Increasing returns and perfect competition
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Increasing Returns and Perfect Competition: 
The Role of Land*

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ABSTRACT

The classical inconsistency between increasing returns and perfect competition is addressed. For example, if firms must pay a fixed cost of entry but then can produce using a constant returns to scale technology, they will generally operate at a loss, necessitating a government subsidy in order to attain an efficient allocation. Here we show that perfect competition and increasing returns can be consistent, in the sense that equilibria exist and are efficient without government intervention, provided that units of some input such as land can be identified and priced separately. The Alonso model with a finite number of agents is extended to include production under increasing returns, where all agents are mobile. The key is that producers use intervals of land, and the price they pay for land interior to the parcels can be adjusted to provide an implicit subsidy. Input price discrimination extends the sway of the free market to monopoly and monopsony. The relevance to the recent minimum wage debate is discussed.

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I. Introduction

Our goal is to reconcile the notions of increasing returns and perfect competition. We demonstrate in our model that equilibria can exist and can be efficient without government intervention. Land plays a key role in this analysis.

It is well-known that increasing returns (say a fixed cost followed by constant returns to scale production) and perfect competition are not compatible, since at an equilibrium, the first order condition for profit maximization – price equals marginal cost – implies negative profits. Although substantial progress has been made using models in which price is set at marginal cost but firms are subsidized, or multipart tariffs are employed, problems still remain; see Bonnisseau and Cornet (1988) [as well as other papers in the symposium issue], Bonanno (1990) or Vassilakis (1989, 1993) for discussion.¹

Our initial goal was to prove a second welfare theorem. Here transfers have generally been employed in the literature. They can obviously mitigate the problem of negative profits for producers by simply providing a subsidy to producers who are operating at a Pareto optimum but who would otherwise make a loss at supporting prices. The idea that firms yielding increasing returns to scale should be subsidized in order to obtain an efficient allocation goes back at least to Marshall (1953, Book V, Chapter XIII), the first edition of which was published in 1890. A precursor can be found in Whitaker (1975, pp. 88–89, pp. 228–230), who published writings of Marshall dating from the 1870’s. Pigou (1962, Part II, Chapter XI), first published in 1920, touches on this subject in passing. Pigou (1927, p. 197) is particularly explicit:²

¹For instance, marginal cost pricing relates only to the first order conditions for optimization for the firms, so at a marginal cost pricing equilibrium, a firm may not be maximizing profits. Further, a marginal cost pricing or multipart tariff equilibrium allocation is not necessarily Pareto optimal. (Marginal cost pricing reflects the first order conditions for Pareto efficiency, but the second order conditions might not hold.)
²Pigou (1927) is part of a far-ranging discussion about "Empty Boxes" in the Economic Journal addressing this topic; see in particular Robertson (1924, p. 22).
In order to maximize satisfaction – inequalities of wealth among different people and so on being ignored – it is necessary, except in the special case where satisfaction is maximised by a nil output, for that quantity of output to be produced which makes demand price equal to marginal costs, i.e. which corresponds to the point of intersection of the demand curve and the curve of marginal costs. [...] Output, however, tends to be carried to the point in respect of which the demand curve intersects with the supply curve. [...] But in conditions of decreasing costs, where the supply curve coincides with the curve of average costs, it will not be the right point. Unless the State intervenes by a bounty or in some other way, output will be carried less far than it is socially desirable that it should be carried.

Others involved in this discussion are Clapham (1922), Pigou (1922), Sraffa (1926), Shove (1928), Pigou (1928), Robbins (1928), Schumpeter (1928), Young (1928), Robertson (1930), Sraffa (1930), and Shove (1930). It is important to note that the work of Marshall and Pigou confused scale economies with externalities internal to an industry but external to each firm, and consequently they recommended a misplaced Pigouvian remedy for scale economies. Our reconciliation of increasing returns and perfect competition is direct and invokes no externality argument.

The use of transfers would be an easy way out of the conflict between increasing returns and a perfectly competitive equilibrium by essentially assuming the conflict away. Instead, we focus on existence of a competitive equilibrium and the first welfare theorem. The latter is proved using standard techniques.

This research has applications to the theory of agglomeration and city formation. Increasing returns is often used as an agglomerative force in models seeking to explain how, where, and why cities form. For example, Fujita (1988), Fujita and Krugman (1993, 1995), and Krugman (1991, 1993a, 1993b) use a Dixit–Stiglitz (1977) framework and increasing returns to generate city formation in a monopolistic competition context. Since our model will employ increasing returns in a spatial context, it offers the prospect of addressing questions and generating testable hypotheses about cities. This
is discussed further in the conclusion.

In what follows, we stick as closely as possible to the perfectly competitive ideal, since it is simplest to analyze, it is a very standard and convenient benchmark, it allows us to develop proofs of existence of equilibrium (perhaps useful in the imperfect competition context) without having to worry about other distractions, it may be a good approximation to reality in large economies, and it will tell us when the welfare theorems are likely to hold and why. Moreover, it enables us to separate problems due to the spatial context from those attributable to imperfect competition. Notice that models of marginal cost pricing, multipart tariffs, and subsidization of firms under increasing returns all employ close relatives of perfect competition.

Of course, one can take a more positive approach to the problem of increasing returns if perfect competition is deemed unreasonable in such environments, and simply employ imperfect competition models. However, such models usually use rather special assumptions about game forms (e.g., Cournot or Bertrand competition), as in the literature we have already cited. Moreover, equilibrium allocations generated by these models are generally not first best. Monopsony and minimum wages are discussed in the conclusion.

We investigate whether a government ought to intervene in markets for commodities subject to increasing returns in production. A spatial model with finite numbers of producers and consumers (rather than a continuum) is examined both because in the arguments we use, agents employ intervals rather than densities of land, and for several other reasons. Berliant (1985) shows that the usual approximation of continuum economies by finite economies does not work when land plays a role in the models, so demand and equilibria of the continuum models may not

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3We mean that agents own land parcels represented by sets of positive Lebesgue measure in a Euclidean space (R) rather than owning land parcels represented by a quantity at a point. The latter is more common in urban economics, and is usually called a density.
be close to those of any interesting finite model. It is then reasonable to ask if the continuum models make any sense. Examples in Berliant and ten Raa (1991) show that equilibrium can fail to exist in the monocentric city model under standard assumptions on preferences. Examples in Berliant, Papageorgiou and Wang (1990) show that the welfare theorems can fail in the monocentric city model. Berliant and Wang (1993) show that even utilitarian social optima might fail to exist in continuum models with land. The implication of these examples is that the use of a continuum of consumers solves some of the problems associated with the indivisibility of location, but creates others.

The key to the analysis is provided by Berliant and Fujita (1992), who show that for Alonso's urban economic model, a model of pure exchange on the real line where agents are required to own intervals that represent land parcels, there is generally a continuum of equilibria under perfect competition. Infra-marginal land (that is, land not at the endpoints of an interval owned by an agent) is not priced uniquely, thus allowing a kind of indeterminacy in the expenditure of agents on land. It is this kind of indeterminacy that we exploit below to effect implicit transfers to producers (by keeping the infra-marginal price of land low) who would otherwise have negative profits.

Section II presents the notation and model, section III details a general version of the first welfare theorem, and section IV introduces a model with one producer and one consumer, solving for two different classes of equilibria. Section V shows how these equilibria can be extended to a model with two producers and multiple consumers, while section VI concludes. An appendix contains all of the proofs.
II. The Model

We introduce production into Alonso's (1964) model of pure exchange. The model of pure exchange was developed further by Asami (1988), Asami, Fujita and Smith (1991), Berliant (1991), and Berliant and Fujita (1992).

Consider a long narrow city represented on the real line. Land is given by $X = [0, l]$, where $l$ is the length of the city. The density of land available is 1 at each point $x \in X$.

