Essays on bid rigging

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Macht's gut, und danke für den Fisch!
Viszlát, és kösz a halakat!

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Chapter 1

Introduction

What really matters in auction design are attracting entry and preventing collusion, claims Klemperer (2002). Understanding collusion in auctions is important for a number of reasons. The most important claim is that cartels are welfare-damaging. Since manipulating prices raises antitrust concerns, a bidding ring shades its activities, creating information rents. Changing prices lowers the revenue of the auctioneer.

Bid rigging is a prevalent phenomenon. Kawai and Nakabayashi (2015) estimate that about 20 percent of Japanese procurement auctions have been non-competitive in the mid 2000's. While there is no systematic international estimate on the overall effect of bid rigging, the affected market is enormous. Public procurement amounts to between 10 and 25 percent of national GDP in industrialized countries. Connor (2007) presents a large database of long-running cartels and shows that 20 percent of the prosecuted cases in the sample involved bid rigging.

This doctoral thesis contributes to the literature by showing that the source of information asymmetry between cartel members has a profound effect on the feasibility and form of collusion. This point is not without its policy implications. These results can contribute to our understanding on how to combat collusion and promote allocative efficiency.

Bidding markets do not have a uniformly accepted definition. Klemperer (2008) cites a number of criteria usually associated with them. In order to provide either a legal or scientific definition, one has to apply a range of structural assumptions. However, not all markets usually perceived as bidding markets satisfy these criteria. While we always have a greater set in mind, in each chapter of this thesis, we use a structured model limiting our attention to a particular set of markets. We focus on two key features. First, competition results in sharp differences in market outcome. In extreme case, a "winner-take-it-all" rule applies. Second, a set of rules determines the outcome. These points can be effectively summarized by showing that efficiency in a bidding market is mainly a matter of allocation. The rules determine the winning players and this outcome implies allocative efficiency.

It is commonly accepted that competition between players is welfare improving. In protecting competition, the seller has an important role. Previous literature focuses on the rules of the bidding process. While it is commonly acknowledged that this toolbar can be limited by legal and practical barriers or by agency problems, several effective methods are identified by which the bid-taker can increase competition. In a single auction, the seller might be able to apply different auction mechanisms, discrimination between participants or reserve prices. Public disclosure of information is a part of this toolbar. We define it as a particular form of information revelation, by which the seller provides verifiable information to all bidders, which reduces the uncertainty regarding valuations.

Public disclosure is endorsed in economic literature and policy making alike. Milgrom and Weber (1982) claim its effect is always positive in a competitive setting. The argument is referred to as the Linkage Principle and cited as one of the fundamental results of auction theory. The intuition behind it is that bidders should be provided with all available information by the auctioneer, so that information rents are lower and winning bids are higher.

Disclosure has policy relevance. The seller or the agent is able to change the setting by revealing private information. In the context of a construction procurement tender, disclosure can take a range of forms, including site visits, cost estimates, formal meetings, contractual terms and conditions. This thesis challenges the Linkage Principle and shows that full disclosure can be harmful. In the spirit of Motta (2004), we focus on the question: How to prevent collusion?

Economics of antitrust addresses the issue of its underlying causes and potential means of prevention. Previous literature contributes to our understanding of many facilitating factors. A number of aspects affecting cartel formation have been identified including the auction mechanism (Klemperer, 1999; Lopomo et al., 2011a; Klemperer, 2002, 2007), bidder registration (Samkharadze, 2012), reserve price (Graham and Marshall, 1987), round-up rules (Salant, 2000; Cramton and Schwartz, 2000), auctioning of entry licenses (Offerman and Potters, 2006), revelation of reference prices (Armantier et al., 2013) and the symmetry of bidders (Mailath and Zemsky, 1991).

When it comes to cartels, the source of information asymmetry attracts little attention. However, theoretical and empirical research confirm that the information structure affects expected revenue, efficiency and even the form of collusion, confirming the relevance of our research topic. The distinction between idiosyncratic and common effects on valuations has been studied by the economic literature for decades. Extreme cases, the private value (PV) and common value (CV) models are widely used as models of typical auction markets. The novelty of my approach is that the relationship between these two are analyzed. The bottom line in using this framework is that it allows us to study information disclosure and efficiency concerns in the same model.

The first paper studies the effect of changing this information structure in a theory model. Full public information disclosure by the seller is supported in the literature. This chapter models disclosure as elimination of CV uncertainty. We conclude that this process makes collusion incentive compatible and reduces expected revenue. The second paper extends this to a broader setting and provides a reasoning for the existence of actual collusive mechanisms using pre-auction bids. Finally, the last chapter addresses efficiency in a laboratory experiment. We conclude that collusion reduces efficiency, even if explicit communication is feasible between cartel members.

In the single-authored Chapter 2, I study the question of optimal disclosure and the existence of collusive equilibria in sealed-bid auctions. Several antitrust cases involve only a single auction or procurement. The paper models bid rigging in which cartel members are able to communicate and send side-payments prior to the auction. My research investigates the existence of incentive compatible collusive mechanisms under different levels of common value uncertainty. I show that no collusive mechanism is incentive compatible if the relative weight of common value uncertainty with respect to private value uncertainty is sufficiently large. The Linkage Principle claims that expected revenue of the auction is a decreasing function of common value uncertainty. I find that this monotonicity result is not guaranteed under collusion, and that disclosure can be harmful.

In Chapter 3 (single-authored), I extend the focus of this research by addressing the model of collusive mechanisms. Previous literature builds models assuming a ring is only formed if the mechanism is incentive compatible. Actual antitrust cases show that rings typically conduct a secondary auction instead of sharing all private information. Unless the cartel is strong enough to control the bids, which rarely happens, ring members can enforce compliance with agreement by credibly committing to a high bid. Under individual private values, this is an easy task, and there exists an incentive compatible collusive mechanism. However, if common value uncertainty is present, I show that this is not possible, since there are bidder types who misreport their values. The problem is solved by the novel Bayesian bid coordination mechanism (BBCM). The chapter shows that if not all private information is revealed within the cartel, a knockout auction is supported as a collusive equilibrium, through which ring members are able to manipulate prices, but they do not always succeed in coordinating their bidding strategy.

It is well known that losses in revenue by collusion can be canceled out by efficiency gains. The ring can potentially pool information which ultimately results in higher winning bids. The prediction is that this effect is counterbalanced by suppressed competition. In Chapter 4 (Co-authored with Charles N. Noussair) we use an experimental design to identify these parameters. The model is related to the one in Chapter 3, except for the specifics about the strength of the ring. After an agreement has been reached, participation in the auction is enforced, and only the designated bidder is allowed to submit a bid. Three treatments concern the relative sources of information asymmetry. Valuation of subjects includes an individual private value (PV) component with a given finite support and uniform distribution. Treatments differ in an additively separable second component, like in Chapter 3. The support is either 0 forming a pure PV setting; small, or large. Cartel formation is endogenous and only takes place if both randomly matched players agree after privately observing types. An additional treatment precludes collusion. This design allows for comparing collusion and competition.

Despite the possibility of unrestricted communication between ring members, we find that collusion reduces efficiency. While prices are lower in auctions with colluding subjects across all treatments, the cartels are unable to reach the maximal collusive gain due to the lower level of allocative efficiency. That is, the cartel tends to fail in helping the subject with the higher valuation win the game. Experimental data shows that the majority of subjects truthfully reveal private information and update beliefs. However, the improved information set does not translate into improved allocation. The conclusion is that the overall effect of collusion is negative. The main reason behind this failure is that subjects fail to use the available extra information and bid non-strategically in the pre-auction knockout.

In all chapters, auctions are modeled in the standard way in which the auctioneer assumes the role of the seller, and bidders are the buyers. We have in mind that many of our motivating examples are reverse auctions, and the results are applicable to both settings. Notations follow an analogous convention in all chapters. Bidders are denoted by lower indices, in general by i. Index $-i$ refers to cartel members other than i. Types are denoted by x_i for private and y_i for common value signals. Elicited values are distinguished by an asterisk superscript *. Expected value is noted by $\mathbb{E}(\cdot)$.

CHAPTER 2

ON THE FAILURE OF THE LINKAGE PRINCIPLE WITH COLLUDING BIDDERS

Abstract

Previous literature has shown that public information disclosure increases expected revenue of the seller in non-cooperative auctions. The Linkage Principle has been contested in collusive settings where information release is conditional on cartel action. We extend this result by showing that *ex ante* provision of private information can sustain collusion and decrease expected revenue. This observation is derived in a model with private and common value components. We show that the existence of an incentive compatible bid coordination mechanism depends on the source of information asymmetry between bidders. While an incentive compatible mechanism exists in a pure private value model, it fails to exist if valuations are sufficiently correlated. Thus, full public disclosure might not be optimal for the seller.

2.1. Introduction

In auctions with non-cooperative bidders, increased transparency decreases information asymmetry between market actors, and thereby increases competition. This result, attributed to Milgrom and Weber (1982), is often cited as the Linkage Principle. The picture is less clear if players can collude, since disclosure can be a coordinating tool of a price-fixing cartel. Disclosure also provides a convenient tool for monitoring compliance with a cartel agreement. Suppose a seller has private information about the buyers' valuations and is able to reveal it. In an auction with non-cooperative bidders, the optimal choice of a risk-neutral seller is full disclosure. Milgrom and Weber (1982), Goeree and Offerman (2003) and Fatima et al. (2005) point out that lower common value (CV) uncertainty increases expected revenue. They argue that reduction of uncertainty by public disclosure results in lower information rents and reduces the extent of the winner's curse.

We focus on disclosure in the context of sealed-bid auctions. Revelation of private information of the seller increases the expected revenue in non-cooperative settings. Transparency is not necessarily the best policy and it is discouraged in procurement auctions in order to combat collusion (OECD, 2014). While international procurement guidelines recognize the tension between maximizing competition and deterring collusion, there is no formal theory regarding this matter. The main contribution of our model is to provide a theoretical framework. We show that public disclosure of information by the seller helps cartel formation by facilitating an incentive compatible mechanism. Collusion reduces revenue by bid suppression, so that the effect of public disclosure decreases seller profit.

In an auction context, public disclosure reduces the uncertainty about the valuation of all bidders. A long-standing bidding ring studied by Asker (2010) was facing resale opportunities. In procurement, tenders involve potential costs. These are factors the auctioneer might have private information about. Bidders can also possess idiosyncratic preferences.

We model this dichotomy with CV and private value (PV) uncertainty.[1] A model including PV and CV information asymmetries allows us to consider a seller optimizing over disclosure. The pure independent PV model assumes that valuations are conditionally independent. Thus, a seller possesses no private information about buyers' valuations. In a CV framework, collusion is only possible under fairly strong conditions. There exists a collusive mechanism if the cartel is able to communicate, members weakly prefer collusion and the cartel has full control over members' bids (McAfee and McMillan, 1992).

Our model considers a one-shot sealed-bid auction setting for a single, indivisible good.[2] The valuation of bidders is modeled by additively separable, binary private and common value elements. Disclosure is modeled as changing the distribution of common values while keeping *ex ante* expected valuation constant.

Related literature emphasizes the negative effects of disclosure in auctions by focusing exclusively on its dynamic effects. Marshall and Marx (2009) analyze a one-shot independent private values (IPV) auction with a registration process. They point out that a seller is able to reduce the cartel's revenue by choosing a less transparent regime of participant registration. Ascending-bid auctions are susceptible to collusion if participants are identifiable. Samkharadze (2012) addresses the problem in a two-stage procurement

[1]While the vast majority of research employs only pure models, the assumption that values are exclusively private or common rarely holds in actual auctions (Laffont, 1997; Goeree and Offerman, 2003).

[2]One-shot auctions are often subjected to collusive schemes, as evidenced in US v. WF Brinkley & Son Const. Co. (1986), US v. AAA Elec. Co. Inc. (1986), US v. Metropolitan Enterprises Inc. (1984), US v. Reicher (1992), US v. MMR Corp. (1992), US v. Rose (2006), US v. Green (2010) and US, EX REL. McGEE v. IBM Corporation (2015).

setting in which the buyer is able to reveal private information to the sellers between stages. The policy of public information revelation decreases expected payoff if bidders form a ring.

We borrow the concept of bid coordination mechanism (BCM) from Marshall and Marx (2007). That is, ring members are able to communicate and send side-payments to each other. We show that, in an IPV model, the cartel is able to form and bidders truthfully reveal their types. Such mechanisms are available in the form of a pre-auction knockout, in which members bid for the right to bid for the good and determine side-payments. In a knock-out auction, the member with the highest valuation will be the designated bidder. Incentive compatibility is ensured by the availability of side-payments and the lower price resulting from the other bidders suppressing their bids. For CV, no incentive compatible mechanism is available. In this setting, colluding players share the same information set and the same expected valuation. Therefore, there are strong incentives to misreport one's type. Information pooling does not help in choosing the efficient buyer. Consequently, incentive compatibility of any mechanism is problematic.

If an incentive compatible mechanism is available, the ring is able to maximize its surplus by choosing the member with the highest PV to bid. The other advantage of a collusive agreement is information pooling. Accordingly, the possibility of sharing CV signals is increasing the incentives to form a bidding ring. Although we do not dispute this notion, we point out that higher CV variance is able to destroy a collusive agreement. With higher variance of the CV term, a bidder is able to alter its report to the cartel to a larger extent. Hence, it is able to manipulate the designated bid. The effect of higher CV uncertainty destroys the incentive compatibility of the mechanism, since participants can anticipate this behavior.

In the framework of Milgrom and Weber (1982), disclosure not only allows the seller to reduce common uncertainty, but also brings the auction market closer to the IPV model. Our model illustrates that it also facilitates collusion, and claims that the Linkage Principle does not hold if bidders can engage in a conspiracy. Section 2.2 builds up the framework of a hybrid auction model and derives the necessary and sufficient conditions for the existence of incentive compatible collusive mechanisms. The results apply to a class of sealed-bid auction mechanisms. We focus on the ability of the bidding ring to suppress internal competition. In Section 2.3, we apply these results to second-price auctions, and we provide an example of the disclosure effect on collusion.

In our model with collusive bidders, revenue is not an increasing function of the information available, as opposed to the non-cooperative model analyzed by Milgrom and Weber (1982) and Goeree and Offerman (2003). In Section 2.4 we prove that the pure private and common value models are robust to perturbation by the other source of information asymmetry. Thus, if we introduce small common value perturbations in a pure private value model, collusion remains feasible. Similarly, there is no incentive compatible bid coordination mechanism in the neighborhood of a pure common value model. Section 2.5 shows that the well-established result of increasing revenue with respect to common value uncertainty does not hold if bidders can form a bidding ring. Finally, Section 2.6 concludes.

2.2. Collusion in a hybrid model

This section constructs an auction model with additively separable values[3] applying the model of McLean and Postlewaite (2004). In the present context, we refer to this setup as the hybrid model, following Milgrom and Weber (1982). Common uncertainty experienced by all players can be modeled by the distribution of the common value, which is taken by bidders as given. A number of bidders may form a bidding ring before participating in a sealed-bid auction. Subsection 2.2.1 constructs the model. The concept of incentive compatible collusive mechanisms is introduced in Subsection 2.2.2 where we characterize its existence. In all cases, we consider perfect Bayesian Nash equilibria (PBNE).

2.2.1. Valuation of bidders

We assume that risk-neutral and symmetric players bid for a single commodity. Bidder i receives a two-dimensional signal (x_i, y_i), where x_i is the independent private value (PV) component of a bidder's valuation, and it is a random variable with discrete probability distribution $x_i \in \{x_L, x_H\}$ with equal probabilities, $x_L \leq x_H$.

The common value (CV) is denoted by y and $y \in \{y_L, y_H\}$ with equal probabilities, where $y_L \leq y_H$. Signal y_i is observed by bidder i, which can take $y_i \in \{y_L, y_H\}$ where $y_i = y$ with probability $\delta \in \left(\frac{1}{2}, 1\right)$. For any y and for $i \neq j$, y_i and y_j are conditionally independent. Valuation v_i of bidder i is equal to the sum of her PV signal and the CV, $v_i = x_i + y$. Hence, bidders face common uncertainty about their valuations and individual valuations can be different.[4]

[3] This is a standard assumption for private and common values. Pesendorfer and Swinkels (2000), Goeree and Offerman (2003) and Fatima et al. (2005) also analyze models with additively separable values.

[4] This paper models CV uncertainty as the spread of CV types. We can note that δ also captures uncertainty and it is able to model disclosure. Changing δ has two effects. Extreme values $\delta = 0.5$ and $\delta = 1$ both represent a signal carrying no CV asymmetry between bidders and expected payoffs are identical to risk-neutral bidders. So, there is collusion in the neighborhood of extreme values, as illustrated in Figure 2.1. Most importantly, it also has a non-monotonic effect on expected revenue. Too high δ induces collusion and results in a negative drop at this point of discontinuity.

This model can be linked to the hybrid model by Milgrom and Weber (1982). They assume that bidders' private information can be expressed by single real-valued informational variables which have affiliated densities.[5] The construction of parameter δ ensures positive affiliation. If someone receives a high signal, the conditional expected value of another player's signal is also higher. We can define an informational variable simply as the sum of signals, which generally identifies both components.[6] Discrete distribution of both components allows for a solution for the single-valued representation.

We consider two standard auction mechanisms. Both are sealed-bid formats, bids are submitted simultaneously. A bid is a non-negative value b_i by which the player submitting the highest bid wins a non-divisible commodity. The price is paid only by the winner. In the first-price auction, this equals the bid of the winner. In the second-price auction, this is equal to the second highest bid.[7]

2.2.2. Equilibrium with bid coordination mechanism

A collusive mechanism is a function determining a bidding strategy and side-payments among ring members, conditional on signals they send to each other prior to the auction. Models of collusive mechanisms in auctions distinguish cartel types according to their ability to communicate, to verify information, to make transfer payments and to control bids (McAfee and McMillan, 1992). We apply the concept of bid coordination mechanism (BCM) (Marshall and Marx, 2007; Lopomo et al., 2011b). A BCM allows for pre-auction side-payments[8] and a set of recommended bids as a function of signals about ring member types. Side-payments serve as an incentive device in setting lacking repeated interaction.[9]

Surplus of bid rigging comes from suppressing competition and pooling available information. Pre-auction transfers are necessary in order to incentivize ring members.[10]

[5]We define affiliation following the simple definition of Castro (2010). Although this defines affiliation for density functions, we can generalize it for any probability distribution. We say that the density function $f : [\underline{t}, \bar{t}] \to \mathbb{R}_+$ is affiliated, if for any t, t', we have $f(t) f'(t) \leq f(t \wedge t') f(t \vee t')$, at which $t \wedge t' = (min\{t_1, t_1'\}, \ldots, min\{t_n, t_n'\})$ and $t \vee t' = (max\{t_1, t_1'\}, \ldots, max\{t_n, t_n'\})$. The concept is called Multivariate Total Positivity of Order 2 (MTP2) for the multivariate case by Karlin (1968).

[6]Milgrom and Weber (1982) assume that individual valuations are determined by informational variables and a number of non-observed variables. We can construct these non-observables as the difference between informational variables and real individual valuations resulting in: $w_i = y - y_i$.

[7]We do not directly address the problem of setting a reserve price. While it is relevant in a pure independent PV model, Levin and Smith (1996) show that the revenue-maximizing reserve price monotonically and often rapidly converges to the seller's valuation as the number of bidders grows.

[8]McAfee and McMillan (1992) model *ex post* knockout auctions, which are common in practice. However, they are subjected to *ex post* inefficiency, that is, the designated bidder might post a bid higher than any bid in the knockout.

[9]For a list of cartel cases involving side-payments see Marshall and Marx (2009).

[10]In practice, transfers typically come indirectly. Kovacic et al. (2006) emphasizes they often come in the form of subcontracts.

On the other hand, punishment for deviating from the cartel agreement can be costly.[11] We also assume recommended bids cannot be enforced by the ring.[12]

The majority of theoretical models consider incentive compatible collusive mechanisms with symmetric bidders (Graham and Marshall, 1987; McAfee and McMillan, 1992; Marshall and Marx, 2007). We also also assume players are *ex ante* symmetric with respect to information variables (x_i, y_i). Following Marshall and Marx (2007), we consider an exogenously determined ring of $n \geq 2$ members, where the set of cartel members is denoted by N. There are $k \geq 0$ outsiders. If a ring faces at least one outsider, it is called a non-inclusive ring. All-inclusive rings encompass all players. We assume the set of players and ring membership are exogenously given.

An outsider bidder j, if any, plays according to a given pure strategy $\alpha_j(x_j, y_j)$. We assume the only arguments of this function are the informational variables observed by the player. Also, the outsider is not a strategic player in the sense that her strategy is independent of the existence of the ring and it is not necessarily a best response to the ring members' strategies.[13] Nevertheless, cartel members' strategy maximize their expected payoff considering the outsider strategy. In what follows we apply the simplified notation of $\alpha(\cdot)$.

Formally, a BCM is a function

$$\mu(x^*, y^*) = (\beta(x^*, y^*), p(x^*, y^*))$$

where (x^*, y^*) denotes the vector of signals simultaneously shared within the ring, indicating PV and CV signals. Vector $\beta(\cdot)$ represents recommended bids and $p(\cdot)$ is the normalized side-payment vector. That is, $p_i(\cdot)$ is the amount ring member i receives from other members, and the sum of components is $\sum p_i(\cdot) = 0$, satisfying *ex post* budget balance. The timing of the game is as follows.

1. Ring members learn mechanism $\mu(\cdot)$.
2. They make a decision about participation.[14]
3. They learn their types (x_i, y_i).

[11] For a study on the applied model with possibility of *ex post* actions, see Marshall and Marx (2009).

[12] Enforcement can come from punishment mechanism either externally (organized crime) or in the form of a grim trigger strategy (McAfee and McMillan, 1992; Mailath and Zemsky, 1991). Since all these examples stem from a repeated game, it is arguable that such tools are not available for the ring in a one-shot setup. Another standard way is to employ an agent submitting all bids. This is difficult to organize, since it assumes anonymity. Asker (2010) demonstrates this on a stamp-dealer cartel which participated in open auctions with no legal entry barriers.

[13] An outsider with hard evidence can turn to the authorities.

[14] Due to *ex ante* symmetry, they make a unanimous decision.

4. If a ring is formed, members share signals (x^*, y^*) simultaneously. Following the mechanism, members learn the recommended bids $\beta(x^*, y^*)$, side-payments $p(x^*, y^*)$ are enforced and implemented.[15]

5. Players submit their bids in the auction.

Conditions are detailed below.

Function Π_i denotes the expected payoff (with side-payments) of ring member i. We say that $\mu(\cdot)$ is a BCM against outside bid function $\alpha(\cdot)$, if conditions (2.1), (2.2) and (2.3) hold. We denote expected values over all bidder types with $E(\cdot)$, all payoffs and subscripts refer to ring members, subscript $-i$ refers to members of the ring other than member i.

$$(x_i, y_i) \in arg\, max_{x_i^*, y_i^*}\, \mathbb{E}\left(\Pi_i(\cdot)\,|\mu(\cdot), x_{-i}^* = x_{-i}, y_{-i}^* = y_{-i}, \alpha(\cdot)\right), \forall i \tag{2.1}$$

$$\beta_i(x^*, y^*) \in arg\, max\, \mathbb{E}\left(\Pi_i(\cdot)\,|x^*, y^*, \beta_{-i}, \alpha(\cdot)\right) \tag{2.2}$$

$$\beta(x^*, y^*) \in \arg\max\, \mathbb{E}\left(\sum_i \Pi_i(\cdot)\,|x^*, y^*, \alpha(\cdot)\right) \tag{2.3}$$

Thus, incentive compatibility has the following requirements. Condition (2.1) requires that members find it optimal to truthfully reveal their types. Condition (2.2) captures the idea that recommended bids are not enforced, following them must be optimal for ring members. Finally, condition (2.3) concerns the optimal collusive strategy, which is achieved if the sum of their payoffs is maximal. We say that the ring is able to suppress all ring competition in that case. Our definition of BCM differs from Marshall and Marx (2007), in that we also consider all-inclusive rings.[16]

In the spirit of Marshall and Marx (2007), the mechanism does not involve randomization, except for a tie-breaking rule. If a number of mechanisms $\mu(\cdot)$ are permutations of side-payments and recommended bids, and they provide the same expected payoff for the ring, a mixed mechanism is applied, in which one of them are chosen randomly with the same probability. It is easy to see there is a finite number of permutations and this

[15] *Ex ante* implementation of side-payments is important to avoid costly re-negotiations and rent-seeking Marshall and Marx (2012).

