion. The sending-off caused a major turmoil, partly due to language problems, but also partly due to the South-American suspicion of dubious West-European ‘home refereeing’. Eventually, Rattin left the pitch, escorted by policemen. Moreover, Kreitlein also officially warned both Bobby Charlton, England’s skillful attacking midfielder, and his elder brother Jack, a tough central defender. However, Alf Ramsey, the England manager, had to ask FIFA for confirmation, as few people were aware of these cautions. In the aftermath of this match, Ken Aston, an English ex-referee who then presided the FIFA’s referee committee, decided that something had to be done. He proposed a card system for cautions (yellow) and expulsions (red). Rumor has it that this idea took shape

in his mind when he stood in front of traffic lights in the British capital. FIFA introduced the card system at the 1970 World Cup in Mexico.

The rules for referees to decide whether or not a yellow card or a red card should be imposed are provided by the FIFA.¹ The following seven offenses ought to lead to football players receiving a yellow card: unsporting behavior, dissent by word or action, persistent infringement of the Laws of the game, delaying the restart of play, failure to respect the required distance when play is restarted with a corner kick, free kick or throw-in, entering or re-entering the field of play without the referee's permission, and deliberately leaving the field of play without the referee's permission. A substitute or substituted player is to receive a yellow card if she/he commits any of these three offenses: unsporting behavior, dissent by word or action and, finally, delaying the restart of play.

A player, substitute or substituted player is to be sent off in case of one of these seven offenses: serious foul play, violent conduct, spitting at an opponent or any other person, denying the opposing team a goal or an obvious goal-scoring opportunity by deliberately handling the ball (this does not apply to a goalkeeper within his own penalty area) denying an obvious goal-scoring opportunity to an opponent moving towards the player's goal by an offense punishable by a free kick or a penalty-kick using offensive, insulting or abusive language and/or gestures and receiving a second caution in the same match. After a sending-off, the squad member in question (player, substitute or substituted player) must stay away from the field.

Prior to every World Cup tournament, the selected referees obtain instructions to focus on certain fouls and/or types of misconduct. In 1998, the emphasis was on the tackle from behind. In an early stage of this tournament, however, FIFA demanded the arbiters not to send off players too lightly. Expelling many world-class players would lower the level and, thus, the attractiveness of the tournament. Four years later, alertness regarding simulation ('flopping' or 'diving') was given top-priority. During the 2006 World Cup, the referees were supposed to pay special attention to elbowing, off-side and, once more, simulation. Four years later, they were supposed to simply focus on 'the rules of the game'. Prior to the 2014 World Cup, the arbiters were requested, like in 1998, not to expel players too easily.

¹This subsection is largely based on FIFA documentation. The rules are regularly updated; see for the latest version of the rules FIFA (2015).

However, in case of elbowing, or any similar offense, they should still send off the culprit instantaneously.

2.2 Theoretical background

The effects of red cards in association football on the performance of football teams is interesting from an economist's point of view. Several theories have been developed to study the performance and the motivation of a team or group of members. The seminal work of French agricultural engineer Maximilien Ringelmann (1913) shows that in a rope-pulling experiment individual effort decreases as the number of people involved increases. This phenomenon was later labeled as "social loafing".

Further work on the subject by Latané (1981) evolved into the social impact theory. This theory proposes that increasing the size of a group results into a decreased perceived pressure. As noted by Mechtel, Bäker, Brändle, and Vetter (2011), translating this into an association football setting, an expulsion of a player should increase the pressure on the remaining players of the team. This in turn should increase the motivation and effort, leading to a better performance. This mechanism is sometimes referred to as the "ten do it better" myth (Caliendo and Radic (2006)).

According to Steiner (1972), increasing the size of a group may potentially lead to problems in cooperation and coordination of the members. Hence, high individual skill levels do not necessarily translate into a better performance if the task requires a substantial degree of cooperation between group members. A successful performance of a football team undoubtedly requires the combined effort of all players. It is therefore conceivable that the cooperation of the remaining players on the pitch after a red card sanction increases.

Football matches with their red card rules present natural experiments to investigate the theoretical predictions. One of the performance indicators of a team is the goal-scoring rate. The exogenous reduction in the size of a team resulting from a red card presents an excellent opportunity to investigate whether smaller teams perform better. For this, we study the effects of a red card on the goal-scoring rates of both the sanctioned team and the non-sanctioned team.

TABLE 1A: RED CARDS

Tournament	Number	Minutes until first RC		
		Average	Min	Max
1998	22	62	19	89
2002	17	64	22	103
2006	28	65	28	120
2010	17	66	24	120
2014	10	56	37	94
Total/Avg.	94	63	19	120

TABLE 1B: GOALS PER MATCH

Tournament	Average	No RC	Matches with RC		
			Average	Before RC	After RC
1998	2.67	2.58	2.81	1.25	1.56
2002	2.51	2.62	2.07	1.07	1.00
2006	2.30	2.34	2.20	1.45	0.75
2010	2.27	2.36	2.00	1.12	0.88
2014	2.67	2.74	2.30	1.20	1.10
Total/Avg.	2.48	2.53	2.30	1.22	1.06

Note: 320 matches; RC = red card

2.3 Data and summary statistics

Our dataset consists of 320 FIFA matches, from five World Cup tournaments from 1998 (France) to 2014 (Brazil). In 1998, the FIFA World Cup tournament in France hosted 32 teams for the first time in history. Ever since then, the tournament consists of 64 matches, viz. 48 matches in the group stage, eight matches in the round of sixteen, four quarter finals, two semi-finals, a position match (for the third place) and the final. Moreover, starting from the 1998 FIFA World Cup, teams were allowed to bring on three substitutes, as compared to two players and one goalkeeper in 1994, now without any restrictions. This gives coaches ample opportunities to substitute players who have already received a yellow card thus avoiding that these players receive a second yellow card, i.e. a red card.