There are $i = 1, \ldots, I$ consumers and $j = 1, \ldots, J$ producers. Each consumer has an endowment of 1 unit of labor, which will be supplied inelastically. Labor is not necessarily homogeneous, so labor income can differ among consumers. This can be seen as skill or time endowment differences. Consumers all have the same preferences, and will get utility from a composite consumption good and land. Thus, $u: \mathbb{R}^2 \rightarrow \mathbb{R}$. Consumers are not endowed with composite good or land. Composite good is produced, while an absentee landlord is endowed with land. We write $u(c, s)$, where $c$ is the quantity of consumption good and $s$ is the quantity of land consumed; the latter is equal to the length of the interval owned by the consumer. For consumer $i$, $c_i$ is composite good consumption, $s_i$ is land consumption, $w_i$ is wage rate, and $[a_i, a_i + s_i] \subseteq X$ is the parcel of land owned by $i$. Define $w = [w_1, \ldots, w_I]$.

Notice that $w$ is assumed to be independent of the location of labor. This is an assumption of perfect competition, that each agent takes prices as given independent of their own actions and the actions of other agents, particularly firms' locations. Without such an assumption, equilibrium allocations are not likely to be Pareto optimal. Since our purpose is to reconcile increasing returns with perfect competition, we must take prices as parametric. Of course, for other purposes, imperfect competition is a more suitable premise. If wages are allowed to vary with location in the context of perfect competition, then the constant wage gradient equilibrium that we study here naturally becomes a special case. Consumers have no intrinsic preference for
location.

Composite consumption good, assumed to be freely mobile, is taken to be numeraire. The price of land is denoted by an integrable function $p: X \to \mathbb{R}$. The price of consumer $i$'s parcel is $\int_{a_i}^{a_i+s_i} p(x) \, dm(x)$. Throughout, $m$ is the Lebesgue measure on the real line. Since we shall be dealing with examples rather than functional analytic techniques, we shall be quite explicit about what $p$ is, so there is no reason to be very technical about function spaces.

Since the labor market is competitive and consumers pay their own commuting cost, consumers will be employed by the closest producer. Let producer $j$ use land parcel $[b_j, b_j+\sigma_j] \subseteq X$. Then commuting distance to the closest producer is given by $d(a_i, s_i) = \min \inf \{\|x-y\| \mid x \in [a_i, a_i+s_i], \ y \in [b_j, b_j+\sigma_j]\}$, the closest point distance between consumer $i$ and the nearest employer. In Section IV, commuting cost for consumer $i$ is given by $t \cdot d(a_i, s_i)$, where $t > 0$. This is the example used by Alonso (1964) and Berliant and Fujita (1992); it incorporates a constant marginal cost of transport per unit distance, to the closest firm. Notice that commuting cost depends on both the consumer location and the location of the nearest employer.

In general, commuting cost for agent $i$ is taken to be a function $T_i: X^{2I+2J} \to \mathbb{R}^+$. We write $T_i(\{a_1, s_1, \ldots, a_I, s_I; b_1, \sigma_1, \ldots, b_J, \sigma_J\})$. Thus, commuting cost for consumer $i$ can, but does not necessarily, depend on the actions of all agents in the model. For notational simplicity, define $A = [a_1, s_1, \ldots, a_I, s_I]$ and $B = [b_1, \sigma_1, \ldots, b_J, \sigma_J]$, and write $T_i(A, B)$.

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4Of course, $d$ depends on the parcels and locations chosen by every producer in addition to $a_i$ and $s_i$, but to keep notation simple, we suppress these additional arguments.

5Our framework allows us to examine more general commuting cost functions. For instance, another example that we have studied involves a fixed cost for road construction. Each agent takes a cost-minimizing route to work, but incurs fixed costs (per unit distance) for road construction between their parcel and the next consumer on the route to their job. The next consumer pays the cost for connecting to the next
Notice that the Alonso commuting cost function is a special case that is attained by setting $T_i(A,B) = t \cdot d(a_i,s_i)$. Moreover, the fact that $T_i$ can depend on the allocation of land to every agent creates an externality, in that the choice of land parcel by an agent can affect the budget constraint of others. What is fascinating about this observation is that, as we shall see in the next section, this externality does not create a market failure.

Consumer $i$'s optimization problem is

$$\max \ u(c_i,s_i) \text{ subject to } c_i \leq \int_{a_i}^{a_i+s_i} p(x) \ dm(x) + T_i(A,B) \leq w_i.$$  

Producers use land and labor to produce composite good. All producers have the same production function $g: \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$. Let producer $j$ use land parcel $[b_j,b_j+\sigma_j] \subseteq X$. The vector $q_j \in \{0,1\}^I$ contains a 1 in position $i$ if consumer $i$ works at firm $j$, and a 0 otherwise. We write $z_j = g(\sigma_j,q_j)$. The profit optimization problem of firm $j$ is:

$$\pi_j = \max \ g(\sigma_j,q_j) - \int_{b_j}^{b_j+\sigma_j} p(x) \ dm(x) - q_j \cdot w.$$  

Define $\pi = [\pi_1, \ldots, \pi_J]$.

Since land and labor are differentiated and indivisible commodities, it is not clear what "increasing returns to scale" means. One might argue that since land has a fixed density but can be priced differently at different locations, land input should be modeled as an interval rather than just the size of an interval. If so, it would be impossible to alter the use of any input without jumping to another commodity; quantities would be zero or one. We have assumed, implicitly, that only the size of an interval matters in production. In the equilibrium existence analysis (sections IV and V) we shall also assume that only the total amount of labor input, the sum of the closest agent, and so forth. The intuitive interpretation is that the road through the next consumer's parcel is already built. Total commuting cost will be the sum of the marginal commuting costs along the entire route.
components of vector \( q_j \) matters. Thus, output is a function of land and labor where both inputs are represented by scalars and, therefore, returns to scale can be defined as usual.

Following Alonso (1964) and the new urban economics literature, an absentee landlord is endowed with all of the land, but gets utility only from composite good. For simplicity, we also endow the absentee landlord with all of the shares in all of the firms.\(^6\) In equilibrium, the absentee landlord collects all of the land rent. Taking \( p(x) \) and \( \pi \) as given, the landlord consumes \( \int_0^t p(x) \, dm(x) + \sum_{j=1}^J \pi_j \). The composite good consumption of the landlord will be denoted by \( c_L \).

Notice that, as in the Alonso model, preferences and production are location independent.

We continue with the analogs of standard definitions for this model. An allocation is a list \( \{\{c_i, s_i, a_i\}_{i=1}^I, c_L, \{z_j, b_j, \sigma_j, q_j\}_{j=1}^J\} \), where for every \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \), \( c_i, z_j, c_L \in \mathbb{R}_+, s_i, a_i, b_j, \sigma_j \in X, q_j \in \{0,1\} \). An allocation \( \{\{c_i, s_i, a_i\}_{i=1}^I, c_L, \{z_j, b_j, \sigma_j, q_j\}_{j=1}^J\} \) is called feasible if\(^7\)

\[
\sum_{i=1}^I [c_i + T_i(A,B)] + c_L \leq \sum_{j=1}^J z_j
\]

(3)

\[
z_j = g(\sigma_j, q_j) \text{ for } j = 1, \ldots, J
\]

(4)

\[
\sum_{j=1}^J q_j = [1, \ldots, 1]
\]

(5)

\[
\{[a_i, s_i + s_i]\}_{i=1}^I, ([b_j, b_j + s_j])_{j=1}^J \text{ form a partition of } X.
\]

(6)

A feasible allocation \( \{\{c_i, s_i, a_i\}_{i=1}^I, c_L, \{z_j, b_j, \sigma_j, q_j\}_{j=1}^J\} \) is called Pareto Optimal if

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\(^6\)It seems clear that one could allow consumer ownership of stock in the firms without altering the results much, but at the cost of complicating the arguments and notation.

\(^7\)Condition (5) requires that all people work. Strictly speaking, this is not necessary. However, since we will assume that there is no disutility of work and utility is increasing in consumption, (5) will hold in equilibrium. Also, condition (6) requires that all land is used. This will hold in equilibrium since we will assume that utility is increasing in land consumption.
there is no other feasible allocation \( \{c_i', s_i', a_i': i = 1, \ldots, I\} \cup \{z_j, b_j, \sigma_j, q_j: j = 1, \ldots, J\} \) such that \( c_L' \geq c_L \) and for each \( i = 1, \ldots, I \), \( u(c_i', s_i') > u(c_i, s_i), \) with a strict inequality holding for at least one of these relations.