[16] In case there are no outsiders, we shall assume an outside bid function $\alpha(\cdot) = 0$.

happens if and only if the permutation is between members who report the same PV, following Condition (2.3).

Additionally, we only consider individually rational mechanisms.[17] That is, in which participation provides higher *ex ante* expected payoff than the competitive game.[18] Ring members coordinate their bids. This includes the possibility of the competitive equilibrium strategy. Therefore, the sum of the ring members' payoffs is at least as high as without the collusive mechanism.[19] Consequently, any BCM satisfies the *ex ante* participation constraint.

BCM is a direct mechanism. This consideration is followed by the Revelation Principle for BNE (Fudenberg and Tirole, 1991). That is, if a BNE implements a certain choice, it is also truthfully implementable. At this point we assume distribution of CV signals is given. The problem of the seller is addressed in Section 2.5.

It follows from Condition (2.3) that, if there is a BCM, there exists a mechanism in which all members submit 0, except for one. We can further restrict our attention to a subset of incentive compatible mechanisms, as pointed out in Lemma 2.1.

Lemma 2.1. *Suppose there exists an incentive compatible BCM. Then, there exists an incentive compatible BCM, in which the designated bidder is the member with the highest PV.*

Proof. A BCM $\mu(\cdot)$ allots recommended bids to the ring. We refer to the ring member with a positive bid as designated bidder. This is the member with the highest PV (without loss of generality, member 1). Consider $\mu(\cdot)$, incentive compatible, in which there is at least one pair $(x^*, y^*), (x^*, y^{*\prime})$, such that the designated bidders are different. That is, there is at least one of these pairs in which the designated bidder is not the one with the highest PV (member 2). Payoffs can be weakly improved by switching the designated bidder's role to bidder 1 in mechanism $\mu'(\cdot)$. The side-payment of player 2 shall be equal to the expected profit of being a designated player according to mechanism $\mu(\cdot)$. That is, all surplus from choosing the efficient buyer goes to player 1 in mechanism $\mu'(\cdot)$. Thus, $\mu'(\cdot)$ also results in an incentive compatible solution, since all constraints remain identical. *Q.E.D.*

[17] In other words, it needs to satisfy the weak participation constraint, as defined by Borgers et al. (2015).

[18] For the latter one we consider a Bayesian Nash equilibrium (BNE) in which outsiders are non-strategic. As define before, they play according to $\alpha(x_j, y_j)$. In Section 2.3, we show an example in which one outsider plays according to the symmetric BNE of the competitive game, but in what follows we do not make this assumption.

[19] The participation constraint holds, if outsiders are strategic and they bid their best response with respect to the ring. In that case, outsiders bid less aggressively, which increases the ring's payoff.

This consideration of restricting attention to the PV here stems from the information pooling function of the ring. If someone with a strictly lower PV becomes the designated bidder, her partner with higher value would have higher payoff, consequently, she would have incentives to bid higher than her partner in the auction. We point out that there is usually a continuum of BCMs, if any. The non-designated player can submit a sufficiently low bid, which does not increase the expected payment, conditional on winning.[20]

We can see that a BCM defines a side-payment vector p of dimension $n \cdot 2^{2n}$. The n ring members send $2n$ signals, and all of them can attain two possible values. This defines 2^{2n} profiles, which are applied to all ring members. Similarly, the recommended bid vector can be expressed as a vector of $n \cdot 2^{2n}$ dimensions. There is a designated bidder who is randomly chosen among the bidders with the highest PV.

Lemma 2.2. *Suppose that the set of BCMs M_D is non-empty. Then, if we take $\mu = \left(\hat{p}, \hat{\beta}\right) \in M_D$, the set of p for which $\mu = \left(p, \hat{\beta}\right) \in M_D$ is convex.*

Proof. Given that there is a designated bidder receiving an optimal recommended strategy, all types shall be truthfully revealed according to Condition (2.1). That is, no type finds it better to misreport. There are 4 possible types, each defining 3 incentive compatibility constraints, together 12. On both sides of this equation, the payoffs are a linear function of side-payment components in p. Since $\mathbb{R}^{n \cdot 2^{2n}}$ is convex, the resulting set is also convex.[21] *Q.E.D.*

Lemma 2.2 characterizes the set of incentive compatible mechanisms. Convexity implies that if two BCMs with the same recommended bid function are incentive compatible, so are their convex combinations. Ring members are *ex ante* symmetric, so incentive compatibility is maintained if we permute them. The linear combination of such mechanisms results in a symmetric mechanism with respect to the ring members. That is, if M_D is non-empty, there exists a mechanism so that the side-payment only depends on the number of high private and common value reports within the ring and the own type.

Accordingly, we can apply the notation $p\left(|x_H|, |y_H|\right)$, which is the sum of the amounts that bidders with low reported PV receive. Similarly, an equal aggregate amount is subtracted from those who have high PV. Since there are $n - |x_H|$ members with low and $|x_H|$ members with high PV, the side-payment of each ring member with high PV equals $-\frac{1}{|x_H|} p\left(|x_H|, |y_H|\right)$. Similarly, the same amount for members with low PV report

[20] Any value between 0 and the lowest equilibrium outside bid can be a complementary or cover bid if it does not affect the price.

[21] Intersection of convex sets is always convex (Simon et al., 1994).

equals $\frac{1}{n-|x_H|}p\left(|x_H|,|y_H|\right)$. The sum of these values equals zero, so *ex post* budget balance is satisfied.

To sum up, if there is an incentive compatible mechanism, there is one in which the side-payment and the recommended bid of a member only depends on the number of certain signals within the group and the own PV. In addition to the PV of the designated bidder, distribution of CV signals determine the maximal expected gain from participating in the auction.

The existence of a BCM depends on the auction format and the extent of CV uncertainty. Our points are formally stated in Proposition 2.1 and 2.2, which serve as the main results of our paper for second- and first-price auctions, respectively. Our propositions serve as an extension of the results of Marshall and Marx (2007). First, they point out that CV variance affects the existence of BCM. Second, conditionally on the existence of incentive compatible BCM, they confirm the results hold for positive CV variance.

Without loss of generality, the designated bidder is denoted by 1, whereas index -1 refers to non-designated ring members. The expected payoffs always use the following notations. Function $\pi\left(x_1,|y_H|,\alpha\left(\cdot\right)\right)$ represents the expected payoff of a bidder as a function of her own PV and the number of high CV signals among $n-1$ other bidders, who submit zero bid. The remaining bidders are assumed to follow strategy $\alpha\left(\cdot\right)$, as defined earlier. Function $\pi\left(x_1,|y_H|,\alpha\left(\cdot\right)\right)$ only takes the outcome of the auction into account. That is, the designated bidder's total expected payoff is $\pi\left(x_1,|y_H|,\alpha\left(\cdot\right)\right)-p\left(|x_H|,|y_H|\right)$, conditional on truthfully reported signals.

Proposition 2.1. *Suppose there is a given bidding ring with n members and $k \geq 0$ outsiders in a sealed-bid second-price auction. There exists a BCM if and only if* (2.4) *is satisfied.*[22]

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|=i,|y_H|=j \;\middle|\; y_1=y_L\right)\frac{1}{n-i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)\geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|=i,|y_H|=j \;\middle|\; y_1=y_H\right)\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right) \tag{2.4}$$

Proof. See Appendix A.[23] *Q.E.D.*

Proposition 2.1 provides a necessary and sufficient condition for the existence of an

[22] On the RHS we can see that the expected revenue function captures a case when the designated bidder has low PV while the number of ring members with high PV's is positive. This side of the inequality comes from an incentive compatibility constraint capturing misreported PV type. See Appendix A.

[23] Appendix A also provides an equivalent form of the inequality (2.4), with probability values written explicitly.

incentive compatible BCM in second-price auctions. This result can be interpreted as follows. There exists a BCM if and only if the relative CV uncertainty is greater than the PV uncertainty. This is highlighted by the two sides of (2.4). The left-hand side (LHS) of the inequality employs high own PV and lower CV signals, while the right-hand side (RHS) has low PV and higher CVs. If the relative CV uncertainty becomes greater, collusion breaks down.

On the contrary, if (2.4) is not satisfied, there is no incentive compatible side-payment vector. Here side-payments must satisfy a two-fold role: providing incentives not to overreport if CV or PV is low, and not to underreport if it is high. Higher CV uncertainty makes this more difficult. Members with low PV can receive high CV signals, making the role of designated bidder more attractive. Also, a member with high PV and low CV signal perceives the role of the designated bidder as less attractive if the CV variance is higher.

If (2.4) holds, there exists a respective recommended bid function, $\beta(\cdot)$ which ring members follow, and which maximizes collusive gains. For an all-inclusive cartel, the existence is clear, a sufficiently high bid of the designated bidder deters other members from bidding higher. Proposition 2.1 extends this to non-inclusive cartels. The supremum of the set of best responses of the designated bidder to outsider strategies is a solution. The other bidders have lower valuation than the designated bidder. Bidding higher than the designated bidder's optimal bid results in a positive payoff if and only if the designated bidder could increase her own payoff by bidding higher. This contradicts best-response bidding.

Inequality (5) shows an explicit example for Proposition 2.1 with $n = 2$, $x_L = 0$, $x_H = 1$, $y_L = -z$ and $x_L = z$ with $z \geq 0$. This set of cases covers the normalization of the entire parameter space, and by z we can model the effect of CV information asymmetry by keeping the *ex ante* expected value of the CV term constant. As before, $\pi(\cdot)$ captures expected payoff of the designated bidder without side-payments, with a given information set. The first argument refers to the PV of the designated bidder, the second is the number of high CV signals of the ring. In the example, we write the probabilities explicitly using exogenous parameter δ, which expresses the quality of CV signals. Higher values mean better signals.

$$\begin{aligned}&\left(\delta^2 + (1-\delta)^2\right)\pi\left(1, -z, -z, \alpha(\cdot)\right) + \left(1 - \delta^2 - (1-\delta)^2\right)\pi\left(1, z, -z, \alpha(\cdot)\right) \geq \\ &\left(\delta^2 + (1-\delta)^2\right)\pi\left(0, z, z, \alpha(\cdot)\right) + \left(1 - \delta^2 - (1-\delta)^2\right)\pi\left(0, z, -z, \alpha(\cdot)\right)\end{aligned} \tag{2.5}$$

The interpretation of our result stems from the relative importance of the PV as in the case of (2.4). Collusion is feasible, if payoff generated by high PV's is higher than the payoff for low PV with higher CV signals. If CV uncertainty decreases, in other words, z is lower, the LHS becomes relatively higher, making collusive agreements incentive compatible. Note that the value of z does not change the *ex ante* expected valuation of bidders.

Above we discussed second-price auctions in which price is independent of the highest submitted bid. In a first-price auction the designated bidder faces a threat that other ring members might outbid her. This threat results in a suboptimal collusive outcome. Let us see an example. Suppose there are two members of an all-inclusive ring with valuations equal to $x_1 = \frac{3}{2}$ and $x_2 = \frac{1}{2}$ in a pure PV auction with $z = 0$, so without CV uncertainty. Bidders truthfully reveal their types and make bidder 1 designated bidder. In a second-price auction after side-payments are paid, it is an equilibrium that bidder 1 submits 1, or any value higher than $\frac{1}{2}$ and bidder 2 submits 0. It is clear that they will comply with the agreement and they pay 0, maximizing the collusive gain. In a first-price auction, the ring is unable to achieve the first-best outcome. Player 2 only follows the recommended bid if bidder 1 submits more than $\frac{1}{2}$, similarly to the previous case, but in a first-price auction this results in a selling price $\frac{1}{2} > 0$.

This is reflected in the incentive compatibility constraints. In the last stage of the game, the designated bidder's expected payoff in the auction depends on the PV of the second highest PV in the ring. We denote this by function $\pi^* (x_1, \max x_{-1}, |y_H|, \alpha(\cdot))$, which is analogously defined as $\pi(\cdot)$. The second argument denotes the highest opposing PV within the ring.

Proposition 2.2. *Suppose there is a given bidding ring with n members and $k \geq 0$ outsiders in a sealed-bid first-price auction. There exists a PNBE in which a bidding ring is formed, types are truthfully revealed (Condition* (2.1)*) and members comply with recommended bids (Condition* (2.2)*), if and only if constraint* (2.6) *is satisfied. In equilibrium, not all ring competition is suppressed.*

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right) \frac{1}{n-i}\pi^* \left(x_1 = x_H, \max x_{-1}, |y_H|= j, \alpha(\cdot)\right) \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right) \frac{1}{i+1}\pi^* \left(x_1 = x_L, \max x_{-1}, |y_H|= j, \alpha(\cdot)\right) \quad (2.6)$$

Proof. Constraint (2.6) is a necessary and sufficient condition for the existence of an incentive compatible mechanism. The derivation is identical to that of Proposition 2.1, but the expected payoff function is different, payoff depends on PV of other ring members. Function $\pi^*(\cdot)$ takes this into consideration, and expresses that expected payoff depends on the highest other PV.

However, there is no BCM. An incentive compatible PBNE means that ring members truthfully reveal their types (Condition 2.1) and they follow recommended bids (Condition 2.2). With a positive weight on the highest bid in the selling price function, the aggregate payoff of the ring is not maximal, Condition 2.3 is violated. The designated bidder shall increase her bid in order to avoid that other ring members violate the agreement by bidding higher. In contrast with second-price auctions, this increases the expected price conditional on winning, since the price is a strictly increasing function of the highest bid.

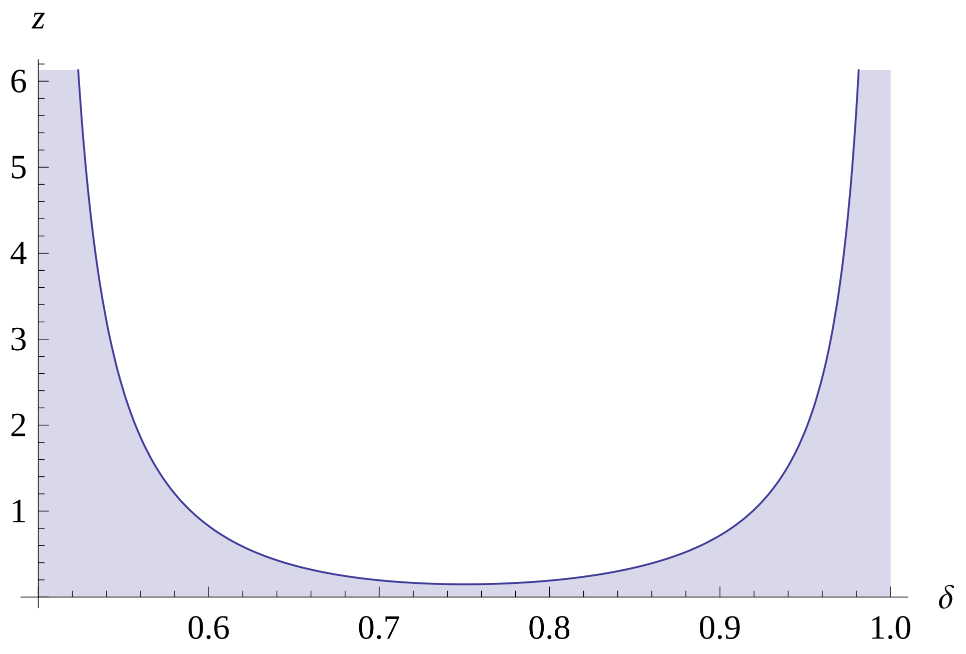
Suppose that all ring competition can be eliminated and the designated bidder bids her best response to outside bid functions (for an inclusive ring, that is 0). In that case, a non-designated bidder with identical PV can be better off by bidding marginally higher, obtaining a positive expected payoff. As such, the designated bidder's bid will be higher than optimal. Positive *ex ante* payoff of the designated bidder is guaranteed by the positive probability of having no other ring member with identical PV. This argument also highlights why the expected revenue $\pi^*(\cdot)$ depends on the type of the highest opposing ring member. *Q.E.D.*

This conclusion corresponds to Marshall and Marx (2007), who concluded that an equilibrium BCM in first-price auctions with individual PVs is not able to suppress all ring competition. Our model extends their results to all-inclusive rings and adds the insights regarding the feasibility of collusion if CV uncertainty is present.

2.3. Example: Second-price auctions

In order to motivate the intuition of Proposition 2.1, we illustrate our results for the case of second-price auctions. The incentive compatibility constraint (2.4) is quite general and the expected payoff function $\pi(\cdot)$ depends on multiple factors. In order to examine the existence of an incentive compatible BCM, an outsider bid function has to be specified. Subsection 2.3.1 concerns the non-cooperative equilibrium in a second-price sealed-bid auction. The existence of an incentive compatible BCM for the example is derived in Subsection 2.3.2.

Figure 2.1: Existence of incentive compatible BCM, two ring members and one outsider, second-price sealed-bid auction.



2.3.1. Non-cooperative equilibrium

As before, we look for a pure-strategy BNE. A numerical example is provided in the Web Appendix for two ring members and one outsider bidder.

There exists a unique symmetric BNE in a bigger class of auctions. Milgrom and Weber (1982) show the existence of a symmetric equilibrium in second-price sealed-bid auctions. Equilibrium bids satisfy that bidders are indifferent between winning and not-winning where the highest opposing bid is identical. This solution comprises the pure PV equilibrium as a special case at which bidders submit their values. Levin and Harstad (1986) also demonstrate that this is the unique symmetric equilibrium.

In the example below, we consider again the case $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$, where $z \geq 0$. Let us determine the expected revenue of the seller in case of three bidders. Given the unique symmetric equilibrium strategy, this value can be determined by the probability distribution of the second highest bid (b_2^*). Now we focus on the case, where $b^*(1, -z) < b^*(0, z)$. The other one can be calculated accordingly. The probability of the second highest bid being equal to the highest possible value can be calculated in the following way. If this is the case, the two highest bids are both equal to $b^*(1, z)$.

The *ex ante* probability of high and low CV is $\frac{1}{2}$. Let us consider $y = z$, and calculate probabilities conditional on that. One can distinguish cases in which the lowest bid takes four different values. It is associated with the highest signal $(1, z)$ with probability $\frac{1}{8}\delta^3$. For signal $(1, -z)$, the probability of this being the lowest bid equals $\frac{3}{8}\delta^2(1-\delta)$, where numerator 3 refers to the three possible bidders having lower bids. Similarly, signals $(0, z)$ and $(0, -z)$ give probabilities $\frac{3}{8}\delta^3$ and $\frac{3}{8}\delta^2(1-\delta)$.

In case $y = -z$, conditional probabilities of high ($y_i = z$) and low ($y_i = -z$) signals are reversed, such that the lowest bid is maximal if all signals are $(1, z)$, which occurs with probability $\frac{1}{8}(1-\delta)^3$. Similarly, the lowest bid with signals $(1, -z)$, $(0, z)$ and $(0, -z)$ give probabilities $\frac{3}{8}\delta(1-\delta)^2$, $\frac{3}{8}(1-\delta)^3$ and $\frac{3}{8}\delta(1-\delta)^2$, respectively. Adding up probability values of the lowest value yields the result in equation (2.7).

$$\begin{aligned} Pr\left(b_2^* = b^*(1, z)\right) &= \frac{1}{16}\left(\delta^3 + 3\delta^2(1-\delta) + 3\delta^3 + 3\delta^2(1-\delta) + (1-\delta)^3\right. \\ &\quad \left. + \; 3\delta(1-\delta)^2 + 3(1-\delta)^3 + 3\delta(1-\delta)^2\right) \\ &= \frac{1}{8}\left(2 - 3\delta + 3\delta^2\right) \end{aligned} \tag{2.7}$$

The probability of the lowest value being the second highest bid is identical. It takes the other two values with equal probabilities, $\frac{1}{8}(2 + 3\delta - 3\delta^2)$. This symmetry is implied by the fact that there are three bidders.

That is, expected revenue with 2 ring members and 1 outsider is expressed as:

$$\begin{aligned} \mathbb{E}R(z) &= \frac{1}{8}\left[\left(2 - 3\delta + 3\delta^2\right)\left(b^*(1, z) + b^*(0, -z)\right)\right. \\ &\quad \left. + \; \left(2 + 3\delta - 3\delta^2\right)\left(b^*(1, -z) + b^*(0, z)\right)\right] \end{aligned} \tag{2.8}$$

2.3.2. Existence of a BCM

We show that the result of Milgrom and Weber (1982) about increasing revenue with respect to CV information asymmetry depends on the non-cooperative behavior of bidders, and does not hold if players are allowed to form a bidding ring. Collusion does not occur on the whole range of parameters. Since the cooperative and non-cooperative outcome differs, the expected revenue function is non-increasing and discontinuous. Throughout this setting we assume the participation of 2 ring members and 1 outsider to illustrate this point.

A crucial result for this conclusion is the set of parameters on which there is an incentive compatible BCM. Values of the side-payment function $\pi(\cdot)$ and the designated bid function $\beta(\cdot)$ depend on the order of non-cooperative equilibrium bids. While the highest value always occurs at information set $(1, z)$, and the lowest at $(0, -z)$, the order of the other two depends on the sign of q defined in equation (2.9), which is simply the difference of the two equilibrium bids in question.

$$q \equiv \frac{6 - 14\delta + 12\delta^2 - \delta^3}{3 - 7\delta + 7\delta^2} + \frac{-3 + 7\delta - 3\delta^2 + 2\delta^3}{3 - 7\delta + 7\delta^2} z - \left(\frac{2\delta^2 - \delta^3}{1 - \delta + \delta^2} + \frac{1 - \delta - 3\delta^2 + 2\delta^3}{1 - \delta + \delta^2} z \right) \tag{2.9}$$

Now we need to determine the designated bid, which maximizes the expected payoff of the player, conditional on truthfully revealed type. Here we can simplify further, since the x_2 is irrelevant here: ring member 2 submits a cover bid, so she does not affect the selling price. By parameter δ, the ring can calculate the conditional probability distribution of y and (x_3, y_3), and choose an optimal bid.

Bids must be sufficiently high such that the non-designated player finds it optimal not to overbid. The optimal bids providing maximal payoff for the ring have interval-valued solutions. We choose the supremum of these intervals. For these values, it is always satisfied that the non-designated player does not find it profitable to overbid.[24] We can say that the designated ring member submits a value very close to the non-cooperative equilibrium bids.