Table 1A and Table 1B provide summary statistics of the dataset. In total, our dataset includes 94 red cards. The highest number of 28 red cards was awarded in 2006 (Germany), followed by 22 red cards in 1998. In both the 2002 and 2010 tournaments played in South Korea/Japan and South Africa, respectively, 17 red

cards were awarded. Finally, the lowest number of only 10 red card sanctions occurred in 2014. As can be seen, most red cards are issued in the second half of the match, on average around the 65th minute. The only deviation is the 2014 tournament, where on average a red card was awarded in the 56th minute. The earliest red card was given in the 19th minute, the first red card that was given in the latest stage of match was in the 120th minute in 2006 and 2010.

As shown in the bottom part of Table 1B, the average number of goals per match has fallen from 2.67 in 1998 to 2.27 in 2010. The 2014 tournament reversed this trend with 2.67 goals per match scored. On average, more goals were scored in matches without a red card. Also, in matches in which at least one red card, fewer goals were scored after the red card is issued than before. However, the average duration of the match after a red card is much shorter than the duration up to the first red card. Taken this difference in duration into account, it is clear that the goal-scoring intensity is higher after a red card than before.

Table 2 gives an overview of the characteristics of our sample (the definition of the variables is given in the Appendix). The rank of the team ranges from one to 105 with an average of 21.3. The goal difference at the time the first red card was issued ranges from minus three to plus two with an average of -0.44. Tactics of teams usually differ between group and elimination rounds of a tournament. Also, team strength is usually more equal as weak teams are already out. Therefore, in our analysis we also include a dummy variable indicating whether a match is played in the group phase or later stages of the tournament. The total number of goals scored per match ranges from zero to eight. In the analysis we will investigate to what extent the referee matters. For this, we use two indicators. First, whether the referee is from the same continent as the team for which we analyze the goal-scoring rate or the red card sanction rate. Second, we take the rank of the referee one year prior to the tournament into account. The bottom rows of Table 2 shows which type of players received a first red card in the game.

Table 3 gives the distribution of the red cards by 15 minute intervals in the match. No red card was issued in the first 15 minutes of any match. Most red cards were given in the last 15 minutes of regular time; only five red cards were issued in extra time. This means that in about 25 percent of the matches at least one red card was issued while in the extra time matches this was less than 20 percent. The focus of our analysis will be on the 84 first red cards, ignoring eight

TABLE 2: SAMPLE CHARACTERISTICS

Variable	Mean	Min	Max
<i>Match characteristics</i>			
Rank	21.3	1	105
Group stage	0.75	0	1
16-finals	0.12	0	1
Quarterfinals	0.06	0	1
Semifinals	0.03	0	1
Final/3rd place	0.03	0	1
Goal difference at RC	-0.44	-3	2
Goals scored	1.24	0	8
Same continent referee	0.06	0	1
Referee rank	5.28	1	10
Referee rank missing	0.54	0	1
<i>Player characteristics</i>			
Age	2.74	2.06	3.54
Caps	4.17	0.1	11.3
Midfielder	0.43	0	1
Forward	0.22	0	1
Defender	0.35	0	1
Midfielder	0.44	0	1
Forward	0.21	0	1

Note: 320 matches; RC = Red Card; player characteristics refer to players receiving a red card. Defender includes goalkeeper.

TABLE 3: DISTRIBUTION OF RED CARDS PER 15 MINUTE INTERVALS

Minute	First	Later	Total
0-15	0	0	0
16-30	9	0	9
31-45	19	0	19
46-60	14	1	15
61-75	16	1	17
76-90	22	6	28
91-105	1	0	1
106-120	3	0	3
Total	84	8	92

Note: 320 matches of which 27 matches with extra time. Two red cards were given to players on the bench and are thus not included in the analysis.

later red cards and two red cards that were presented to players on the bench.

2.4 Exploratory analysis

Before analyzing the effect of a red card on the goal-scoring rate, we present a preliminary analysis of the effect of a red card on the outcome of a match. This is a similar approach as used by Mechtel, Bäker, Brändle, and Vetter (2011). We first estimate a simple probit model which takes the following form:

$$\Pr(y_i = 1 \mid z'_i) = \Phi(z'_i \xi + d_{rc} \gamma_1 + d_{rc}^o \gamma_2) \quad (1)$$

where y_i is the outcome variable taking a value of 1 if the team won a match, z'_i is vector of explanatory variables, the parameters γ_1 and γ_2 capture the effect of an opponent's and an own red card on the outcome of a match and Φ is the cumulative standard normal distribution function. We estimate the probability of winning the match for both teams. From the total of 320 matches, this results in 640 observations used in the analysis. In addition to this, we estimate a linear model including the same explanatory variables, with a dependent variable taking