A competitive equilibrium consists of a feasible allocation
\[ \{c_i, s_i, a_i: i = 1, \ldots, I\} \cup \{z_j, b_j, \sigma_j, q_j: j = 1, \ldots, J\}, \]
an integrable land price function \( p: X \rightarrow \mathbb{R} \), a vector of profits \( \pi \in \mathbb{R}^J \) and a vector of wages \( w \in \mathbb{R}^I \) (the freely mobile composite consumption commodity is taken to be numeraire), such that
\[
\begin{align*}
(7) & \quad c_L = \int p(x) \, d\mu(x) + \sum_{j=1}^J \pi_j \\
(8) & \quad \{c_i, s_i, a_i\} \text{ solves (1) for } i = 1, \ldots, I. \\
(9) & \quad \{\pi_j, z_j, b_j, \sigma_j, q_j\} \text{ solves (2), for } j = 1, \ldots, J.
\end{align*}
\]

The allocation component of a competitive equilibrium is called an equilibrium allocation.

Notice that agents take into account the total supply of land when solving their optimization problems. This constriction of the commodity space is essential to our results, and appears in the spatial economic literature more generally.

III. The First Welfare Theorem

This will be proved for a very general version of the model in the standard way, for example as in Berliant and Fujita (1992, Proposition 2). The most interesting aspect of this result is that, in spite of the externality present in the commuting cost function of consumers, equilibrium allocations are first best, so there is no market failure.

We say that the utility function \( u(c, s) \) is increasing in consumption good if for all \( c \geq 0, s > 0 \), for all \( c' > c \), \( u(c', s) > u(c, s) \).

Define \( u = \inf \{u(c, s): (c, s) \in \mathbb{R}^2_+\} \). We say that land is a necessity when \( s = 0 \).
implies $u(c,s) = u$.

**Theorem 1:** If $u$ is increasing in consumption good and land is a necessity, then any equilibrium allocation such that each consumer's utility is greater than $u$ is Pareto optimal.

**Proof:** See Appendix.

Theorem 1 holds even when the utility functions $u_i(c_i,s_i,a_i)$ differ by consumer and are location dependent (provided they satisfy analogs of the assumptions), and when the production functions $g_j(q_j, r_j, b_j)$ differ by producer and are location dependent.

IV. Existence of Equilibrium with One Producer and One Consumer

As described in Berliant and Fujita (1992), demand (and in the present model, supply) correspondences are not convex-valued. In fact, the contract curve in the pure

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8The assumption of monotonicity in consumption good requires as a hypothesis that consumer land consumption be non-zero; the purpose is to include examples such as Cobb–Douglas utilities, which would not satisfy the assumption if it were imposed where land consumption is zero. Usually, local non-satiation is used in place of a monotonicity assumption. Local non-satiation would be sufficient here if the commuting cost function were upper semicontinuous. Unfortunately, we must employ a commuting cost function that is not upper semicontinuous in section V below, so we weaken the assumption on the commuting cost function and strengthen the assumption on utility. Finally, with commuting cost dropping discontinuously when parcel size tends to zero, it is possible to have equilibria where some consumer owns no land. If such a consumer owns some consumption good, then the equilibrium allocation might not be Pareto optimal, for example with Cobb–Douglas utility, since the consumer gets no utility from the consumption good, and a transfer to the absentee landlord would result in a Pareto improvement. Thus, the assumption that utility exceeds its infimum is necessary. Hence, for the purposes of this Theorem, we assume that land is a necessity and that utility exceeds its infimum at an equilibrium allocation. The implication is that land consumption is positive and hence the monotonicity assumption can be applied. Every equilibrium that we shall construct has positive land consumption and utility exceeding its infimum.
exchanger model is disconnected; see figure 2 of that paper. Due to the discreteness and nonconvexities inherent in the model, we prove that an equilibrium exists by actually finding one. In fact, for the simple case examined in this section, we find two classes of equilibria.

In this section we examine the following set of examples. Let $I = 1$ and $J = 1$, and for notational simplicity, drop the subscripts referring to agents. An equilibrium will belong to one of two classes, I or II, depending on its characteristics. Equilibria not in either class exist as well.

**Definition:** We say that the *functional form restrictions for classes I and II* hold when utility, production, and commuting cost satisfy the following conditions: $u(c,s) = c + \alpha \cdot \ln(s) \ (\alpha > 0)$, $g(q) = \beta \cdot \min (s, q) - f$, where $f$ is a fixed cost in terms of composite good, and $T_i(A,B) = t \cdot d_1(a_1, s_1)$; we use the Alonso commuting cost in this section.

Next, let us give bounds on exogenous parameters for class I.

**Definition:** We say that the *parameter restrictions for class I* hold when the following conditions are met: $l \geq 2.87$, $f \leq \alpha \cdot \{1/(l-2) - \ln((l-1)/(l-2))\}$, $\beta \geq \alpha/(l-2) + \alpha \cdot \{2 \cdot \ln((l-1)/(l/2)) - 1\} - \ln((l-1)/(l-2))$.

It is easy to see that the bounds on $f$ and $\beta$ are positive. In essence, what is needed is that total land $l \geq 2.87$, fixed cost $f$ is small relative to the marginal utility of land ($\alpha$), and returns to scale ($\beta$) are large relative to $\alpha$. Clearly, these restrictions represent a set of parameters with nonempty interior.

**Theorem 2:** Under the functional form restrictions for classes I and II and the parameter restrictions for class I, there exists an equilibrium. Moreover, Theorem 1 applies, so the equilibrium allocation is Pareto optimal.

**Proof:** See Appendix.
Figure 1 provides a picture of the equilibrium. The horizontal axis represents the location space, while the vertical axis is used for the land price density (in dollars per foot or inch). The horizontal axis is located not at height zero, but at height \( \alpha/(l-1) \), the equilibrium marginal utility of land for the consumer. The firm is located on the parcel \([0,1)\) while the consumer buys the remainder of the land. The shaded area is the implicit subsidy from the landlord to the producer, in dollars. The price density is in fact the minimum of two curves representing marginal willingness to pay for land of the consumer. The first represents marginal willingness to pay for quantities of land starting at the left endpoint of the interval \( X \), while the second is the marginal willingness to pay for quantities of land starting at the right endpoint. The minimum of the two is the symmetric curve \( p(x) \).

Next we shall study another class of equilibria for this same model, one that is motivated by the observation that marginal commuting cost is discontinuous when the consumer and producer are adjacent. Marginal commuting cost drops from \( t \) to zero when the consumer and producer touch, thus allowing a discontinuity in land price at the boundary. We call this class of equilibria class II.

**Definition:** We say that the **parameter restrictions for class II** hold when the following conditions are met:

\[
\begin{align*}
l &\geq 3.19, \\
f &\leq \alpha \cdot \frac{1}{((l-1)-1)} - 1/(l-2), \\
\beta &\geq \frac{\alpha}{(l-2)} + \alpha \cdot \frac{1}{(l-1)} + 2 \cdot \ln(\frac{(l-2)}{(l-1)})
\end{align*}
\]

Once again, total land \( l \) needs to be large enough, while fixed cost \( f \) must be small relative to the marginal utility of land \( \alpha \) and returns to scale \( \beta \) large relative to \( \alpha \). Again, these restrictions represent a set of parameters with nonempty interior.

**Theorem 3:** Under the functional form restrictions for classes I and II and the parameter restrictions for class II, there exists an equilibrium. Moreover, Theorem 1 applies, so the equilibrium allocation is Pareto optimal.
Proof: See Appendix.

Figure 2 provides a picture of the equilibrium. The horizontal axis represents the location space, while the vertical axis is used for the land price density (in dollars per foot). The horizontal axis is located not at height zero, but at height $\alpha/(l-1)$, the equilibrium marginal utility of land for the consumer. The firm is located on the parcel $[0,1)$ while the consumer buys the remainder of the land. The shaded area is the implicit subsidy from the landlord to the producer, in dollars.