The solution follows equation (2.10) for the unique symmetric equilibrium in second-price sealed-bid auction if $b^*(0, z) \leq b^*(1, -z)$ and (2.11). If the opposite is true, $b^*(0, z) \geq b^*(1, -z)$ occurs.

$$\beta(\cdot) = \begin{cases} b^*(1, z) + 1, & \text{if } x_1 = 1,\ y_1 = y_2 = z; \\ b^*(1, z), & \text{if } x_1 = 1,\ y_1 \neq y_2; \\ b^*(1, -z), & \text{if } x_1 = 1,\ y_1 = y_2 = -z; \\ b^*(1, -z), & \text{if } x_1 = 0,\ y_1 = y_2 = z; \\ b^*(1, -z), & \text{if } x_1 = 0,\ y_1 \neq y_2; \\ b^*(0, -z), & \text{if } x_1 = 0,\ y_1 = y_2 = -z; \end{cases} \tag{2.10}$$

[24]In case of information set $(x_1 = 1, y_1 = y_2 = z)$, the set has no supremum, any bid higher than the highest possible outside bid is optimal. For example, $1 + z$ is optimal, since it is the highest possible valuation of any bidder.

$$\beta(\cdot) = \begin{cases} b^*(1,z)+1, & \text{if } x_1 = 1,\ y_1 = y_2 = z; \\ b^*(1,z), & \text{if } x_1 = 1,\ y_1 \neq y_2; \\ b^*(0,z), & \text{if } x_1 = 1,\ y_1 = y_2 = -z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 \neq y_2; \\ b^*(0,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = -z; \end{cases} \tag{2.11}$$

Two possible versions of inequality (2.5) can be found in the Web Appendix. The parameter set on which there is an incentive compatible BCM is depicted in Figure 2.1. There exists a BCM for points of the shaded area in the space of (z, δ). For all $\frac{1}{2} < \delta < 1$, there exists an incentive compatible BCM, if z is sufficiently low. Equation (2.9) defines the switch between the two parametric forms.

As an illustration, let us consider a few examples.[25] We apply the notation $\bar{z}(\delta)$ for the supremum of the set on z given δ on which collusion is feasible. If $\delta = 0.6$, there is an incentive compatible BCM if and only if

$$z \geq \bar{z}(0.6) \approx 0.826109$$

For the example above, there is a critical value of $\bar{z}(\delta)$ for every $\delta \in \left(\frac{1}{2}, 1\right)$ such that there is an incentive compatible BCM for a given signal quality δ if and only if $z \leq z(\delta)$.

The example above supports the claim that the existence of an incentive compatible mechanism depends on the proximity of the pure private model, which has a neighborhood satisfying this criterion on the whole range of parameter δ. Less available information (higher z) about the commodity makes collusion infeasible.

2.4. Perturbed games and robustness

Proposition 2.1 appears to be robust against perturbations in information asymmetry. Perturbed games in which private information is introduced to a CV model focus on the effect of private information on the bidding behavior of the informed bidder in affiliated (Klemperer, 1998) and non-affiliated (Larson, 2009) settings. Perturbation can be used to examine the robustness of our conclusions with respect to the pure models.

The significance of these results is highlighted by the expected revenue function, for

[25] Values are calculated and figures are created by Wolfram Mathematica. See Web Appendix.

which we provide an example in Section 2.5. If the existence of BCM is robust with respect to perturbations for pure private and CV models, there exists an interior cut-off point. That is, if we consider a range of settings with respect to CV uncertainty by keeping everything else constant, including *ex ante* expected CV, there is an interior point at which the existence of BCM changes. If CV uncertainty decreases, it induces a positive downward jump in expected revenue. That is, expected revenue is not an increasing function of the availability of public information, in contrast with the non-cooperative model of Milgrom and Weber (1982).

The neighborhood of pure PV models supports collusion. On the contrary, the neighborhood of pure CV models does not. Lemma 2.3 concerns robustness of the pure PV model. In order to capture CV uncertainty, we apply the normalization $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$. Outsider strategy is denoted as $\alpha(x_{out}, y_{out}, z)$, where x_{out} and y_{out} are the signals observed by outsiders.

Lemma 2.3. *Suppose a bidding ring is formed in a second-price sealed-bid auction in which the selling price is independent of the highest bid. Assume that $\alpha(x_{out}, y_{out}, z)$ is continuous with respect to y_{out} at a neighborhood of $(x = 0, z = 0)$. Then, there exists a right-side neighborhood of* 0 *on the range of z, on which there is an incentive compatible BCM.*

Proof. First, we point out that continuity of outsider strategy $\alpha(\cdot)$ implies that $\pi(\cdot)$ is also continuous at a neighborhood of $(x_1 = 0, |y_i = 0| = i)$ for any $i \in \{0, \ldots, n\}$, since the price function is a linear combination of bids. Let us consider (2.4) and substitute $z = 0$. Then the inequality simplifies to

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr(|x_H| = i)\frac{1}{i+1}\pi(x_1 = 1, \cdot) \geq$$
$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr(|x_H| = i)\frac{1}{n-i}\pi(x_1 = 0, \cdot) \tag{2.12}$$

However, this is always satisfied. Consider the optimal strategy for information set $x_i = 0$. The same strategy yields higher payoff for information set 1, and strictly higher, if there is a positive probability of winning with 0. The latter condition is relevant for the existence of the above-defined neighborhood. If the designated player also bids according to the outsider strategy, that provides a positive probability of winning with a non-negative payoff, so these results apply to the optimal choice as well. Thus, for any sealed-bid auction, constraint (2.12) is satisfied as a strict inequality. With continuity of $\pi(\cdot)$, we can conclude. *Q.E.D.*

Lemma 2.3 shows that, in accordance with earlier findings, the pure PV model always supports collusion. Moreover, the result is robust to small CV perturbations. In other words, for low levels of common uncertainty, an incentive compatible bid coordination mechanism is always sustained.

This result also holds for the opposite direction, as it is formalized in Lemma 2.4. Here we examine the environment of the point $x_L = 1$, $x_H = 1$, $y_L = -z$ and $y_H = z$ in order to examine the neighborhood of the pure CV model.

Lemma 2.4. *Suppose a bidding ring is formed with $n \geq 2$ members and there are $k \geq 0$ outsiders in a second-price auction. Assume that $\alpha\left(x_{out}, y_{out}, z\right)$ is continuous in the neighborhood of $(x_i = 1, |x_H| = i)$ for any $i \in \{0, \ldots, n\}$. Then, for every $\delta \in \left(\frac{1}{2}, 1\right)$, there exists a right-side neighborhood of 1 on the range of x_H, on which there is no BCM.*

Proof. The proof is analogous to Lemma 2.3. *Q.E.D.*

Lemma 2.3 and 2.4 has an important implication, expressed in Corollary 2.1 in terms of x_L, x_H, y_L and y_H. For any given $x_L \neq x_H$, CV uncertainty is defined as $y_H - y_L$.

Corollary 2.1. *Suppose the assumptions of Lemma 2.3 and 2.4 hold. Then, there exists a BCM in a pure PV model ($y_L = y_H$) and there exists no BCM in a pure CV model ($x_L = x_H$). Moreover, there exists an interior cutoff point on the set of CV uncertainty with respect to the existence of BCM.*

Proof. The set $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$ with $z \geq 0$ and $x_H > 0$ is a normalization of the set above. We address the existence of BCM on the domain of z. Following Lemma 2.3, there exists a BCM if z is sufficiently close to 0, thus, the model is close to IPV. Similarly, following Lemma 2.4, there exists no BCM if z is sufficiently high. *Q.E.D.*

This result is illustrated in Section 2.5. We note that analogous results can be derived for first-price auctions. To illustrate this, Constraint (2.6) simplifies to (2.13).

$$\sum_{i=0}^{n-1} \sum_{j=1}^{n} Pr\left(|x_H| = i\right) \frac{1}{i+1} \pi^* \left(x_1 = 1, \max x_{-1}, \cdot\right) \geq$$
$$\sum_{i=1}^{n} \sum_{j=0}^{n-1} Pr\left(|x_H| = i\right) \frac{1}{n-i} \pi^* \left(x_1 = 0, 0, \cdot\right) \tag{2.13}$$

2.5. Expected revenue and CV variance

The negative effect of CV uncertainty in non-cooperative auctions is a robust result. Less uncertainty results in higher expected revenue. We illustrate that this is not the case for a collusive setting. Lower common uncertainty makes collusion incentive compatible. Hence, it decreases revenue on a part of the domain.

Milgrom and Weber (1982) state that publicly revealed information has a non-negative effect on expected revenue. Goeree and Offerman (2003) have the same conclusion with non-affiliated values. It is also supported by experimental (Goeree and Offerman, 2002; Kagel et al., 1995) findings. Also, Silva et al. (2008) found the same conclusion in an empirical model directly testing the effect of greater public information in procurement auctions.

Expression (2.14) determines the first-order derivative of the expected revenue with respect to z in second-price auctions with 3 bidders, for all $\delta \in \left(\frac{1}{2}, 1\right)$, for the normalized case $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$.[26]

$$\frac{\partial \mathbb{E}R\left(z\right)}{\partial z} = -\frac{4\left(\delta-1\right)^2\delta^2\left(2\delta-1\right)\left(-3+12\delta-20\delta^2+10\delta^3+10\delta^4-18\delta^5+6\delta^6\right)}{\left(1-\delta+\delta^2\right)\left(1-3\delta+3\delta^2\right)\left(3-5\delta+5\delta^2\right)\left(3-7\delta+7\delta^2\right)} \tag{2.14}$$

The derivative $\frac{\partial \mathbb{E}R(z)}{\partial z}$ is always positive on the range of δ. Thus, the expected revenue is decreasing in z. In other words, lower variance has a positive effect on the expected revenue and $z = 0$ provides the highest possible revenue for any given δ. This is consistent with earlier findings cited above, lower CV variance results in higher expected revenue.

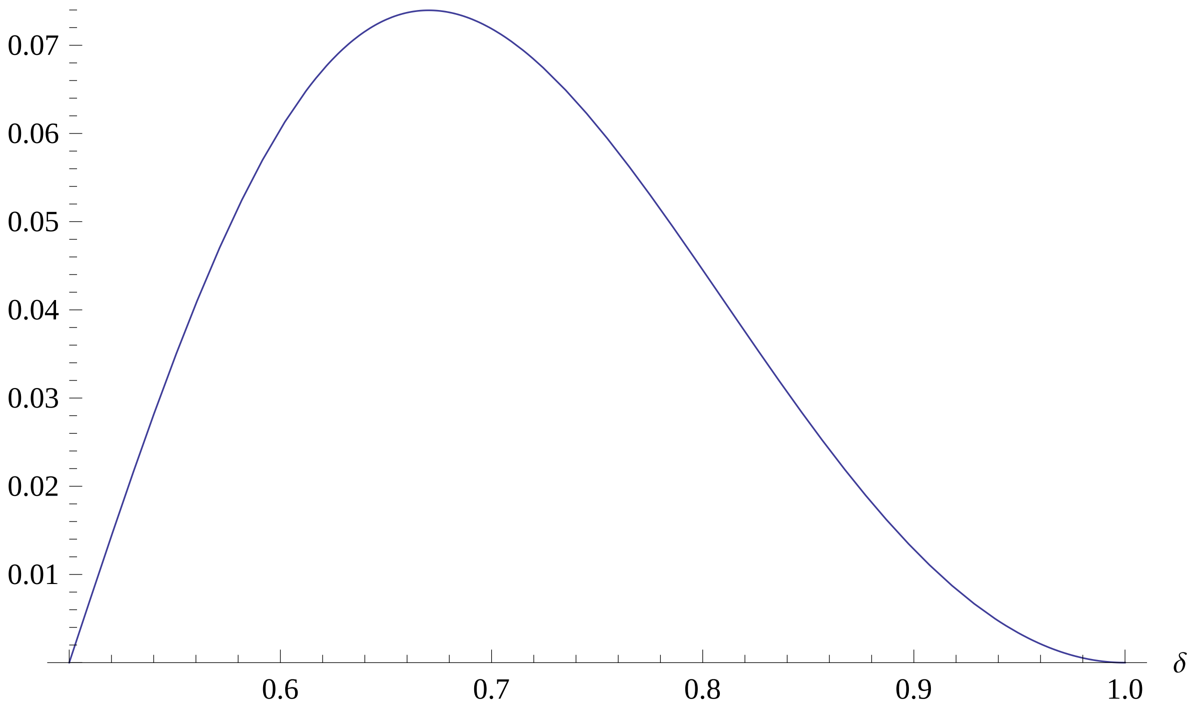
Figure 2.2 depicts the marginal effect for all possible δ. It can be noted that the effect of CV uncertainty diminishes for extreme values of δ, which determines the probability of a CV signal being correct. First, if δ is close to $\frac{1}{2}$, the CV signal is nearly pure noise. If it is close to 1, the signal is almost perfect, common uncertainty has again no role in the limit.

This result is ambiguous if bidders are able to collude. At $z = \bar{z}\left(\delta\right)$, BCM decreases the expected revenue, and the expected revenue function is discontinuous at that point. If there is no incentive compatible BCM, potential ring members correctly anticipate that type signals are not credible, so they do not form a ring.[27] Thus, players bid according to the unique symmetric BNE, and expected revenue follows equation (2.8). If there

[26] We get the expected revenue from the non-cooperative equilibrium strategies and the probability distribution of the types of bidders.

[27] This is a standard assumption in the literature (Marshall and Marx, 2009).

Figure 2.2: Marginal effect of CV uncertainty on expected revenue, with competing bidders



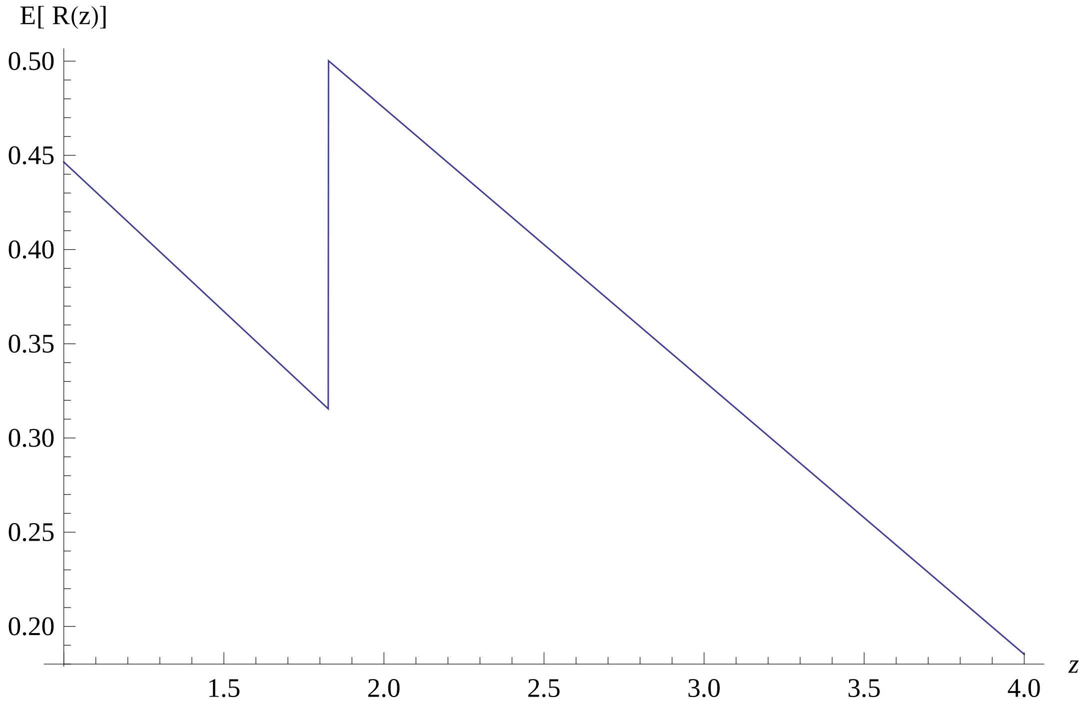
is an incentive compatible BCM, they engage in a collusive agreement. The expected revenue of this case depends also on the sign of q. Derivations can be found in the Web Appendix.

Let us consider an example, and set $\delta = 0.6$. The expected revenue function $\mathbb{E}R(z)$ is depicted in Figure 2.3 for the case when the seller is able to reduce z to 1. In the non-cooperative outcome, revenue is a decreasing function of z. Collusion is feasible, if $z \geq \bar{z}(0.6) \approx 0.826109$. As such, ring members can engage in a collusive mechanism at $\bar{z}(0.6)$, which decreases the revenue, and results in a discontinuous function.

Discontinuity is a consequence of the lack of enforced collusive agreement for low z. Collusion does not always emerge if ring members are able to increase their payoff. They also need to provide sufficient incentives by side-payments to prevent the misreporting of types. Consequently, at the point by which sufficient information is revealed to form a bidding ring, they have a strictly positive gain, resulting in a discontinuity point of the expected revenue function. This is an interior point, followed by Proposition 2.1.

However, revenue is decreasing with respect to z to the right from the point of discontinuity as well. So, this is the only point on the domain at which $\mathbb{E}R(z)$ is non-increasing or discontinuous. The negative slope to the right of $\bar{z}(0.6)$ has a similar intuition as the non-cooperative outcome. If a bidding ring is formed, the number of bidders is reduced to two, which are informed asymmetrically. Lower values of z reduce the common uncertainty, and increase the lower bids, resulting in higher expected revenue.

Figure 2.3: Expected revenue as a function of z, $\delta = 0.6$



Consequently, a revenue-maximizing seller might not find it optimal to reduce common uncertainty about the commodity. We can expect that it is not possible to completely eliminate common uncertainty, so that the seller is only able to choose from a constrained set. In Figure 2.3 this is illustrated, full disclosure is not optimal. Since expected revenue is not a monotonic function of CV uncertainty, an interior solution might be optimal. While lower variance increases the expected revenue conditional on non-cooperative behavior, it enhances cartel stability and might lead to lower revenue, as in our numerical example.

2.6. Conclusion

The role of common value uncertainty in cartel coordination is a crucial one. Private information about market demand can destroy collusive equilibria (Kandori and Matsushima, 1998). Variance of the stochastic demand component decreases the excess profit of a cartel (Porter, 1983). Theoretical literature emphasizes that common uncertainty weakens the effect of punishment mechanisms. Our paper, focusing on an auction setting, adds the notion that the latter effect can hold for cartels without repeated interaction or punishment.

Collusion in auctions is a prevalent phenomenon.[28] Related theory literature ad-

[28]Marshall et al. (2014) cite a number of recent bid rigging cases by the US Department of Justice.

dresses the role of a strategic seller, emphasizing the significance of information disclosure. This paper contributes to the debate by adding that reducing uncertainty about the commodity in a collusive market can be damaging. In some cases, it can help cartel stability and reduce revenue.

This paper builds up an auction model with additively separable common and private value elements, and symmetric, risk-neutral bidders. Moreover, we assume that an exogenous subset of bidders can engage in a collusive agreement without the possibility of enforced bids. Milgrom and Weber (1982) prove that revenue is a non-decreasing function of the publicly available information if players bid competitively, known as the Linkage Principle. We conclude, that information revelation supports collusion and can result in a negative effect on revenue. This result is robust for sealed-bid auction mechanisms. Reducing common uncertainty by the seller helps to sustain collusive mechanisms, which reduces expected revenue. For sealed bid auctions in which price is not increasing in the highest bid, most notably in second-price auctions, this drop occurs for a partial reduction of uncertainty. That is, expected revenue is not a monotonic function of common value variance in collusive auctions. Consequently, a seller who is unable to completely eliminate common value uncertainty might find it optimal to partially reduce it. That is, the Linkage Principle fails if collusion can occur.

Our paper contributes to the collusion literature. Our model implies that the effect of reducing information asymmetry is ambiguous, similarly to the case of production markets. It helps reduce uncertainty, and results in more efficient outcomes. On the other hand, it might sustain coordination of colluding players if it signals compliance with a collusive agreement. What our paper adds is that the collusion inducing effect persists even if it does not help coordination. Lower uncertainty about valuations helps sustain cartels by increasing incentive compatibility of collusive mechanisms. As a result, it is able to reduce expected revenue in auctions. We can conclude that separating the effect of private and common value information asymmetry can help us better understand the mechanism behind this phenomenon.

Appendices

Appendix 2.A Existence of BCM

First, we derive the necessary and sufficient conditions for the existence of a BCM, which satisfies truthful revelation, and such that bidders comply with recommended bids. Then, we derive that this is sufficient for the existence of a mechanism which also maximizes the aggregate expected payoff of the ring.

A bidder has four possible information sets depending on the private and CV types. The side-payment $p(|x_H|, |y_H|)$ is a function of these reports. This is the amount that members with low PV receive if there is at least one member with high CV. The arguments $|x_H|$ and $|y_H|$ refer to the number of high private and CVs among ring members. Thus, $p(|x_H|, |y_H|)$ can take $(n+1)^2$ values. We apply the notation $p(|x_H|, |y_H|) = p_{|x_H|,|y_H|}$. For example, $p_{1,2}$ refers to the side-payment a player with low PV receives if the number of high private and CV signals among members is 1 and 2, respectively. Thus, the side-payment payed by players with high PV amounts to $\frac{|x_H|}{n-|x_H|} p_{|x_H|,|y_H|}$.

Each of the four information sets defines three incentive compatibility constraints determining that the ring member is better off with a truthfully revealed type, than with a misreport. In each case, $\pi(x_1, |y_H| = j, \alpha(\cdot))$ denotes the expected payoff of the designated bidder (without loss of generality, member 1) who participates in the auction, depending on the learned PV type and the number of high value signals of the ring, not taking side-payments into account. The constraints are applied to unilateral deviations. Thus, the misreporting member learns the type of the other ring members, and these expected profits are unconstrained. The 12 incentive compatibility constraints (ICC) are indexed. As before, L refers to low and H refers to high values. For example, in $(ICC_{LL,LH})$, the first two characters show the information set of the bidder, low PV and low CV signal. The last two denotes the misreport in the same order, low private and high CV.

First we consider the four constraints at which only the CV signal is misreported. A misreport of this type does not affect the choice of the designated bidder as derived in Lemma 2.1, so that we can simplify the expected designated revenue term $\pi(\cdot)$. On the other hand, it can change the side-payments, since it depends on the number of high CV reports, as derived above. In $(ICC_{LL,LH})$, the bidder can only receive side-payment, never pays it, since the PV report is low on both sides. The side-payments depend on the number of CV reports, which is altered on the right hand side (RHS). The probability distribution remains the same, since it refers to the distribution of types of the other ring members. All four constraints are calculated analogously. It can be noted that the expected payoff function $\pi(\cdot)$ does not appear here, since CV misreport does not change the choice of the designated bidder. That is, it cancels out in all four cases. In constraints $(ICC_{HL,HH})$ and $(ICC_{HH,HL})$, negative signs are explained by that the player reports high PV.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right) \frac{1}{n-i} p_{i,j} \quad \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right) \frac{1}{n-i} p_{i,j+1} \quad (ICC_{LL,LH})$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right) \frac{1}{n-i} p_{i,j} \quad \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right) \frac{1}{n-i} p_{i,j-1} \quad (ICC_{LH,LL})$$

$$-\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right) \frac{1}{i} p_{i,j} \quad \geq$$
$$-\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right) \frac{1}{i} p_{i,j+1} \quad (ICC_{HL,HH})$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right) \frac{1}{i} p_{i,j} \quad \geq$$
$$-\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right) \frac{1}{i} p_{i,j-1} \quad (ICC_{HH,HL})$$

Let us take inequality $(ICC_{HL,HH})$ and multiply it by -1. This way both the LHS and RHS of constraints $(ICC_{LL,LH})$ and $(ICC_{HL,HH})$ are identical due to symmetry of the binomial probabilities. The same holds for $(ICC_{LH,LL})$ and $(ICC_{HH,HL})$. That is, if all ICCs hold, all of them are binding, they hold with equality.

The remaining eight constraints address deviations involving misreported PV affecting winning probabilities. Firstly, we point out, that these inequalities can be grouped into four pairs, which are pairwise identical if the constraints above involving only CV misreport hold. This point is demonstrated for the cases of $ICC_{LL,HL}$ and $ICC_{LL,HH}$.

