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Output Dynamics, Technology, and Public Investment*

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Abstract

The paper studies the dynamic output effects of public infrastructure investment in a small open economy. We develop an overlapping generations model that includes a production externality of public capital and a wealth effect on labor supply. Public capital enters the firm's production function under various technological scenarios. We show that if factors of production are gross complements and public capital is Solow neutral, which is the empirically plausible case, the long-run output multiplier falls short of its Hicks-neutral value. The way in which public capital augments factor productivity crucially affects the dynamics of private capital and net foreign assets, but yields qualitatively similar output dynamics. In contrast to conventional results obtained from hysteretic models, we find non-monotonic output dynamics of a public investment impulse in the non-hysteretic model. Schmitt-Grohe and Uribe's (2003) finding of identical impulse responses across the two model types is thus not robust to the inclusion of spillovers of public capital.

JEL codes: E62, F41, H54
Keywords: Infrastructure capital, public investment, fiscal policy, output multipliers, transitional dynamics, technology

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1 Introduction

To address the growing economic and financial crisis, many industrialized countries have adopted fiscal stimulus measures. Most countries increased public spending on infrastructure, in particular transportation networks (e.g., highways, mass transit, and airports).[1] These developments have revived interest in the debate on the macroeconomic effects of public infrastructure investment. The present paper contributes to this literature by analyzing the short-run, transitional, and long-run output effects of public investment for a small open economy. More specifically, we analyze how various assumptions on the firm's production technology and the household's labor supply response affect the impulse responses and the size of long-run output multipliers of public investment.

The notion that public capital generates beneficial spillover effects to the private sector is widely accepted in the empirical literature.[2] The theoretical literature on the dynamic allocation effects of public spending on infrastructure is less well developed. Most contributions employ a Ramsey framework in which households are infinitely lived. Baxter and King (1993) and Turnovsky and Fisher (1995), for instance, employ closed-economy models with inelastic labor supply to analyze transitional and welfare effects of public investment shocks under both distortionary and non-distortionary financing scenarios. Fisher and Turnovsky (1998) and Rioja (1999) explicitly focus on the effects of public capital congestion.[3] The open economy implications of public investment are little researched. A notable exception is the finite-horizon model of Heijdra and Meijdam (2002), to which our work is most closely related.

Public capital gives rise to positive spillover effects in private production and therefore may enter the firm's production function in two ways: (i) as a separate input (direct effect); and (ii)

[1]Public infrastructure capital is defined to include, among others, highways, railways, airports, sewerage and water systems, dams and other flood control structures, and lighthouse services. Public capital in a broad sense also includes hospitals and educational buildings and other public buildings.

[2]Aschauer's (1989) seminal paper—which estimates an output elasticity of public capital of 0.39—gave a strong boost to the empirical literature on public capital. The meta-analysis by Bom and Ligthart (2008) finds estimates of the output elasticity of public capital in the range 0.08–0.15.

[3]A second strand of literature considers models in which the assumption of constant returns to scale to reproducible factors of production generates endogenous growth. Barro (1990) and Glomm and Ravikumar (1994, 1997) use endogenous growth models featuring infinitely-lived households without a leisure-labor choice to derive the conditions of optimal fiscal policy along the balanced growth path. Chatterjee (2008) focuses rather on the short-run and long-run interhousehold distributional effects of public investment.

via the index of factor productivity (indirect effect). The majority of studies, however, assume a Cobb-Douglas production function, in which case the direct and indirect effects cannot be disentangled.[4] More important, public capital is always assumed to enter production in a Hicks-neutral fashion, that is, it affects labor and capital productivity to the same extent. The change in factor productivity in the US economy during the 20th century has neither been Hicks neutral nor has it been Harrod neutral (i.e., labor augmenting).[5] In addition, the elasticity of substitution between private capital and labor is not necessarily unity. Empirical evidence on this elasticity is mixed, ranging from 0 to 3.4 with the majority of estimates falling into the range of 0.40–0.60 (cf. Chirinko, 2008). By choosing a more general functional form, which embeds various elasticities of substitution, we are able to meaningfully analyze the factor-augmenting role of public capital. We distinguish between Harrod-neutral public capital and Solow-neutral (capital-augmenting) public capital. To our knowledge, the factor-augmenting role of public capital has not been analyzed in a dynamic macroeconomic context yet.[6]

We develop a microfounded dynamic macroeconomic model for a small open economy. The household sector extends the Yaari (1965)–Blanchard (1985) framework of overlapping generations—which assumes that households face a constant probability of death—by modeling endogenous labor supply.[7] This extension is not only important in view of the emphasis in the Real Business Cycle literature on the *inter*temporal labor supply effect for shock propagation (cf. Prescott, 2006), but also because recent studies have demonstrated its empirical relevance (e.g., Kimball and Shapiro, 2008). The model features an internationally traded bond, ensuring that households can use the current account of the balance of payments to smooth private consumption. Firms operate under perfect competition and enjoy production spillovers from public capital. The presence of public capital yields a suboptimal market outcome, providing a justification for government intervention. To limit the international

[4]Because the Cobb-Douglas function yields constant output elasticities of inputs, the literature has strongly focused on this case (cf. Bom and Ligthart, 2008). Otto and Voss (1998) is a notable exception.

[5]See David and Van de Klundert (1965) and Boskin and Lau (2000).

[6]Feehan (1998) touches upon the issue, but employs a static two-factor, two-goods model of a small open economy. In addition, he focuses on the flow rather than the stock of infrastructure.

[7]Heijdra and Meijdam (2002) neither model endogenous labor supply nor take into account different production technologies.

mobility of physical capital, and thus to avoid trivial capital dynamics, we postulate adjustment costs of both private and public investment. The government balances its budget by employing lump-sum taxes.

Our paper develops a simple graphical framework to analyze the qualitative steady-state effects of a public investment shock. The framework is versatile because it incorporates the key specifications employed in the literature (e.g., exogenous labor supply, Hicks-neutral public capital, a Cobb-Douglas production technology, and infinitely-lived households) as special cases. To get insight into the quantitative effects of public investment shocks over time, we numerically simulate the model using empirically plausible parameter values taken from the literature on small open economy models. We go beyond the standard practice of using numerical impulse response functions by deriving analytical expressions for the transition paths.

Infinite-horizon models of a small open economy typically suffer from the knife-edge property (i.e., the rate of interest should equal the pure rate of interest for a meaningful steady state to exist), thus yielding hysteresis. Introducing Yaari-Blanchard overlapping generations is a convenient way to arrive at an endogenously determined (non-hysteretic) steady state. Besides serving this technical objective, overlapping generations provide a realistic description of the demographic structure of the household sector. A key question is whether our hysteresis-eliminating device affects the impulse responses of a fiscal shock. Therefore, another objective of our paper is to compare the impulse responses in the non-hysteretic model with those found in the hysteretic version of our model. More specifically, we investigate the robustness of Schmitt-Grohe and Uribe's (2003) central result, that is, non-stationary and stationary models yield virtually identical impulse responses.[8] Schmitt-Grohe and Uribe (2003), however, neither incorporate externalities in their model nor employ overlapping generations as a stationarity-inducing device.

We show that while the type of factor-augmenting public capital does not matter for the sign of the output multiplier, it affects its size. If public capital is Solow neutral and factors of

[8]Schmitt-Grohe and Uribe (2003) employ a dynamic stochastic general equilibrium model for a small open economy and therefore speak of stationary and non-stationary models. In our deterministic setting, this terminology corresponds to non-hysteretic and hysteretic models.

production are gross complements, which is the empirically relevant case, the long-run output multiplier falls short of its Hicks-neutral value. Conversely, the Solow-neutral case yields a long-run output multiplier that exceeds the Hicks-neutral value if factors of production are gross substitutes. Harrod-neutral public capital always yields a long-run output multiplier equal to that found in the Hicks-neutral scenario. Second, the way in which public capital affects factor productivity crucially matters for the dynamics of private capital and net foreign assets, but yields qualitatively similar output dynamics. Finally, we show that the impulse responses of a public investment shock in the finite-horizon model are very different from those found in the infinite-horizon model. The combination of a public capital externality, endogenous labor supply, and finite horizons gives rise to non-monotonic output dynamics. If any of these three elements is dropped from the analysis, we find the conventional result of monotonic transition paths. Schmitt-Grohe and Uribe's (2003) result is not robust to the inclusion of production externalities and is therefore not as generally valid as suggested.

The paper is organized as follows. Section 2 develops a dynamic macroeconomic framework for a small open economy in which public capital enters the production function in a factor-augmenting fashion. Section 3 studies the steady state and develops a graphical framework. Section 4 analyzes analytically and graphically the long-run effects of a public investment shock financed by lump-sum taxes. Section 5 studies numerically the dynamic macroeconomic effects of an unanticipated and permanent increase in public investment. Section 6 summarizes and concludes.

2 The Model

Consider a small open economy populated by overlapping generations of finitely-lived households and infinitely-lived representative firms. The household section of the model extends Yaari (1965) and Blanchard (1985) by incorporating an endogenous labor-leisure choice along the lines of Heijdra and Ligthart (2007). Firms enjoy positive spillover effects from the stock of public infrastructure capital.

2.1 Households

Individual households face a constant probability of death $\beta \geq 0$, which is assumed to equal the rate at which new agents are born. Because population growth is absent, the size of the population can be normalized to unity. Households are disconnected and therefore do not leave bequests. Efficient financial markets allow households to borrow and lend at the exogenously given world rate of interest (denoted by r) adjusted for the probability of death.

The utility functional at time t of a household born at time $v \leq t$ is:

$$\Lambda(v,t) \equiv \int_t^\infty \ln U(v,\tau) e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{1}$$

where α is the pure rate of time preference and the sub-utility index $U(v,t)$ is a Cobb-Douglas utility index defined over private consumption $C(v,t)$ and leisure $1 - L(v,t)$:

$$U(v,t) \equiv C(v,t)^{\varepsilon_C} \left[1 - L(v,t)\right]^{1-\varepsilon_C}, \qquad 0 < \varepsilon_C < 1, \tag{2}$$

where $L(v,t)$ denotes labor supply. Note that total time available to the household has been normalized to unity. Equation (1) implies a unitary intertemporal elasticity of substitution and (2) imposes a unitary intratemporal elasticity of substitution between private consumption and leisure. The household's flow budget constraint is:

$$\dot{A}(v,t) = (r+\beta)A(v,t) + w(t)L(v,t) - T(t) - C(v,t), \tag{3}$$

where $\dot{A}(v,t) \equiv dA(v,t)/dt$, $A(v,t)$ denotes real financial wealth, $w(t)$ is the (age-independent) real wage rate, and $T(t)$ are lump-sum taxes. Private consumption is used as numeraire commodity whose price has been normalized to unity. Households can contract actuarially fair 'reverse' life insurance (cf. Blanchard, 1985), implying an effective return on financial wealth equal to $r + \beta$.

The representative household of cohort v, who is endowed with perfect foresight, maximizes lifetime utility (1)–(2) subject to its budget identity (3) and a no-Ponzi game solvency condition. We solve the household's problem by two-stage budgeting. In the first stage, the household decides on its consumption over time, yielding the Euler equation for individual 'full' consumption:[9]

$$\frac{\dot{X}(v,t)}{X(v,t)} = r - \alpha > 0, \tag{4}$$

where full consumption is defined as:

$$X(v,t) \equiv w(t)\left[1 - L(v,t)\right] + C(v,t). \tag{5}$$

We study the case of a patient nation (i.e., $r > \alpha$), which yields rising individual consumption profiles. In the second stage, full consumption is allocated over private consumption and leisure. The first-order conditions yield an expression for the consumption-leisure ratio: $\frac{C(v,t)}{1-L(v,t)} = \frac{\varepsilon_C}{1-\varepsilon_C} w(t)$. Together with (5), this expression gives rise to demand functions for goods consumption and leisure:

$$\begin{aligned} C(v,t) &= \varepsilon_C X(v,t), &(6)\\ 1 - L(v,t) &= (1-\varepsilon_C)\frac{X(v,t)}{w(t)}. &(7) \end{aligned}$$

Variables at the aggregate level can be calculated as a weighted sum of the values for different generations. For example, aggregate financial wealth is $A(t) \equiv \int_{-\infty}^{t} A(v,t)\beta e^{\beta(v-t)} dv$, where $\beta e^{\beta(v-t)}$ is the size of cohort v at time t. By aggregating (4) over all existing generations, we arrive at aggregate full consumption:

$$\begin{aligned} \frac{\dot{X}(t)}{X(t)} &= r - \alpha - \beta(\alpha+\beta)\frac{A(t)}{X(t)} \\ &= \frac{\dot{X}(v,t)}{X(v,t)} - \beta \cdot \frac{X(t) - X(t,t)}{X(t)}. &(8) \end{aligned}$$

The second line of equation (8) says that aggregate consumption growth equals individual consumption growth (the first term) minus the 'generational turnover effect' (the second term), that is, the wealth redistribution caused by the passing away of generations. Intuitively, old generations have accumulated wealth over the course of their life, whereas new generations

[9]Further details on the mathematical derivations can be found in Bom, Heijdra, and Ligthart (2010).

are born without financial wealth (i.e., $A(t,t)=0$). Consequently, the consumption level of new generations $X(t,t)$ falls short of the average consumption level $X(t)$.[10]

2.2 Firms

The representative firm produces a homogeneous good $Y(t)$ under perfect competition. Technology is described by a constant elasticity of substitution function, which is linearly homogeneous in private capital $K(t)$ and labor $L(t)$. The public capital stock $K_G(t)$ enters private production in a factor-augmenting fashion:

$$
\begin{aligned}
Y(t) &= Y\left[K(t), L(t), K_G(t)\right] \\
&= \left[\varepsilon_Y \left[A_K(t)K(t)\right]^{\frac{\sigma_Y-1}{\sigma_Y}} + (1-\varepsilon_Y)\left[A_L(t)L(t)\right]^{\frac{\sigma_Y-1}{\sigma_Y}}\right]^{\frac{\sigma_Y}{\sigma_Y-1}}, \qquad (9)
\end{aligned}
$$

where $0 < \sigma_Y \ll \infty$ is the elasticity of substitution between private capital and labor, $0 < \varepsilon_Y < 1$ is the efficiency parameter of capital, and $A_K(t)$ and $A_L(t)$ are technology functions:

$$
A_i(t) \equiv \rho_i K_G(t)^{\eta_i}, \qquad i = \{K, L\}, \qquad (10)
$$

where η_i represents the elasticity of the technology function and $\rho_i > 0$ is a scaling factor. Hicks-neutral public capital can be represented by $\eta_K = \eta_L = \eta > 0$,[11] Harrod-neutral (or labor-augmenting) public capital assumes $\eta_L > \eta_K = 0$, and Solow-neutral (or capital-augmenting) public capital is described by $\eta_K > \eta_L = 0$.[12] If $\sigma_Y = 1$, then (9) collapses to a Cobb-Douglas function, in which case the distinction between the various types of factor-augmenting public capital is immaterial.