TABLE 4: PARAMETER ESTIMATES PROBABILITY TO WIN A MATCH

	Probit model		Linear model	
	(1)	(2)	(3)	(4)
Opponent RC	0.23** (3.2)	0.24*** (3.3)	0.47*** (5.3)	0.50*** (5.6)
Own RC	-0.28*** (7.8)	-0.31*** (8.6)	-0.37*** (4.0)	-0.41*** (4.2)
Rank opponent	0.06*** (5.0)	0.06*** (5.0)	0.12*** (6.8)	0.12*** (6.6)
Rank own	-0.08*** (5.6)	-0.08*** (5.5)	-0.12*** (6.8)	-0.12*** (6.6)
16-finals	-0.07 (1.1)	0.05 (0.8)	-0.01 (0.1)	-0.01 (0.1)
Quarterfinals	-0.06 (0.7)	-0.03 (0.3)	-0.03 (0.2)	-0.02 (0.2)
Semifinals	-0.04 (0.4)	0.01 (0.1)	-0.03 (0.2)	-0.03 (0.1)
Final/3rd place m.	-0.05 (0.4)	0.06 (0.5)	-0.05 (0.2)	-0.05 (0.2)
Goal diff at RC	0.21** (2.5)	0.20** (2.3)	0.22** (3.0)	0.20** (2.8)
Constant	-0.24* (1.8)	-0.24* (1.8)	1.00*** (13.3)	1.01*** (13.1)
-Log likelihood	355.0	360.2	725.6	743.9
Extra time included	Yes	No	Yes	No

640 observations; Absolute t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.001$

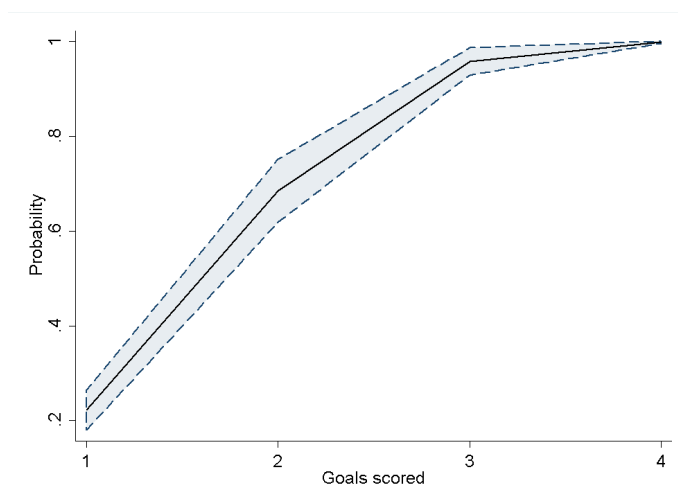
Note: Probit model: probability of winning a match, marginal effects; linear model: 0 for a loss, 1 for a draw, 2 for a win; rank divided by 10.

values of 0 if the team loses, 1 for a draw and 2 for a win.² The results are summarized in Table 4. Columns (1) and (2) present estimates from the probit model, columns (3) and (4) from the linear model. Columns (2) and (4) are excluding extra time results.

It is clear that a higher number of goals scored increases the probability of winning a match substantially. Figure 1 plots the probability of winning as a function of the number of goals scored (the gray area represents the 95 percent confidence interval). After scoring the first goal, the probability of winning is equal to 22 percent. Scoring a second goal increases the probability of winning to 68 percent, whereas the third goal takes the odds of winning up to 95 percent. Similarly, the probability of winning a match increases if the opposing team receives red card. For the team penalized with a red card sanction, the odds of winning decrease substantially. The lower the ranking of the opponent, the higher the probability of winning a match. Conversely, the lower the ranking of the team itself, the lower the odds of winning. The stage of tournament does not seem to have any effect. The more goals a team is ahead of its opponent before the red card, the higher the

²We also investigate an ordered probit specification finding very similar results.

FIGURE 1: PROBABILITY TO WIN AS FUNCTION OF GOALS SCORED



Note: the dotted lines represent the 95 percent confidence interval.

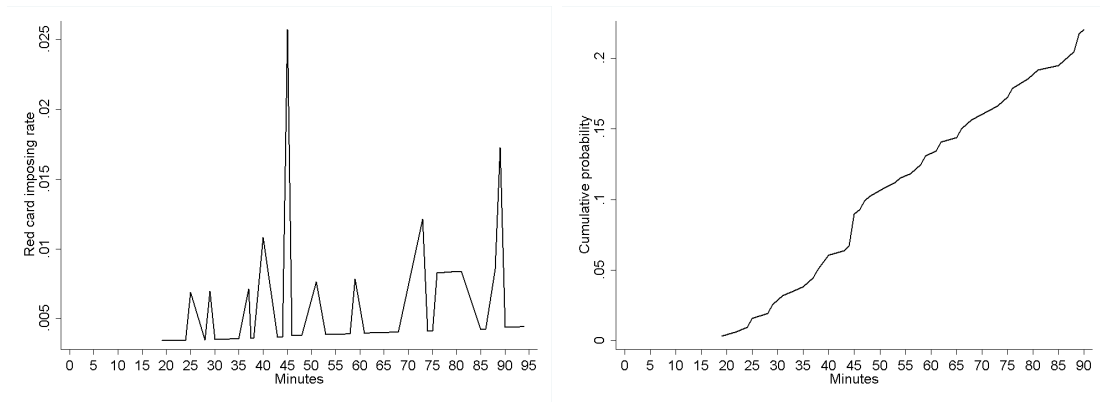
probability of winning the match. At the mean value of the explanatory variables, the marginal probability of winning a match before a red card sanction is equal to 29 percent. After an opponent's red card sanction, the odds of winning increase by 23 percentage points to almost 52 percent. After the team itself receives a red card, the odds of winning decrease from 35 percent to only seven percent. The results shown in columns (3) and (4) make it clear that these are not dependent on the specific functional form assumptions underlying the probit model and the linear model.