The LHS of the constraints are identical, whereas the RHS is different by the report of the CV. The designated player's payoffs and respective probabilities are the same, since in both cases the PV reports and the information sets are identical. The side-payments are different, but if the first four constraints above hold, $ICC_{HH,HL}$ is binding. Accordingly, if all constraints are satisfied, $ICC_{LL,HL}$ and $ICC_{LL,HH}$ are identical. Similarly, this result holds for pairs $(ICC_{LH,HL}, ICC_{LH,HH})$, $(ICC_{HL,LL}, ICC_{HL,LH})$ and $(ICC_{HH,LH}, ICC_{HH,LL})$. Consequently, in order to consider incentive compatibility, we only need to examine $ICC_{LL,HL}$, $ICC_{LH,HL}$, $ICC_{HL,LH}$ and $ICC_{HH,LH}$.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_L\right) \frac{1}{n-i} p_{i,j} \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_L\right) \frac{1}{i+1} \left(\pi\left(x_1 = x_L, |y_H|=j, \alpha\left(\cdot\right)\right) - p_{i+1,j}\right) \qquad (ICC_{LL,HL})$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_H\right) \frac{1}{n-i} p_{i,j} \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_H\right) \frac{1}{i+1} \left(\pi\left(x_1 = x_L, |y_H|=j, \alpha\left(\cdot\right)\right) - p_{i+1,j-1}\right) \qquad (ICC_{LH,HL})$$

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_L\right) \frac{1}{n-i} \left(\pi\left(x_1 = x_H, |y_H|=j, \alpha\left(\cdot\right)\right) - p_{i,j}\right) \geq$$
$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_L\right) \frac{1}{i-1} p_{i-1,j+1} \qquad (ICC_{HL,LH})$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_H\right) \frac{1}{n-i} \left(\pi\left(x_1 = x_H, |y_H|=j, \alpha\left(\cdot\right)\right) - p_{i,j}\right) \geq$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|=i, |y_H|=j \;\middle|\; y_1 = y_H\right) \frac{1}{i-1} p_{i-1,j} \qquad (ICC_{HH,LH})$$

Let us rearrange the constraints according to side-payment components. It can be seen that they are all equal. The reason is that the probability distribution of types, (hence, payments) is binomial. As such, it is symmetric. This value is denoted by P in equation (2.15), while probability values are expressed explicitly. Let θ be a short-hand notation for the probability that another CV signal equals a bidder's own CV signal, such that

$\theta = \delta^2 + (1-\delta)^2$.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right)\left(\frac{1}{n-i}p_{i,j} + \frac{1}{i+1}p_{i+1,j}\right) =$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right)\left(\frac{1}{n-i}p_{i,j} + \frac{1}{i+1}p_{i+1,j-1}\right) =$$
$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_L\right)\left(\frac{1}{i-1}p_{i-1,j+1} + \frac{1}{n-i}p_{i,j}\right) =$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|= i, |y_H|= j \;\middle|\; y_1 = y_H\right)\left(\frac{1}{i-1}p_{i-1,j} + \frac{1}{n-i}p_{i,j}\right) =$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \binom{n}{i}\left(\frac{1}{2}\right)^n \binom{n}{j} (1-\theta)^j\,\theta^{n-j}\left(\frac{1}{n-i}p_{i,j} + \frac{1}{i+1}p_{i+1,j}\right) = P$$

Equation (2.15) implies that the constraints $ICC_{LL,HL}$, $ICC_{LH,HL}$, $ICC_{HL,LH}$ and $ICC_{HH,LH}$ can be summarized according to (2.15) and (2.16). Explicit representation of probabilities are:

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} \binom{n}{i}\left(\frac{1}{2}\right)^n \binom{n}{j} (1-\theta)^j\,\theta^{n-j}\frac{1}{n-i}\pi\left(x_1 = x_H, |y_H|= j, \alpha(\cdot)\right) \geq$$
$$P \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \binom{n}{i}\left(\frac{1}{2}\right)^n \binom{n}{j} (1-\theta)^j\,\theta^{n-j}\frac{1}{i+1}\pi\left(x_1 = x_L, |y_H|= j, \alpha(\cdot)\right) \tag{2.15}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \binom{n}{i}\left(\frac{1}{2}\right)^n \binom{n}{j}\theta^j\,(1-\theta)^{n-j}\,\frac{1}{n-i}\pi\left(x_1 = x_H, |y_H|= j, \alpha(\cdot)\right) \geq$$
$$P \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} \binom{n}{i}\left(\frac{1}{2}\right)^n \binom{n}{j}\theta^j\,(1-\theta)^{n-j}\,\frac{1}{i+1}\pi\left(x_1 = x_L, |y_H|= j, \alpha(\cdot)\right) \tag{2.16}$$

Constraints (2.15) and (2.16) have a solution if and only if inequality (2.17) holds. They define two upper and two lower bounds for the possible values of P. It can be noted that the LHS of (2.15) is greater than the LHS of (2.16), because the CV signals are lower in the latter expression. A similar result applies to the RHS of both, CV signals are higher on the RHS of (2.15). As such, bounds are the tightest when the LHS of (2.15) and the RHS of (2.16) are equal. In order to have a solution for P, the latter must not be greater than the former. This way, we have defined a necessary and sufficient condition for the existence of an incentive compatible collusive equilibrium.

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}(1-\theta)^{j}\,\theta^{n-j}\frac{1}{n-i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)\geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}\theta^{j}\left(1-\theta\right)^{n-j}\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right) \tag{2.17}$$

Equivalently, the same result can be written as in inequality (2.18).[29]

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1}Pr\left(|x_H|=i,|y_H|=j\;\middle|\;y_1=y_L\right)\frac{1}{n-i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)\geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n}Pr\left(|x_H|=i,|y_H|=j\;\middle|\;y_1=y_H\right)\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right) \tag{2.18}$$

What remains is to prove that the latter condition guarantees that no other bidder finds it profitable to outbid the designated bidder in a second-price auction. Again, without loss of generality, the designated bidder is denoted by 1. In order to bid optimally, we need that

$$\beta_1=\arg\max\left[\sum\left[v_1-\max\left\{\alpha\left(x_N,y_N\right)\right\}\right]Pr\left(\beta_1>\max\left\{\alpha\left(x_N,y_N\right)\right\}\right)\right]$$

where $Pr\left(\beta_1>\max\left\{\alpha\left(x_N,y_N\right)\right\}\right)$ is the probability that the designated bid is higher than the maximal outsider bid, given the information set (x_N,y_N) of the ring. The types are truthfully reported, so that the distribution of valuations of the designated bidder and other ring members only differ in the PV. There are two cases. The PV of the other member with the highest PV is either lower or identical to the designated bidder's value. It is clear that it is enough to consider the latter case.

Consider the supremum of the set of optimal designated bids.[30] If deviation is profitable for a non-designated member, it would also be profitable to the designated bidder, contradicting that she submitted an optimal bid. In that case, the expected payoff of the non-designated bidder is less than or equal to the excess expected payoff the designated bidder could receive with the same bid, since the list of bids affecting price is strictly greater for the designated bidder, and the designated player's valuation is weakly greater.

[29] Identical to (2.4).

[30] In some cases, it has no supremum, we can set the designated bid to any value greater than $a_2\left(x_H+y_H\right)$, making deviation never profitable.

Chapter 3

Auction Cartels and the Absence of Efficient Communication

Abstract

This chapter examines the feasibility of collusive mechanisms in single-unit auctions. A model is constructed with private and common value information asymmetry and continuous type space. We show that an incentive compatible bid coordination mechanism (BCM) does not exist if common value uncertainty is present. This result contradicts actual antitrust cases, where common effects or resale opportunities created uncertainty about valuations, but a price-fixing cartel was formed. We solve the puzzle by relaxing the assumption that all bidder types truthfully reveal their private information. The introduced Bayesian bid coordination mechanism (BBCM) exists if the main source of information asymmetry is private value. In that case, a designated ring member can signal high valuation and suppress competition. Our results demonstrate the rationale behind cartel mechanisms with pre-auction knockouts.

3.1. Introduction

Bid rigging is a violation of the Sherman act and EU competition law and attracts substantial research interest.[31] A large fraction of related literature addresses the existence of collusive mechanisms and looks for efficient policy or detection tools to combat them. We focus on the collusive mechanism itself in the context of auctions for a single commodity in a second-price sealed-bid auction. As we show, current theory models, most notably the bid coordination mechanism (BCM) introduced by Marshall and Marx (2007) cannot be applied to certain information asymmetry settings.

If the type space of bidders' valuations is continuous and valuations are independent private values (PV), a BCM exists in second-price auctions. We show that additional common value uncertainty leads to the breakdown of the cartel. The reason behind this

[31] This article uses the expressions of auction cartel, collusion, bidding ring and bid rigging interchangeably. All refers to a horizontal price-fixing conspiracy.

result is the assumption of incentive compatibility. A mechanism with truthfully revealed private information fails to exist if any of the bidder types fail to comply with the cartel agreement. Actual cartels often solve coordination of their actions by a pre-auction knockout auction determining their bidding strategy and the way to share the collusive gain. The timing of the knockout can be argued to prevent costly *ex post* negotiations. Simple allocation mechanisms are common, which do not allow for revealing all of the private information. We introduce a new collusive mechanism concept showing that knockout auctions can emerge in equilibrium.

The choice of assumptions has a significant effect on the existence of collusive equilibria. We argue that the literature applies requirements that are too strong with respect to collusive agreements. More specifically, existing concepts seek for collusive mechanisms in which all private information is truthfully revealed. This point can be questioned in two ways. First, it does not necessarily correspond with practice. Second, it restricts the scope of models to ones dealing with one-dimensional information asymmetry. As Graham and Marshall (1987) define, in a collusive equilibrium, private information is truthfully revealed and bidders follow the coordinated bidding strategy. This is also assumed by Mailath and Zemsky (1991), McAfee and McMillan (1987) and Marshall and Marx (2007). Harrington (2008) classifies the empirical methods of identifying cartels. Two robust approaches either test the difference between behavior of presumably competing and colluding firms, or compare the fit of a collusive and a competitive model. Since communication between cartel members, including revealing private information, is usually only partially observable, these empirical models do not control for them. The most comprehensive empirical study by Kawai and Nakabayashi (2015) identifies the prevalence of collusion in Japanese construction procurement auctions between 2003-2006 from bidding behavior. Similarly, the seminal paper of Pesendorfer (2000) does not control for information exchange, only market characteristics and cartel membership.

While the theory literature endorses incentive compatibility, there is little evidence bidders actually reveal all private information to other ring members. This is especially apparent, when we consider procurement auction cases where sources of information can be multidimensional and complex.[32] Asker (2010) cites a postal stamp cartel operating in the 1990s. Ring members conducted a pre-auction knockout before each auction in order to determine side-payments and the person of the designated bidder. Others abstained from bidding for the stamp, suppressing competition. In the stamp auction market, one can think of factors relevant to valuation of all bidders or the valuation of

[32] Some recent bid rigging cases pursued by the U.S. Department of Justice include cases involving military equipment (2008, http://www.justice.gov/atr/public/press_releases/2008/231569.pdf and http://www.justice.gov/atr/public/press_releases/2008/234542.pdf) and natural gas leases (2014, http://www.justice.gov/atr/cases/f280200/280290.pdf). (All accessed January 22, 2015.)

a certain bidder. Buyer-specific factors can be liquidity, ownership of related stamps or taste. On the other hand, resale possibilities were the same for all bidders, but they might have had different estimates on them. Sharing this info could have improved the allocation of the good and provided a better value estimate for the designated bidder.[33] Hence, full information pooling could have increased the cartel's payoff and allocative efficiency. However, communication only occurred by knockout bids, indirectly, through agents. This is not unusual in posted-price markets. A well-known cartel case of complex information sharing is the Banana cartel operating between 2000-2002. It was concluded that Chiquita, Dole and Weichert engaged in pre-pricing communications, disclosing information relevant for setting of quotation prices of banana, including price trends and indications of quotation prices for the upcoming week. As the European Commission (2009) argues, "The decision concludes that the object of pre-pricing communications was to reduce uncertainty as to the conduct of the parties with respect to the quotation prices to be set by them."

Our model explains the puzzle of the lack of full information sharing within a cartel. We derive that an incentive compatible bid coordination mechanism only exists in a pure private value setting. The implication of this result is that the seller is able to effectively combat bid rigging by creating common value uncertainty, which affects all bidders. This is an optimistic result, but our motivating examples show that this is not necessarily enough.

Throughout our paper, we consider pure strategies and risk-neutral bidders participating in a second-price sealed-bid auction. Our approach is reduced to the key features of collusion. Following the legal definition, we describe bid rigging as an attempt to manipulate the auction outcome by coordinating bidding strategy.

The newly introduced concept is the Bayesian bid coordination mechanism (BBCM). Keeping the condition of *ex ante* individual rationality, we introduce a model of collusion in which private information is partially revealed and within-cartel competition is suppressed. As Goeree and Offerman (2003) mention, information sharing can improve the collusive gain by pooling scattered information. The model provides insights as to why actual cartels conduct so-called pre-auction knockout auctions to determine their coordinated bidding strategy. Instead of revealing all private information, ring members only signal their willingness-to-pay for the right to be the designated bidder. We derive the formal conditions under which BBCM exists. A collusive mechanism fosters information sharing between ring members. In this case, it is partial. In a knockout, the bids provide signals about the information set of cartel members. If the cartel is weak and unable

[33]We use the term designated bidder for the ring member who submits a serious bid. In actual rings, it is a common practice to choose one member of the cartel to do so.

to enforce coordinated bidding in the auction, the designated bidder's best interest is to signal high PV's. On the other hand, there is no gain from disclosing information valuable to other ring members, and giving up information rents. In the absence of full information revelation, disclosing information has an ambiguous effect. Signaling higher PV comes with a higher probability of being the designated bidder. On the other hand, disclosing common value (CV) signals implies giving up information rents. Consequently, collusion is feasible if the first effect dominates. We derive that this occurs if the main source of information asymmetry is PV.

Chapter 2 shows that the existence of BCM depends on the main source of information asymmetry. That is, it exists in pure PV setting and if there is small CV uncertainty, if type space of valuations is discrete. Consequently, public disclosure of information regarding the commodity, by reducing CV uncertainty, has a profound effect on the existence of bidding rings. In this paper we extend this result to a continuous type space. It is a reasonable assumption in a range of cases that the seller possesses secret information affecting valuation of bidders.[34] We show that a BBCM exists if the main source of information asymmetry is PV. That is, we can confirm that public disclosure fosters forming a cartel with a pre-auction knockout. Hence, it can reduce expected revenue of the seller, contradicting the Linkage Principle introduced by Milgrom and Weber (1982).

Modeling bidding rings is not without methodological difficulties. Auction theory literature tends to apply highly structural models specifying the timing of the game and the strength of the cartel. As we argue above, the distinction between PV and CV information asymmetry is relevant in our setting. Therefore, in order to address our points on the role of the source of information asymmetry, we use a model with additively separable PV and CV elements with symmetric, independent and identically distributed values.[35] The use of two-dimensional type space allows for modeling information disclosure and heterogeneity of information asymmetry between bidders.

The paper is structured as follows. In Section 3.2, we construct the model and we find that BCM does not exist on a continuous type space. The main results are provided in Section 3.3, in which we introduce the concept of Bayesian bid coordination mechanism. Public disclosure and the problem of cartel size are addressed in Section 3.4. Finally, we conclude in Section 3.5.

[34]See OECD (2014).

[35]Recent empirical models applying a different setup entail Asker (2010), who considers a specific long-standing cartel about which much information is available. Further empirical studies concern public procurement auctions (Kawai and Nakabayashi, 2015) and the consulting sector (Ishii, 2009) in Japan, construction contracts in Italy (Conley and Decarolis, 2015), Japan (Padhi and Mohapatra, 2011) and wheat procurement in India (Banerji and Meenakshi, 2004).

3.2. Collusion and truthful revelation of private information

In this chapter, we consider the model of Goeree and Offerman (2003) for modeling the valuation of bidders who participate in a second-price sealed-bid auction. The use of both types of information signals, private values (PV) and common values (CV), is justified in most applications. We argue that it has an effect on the stability of collusive agreements and on expected revenue. This section describes the model of valuation of bidders and of the collusive mechanism. We show that an incentive compatible bid coordination mechanism (BCM) exists only if PV is the only source of information asymmetry.

3.2.1. Model

Bidders' valuation is modeled using a symmetric setup applying independently drawn two-dimensional signals. PV signals are drawn from support $[x_L, x_H]$ according to cumulative density function (CDF) $F(x)$, and CV signals are drawn from $[y_L, y_H]$ following $G(y)$. Both are continuously differentiable. The respective density functions are denoted by $f(x)$ and $g(y)$. End points are bounded away from zero, $x_L > 0$ and $y_L > 0$. Common value is determined by all CV signals as $Y \equiv \sum \frac{y_i}{n}$, where n is the number of bidders. That is, the winner's payoff is given as $Y + x_i - P = \sum \frac{y_i}{n} + x_i - P$, where P is the selling price.

As Goeree and Offerman (2003) derive, there exists a symmetric Bayesian equilibrium in pure strategies of the second-price sealed-bid or of the Vickrey auction with non-cooperating bidders. In this auction format, bidders simultaneously submit bids B_i. The player submitting the highest value wins and pays a price equal to the second highest bid. In equilibrium, players bid their expected valuation, conditional on having the highest surplus and the opponent having identical surplus, where the surplus is $s_i = \frac{y_i}{n} + x_i$. This expression is analogous to Milgrom and Weber (1982). That is, the equilibrium bid is expressed following Equation (3.1).

$$B_i(x_i, y_i) = \mathbb{E}\left(Y + x_i | s_i = \max_{j \neq i} s_j\right) \tag{3.1}$$

Surplus expresses the fraction of the valuation which is known by bidder i. The remaining term $\frac{\sum y_{-i}}{n}$ is independent of surplus s_i, since types are independently drawn. The intuition behind (3.1) is that bidders take into account, that conditional on winning, other bidders' CV signals are lower, so their valuation is lower as well. That is, they shade their bids not to fall prey to the winner's curse.

Bid coordination mechanism (BCM) introduced by Marshall and Marx (2007) is defined as in Chapter 2, with $m \leq n$ ring members, where $m \geq 2$. The set of players and ring members are denoted by N and M, respectively. A BCM is a function $\mu(x^*, y^*) = (\beta(x^*, y^*), p(x^*, y^*))$ of shared information of types allotting recommended strategies β and a side-payment vector p to ring members. The timing of the game is as follows. Bidders learn mechanism μ and decide about participation. Then, they learn their types. Subsequently, they share signals (x^*, y^*) with each other simultaneously. In all cases, (x^*, y^*) refers to the set of ring members.[36] Following the mechanism, members learn the recommended bids β and side-payments p. The side-payments satisfy *ex post* budget balance, they are enforced and implemented.[37] Finally, the auction takes place, and compliance with the recommended bids is not enforced. Conditions are detailed below.

Suppose Π_i denotes the expected payoff (with side-payments) of ring member i. We say that $\mu(\cdot)$ is an incentive compatible BCM against outside bid function $\alpha(\cdot)$ if conditions (3.2), (3.3) and (3.4) hold. We denote expected values over all bidder types with $\mathbb{E}(\cdot)$, all payoffs and subscripts refer to ring members, subscript $-i$ refers to members of the ring other than member i.

$$(x_i, y_i) \in \arg\max_{x_i^*, y_i^*} \mathbb{E}\left(\Pi_i(\cdot) \,|x_{-i}^*(x_{-i}, y_{-i}), y_{-i}^*(x_{-i}, y_{-i}), \mu(\cdot), \alpha(\cdot)\right), \forall i \tag{3.2}$$

$$\beta_i(x^*, y^*) \in \arg\max \mathbb{E}\left(\Pi_i(\cdot) \,|x^*, y^*, \beta_{-i}, \alpha(\cdot)\right), \forall i \tag{3.3}$$

$$\beta(x^*, y^*) \in \arg\max \mathbb{E}\left(\sum_i \Pi_i(\cdot) \,|x^*, y^*, \alpha(\cdot)\right) \tag{3.4}$$

Thus, an incentive compatible BCM has the following requirements. Condition (3.2) requires that members find it optimal to truthfully reveal their types. Condition (3.3) captures the idea that recommended bids are followed in equilibrium. Finally, Condition (3.4) concerns the optimal collusive strategy, which is achieved if the sum of their expected payoffs is maximal, given their information set. We say that the ring is able to suppress all ring competition in that case.[38]

[36] The action space of ring members here is $(x^*, y^*) \in [x_L, x_H] \times [y_L, y_H]$.

[37] *Ex ante* implementation of side-payments is important to avoid costly re-negotiations and rent-seeking (Marshall and Marx, 2012).

[38] Our definition of BCM differs from that of Marshall and Marx (2007), in that we also consider all-inclusive rings. In case there are no outsiders, we shall assume an outsider bid function $\alpha(\cdot) = 0$.

In what follows, an incentive compatibility constraint (ICC) expresses that a ring member has higher expected payoff by truthfully revealing type than reporting another type. All ICC is a special case of Condition (3.2) for a given information set (x_i, y_i).

3.2.2. Existence of BCM

We proceed in a number of steps. Lemmas 3.1 and 3.2 prove that our attention can be restricted to a certain subset of mechanisms, in which a designated bidder submits a bid in the auction, while all other ring members submit a cover bid not influencing the final payoffs, but receive a side-payment from the designated bidder. We also show that this bidder is the one with the highest PV. We consider non-discriminative mechanisms. That is, side-payments and recommended bids are index-invariant.

There are two sources of payoff in the game, side-payment and auction payoff. Side-payment of ring member i is denoted by $p_i(x^*, y^*)$. The auction payoff is captured by $\pi(x, y) = \pi(x_i, x_{-i}, y_i, y_{-i})$, where this value is the expected payoff of designated bidder i if her type is (x_i, y_i) and $m-1$ players with vector (x_{-i}, y_{-i}) of types submit zero. Outsider bidders $(j \notin M)$ follow strategy $\alpha(x_j, y_j)$.

Lemma 3.1. *Suppose there exists a BCM. Then, there exists a BCM, in which there is a designated bidder while the other ring members submit a bid of* 0 *and this ring member has the highest PV.*

Proof. Suppose BCM $\mu(\cdot)$ allots a number of recommended bids to the ring, and more than one of them is positive. Lowering all the bids except for the highest one does not change the payoff of those who lowered these values. On the other hand, this change either increases or does not change the payoff of the one with the highest bid. An increase contradicts Condition (3.4). That is, decreasing all recommended bids except for the highest one to 0 does not change payoffs and it is also a BCM. In what follows, we refer to the highest-bidding ring member as designated or efficient bidder.[39]

The designated bidder must be the bidder with the highest expected valuation in order to satisfy Condition (3.4). This is equivalent to having the highest PV among the ring members. *Q.E.D.*

Lemma 3.2. *Suppose there exists a BCM. Then, there exists a BCM, in which the expected side-payments are independent of the CV reports. That is,* $\mathbb{E}p(x^*, y^{1*}) = \mathbb{E}p(x^*, y^{2*})$ *for all* x^*, y^{1*}, y^{2*}.[40]

[39]Efficiency is captured by the PV, since the CV component of valuations is identical to all bidders and it is always assumed the seller has 0 valuation. The expression refers efficient allocation *within* the ring.

[40]Expectation is over the type of other ring members.

Proof. If there is a BCM, following Lemma 3.1, there is a BCM in which the ring member with the highest reported PV is the designated bidder while others submit 0. In this BCM, $\beta\left(x^*, y^{1*}\right)\beta\left(x^*, y^{2*}\right)$ for any x^*, y^{1*}, y^{2*}, that is, the recommended bids do not depend on the CV report. Consequently, ring members choose to report the CV providing the highest expected side-payment. We have that $\mathbb{E}p\left(x^*, y^*\right)$ is identical for any y^* for all x^*. *Q.E.D.*

We assume for technical reasons that $p\left(x^*, y^*\right)$ is integrable. The ICCs express that ring members are better off by reporting their true types. Misreporting the CV signal does not change the payoff of a ring member. It does not affect the side-payments and in case of being the designated bidder, there is no binding agreement about the bidding strategy.