V. Existence of Equilibrium with Two Producers and Many Consumers

This extension of the model is not as easy as it may appear. Consider first a model with one producer and 2I consumers, and a class I equilibrium. To keep the model as close as possible to the one in the last section, let us change the technology to $g(\sigma,q) = \beta \cdot \min (\sigma_i (q \cdot 1)/I) - f$ where 1 is the vector of I ones, and let $X = [-l+1,l]$. One way to construct a class I equilibrium is illustrated in Figure 3. Again, the horizontal axis represents location space while the vertical axis gives the price density for land in dollars per foot. The horizontal axis is located at height $\alpha/(l-1)$ rather than at zero on the vertical axis. The price density is the same as in the previous section for the consumer to the right of the firm. We replicate the same density for the consumer to the left of the firm. This necessitates an alteration of the density on the firm's parcel, due to the presence of land to the left of the firm that it would want to buy unless the price were raised (this is justified by the first order condition for firm optimization with respect to b). Thus, we take the maximum of these two price densities. However, land at the extreme left and extreme right in X is cheapest under this new density, so the firm would move out to an extreme. To prevent this,
we must raise the price of land in the extremes by replicating a shifted price density once again, and taking the maximum of all price densities. This will violate the first order conditions for the consumers, which state that the price of the edge of a parcel closer to the firm must be higher than the edge further away from the firm (as in Berliant and Fujita (1992)). This statement does not apply to the innermost two consumers, since there is a discontinuity in their marginal commuting cost at zero distance; there is no such discontinuity for consumers not adjacent to the firm, so this statement must apply to them. Moreover, given that the price density on each consumer parcel is the same, the total cost of each consumer parcel is the same, so why would any consumer choose to live on a parcel not adjacent to the firm? They would pay the same total land rent, but incur a higher commuting cost further out, thus attaining a lower level of utility. Figure 3 does not represent an equilibrium.

So how can we solve this problem and obtain an equilibrium?

The answer to this question lies in noticing that the problem we have is overconstrained. We are asking too much of the rent density, in that it reflects differences in commuting cost among parcels as stated above (essentially the Mills (1967) – Muth (1969) condition for our model), but at the same time, reflects the fact that the profit function only accounts for the cost and not the location of the parcel, so the producer will always choose the cost minimizing one. In other words, consumer optimization requires that rent decreases as distance from a producer increases, to compensate for commuting costs, while the producer will always find the lowest cost parcel, located as far as possible from its current spot.

If prices are low on the producer parcel, then consumers will move there to reduce commuting cost. If prices are low on consumer parcels distant from the consumers to compensate for commuting cost, then producers will move there to reduce land cost.

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9See, for instance, Fujita (1989, p. 25, equation 2.37) for a nice statement and explanation.
Equilibrium is not likely to exist. This is in essence the problem discovered by Koopmans and Beckmann (1957) in their investigation of the quadratic assignment problem. Although their model is different from ours, this kind of problem pertaining to existence of equilibrium arises in most location models where all agents and resources are mobile.

We must specify out-of-equilibrium commuting costs properly.

In the pure exchange version of the Alonso model, the location to which consumers commute, the central business district or CBD, is given and occupies no land. Commuting cost is given by the "front location" or "front door" (closest point) distance from the consumer's parcel to the CBD. See Asami, Fujita and Smith (1991) for elaboration. However, if a producer (or the CBD) occupies space, it is unclear, especially out of equilibrium, where the consumer must commute to. For instance, if the consumer decides to buy a subset of the parcel used by a producer, clearly a disequilibrium situation, what is its commuting distance and cost? This must be specified, even out of equilibrium, in order to verify whether a particular situation represents an equilibrium or not.

We assume that if a consumer outbids a producer, he or she can no longer work at that location, since the producer will no longer be there. Consumers and producers remain price takers; this is simply a specification of disequilibrium commuting costs. Formally, it amounts to defining commuting distance as

\[ \delta(a_i, s_j) = \min_{(a_i, a_i+s_i) \cap (b_j, b_j+s_j) = \emptyset} \inf_{x \in (a_i, a_i+s_i), y \in (b_j, b_j+s_j)} \|x - y\|. \]

Commuting cost is defined to be \( T_1(A, B) = t \cdot \delta(a_i, s_j) \), analogous to the Alonso model.

We say that **commuting cost satisfies the functional form restriction** when this

10 The quadratic assignment problem is distinct from, but related to, the linear assignment problem (or one sided matching problem) that is generally more familiar to economists. The quadratic assignment model allows flows of (intermediate) goods between agents, at some cost.
commuting cost function is used. Notice that this commuting cost function is not upper semicontinuous in consumer location; it can drop discontinuously as the intersection of consumer and producer parcels tends to the empty set. This is what necessitates the assumptions of Theorem 1.

Figure 4 illustrates what an equilibrium will look like. The horizontal axis represents the location space $X = [-2l, 2l]$, while the vertical axis is used for the land price density (in dollars per foot). The horizontal axis is located not at height zero, but at height $p(2l)$, the equilibrium marginal utility of land for the consumers located furthest from a firm. Equilibrium configurations consist of individual producers surrounded by commuting consumers. This configuration involves agglomeration around a producer, essentially a company town. The configuration appears to be unique, though the equilibrium allocation of mobile good and land is not (as in the Alonso exchange model). Notice that the freedom in choosing land price levels as distance from the firm increases allows us to let parcels get cheaper as we move out. This is necessary in equilibrium in order to compensate for the increased cost of commuting as distance from the firm increases, for otherwise nobody would live in the hinterlands. Notice also that we can do this while still making the firm's parcel the cheapest per unit cost of land, so the firm has no incentive to move. The modification of the commuting cost function implies that no consumer will encroach on a producer's parcel, since encroachment means that the consumer must commute to the next closest producer, requiring a large jump in expenditure on commuting. Thus, the commuting cost deters consumer encroachment into a firm's parcel, and the low price of land on a firm's parcel keeps the firm there.

We intend to attack the Koopmans–Beckmann quadratic assignment problem head on, using the same modification of out-of-equilibrium transport costs that we have used here for commuting costs. If an agent wants to cohabit a parcel with another, then it must go elsewhere for supplies (or more generally, transactions). In closing, we note that the quadratic programming disease is present in many location models.
There will be some restrictions on the parameters. The equilibrium will have the same pattern as equilibrium in the Alonso model, that consumers with higher wages live further from the firm and buy more land. As in Berliant and Fujita (1992), if land is a normal good, consumers with higher wages and thus more income will buy more land and, in any efficient allocation, consumers purchasing more land must be located further from the producer, for otherwise we can switch positions of the consumers, save on commuting costs, and create a Pareto improvement, contradicting Theorem 1. (Although land is not strictly normal in the example we considered in section IV, it is weakly normal in the sense that the income derivative of demand for land is zero, so the argument applies.) For simplicity, we shall only examine the case when all consumers are identical.

To make notation simpler, let $X = [-21, 21]$. We focus on the part of the economy to the right of 0 in $X$; the part to the left will be symmetric. There are $4I$ consumers. In contrast with the assumptions of the preceding section, we allow a general utility function. The utility function of every consumer is $u(c, s)$, where $u: \mathbb{R}^2_+ \rightarrow \mathbb{R}$ satisfies the following conditions, the first three of which are adapted from Berliant and Fujita (1992, Assumption 1).

**Definition:** A utility function $u$ is said to be well-behaved if it satisfies the following.

(i) On $\mathbb{R}^2_+$, $u$ is twice continuously differentiable, strictly quasi-concave, and increasing in both $c$ and $s$.

(ii) No indifference curve intersecting $\mathbb{R}^2_+$ cuts an axis, and every indifference curve intersecting $\mathbb{R}^2_+$ has the $c$-axis as an asymptote.

(iii) Lot size (or land) $s$ is a normal good on $\mathbb{R}^2_+$.

(iv) The composite consumption commodity is a normal good on $\mathbb{R}^2_+$.

Cobb–Douglas utilities are an example.

**Definition:** Production satisfies the functional form restriction if $g(c, q) = \beta \cdot \min$
Definition: The parameter restrictions are said to be satisfied if the following hold. 