3 Set-up of the analysis

3.1 Imposition of a red card

A red card foul is a phenomenon that usually occurs later in a football match but very rarely within the first 20 minutes of a match. In our dataset, the earliest red card awarded was in the 19th minute. This can be seen in Figure 2 which plots the rate at which red cards were issued in a match. The highest occurrence of red card sanctions is around the 45th minute of a match, with another peak around the 75th minute. Figure 2 also plots the cumulative probability that a red card is imposed. Within the first half of the match, the probability of committing a red

FIGURE 2: RED CARD IMPOSING RATE AND CUMULATIVE RED CARD PROBABILITY



card offense is less than 10 percent. In the course of the second half of a match, this figure rises to slightly more than 20 percent.

We model the rate by which a red card is imposed using a mixed proportional hazard model. The red card sanction rate at time t conditional on observed characteristics x and unobserved characteristics v , is specified as follows:³

$$\theta_r(t | x, v) = \lambda_r(t_i) \exp(x' \beta_r + v), \quad (2)$$

where β_r is a vector of parameters. The duration dependence is flexibly modeled using step function:

$$\lambda_r(t_i) = \exp \left(\sum_k \lambda_{k,r} I_k(t) \right), \quad (3)$$

where $k (= 1, \dots, K)$ is a subscript for match-intervals and $I_k(t)$ are time-varying dummy variables for subsequent match-intervals. The match is divided into two halves. We estimate a constant and normalize $\lambda_{0,r} = 0$. Extra time matches include one additional interval. The conditional density function for the completed

³The end of the first half of the match is assumed to be at the 45th minute. The end of the second half of the match is assumed to have an injury time of five minutes for all matches included in the analysis. The extra time matches are modeled as follows: the first part of the extra time is assumed to be 15 minutes, while the second part is assumed to be 17 minutes, so including two minutes of injury time. In some tournaments matches in extra time could have been concluded by a golden goal. Then, match time stops at the moment a golden goal is scored.

durations until a red card was issued can be written as:

$$f(t | x, v) = \theta_r(t | x, v) \exp \left(- \int_0^t \theta_r(s | x, v) ds \right). \quad (4)$$

Several unobserved factors such as coach tactics, weather or match attendance might affect the goal-scoring rate. We assume that these unobservables v come from a discrete distribution G with two points of support (v_1, v_2) , representing two types of teams, one with a low red card rate and another with high red card rate. The associated probabilities are denoted as follows: $\Pr(v = v_1) = p_1$, $\Pr(v = v_2 - v_1) = p_2$, where p_j ($j = 1, 2$) is assumed to have a logistic distribution. We remove the unobserved heterogeneity distribution through integration:

$$f(t | x) = \int_v f(t | x, v) dG(v). \quad (5)$$

In the analysis for matches without a red card the duration of the red card is assumed to be censored at the end of the match. The likelihood function is optimized using the method of maximum likelihood.

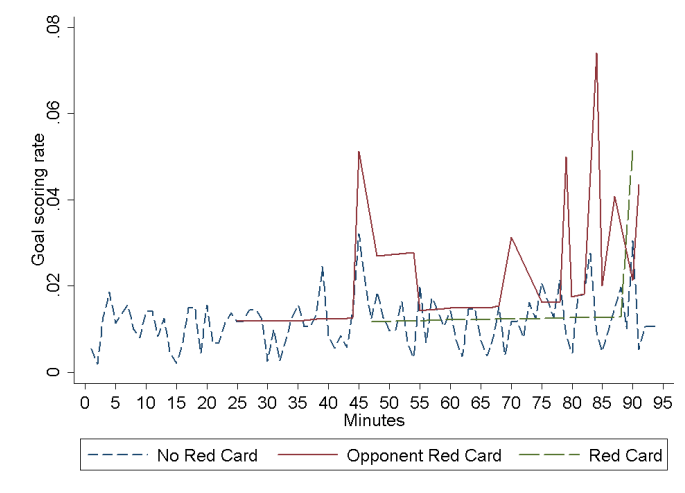
3.2 The effects of a red card

Figure 3 plots the goal-scoring rate before and after a first red card was issued, focusing on the first goal scored before the red card (this includes also matches with no red cards given) and the first goal scored after a red card sanction. The goal-scoring rate is higher after a red card has been issued. The goal-scoring rate after a team receives a red card appears to be lower compared to a situation when the opponent receives a red card. There seems to be an increase in the goal-scoring rate towards the end of a match, a well-known fact found also in previous studies (Van Ours and van Tuijl (2011)).

The focus of our analysis is on the effect of a first red card on the goal-scoring rate of both the sanctioned and the non-sanctioned team. Each match is divided into several periods by the goals scored and the red cards given, with episodes from a goal to a red card or to the end of a match being right-censored.⁴ The effect of

⁴Only one red card in the dataset resulted into a penalty kick. Excluding this observation has no effects on the results.