Equation (9) incorporates a public capital externality, which gives rise to $\theta_K(t) + \theta_L(t) + \theta_G(t) > 1$, where $\theta_j(t) \equiv \frac{\partial Y(t)}{\partial j(t)} \frac{j(t)}{Y(t)} > 0$ represents the output elasticity of factor $j =$

[10] We use $X(t) = (\alpha+\beta)\left[A(t) + H(t)\right]$ and $X(t,t) = (\alpha+\beta)H(t)$, where $H(t)$ is 'full' human wealth, that is, the after-tax value of the household's time endowment: $H(t) \equiv \int_t^\infty \left[w(\tau) - T(\tau)\right] e^{(r+\beta)(t-\tau)} d\tau$.

[11] Equation (10) boils down to $A_L(t) = A_K(t) = \rho K_G(t)^\eta$, where $\rho_L = \rho_K = \rho$. If we also set $\sigma_Y = 1$, equation (9) reduces to Heijdra and Meijdam's (2002) production technology.

[12] Of course, intermediate cases such as $\eta_K > 0$ and $\eta_L > 0$ (with $\eta_K \neq \eta_L$) are feasible. Here, we focus on the pure types of factor-augmenting public capital only.

$\{K, L, K_G\}$.[13] The output share of public capital can be written as

$$\theta_G(t) = \theta_K(t)\eta_K + \theta_L(t)\eta_L. \tag{11}$$

To ensure diminishing returns with respect to broad capital (and thus exclude endogenous growth), we impose $\eta = \theta_G < 1 - \theta_K$ in the case of Hicks-neutral public capital. The Solow-neutral case requires $\theta_K(1+\eta_K) < 1$, whereas no conditions are needed for the Harrod-neutral case. For plausible parameter combinations, these conditions are easily met (see Section 5.1).

To avoid trivial capital dynamics, we introduce adjustment costs in private investment. Net capital formation is linked to gross investment $I(t)$ according to a concave accumulation function (cf. Uzawa, 1969):

$$\dot{K}(t) = \left[\Phi\left(\frac{I(t)}{K(t)}\right) - \delta\right] K(t), \tag{12}$$

where δ is the (constant) rate of capital depreciation and $\Phi(\cdot)$ is the installation cost function of private capital accumulation, which satisfies $\Phi(0) = 0$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$ (where primes denote derivatives). The degree of physical capital mobility of private capital is given by $0 < \sigma_A \equiv -\frac{I}{K}\frac{\Phi''(\cdot)}{\Phi'(\cdot)} \ll \infty$. A small σ_A—representing a less concave installation cost function—characterizes a high degree of physical capital mobility.

The representative firm is infinitely lived and maximizes the net present value of its cash flow:

$$V(t) \equiv \int_t^\infty [Y(\tau) - w(\tau)L(\tau) - I(\tau)]\, e^{r(t-\tau)} d\tau, \tag{13}$$

subject to the capital accumulation constraint (12) and the economy-wide stock of public capital. The prices of output and investment goods are normalized to unity. To allow for meaningful production spillovers, we require that the government cannot charge a user fee on

[13] Some authors assume constant returns to scale across all inputs (e.g., Aschauer, 1989) with a view to model congestion effects. In our context, however, public capital is modeled as a pure public good.

the services of public capital, for example, road toll fees. The optimality conditions are:

$$\begin{aligned} w(t) &= Y_L\left[K(t), L(t), K_G(t)\right], && (14)\\ 1 &= q(t)\Phi'\left(\frac{I(t)}{K(t)}\right), && (15)\\ \frac{\dot{q}(t)}{q(t)} &= r + \delta - \Phi\left(\frac{I(t)}{K(t)}\right) + \frac{I(t)}{q(t)K(t)} - \frac{Y_K\left[K(t), L(t), K_G(t)\right]}{q(t)}, && (16) \end{aligned}$$

where $q(t)$ denotes Tobin's q, which is defined as the market value of private capital relative to its replacement costs, and $Y_j(t) \equiv \frac{\partial Y(t)}{\partial j(t)} > 0$ represents the marginal productivity of factor $j = \{K, L, K_G\}$. Equation (14) describes labor demand, (15) represents investment demand, and (16) shows the evolution of Tobin's q.[14] The ratio of marginal products of the private factors of production is given by:

$$\frac{Y_K(t)}{Y_L(t)} = \frac{\varepsilon_Y}{1-\varepsilon_Y}\left(\frac{\rho_K}{\rho_L}\right)^{\frac{\sigma_Y-1}{\sigma_Y}} K_G(t)^{\frac{(\eta_K-\eta_L)(\sigma_Y-1)}{\sigma_Y}}\left(\frac{K(t)}{L(t)}\right)^{-1/\sigma_Y}. \qquad (17)$$

If private production factors are *gross substitutes* (i.e., $\sigma_Y > 1$) and public capital is capital-augmenting (i.e., $\eta_K - \eta_L > 0$), an increase in $K_G(t)$ increases the relative marginal product of private capital. Thus, public capital is biased toward private capital. If private factors are *gross complements* (i.e., $\sigma_Y < 1$) and $\eta_K - \eta_L > 0$, an increase in $K_G(t)$ increases the relative marginal product of labor. Thus, public capital is biased toward labor.

2.3 Government and Foreign Sector

The government invests in public capital $I_G(t)$ and consumes goods $C_G(t)$. To focus solely on spillovers of public investment on the production side, we assume that public consumption does not give rise to spillovers on the consumption side. Total public spending is financed by lump-sum taxes, implying that $C_G(t) + I_G(t) = T(t)$ holds at each instant of time. Just like

[14] Without adjustment costs, we have $\Phi(\cdot) = I(t)/K(t)$ and $\Phi'(\cdot) = 1$, which yields $\sigma_A = 0$. Equation (15) then reduces to $q = 1$. In this case, $K(t)$ adjusts instantaneously to its steady-state level. Consequently, equation (16) reduces to $Y_K = r + \delta$, which is the familiar rental rate derived in a static framework.

private capital, public capital accumulates according to a concave function:

$$\dot{K}_G(t) = \left[\Phi_G\left(\frac{I_G(t)}{K_G(t)}\right) - \delta_G\right] K_G(t), \qquad 0 < \sigma_G \equiv \frac{I_G \Phi_G'(\cdot)}{K_G} \ll \infty, \tag{18}$$

where δ_G is the rate of depreciation of public capital and $\Phi_G(\cdot)$ is the installation cost function of public capital, which satisfies $\Phi_G(0) = 0$, $\Phi_G'(\cdot) > 0$, and $\Phi_G''(\cdot) < 0$. The parameter σ_G represents the elasticity of the public capital installation cost function.

Foreign financial capital is perfectly mobile across borders. The change in net foreign assets $\dot{F}(t)$ follows from the current account of the balance of payments: $\dot{F}(t) = rF(t) + Z(t)$, where $rF(t)$ denotes the return on net foreign assets and net exports are given by: $Z(t) \equiv Y(t) - C(t) - C_G(t) - I(t) - I_G(t)$.

2.4 Market Equilibrium

The domestic labor market clears at each instant of time. Similarly, goods markets do not feature any rigidities, so that $Y(t) = C(t) + C_G(t) + I(t) + I_G(t) + Z(t)$. Portfolio equilibrium amounts to $A(t) = V(t) + F(t)$, where $V(t) = q(t)K(t)$ denotes the firm's stock market value. Assets in the household's portfolio are assumed to be perfect substitutes. Initially, $A(0) = V(0) > 0$ because $K(0) > 0$ and $F(0) = 0$, implying that the trade account of the balance of payments is initially balanced; physical capital is thus fully domestically owned.

3 Solving the Model

This section studies the steady state and develops a graphical framework to analyze the dynamic effects of a public investment impulse.

3.1 Steady State

To solve the model, we log-linearize it around an initial steady state in which $F(0) = 0$. A tilde ($\tilde{}$) denotes a relative change, for example, $\tilde{X}(t) \equiv dX(t)/X_0$, where X_0 denotes the initial steady-state value of full consumption.[15] Notable exceptions are financial assets

[15]We use the subscript 0 to denote the initial steady-state value of a given variable.

and lump-sum taxes. For financial assets $A(t)$ and $F(t)$, we use $\tilde{A}(t) \equiv rdA(t)/Y_0$ and $\dot{\tilde{A}}(t) \equiv rd\dot{A}(t)/Y_0$, whereas the change in lump-sum taxes is scaled by steady-state output Y_0 only, that is, $\tilde{T}(t) \equiv dT(t)/Y_0$.

The reduced-form dynamic model contains two predetermined variables (i.e., the private capital stock and financial assets) and two non-predetermined variables (i.e., Tobin's q and full consumption). By collecting variables in the vector $\tilde{\mathbf{z}}(t) \equiv [\tilde{K}(t)\ \tilde{q}(t)\ \tilde{X}(t)\ \tilde{A}(t)]'$ and shock terms in the vector $\mathbf{\Gamma}(t) \equiv [0\ \gamma_q(t)\ 0\ \gamma_A(t)]'$ we can write the dynamic system as:

$$\dot{\tilde{\mathbf{z}}}(t) = \Delta\tilde{\mathbf{z}}(t) - \mathbf{\Gamma}(t), \tag{19}$$

where the Jacobian matrix (with element $\bar{\delta}_{ij}$) on the right-hand side of (19) is given by:

$$\Delta \equiv \begin{bmatrix} 0 & \frac{r\omega_I}{\sigma_A\omega_A} & 0 & 0 \\ \frac{r\theta_K}{\sigma_Y\omega_A}(1-\xi_{yk}) & r & -\frac{r\theta_K}{\sigma_Y\omega_A}\xi_{yx} & 0 \\ 0 & 0 & r-\alpha & -\frac{r-\alpha}{\omega_A} \\ r\omega_w\xi_{wk} & 0 & r(\omega_w\xi_{wx}-\omega_X) & r \end{bmatrix},$$

where $\omega_A \equiv rA_0/Y_0$ denotes the output share of asset income, $\omega_I \equiv I_0/Y_0$ is the output share of private investment, $\omega_w \equiv w_0/Y_0$ is the output share of wages, $\omega_X \equiv X_0/Y_0$ denotes the output share of full consumption, and $\xi_{yk} > 0$, $\xi_{yx} < 0$, $\xi_{wk} > 0$, and $\xi_{wx} > 0$ are defined in Appendix A.1. The policy shock terms are given by:

$$\gamma_q(t) \equiv \frac{r\theta_K}{\sigma_Y\omega_A}\left[\xi_{yg} + (\sigma_Y - 1)\eta_K\right](1 - e^{-\sigma_G t})\tilde{I}_G, \tag{20}$$

$$\gamma_A(t) \equiv -r\left[\omega_w\xi_{wg}(1 - e^{-\sigma_G t}) - \omega_G^I\right]\tilde{I}_G + r\omega_G^C\tilde{C}_G, \tag{21}$$

where $\omega_G^C \equiv C_{G0}/Y_0$, $\omega_G^I \equiv I_{G0}/Y_0$, and $\xi_{yg} \gtreqless 0$ and $\xi_{wg} \gtreqless 0$ are defined in Appendix A.1. Because we are studying a public investment impulse, we set $\tilde{I}_G > 0$ and $\tilde{C}_G = 0$.