$I \geq 2, l \geq 2l^2 + 1,$ 

$$\beta > \max\{3f/[4(l + 1)], f/(2l) + f[l/(2l) - 1/4][l(l + 1)]/[l(l + 1)] + (1 - 1/l^2)(l - l)\},$$ 

$$f(l - 1/2)/[2I(l + 1)] < t.$$ Finally, the marginal rate of substitution of composite good for land, or the marginal willingness to pay for land, satisfies the following inequality at a particular (given) allocation. Define 

$$\bar{c} \equiv \min \{\beta - f/(2l) - f[l/(2l) - 1/4][l(l + 1)] - (1 - 1/l^2)(l - l),$$ 

$$f[l/(2l) - 1/4][l(l - 1)(l - l - 1)]/[l(l + 1)(l - 1)]\}$$ 

and $$\bar{s} = f(2l/l - 1)/[(l + 1)(l - 1)].$$ The first argument of the min in the expression for $$\bar{c}$$ is positive due to the parameter restriction on $$\beta.$$ The second argument is positive because $$\bar{s} < (l - 1)/l$$ due to the parameter restrictions on $$l, l,$$ and $$t/l.$$ Hence 

$$\bar{c} > 0. \ \bar{s} > 0$$ due to the parameter restriction on $$l.$$ Then 

$$\left. \frac{\partial u}{\partial s} \right|_{(\bar{c}, \bar{s})} \left. \frac{\partial u}{\partial c} \right|_{(\bar{c}, \bar{s})} > \beta - 3f/[4(l + 1)] + (l - 1)l.$$

For example, a CES utility function will satisfy the last inequality if parameters are chosen appropriately.

These parameter restrictions imply that the total land available ($l$) is large relative to the number of consumers, marginal productivity ($\beta$) is large relative to fixed costs (or that land and the number of consumers are large relative to fixed costs), and that commuting costs are large relative to fixed costs. The condition on marginal willingness to pay for land at a particular bundle implies that one consumer's land consumption cannot become too small relative to another's.

**Theorem 4:** If utility is well-behaved, production satisfies the functional form restriction, commuting cost satisfies the functional form restriction, and the parameter restrictions hold, then there exists an equilibrium. Moreover, if utility is increasing in
consumption good and land is a necessity for every consumer, then Theorem 1 applies, so the equilibrium allocation is Pareto optimal.

Proof: See Appendix. Figure 4 provides a picture of the equilibrium, and was explained earlier in this section.

The strategy of the proof is as follows. Guess that the firms' parcels are $[-(l+1),-l]$, $[-l,-(l-1)]$, $[l-1,l+1]$. Then we fix a wage rate, and then solve the consumer equilibrium problem on the parcels not occupied by firms, exploiting the results of Berliant and Fujita (1992) to construct an equilibrium. We set the firm land price lower than the lowest consumer price, the difference depending only on fixed costs, total land available, and the number of consumers. Then we set up the zero profit condition of the firm in equilibrium, and find a wage rate that solves it. This wage rate, the implied rent density, the allocation of land, and the allocation of consumption good form an equilibrium. The hard part of the proof is to show that no consumer would intrude on a firm's parcel, and vice-versa.

The details of the proof can be found in the appendix.

VI. Conclusions and Extensions

Using some interesting classes of examples, we have examined how land can reconcile increasing returns and perfect competition in the following sense. In a model without location, production of a commodity using a technology requiring a fixed cost followed by constant returns to scale will imply that only one firm producing this good

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12 Even though it has already been assumed that utility is well-behaved, these assumptions are applied in order to limit the behavior of the utility function on the boundary of $\mathbb{R}_+^2$. 

will operate in an efficient allocation. However, in a spatial model with commuting cost, such as the one examined here, there is a trade-off between returns to scale and the cost of accessing a firm, thus limiting the extent of the market served by any single firm, and therefore allowing multiple active firms in an efficient allocation. A perfectly competitive equilibrium can result in a land price scheme that limits firm size optimally and provides a subsidy to active firms consistent with efficiency.

The questions we have studied seem important not only in the theory of industrial organization, in that government intervention in markets for goods produced under an increasing returns to scale technology may not be justified, but also in the theory of spatial economics. For example, we can separate results due to imperfect competition from those due to the presence of location in models. These questions are of central interest to urban economics and location theory as well. The Spatial Impossibility Theorem of Starrett (1978), as interpreted by Fujita (1986), tells us that some assumption of neoclassical economics must not hold if we are to generate equilibrium models of agglomeration. Here we have used increasing returns and perfect competition, but we are able to generate agglomeration and factory towns in equilibrium without imperfect competition. Unlike much of the other work on agglomeration, our equilibrium configurations are first best.

The model can accommodate more firms (and any number of consumers) just by replicating the example of section V.

Notice that when the number of firms is fixed at the equilibrium number (allowing entry), the arguments of the previous sections apply. Entry is determined by the zero profit condition. It seems to us that the assumption of perfect competition in our model might be justified by competition from potential entrants rather than by spatial competition from extant firms, but a formal theory of such a justification is beyond the scope of this paper. Here we have assumed perfect competition, but have not justified this assumption formally. The latter should be the subject of future work; the tests of
Gretsky, Ostroy and Zame (1996) for perfect competition should be useful.

One testable implication derived from the model is that the unit land price of a firm's parcel should be low relative to the unit price of residential land surrounding the producer. Of course, the hazards involved in testing this hypothesis include the difficulty in separating the value of land from structures as well as zoning laws.

We propose to examine the comparative statics of the model, particularly the relation between the exogenous parameters (such as returns to scale or commuting cost) and the number of firms. The number of firms would be endogenous and determined by the zero profit condition. We hope to generate testable hypotheses in this way.

It is interesting that the arguments used here seem inapplicable to a model with a continuum of agents who own points rather than parcels of land, since the model relies on charging different prices for land within a parcel. The use of a finite number of agents contrasts with much of the other literature on spatial economics.

Another issue of interest is the conjecture that, in both this model and the simpler Alonso exchange model, even though equilibria exist and equilibrium allocations are Pareto optimal (see Berliant and Fujita (1992) for the exchange case), the core can be empty. Thus far, we have a quasi-linear example where the emptiness or non-emptiness of the core depends on endowments. We intend to look at this more generally, and examine the implications for core convergence.

Our approach to the problem of competitive price support for local monopolies is applicable whenever units of an input can be identified and priced separately. Land is one commodity that fits naturally, but this approach is also quite relevant to the recent minimum wage debate. Here proponents argue that a monopsonist has an incentive to restrain employment even when the marginal worker has a reservation wage below marginal productivity. The consequent reduction of the wage rate spreads to the other workers and the cut in the wage bill (more than) compensates for the foregone profit opportunity on the marginal worker. This source of inefficiency can be reversed.
by a minimum wage rate, just as monopoly power can be checked by a maximum product price.

What plagues the efficiency problem of the monopsonist is the law of one price. The monopsonist refrains from hiring an additional worker, even when marginal revenue product exceeds cost, out of fear that the new wage spills over to the incumbent workers. If, however, workers can be identified and rewarded individually, the law of one price no longer interferes with efficiency and the free market can be relied upon. Such deviations from the law of one price can be hidden. For example, employers can offer a bonus on accepting a job (differential moving expenses or other favorable fringe benefits, for example). The bottom line of our analysis is that input price discrimination can potentially reconcile the first order condition for profit maximization under perfect competition — price equals marginal cost — with financial viability — total revenue is at least equal to total cost — when there are increasing returns to scale.
APPENDIX

Proof of Theorem 1: Take an equilibrium allocation

\[ \{c_i, s_i, a_i\}_{i=1}^J, \{z_j, b_j, \sigma_j, q_j\}_{j=1}^J \], and suppose that it is Pareto dominated by another feasible allocation, \[ \{c'_i, s'_i, a'_i\}_{i=1}^J, \{z'_j, b'_j, \sigma'_j, q'_j\}_{j=1}^J \]. So \( u(c'_i, s'_i) \geq u(c_i, s_i) \) for all \( i \), and \( c'_L \geq c_L \), with strict inequality holding for at least one relation. Define \( A' \) and \( B' \) analogous to \( A \) and \( B \). Since equilibrium utility exceeds \( u \) by assumption, apply the postulate that land is a necessity to obtain that the Pareto dominating allocation has \( s'_i > 0 \) for all \( i \).