FIGURE 3: GOAL-SCORING RATE BEFORE AND AFTER RED CARD



the red card is modeled as a shift in the goal-scoring (hazard) rate. To estimate the effect, we use a mixed proportional hazard framework which allows the rate of transition from one state to the other to be affected by individual observed and unobserved characteristics as well as the duration of the stay in a state (Heckman and Singer (1984)). In our case, the state is defined by the current number of goals scored by the team, considering also whether or not a red card was given. The transition rate is equivalent to the goal-scoring rate. As additional observed characteristics we use the pre-tournament FIFA ranking of both teams and the current stage of the tournament. We also take a possible referee bias into account. Thus, the goal-scoring rate at the time of a match t_i conditional on the observed characteristics x , the time of the opponent's red card sanction t_{rc} , time of the own red card sanction t_{rc}^o and the unobserved characteristics u , the model is specified as follows:

$$\theta(t_i | x, t_{rc}, t_{rc}^o, u) = \lambda(t_i) \exp(x' \beta + \delta_1 I(t_i > t_{rc}) + \delta_2 I(t_i > t_{rc}^o) + u), \quad (6)$$

where β is a vector of parameters. The parameter δ_1 describes how the hazard rate shifts at the moment a red card was issued and thus measures the effect of an opponent's red card on the goal-scoring rate. Similarly, the parameter δ_2 measures the shift of the goal-scoring rate after a team receives a red card.⁵ The subscript

⁵Note that we model only one of these events. If a second red card is imposed match time

i denotes the period from a goal scored or a red card awarded. The first period from the beginning of the match up until the first event (a goal scored or a red card issued) is denoted as $i = 1$. The duration dependence is flexibly modeled using step function:

$$\lambda(t_i) = \exp \left(\sum_k \lambda_k I_k(t_i) \right), \quad (7)$$

where $k(= 1, \dots, K)$ is a subscript for match-intervals and $I_k(t)$ are time-varying dummy variables for subsequent match-intervals. The match is divided into two halves, assuming an injury time of five minutes. Extra time matches include one additional interval, allowing for a third injury time of two minutes.⁶ We estimate a constant and normalize $\lambda_0 = 0$. The conditional density function for the completed durations until goal scored can be written as:

$$f(t_i | x, t_{rc}, t_{rc}^o, u) = \theta(t_i | x, t_{rc}, t_{rc}^o, u) \exp \left(- \int_{t_{i-1}}^{t_i} \theta(t_i | x, t_{rc}, t_{rc}^o, u) ds \right). \quad (8)$$

Note that t_{i-1} denotes the time of a previous event in the match, either a goal scored or a red card issued. Several unobserved factors such as coach tactics, the weather or match attendance might affect the goal-scoring rate. We assume that the random effects u come from a discrete distribution G with two points of support (u_1, u_2) , representing two types of teams, one with a low goal-scoring rate and another with a high goal-scoring rate. The associated probabilities are denoted as follows: $\Pr(u = u_1) = p_1$, $\Pr(u = u_2 - u_1) = p_2$, where p_n ($n = 1, 2$) is assumed to have a logistic distribution. We remove the unobserved heterogeneity distribution through integration:

$$f(t_i | x, t_{rc}, t_{rc}^o) = \int_u f(t_i | x, t_{rc}, t_{rc}^o, u) dG(u). \quad (9)$$

The resulting log-likelihood function equals:

$$\mathcal{L} = \sum_{j=1}^N \log \left\{ \prod_{i=1}^I \theta(t_i | x, t_{rc}, t_{rc}^o)^{d_i} \exp \left(- \int_{t_{i-1}}^{t_i} \theta(t_i | x, t_{rc}, t_{rc}^o) \right) \right\}. \quad (10)$$

stops at which point the goal scoring rate is assumed to be right-censored.

⁶We also test for an alternative specification of 15-minute intervals finding that our main parameter estimates are not affected by this.

TABLE 5: PARAMETER ESTIMATES RED CARD SANCTION RATE

	(1)		(2)		(3)		(4)		(5)	
Rank opponent	-0.04	(0.6)	-0.04	(0.6)	-0.14	(1.2)	-0.04	(0.7)		
Rank own	-0.03	(0.4)	-0.02	(0.3)	-0.11	(0.8)	-0.03	(0.5)		
Expected no. of points									-0.18	(0.9)
16-finals	-0.37	(1.1)	-0.15	(0.5)	-0.34	(0.7)	-0.33	(0.9)	-0.40	(1.0)
Quarterfinals	0.31	(0.8)	0.58	(1.5)	-0.07	(0.1)	0.39	(1.0)	0.48	(1.1)
Semifinals	-1.07	(1.0)	-0.78	(0.8)	-1.10	(1.0)	-0.75	(0.7)	-0.59	(0.6)
Final/3rd place m.	0.06	(0.1)	0.34	(0.6)	-0.01	(0.0)	0.23	(0.4)	0.43	(0.7)
Score before RC	-1.42***	(9.9)	-1.40***	(9.8)	-1.38***	(6.5)	-1.50***	(10.0)	-1.66***	(9.8)
Same continent referee	-0.52	(1.1)	-0.49	(1.1)	-0.58	(0.9)	-0.45	(1.0)	0.00	(0.0)
Referee rank					0.03	(0.5)	0.01	(0.2)	0.02	(0.4)
Referee rank missing							0.61	(1.5)	0.68	(1.5)
Constant	-7.08***	(23.1)	-7.14***	(23.2)	-7.55***	(11.7)	-7.51***	(15.6)	-7.60***	(12.8)
Second half	0.76**	(3.2)	0.84***	(3.6)	1.37**	(3.2)	0.80***	(3.3)	0.93***	(3.5)
Extra time	1.72**	(3.2)			2.47**	(3.1)	1.77**	(3.3)	2.03***	(3.6)
Observations	640		640		294		640		590	
-Log likelihood	610.4		609.4		240.3		607.7		517.9	
Extra time included	Yes		No		Yes		No		Yes	

Absolute t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.001$

Note: Rank divided by 10. RC = red card. Tournament year dummies for column (5) not reported.

considering a dataset consisting of $j = 1, \dots, N$ matches, $i = 1, \dots, I$ denotes the period in a match from the first goal scored or red card issued, with I being the last part from a goal scored or a red card awarded until the end of match, while d_i denotes indicator whether a goal was scored. The parameters of the likelihood function are estimated using the method of maximum likelihood.