The model has a unique and locally saddle-point stable steady state, featuring two positive real roots and two negative real roots (Appendix A.1). The system (19) embeds various special cases. First, if labor supply is exogenous (i.e., $\bar{\delta}_{23} \equiv -\frac{r\theta_K}{\sigma_Y\omega_A}\xi_{yx} = 0$), the system is recursive, meaning that the investment subsystem $[\tilde{q}(t), \tilde{K}(t)]$ can be solved independent of the savings subsystem $[\tilde{X}(t), \tilde{A}(t)]$. Second, if households are infinitely lived (imposing $\beta = 0$ and thus $r = \alpha$ must hold for a steady state to exist) then the third row of Δ consists of zeros only. The four characteristic roots in the infinite-horizon case are: $-h_1^* = (r - \sqrt{r^2 + 4\bar{\delta}_{12}\bar{\delta}_{21}})/2 < 0$, $r_1^* = r$, $h_2^* = 0$, and $r_2^* = (r + \sqrt{r^2 + 4\bar{\delta}_{12}\bar{\delta}_{21}})/2 > 0$. A hysteretic steady state is obtained if a zero root is present.

3.2 Graphical Framework

Figure 1 graphically summarizes the model's long-run equilibrium, which is simultaneously determined in four panels. The economy is initially at the steady-state equilibrium E_0. Panel (a) shows the equilibrium on the private capital market. The long-run supply of private capital $K^s(r)$ is perfectly elastic in a small open economy, and can thus be graphically represented by a horizontal line. Using the steady-state versions of (12) and (16), the long-run capital demand curve $K^d(r; L_0, K_{G0}, q_0)$—given $L_0 > 0$, $K_{G0} > 0$, and $q_0 > 0$—can be derived as:

$$r = \frac{1}{q_0}\left[Y_K(K, K_{G0}, L_0) - \Phi^{-1}(\delta)\right], \tag{22}$$

which is downward sloping since $\partial Y_K / \partial K < 0$. The intersection of capital supply $K^s(r)$ and capital demand $K^d(r; L_0, K_{G0}, q_0)$ determines the initial steady-state level of capital $K(0) = K_0 > 0$. The effect of public capital on private capital demand is given by:

$$\frac{\partial Y_K}{\partial K_G} = \frac{Y_K}{\sigma_Y K_G}[\theta_G + \eta_K(\sigma_Y - 1)]. \tag{23}$$

Hence, an increase in public capital shifts the capital demand curve to the right if $\theta_G + \eta_K(\sigma_Y - 1) > 0$. In the Solow-neutral case, this condition implies that $\sigma_Y > \theta_L$. We label the shift in the capital demand curve the *capital-productivity effect*, which consists of the *pure externality effect* θ_G and the *Solow-substitution effect* $\eta_K(\sigma_Y - 1)$. Only for $\eta_K > 0$ and $\sigma_Y \neq 1$ does the Solow substitution effect play a role; its value is small for complementary private inputs. Because $\partial Y_K / \partial L > 0$, the capital demand curve always shifts to the right if employment increases.

Panel (b) depicts the equilibrium in the labor market. For a given level of full consumption, the upward-sloping labor supply curve $L^s(w; X_0)$ is given by:

$$w = \frac{(1-\varepsilon_C)X_0}{1-L}, \tag{24}$$

which follows from (7) in aggregate form. A rise in full consumption induces households to work less—the *wealth effect* on labor supply—and thus shifts the labor supply curve to left, thereby pushing wages up and depressing employment. The labor demand curve $L^d(w; K_0, K_{G0})$ is derived from the marginal productivity of labor (14):

$$w = Y_L(L, K_0, K_{G0}), \tag{25}$$

which is downward sloping because $\partial Y_L/\partial L < 0$. For $K = K_0$, $K_G = K_{G0}$, and $X = X_0$, the equilibrium employment level and wage rate are L_0 and w_0, respectively. The effect of public capital on the marginal productivity of labor is given by:

$$\frac{\partial Y_L}{\partial K_G} = \frac{Y_L}{\sigma_Y K_G}[\theta_G + \eta_L(\sigma_Y - 1)]. \tag{26}$$

Thus, an increase in public capital shifts the labor demand curve to right if $\theta_G + \eta_L(\sigma_Y - 1) > 0$, causing wages and employment to rise. We call this the *labor-productivity effect*, which consists of the *pure externality effect* θ_G and the *Harrod-substitution effect* $\eta_L(\sigma_Y - 1)$. Since $\partial Y_L/\partial K > 0$, a larger private capital stock also pushes the labor demand curve to the right.

Panel (d) determines initial full consumption and the stock of assets for given wages and lump-sum taxes. The modified Keynes-Ramsey schedule $X^{kr}(A)$ gives the steady-state relationship between full consumption and financial assets [from (8)]:

$$X = \frac{\beta(\alpha+\beta)}{r-\alpha}A. \tag{27}$$

The $X^{kr}(A)$ schedule is upward sloping because of the generational turnover effect. The variables X and A must also satisfy the steady-state household budget identity $X^{hb}(A; w_0, K_{G0})$

that follows from (3):

$$X = rA + w_0 - C_{G0} - \Phi^{-1}(\delta_G)K_{G0}, \tag{28}$$

where we used the steady-state version of (18) and the government's budget constraint. Given $K_G = K_{G0}$ and $w = w_0$, the intersection of the $X^{kr}(A)$ and $X^{hb}(A; w_0, K_{G0})$ schedules determines the initial steady-state levels of full consumption X_0 and financial assets A_0. A rise in the wage rate shifts the $X^{hb}(\cdot)$ curve up, resulting in a higher equilibrium level of assets.

Finally, Panel (c) shows the production function $Y(\cdot)$, which relates output to private capital given labor and public capital. With initial levels of inputs of L_0, K_0, and K_{G0}, initial output is Y_0. Panel (c) also depicts the line qK. Because $A_0 = qK_0$ (point G), foreign assets in the initial steady state are zero, that is, $F_0 = 0$.

As a result of an investment shock, the value of Tobin's q deviates from $q(0) = q(\infty)$ in the short and medium run. To graphically analyze the dynamics during this time period, we make use of Figure 2. The $\dot{q} = 0$ locus in Panel (a) shows combinations of $q(t)$ and $K(t)$ for which Tobin's q is constant over time; it is also given by (22) with fixed r and variable q. The schedule is downward sloping because a higher capital stock reduces the marginal product of capital and thus yields lower dividends to shareholders. The $\dot{K} = 0$ locus denotes combinations of $q(t)$ and $K(t)$ for which net investment is zero. The schedule is horizontal at the unique value of Tobin's q for which $\Phi(\cdot) = \delta$. Panel (b) illustrates the labor market dynamics by adding the short- and medium-term labor-productivity and wealth effects to Panel (b) of Figure 1. Recall that because the dynamic system is not recursive the dynamic equilibrium paths are simultaneously determined in the investment subsystem [Panel (a)] and saving subsystem (not shown), which are connected by the endogenously determined level of employment (see Section 3.1).

4 Analytical Long-Run Effects of Public Investment

This section analytically investigates the long-run effects of a permanent and unanticipated increase in public investment. The fiscal shock is unanticipated in the sense that it is simultaneously announced and implemented. To keep its budget balanced, the government raises lump-sum taxes.

The increase in public investment expands the steady-state stock of public capital from $K_G(0) = K_{G0}$ to $K_G(\infty) = K_{G\infty}$ via the capital accumulation function:

$$\frac{dK_G(\infty)}{dI_G} = \frac{1}{\Phi_G^{-1}(\delta_G)} > 0. \tag{29}$$

Panel (b) of Figure 1 shows that the increase in public capital has two opposite effects on the labor market. On the one hand, the labor-productivity effect shifts the labor demand curve to the right from $L^d(w; K_0, K_{G0})$ to $L^d(w; K_\infty, K_{G\infty})$, thereby boosting employment and wages. On the other hand, the wealth effect on labor supply causes a leftward shift of the labor supply curve from $L^s(w; X_0)$ to $L^s(w; X_\infty)$, further increasing wages but partially offsetting the increase in employment.[16] The new steady-state equilibrium is denoted by E_∞. The labor-productivity effect always dominates the wealth effect, resulting in a net increase in employment to L_∞ and wages to w_∞:[17]

$$\begin{aligned} \frac{dL(\infty)}{dI_G} &= \omega_{LL}\frac{Y_{G0}}{Y_{L0}}\frac{1}{\Phi_G^{-1}(\delta_G)} \\ &\quad -\frac{\omega_{LL}}{\omega_X}\frac{\theta_L}{w_0}\frac{\beta(\alpha+\beta)}{\beta(\alpha+\beta)-r(r-\alpha)}\left(\frac{Y_{G0}}{L_0}\frac{1}{\Phi_G^{-1}(\delta_G)}-1\right), \qquad (30) \\ \frac{dw(\infty)}{dI_G} &= \frac{Y_{G0}}{L_0}\frac{1}{\Phi_G^{-1}(\delta_G)}, \qquad (31) \end{aligned}$$

where $\omega_{LL} \equiv (1-L_0)/L_0$ is the leisure-labor ratio (or intertemporal substitution elasticity of labor supply).[18] The first term of (30) represents the labor-productivity effect and the second

[16] The wealth effect derives from the fact that—for plausible parameter values—the rise in gross wages dominates the increase in lump-sum taxes necessary to balance the government budget. In Panel (d) of Figure 1, the $X^{hb}(\cdot)$ curve shifts up to $X^{hb}(A; w_\infty, K_{G\infty})$, raising full consumption to X_∞ and financial assets to A_∞.

[17] Using equation (8) in steady state yields $\beta(\alpha+\beta) - r(r-\alpha) = \beta(\alpha+\beta)(\omega_X - \omega_A)/\omega_X > 0$, where $\omega_X - \omega_A > 0$.

[18] Note that the intertemporal substitution elasticity of labor supply is equivalent to the intratemporal

term captures the wealth effect. Since both effects contribute to raise the wage rate, the wage multiplier given by (31) is positive. Of course, in the absence of a public capital externality (i.e., $Y_{G0} = 0$ and thus $\theta_G = 0$), we find a zero effect on wages.

Public capital has a direct effect on private capital demand, which is captured by the capital productivity effect described by (23). Also, because labor and private capital are cooperative factors, public capital has an indirect effect on private capital through its effect on employment. The private capital multiplier is:

$$\begin{aligned} \frac{dK(\infty)}{dI_G} &= \frac{y_G}{\theta_{L} y}\left[\theta_G + \eta_K(\sigma_Y - 1)\right]\frac{1}{\Phi_G^{-1}(\delta_G)} + \omega_{LL}\frac{y_G}{\theta_{L} y}\frac{1}{\Phi_G^{-1}(\delta_G)} \\ &\quad -\omega_{LL}\frac{1}{\omega_X y}\frac{\beta(\alpha+\beta)}{\beta(\alpha+\beta) - r(r-\alpha)}\left(\frac{Y_{G0}}{L_0}\frac{1}{\Phi_G^{-1}(\delta_G)} - 1\right), \end{aligned} \tag{32}$$

where $y \equiv Y_0/K_0$ and $y_G \equiv Y_0/K_{G0}$. The first term of (32) is the capital-productivity effect. The second and third terms capture the effect of public investment on employment and correspond to the labor-productivity and wealth effects, respectively. If public capital is Harrod neutral (i.e., $\eta_K = 0$) or factors of production are gross substitutes (i.e., $\sigma_Y > 1$) or both, then the multiplier of private capital tends to be positive (see also Section 5). Graphically, public investment shifts the capital demand curve to the right for a given interest rate—as depicted in Panel (a) of Figure 1—causing an increase in the long-run stock of private capital from K_0 to K_∞. However, if public capital is Solow neutral and factors of production are strong gross complements, then the negative Solow-substitution effect dominates the positive pure externality effect so as to render the capital-productivity effect also negative. In this case, the net effect of public investment on private capital may be negative if the negative capital-productivity effect more than offsets the positive indirect effect spilling over from the labor market.

If employment, private capital, and public capital increase, then output also rises to Y_∞.

compensated wage elasticity of labor supply.

The expression for the long-run output change is:

$$\begin{aligned}\frac{dY(\infty)}{dI_G} &= \frac{y_G}{\theta_L}\left[\theta_G + \theta_K\eta_K(\sigma_Y - 1)\right]\frac{1}{\Phi_G^{-1}(\delta_G)} + \frac{y_G\theta_G\omega_{LL}}{\theta_L}\frac{1}{\Phi_G^{-1}(\delta_G)} \\ &\quad -\frac{\omega_{LL}}{\omega_X}\frac{\beta(\alpha+\beta)}{\beta(\alpha+\beta) - r(r-\alpha)}\left(\frac{Y_{G0}}{L_0}\frac{1}{\Phi_G^{-1}(\delta_G)} - 1\right),\end{aligned} \tag{33}$$

where we have totally differentiated (9) and made use of (29), (30), and (32). The two terms in the first line of (33) correspond to the capital-productivity and labor-productivity effects, whereas the negative term in the second line represents the wealth effect. The most important result from (33) is that the output multiplier is smaller if private inputs are gross complements (i.e., $\sigma_Y < 1$) and public capital is Solow neutral (i.e., $\eta_K > 0$). In this case, public capital is biased toward the relative expensive factor (labor) and substitutability to the relative cheap factor (private capital) is rather limited, which leads to lower private capital accumulation. In contrast, if public capital is labor-augmenting (i.e., $\eta_L > 0$ and $\eta_K = 0$), then the elasticity of substitution between private inputs is irrelevant to the size of the multiplier, reflecting the availability of private capital at a fixed user cost.