Misreporting the PV type changes the expected side-payments. It does not change the expected payoff conditional on being the designated bidder, but the misreport x_i' can change the selection of the designated bidder. These points are reflected in the generic ICC in Constraint (3.5), where $x_i' \geq x_i$. It is simplified by payoffs of the auction in case ring member i is the designated bidder for both reports. In all cases, index $-i$ refers to the set of ring members other than i and $\max\{x_{-i}\}$ denotes the highest PV in the ring other than x_i. Functions $\bar{F}\left(x_{-i}\right)$ and $\bar{G}\left(y_{-i}\right)$ refer to the joint CDFs of PVs and CVs of the respective ring members.

$$\int p_i\left(x_i, x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) \geq \int p_i\left(x_i', x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) + \iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i, x_{-i}, y_i, y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right) \tag{3.5}$$

Let us change the role of x_i and x_i'. That is, the PV is x_i' while the misreport x_i is lower. We get Constraint (3.6).[41]

$$\int p_i\left(x_i, x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) \leq \int p_i\left(x_i', x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) + \iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i', x_{-i}, y_i, y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right) \tag{3.6}$$

From (3.5) and (3.6) we get (3.7).

[41] Note, that x_i and x_i' both take the same values in Constraints (3.5) and (3.6).

$$\begin{aligned}
&\iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i, x_{-i}, y_i, y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right) \\
&\leq \int p_i\left(x_i, x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) - \int p_i\left(x_i', x_{-i}\right) d\,\bar{F}\left(x_{-i}\right) \\
&\leq \iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i', x_{-i}, y_i, y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right)
\end{aligned} \tag{3.7}$$

Constraint (3.7) must be satisfied for any y_i. Let us rewrite expression (3.7) such that y_i is replaced by y_i'. Using the left-hand side (LHS) of (3.7) and the right-hand side (RHS) of the new expression, we get (3.8). That is, we have a necessary condition for the existence of a BCM expressed in constraint (3.8).

$$\begin{aligned}
&\iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i, x_{-i}, y_i, y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right) \\
&\leq \iint\limits_{x_i \leq \max\{x_{-i}\} \leq x_i'} \pi\left(x_i', x_{-i}, y_i', y_{-i}\right) d\,\bar{F}\left(x_{-i}\right) d\,\bar{G}\left(y_{-i}\right)
\end{aligned} \tag{3.8}$$

We can state Lemma 3.3.

Lemma 3.3. *A BCM exists only if* (3.8) *is satisfied for any* y_i, y_i', x_i, x_i' *with* $x_i \leq x_i'$.

Consider $y_i = y_H$ and $y_i' = y_L$, in a setting with $x_H > x_L$ and $y_H > y_L$. The CDF $F(x)$ of PV types is continuous, so that $x_i' = x_i + \varepsilon$ always exists for $x_i < x_H$ with a sufficiently small ε. Conditional on winning, the information set with y_i' provides a lower payoff. Since $F(\cdot)$ is continuous, there exists x_i' such that the LHS is greater.

Consequently, there is no BCM if both sources of information asymmetry are present. A high PV-type ring member is willing to reveal her type if the side-payments are sufficiently low. The opposite holds for low types. Reporting lower PV is more desirable for a member with low CV signal. Similarly, reporting higher PV is more desirable for a member with high CV signal. Both constraints cannot be satisfied if the difference in CV types is sufficiently large compared to the difference in PV types. However, this always occurs in a hybrid model with continuous type space.

The existence of BCM a is shown for the pure PV model by Marshall and Marx (2007). Direct revelation of PV is incentive compatible, since in a second-price auction, a non-designated bidder always follows the recommended bids. Our results are summarized in Proposition 3.1.

Proposition 3.1. *There exists a BCM if and only if the setting is a pure PV model with* $y_L = y_H$.

That is, if the type space is continuous, any positive CV uncertainty makes revelation of private information not incentive compatible, violating Condition (3.2). Proposition 3.1 implies that combating collusion is possible by introducing any extent of CV uncertainty. In the context of a procurement auction with a building project, omitting a site visit can prevent bid rigging.

Proposition 3.1 contradicts empirical evidence. Milgrom (1989) argues that the independent PV assumption does not hold in most procurement settings. However, Kawai and Nakabayashi (2015) estimate that over 20 percent of Japanese procurement auctions were non-competitive. The subsequent sections cope with this issue by introducing an alternative model of collusion.

Marshall and Marx (2007) also address strong cartels. A bid submission mechanism (BSM) is identical to BCM except that it satisfies the enforcement of recommended bids in β. That is, BSM is a mechanism $(\beta(x^*, y^*), p(x^*, y^*))$ satisfying Conditions (3.2) and (3.4) but not (3.3). Non-existence of a BSM in non-generic cases directly comes from Jehiel and Moldovanu (2001) who derive this result for a more general setting and claim that no incentive compatible Bayesian equilibrium exists if types are multi-dimensional and cannot be represented by a one-dimensional informational variable. Consequently, we can summarize this point in Corollary 3.1.

Corollary 3.1. *There exists a BSM if and only if the setting is a pure PV model with* $y_L = y_H$.

3.3. Bayesian bid coordination mechanism

If the type space is continuous, BCM only exists in the pure PV model. We address this problem by relaxing the assumptions of BCM. Subsection 3.3.1 introduces the concept of Bayesian bid coordination mechanism (BBCM). Ring members do not reveal all private information truthfully and they take this into account using Bayesian updating. Subsection 3.3.2 shows that BBCM exists if ring members can credibly signal high PV so that they can suppress competition within the cartel.

3.3.1. Model

In BCM, ring members learn the type of other ring members conditional on reports (x^*, y^*). Actual bidding rings do not follow this pattern and there is little evidence they share all the private information with each other.

The postal stamp cartel analyzed by Asker (2010) was able to communicate and share side-payments by a knockout auction. Pre-auction knockout bids were collected by an agent who calculated the outcome and the side-payments, but no other form of information was pooled for a particular auction. This practice is not an efficient collusive mechanism. Cartel members were professional dealers who faced resale possibilities. Sharing information about resale could have potentially increased their aggregate profit by improving allocative efficiency.

In what follows we build up a model which shows that a pre-auction knockout can be supported in equilibrium if BCM is not. The model relaxes the conditions of BCM. Consider the same model as before with respect to type space. Similarly, as for BCM, we seek a mechanism which satisfies that bidders follow recommended strategies and satisfies *ex ante* individual rationality. We refer to a symmetric collusive mechanism, $\mu\left(x_i^*, y_i^*\right) = \left(\beta\left(x_i^*, y_i^*\right), p\left(x_i^*, y_i^*\right)\right)$ satisfying Conditions (3.9), (3.10) and (3.11) as a Bayesian bid coordination mechanism (BBCM). Variables x_i^* and y_i^* denote reports while $x_i^*\left(x_i, y_i\right)$ and $y_i^*\left(x_i, y_i\right)$ refer to the reports as a function of type.[42]

$$\left(x_i^*\left(x_i, y_i\right), y_i^*\left(x_i, y_i\right)\right) \in \underset{x_i^*, y_i^*}{\arg\max}\ \mathbb{E}\left(\Pi_i\left(\cdot\right) | x_{-i}^*\left(x_{-i}, y_{-i}\right), y_{-i}^*\left(x_{-i}, y_{-i}\right), \mu\left(\cdot\right), \alpha\left(\cdot\right)\right), \forall i \tag{3.9}$$

$$\beta_i\left(x^*, y^*\right) \in \arg\max\ \mathbb{E}\left(\Pi_i\left(\cdot\right) | x^*, y^*, \beta_{-i}, \alpha\left(\cdot\right)\right), \forall i \tag{3.10}$$

$$\beta\left(x^*, y^*\right) \in \arg\max\ \mathbb{E}\left(\sum_i \Pi_i\left(\cdot\right) | x^*, y^*, \alpha\left(\cdot\right)\right) \tag{3.11}$$

They are different from the set of conditions of BCM in the following aspects. According to (3.9), ring members play best response to other ring members' signaling strategy. In comparison, Condition (3.2) requires that ring members truthfully reveal their types. Condition (3.10) and (3.11) are identical to (3.2) and (3.3), respectively. Consequently, the signaling functions form a Bayesian equilibrium, where recommended bids are followed by ring members and maximize the sum of their expected payoff conditional on signals shared. In BCM, types are revealed, so the recommended bid function $\beta\left(\cdot\right)$ also satisfies that it maximizes the expected ring payoff with respect to the actual type $\left(x_i, y_i\right)$ of members.

Distribution of PV and CV defines a space of auction games. The subset of type

[42] Functions $x_{-i}^*\left(x_{-i}, y_{-i}\right)$ and $y_{-i}^*\left(x_{-i}, y_{-i}\right)$ refer to the vector of reporting functions for ring members other than i.

space on which there exists a BCM is smaller than or equal to the same for BBCM, since the conditions are stricter.

Corollary 3.2. *Suppose there exists a BCM in an auction. Then, there exists a BBCM satisfying Conditions* (3.9)-(3.11).

The question remains what happens if both PV and CV information asymmetry are present. First we point out that attention can be focused on cases in which the mechanism determines a designated bidder who submits a positive bid while others submit zero. The designated bidder is also the efficient one within the ring, having the highest valuation. Now, we consider mechanisms utilizing surplus s, so that a mechanism is denoted by $\mu(s^*) = (\beta(s^*), p(s^*))$, Conditions (3.9)-(3.11) are changed accordingly.[43]

Lemma 3.4. *Suppose there exists a BBCM. Then, there exists a mechanism satisfying Conditions* (3.9)-(3.11), *in which ring members can signal their surplus s_i, the one with the highest reported surplus becomes designated bidder and others submit zero bid in the auction.*

Proof. The proof for the designated bidder is analogous to Lemma 3.2. Condition (3.11) requires that recommended bids maximize the sum of ring members' expected payoffs. If there is such a recommended bid function, the one in which non-designated ring members submit zero is also optimal in this respect.

If a BBCM exists, the designated bidder i must be able to credibly signal that her expected valuation conditional on shared signals $x_i + \mathbb{E}\left(\frac{\sum_{i\in M} y_i}{n}|x^*_{-i}, y^*_{-i}\right) = s_i + \mathbb{E}\left(\frac{\sum y_{-i}}{n}|x^*_{-i}, y^*_{-i}\right)$ is maximal in the cartel, where $-i$ refers to ring members other than i.[44] Otherwise, there is another cartel member who believes her expected valuation is higher since beliefs about the expected CV are identical, so she will bid higher in the auction. Such overbidding is not possible since (3.11) requires that their aggregate payoff is maximal conditional on shared signals.

Ring members form posterior beliefs about the distribution of a bidder's type conditional on signals. In a symmetric equilibrium, this means that ring member becomes designated bidder, whose expected valuation is the highest conditional on reported types.

Q.E.D.

In what follows our focus of interest is on whether surplus can be truthfully revealed. The ICC of ring member i takes the form of Condition (3.12). Values s_i and s'_i are the

[43]Note that we distinguish functions $\mu(x^*, y^*)$ and $\mu(s^*)$ as the latter is a function of surplus, in what follows we use this notation.

[44]The expression is simplified by omitting the CV types of outsiders, which are conditionally independent.

surplus and the misreported surplus of ring member i, respectively. The vector of surplus of ring members other than i is denoted by s_{-i}. Total payoff of a ring member consists of two components: auction payoff and side-payment.

Expression $\hat{\pi}(s_i, s_{-i})$ refers to the expected auction payoff of bidder i whose surplus is s_i and knows the surplus vector s_{-i} of $m-1$ other bidders who submit zero in the auction. That is, function $\hat{\pi}(\cdot)$ expresses the expected auction payoff that bidder i can obtain by being the designated bidder with a given information set, not taking any side-payment into account.[45] One has to see that s_i is a proper information variable for the calculation of the expected payoff here, since $x_i + \frac{\sum y_j}{n} = s_i + \frac{\sum y_{N/\{i\}}}{n}$. As above, outsiders $(j \notin M)$ follow bidding strategy $\alpha(\cdot)$.

The side-payment function of bidder i is denoted by $\hat{p}_i(s_i, s_{-i})$ and we assume it is differentiable. Cumulative distribution function of the surplus $J(s)$ is derived from distributions $F(x)$ and $G(y)$ and notation $\bar{J}(s_{-i})$ refers to the joint CDF of the surplus values of ring members other than i.[46]

$$\begin{aligned}
&\int_{s_{-i}} \cdots \int \hat{p}_i(s_i, s_{-i})\, d\bar{J}(s_{-i}) + \int_{s_i > \max\{s_{-i}\}} \cdots \int \hat{\pi}(s_i, s_{-i})\, d\bar{J}(s_{-i}) \geq \\
&\int_{s_{-i}} \cdots \int \hat{p}_i(s_i', s_{-i})\, d\bar{J}(s_{-i}) + \int_{s_i' > \max\{s_{-i}\}} \cdots \int \hat{\pi}(s_i, s_{-i})\, d\bar{J}(s_{-i})
\end{aligned} \tag{3.12}$$

3.3.2. Existence of BBCM equilibrium

This subsection characterizes the existence of a BBCM. Reports on surplus are denoted by s^*. That is, the mechanism $\mu(s^*) = (\beta(s^*), p(s^*))$ determines the designated bidder and the side-payments as a function of s^*. The ring member with the highest surplus becomes designated bidder while the others submit a cover bid of 0 in the auction.

As before, we can follow the revelation principle and consider a direct mechanism (Myerson, 1979). The role of the direct signal from the designated bidder's point of view is to credibly claim a high PV. If such a signal is credible, the other ring members will comply with the mechanism, since the designated bidder has a higher expected PV, hence, a higher expected valuation. If the signal is not credible, non-designated bidders may compete in the auction, since they anticipate they can bid higher than the

[45] Note that this function is different from $\pi(x, y)$, it has different arguments. Also, $\pi(\cdot)$ is the expected payoff with respect to outsider bidding strategies. In case of $\hat{\pi}(\cdot)$, the payoff also depends on the realization of the vector of CV components y_{-i}.

[46] The cumulative distribution function is given as $J(s) = Pr\left(x + \frac{y}{n} < s\right)$.

designated bidder and earn a positive expected payoff.

If the main source of information asymmetry is PV, the signals provide little information about the CV but it makes the designated bidder credibly commit to a PV type. However, if the main source of information asymmetry is CV, the surplus conveys less information about the PV, and provides a more precise signal regarding the CV, if the signal is credible. Credibly signaling high PV is important to deter within-cartel competition. On the other hand, this is not the case with CV, signaling it improves the information set of others and reduces information rents. In what follows, we prove these points and illustrate them with examples.

Proposition 3.2 shows that it is sufficient for the existence of a BBCM that ring members can signal their PV in an indirect way. That is, if higher surplus signals higher conditional expected PV.

Proposition 3.2. *There exists a BBCM if the conditional expected PV function*

$$\mathbb{E}\left(x_i | x_i + \frac{y_i}{n} = s_i\right)$$

is a strictly increasing function of surplus for the entire range of s.[47]

Proof. We prove the Proposition by constructing a mechanism. The designated bidder is the ring member with the highest reported surplus. Let us assume ring members follow recommended bids by which the designated bidder submits a bid, which is her best response to outsider bidding strategies given her information set, whereas other members submit zero. We consider a mechanism with the following side-payment function. Let us consider the expected auction payoff function $\hat{\pi}(s_i, s_{-i})$ as defined in Subsection 3.3.1. Suppose bidder i has the highest reported surplus in the ring $s_i^* = \max_{j \in M} \left\{s_j^*\right\}$, and becomes designated bidder. Let the side-payment be equal to

$$\frac{1}{m}\,\mathbb{E}_{s_{-i}^*}\left[\hat{\pi}\left(s_i^*, s_{-i}^*\right) \,\middle|\, s_i^* > \max\left\{s_{-i}^*\right\}\right]$$

for the $m-1$ non-designated bidders. That is, the side-payment is the equal share of the expected auction payoff of the designated bidder, conditional on having the highest reported surplus. This value does not depend on the realization of lower surplus reports, but the expectation is over the distribution of these $m-1$ surplus values, given that they are smaller than the reported surplus of the designated bidder s_i^*. In order to shorten notation, let $\mathbb{E}_{s_{-i}^*}\left[\hat{\pi}\left(s_i^*, s_{-i}^*\right) | s_i^* > \max\left\{s_{-i}^*\right\}\right] = \bar{\pi}(s_i^*)$, where s_i^* refers to the reported surplus of the designated bidder. That is, the side payment for each non-designated

[47]From the range of PV and CV we have that $s \in \left[x_L + \frac{y_L}{n}, x_H + \frac{y_H}{n}\right]$.

bidder is equal to $\frac{1}{m}\bar{\pi}\left(s_i^*\right)$, and satisfying *ex post* budget balance, the side-payment is $-\frac{m-1}{m}\bar{\pi}\left(s_i^*\right)$ for the designated bidder i.[48]

The ICC takes the form of Condition (3.13). Terms capture the expected payoff and side-payment functions. Calculation of the latter vector is as above. The two sides of the ICC of ring member i compare the total expected payoff of reporting the true value s_i and a misreport $s_i' \neq s_i$. Value s_{-i} denotes the vector of surplus of other ring members. As above, $J\left(s\right)$ refers to the distribution of surplus s. The highest ranked surplus in the ring other than s_i is denoted by $\max\left\{s_{-i}\right\}$. The first two terms on the LHS and RHS refer to the side-payments for being the non-designated and the designated bidder. The third term expresses the expected payoff for being the designated bidder.

$$\begin{aligned} &\frac{1}{m}\underset{s_i<\max\{s_{-i}\}}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)\\ -\frac{m-1}{m}J^{n-1}\left(s_i\right)\bar{\pi}\left(s_i\right)+&\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)\geq\\ &\frac{1}{m}\underset{s_i'<\max\{s_{-i}\}}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)\\ -\frac{m-1}{m}J^{n-1}\left(s_i'\right)\bar{\pi}\left(s_i'\right)+&\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right) \end{aligned} \tag{3.13}$$

Condition (3.14) expresses constraint (3.13) in a simplified form for the case when $s_i < s_i'$.

$$\begin{aligned} \frac{1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)-\frac{m-1}{m}J^{n-1}\left(s_i\right)\bar{\pi}\left(s_i\right)\geq\\ -\frac{m-1}{m}J^{n-1}\left(s_i'\right)\bar{\pi}\left(s_i'\right)+\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right) \end{aligned} \tag{3.14}$$

Let us rearrange (3.14) in Condition (3.15). Recall that $\bar{\pi}\left(\cdot\right)$ is the expected auction payoff of the designated bidder conditional on having the highest surplus.

[48] The mechanism is related to the solution of Graham and Marshall (1987) who uses an independent PV model and similar to the one proposed by Pagnozzi (2004).

$$\frac{1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)+\frac{m-1}{m}\left(J^{n-1}\left(s_i'\right)\bar{\pi}\left(s_i'\right)-J^{n-1}\left(s_i\right)\bar{\pi}\left(s_i\right)\right)=$$
$$\frac{1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)+\frac{m-1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(s_i'\right)d\,\bar{J}\left(s_{-i}\right)+$$
$$\frac{m-1}{m}\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\left(\bar{\pi}\left(s_i'\right)-\bar{\pi}\left(s_i\right)\right)d\,\bar{J}\left(s_{-i}\right)\geq$$
$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)\qquad(3.15)$$

Condition (3.15) captures the effects of overreporting s_i. The three terms on the LHS inform us about the negative effects. Bidder i loses the side-payment from the highest opposing bidder who fails to be the designated bidder now (Effect 1); there is a range of new cases when she is required to pay side-payment (Effect 2); and she is required to send higher side-payment due to the higher report if all opposing ring members have lower surplus than s_i (Effect 3). The RHS informs us about the positive component. There is a higher probability of being the designated bidder with the misreport (Effect 4).

We can note the RHS can be divided into two parts as follows. Recall that $\bar{\pi}\left(s_i^*\right)=\mathbb{E}_{s_{-i}^*}\left[\hat{\pi}\left(s_i^*,s_{-i}^*\right)|s_i^*>\max\left\{s_{-i}^*\right\}\right]$ is a conditional expected value, following the definition, $\bar{\pi}\left(s_i\right)=\frac{1}{J^{m-1}(s_i)}\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)$. This is expressed in (3.16).

$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\,\bar{J}\left(s_{-i}\right)=\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)\Leftrightarrow$$
$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)=\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)-\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\,\bar{J}\left(s_{-i}\right)\qquad(3.16)$$

By substituting (3.16) into (3.15) we get (3.17).

$$\frac{1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)+\frac{m-1}{m}\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(s_i'\right)d\,\bar{J}\left(s_{-i}\right)+$$
$$\frac{m-1}{m}\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\left(\bar{\pi}\left(s_i'\right)-\bar{\pi}\left(s_i\right)\right)d\,\bar{J}\left(s_{-i}\right)\geq$$
$$\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)-\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\,\bar{J}\left(s_{-i}\right)\Leftrightarrow$$
$$\frac{1}{m}\left(\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\,\bar{J}\left(s_{-i}\right)\right)+$$
$$\frac{m-1}{m}\left(\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(s_i'\right)d\,\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i'\right)d\,\bar{J}\left(s_{-i}\right)\right)\geq$$
$$\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)\qquad(3.17)$$

We can notice the LHS is an affine combination of two terms since the coefficients add up to $\frac{1}{m}+\frac{m-1}{m}=1$. That is, if both terms are greater than or equal to the RHS, Condition (3.17) holds. Clearly the second term is greater than or equal to the first one, that is,

$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(s_i'\right)d\,\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i'\right)\geq$$
$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\,\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\,\bar{J}\left(s_{-i}\right)$$

So that, it is sufficient if

$$\underset{s_i<\max\{s_{-i}\}<s_i'}{\int\cdots\int}\bar{\pi}\left(\max\left\{s_{-i}\right\}\right)d\bar{J}\left(s_{-i}\right)+\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\bar{\pi}\left(s_i\right)d\bar{J}\left(s_{-i}\right)\geq\underset{\max\{s_{-i}\}<s_i'}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\bar{J}\left(s_{-i}\right)$$

holds. Let us consider both components of the LHS. First, take any s_{-i} with $\max\left\{s_{-i}\right\}<s_i'$. If $s_i<\max\left\{s_{-i}\right\}<s_i'$, then $\hat{\pi}\left(\max\left\{s_{-i}\right\},s_{-i}\right)\geq\hat{\pi}\left(s_i,s_{-i}\right)$. For $\max\left\{s_{-i}\right\}<s_i$, recall that $\bar{\pi}\left(s_i\right)=\frac{1}{J^{m-1}(s_i)}\underset{\max\{s_{-i}\}<s_i}{\int\cdots\int}\hat{\pi}\left(s_i,s_{-i}\right)d\,\bar{J}\left(s_{-i}\right)$, which is a conditional expected value. Consequently, Condition (3.15) holds and reporting $s_i'>s_i$ is not incentive compatible. The proof for $s_i'<s_i$ is analogous.

We also need that all ring members follow the recommended bids and non-designated bidders do not submit a serious bid in the auction. Suppose the condition in the Proposition holds and the expected PV is a strictly increasing function of surplus. Then, other ring members believe competing against the designated bidder is not profitable. The ring member with the highest surplus, the designated bidder, has the highest expected PV in the ring, following the condition of a strictly increasing expected PV function. Consequently, this ring member has the highest expected valuation. *Q.E.D.*

Existence of BBCM is guaranteed if a ring member can credibly signal the PV. Otherwise, this is not necessarily the case. In what follows we focus on knockout mechanisms in which bidders submit a bid, or equivalently, signal their surplus.

Proposition 3.3. *There exists no BBCM with a pre-auction knockout if the conditional expected PV function* $\mathbb{E}\left(x_i|x_i + \frac{y_i}{n} = s_i\right)$ *is not a strictly increasing function of surplus for the entire range of* s_i.