4 Parameter estimates

4.1 Red card sanction rate

Table 5 presents the parameter estimates of the red card sanction rate. The FIFA ranking of teams does not seem to have an effect on the rate at which red cards are issued during a match. Compared to the group stage, later stages of a tournament also do not have an effect. As expected, the more goals ahead, the lower the probability of conducting a red card foul. With each goal ahead, the probability decreases by $(\exp(-1.42)-1) \times 100 \approx 75$ percent. Column (2) presents estimates excluding extra time matches.

From the first two columns, it is clear that the imposition of a red card is not related to the rank of a team or its opponent. The stage of tournament is not important either. The only determinant of the red card sanction rate is the goal difference. If a team has conceded more goals than it scored, it is more likely to receive a red card. Furthermore, a red card is more likely to be issued in the second half of a match and in particular in extra time. In column (3) we add the rank of the referee as an additional explanatory variable. The number of observations drops substantially since there were many referees for whom no ranking is available. The ranking of the referee has no effect on the red card sanction rate. In column (4) we introduce a dummy variable for matches in which no referee ranking is available. This dummy variable has a positive effect suggesting that less skillful referees are more likely to issue a red card. Nevertheless, the dummy variable is not different from zero at conventional levels of significance.

Finally, as an alternative to the FIFA-ranking, we use bookmaker data as an indication of the difference in strength between two teams in a match. We translate pre-match betting odds into an expected number of points.⁷ The relevant parameter estimates are presented in column (5). There is no significant effect of the expected number of points on the red card sanction rate. In an additional sensitivity analysis, we investigated whether unobserved heterogeneity is important but we did not find evidence of this.

4.2 The effects of a red card on the goal-scoring rate

Table 6 shows the parameter estimates of the goal scoring rate. Initially, we ignore potential unobserved heterogeneity in the goal-scoring rate. The first two columns of the table present the effects making a distinction between the own team's and the opponent's red card. Columns (3) and (4) extend the analysis by introducing heterogeneity in the red card effect according to the time the card was issued. Clearly, there is a highly significant positive effect on the goal-scoring rate after a red card is imposed to the opposing team. The probability of scoring a goal increases by almost $(\exp(0.79)-1)\times 100 \approx 120$ percent. In line with this,

⁷See Van Ours and van Tuijl (2016) and the appendix for details. The number of observations decreases slightly since pre-match betting odds are not available for all matches. Because we were unable to find odds from a single source we use tournament-specific dummies to control for possible differences in the structure of betting odds between tournaments.

TABLE 6: PARAMETER ESTIMATES GOAL-SCORING RATE

	(1)	(2)	(3)	(4)
Rank opponent	0.10*** (5.6)	0.10*** (5.5)	0.10*** (5.4)	0.10*** (5.3)
Rank own	-0.16*** (6.3)	-0.16*** (6.4)	-0.16*** (6.4)	-0.16*** (6.5)
16-finals	-0.12 (1.0)	-0.19* (1.7)	-0.13 (1.1)	-0.21* (1.8)
Quarterfinals	-0.39** (2.3)	-0.28 (1.6)	-0.40** (2.4)	-0.28* (1.7)
Semifinals	-0.33 (1.5)	-0.31 (1.3)	-0.33 (1.5)	-0.31 (1.3)
Final/3rd place m.	0.05 (0.2)	0.12 (0.6)	0.02 (0.1)	0.12 (0.6)
Same continent ref.	0.05 (0.4)	0.07 (0.5)	0.04 (0.3)	0.06 (0.4)
Constant	-4.37*** (48.1)	-4.36*** (47.7)	-4.35*** (47.8)	-4.34*** (47.5)
Second half	0.16** (2.2)	0.17** (2.2)	0.16** (2.2)	0.16** (2.2)
Extra time	0.42* (1.7)		0.39 (1.6)	
Effect RC	0.79*** (6.8)	0.80*** (6.7)		
Effect own RC	-0.64** (2.4)	-0.66** (2.4)		
Effect RC 0-45			0.80*** (5.3)	0.80*** (5.3)
Effect RC 45-75			0.49** (2.3)	0.56** (2.6)
Effect RC 75-90			1.38*** (5.5)	1.35*** (4.9)
Effect own RC 0-45			-1.19** (2.6)	-1.19** (2.6)
Effect own RC 45-75			-0.30 (0.7)	-0.40 (1.0)
Effect own RC 75-90			0.33 (0.6)	0.52 (0.9)
-Log likelihood	4294.1	4192.1	4288.3	4187.2
LR test χ^2			11.6**	9.8**
LR test DF			4	4

640 observations; absolute t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.001$

Note: Rank divided by 10. Tournament year dummies for column (5) not reported.

receiving a red card sanction has significantly negative effects on the goal-scoring rate, lowering the chance by 47 percent. The estimated coefficients of the FIFA ranking variables suggest that a lower ranking of a team translates to a lower goal-scoring rate. The lower the ranking of the opponent, the higher the goal-scoring rate. The magnitude of this effect is $(\exp(0.06)^{10} - 1) \times 100 \approx 82$ percent with a 10 place increase/decrease in the ranking. The stage of a tournament does not seem to have much of an effect. The only difference being the quarterfinal stage where the goal-scoring rate is to be lower as compared to the group stage. Whether or not a referee was from the same continent does not influence the goal-scoring rate. The goal scoring rate is higher in the second half of a match than in the first half. In extra time the goal scoring rate is highest.