Equation (33) embeds several other special cases. First, if labor supply is exogenous (i.e., $\omega_{LL} = 0$) the second and third terms of (33)—whose net effect is positive—drop out, implying that the output multiplier is larger if labor supply is endogenous.[19] Second, if agents are infinitely-lived (i.e., $\beta = 0$) only the negative wealth effect drops out, which implies an even larger output multiplier. Finally, if public capital is not productive (i.e., $Y_G = \theta_G = \eta_K = 0$), then only the negative component of the wealth effect remains; we thus obtain the output multiplier of a public consumption shock: $\frac{dY(\infty)}{dC_G} = \frac{dY(\infty)}{dI_G} = \frac{\omega_{LL}}{\omega_X - \omega_A} \geq 0$.[20]

Panel (c) of Figure 1 shows the long-run effect of public investment on output. The increase in employment and public capital shifts up the production function $Y(K; L_0, K_{G0})$ to $Y(K; L_\infty, K_\infty)$. The new steady-state equilibrium is denoted by E_∞, with a larger private capital stock K_∞ and higher output Y_∞. The slope of the production function is the same

[19] It can easily be shown that since the labor-productivity effect dominates the wealth effect in equation (30), so does it in (32) and (33).

[20] The macroeconomic effects of unproductive public investment and public consumption are identical. In this case, the positive output multiplier stems entirely from the wealth effect; intuitively, the lump-sum tax increase causes households to feel poorer, inducing them to supply more labor. If $\omega_{LL} = 0$, then $\frac{dY(\infty)}{dC_G} = \frac{dY(\infty)}{dI_G} = 0$.

at E_∞ and E_0, since I/K, I_G/K_G, Tobin's q, and thus $Y_K(\cdot)$ [see (16)] are fixed in the long run. The stock market value of domestic firms increases to qK_∞ (point N). However, Panel (d) shows that public investment also raises total domestic financial assets to A_∞ (point M). Thus, the difference (represented by the distance between N and M) gives the long-run effect of public investment on net foreign assets, which may be either positive or negative, depending on the long-run change in the stock of private capital relative to the change in domestic financial assets.[21]

5 Quantitative Analysis of the Effects of Public Investment

To quantify and visualize the dynamic macroeconomic effects of an increase in public investment, a simulation is performed. Section 5.1 describes the parameters used in the simulation, Section 5.2 illustrates the transitional dynamics, and Section 5.3 provides numerical results on both the short-run and long-run effects.

5.1 Parameters

Table 1 shows the parameter values that are taken from the literature. The time unit represents a year. We follow Mendoza (1991), who calibrates a dynamic general equilibrium model for the Canadian economy, in assigning values to ω_{LL}, r, and δ. In the benchmark model, the intertemporal substitution elasticity of labor supply ω_{LL} is set to 2.00, the rate of interest takes on a value of 0.04, and the rate of depreciation of private and public capital is 0.1. Based on an average expected life span of 55 (working) years, we assume a probability of death β of 1.82 percent. Following Baxter and King (1993), the ratio of public consumption to GDP is set to 20 percent, whereas the ratio of private consumption to GDP is 0.55, which is in line with the average value for OECD countries. The ratio of public investment to GDP takes on a value of 0.05, which is slightly above the OECD average. Based on Bom and Ligthart's (2008) analysis, we use $\theta_G = 0.08$. Initially, we set σ_Y to unity, which is in line with the Cobb-Douglas specification employed in most empirical studies.

[21] Panel (c) of Figure 1 shows the case $dF(\infty)/dI_G < 0$, which holds for reasonable parameter values in the cases of Hicks-neutral and Harrod-neutral public capital. See Section 5.3.

We employ a logarithmic specification for both the private and public installation cost function:

$$\Phi(x) \equiv \bar{z} \ln \frac{x+\bar{z}}{\bar{z}}, \qquad \Phi_G(x) \equiv \bar{z}_G \ln \frac{x_G+\bar{z}_G}{\bar{z}_G}, \tag{34}$$

where $\bar{z}$ and $\bar{z}_G$ are exogenous constants, $x \equiv I/K$, and $x_G \equiv I_G/K_G$. From (34) and the definitions of σ_A and σ_G, we derive $\sigma_A = x/(x+\bar{z})$ and $\sigma_G = \bar{z}_G x_G/(x_G+\bar{z}_G)$. We set the steady-state values for x at 0.11 and choose $\bar{z} = 0.532$, implying steady-state adjustment costs of about 0.2 percent of output. Similarly, we use $x_G = 0.11$ and pick $\bar{z}_G = 0.532$ to yield identical adjustment costs for public capital as for private capital. The degree of private capital mobility is $\sigma_A = 0.171$ and the elasticity of the public installation function is $\sigma_G = 0.091$.

Given the fixed rate of interest, our parametrization under finite horizons yields rising individual consumption profiles, that is, $r > \alpha = 0.0388$, where α is used as a calibration parameter to arrive at $A_0 = q_0 K_0$.[22] Once the parameters are set, all other information on the spending shares, output elasticities, and the output-capital ratio can be derived: $\omega_I = 0.20$, $\theta_L = 0.71$, $\theta_K = 0.29$, $\varepsilon_C = 0.28$, $\varepsilon_Y = 0.29$, $y = 0.55$, and $y_G = 2.20$. The conditions $\eta < 1 - \theta_K = 0.71$ and $\theta_K(1+\eta_K) = 0.37 < 1$ are thus easily met. Depending on the type of factor-augmenting public capital, we can derive η_K or η_L using (11); that is, $\eta_K = \theta_G/\theta_K = 0.276$ and $\eta_L = 0$ for Solow-neutral public capital and $\eta_L = \theta_G/\theta_L = 0.113$ and $\eta_K = 0$ for Harrod-neutral public capital, and $\eta_K = \eta_L = 0.08$ for Hicks-neutral public capital. Of the four real roots, two are stable and two are unstable; we find $-h_1^* = -0.1576$, $-h_2^* = -0.0172$, $r_1^* = 0.0584$, and $r_2^* = 0.1976$.

5.2 Impulse Responses

To visualize the impulse responses of a permanent and unanticipated rise in public investment financed by lump-sum taxes, we use the analytical expressions (A.23)–(A.26) in Appendix A.2 together with the steady-state log-linearized equations of Table A1. We plot the im-

[22] For the special case of infinite horizons, we set $r = \alpha$.

pulse response functions for 200 time periods, which allows us to accommodate differences in transition speed of variables. The impact (or short-run) effect of the fiscal shock occurs at $t = 0$ and the steady-state (or long-run) effect materializes at $t \rightarrow \infty$. The size of the fiscal shock amounts to $\tilde{I}_G = 0.1$. We first discuss the dynamic linkages between the variables for Hicks-neutral public capital, which allows us to focus on the effect of endogenous labor supply. Subsequently, we relax the Hicks-neutrality assumption and turn to our parametrization of interest.

5.2.1 Hicks-Neutral Public Capital

The dotted lines in Figure 3 show the transitional dynamics for Hicks-neutral public capital. On impact, Tobin's q jumps up, in anticipation of the future increase in the marginal productivity of private capital. Consequently, private investment rises. In Panel (a) of Figure 2, the economy moves from E_0 to E_1^H. Given the (future) increase in wages, households experience a rise in total wealth, inducing them to reduce labor supply.[23] As a result, wages increase and employment falls on impact. Graphically, in Panel (b), the labor supply curve shifts to the left to L_1^s, while the labor demand schedule remains unaffected, yielding the short-run equilibrium E_1. Given that private capital is predetermined, the private capital-labor ratio rises and output falls in the short run. The rise in short-run domestic absorption (denoted by $\tilde{C}(0) + \tilde{I}(0) + \tilde{I}_G > 0$) together with the fall in output implies that the country's net exports fall (i.e., $\tilde{Z}(0) < 0$).

The increase in public investment expands the public capital stock over time. Similarly, the private capital stock rises gradually, owing to the larger private investment rate. In terms of Panel (a) of Figure 2, the economy gradually moves from E_1^H to E_2^H along the dynamic path DP^H. Because private capital and labor are cooperative factors of production, the marginal productivity of labor rises. In Panel (b), the labor demand curve shifts to the right to L_2^d. Labor supply shifts further to the left, reflecting a rise in full consumption. The employment path during transition is non-monotonic. Initially, during periods 10-30, the labor-productivity effect of both private and public capital is rather strong and dominates

[23] Private investment 'crowds in' private consumption; this result is in contrast to the conventional negative effect found by Heijdra and Ligthart (2007).

the wealth effect. Graphically, the rightward shift of the labor demand curve dominates the leftward shift of the labor supply schedule, causing a net increase in wages and employment. Once the economy reaches the temporary equilibrium E_2, employment attains its maximum. Eventually, during periods 30 and beyond, the rise in wages boosts the wealth effect on labor supply. Graphically, the labor supply curve shifts to the left to a greater extent than in the initial periods—which is represented by a move from L_2^s to L_∞^s—pushing wages further up, but depressing the employment increment. However, both employment and wages increase in the long run, as indicated by the location of the new steady state E_∞ to the Northeast of E_0.

The drop in the employment increment in the medium run causes the capital-labor ratio to rise. As a result, Tobin's q gradually decreases over time, eventually returning to its initial steady-state value at E_∞^H. Therefore, the transition in Tobin's q—and thus in the private capital stock—is also non-monotonic. Private and full consumption, however, increase monotonically during transition, reflecting a continuous rise in wage income. The rise in domestic absorption boosts imports, deteriorates the trade balance, and creates a short-run current account deficit. Hence, the country accumulates net foreign debt. During transition, the current account deficit shrinks, giving rise to a non-monotonic path of net foreign assets. In the new steady state, the current account of the balance of payments is balanced again (i.e., $\dot{F}(\infty) = 0$), where a surplus on the trade account offsets the interest payments on net foreign debt.

5.2.2 Solow-Neutral and Harrod-Neutral Public Capital

To distinguish between different types of factor-augmentation, we focus on the case of $0 < \sigma_Y < 1$. The dashed and solid lines in Figure 3 present impulse responses of a public investment shock for Harrod-neutral and Solow-neutral public capital, respectively. Section 4 showed that the elasticity of substitution is irrelevant in the long run if public capital is Harrod neutral; apart from slight short-run differences, it turns out that also the transitional dynamics are virtually identical to the Hicks-neutral case.[24] In contrast, under Solow neutrality the transitional dynamics differ qualitatively for private capital and foreign assets, but

[24] In both technology cases, the Solow-substitution effect $\eta_K(\sigma_Y - 1)$ of equation (23) drops out, leaving only the pure externality effect on the medium-run marginal productivity of private capital.

are only quantitatively different for output.

In the Solow-neutral case, Tobin's q jumps down to E_1^S, resulting in a decrease in private investment in the short run. Consequently, the stock of private capital gradually reduces. As the public capital stock expands, the marginal productivity of labor increases, which boosts employment (recall from (17) that in this case public capital is biased toward labor). The increase in labor use raises the marginal productivity of capital, Tobin's q, and thus private investment. Eventually, net private investment turns positive so that the private capital stock expands back to the pre-shock level, causing the graphical swing in Panel (a) from E_1^S to E_2^S along the dynamic path DP^S. However, over time the decrease in the employment increment causes Tobin's q and private investment to fall again, which creates a drop in long-run private capital accumulation. Graphically, the economy follows the dynamic path DP^S from E_2^S to the new steady state E_∞^S.

In the Harrod-neutral case, Tobin's q jump down from E_0 to some point (not shown) above E_1^S in Panel (a) of Figure 2, which temporarily depresses private investment to a smaller extent than in the Solow-neutral case. As public capital accumulates, the marginal productivity of private capital increases, thereby pushing Tobin's q up. As a result, private investment rises and the private capital stock expands, which moves the economy in the direction of E_2^H, following a dynamic path similar to DP^H. As in the Hicks-neutral case, once employment attains its maximum at E_2 in Panel (b), Tobin's q starts adjusting back to its original level. In Panel (a), the economy moves along the dynamic path DP^H from E_2^H to the new steady-state equilibrium E_∞^H.

The dynamics of net foreign assets also depend on the technology scenario. Under Solow-neutral public capital, the stock of net foreign assets increases in the new steady state, whereas it drops in the Harrod-neutral case. The reason lies in the larger rise in imports due to the higher private investment rate in the latter case. In contrast to the dynamics for capital and net foreign assets, the impulse responses for labor are nearly identical across specifications. Although the short-run effects on labor are slightly different for different values of σ_Y and η_i, the three lines for employment coincide in the long run. Intuitively, the steady-state effect on wages and employment is only affected by $Y_G(\cdot)$, that is, the size of the pure public capital externality [see equations (30)–(31)].