\[
\begin{align*}
\int_{x}^{a_i} p(x) \, dm(x) + T_i(A', B') &\geq \int_{x}^{a_i} p(x) \, dm(x) + c_L \leq \sum_{j=1}^{J} \pi_j, \\
\text{with strict} &\text{ inequality holding for one relation by monotonicity in consumption good. Summing these relations,} \\
\sum_{i=1}^{J} [c_i + \int_{x}^{a_i} p(x) \, dm(x) + T_i(A', B')] &> \sum_{i=1}^{J} [c_i + \int_{x}^{a_i} p(x) \, dm(x)] + T_i(A, B) \\
c_L &= \sum_{i=1}^{J} \pi_j, \quad \text{(using landlord monotonicity). Now} \\
\sum_{j=1}^{J} g(\sigma_j, q_j) - \int_{b_j}^{b_j} p(x) \, dm(x)] &> \sum_{i=1}^{J} \pi_j. \quad \text{Rearranging,}
\end{align*}
\]

inequality is equilibrium profits. The left hand side of this inequality are profits under the alternative production plan at equilibrium prices. Since the latter is larger, some firm must make more profits under the alternative plan than under the equilibrium plan, contradicting that (9) holds at equilibrium. So the hypothesis is false, and the equilibrium allocation is Pareto optimal.
Proof of Theorem 2: Let \( p(x) = \alpha/(l-x-1) \) for \( x \leq l/2 \), \( p(x) = \alpha/(x-1) \) for \( x > l/2 \), \( b = 0 \), \( \sigma = 1 \), \( q = 1 \), \( z = \beta - f \), \( a = 1 \), \( w = \beta - \alpha/(l-2) \), \( \pi = \beta - f - w - \alpha \cdot \ln \left( \frac{l-1}{l-2} \right) \), \( c = w - \left[ \alpha \cdot 2 \cdot \ln \left( \frac{l-1}{\left\{ l/2-1 \right\}} \right) - \alpha \cdot \ln \left( \frac{l-1}{l-2} \right) \right] \) (which is non-negative by the assumption on \( \beta \)), and \( c_L = \alpha \cdot 2 \cdot \ln \left( \frac{l-1}{\left\{ l/2-1 \right\}} \right) + \pi \). We claim that this is an equilibrium. Figure 1 provides a sketch of the price density.

First, we verify that this is indeed a feasible allocation. To verify (3), note that commuting cost is zero in this allocation, and calculate
\[
c + c_L = w - \left[ \alpha \cdot 2 \cdot \ln \left( \frac{l-1}{\left\{ l/2-1 \right\}} \right) - \alpha \cdot \ln \left( \frac{l-1}{l-2} \right) \right] + \alpha \cdot 2 \cdot \ln \left( \frac{l-1}{\left\{ l/2-1 \right\}} \right) + \beta - f - w - \alpha \cdot \ln \left( \frac{l-1}{l-2} \right) = \beta - f = z.
\]
(4) and (5) are obvious. Finally, note that \([0,1),[1,\ell] \) is indeed a partition of \( X \), so (6) holds.

Regarding the equilibrium conditions (7), (8), and (9), (7) can be verified simply by calculating the total area under the price density, \( \alpha \cdot 2 \cdot \ln \left( \frac{l-1}{\left\{ l/2-1 \right\}} \right) \), and adding to it profits \( \pi \).

Problem (1) can be written as the following unconstrained optimization problem by substituting the budget constraint for \( c \):
\[
\max_{a,s} \alpha \cdot \ln(s) + w - \int_{a}^{a+s} p(x) \, dm(x) - t \cdot \max(0,a-1)
\]

The first order condition with respect to \( s \) is \( p(a + s) = \alpha/s \); this is verified for our price density at \( a = 1 \) and \( s = \ell - 1 \). The first order condition with respect to \( a \) is \( p(a) - p(a + s) = t \) if \( a > 1 \), \( p(a) - p(a + s) \in [0,t] \) if \( a = 1 \), \( p(a) - p(a + s) = 0 \) if \( a < 1 \). This is an interesting and important fact. Notice first that \( p(a) - p(a + s) = t \) if \( a = 1 \), so our equilibrium satisfies the first order condition. Second, this first order condition is a result of the assumption that closest point distance is all that matters when computing commuting cost, so discontinuous marginal commuting cost is the consequence. Total commuting cost is continuous.
Regarding second order conditions for the consumer, it is rather evident that the consumer cannot do better by decreasing its parcel size, since the rent curve is less than or equal to the marginal willingness to pay for land of the consumer. The consumer cannot do better by increasing its parcel size since for larger parcels, the rent curve is greater than the marginal willingness to pay for land. Due to the symmetry of the rent curve, the consumer cannot do better by owning a parcel containing \{0\} rather than \{l\}. Thus, the equilibrium allocation solves (1) for the consumer.

With regard to the firm, notice that optimization will imply that \( q = \sigma \) and optimization problem (2) reduces to:

\[
\max_{b, \sigma} \beta \cdot \sigma - f - \int_{b}^{b+\sigma} p(x) \, dm(x) - w \cdot \sigma
\]

The first order condition with respect to \( \sigma \) is \( \beta - p(b + \sigma) - w = 0 \), and \( w \) was chosen to satisfy this equality for \( b = 0 \) and \( \sigma = 1 \). The first order condition with respect to \( b \) is \( p(b) = p(b + \sigma) \), which can either be ignored since the producer hits the land boundary at zero, or we can set \( p(0) = \alpha/(\ell-2) \), altering \( p \) on a set of measure zero.

Turning next to second order conditions for the firm, notice first that if the firm uses a parcel of any size, it is indifferent about its location, so it will choose one of the cheapest parcels, and \([0, \sigma)\) is among these. The first order condition with respect to \( \sigma \) will imply that it will choose \( \sigma = 1 \). Beyond this, up to \( \sigma = \ell/2 \), the marginal cost of land exceeds the marginal benefit net of labor cost. If the firm can make higher profits from expanding the scale of its operations beyond 1, then given the production function and the price density, it will make higher profits when \( b = 0 \) and \( \sigma = \ell \). Profits from such a production plan are given by

\[
\beta \cdot \ell - f - w \cdot \ell - 2 \cdot \alpha \cdot \int_{\ell/2}^{\ell} 1/(x-1) \, dm(x)
\]

What this means is that the production function is location independent.
Profits from the equilibrium production plan are given by

\begin{equation}
\beta - f - w - \alpha \cdot \int_{l-1}^{l} \frac{1}{(x-1)} \, dm(x)
\end{equation}

Following some calculations, it can be shown that (11) always exceeds (10) if 
\[\frac{[(l-1)/(l-2)]}{[(l-1)/(l-2)]} \leq 2 \cdot \ln(2) + \ln\left[\frac{(l-1)/(l-2)}{(l-1)/(l-2)}\right]\] or, as assumed above, \(l \geq 2.87\).

Finally, it is necessary to show that (11) is non-negative, in order to ensure that the producer will not exit. Again, following some calculations, the assumption that \(f \leq \frac{\alpha}{l-2} - \alpha \cdot \ln\left[(l-1)/(l-2)\right]\) implies that (11) is always non-negative.

Q.E.D.

Proof of Theorem 3: Let \(p(x) = \alpha/(l-x-1)\) for \(1 \leq x \leq l/2\), \(p(x) = \alpha/(l-1)\) for \(l-1 \geq x \geq l/2\), \(p(x) = \alpha/(l-1)\) for \(0 \leq x < 1\), \(p(x) = \alpha/(l-1)\) for \(l-1 < x \leq l\), \(b = 0\), \(\sigma = 1\), \(q = 1\), \(z = \beta - f\), \(a = 1\), \(s = l-1\), \(w = \beta - \alpha/(l-2)\), \(\pi = \beta - f - w - \alpha/(l-1)\), \(c = w - \left[\alpha/(l-1) + \alpha \cdot 2 \ln((l-2)/((l/2)-1))\right]\) (which is non-negative by the assumption on \(\beta\)), and \(c_L = 2\alpha/(l-1) + \alpha \cdot 2 \ln((l-2)/((l/2)-1)) + \pi\). We claim that this is an equilibrium. Figure 2 provides a sketch of the price density.

First, we verify that this is indeed a feasible allocation. To verify (3), note that commuting cost is zero in this allocation, and calculate
\[c + c_L = w - \left[\alpha/(l-1) + \alpha \cdot 2 \ln((l-2)/((l/2)-1))\right] + 2\alpha/(l-1) + \alpha \cdot 2 \ln((l-2)/((l/2)-1)) + \beta - f - w - \alpha/(l-1) = \beta - f = z.\]

(4) and (5) are obvious. Finally, note that \([0,1],[1,l]\) is indeed a partition of \(X\), so (6) holds.