From columns (3) and (4) it appears that the effect of a red card given between

the 45th and 75th minute of a match lowers the goal-scoring rate when compared to red cards given either earlier or later in a match. The parameter estimate of a red card received between the 75th and the 120th minute shows some evidence of the “ten do it better” myth. However the parameter estimate is statistically not significantly different from zero. The result of the likelihood ratio test confirms that the additional time-specific red card effects significantly improve the model fit. Apparently, the timing of the red card is important. Columns (2) and (4) present estimates excluding extra time matches. The effect on the magnitude of the parameter estimates is negligible. It was not possible to identify unobserved heterogeneity in the goal scoring rates.

4.3 Sensitivity analysis

To explore the robustness of our findings, we present a wide range of sensitivity analysis of which the main results are summarized in Table 7. To investigate possible heterogeneity in the effects of a red card on the goal-scoring rate, we introduce interaction effects between a red card and the experience or the age of a player sent off. Such an interaction is also investigated with the line in which the player was deployed (attack, midfield or defense). Interaction effects of both an own and an opponent’s red card with the age and experience shown in column (1) suggest that the loss of an older or experienced player does not present a significant advantage.⁸ Column (2) introduces an interaction with the line of play of the player sent off finding no significant effects. Column (3) investigates whether there is a difference in the shift of the goal-scoring rate resulting from a second-yellow red card or straight red card. A foul or misconduct resulting into a straight red card does not have a significantly different impact on the goal-scoring rate either.

We also analyze whether there is a difference in the effect of receiving a red card on the goal-scoring rate of an experienced team. A national team is defined as experienced if a country participated in every World Cup since 1982. Six countries fulfill this criterion – Argentina, Brazil, England, Italy, Germany and Spain – accounting for 12 red cards in the analysis. The results shown in column (4) indicate that these teams have higher goal-scoring rate after the sending-off of an own player compared to the others. However, the estimated parameter narrowly

⁸Introducing age-squared does not improve the estimation results.

misses the 10-percent significance level. Finally, we investigate whether a different measure of team strength has any effects on the results, as FIFA ranking has been criticized as unreliable. Instead, as before we employ betting odds translated into the expected number of points from a match. As shown in column (5), including the expected number of points as a measure of team strength results in a negligible change in magnitude of the estimates compared to the baseline. The main conclusion holds, as teams with higher expected number of points have higher goal-scoring rate.

5 Conclusions

We analyze 320 World Cup matches to investigate both the determinants of the issuing of a red card and the effects of a red card on goal scoring. We find the rate at which a red card is issued to depend on the goal difference that evolves during a match. Teams who are behind in terms of goals scored are more likely to be confronted with a red card. We also find that the sending-off of a player presents a significant disadvantage for the team that is confronted with a red card. The size of this effect depends on the time the red card was issued.

Our results suggest that the sending off of a player does not translate into a better performance of the sanctioned team. Hence, our findings are at odds with the “ten do it better” myth. It may be that the loss of a player results in increased motivation of the remaining players on the pitch. However, on balance smaller teams do worse. Apparently, the loss of a player in a sports competition presents a significant disadvantage which is difficult to overcome through increased effort.

TABLE 7: PARAMETER ESTIMATES GOAL-SCORING RATE; SENSITIVITY ANALYSIS

	(1)	(2)	(3)	(4)	(5)
Rank opponent	0.10*** (5.6)	0.10*** (5.7)	0.10*** (5.7)	0.10*** (5.7)	
Rank own	-0.16*** (6.5)	-0.16*** (6.5)	-0.15*** (6.3)	-0.15*** (6.2)	
Expected no. of points					0.59*** (9.7)
16-finals	-0.12 (1.1)	-0.13 (1.1)	-0.11 (1.0)	-0.12 (1.0)	-0.12 (1.0)
Quarterfinals	-0.33* (1.9)	-0.32* (1.9)	-0.40** (2.4)	-0.41** (2.4)	-0.33** (2.0)
Semifinals	-0.33 (1.5)	-0.34 (1.5)	-0.33 (1.5)	-0.34 (1.5)	-0.22 (1.0)
Final/3rd place m.	0.13 (0.7)	0.13 (0.7)	0.05 (0.3)	0.05 (0.3)	0.14 (0.8)
Constant	-4.36*** (47.9)	-4.36*** (47.8)	-4.37*** (48.1)	-4.40*** (47.9)	-5.22*** (38.4)
Second half	0.17** (2.3)	0.17** (2.2)	0.17** (2.2)	0.22** (2.9)	0.15** (2.0)
Extra-time	0.41 (1.5)	0.42 (1.6)	0.42* (1.7)	0.52* (2.0)	0.24 (1.0)
Effect RC	0.67** (3.1)	0.77*** (4.0)	0.74*** (4.1)	0.99*** (8.2)	0.81*** (6.5)
Effect own RC	-0.99 (1.6)	-0.61 (1.2)	-1.17** (2.0)	-0.65** (2.0)	-0.54* (1.8)
Effect RC×caps	0.04 (0.8)				
Effect own RC×caps	0.11 (0.7)				
Effect RC×age	-0.40 (1.0)				
Effect own RC×age	-1.24 (1.0)				
Effect RC×M		-0.01 (0.0)			
Effect own RC×M		0.18 (0.3)			
Effect RC×F		0.22 (0.7)			
Effect own RC×F		-0.62 (0.7)			
Effect RC×SR			0.08 (0.4)		
Effect own RC×SR			0.74 (1.1)		
Effect own RC×top team				0.93 (1.6)	
Observations	632	632	640	640	590
-Log likelihood	4243.9	4244.0	4293.3	4291.1	3939.7
LR test χ^2	2.12	1.89	1.60	5.96**	
LR test DF	4	4	2	1	

Absolute t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.001$.