5.2.3 Comparison of Hysteretic and Non-Hysteretic Models

Output shows a non-monotonic transition path in the benchmark scenario, which crucially depends on the assumptions made on the labor supply elasticity, the life span of agents, and the presence of a public capital externality. The dotted lines in Figure 4 show monotonic transition paths for output if labor supply is exogenous, households have infinite horizons, and public capital does not give rise to a production externality. If labor supply is exogenous (i.e., $\omega_{LL} = 0$), there is no wealth effect on labor supply. In this case, output and private consumption do not react on impact.[25] In the long run, consumption and output multipliers are both positive. If households have infinite life spans (i.e., $\beta = 0$ so that $r = \alpha$), full consumption dynamics is degenerate, that is, $\dot{X} = 0$. In that case, full consumption jumps on impact and stays constant over time. Intuitively, all future changes in disposable income are fully anticipated and already reflected in the initial jump in full consumption, so that there are no further leftward shifts in the $L^s(w; X)$ curve during transition. Consequently, no wiggle is present in the output path.

Although not analyzed by Schmitt-Grohe and Uribe (2003), finite horizons can be used to arrive at an endogenously-determined steady state.[26] A key question is whether using a hysteresis-eliminating device, such as overlapping generations, affects the impulse responses. Panels (a)–(c) of Figure 4 show that the impulse responses of the hysteretic (infinite-horizon) model—as represented by the dotted lines—are very different from those found in the non-hysteretic (finite-horizon) model. This result contradicts Schmitt-Grohe and Uribe's (2003) primary finding that the non-stationary model and the stationary model yield nearly identical impulse responses. Only in the absence of a production externality can we replicate their result. Schmitt-Grohe and Uribe's (2003) result is thus not as generally valid as suggested.

[25] The rise in wages without a wealth effect is smaller than in the benchmark case with a wealth effect present.

[26] Schmitt-Grohe and Uribe (2003) discuss five ways to 'close small open models:' (i) an endogenous discount factor; (ii) a debt-elastic interest rate premium; (iii) convex portfolio adjustment costs; and (iv) complete asset markets. They show that the type of device does not matter.

5.3 Quantitative Short-Run and Long-Run Effects

Table 2 presents macroeconomic multipliers for the benchmark scenario and alternative values of the output elasticity of public capital θ_G and substitution elasticity in production σ_Y. The long-run output multiplier in the benchmark scenario is 2.71, whereas the short-run multiplier is negative, suggesting substantial long-term output gains from investment in public capital. The case of exogenous labor supply (i.e., $\omega_{LL} = 0$) and Hicks-neutral public capital—which corresponds to Heijdra and Meijdam's (2002) model, but is not shown in the table—yields a long-run output multiplier of 2.25. Abstracting from endogenous labor supply thus underestimates the long-run output multiplier. Output multipliers do not change sign across various values of θ_G and σ_Y. A larger output elasticity of public capital—and thus a larger production externality—increases the marginal productivity of private capital. Consequently, the steady-state investment and output multipliers both rise. If the public capital externality is absent, the long-run output multiplier is only marginally above unity.[27] In contrast to the case with a public capital externality, short-run employment rises, reflecting a fall in aggregate household wealth induced by the rise in lump-sum taxes necessary to balance the government budget.

Although the sign of the long-run output multiplier is independent of the type of factor-augmenting public capital, its size is substantially affected. The elasticity of substitution plays an important role if public capital is Solow neutral. We find a long-run output multiplier that falls short of that found in the Hicks-neutral scenario if factors of production are gross complements; the output multiplier amounts to 1.59 compared to 2.71 in the benchmark scenario. Both short-run and long-run investment multipliers may take on negative values. Conversely, if factors of production are gross substitutes and public capital is Solow neutral, the long-run output multiplier exceeds that found in the Hicks-neutral case. In absolute terms, the response of net foreign assets is much larger than that of physical capital. Under Harrod neutrality, the long-run output multiplier is equal to that found under Hicks neutrality. As expected from the analytical results, both long-run wage and employment multipliers are

[27] Note that the output multiplier may fall below unity if the labor supply elasticity is small. For example, for $\theta_G = 0$ and $\omega_{LL} = 1$, we find an output multiplier of 0.85.

independent of σ_Y and η_i.

6 Conclusions

The paper develops a dynamic microfounded macroeconomic model of a small open economy to study the transitional dynamics of a balanced-budget increase in public investment. Public capital gives rise to an externality by entering private production in a factor-augmenting fashion. Various forms of factor-augmenting public capital are distinguished. The household side of the model extends a Yaari-Blanchard overlapping generations model—which gives rise to a non-hysteretic steady state—by introducing a wealth effect on intertemporal labor supply.

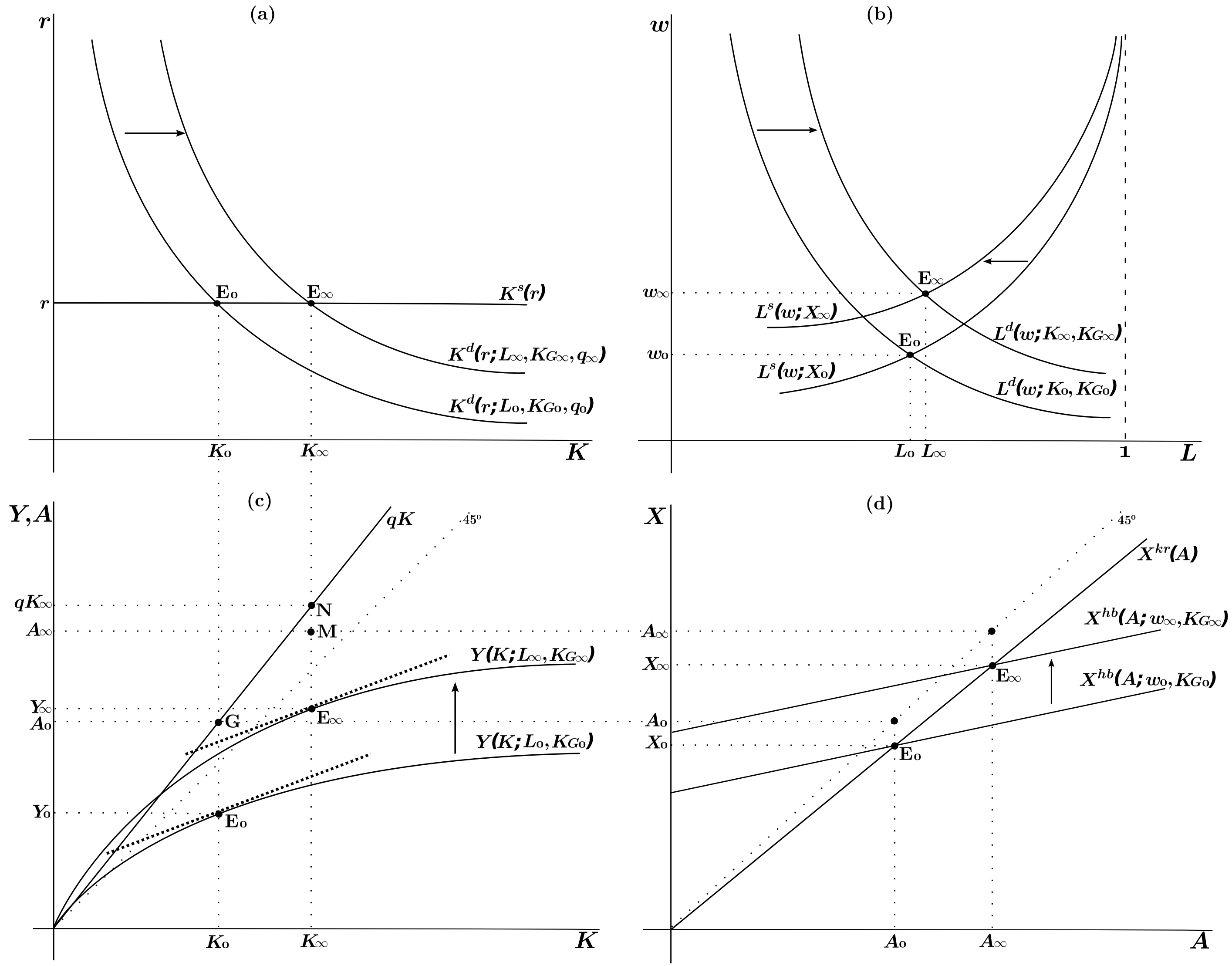
The paper shows that the type of factor-augmenting public capital matters for the size of the output multiplier, but does not affect its sign. The long-run output multiplier for the empirically plausible case of Solow-neutral public capital and low input substitutability amounts to 1.59, which is substantially smaller than the value of 2.71 that is found if public capital is Hicks neutral. The size of the long-run output multiplier under Harrod-neutral public capital is identical to that under Hicks neutrality. Endogenous labor supply and larger elasticities of substitution between private capital and labor boost the long-run output multiplier.

The type of factor-augmenting public capital matters to the transitional dynamics of a public investment shock. Qualitatively similar output dynamics of a public investment impulse are found for various types of factor-augmenting public capital. Transition paths for private capital and net foreign assets, however, differ substantially. If factors of production are gross complements and public capital is Solow neutral (i.e., public capital is labor biased), the path of the private capital stock shows an initial fall, followed by a rise, and a subsequent decline in the new steady state. Long-run net foreign assets increase, sustaining a long-run current account surplus. If public capital is Harrod or Hicks neutral, then the non-monotonic path for private capital always lies above the zero axis, whereas the stock of net foreign assets always falls.

In contrast to conventional results obtained in hysteretic (infinite-horizon) models for a small open economy, the output dynamics of a public investment shock in the non-hysteretic (finite-horizon) model are non-monotonic. A necessary condition for this result is the simultaneous presence of a public capital externality, endogenous labor supply, and finite horizons of households. Our results demonstrate that Schmitt-Grohe and Uribe's (2003) central finding—i.e., the impulse responses of hysteretic and non-hysteretic models are virtually identical—is not as generally valid as suggested. Impulse responses of the hysteretic and the non-hysteretic model are only identical without a production externality.

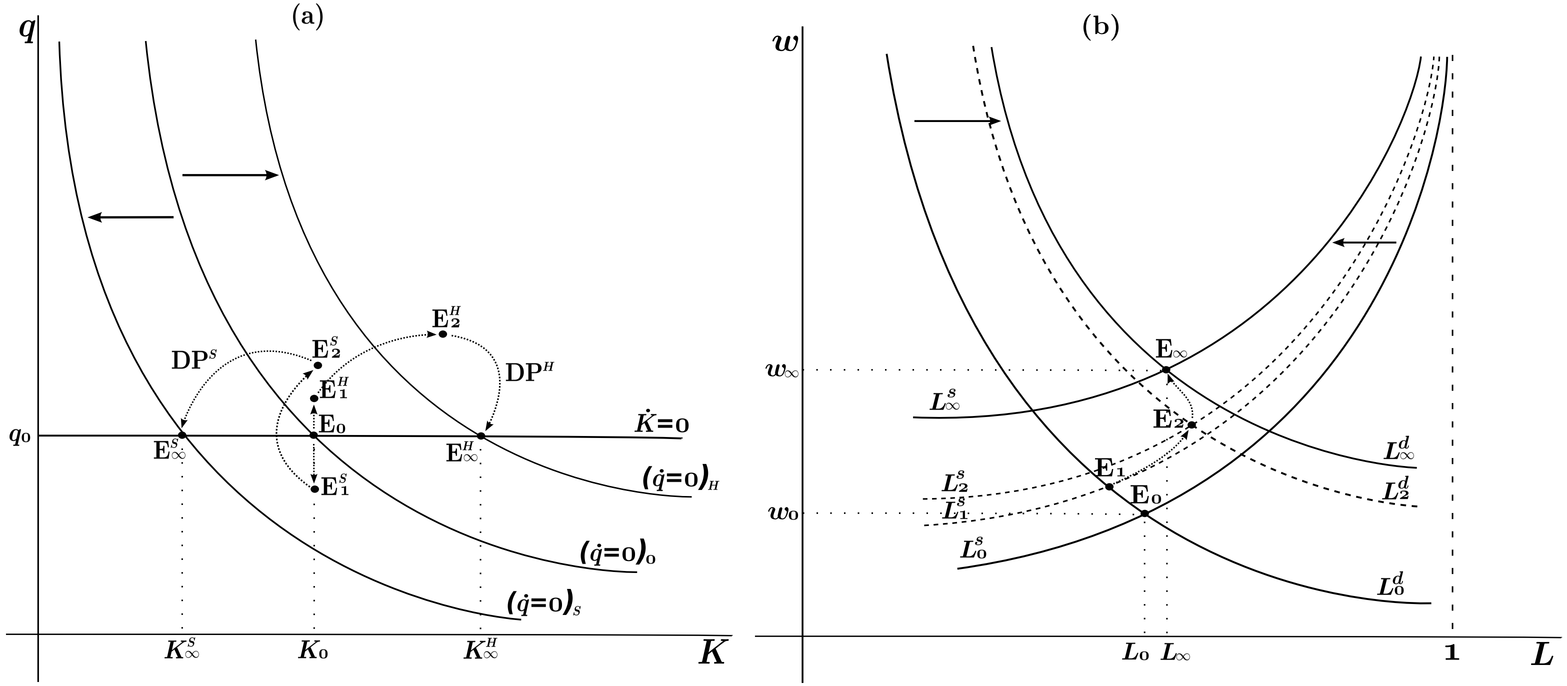
Our study does not analyze the intergenerational welfare effects of an increase in public investment. We leave this for further research. In addition, the analysis assumes that the government has access to lump-sum taxes. In future work, we want to study the dynamic macroeconomic effects of alternative ways to balance the government's budget, that is, labor tax and debt financing. Finally, the model can easily be extended to a real business cycle setting.