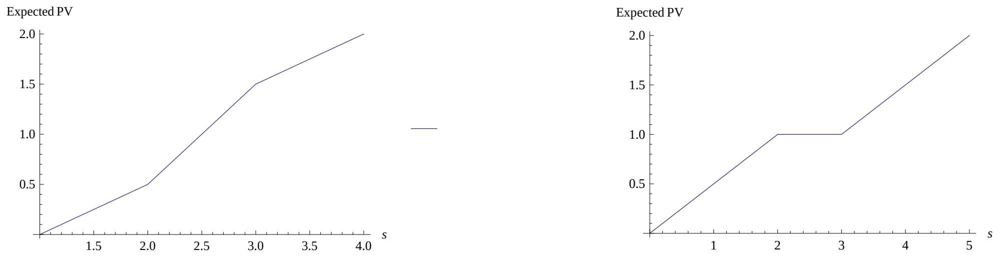
Proof. Note that that density functions $f\left(x_i\right)$ and $g\left(y_i\right)$ of types are continuous, so that this also holds for the joint density of x_i and y_i. That is, the expected PV function $\mathbb{E}\left(x_i|x_i + \frac{y_i}{n} = s_i\right)$ defined above is also continuous with respect to s_i. Suppose the expected PV function is not strictly increasing. Then, it either has a local minimum at $\check{s}$ or there is a range of s_i on which it is constant.

Let us address the former case when $\mathbb{E}\left(x_i|x_i + \frac{y_i}{n} = s_i\right)$ attains a local minimum at $\check{s}$. Consequently, there is a left-side neighborhood $(\check{s} - \varepsilon, \check{s})$ on which the conditional expected PV is higher. If a ring member with $s_i \in (\check{s} - \varepsilon, \check{s})$ does not become designated bidder, the designated bidder with higher surplus $s_i \leq \max\left\{s_{-1}\right\} \leq \check{s}$ is not able to deter her in the auction from bidding higher, having lower expected PV and lower expected valuation. This occurs with a positive probability.

Let us consider the other case. If there is a line segment of s_i on which the expected PV function is constant, deterrence is also not possible, since the probability of facing another ring member with identical expected PV is positive. Consequently, there is a segment of ring member types, which does not truthfully report surplus s_i. This is true for every point of the segment, so there is no mechanism in which surplus is truthfully revealed, which means there is no BBCM following Lemma 3.4. That is, the strict monotonicity condition is necessary for the existence of a BBCM with a knockout. *Q.E.D.*

To sum up, feasibility of a collusive arrangement depends on the ability of ring members to make a credible, noisy report of their PV. For extreme settings, feasibility is known. If only PV uncertainty is present, PV corresponds to valuation. On the other

Figure 3.1: Expected PV as a function of surplus



(a) Example 3.1.

(b) Example 3.2.

hand, this is not possible in a CV auction. By giving up private information, ring members share the same information set. Sending a credible signal intensifies the competition between ring members, leading to the inability to form a ring. The mechanism described in Proposition 3.2 does not necessarily lead to a first-best payoff for the ring. Since CV types are not truthfully revealed, the designated bidder i does not know y_{-i} with certainty.

Example 3.1 depicts an auction game in which BBCM is feasible.

Example 3.1. *Suppose PV and CV follow a uniform distribution with support* $[0, 2]$ *and* $[2, 4]$*, respectively, and there are* $m = n = 2$ *ring members. Higher surplus corresponds to higher expected PV, so that suppressing within-cartel competition is feasible. Figure 3.1a shows the expected PV as a function of surplus.* [49]

If ring members are not able to signal high PV, there is no BBCM with a knockout. This is illustrated in Example 3.2.

Example 3.2. *Similarly to the previous example, PV and CV follow a uniform distribution. Support is* $[0, 2]$ *for PV and* $[0, 6]$ *for CV. There are* $m = n = 2$ *ring members. There is a range, where a ring member, conditional on bidding, signals the same expected PV. The intuition behind this is that for all* $s \in [2, 3]$*, all CV types are possible. Figure 3.1b shows the expected PV as a function of surplus.*

3.4. Extensions

Sections 3.2 and 3.3 show that the source of information asymmetry between bidders matters. One can point out that these results have policy relevance, since an auctioneer possessing secret information is able to change the information setting. Milgrom and Weber (1982) argue that reducing the variance of the CV by public disclosure always

[49] All Figures were created by Wolfram Mathematica. All codes are provided in the Web Appendix.

increases the expected revenue of the seller. This sharp result is one of the fundamental results of auction theory and it is known as the Linkage Principle (LP).

In the followings we test LP in our setting and show that it does not hold if we allow for collusion. This is addressed in Subsection 3.4.1. Also, we address the problem of endogenous cartel formation by analyzing the effect of additional cartel members on ring stability in Subsection 3.4.2.

3.4.1. Public disclosure

The seller might possess private information affecting the valuation of all bidders. Public disclosure increases the expected revenue of the seller in non-cooperative auctions following Milgrom and Weber (1982) and Goeree and Offerman (2003). The Linkage Principle does not hold if we allow for collusion. We demonstrate it with an example, since disclosure provides incentives for collusion, and also results in a drop in expected revenue.

Public disclosure of information can be modeled in different ways. We apply the interpretation that the seller can alter the CV distribution by decreasing its variance. Public disclosure is defined as a mean-preserving contraction (MPC) of the CV distribution.[50] Let us take the mean of the distribution $y_M = \int_{y_L}^{y_H} y\, g(y)\, d\, y$. Disclosure is defined as an MPC from $g(y)$ to $\hat{g}(y)$ in which y_M remains constant while the respective density function contracts proportionally, $\hat{g}(y_M + ay) = \frac{1}{a} \cdot g(y_M + y)$ with $0 < a < 1$, for all $y_L - y_M < y < y_H - y_M$.[51]

The effect of disclosure is ambiguous. Examples 3.1 and 3.2 suggest that for uniform distribution there is a BBCM if and only if $\frac{y_H - y_L}{x_H - x_L} \leq n$. That is, sufficient disclosure makes collusion feasible. Below we focus on a case when $\frac{y_H - y_L}{x_H - x_L} > n$ with $a = 1$, so disclosure can change the existence of BBCM.

Example 3.3. *The shift between the previous examples can be interpreted as public disclosure, since the support of the CV component of 3.1 is a MPC of 3.2, where MPC is defined as above. Consider the following setting. The PV support is* $[0, 2]$*, whereas the CV support is* $[3 - d, 3 + d]$*, where* d *is the disclosure parameter. We can see that the preceding examples are special cases applying* $d = 1$ *and* $d = 3$*. Lower values of* d *refer to a greater extent of disclosure. Suppose there are* $m = 2$ *ring members and no outsider,* $n = 2$*. One can find that there is a BBCM if* $d \leq 2$*. In that case, the designated bidder*

[50] In the model of Milgrom and Weber (1982), the seller possesses an informational variable which can be released. In our setting, this would not change the feasibility of collusion, since the type space of a bidder is independent of that choice.

[51] We can see $\hat{g}(y_i)$ is a density function and $\int_{y_L - y_M}^{y_H - y_M} \hat{g}(y_M - ay) = \int_{y_L}^{y_H} \hat{g}(ay) = \frac{1}{a} \int_{y_L}^{y_H} \hat{g}(y) = 1$. Note that our definition of MPC is narrower than that of Rothschild and Stiglitz (1970), we impose symmetry of the contraction.

gets the good for a price of 0*, since there is no competition. On the other hand, there is no such mechanism if* $d > 2$*. Here we make the assumption that if there is no BBCM, potential ring members do not form a ring and compete instead.*[52]

There is a BBCM if $d \leq 2$*, in that case, the second highest bid is* 0*, consequently, the expected revenue is also* 0*. If* $d > 2$*, the equilibrium is competitive. Following Goeree and Offerman (2003), the symmetric Bayesian equilibrium bids satisfy Equation* (3.18)*.*

$$B_i\left(x_i, y_i\right) = \mathbb{E}\left(Y + x_i | s_i = \max_{j \neq i} s_j\right) \tag{3.18}$$

In terms of surplus, the equilibrium bid is as in Equation (18) *on the possible range* $\frac{3-d}{2} \leq s \leq 2 + \frac{3+d}{2}$*.*

$$b^*\left(s\right) = \begin{cases} \frac{3}{2}s + \frac{3-d}{4}, & \textit{if } \dfrac{3-d}{2} \leq s \leq 2 + \dfrac{3-d}{2}; \\ 2s - 1, & \textit{if } 2 + \dfrac{3-d}{2} \leq s \leq \dfrac{3+d}{2}; \\ \frac{3}{2}s + \frac{d-1}{4}, & \textit{if } \dfrac{3+d}{2} \leq s \leq 2 + \dfrac{3+d}{2}; \end{cases} \tag{3.19}$$

The expected revenue of the seller is the expected value of the second highest bid, $\mathbb{E}\left[\min\left\{b_1^*\left(s_1\right), b_2^*\left(s_2\right)\right\}\right]$*. As a function of d, it is depicted in Figure 3.2. Expected revenue is an increasing function of disclosure of bidders who compete but drops if disclosure is sufficiently great, that is, if d is sufficiently small.*

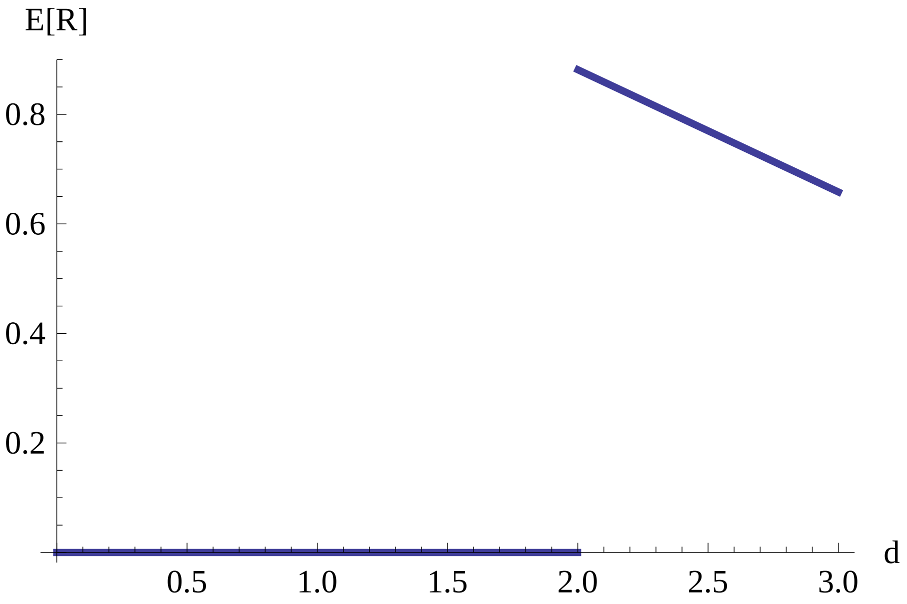
3.4.2. Cartel formation

Feasibility of collusion also depends on the number of cartel members. Inclusion of additional members can further reduce competition and allows for pooling more information about the CV. Existence of BBCM depends on the way a cartel is able to expand. Suppose the ring recruits members who were not players before.[53] Paradoxically, bigger cartels are more stable in this case, since additional members decrease the relative weight of the CV component in surplus, $\frac{y_i}{n+1} < \frac{y_i}{n}$. This result is in contrast with the theory literature. Porter (2005) provides a review, claiming "There is no honor among thieves." Bigger cartels deviate more from competitive bids, which provides additional incentives

[52] One can consider alternative conducts, including strategic cartel formation or reporting the bidding ring in a leniency program. While we do not explicitly model other cases, we can expect our results still apply. In general, an interior switch of the feasibility of collusion results in discontinuity of the expected revenue function.

[53] Consider another form of expansion by including bidders who are already participants in the auction. Clearly, this does not change the monotonicity of the expected PV function $\mathbb{E}\left(x_i | x_i + \frac{y_i}{n} = s_i\right)$. Thus, cartel expansion of this type does not affect the existence of BBCM.

Figure 3.2: Expected revenue as a function of public disclosure.



for deviation from the coordinated strategy. Consequently, collusion is harder with more ring members. This is also consistent with several actual cases.[54]

Our result relies on the assumption that a player entering the auction possesses relevant information. The intuition behind this idea is that with more participants, the information regarding the CV is more spread out among players and the weight of a particular signal y_i is smaller accordingly.

The existence of a BBCM with a knockout depends on the monotonicity of expected PV as a function of surplus, as shown in Proposition 3.2 and Proposition 3.3. Formally, if $\mathbb{E}\left(x_i | x_i + \frac{y_i}{n} = s_i\right)$ is monotone with respect to surplus s_i. To illustrate the significance of cartel expansion, we point out that there is a minimal number of bidders. If $\frac{x_H - x_L}{y_H - y_L} \geq \frac{1}{n}$ is not satisfied, $\mathbb{E}\left(x_i | x_i + \frac{y_i}{n} = s_i\right)$ is constant between x_H and $\frac{y_H}{n}$. That is, $\left\lfloor \frac{y_H - y_L}{x_H - x_L} \right\rfloor = n^*$ is a lower bound for the minimum, and one can see n^* can take values greater than 2. Example 3.4 illustrates our point for uniform distribution.

Example 3.4. *We consider a model with uniform type distributions $F(x_i)$ and $G(y_i)$ as in the previous examples. There exists a BBCM if and only if $\frac{x_H - x_L}{y_H - y_L} \geq \frac{1}{n}$ is satisfied. If this constraint is satisfied for n, it is also satisfied with the inclusion of new bidders. Moreover, there exists a minimal number of bidders such that the constraint is satisfied.*

[54]For a list of examples see Marshall and Marx (2009).

3.5. Discussion

The main contribution of this paper is that its results help to understand the practices of actual bidding rings. Sharing all private information between ring members enables the efficient allocation of the good within the cartel and maximizes the sum of their expected payoff. Actual cartels do not follow this pattern. Simple mechanisms including pre-auction knockouts are widespread forms of conspiracy. We characterize collusive equilibria for the case if bidders possessing multidimensional private information signals, which cannot be represented by a single variable. We also show why knockout auctions are prevalent despite not maximizing the collusive gain.

Our results show that in general, sharing all private information within a bidding ring is not incentive compatible. By relaxing the assumptions of incentive compatibility, the introduced Bayesian bid coordination mechanism (BBCM) shows why cartels conduct knockout auctions, sacrificing some of their potential collusive gains. Since not all claims are credible, coordinating bidding strategy by a simple auction mechanism allows members to credibly share partial information while keeping participation individually rational. The cartel can be successful if there is a positive gain which can be shared. Our results emphasize that a cartel can successfully implement a collusive mechanism if it allows ring members to signal high private values, hence, higher valuations. If the main source of information asymmetry between bidders is independent private values, collusion is feasible by using a knockout determining side-payments and the designated bidder.

Antitrust efforts are necessary to combat collusion. Most research papers focus on the auction mechanism and bidder's registration. This paper emphasizes the importance of the source of information asymmetry claiming that it has a profound effect on the form and feasibility of bid rigging. Our paper has some practical implications. First, efficient collusion with full information sharing between ring members is not likely to occur. On the other hand, this weaker form of bid rigging with a pre-auction knockout mechanism can be supported in equilibrium in a wide range of settings. Second, transparency can support a conspiracy. We argue that the seller should not always disclose all private information. That is, we challenge the Linkage Principle of Milgrom and Weber (1982). While disclosure has its merits by reducing information rents, the overall effect can be negative. In this respect, this paper extends the results of Chapter 2 to a different form of collusive mechanism.

Chapter 4

The Effect of Collusion on Efficiency in Experimental Auctions

Abstract

This chapter examines the effect of collusion on allocative efficiency in single-unit auctions. Explicit collusion may improve efficiency if cartel members can freely communicate. We test this hypothesis in a laboratory design. Subjects can choose whether they compete or form a cartel. Ring members can communicate and conduct a knockout auction for the allocation of the good and the share of the payoff. We employ three treatments allowing collusion with varying common value uncertainty, and an additional treatment precluding collusion. Our results show that collusion has a negative impact on efficiency. Communication within the cartel could allow for an efficient allocation by pooling information, but subjects fail to do so on average. While the majority of members truthfully report their private information and also update their beliefs elicited in a control procedure, they do not implement it completely in their bidding strategy. The cartel is able to achieve a significant gain compared to the competitive outcome, but this is negatively affected by efficiency loss. Seller's revenue suffers from limited competition. Subjects with higher type are less willing to join a ring. Also, common value variance has a negative effect on efficiency in competitive auctions, whereas this effect is not significant if a bidding ring is present.

4.1. Introduction

In an auction market for a single good, full *ex post* allocative efficiency is achieved if the player with the highest valuation wins. Reaching this outcome is hindered by information asymmetry and incomplete information on the part of the bidders. It is commonly accepted that cartels have a detrimental effect on the seller's revenue. As we show, it is less clear whether collusion improves efficiency.

This paper examines this problem by presenting a laboratory experimental study on second-price auctions. There is a non-negative effect in a perfect Bayesian equilibrium with rational bidders. The intuition stems from information sharing. If ring members can explicitly communicate, we can expect that the possibility of reducing information asymmetry can reduce inefficiency. This hypothesis is tested in a setting where players can form a bidding ring.[55] Surprisingly, our estimates show that explicit collusion has a negative effect on efficiency.

In a generalized mineral rights auction defined by Milgrom and Weber (1982), the valuation of all bidders is identical. In other markets, the structure of information asymmetry plays a crucial role. Art markets are the often-cited examples for a market in which valuations of bidders are independent. While inefficient allocation is possible, the bidder with the highest valuation always wins in a symmetric equilibrium. In procurements, bidders have different individual-specific characteristics. A standard simplification of this structure is to distinguish idiosyncratic informational variables as private values (PV), and shared ones as common values (CV). An efficient bidder is always the one with the highest PV. A bidder with low PV and high CV signal can bid higher in equilibrium than one with high PV and low CV signal, creating welfare loss.

Lopomo et al. (2005) argue that collusion can create inefficiency if cartel members are unable to communicate. Groenewegen (1994) as well as Goeree and Offerman (2003) raise the idea that collusion, with the possibility of explicit communication is actually beneficial from efficiency considerations. Communication makes information pooling feasible and the ring can achieve a higher payoff by increasing efficiency. This concern has been raised by members of the long-standing Dutch construction cartel which testified in 2002 that the primary motivation of their operation was information pooling (Boone et al., 2009).[56] We address the latter argument by constructing an experimental design in which a bidding ring can potentially increase efficiency compared to the competitive equilibrium.

We study one-shot interaction between bidders. The structure of the game is akin to Goeree and Offerman (2003), accompanied by the possibility of cartel formation and additional control procedures. Our design examines collusion by controlling for the source of information asymmetry. This paper is closely related to the work of Hinloopen and Onderstal (2010). They show that with exogenously determined side-payments, cartels break down in first-price and English auctions if players interact only once. Our study allows for endogenously determined side-payments.

[55] Since there is no single commonly accepted expression, we use the terms bidding ring and cartel interchangeably. The same applies to collusion and bid rigging.

[56] For details of the parliamentary hearings, see Parlementaire Enquêtecommissie Bouwnijverheid (2003).

In our design, randomly matched pairs of subjects participate in a second-price auction facing a weak computerized bidder.[57] If both bidders agree, they form a cartel, which allows them to communicate in a chat window and conduct a pre-auction knockout auction or simply knockout (KO). That is, a secondary auction takes place before the principal auction. The knockout allows them to send side-payments and choose a designated bidder. In the absence of collusion, both players enter the auction. In each round, pairs are rematched to approximate the one-shot conditions of the model. Risk aversion is measured applying the Holt and Laury (2002) protocol. In order to estimate the effect of collusion, we employ an additional treatment precluding collusion.

The effect of CV uncertainty on expected revenue is positive in a competitive auction. Milgrom and Weber (1982), Goeree and Offerman (2003) and Fatima et al. (2005) argue that publicly revealed information about the commodity increases expected revenue and efficiency. Lower uncertainty results in higher equilibrium bids on average, hence, higher revenue. Chapter 2 shows that this is not robust in a collusive setting. Experimental evidence supports the level of noise having a negative overall impact on cooperation (Aoyagi and Fréchette, 2009). Literature on laboratory auctions supports the claim that symmetry and complete information support collusion in private value games (Sherstyuk, 2002). Since this body of literature suggests the information setting has a profound effect on feasibility of collusion and on efficiency, we apply three treatments allowing for collusion, involving a given PV uncertainty, but different levels of CV uncertainty.

In the three treatments allowing for collusion, subjects self-select into a bidding ring. A ring is formed if both subjects simultaneously agree. We find that bidders' type has a negative effect on the probability of agreeing to do so.

We apply two different concepts for allocative efficiency. First, a dummy variable indicating that the commodity is allocated to the bidder with the highest valuation. Second, we consider welfare loss compared to the *ex post* efficient outcome. We find that allocation of the commodity is worse, either by estimating the effect of endogenous cartel formation or the exogenously given possibility of bid rigging. It is possible that this outcome is induced by biased beliefs, that is, subjects strategically misreport their own types but have optimistic assumptions about the same regarding the other ring member. The hypothesis of consistent beliefs cannot be rejected. Also, we show that communication significantly improves the beliefs of colluding bidders about types. The loss is implied by the knockout bidding strategy, which fails to take this information into account. The model estimates that players submit higher values if their expected

[57] In a second-price auction, a bidder with pure private values has a dominating strategy. This provides us with a natural candidate for the computerized bidder's strategy. A weak player might bid aggressively in an experimental auction, as estimated by Güth et al. (2005).

valuation is higher, but the coefficients are well below equilibrium levels.

We also address the distribution of welfare in the game. The hypotheses tested involve payoffs of the players and the seller. Equilibrium predictions of our model suggest that collusion increases payoff of ring members, but reduces the seller's revenue. In this design, one cartel member is chosen to be the designated bidder and compete in the auction. It is predicted that she has at least as high a payoff as the other member. Estimates show that there is no significant difference in payoffs.

While the cartel is able to obtain a significant gain, due to inefficient allocation of the designated bidder role and low level of competition if they do not collude, the increase compared to competition is lower than in equilibrium. Revenue soars under collusion. The treatment effects listed are significant and robust. In treatments allowing for collusion, CV uncertainty has a negative effect on efficiency, which replicates the results of Goeree and Offerman (2002). On the other hand, the same is rejected for auctions having a bidding ring.

The chapter is structured as follows. Section 4.2 describes the model, characterizes the equilibrium with risk neutral, rational bidders and formally states our hypotheses. Details of the experimental design are given in Section 4.3. Section 4.4 describes the estimations and summarizes our results. Finally, Section 4.5 concludes with a discussion.

4.2. Model and equilibrium

In what follows, valuation of bidders, the auction format and the collusive mechanism are described. A perfect Bayesian equilibrium is derived assuming rational, risk-neutral players.

4.2.1. Model

In our model, we distinguish two strong bidders and a weak one.[58] Valuation functions follow the hybrid model of Goeree and Offerman (2003). Strong bidders' valuation consists of two additively separable components, which will be referred to as private value (PV) and common value (CV). The first component PV is drawn independently and individually from a support $x_i \in [x_L, x_H]$ with uniform distribution, where $x_H \geq x_L$. Component CV is the average of bidders' independently and identically distributed CV signals from support $y_i \in [y_L, y_H]$ with uniform distribution, where $y_H \geq y_L$. That is, the valuation of bidder i is given as in Equation (4.1), in which y is the CV.

[58]This setup is relevant since it provides sufficient incentives for the strong bidders to form a ring. A cartel can only suppress competition if the two strongest players are involved.

$$v_i = x_i + y = x_i + \frac{\sum y_i}{2} \quad (4.1)$$

The third bidder is weak in the following sense. Her valuation is PV drawn from a uniform distribution with support $[c_L, c_H]$, where $c_L \leq x_L + y_L$, $c_H \leq x_H + y_H$ and we assume first-order stochastic dominance. All lower values are non-negative. Bidders participate in a second-price auction. That is, bids are submitted simultaneously and the highest bid wins. The highest-bidding player obtains the commodity for a price equal to the second highest bid. The two strong bidders can form a cartel with a mutual agreement.