Regarding the equilibrium conditions (7), (8), and (9), (7) can be verified simply by calculating the total area under the price density, \(2\alpha/(l-1) + \alpha \cdot 2 \ln((l-2)/((l/2)-1))\), and adding to it profits \(\pi\).

As the reader might suspect, the remainder of the proof that class II is in fact an equilibrium is quite analogous to the proof for class I, so we shall not bother to repeat it here. The proof that equilibrium profits are larger than profits using all land
Proof of Theorem 4: We begin by fixing \( w \), the wage rate, in \([0, \beta]\). Apply Proposition 4 of Berliant and Fujita (1992) to the exchange economy where consumers have an endowment of consumption good \( w \) and land is limited to the interval \((l+1,2l]\), to obtain an equilibrium price density \( p_w(x) \), where \( p_w(2l) \) is uniquely determined (and is the same for all equilibria). Using the assumption that land is a normal good, \( p_w(2l) \) is increasing in \( w \). Using upper hemi-continuity of the equilibrium correspondence of the exchange economy in \( w \), \( p_w(2l) \) is continuous in \( w \). We want to solve

\[
\beta - w - f/(2l) - \max\{p_w(2l) - f[l/(2l) - 1/4]/(l + 1), 0\} = 0
\]
on \( 0 \leq w \leq \beta \). This will be the zero profit condition for the firms.

As \( w \) tends to zero, \( p_w(2l) \) tends to zero, so the left hand side of (12) tends to \( \beta - 3fl/[4(l + 1)] \), which is positive by assumption. Note that at \( w = \beta \), the left hand side is \( -p_\beta(2l) - 3fl/[4(l + 1)] \), which is negative. By the intermediate value theorem, there is a \( w^* \) strictly between 0 and \( \beta \) solving the equation. Define \( p = p_{w^*} \), and define \( s_i, c_i, \) and \( a_i \) (\( i = 1, \ldots, I \)) to be the equilibrium land consumption, composite good consumption, and parcel front location of consumer \( i \), respectively, derived from the exchange economy with consumer endowments \( w^* \). This is done first on the interval \((l+1,2l]\), and mirrored on the interval \((0,l-I)\). The allocation on the intervals \((-2l,-l-I)\) and \((-l+1,0)\) is defined analogously.

For \( l-I \leq x \leq l+I \), define \( p(x) = \max\{p(2l) - f[l/(2l) - 1/4]/(l + 1), 0\} \). The price density on the firm's parcel is less than the lowest price on any consumer's parcel. For \( 0 \leq x \leq l-I \), define \( p(x) = p(2l-x) \). For \(-2l \leq x \leq 0 \), define \( p(x) = p(-x) \).

Let \( b_1 = -l-I, b_2 = l-I \). For \( j = 1,2 \) let \( \sigma_j = 2I, q_j \) consists of \( 2I \) ones and \( 2I \) zeros with \( q_1 + q_2 = 1, z_j = 2I\beta - f, \pi_j = 0 \). For consumers residing in the interval
\[(l+1,2l), \quad c_i = w^* - t \cdot \text{cf}(a_i-l-1) - \int_{a_i}^{a_i+s_i} p(x) \, dm(x) \geq 0 \text{ by construction of the exchange economy allocations.} \]

The consumption of other consumers is defined analogously. \[c_L = \int_{-2l}^{2l} p(x) \, dm(x) \geq 0.\]

We claim that this is an equilibrium. (3) is verified by substitution of the expressions above for consumption and output (note that the transportation cost terms cancel). (4), (5), (6) and (7) hold by construction.

Next, we argue that the allocation we have specified solves the consumers' problems (1). By construction of the exchange economy equilibrium, no consumer has an incentive to relocate within the intervals occupied by the consumers. The land occupied by producers is less expensive than any land occupied by consumers, but always requires more transport cost. Consider a consumer parcel \((a,a+s)\) containing part of the land parcel of the firm located at \((l-I, l+I)\). We may assume that \(a + s/2 \leq l\). For if \(a + s/2 > l\), then we can flip the consumer parcel symmetrically about \(l\), save on commuting cost, and obtain the same quantity of land.

First we consider the case \(a + s > l + I\). The idea is to shift the parcel towards the left. This saves commuting cost. It also saves rent, as long as \(p(a) \leq p(a + s)\). By symmetry about \(l\), rent density \(p(a + s)\) is also attained at \(2l - (a + s)\), but \(a\) is to the left of this point, since \(a + s/2 \leq l\). The next point leftward where rent density \(p(a + s)\) is attained is \(-2l + (a + s)\), by symmetry about 0. As long as \(s \leq 2l\), \(a\) is to the right of \(-2l + (a + s)\) and we can shift the parcel towards the left, saving both commuting cost and rent. If \(s > 2l\), then since \(a + s/2 \leq l\), \(a < 0\); now we will show that the utility associated with such a big parcel is below the equilibrium utility level of consumers. We distinguish two sub-cases. Call the rightmost consumer commuting to the left producer consumer \(i\). In the first case, \(a \leq a_i\). The encroaching consumer is spending at least as much on land as any consumer in equilibrium, is
consuming at least as much land, and is facing the same marginal commuting cost. Therefore, using strict quasi-concavity, the marginal willingness to pay of this encroaching consumer for land to the left of \(a_i\) is no more than the marginal willingness to pay of consumer \(i\). So parcels containing points to the left of \(a_i\) will yield lower utility. Now consider the second sub-case, \(a_i < a < 0\). By shifting the parcel to the left, towards the left producer, the quantity of land consumed is the same, and the savings in commuting cost (\(t\) per unit distance) exceed the additional rent, \(p(a) - p(a+s)\). This inequality follows from three facts. First, since we are in the declining rent region, \(p(a) < p(a_i)\). Second, \(p(a + s) > p(0)\), the minimum consumer rent density (recall that \(a + s > l + I\), so \(a + s\) is in a consumer's parcel). Third, \(p(a_i) - p(0) = t\), the first order condition of consumer \(i\) with respect to \(a\). Thus, a shift to the left increases utility and we conclude that it suffices to consider \(a + s \leq l + I\).

Summarizing, and using the fact that very small consumer parcels will only be located on the left of the firm's parcel to save commuting cost, the only choices that might be optimizing and yielding higher utility than equilibrium utility for any consumer are:

for \(s < 2I\), \((l - I, l - I + s)\)

for \(2I \leq s \leq l + I + s_i\) (or \(a_i \leq a \leq l - I\)), \((a, l + I)\).

In the first case, by assumption, \(l \geq 2I^2 + I\), \(s < 2I \leq (l - I)/I \leq s_i\). If the encroaching consumer has a greater utility level than consumer \(i\), then we reduce his composite good consumption until the utility levels are the same. By strict quasi-concavity, the marginal willingness to pay for land is greater for the encroaching consumer. By the first order conditions the rent density he faces on the right hand side of his parcel must exceed that of consumer \(i\). This contradicts the construction of the rent schedule.

In the second case the parcel is \((a, l + I)\). If \(a > 0\), let us compare this parcel
to an alternative parcel, \((a - 2I, l - 1)\), that is the same size but just does not encroach on the producer. Since \(a > 0\) and the alternative parcel does not encroach, the consumer saves at least \((l - 1)t\) in commuting cost by moving to the alternative, which is adjacent to a producer. An upper bound on the additional cost of land is the difference between the maximal and minimal prices of land over a parcel of size \(2I\),
\[2I(1-I)t + f(l/2)/(l + 1).\]
Notice that the assumption \(f(l/2)/(2I(l + 1)) < t\) implies \(f(l/2)/(t(l + 1)) < 2I\). Hence, by assumption, \(l \geq 2I^2 + 1 = 2I^2 - I + 2I > 2I^2 - 1 + f(l/2)/(t(l + 1))\). Hence \((l - 1)t > 2lt(l - 1) + f(l/2)/(l + 1)\).