Figure 1: Long-Run Effects of Public Infrastructure Investment



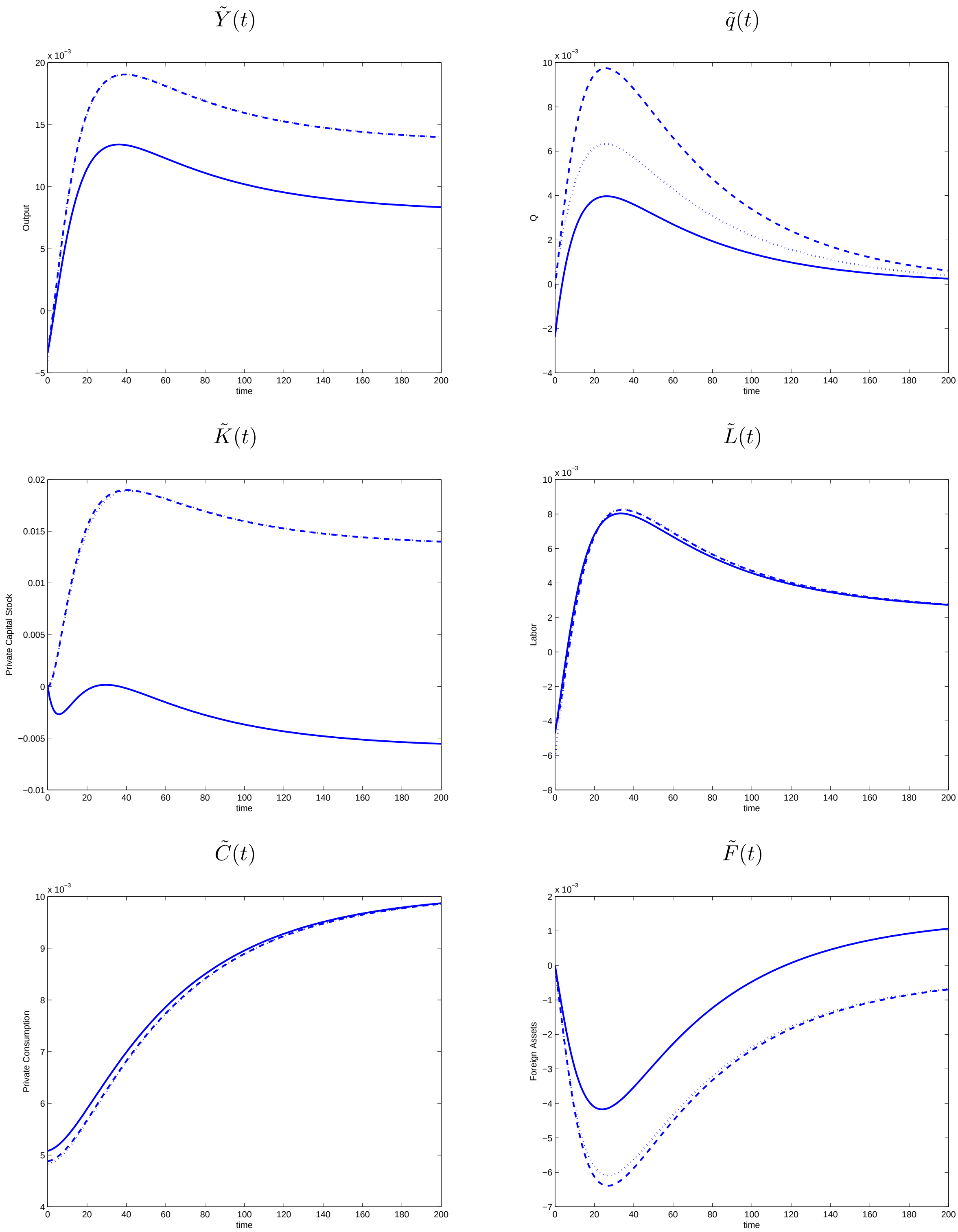
Notes: Panels (a)–(d) depict the capital market, labor market, production function, and savings subsystem, respectively. E_0 denotes the initial (pre-shock) steady state and E_∞ represents the new long-run equilibrium after the public investment shock. Panel (a) also shows the case of a positive long-run private capital multiplier, as found under both Hicks and Harrod neutrality. Similarly, Panel (c) depicts the production function, where the case of a negative net foreign assets multiplier (distance between points M and N).

Figure 2: Short-Run and Medium-Run Effects of Public Infrastructure Investment: The Capital and Labor Markets



Notes: E_0 denotes the initial (pre-shock) steady state, E_1 shows the short-run equilibrium after the public investment shock, E_2 gives the medium-run equilibrium associated with the maximum level of employment, and E_∞ denotes the new steady state. Since the dynamics of private capital are qualitatively different under Harrod/Hicks neutrality and Solow neutrality, we use the superscripts 'H' and 'S' to distinguish the respective equilibria in Panel (a); this distinction is unnecessary in Panel (b).

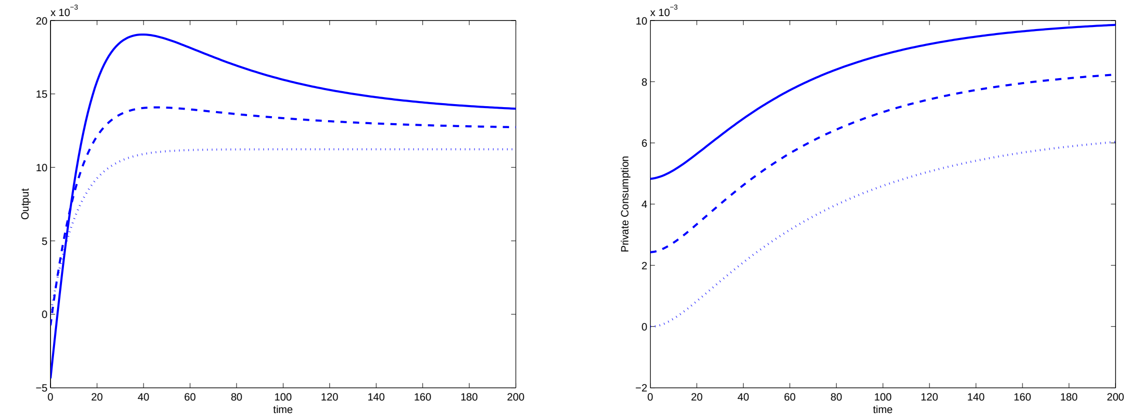
Figure 3. Permanent Public Investment Shock under Various Types of Factor-Augmenting Public Capital



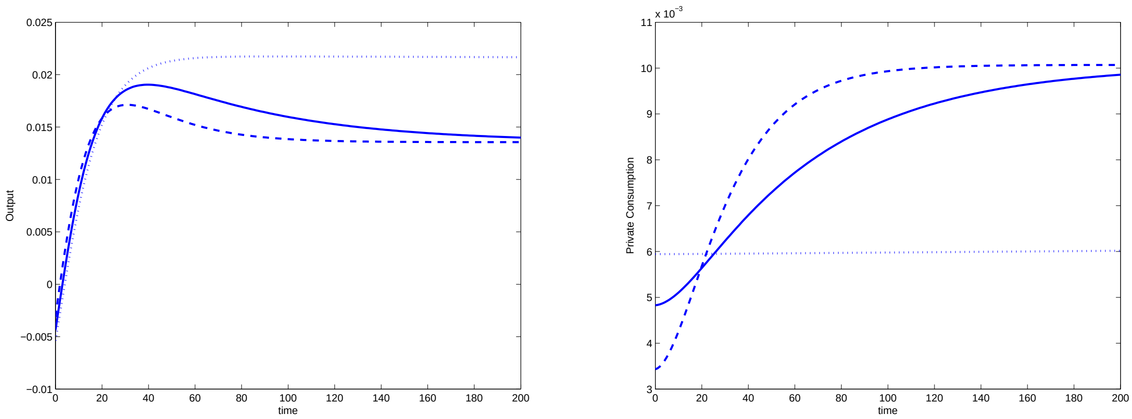
Notes: The dashed line denotes the scenario of $\sigma_Y = 0.5$ and $\eta_K = 0$ (Harrod-neutral case), the solid line represents $\sigma_Y = 0.5$ and $\eta_L = 0$ (Solow-neutral case), and the dotted line represents $\sigma_Y = 1$ and $\eta_K = \eta_L$ (Hicks-neutral case). The other parameters are set at their benchmark values.

Figure 4. Impulse Responses of a Permanent Public Investment Shock:
Various Values of ω_{LL}, β, and θ_G

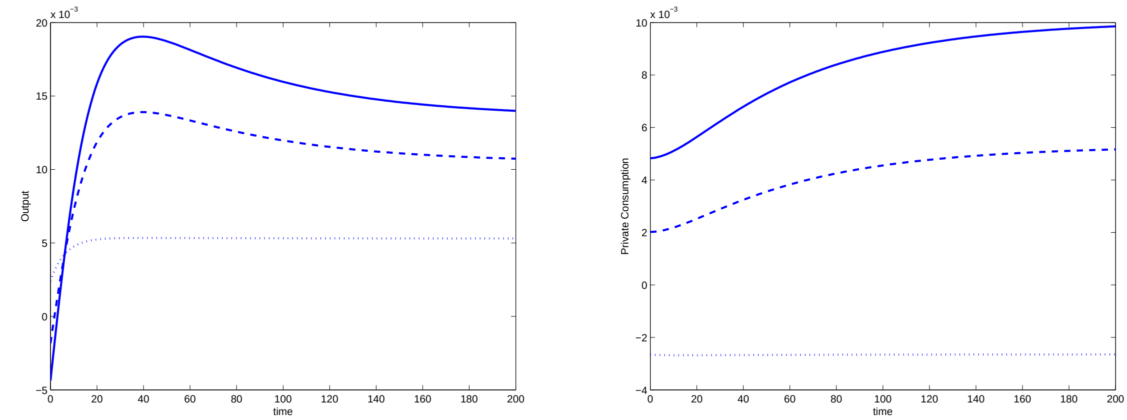
Panel (a): $\tilde{Y}(t)$ and $\tilde{C}(t)$ for various ω_{LL} values



Panel (b): $\tilde{Y}(t)$ and $\tilde{C}(t)$ for various β values



Panel (c): $\tilde{Y}(t)$ and $\tilde{C}(t)$ for various θ_G values



Notes: In Panel (a), ω_{LL} takes on the values 0 (dotted line), 0.50 (dashed line), and 2.00 (solid line). In Panel (b), β takes on the values 0 (dotted line), 0.0182 (solid line), and 0.05 (dashed line), respectively. In Panel (c), θ_G takes on the values 0 (dotted line), 0.05 (dashed line), and 0.08 (solid line). The other parameters are set at their benchmark values.