Note: RC = Red Card, SR = Straight red card; M = Midfielder; F = Forward. Rank, age and caps divided by 10. age = age - $\bar{\text{age}}$, caps = caps - $\bar{\text{caps}}$. For column (1) and (2), LR test uses model (1) from Table 6 estimated with 632 observations as baseline. Tournament year dummies for column (5) not reported.

References

- ANDERSON, C., AND D. SALLY (2013): *The Numbers Game: Why Everything You Know about Football is Wrong*. Penguin Books, Limited.
- BAR-ELI, M., G. TENENBAUM, AND S. GEISTER (2006): “Consequences of Players’ Dismissal in Professional Soccer: A Crisis Related Analysis of Group Size Effect,” *Journal of Sport Sciences*, 24(10), 1083–1094.
- BURAIMO, B., D. FORREST, AND R. SIMMONS (2010): “The 12th man? Refereeing bias in English and German soccer,” *Journal of the Royal Statistical Society: Series A*, 173(2), 431–449.
- CALIENDO, M., AND D. RADIC (2006): “Ten Do It Better, Do They? An Empirical Analysis of an Old Football Myth,” IZA Discussion Papers 2158.
- DAWSON, P., S. DOBSON, J. GODDARD, AND J. WILSON (2007): “Are Football Referees Really Biased and Inconsistent? Evidence on the Incidence of Disciplinary Sanction in the English Premier League,” *Journal of the Royal Statistical Society: Series A - Statistics in Society*, 170(1), 231–50.
- FIFA (2015): *Rules of the Game 2015/16*. Zurich: FIFA.
- HECKMAN, J., AND B. SINGER (1984): “A Method for Minimizing the Impact of Distributional Assumptions in Econometric Models for Duration Data,” *Econometrica*, 52(2), 271–320.
- LATANÉ, B. (1981): “The Psychology of Social Impact,” *American Psychologist*, 36(4), 343–356.
- MECHTEL, M., A. BÄKER, T. BRÄNDLE, AND K. VETTER (2011): “Red Cards: Not Such Bad News For Penalized Guest Teams,” *Journal of Sports Economics*, 12(6), 621–646.
- POPE, B. R., AND N. G. POPE (2015): “Own-Nationality Bias: Evidence From UEFA Champions League Football Referees,” *Economic Inquiry*, 53(2), 1292–1304.
- RIDDER, G., J. S. CRAMER, AND P. HOPSTAKEN (1994): “Down to Ten: Estimating the Effect of a Red Card in Soccer,” *Journal of the American Statistical Association*, 89(427), 1124–1127.
- RINGELMANN, M. (1913): “Recherches sur les Moteurs Animes: Travail de Homme (Research on Animate Sources of Power: The Work of Man),” *Annales de l’Institut National Agronomique*, 2e serie tome XII, 1–40.

- STEINER, I. D. (1972): *Group Processes and Productivity*. New York: Academic Press.
- TITMAN, A., D. COSTAIN, P. RIDALL, AND K. GREGORY (2015): “Joint Modelling of Goal and Bookings in Association Football,” *Journal of the Royal Statistical Society: Series A*, 178(3), 659–683.
- VAN OURS, J. C., AND M. A. VAN TUIJL (2011): “Country-specific Goal-scoring in the “Dying seconds” of International Football Matches,” *International Journal of Sport Finance*, 6(2), 138–154.
- VAN OURS, J. C., AND M. A. VAN TUIJL (2016): “In-season Head-coach Dismissals and the Performance of Professional Football Teams,” *Economic Inquiry*, 54(1), 591–604.
- VECER, J., F. KOPRIVA, AND T. ICHIBA (2009): “Estimating the Effect of the Red Card in Soccer: When to Commit an Offense in Exchange for Preventing a Goal Opportunity,” *Journal of Quantitative Analysis in Sports*, 5(1), 1–20.

Appendix: Definition of variables

- Rank own: FIFA ranking of the team of which the goal-scoring rate and the red card sanction rate is estimated. The ranking is established before the start of each tournament.
- Rank opponent: FIFA ranking of the opposing team.
- Expected number of points is calculated using pre-match betting odds. These odds are from various websites including for example sports.williamhill.com and www.betexplorer.com. The calculation is done as follows: $\Pr(\text{Team 1 win}) = \left(\frac{\text{Odds team 1 wins}}{\text{Premium}}\right)^{-1}$, $\Pr(\text{Draw}) = \left(\frac{\text{Odds draw}}{\text{Premium}}\right)^{-1}$. The expected number of points for team 1 = $3 \times \Pr(\text{Team 1 wins}) + \Pr(\text{Draw})$.
- Stage of a tournament: group stage is reference category.
- Goal difference before red card: number of goals scored minus the number of goals conceded at the time the first red card was issued.
- Goals scored: number of goals scored.
- Same continent referee: dummy variable; takes value of one if the team is from the same continent as the referee. If both teams are from the same continent as the referee the variable takes value of zero.
- Referee rank: based on International Federation of Football History & Statistics annual ranking. We use the ranking of a referee one year prior to the tournament (if available).
- Age: age of a player at the start of a tournament.
- Caps: number of caps of a player at the start of a tournament.
- Position of a player: line of a play on the pitch - midfield, attack. Reference category: defense, including goalkeeper.