Summarizing, the alternative parcel (that does not encroach on a producer), \((a - 2I, l - 1)\), is the same size as the original parcel, \((a, l + 1)\), and after paying for commuting cost, there is at least as much consumption good remaining. Thus, the only parcel choices that might be optimizing and yielding higher utility than equilibrium utility are \((a, l + 1)\) where \(a_i \leq a \leq 0\).

If \(a_i \leq a \leq 0\), then the amount of land purchased exceeds \(l - 1\), hence \(s_i\), and therefore the marginal willingness to pay for land is less than \(p(2I)\). Hence the consumer must therefore be willing to purchase more land, beyond the point \(0\), only if
\[
\int_{l-I}^{l+I} p(x) \, dm(x) + \int_{l-I}^{l+I} p(x) \, dm(x) \leq \int_{l-I}^{l+I} p(2I) \, dm(x) + \int_{l-I}^{l+I} p(2I) \, dm(x) \text{ or}
\]
\[
\int_{l-I}^{l+I} [p(x) - p(2I)] \, dm(x) \leq 2If[l/(2l) - 1/4]/(l + 1) .
\]

Next, we contradict this inequality by using our assumptions. Focus on consumer 1, a consumer adjacent to a firm and who consumes the smallest parcel of land among all consumers. Now if \(c_1 \geq \bar{c}\) and \(s_1 \leq \bar{s}\), then the final assumption made for this example, on the marginal rate of substitution, normality of both goods, and equation \((12)\) imply that the price faced by consumer 1 in equilibrium\(^{14}\) is less than that

\(^{14}\text{Notice that in the equilibria we construct, the price function is constant on consumer 1's parcel. The details of the argument in the body of the proof are as follows. As}\)
consumer's marginal willingness to pay for land, so this could not be part of an exchange economy equilibrium allocation. In essence, the assumption on the marginal rate of substitution ensures that the smallest parcel owned by a consumer in equilibrium is bounded below in size, so that the additional cost incurred in owning the land, say, between 0 and \( l\), is at least \( t \) times this length, and thus bounded below. If this parcel is large enough, then this cost is not offset by the discounted price available on the consumer parcel \((l-1, l+1)\). Two possibilities remain: \( s_1 > \bar{s} \) or \( c_1 < \bar{c} \). If \( s_1 > \bar{s} \), then \( s_1 > 4f[l/(2l) - 1/4]/[(l + 1)(l - 1)t] \), so \( s_1(l-1)t > 4f[l/(2l) - 1/4](l + 1) \). Now \( s_2(l - 2)t \geq s_1(l - 2)t, ..., s_{l-1}t \geq s_1t \). Summing these inequalities and using \( 1 + 2 + ... + I-1 = (I-1)I/2 \), we obtain \( \int_0^{l-1} (p(x) - p(2l)) \, dm(x) > s(I-1)It/2 = 2It[l/(2l) - 1/4](l + 1) \), contradicting inequality (13).

Now consider the remaining case, \( c_1 < \bar{c} \) and \( s_1 < \bar{s} \). We consider two separate sub-cases.

If \( p(2l) < f[l/(2l) - 1/4]/(l + 1) \), then by equation (12), \( w^* = \beta - f/(2l) \). Note that an upper bound for transport cost on \([0,l-1]\) is obtained when all \( s_i = (l-1)/I \):

\[
t(l - I)/I + ... + t(l - 1)(l - 1)/I = t(I - 1/I)(l - I)/(2l).
\]

Similarly, an upper bound for rent on \([0,l-1]\) is obtained when all \( s_i = (l-1)/I \):

\[
p(2l)(l - I) + (I - 1)t(l - I)/I + (I - 1)t(l - I)/I + ... + t(l - 1)/I = p(2l)(l - I) + (I - 1)(1 + 1/2)t(I - I)/I.\]

Subtracting, a lower bound for mean consumption is \( \beta - f/(2l) - p(2l)(l - I) - (I - 1)(1 + 1)(l - I)t/I^2 > \beta - f/(2l) - f[l/(2l) - 1/4](l - 1)/[l(l + 1)] - (1 - 1/l^2)(l - I)t \geq \bar{c} \) by definition of \( \bar{c} \).

\( c_1 \geq \bar{c} \), the marginal rate of substitution of land relative to the consumption good is higher at \( c_1 \) than at \( \bar{c} \) because land is normal. As \( s_1 < \bar{s} \), the marginal rate of substitution of land relative to the consumption good is higher at \( s_1 \) than at \( \bar{s} \) because the consumption good is normal.
If \( p(2l) \geq f[l/(2l) - 1/4]/(l + 1) \), use the lower bounds for transport cost and rent on \([0, l - 1)\): 
\[
\text{ts}_1 + \ldots + t(l-1)s_1 = t(I-1)s_1/2 \quad \text{and} \quad p(2l)(l - 1) + (I - 1)ts_1 + \ldots + ts_1 = p(2l)(l - 1) + (I - 1)Its_1/2,
\]
respectively. Then using \( c_1 \) (the consumption of the first consumer) as a lower bound on the consumption on the interval \([0, l - 1)\), 
\[
c_1 + p(2l)(l - 1) + (I - 1)Its_1 \leq lw^* = Ic_1 + I[p(2l) + (I - 1)t]s_1.
\]
Hence \( c_1 \geq [p(2l)(l - 1) - Ip(2l)s_1]/(I - 1) \geq f[l/(2l) - 1/4](l - 1 - Is)/(l + 1)(I - 1) \geq \bar{c} \) by definition of \( \bar{c} \).

Hence, in either sub-case, we contradict \( c_1 < \bar{c} \).

Thus when transport costs are taken into account, the willingness to pay of a consumer for any land occupied by a producer falls short of the cost. A consumer purchasing land used by a producer will have utility lower than a consumer furthest away from a producer. Since all consumers are at the same utility level in equilibrium, such a purchase would reduce the utility level of the consumer, and therefore will not be made.

With regard to the firms, notice that optimization will imply that the labor input quantity will be set equal to the land input quantity, and optimization problem (2) reduces to:
\[
\max \beta \cdot \sigma - f - \int_{b}^{b+\sigma} p(x) \, dm(x) - w^* \cdot \sigma
\]

The first order condition with respect to \( \sigma \) is \( \beta - w^* = p(b + \sigma) \in [p(2l), p(l+1)] \). Marginal revenue net of labor cost equals the marginal cost of land. Since there is a discontinuity in the price of land, this net marginal revenue need only be between the bounds of the discontinuity. \( w^* \) was chosen to satisfy this condition for \( b_1 = -l-I, \sigma_1 = 2l, b_2 = l-I, \sigma_2 = 2l \). The first order condition with respect to \( b \) is \( p(b) = p(b + \sigma) \); this is fulfilled by symmetry. Equilibrium profits are zero by construction of \( w^* \); see equation (12).

Turning next to second order conditions for the firm, notice first that if the firm
uses a parcel of any size $\sigma$, it is indifferent about its location, so it will choose one of the cheapest parcels. For $\sigma \leq 2I$, these are contained in $(b_1, b_1 + \sigma_1), (b_2, b_2 + \sigma_2)$. The first order condition with respect to $\sigma$ will imply that it will choose $\sigma = 2I$. If it occupies a parcel at an extreme of $X$ and $\sigma$ is slightly larger than $2I$, then the cost of this parcel is higher than the cost of a similarly slight extension of $(b_1, b_1 + \sigma_1)$ or $(b_2, b_2 + \sigma_2)$. If the firm can make higher profits from expanding the scale of its operations beyond $2I$, then given the production function and the price density, it will make still higher profits when $b = -2l$ and $\sigma = 4l$.

Profits from such a production plan are given by

$$4bI - f - w^* \cdot 4l - \int_{-2l}^{2l} p(x) \, dm(x)$$

Profits from the equilibrium production plan are zero by construction of $w^*$. Substituting this into equation (14) by using the definition of $w^*$ given by equation (12), after some tedious calculations, non-positivity of (14) is equivalent to

$$\int_{0}^{l-I} [p(x) - p(2B)] \, dm(x) \geq 0.$$ The integrand is non-negative by construction.

Q.E.D.
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Figure 1

Class I Equilibrium
Figure 2

Class II Equilibrium
Multiple Consumers - Class I
Equilibrium with Multiple Consumers and 2 firms
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