Table 1: The Parameter Values in the Benchmark Model

Description	Parameter	Value
Birth rate	β	0.018
Rate of interest	r	0.040
Depreciation rate of private capital	δ	0.100
Depreciation rate of public capital	δ_G	0.100
Leisure-labor ratio	ω_{LL}	2.000
Elasticity of substitution between labor and private capital	σ_Y	1.000
Ratio of private consumption to GDP	ω_C	0.550
Ratio of public consumption to GDP	ω_G^C	0.200
Ratio of public investment to GDP	ω_G^I	0.050
Output elasticity of public capital	θ_G	0.080
Parameter of the installation function for private capital	$\bar{z}$	0.532
Parameter of the installation function for public capital	$\bar{z}_G$	0.532

Table 2: Macroeconomic Multipliers of a Rise in Public Investment

Multiplier	$\sigma_Y = 1$	θ_G			$\sigma_Y = 0.50$		$\sigma_Y = 0.75$	
	Hicks	0	0.05	0.12	Harrod	Solow	Harrod	Solow
$\frac{dY(0)}{dI_G}$	-0.8729	0.4808	-0.3653	-1.5498	-0.6464	-0.6730	-0.7816	-0.7967
$\frac{dY(\infty)}{dI_G}$	2.7111	1.0600	2.0920	3.5367	2.7111	1.5879	2.7111	2.1495
$\frac{dC(0)}{dI_G}$	0.5310	-0.2925	0.2222	0.9428	0.5368	0.5590	0.5334	0.5437
$\frac{dC(\infty)}{dI_G}$	1.1077	-0.2915	0.5830	1.8073	1.1077	1.1077	1.1077	1.1077
$\frac{dI(0)}{dI_G}$	0.0474	0.3721	0.1691	-0.1150	-0.0484	-0.5522	0.0090	-0.2290
$\frac{dI(\infty)}{dI_G}$	0.5422	0.2120	0.4184	0.7073	0.5422	-0.2384	0.5422	0.1519
$\frac{dF(0)}{dI_G}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\frac{dF(\infty)}{dI_G}$	-1.5292	-3.4883	-2.2639	-0.5497	-1.5292	7.0340	-1.5292	2.7524
$\frac{dL(0)}{dI_G}$	-0.4085	0.2250	-0.1710	-0.7253	-0.3025	-0.3150	-0.3658	-0.3729
$\frac{dL(\infty)}{dI_G}$	0.1549	0.3533	0.2293	0.0557	0.1549	0.1549	0.1549	0.1549
$\frac{dK(0)}{dI_G}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\frac{dK(\infty)}{dI_G}$	4.9287	1.9271	3.8031	6.4295	4.9287	-2.1674	4.9287	1.3806
$\frac{dw(0)}{dI_G}$	0.7536	-0.4150	0.3153	1.3379	1.1160	1.1620	0.8997	0.9171
$\frac{dw(\infty)}{dI_G}$	4.8000	0.0000	3.0000	7.2000	4.8000	4.8000	4.8000	4.8000
$\frac{dq(0)}{dI_G}$	0.0490	0.3847	0.1749	-0.1189	-0.0500	-0.5708	0.0093	-0.2368
$\frac{dq(\infty)}{dI_G}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Notes: Unless indicated otherwise, all parameters are set at their benchmark values (Table 1). Hicks-neutral public capital assumes $\eta = \eta_L = \eta_K = 0.08$. Harrod-neutral public capital is given by $\eta_L = 0.12$ and $\eta_K = 0$, whereas Solow-neutral public capital is defined as $\eta_L = 0$ and $\eta_K = 0.29$.

Appendix

This Appendix derives the log-linearized model, analyzes stability, and solves for the short-run, transitional, and long-run effects of a public investment shock. Further details are provided in Bom, Heijdra, and Ligthart (2010)

A.1 The Reduced-Form Model

We log-linearize the finite-horizon model around an initial steady state in which $F(0) = 0$ so that $A(0) = q(0)K(0)$.[28] The results are reported in Table A.1. A tilde ($\tilde{}$) denotes a relative change, for example, $\tilde{X}(t) \equiv dX(t)/X_0$, where X_0 denotes the initial steady-state value of full consumption. For financial assets $A(t)$ and $F(t)$, we use $\tilde{A}(t) \equiv rdA(t)/Y_0$ and $\dot{\tilde{A}}(t) \equiv rd\dot{A}(t)/Y_0$. Lump-sum taxes are scaled by steady-state output Y_0 only, that is, $\tilde{T}(t) \equiv dT(t)/Y_0$.

A.1.1 The Quasi-Reduced Form of the Static System

Conditional on the state variables and the policy shocks (see below), equations (TA.7)–(TA.9) can be condensed to the following quasi-reduced form expressions:

$$\begin{bmatrix} \frac{1}{\sigma_Y} & -\frac{1}{\sigma_Y} & -1 \\ 1 & -\theta_L & 0 \\ 0 & 1 & -\omega_{LL} \end{bmatrix} \begin{bmatrix} \tilde{Y}(t) \\ \tilde{L}(t) \\ \tilde{w}(t) \end{bmatrix} = \begin{bmatrix} \frac{1-\sigma_Y}{\sigma_Y}\eta_L \tilde{K}_G(t) \\ \tilde{K}^*(t) \\ -\omega_{LL}\tilde{X}(t) \end{bmatrix}, \tag{A.1}$$

where $\omega_{LL} \equiv (1 - L_0)/L_0 \geq 0$ is the leisure-labor ratio and $\tilde{K}^*(t)$ denotes broad capital:

$$\tilde{K}^*(t) = \theta_K \tilde{K}(t) + \theta_G \tilde{K}_G(t). \tag{A.2}$$

Using $\tilde{K}_G(t) = \left(1 - e^{-\sigma_G t}\right)\tilde{I}_G$, we find:

$$\tilde{K}^*(t) = \theta_K \tilde{K}(t) + \theta_G \left[1 - e^{-\sigma_G t}\right]\tilde{I}_G. \tag{A.3}$$

[28] In the special case of infinite horizons ($\beta = 0$ and thus $r = \alpha$), we set $F(0) = -q(0)K(0)$ to arrive at $A(0) = 0$.

The system (A.1) can be solved to yield:

$$\begin{bmatrix} \tilde{Y}(t) \\ \tilde{L}(t) \\ \tilde{w}(t) \end{bmatrix} = \begin{bmatrix} \xi_{yk} & \xi_{yx} & \xi_{yg} \\ \xi_{lk} & \xi_{lx} & \xi_{lg} \\ \xi_{wk} & \xi_{wx} & \xi_{wg} \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{X}(t) \\ \left(1 - e^{-\sigma_G t}\right) \tilde{I}_G \end{bmatrix}, \tag{A.4}$$

where the coefficients for output are:

$$\xi_{yk} \equiv \frac{\theta_K(\sigma_Y + \omega_{LL})}{\sigma_Y + \omega_{LL}\theta_K} > 0, \;\; \xi_{yx} \equiv -\frac{\theta_L \omega_{LL} \sigma_Y}{\sigma_Y + \omega_{LL}\theta_K} < 0, \;\; \xi_{yg} \equiv \frac{\theta_G(\sigma_Y + \omega_{LL}) - (1 - \sigma_Y)\omega_{LL}\theta_L \eta_L}{\sigma_Y + \omega_{LL}\theta_K},$$

and the coefficients for employment are:

$$\xi_{lk} \equiv \frac{\theta_K \omega_{LL}}{\sigma_Y + \omega_{LL}\theta_K} > 0, \quad \xi_{lx} \equiv -\frac{\omega_{LL}\sigma_Y}{\sigma_Y + \omega_{LL}\theta_K} < 0, \quad \xi_{lg} \equiv \frac{\omega_{LL}[\theta_G + \eta_L(\sigma_Y - 1)]}{\sigma_Y + \omega_{LL}\theta_K},$$

and the coefficients for wages:

$$\xi_{wk} \equiv \frac{\theta_K}{\sigma_Y + \omega_{LL}\theta_K} > 0, \quad \xi_{wx} \equiv \frac{\omega_{LL}\theta_K}{\sigma_Y + \omega_{LL}\theta_K} > 0, \quad \xi_{wg} \equiv \frac{\theta_G + \eta_L(\sigma_Y - 1)}{\sigma_Y + \omega_{LL}\theta_K}.$$

A.1.2 Stability of the Dynamic System

Solving the dynamic system (19) gives rise to a fourth-order characteristic polynomial:

$$P(s) \equiv |s\mathrm{I} - \Delta| = \phi(s)\,\psi(s) - \bar{\delta}_{12}\bar{\delta}_{23}\bar{\delta}_{34}\bar{\delta}_{41} = 0, \tag{A.5}$$

where I is the identity matrix and $\phi(s)$ and $\psi(s)$ are:

$$\phi(s) \equiv \left(s - \bar{\delta}_{33}\right)\left(s - \bar{\delta}_{22}\right) - \bar{\delta}_{34}\bar{\delta}_{43}, \tag{A.6}$$

$$\psi(s) \equiv s\left(s - \bar{\delta}_{22}\right) - \bar{\delta}_{12}\bar{\delta}_{21}. \tag{A.7}$$

We can rewrite $P(s)$ as:

$$P(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0, \tag{A.8}$$

where the $a_i's$ are defined as:

$$a_3 \equiv -\mathrm{tr}(\Delta) = -(2\bar{\delta}_{22} + \bar{\delta}_{33}) < 0, \tag{A.9}$$

$$a_2 \equiv \bar{\delta}_{22}^2 - \bar{\delta}_{12}\bar{\delta}_{21} + 2\bar{\delta}_{22}\bar{\delta}_{33} - \bar{\delta}_{34}\bar{\delta}_{43}, \tag{A.10}$$

$$a_1 \equiv \bar{\delta}_{12}\bar{\delta}_{21}(\bar{\delta}_{22} + \bar{\delta}_{33}) + \bar{\delta}_{22}\left[\bar{\delta}_{34}\bar{\delta}_{43} - \bar{\delta}_{22}\bar{\delta}_{33}\right], \tag{A.11}$$

$$a_0 \equiv |\Delta| = (r - \alpha)\frac{r^3\omega_I\theta_K\theta_L}{\sigma_A\omega_A^3(\sigma_Y + \omega_{LL}\theta_K)}(\omega_X - \omega_A) > 0. \tag{A.12}$$

The positive determinant may indicate various cases: (i) two positive roots and two negative roots; (ii) four positive roots (in which case the system is unstable); and (iii) four negative roots, giving rise to an indeterminate steady state (cf. Benhabib and Farmer, 1994, p. 30). The third case is excluded because of $\mathrm{tr}(\Delta) > 0$. To distinguish between cases (i) and (ii), we use Routh's criterion (cf. Shi and Epstein, 1993), which considers the number of sequential sign changes in the Routh scheme as an indicator of the number of unstable roots.[29] Based on the Routh analysis and the numerical results in Section 5, we find that the first is the relevant case. Therefore, the system (19) has a unique and locally saddle-point stable steady state, featuring four characteristic roots; that is, two stable real roots denoted by $-h_1^* < 0$ and $-h_2^* < 0$ and two unstable real roots denoted by $r_1^* > 0$ and $r_2^* > 0$.

A.2 Solving for the Comparative Dynamics

A.2.1 The Transformed Model

We will make use of the Laplace transform technique to analyze the model (cf. Judd, 1982). The Laplace transformation of $x(t)$ evaluated at s is given by

$$\mathcal{L}\{x, s\} \equiv \int_0^\infty x(t)e^{-st}\mathrm{d}t. \tag{A.13}$$

Intuitively, $\mathcal{L}\{x, s\}$ represents the present value of $x(t)$ using s as the discount rate.

[29] See Heijdra and Ligthart (2008) for further details on this analysis.

By taking the Laplace transform of (19) we obtain:[30]

$$\Lambda(s)\begin{bmatrix}\mathcal{L}\{\tilde{K},s\}\\ \mathcal{L}\{\tilde{q},s\}\\ \mathcal{L}\{\tilde{X},s\}\\ \mathcal{L}\{\tilde{A},s\}\end{bmatrix}=\begin{bmatrix}0\\ f\tilde{q}(0)-\mathcal{L}\{\gamma_q,s\}\\ \tilde{X}(0)\\ \omega_A\tilde{q}\left(0\right)-\mathcal{L}\{\gamma_A,s\}\end{bmatrix}, \tag{A.14}$$

where $\Lambda(s)\equiv s\mathrm{I}-\Delta$ and I is the identity matrix. We know that:

$$\Lambda(s)^{-1}\equiv\frac{1}{(s+h_1^*)(s+h_2^*)(s-r_1^*)(s-r_2^*)}\operatorname{adj}\Lambda(s), \tag{A.15}$$

where $\operatorname{adj}\Lambda(s)$ is the adjoint matrix of $\Lambda(s)$. By pre-multiplying both sides of (A.14) by $\Lambda(s)^{-1}$ and rearranging we obtain the following expression in Laplace transforms:

$$(s+h_1^*)(s+h_2^*)\begin{bmatrix}\mathcal{L}\{\tilde{K},s\}\\ \mathcal{L}\{\tilde{q},s\}\\ \mathcal{L}\{\tilde{X},s\}\\ \mathcal{L}\{\tilde{A},s\}\end{bmatrix}=\frac{\operatorname{adj}\Lambda(s)\begin{bmatrix}0\\ \tilde{q}(0)-\mathcal{L}\{\gamma_q,s\}\\ \tilde{X}(0)\\ \omega_A\tilde{q}\left(0\right)-\mathcal{L}\{\gamma_A,s\}\end{bmatrix}}{(s-r_1^*)(s-r_2^*)}. \tag{A.16}$$

The $\operatorname{adj}\Lambda(s)$ matrix is equal to:

$$\operatorname{adj}\Lambda(s)\equiv\begin{bmatrix}\left(s-\bar{\delta}_{22}\right)\phi\left(s\right) & \bar{\delta}_{12}\phi\left(s\right) & \bar{\delta}_{12}\bar{\delta}_{23}\left(s-\bar{\delta}_{22}\right) & \bar{\delta}_{12}\bar{\delta}_{23}\bar{\delta}_{34}\\ \bar{\delta}_{21}\phi\left(s\right)+\bar{\delta}_{23}\bar{\delta}_{34}\bar{\delta}_{41} & s\phi\left(s\right) & \bar{\delta}_{23}s\left(s-\bar{\delta}_{22}\right) & \bar{\delta}_{23}\bar{\delta}_{34}s\\ \bar{\delta}_{34}\bar{\delta}_{41}\left(s-\bar{\delta}_{22}\right) & \bar{\delta}_{12}\bar{\delta}_{34}\bar{\delta}_{41} & \left(s-\bar{\delta}_{22}\right)\psi\left(s\right) & \bar{\delta}_{34}\psi\left(s\right)\\ \bar{\delta}_{41}\left(s-\bar{\delta}_{22}\right)\left(s-\bar{\delta}_{33}\right) & \bar{\delta}_{12}\bar{\delta}_{41}\left(s-\bar{\delta}_{33}\right) & \bar{\delta}_{43}\psi\left(s\right)+\bar{\delta}_{12}\bar{\delta}_{23}\bar{\delta}_{41} & \left(s-\bar{\delta}_{33}\right)\psi\left(s\right)\end{bmatrix}.$$

[30] We have made use of $\mathcal{L}\{\dot{\tilde{q}},s\}=s\mathcal{L}\{\tilde{q}(t),s\}-\tilde{q}(0)$. In addition, we note that $\tilde{K}(0)=0$ and $\tilde{A}\left(0\right)\neq0$ due to unanticipated capital gains/losses, that is, $\tilde{A}\left(0\right)=\omega_A\tilde{q}\left(0\right)$.