We summarize the timing of the game.

1. Players privately learn their values x_i, y_i and c_i.
2. The strong bidders decide about collusion, simultaneously. A cartel is formed if they both choose to join.
3. Colluding pairs can simultaneously directly reveal each other their type. This is denoted by x_i^*, y_i^* and non-verifiable.[59]
4. Cartel members participate in a knockout auction, in which they simultaneously submit their bids. The winner participates in the auction and pays the lower of the two knockout bids to the other ring member, who is forced to bid 0 in the auction. Knockout bids are observable to the ring.
5. The principal auction takes place.[60]

This form of collusion is usually considered to be a strong cartel, since participation in the principal auction is controlled by the ring.[61]

4.2.2. Equilibrium analysis

We characterize the symmetric perfect Bayesian equilibrium, the surplus payoff from collusion and address efficiency. The surplus of a bidder is defined as $s_i = x_i + \frac{y_i}{2}$.

[59] These are restricted to $x_i^* \in [x_L, x_H]$ and $y_i^* \in [y_L, y_H]$.

[60] If a cartel is formed, there are two stages that are denoted as auction. In order to avoid confusion, the auction conducted by the seller is called the *principal auction* or simply the auction. It is distinguished from the *knockout auction* or simply *knockout*, that is organized by the cartel.

[61] The concept has several alternative definitions. Marshall and Marx (2007) call it bid submission mechanism (BSM). Their model entails only pure PV. Here we emphasize the characteristic that the cartel is able to submit bids and prevent individual ring members from participation. In actual rings, this can be implemented by agents who submit bids instead of members themselves.

Goeree and Offerman (2003) show that there exists a symmetric Bayesian equilibrium in the non-cooperative model in which players submit bids equal to the expected valuation v_i, conditional on the highest opposing surplus being equal to s_i. That is, the equilibrium bid of a bidder depends on surplus. To see this, note that valuation of bidders consists of the surplus known by the bidder and the stochastic part y_i which shall be counted conditional on winning, in order to avoid the winner's curse. The pure PV auction is a special case of this solution, in which bidders submit their valuation x_i. Note, that the existence of a dominant strategy does not carry over to auctions with CV uncertainty.[62] The equilibrium bids $b^*(x_i, y_i)$ are given by Equation (4.2).

$$b^*(x_i, y_i) = \mathbb{E}(v_i | s_i = \max\{s_{-i}\}) \tag{4.2}$$

The weak player submits her valuation, which is also the dominating strategy since this value is not conditional on winning.

Suppose the two strong bidders engage in collusion. In equilibrium, not updating beliefs after sharing signals is sustained. Communication does not provide any additional gain, but it can change the knockout bid of the other ring member. If it does, the mechanism is not incentive compatible, bidders send signals minimizing the opposing knockout bid. That is, bidders do not change knockout bids conditional on shared signals.[63]

Let us assume bidder i wins the knockout auction. She faces the computerized bidder and possesses information set (x_i, y_i) and observes the knockout bid of the other ring member k_j. In equilibrium i updates her belief about the type of j. Her equilibrium bid in the auction is related to (4.2) and corresponds to $x_i + \frac{y_i}{2} + \frac{\mathbb{E}(y_j|k_j)}{2}$. To see why this is the perfect Bayesian equilibrium (PBNE) bid, note that the weak bidder has a dominating strategy and submits her valuation in equilibrium. There is no profitable deviation of i. If bidding higher results in overbidding the weak bidder, the expected payoff becomes negative. Underbidding changes a positive expected payoff to negative. If the outcome does not change, the payoff remains unaltered in a second-price auction.

The knockout bid is an increasing function of the surplus. The argument is analogous to that of the competitive equilibrium. This also means that the auction bid of the ring

[62] Consider a bidder with very low surplus. Conditionally on winning, the CV of the other bidder is also low. For higher surplus, the CV of the other bidder is not constrained similarly. Consequently, the equilibrium bidding strategy is typically only piecewise linear function of s_i.

[63] This result is consistent with a number of actual bidding rings with simple commodities. In a long standing stamp auction cartel studied by Asker (2010), participants only carried out a knockout auction without any prior communication. In an auction for a single commodity, this is even more reasonable considering no punishment mechanism is feasible.

member winning the knockout is $x_i + \frac{y_i}{2} + \frac{\mathbb{E}(y_j|k_j)}{2} = s_i + \frac{\mathbb{E}(y_j|k_j)}{2}$. That is, we denote the expected payoff of i in the auction simply with $\bar{\Pi}(s_i) = \mathbb{E}\Pi(s_i|s_i \geq s_j)$, where $\Pi(\cdot)$ refers to the expected revenue of i only facing the weak bidder. The equilibrium knockout bid is half of this value. The reason is that unlike in the standard second-price auction, the loser receives her own bid. That is, in equilibrium, conditionally on facing an identical opposing bid, the bidder must be indifferent between being the winner and receiving the side-payment. So that, a ring member i bids $\frac{1}{2}\mathbb{E}\bar{\Pi}(s_i)$.

Consequently, the expected payoff of a bidder engaging in collusion is

$$\int_{s_j = x_L + \frac{y_L}{2}}^{s_i} \left(\bar{\Pi}(s_i) - \frac{1}{2}\bar{\Pi}(s_j)\right) d\,H(s_j) + \int_{s_j = s_i}^{x_H + \frac{y_H}{2}} \frac{1}{2}\bar{\Pi}(s_i)\; d\,H(s_j) \tag{4.3}$$

conditionally on all bidder types joining the ring. Let us show that this holds. First note that for all $s_j > s_i$, i is better off in a ring. Also, $\int_{s_j = x_L + \frac{y_L}{2}}^{s_i} \left(\bar{\Pi}(s_i) - \frac{1}{2}\bar{\Pi}(s_j)\right) d\,H(s_j) \geq \int_{s_j = x_L + \frac{y_L}{2}}^{s_i} \left(\bar{\Pi}(s_i) - \bar{\Pi}(s_j)\right)\; d\;H(s_j)$. The right-hand side is always greater than the expected competitive payoff of bidder i. This is clearly satisfied for $s_j \geq s_i$. Let us consider having $s_i \geq s_j$. For each s_i, s_j, the expected payoff of i is smaller than or equal to $s_i - s_j$. If $s_i = s_j$, this equals zero by the construction of the equilibrium. If s_j is lower, the price paid shrinks as well as the expected utility. This is smaller than $\bar{\Pi}(s_i) - \bar{\Pi}(s_j)$. If $s_i = s_j$, the expression clearly equals 0. Also, $\frac{\partial \bar{\Pi}(s_j)}{\partial s_j} > 0$, since higher values increase the expected value of the other ring member's CV signal. Our findings are summarized in Lemma 4.1.

Lemma 4.1. *All bidder types join the ring.*

In other words, the mechanism is *ex ante* individually rational. The intuition behind this result comes from the power of the cartel. A strong bidder achieves substantial expected gain from suppressing competition with the strongest opponent.

The predictions with respect to efficiency correspond to previous literature. Efficiency is guaranteed in a pure CV auction, since valuations are identical. In a pure PV auction, equilibrium bids increase in valuation, and this guarantees that the efficient buyer obtains the commodity. The solution is less clear if both types of information asymmetry are present, as Goeree and Offerman (2003) argue.

A bidder with high PV and low CV might bid lower in equilibrium than an opponent with low PV and high CV. Consider two bidders, with types $(x_i = 800, y_1 = 0)$ and $(x_i = 700, y_1 = 400)$, where the supports of private and CV signals are $[0, 400]$ and $[0, 400]$, respectively. The computer bids $c \in [0, 500]$. In equilibrium, player 1 submits $b_1 = 1000$, whereas $b_2 = 1050$. Bidder 2 wins the auction, while her opponent has higher PV, resulting in an efficiency loss of 100.

The question is what happens if we address the effect of collusion on efficiency. We can argue the effect is non-negative. Cartel members fully randomize their information signals in equilibrium. If ring members Bayesian-update their beliefs, ring members choose the signal minimizing the expected opposing knockout bid. As we argued above, the ranking of knockout bids and competitive auction bids coincide.

Additionally, the knockout bids help the designated bidder to improve her beliefs regarding the CV signal of the other ring member, as we have also seen above, bidding $s_i + \frac{\mathbb{E}(y_j|k_j)}{2}$. This bid equals the expected valuation of the bidder, where the distribution second-order stochastically dominates the same of the competitive bidder. That is, the probability of winning while having greater valuation than the weak bidder is also higher than in the competitive auction.

Lemma 4.2. *Collusion has no effect on the allocation of the good between the strong players. The probability of efficient allocation is higher if a bidding ring is formed.*

Lemma 4.2 is consistent with previous theoretical findings.[64] The intuition is that collusion does not serve as a signaling device within the ring if it uses a knockout auction instead of full disclosure and only the designated bidder can participate in the auction.[65]

The effect of CV uncertainty on allocative efficiency is well-known from experimental and theoretical literature. A bidder with high CV and low PV might bid higher than her opponent with low CV and high PV, while the latter is the efficient bidder.

Lemma 4.3. *Mean-reserving spread of the CV distribution increases the probability that the strong bidder with the lower PV submits a higher bid in the auction than the other strong bidder. This results holds for competitive and collusive auctions alike.*

Proof. As we argued above, collusion does not alter the ranking of auction bids. That is, the subject of our interest is the probability of $s_i \geq s_j$ if $x_j \geq x_i$.[66]

$$s_i \geq s_j \iff x_i + \frac{y_i}{2} \geq x_j + \frac{y_j}{2} \iff \frac{y_i}{2} - \frac{y_j}{2} \geq x_j - x_i \tag{4.4}$$

[64] We must keep in mind that this result depends on the rationality assumption. If bidders update their beliefs after communication, efficiency can increase. Suppose players also possess unbiased beliefs, that is, their beliefs are consistent with the actual distribution of types conditionally on signals. In the extreme case, ring members share their types truthfully and they correctly anticipate this. Conditionally on shared signals, they have the same set of beliefs regarding the CV and the one with the higher PV bids higher. Consequently, the allocation of the designated bidder right is efficient. In the literature, this is denoted as efficient collusion.

[65] The latter factor is important. Without enforced participation, the designated bidder should prevent other ring members' entry, hence, discounts her own expected payoff in the auction.

[66] This result for competitive auctions is analogous to iii) of Proposition 2. of Goeree and Offerman (2003).

For any x_j, x_i, the probability that the right-hand-side constraint in (4.4) holds is an increasing function of the support of CV. *Q.E.D.*

This paper examines three hypotheses set by the preceding Lemmas.

Hypothesis 1. *All bidder types join the ring.*

Hypothesis 2. *Forming a bidding ring increases the probability of efficient allocation.*

Hypothesis 3. *Common value variance has a negative effect on efficiency in competitive as well as collusive auctions.*

4.3. Design of the experiment

The experiment was conducted between 10-31 October, 2014 and on 3 March, 2016, in CentERlab, Tilburg. We ran nine sessions with 166 subjects, in four treatments. Subjects were recruited online from a pool of undergraduate students at Tilburg University. Participants' earnings were expressed in experimental currency *Coin*, which was exchanged for Euro at the end of the session at a rate of 100 Coins for 1 Euro. If it is not explicitly mentioned, payoffs are expressed in the experimental currency. In the followings we summarize the design of the treatments.

Each session had an even number of subjects between 16 and 22 who participated in an auction game and in a second part employing the Holt and Laury (2002) protocol measuring risk preferences. Both parts were fully computerized and programmed in z-Tree.[67] In this section we provide a detailed description of our design.

We applied four treatments, R1, R2, R3 and C3. In the names, *R* refers to treatments, in which rings are allowed, whereas *C* means competitive bids in which collusion was precluded. The numbers refer to the CV signals: 1 means no, 2 means low, whereas 3 means high variance.

Subjects received the instructions followed by a number of control questions to ensure their understanding of the rules. The instruction sheet and the control questions can be found in Appendix 4.B.[68] The auction game consisted of eleven periods of which the first one served as practice and a randomly chosen one out of the remaining ten counted towards the final earnings. Pairs of subjects were randomly rematched anonymously in each period.

[67] Fischbacher (2007).

[68] Instruction sheets and control questions were analogous for all treatments except for the distribution of the CV variables and the scoring rule of the belief elicitation stage. Treatment C3 precluded collusion which was there absent from the instructions. The Appendix contains materials for Treatment R2.

Types of different periods were drawn independently from each other. Subjects started each period with an endowment of 800 Coins, independently from previous periods. In order to guarantee non-negative final payoffs, submitted bids were bounded from above such that payoff of the auction game was at least 0. The average payoff of the auction game without the incentivized belief elicitation treatment was 955 Coins. The average total earning per subject amounted to 14.15 EUR.

The auction game is summarized below, stages appearing only for players forming a ring are clearly noted. Since Treatment C3 precluded collusion, it only included stages 1, 6 and 7.

1. Subjects learned values x_i (and y_i, depending on the treatment).
2. They answered a yes/no question regarding their participation in the bidding ring. The ring was formed if both of them replied *yes*.
3. *Ring members*: Free chat.[69] Players could leave the stage by submitting the knockout (KO) bid.
4. Incentivized elicitation of beliefs about PV (and CV) of the other subject.
5. *Ring members*: Feedback on the outcome of the knockout. Side-payments are implemented.
6. Auction stage.
7. Feedback on the outcome of the auction. Subjects learned all three bids and their final payoff, without the payment of the belief elicitation stage.

A laboratory experiment on collusion in auctions is not without methodological difficulties. In particular, the implementation of side-payments is far from obvious (Kagel and Levin, 2011). In order to mitigate the mechanism of actual cartels, we allowed for a KO auction with prior communication. Subjects were not allowed to signal their identity, they could only send messages phrased in English, through the computer.

The weak bidder was computerized and did not interact with the cartel. While valuations of subjects moved in the range 200 to 1200, depending on the treatment, the computer always submitted a bid between 0 and 500 Coins, following a uniform distribution. The reason behind this is that only strong bidders find it rational to collude (Hu et al., 2011). In a second-price auction, a ring can only reach a positive collusive

[69]The communication was not restricted, except subjects were not allowed to reveal their identity. The chat data was decoded by three subjects separately after all sessions who were paid a fixed fee of 70 EUR.

Table 4.1: Summary of treatments

Treatment	PV	CV	Computer	Collusion	Sessions	N
R1	$[400, 800]$	$[0, 0]$	$[0, 500]$	Allowed	2	32
R2	$[200, 600]$	$[0, 400]$	$[0, 500]$	Allowed	3	54
R3	$[200, 600]$	$[0, 800]$	$[0, 500]$	Allowed	2	40
C3	$[200, 600]$	$[0, 800]$	$[0, 500]$	Precluded	2	40

gain in equilibrium if the bidders with the two highest equilibrium bids are members. Low computerized bids guaranteed that this event had a high probability, for example 95.31 percent in Treatment R1, but it was never certain.

Treatments differ in their support of PV and CV. Table 4.1 summarizes the treatments providing the parameters in the form presented in the instructions. All values are non-negative. Valuations of any subject is positive in order to avoid throw-away bids. It is a major concern in experimental auctions that subjects may bid well below equilibrium levels if the probability of winning a positive amount is negligible Harstad (2000).[70]

Our design employed two incentivized control procedures. The first one took place in each period between the chat/knockout stage and the feedback stage in treatments allowing for collusion (R1, R2 and R3). Beliefs on the values of the other ring member were elicited to control for belief updating after entry decision and the chat. This stage appeared for all subjects, regardless of collusion.[71] The payoff was calculated by a simple quadratic scoring rule, providing 200 Coins for a perfect guess. That is, the payoff was $\max\left\{0, \frac{10000-x_{dev}^2}{10000}\right\}$ for R1, $\max\left\{0, \frac{20000-x_{dev}^2-u_{dev}^2}{20000}\right\}$ for R2 and R3, where x_{dev} and y_{dev} denotes the absolute difference between the elicited beliefs and the actual values.[72] A checker was employed in order to make sure that subjects submit values from the feasible range.

Subjects' risk aversion was measured using the Holt and Laury (2002) protocol, identically for all subjects and treatments.[73] It took place after the auction experiment. Subjects made 10 choices between a low-variance and a high-variance lottery. The choices took the form $p \cdot l_1 + (1-p) \cdot l_2$ and $p \cdot h_1 + (1-p) \cdot h_2$, where p varied between 0.1 and 1 with 0.1 increments. Values were set $h_1 = 25$ Coins, $l_1 = 400$ Coins, $l_2 = 500$ Coins and $h_2 = 960$ Coins. The choices were presented on the same screen and subjects

[70] A bid is denoted as "throw-away" if its value is so low that it is losing with certainty. In all regressions, these values are included. Excessively high bids were ruled out by a checker, which were set to avoid negative payoffs.

[71] Rejection of forming a bidding ring provided additional information.

[72] Quadratic scoring rules are widely used in experimental literature. Sonnemans and Offerman (2001) show that they are incentive compatible. Proper Scoring Rules (PSR) have an extensive literature. For an overview, see Armantier and Treich (2013).

[73] The specific form and z-Tree code was borrowed from Breaban and Noussair (2014), using a modified payoff scale with our experimental currency Coin. Instructions can be found in Appendix 4.C.

could submit their choices with a single click. There was no explicit time limit and they were allowed to revise their choices up to the point of submission. The payoff was the outcome of a randomly chosen period. Any combination of choices was permitted.

Chat entries were decoded by three subjects who did not participate in any of the sessions. The corresponding instructions can be found in Appendix 4.D. Those values were added to the data by which at least two of the three coders had the same entry. Otherwise, the value remained empty.

4.4. Results

While we can predict that efficiency loss compared to the efficient outcome is low, experimental data shows this is not the case. In contrast with equilibrium predictions, collusion reduces efficiency. This result is in sharp contrast with both the rational benchmark and efficient collusion. In what follows descriptive statistics are provided and factors playing a role in the puzzle are identified. Subsection 4.4.2 addresses the decision about collusion. Subsequently, we address the effect of collusion on efficiency in Subsection 4.4.3. Finally, we conclude with additional remarks on welfare distribution. In all cases, results of the trial round are omitted. Description of variables can be found in Appendix 4.E.

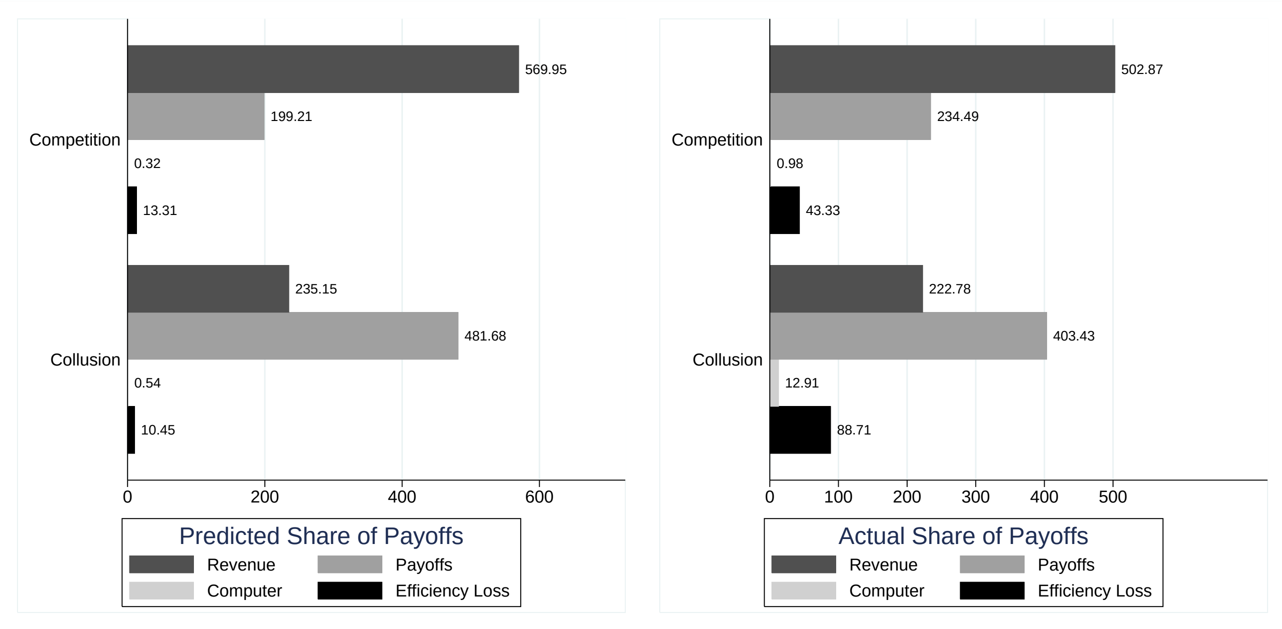
4.4.1. Competition and bid rigging

Figure 4.1 shows the welfare distribution in a scale of payoffs expressed in the experimental currency. The four diagrams depict the sum of payoffs of subjects, the computer's payoff, the seller's revenue and the efficiency loss. Actual and predicted average values are compared in auctions where players compete and where they engage in collusion. Values show the mean for all treatments together.

Efficiency loss is defined as the difference between the welfare-maximizing and actual outcomes. Allocative efficiency is lower under competition and collusion than in equilibrium. While the ring has higher payoff than competing bidders, the difference is not as much as with rational bidders. The ring performs worse than it would in equilibrium, while competing bidders are better off. Predictions and actual outcomes are consistent in the sense that the seller significantly suffers from collusion.

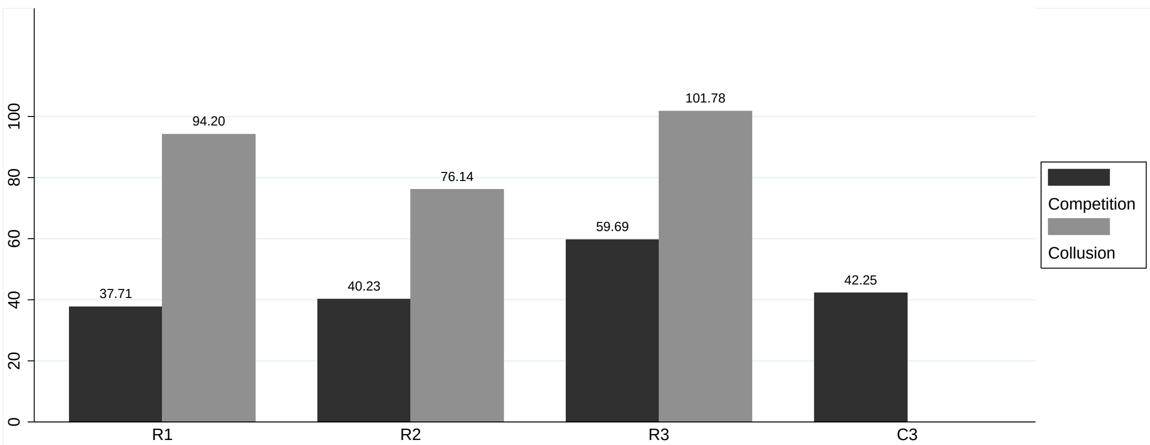
Welfare, which is lower with cartel presence, is distributed between the three bidders and the seller. The computer receives negligible payoff in all settings. Competing bidders' payoff is slightly higher than predicted, at the expense of the seller's revenue, which is lower than predicted. The average revenue for all treatments is 502.87 in competitive and 222.78 in games with a ring.

Figure 4.1: Distribution of welfare



Rings do not achieve the predicted payoff and receive an amount lower by 78.25 on average. The seller's revenue is slightly lower than predicted, 222.78 instead of 235.15. The negative differences are attributed to low welfare. Efficiency loss is much higher under collusion than in competition, the mean values are given as 88.71 and 43.33, respectively. Figure 4.2 shows that the negative effect is persistent across all treatments. This outcome contradicts Hypothesis 2. Comparing Treatments R3 and C3 shows that this difference might not only be attributed to endogenous selection of ring members, since bid rigging is precluded in C3.