A.2.2 Impulse Response Functions

We have two jumping variables [$\tilde{q}(t)$ and $\tilde{X}(t)$] so that we need to impose only two initial conditions. The values of $\tilde{q}\,(0)$ and $\tilde{X}(0)$ are such that the right-hand side of (A.16) is of the $0 \div 0$ type for both unstable roots r_1^* and r_2^*.[31] Using the first row of $\mathrm{adj}\,\Lambda(s)$, for example, we get for $s = r_1^*$ and $s = r_2^*$:

$$\left[\phi\,(s) + \bar{\delta}_{23}\bar{\delta}_{34}\omega_A\right]\tilde{q}\,(0) + \bar{\delta}_{23}\left(s - \bar{\delta}_{22}\right)\tilde{X}\,(0) = \phi\,(s)\,\mathcal{L}\{\gamma_q, s\} + \bar{\delta}_{23}\bar{\delta}_{34}\mathcal{L}\{\gamma_A, s\}. \quad \text{(A.17)}$$

We can solve (A.17) to yield:

$$\begin{bmatrix} \tilde{q}(0) \\ \tilde{X}(0) \end{bmatrix} = \begin{bmatrix} \phi\,(r_1^*) + \bar{\delta}_{23}\bar{\delta}_{34}\omega_A & \bar{\delta}_{23}\left(r_1^* - \bar{\delta}_{22}\right) \\ \phi\,(r_2^*) + \bar{\delta}_{23}\bar{\delta}_{34}\omega_A & \bar{\delta}_{23}\left(r_2^* - \bar{\delta}_{22}\right) \end{bmatrix}^{-1} \begin{bmatrix} \phi\,(r_1^*)\,\mathcal{L}\{\gamma_q, r_1^*\} + \bar{\delta}_{23}\bar{\delta}_{34}\mathcal{L}\{\gamma_A, r_1^*\} \\ \phi\,(r_2^*)\,\mathcal{L}\{\gamma_q, r_2^*\} + \bar{\delta}_{23}\bar{\delta}_{34}\mathcal{L}\{\gamma_A, r_2^*\} \end{bmatrix}.$$

The transitional dynamics follow from the analytical impulse response functions, which can be derived following the steps set out in Bom, Heijdra, and Ligthart (2010). As can be seen from (20)–(21), the most general shock takes the following form:

$$\gamma_i\,(t) = \pi_{ip} + \pi_{it}e^{-\sigma_G t}, \qquad \text{for } i = \{q, A\}, \quad \text{(A.18)}$$

where

$$\begin{aligned} \pi_{qp} &\equiv \frac{r\theta_K}{\sigma_Y\omega_A}\left[\xi_{yg} + (\sigma_Y - 1)\eta_K\right]\tilde{I}_G, \qquad & \pi_{qt} \equiv -\frac{r\theta_K}{\sigma_Y\omega_A}[\xi_{yg} + (\sigma_Y - 1)\eta_K]\tilde{I}_G, \\ \pi_{Ap} &\equiv -r\left(\omega_w\xi_{wg} - \omega_G^I\right)\tilde{I}_G, & \pi_{At} \equiv r\omega_w\xi_{wg}\tilde{I}_G. \end{aligned}$$

We employ the following definitions for the temporary transition terms $\mathbf{T}_l(.)$ for $l = \{1, 2, 3\}$

[31] The denominator on the right-hand side of (A.16) is zero. The only way to obtain bounded solutions for the four key variables is that the numerator on the right-hand side is also zero.

and the adjustment term $\mathbf{A}(.)$:

$$\begin{aligned}
\mathbf{T_1}(x,u,t) &\equiv \frac{e^{-xt}-e^{-ut}}{u-x}, \qquad x\neq u, && \text{(A.19)}\\
\mathbf{T_2}(x,u,t) &\equiv \frac{ue^{-ut}-xe^{-xt}}{u-x}, \qquad x\neq u, && \text{(A.20)}\\
\mathbf{T_3}(x,u,v,t) &\equiv \frac{1}{u-x}\left(\frac{e^{-xt}-e^{-vt}}{v-x}-\frac{e^{-ut}-e^{-vt}}{v-u}\right), \quad x\neq u, \quad v\neq x, \quad v\neq u, && \text{(A.21)}\\
\mathbf{A}(x,u,t) &\equiv \frac{1}{u-x}\left(\frac{1-e^{-xt}}{x}-\frac{1-e^{-ut}}{u}\right), \qquad x\neq u. && \text{(A.22)}
\end{aligned}$$

The path for the private capital stock is:

$$\begin{aligned}
\tilde{K}(t) &= \bar{\delta}_{12}\tilde{q}(0)\,\mathbf{T_1}(h_1^*,h_2^*,t)-\bar{\delta}_{12}\frac{\pi_{qt}\phi(-\sigma_G)+\pi_{At}\bar{\delta}_{23}\bar{\delta}_{34}}{(r_1^*+\sigma_G)(r_2^*+\sigma_G)}\mathbf{T_3}(h_1^*,h_2^*,\sigma_G,t)\\
&\quad+\bar{\delta}_{12}\frac{\pi_{qp}\left(\bar{\delta}_{34}\bar{\delta}_{43}-\bar{\delta}_{22}\bar{\delta}_{33}\right)-\pi_{Ap}\bar{\delta}_{23}\bar{\delta}_{34}}{r_1^*r_2^*}\mathbf{A}(h_1^*,h_2^*,t). && \text{(A.23)}
\end{aligned}$$

The impulse response function for Tobin's q is:

$$\begin{aligned}
\tilde{q}(t) &= \left[\left(r_1^*+r_2^*-\bar{\delta}_{22}-\bar{\delta}_{33}\right)\tilde{q}(0)+\bar{\delta}_{23}\tilde{X}(0)-(\pi_{qp}+\pi_{qt})\right]\mathbf{T}_1(h_1^*,h_2^*,t)\\
&\quad \tilde{q}(0)\,\mathbf{T}_2(h_1^*,h_2^*,t)+\sigma_G\frac{\bar{\delta}_{23}\bar{\delta}_{34}\pi_{At}+\pi_{qt}\psi(-\sigma_G)}{(r_1^*+\sigma_G)(r_2^*+\sigma_G)}\mathbf{T_3}(h_1^*,h_2^*,\sigma_G,t). && \text{(A.24)}
\end{aligned}$$

The paths for full consumption and financial assets are, respectively:

$$\begin{aligned}
\tilde{X}(t) &= \left[\bar{\delta}_{34}\omega_A\tilde{q}(0)+\left(r_1^*+r_2^*-2\bar{\delta}_{22}\right)\tilde{X}(0)\right]\mathbf{T}_1(h_1^*,h_2^*,t)+\tilde{X}(0)\mathbf{T}_2(h_1^*,h_2^*,t)\\
&\quad-\bar{\delta}_{34}\frac{\bar{\delta}_{12}\bar{\delta}_{41}\pi_{qt}+\pi_{At}\psi(-\sigma_G)}{(r_1^*+\sigma_G)(r_2^*+\sigma_G)}\mathbf{T_3}(h_1^*,h_2^*,\sigma_G,t)\\
&\quad+\bar{\delta}_{12}\bar{\delta}_{34}\frac{\bar{\delta}_{21}\pi_{Ap}-\bar{\delta}_{41}\pi_{qp}}{r_1^*r_2^*}\mathbf{A}(h_1^*,h_2^*,t), && \text{(A.25)}
\end{aligned}$$

$$\begin{aligned}
\tilde{A}(t) &= \left[\omega_A\left(r_1^*+r_2^*-\bar{\delta}_{22}-\bar{\delta}_{33}\right)\tilde{q}(0)+\bar{\delta}_{43}\tilde{X}(0)-(\pi_{Ap}+\pi_{At})\right]\mathbf{T_1}(h_1^*,h_2^*,t)\\
&\quad+\omega_A\tilde{q}(0)\mathbf{T_2}(h_1^*,h_2^*,t)+\bar{\delta}_{12}\bar{\delta}_{33}\frac{\bar{\delta}_{41}\pi_{qp}-\bar{\delta}_{21}\pi_{Ap}}{r_1^*r_2^*}\mathbf{A}(h_1^*,h_2^*,t)\\
&\quad+\left(\sigma_G+\bar{\delta}_{33}\right)\frac{\bar{\delta}_{12}\bar{\delta}_{41}\pi_{qt}+\psi(-\sigma_G)\pi_{At}}{(r_1^*+\sigma_G)(r_2^*+\sigma_G)}\mathbf{T_3}(h_1^*,h_2^*,\sigma_G,t). && \text{(A.26)}
\end{aligned}$$

Table A1: Summary of the Log-Linearized Model

(a) Dynamic Equations

$$\dot{\tilde{K}}(t) = \frac{r\omega_I}{\omega_A}[\tilde{I}(t) - \tilde{K}(t)] \tag{TA.1}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - \frac{r\theta_K}{\sigma_Y \omega_A}[\tilde{Y}(t) - \tilde{K}(t) + (\sigma_Y - 1)\eta_K \tilde{K}_G(t)] \tag{TA.2}$$

$$\dot{\tilde{X}}(t) = (r-\alpha)\left[\tilde{X}(t) - \frac{\tilde{A}(t)}{\omega_A}\right] \tag{TA.3}$$

$$\dot{\tilde{A}}(t) = r\left[\tilde{A}(t) + \omega_w \tilde{w}(t) - \tilde{T}(t) - \omega_X \tilde{X}(t)\right] \tag{TA.4}$$

$$\dot{\tilde{K}}_G(t) = \sigma_G[\tilde{I}_G - \tilde{K}_G(t)] \tag{TA.5}$$

(b) Static Equations:

$$\tilde{q}(t) = \sigma_A[\tilde{I}(t) - \tilde{K}(t)] \tag{TA.6}$$

$$\tilde{w}(t) = \frac{1}{\sigma_Y}\left[\tilde{Y}(t) - \tilde{L}(t) + (\sigma_Y - 1)\,\eta_L \tilde{K}_G(t)\right] \tag{TA.7}$$

$$\tilde{Y}(t) = \theta_K \tilde{K}(t) + \theta_L \tilde{L}(t) + \theta_G \tilde{K}_G(t), \tag{TA.8}$$

$$\tilde{L}(t) = \omega_{LL}[\tilde{w}(t) - \tilde{X}(t)] \tag{TA.9}$$

$$\tilde{C}(t) = \tilde{X}(t) \tag{TA.10}$$

$$\tilde{F}(t) = \tilde{A}(t) - \omega_A[\tilde{q}(t) + \tilde{K}(t)] \tag{TA.11}$$

$$\tilde{T}(t) = \omega_G^I \tilde{I}_G(t) + \omega_G^C \tilde{C}_G(t) \tag{TA.12}$$

Notes: The following definitions are used: $\theta_K \equiv (Y_K K/Y)_0$, $\theta_L \equiv (Y_L L/Y)_0$, $\theta_G \equiv (Y_G K_G/Y)_0$, $\omega_A \equiv r(qK/Y)_0$, $\omega_I \equiv I_0/Y_0$, $\omega_G^C \equiv C_{G0}/Y_0$, $\omega_G^I \equiv I_{G0}/Y_0$, $\omega_w \equiv w_0/Y_0$, $\omega_{LL} \equiv (1-L_0)/L_0$, $\omega_X \equiv X_0/Y_0$, $\sigma_A \equiv -(I/K)(\Phi''/\Phi') > 0$, and $\sigma_G \equiv I_G \Phi'_G(\cdot)/K_G > 0$. A tilde ($\tilde{}$) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C_0$ for most variables. Financial assets, however, are scaled by steady-state output and multiplied by r, for example, $\tilde{A}(t) \equiv r dA(t)/Y_0$. Lump-sum taxes are scaled by output, that is, $\tilde{T}(t) \equiv dT(t)/Y_0$.

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