Figure 4.2: Efficiency Loss under competition and collusion



Conditionally on cartel formation, payoff is evenly distributed between the designated and non-designated ring members on average. This is in contrast with the equilibrium outcome in which the designated bidder has a lower payoff. The actual average payoffs without the endowment are 201.29 for the designated ring member and 202.14 for the other subject of the group.

We found no strong evidence in favor of social preferences. The Kolmogorov-Smirnov test for the distribution of payoff differences between collusion and competition is significant at 5 percent level but not significant at 1 percent level in Treatments allowing for collusion.

The performance of competing bidders shows higher payoffs than in equilibrium, 234.49 vs. 199.21, respectively. Regressions in Table 4.2 show what explains the low level of competition for each of the four treatments. Previous experimental literature argues that bidders rarely bid below equilibrium levels in competitive second-price auctions. Kagel and Levin (1993) report that this only happens in 4 percent of the observations. In comparison, this probability value was 79.9 percent in R1 if subjects do not form a cartel. This difference is implied by the difference in experimental designs for treatments allowing for collusion. All subjects in the experiment of Kagel and Levin (1993) play a non-cooperative game. In Treatments R1, R2 and R3, bidders are self-selected into competition and they are exposed to experience in collusion. Regressions (1)-(3) capture the effect of this exposure. Variable *Collusion Percentage* expresses the ratio of preceding periods in which the subject was a cartel member and it takes values between 0 and 1. The coefficient estimate is negative and significant at 0.1 percent level for all treatments, its value ranges from -187.4 to -275.7. That is, exposure to collusion makes subjects bid less aggressively in competitive auctions.

4.4.2. Bidder types and collusion

We test Hypothesis 1 whether all subjects choose collusion, independently of types. The decision regarding collusion takes place after subjects have learned their private information. Other than bidder types, decisions can be influenced by the history of the game, treatment and risk preferences. As it is clear from the probit regression estimates of Table 4.3, the hypothesis can be rejected for the entire length of the session. In specifications (1)-(3), the random effect probit coefficient of surplus is significant at 0.1 or 5 percent level and negative, higher types were less inclined to collude.[74] Risk aversion has an ambiguous effect, the coefficient estimates suggest risk lovers tend to choose to join the ring in Treatment R2 whereas the opposite holds for R3. We can observe a negative learning effect, subjects are less inclined to join in later periods, but the coefficient is only significant at 5 percent level for Treatment R3. These results can be attributed to the complexity of R3. Since it cannot be explained by risk preferences, the higher types'

[74]Recall $s_i = x_i + \frac{y_i}{2}$.

Table 4.2: Competitive Bids

	(1) Bid	(2) Bid	(3) Bid	(4) Bid
Own PV	0.721***	0.605***	0.315	0.494***
	(10.65)	(4.57)	(1.83)	(6.20)
Own CV		0.279**	0.236*	0.318***
		(2.65)	(2.22)	(6.81)
Risk Aversion	-31.24	-6.935	-9.464	-0.920
	(-1.55)	(-0.67)	(-0.60)	(-0.11)
Period	10.83**	12.41**	17.71**	20.87***
	(2.86)	(2.71)	(2.71)	(8.38)
Collusion Percentage	-161.8**	-166.6***	-210.8*	
	(-2.92)	(-3.86)	(-2.29)	
Constant	210.1	290.0**	377.3*	239.9***
	(1.87)	(2.98)	(2.06)	(3.50)
Treatment	R1	R2	R3	C3
Observations	152	210	116	400

note: Random effects panel estimates. Robust standard errors are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

lower willingness to collude might be explained by a mistaken belief that bidding alone provides higher payoffs. It is notable that the decision of the previous round does not have a significant effect on a period. Models (4)-(6) have analogous results by controlling for PV and CV signals separately. All coefficients are significant at 5 percent level, except for that of the CV signal for Treatment R3.

The effect of bidder type has an effect on the estimated probability of a positive answer on collusion, but this diminishes in time. Figure 4.3 illustrates these points by probit estimates of regression model (1)-(3) in Table 4.3 for Treatments R1, R2 and R3 allowing for forming a bidding ring. In all graphs, the horizontal axis refers to the surplus of a bidder and the vertical line depicts the estimated probability that a subject replied 'yes'. For bidders with the lowest surplus, this probability was near 100 percent. For them, initiating collusion is a dominant strategy irrespective of risk preferences, since their chance of winning in equilibrium is zero. The probability of replying 'yes' drops to between 55 and 85 percent for subjects near the maximal surplus.

4.4.3. Efficiency in treatments allowing for collusion

Our main question of interest is in the effect of collusion on allocative efficiency. Since regressions concerning the decision about collusion confirm that selection bias persists,

Table 4.3: Willingness-to-Collude as a function of surplus, random effects

	(1) Coll. A.	(2) Coll. A.	(3) Coll. A.	(4) Coll. A.	(5) Coll. A.	(6) Coll. A.
Risk Aversion L1	0.0410	0.359	0.0892	0.0410	0.354	0.115
	(0.15)	(1.61)	(0.27)	(0.15)	(1.58)	(0.34)
Surplus	-0.00325***	-0.00235***	-0.00151*			
	(-3.82)	(-3.52)	(-2.12)			
Risk Aversion	-0.00183	0.339**	-0.299*	-0.00183	0.352**	-0.291*
	(-0.01)	(3.10)	(-2.27)	(-0.01)	(3.04)	(-2.21)
Period	-0.0323	-0.0199	-0.103*	-0.0323	-0.0222	-0.105*
	(-0.88)	(-0.66)	(-2.44)	(-0.88)	(-0.72)	(-2.48)
Own PV				-0.00325***	-0.00153*	-0.00290**
				(-3.82)	(-2.07)	(-2.83)
Own CV					-0.00307***	-0.0000602
					(-3.88)	(-0.12)
Constant	2.785**	0.807	4.316***	2.785**	0.870	4.569***
	(3.16)	(1.34)	(4.23)	(3.16)	(1.38)	(4.40)
Treatment	R1	R2	R3	R1	R2	R3
Obs.	288	504	342	288	504	342

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

the difference-in-means estimator is biased. First we address endogenous ring formation by comparing, in treatments allowing for collusion, auctions with or without ring. Subsequently, we test for the treatment effect by estimating the effect of allowing for collusion, comparing Treatments R3 and C3.

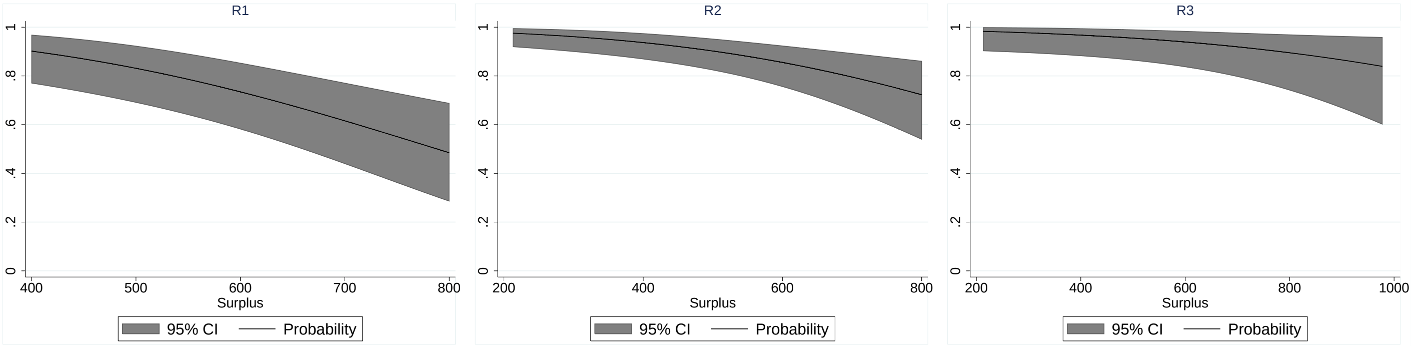
The efficiency levels realized in the experiment are captured in two different ways. Dummy variable *Efficient* takes value 1 if the winner of the auction is the bidder with the highest valuation and 0 otherwise. *Efficiency Loss* is the absolute difference between the maximal and actual welfare. That is, it takes 0 if and only if *Efficient*= 1.[75]

Table 4.4 addresses the effect of collusion on revenue, payoffs and efficiency loss.[76] In

[75]The design of treatments R2 and R3 is comparable with Goeree and Offerman (2002) if the game is non-cooperative. They report efficiency levels analogous to dummy variable *Efficient* and have significant estimates 79 and 54 percent for low and high CV uncertainty, respectively in the first ten periods. The means of efficient outcomes in our sample are 62 and 49 percent. Note that bidders self-select into a bidding ring in three treatments and the two designs are different in several details including the number of players and the bankruptcy rule.

[76]According to the Hausman test statistics the random effects estimator is consistent for both dependent variable, H_0 cannot be rejected at 5 percent significance level.

Figure 4.3: Probability of initiating collusion as a function of surplus, for treatments allowing for collusion, RE probit estimates



order to control for unobserved heterogeneity, the models are applied to Treatments R3 and C3. The coefficient estimates are consistent with the descriptive statistics, allowing for collusion significantly increases efficiency loss and payoff, and reduces the seller's expected revenue, all at 0.1 percent level. The estimates also show a learning effect. Bidders achieve decreasing payoffs while revenue increases in subsequent periods. This effect is more clear for the latter, at 0.1 percent significance level, and suggests the seller is better off by 14.10 Coins per period for the whole game. Risk aversion and bidders' performance expressed by past average payoffs do not play a role in welfare distribution.

The observed difference in efficiency levels does not necessarily correspond to the average treatment effect (ATE) of collusion. Cartels were formed by subjects, and the previous subsection shows that it was not random, so we need to control for the selection bias. Control variables from the latter regression results are used in our study of collusion effects. If we do so, decision about collusion becomes conditionally independent and an unbiased estimate on the treatment effect can be obtained.

The random effects regression outputs in Table 4.5 confirm the descriptive result that collusion reduces efficiency. The treatment effect on *Efficiency Loss* is significant at 1 percent in all specifications and the coefficient estimates are above 40. ATE estimates are consistent with this result showing similar impact and are also significant. The effect on the probability of efficient allocation is a bit more ambiguous. While the treatment effect is significant for all periods, it fails to be significant for the last five. The latter might be the result of a learning effect. Treatment effect is not significant, the result persists irrespectively of the CV uncertainty.

Estimates on the effect of collusion are negative and significant for alternative models, too. The estimated Average Treatment effect for the Treated is 39.89 with four nearest neighbors. The analogous Average Treatment Effect is 35.49. Both coefficients are significant at 0.1 percent significance level.

Table 4.4: The effect of Collusion on Payoff without HL and belief elicitation payments

	(1) Revenue	(2) Rev.	(3) Rev.	(4) Auction Payoff	(5) Auction P.	(6) Auction P.	(7) Efficiency Loss	(8) Efficiency L.	(9) Efficiency L.
Own PV	0.199***	0.184***	0.220**	0.619***	0.616***	0.519***	-0.00591	0.0213	0.0380
	(3.72)	(3.64)	(2.88)	(10.04)	(10.52)	(5.90)	(-0.15)	(0.52)	(0.73)
Own CV	0.0983***	0.100***	0.111**	0.312***	0.315***	0.293***	0.0211	0.0224	0.0274
	(4.08)	(4.07)	(3.14)	(11.14)	(10.36)	(6.63)	(1.15)	(1.17)	(1.16)
Other PV	0.194***	0.181***	0.222**	-0.304***	-0.314***	-0.280**	-0.00591	0.0157	0.0354
	(4.04)	(3.99)	(2.69)	(-4.48)	(-4.77)	(-3.05)	(-0.16)	(0.42)	(0.70)
Other CV	0.101***	0.108***	0.123**	0.0688	0.0609	0.0570	0.0211	0.0221	0.0277
	(3.91)	(4.15)	(3.11)	(1.88)	(1.69)	(1.15)	(1.15)	(1.24)	(1.27)
Collusion Allowed	-295.9***	-306.9***	-322.1***	120.9***	129.0***	116.1***	45.70***	42.08***	44.66***
	(-18.25)	(-18.05)	(-16.17)	(8.12)	(6.88)	(4.76)	(6.43)	(5.84)	(3.92)
Period		14.10***	16.22**		-6.342*	-7.499		-0.445	2.967
		(7.39)	(2.74)		(-2.21)	(-1.18)		(-0.28)	(0.95)
CompBid		0.299***	0.273***		-0.281***	-0.231**		0.180***	0.126**
		(6.21)	(4.52)		(-4.96)	(-3.18)		(4.41)	(2.82)
Payoff Average		0.0688	0.0725		-0.0627	0.0727		0.0538	0.0292
		(1.43)	(0.67)		(-0.75)	(0.52)		(1.60)	(0.56)
Risk Aversion		3.864	6.881		-5.328	-9.075		1.841	3.044
		(0.60)	(0.94)		(-1.14)	(-1.48)		(0.70)	(1.00)
Constant	349.3***	129.8*	65.82	639.7***	829.5***	752.0***	29.90	-92.24	-107.8
	(9.68)	(2.06)	(0.50)	(15.21)	(7.89)	(4.79)	(1.35)	(-1.84)	(-1.76)
Observations	780	702	390	780	702	390	780	702	390
Periods	1-10	1-10	6-10	1-10	1-10	6-10	1-10	1-10	6-10

note: Random effect panel estimates. Robust standard errors are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.5: Efficient outcome and Efficiency Loss, random effects

	(1) Efficient	(2) Efficient	(3) Efficient	(4) Efficiency Loss	(5) Efficiency Loss	(6) Efficiency Loss
Collusion	-0.266***	-0.267***	-0.261***	42.90***	43.06***	43.94***
	(-3.66)	(-3.66)	(-3.57)	(6.58)	(6.61)	(6.72)
Treatment	-0.000134	-0.00011	-0.000019	0.0200	-0.00292	0.00956
	(-1.12)	(-0.50)	(-0.08)	(1.86)	(-0.16)	(0.46)
Own CV		-0.00003	-0.000037		0.0228	0.0213
		(-0.12)	(-0.15)		(1.04)	(0.97)
Other CV		-0.00003	-0.000039		0.0228	0.0219
		(-0.12)	(-0.16)		(1.04)	(1.00)
Own PV			0.00015			0.0269
			(0.54)			(1.05)
Other PV			0.00015			0.0268
			(0.54)			(1.04)
Risk Av.			-0.00440			1.721
			(-0.19)			(0.85)
Constant	0.270***	0.270***	0.114	36.75***	36.72***	0.0943
	(3.77)	(3.77)	(0.51)	(5.78)	(5.77)	(0.00)
Periods	1-10	1-10	1-10	1-10	1-10	1-10
Obs.	1260	1260	1260	1260	1260	1260

note: Models on *Efficient* are RE probit, models on *Efficiency Loss* are RE panel estimates. Robust standard errors are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The question remains: Why does a bidding ring not do a better job in allocating the winning bid? This puzzle has a number of feasible explanations. The allocation of the designated bidder role is determined in the knockout, which takes place after the chat. If bidders do update their beliefs but deceive each other, the outcome might be serious distortion in the knockout bids. In our context this is a form of biased beliefs, following Armantier and Treich (2009), that is often present in auction games with CV information asymmetry. Their concept on subjective beliefs is applied with respect to the information asymmetry regarding bidder types. Belief elicitation can be used to test for the presence of biased beliefs. If the hypothesis holds, the distribution of beliefs under collusion should be significantly different from the actual values.

Neither descriptive statistics nor regression estimates confirm this hypothesis. On the contrary, communication between subjects improves the precision of their beliefs. The two graphs of Figure 4.4 show kernel estimates on the distribution of the precision of elicited beliefs under competition and collusion. The left graph depicts PV while the right one depicts CV. In both cases, values refer to the distribution of absolute differences.

The mean of the distribution of elicited CV and actual CV signals of colluding bidders is not significantly different. The estimated difference is 2.40 with a standard error 6.31. The same test of PV shows a small difference of -13.54, significant at 0.1 percent level.

A possible explanation for the efficiency puzzle could be that while the distribution of beliefs corresponds to the actual one, precision of beliefs is lower on the individual level. Figure 4.4 depicts kernel estimates of the distribution of deviations between elicited beliefs and actual values. In all cases, differences are absolute values. Both graphs compare distributions under collusion and competition, for private and common values, respectively. The mean of difference between actual values and elicited beliefs is improved from 125.4 to 93.4 for the PV and from 157.7 to 125.7 for the CV. For both, we used Kolmogorov-Smirnov test for equality of distribution functions. In both cases we can reject that the distributions are identical at significance level 0.1 percent. That is, performance of subjects in the belief elicitation stage is improved if they engage in collusion and explicitly communicate.

Most ring members make claims about their types, there are 457 observations for PV and 393 for CV. While many of them are false, several statements are accurate. The precision of these claims is estimated by random effects probit. Coefficients of the actual values are highly significant at all specifications, as Table 4.6 reports. The magnitude can be explained by the fact that no direct feedback was provided on the belief elicitation game until the end of the session, nor were the types of other subjects shared. Subjects could infer the truthfulness of claims only from the knockout bids which were disclosed after the elicitation stage, together with the side-payments realized.

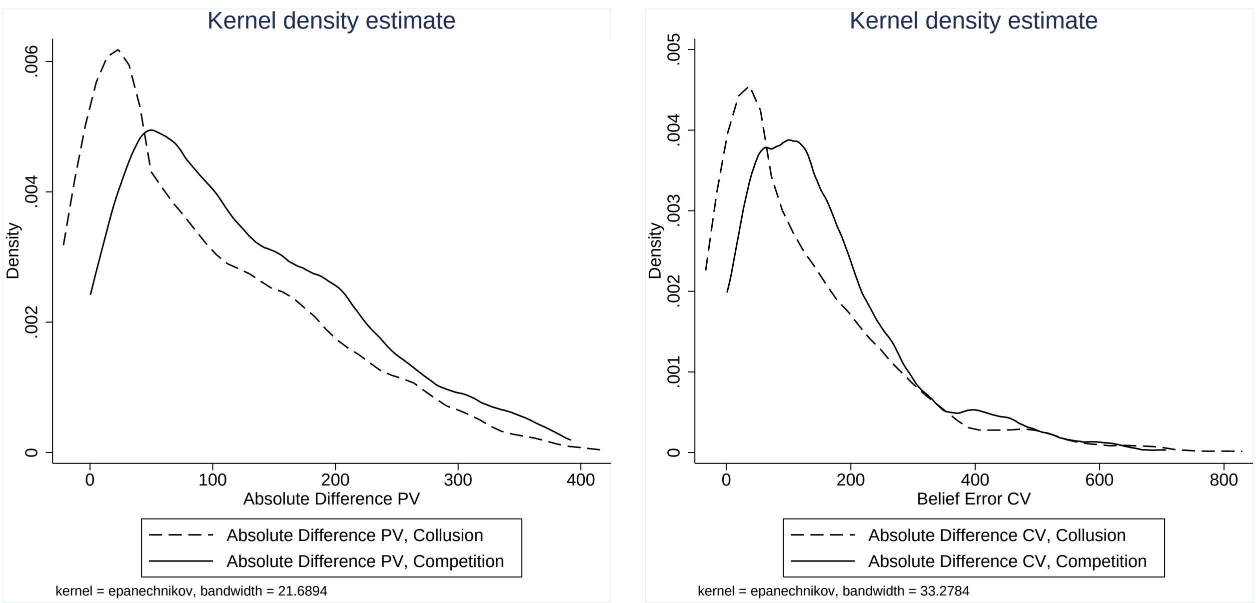
Table 4.6: Claimed private and common values

	(1) Claim Own PV	(2) C. Own PV	(3) C. Own CV	(4) C. Own CV	(5) PV Misreport	(6) PV Mr.	(7) CV Mr.	(8) CV Mr.
Own PV	0.510***	0.508***			0.144**	0.150**		
	(8.22)	(8.16)			(2.70)	(2.79)		
Treatment	-0.0841*	-0.0880*	0.189**	0.177**	0.0406	0.0412	0.144*	0.156*
	(-2.41)	(-2.54)	(2.96)	(2.69)	(1.33)	(1.29)	(2.14)	(2.21)
Period	3.794	4.558	4.263	3.270	-2.140	-2.195	3.485	3.054
	(1.73)	(1.92)	(1.74)	(1.16)	(-1.28)	(-1.08)	(1.82)	(1.40)
Risk Aversion	8.565	9.126	10.20	12.87	1.161	1.001	-9.072	-9.324
	(1.48)	(1.68)	(1.73)	(1.94)	(0.23)	(0.20)	(-1.33)	(-1.32)
Auction Payoff_L1		0.0764**		0.0908*		0.000680		-0.0251
		(2.89)		(2.27)		(0.03)		(-0.91)
Collusion_L1		-18.37		9.645		7.339		4.895
		(-1.56)		(0.73)		(0.60)		(0.41)
Agreement		-11.26		16.05		-7.368		-3.841
		(-1.00)		(1.05)		(-0.78)		(-0.33)
Own CV			0.489***	0.456***			0.0620	0.0612
			(6.17)	(5.45)			(1.35)	(1.24)
Constant	167.6***	104.5*	-40.56	-137.1**	1.612	-1.268	3.371	25.41
	(3.75)	(2.26)	(-1.32)	(-2.71)	(0.04)	(-0.03)	(0.10)	(0.59)
overall R^2	0.3606	0.3737	0.4531	0.4471	0.0309	0.0392	0.1056	0.0991
Observations	457	432	393	371	457	432	393	371

note: Random effect panel estimates. Robust standard errors are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure 4.4: Belief updating and its effects



Moreover, the values of claims depend on previous encounters. The preceding payoff has a small but positive effect. That is, conditional on actual types, subjects tend to make significantly higher claims. Regressions (5)-(8) enlist regression estimates on the absolute difference between claimed and actual values. There is a significant positive effect of the PV on the deviation. That is, subjects with higher PV were less reliable in their reports. The same cannot be said about the CV: There is no significant effect of the CV on the accurateness of claims.

Conditionally on belief updating, strategic behavior would suggest that subjects report actual or lower CV. Lowering reports can be rationalized since low claims make a belief-updating player bid lower in the KO auction. Accurate claims can be explained by positive lying costs (Kartik, 2009; Serra-Garcia et al., 2011). However, our data is not consistent with strategic lying. The mean difference between actual CVs and respective claims is 7.99 and not significantly different from zero, and its standard deviation equals 154.40.

Subjects also give credit to reported types. Table 4.7 reports estimates with the elicited values as dependent variables. Clearly, coefficients of the communicated values are high and significant at 0.1 percent level. The conclusion is the same for elicited PV and CV, belief updating is strong for both.

All these estimations suggest that biased beliefs play no role in inefficiency. Truthfulness of claims with standard errors with normal distribution and no systematic misreport is present. Similarly, beliefs are updated if pairs make claims. Regression results confirm the kernel estimates that collusion increases the precision of beliefs. The improvement is significant at 0.1 percent level both for PV and CV beliefs.

Among groups forming a cartel, 90 percent make an explicit attempt to form an agreement during the chat stage.[77] Among those who make an attempt, 62.5 percent manage to reach an agreement.

Subjects decoding the chat data were instructed to identify certain variables. This included groups in which explicit agreement or disagreement occurred, claims of own type, suggestions for own knockout bid and other subject's knockout bid. Observations, except for reports on own type, are sporadic and thereby omitted from regressions.

Knockout bids are used as dependent variables in random effects regressions in Table 4.8.[78] The use of own type and belief about the type of the other subject are consistent with predictions. Coefficients are positive and significant except for the belief on the PV of the other bidder. Again, this is consistent with the model since this variable does not change valuation. Trend is significant at 1 and 5 percent level, respectively in the two specifications. In model (4), this corresponds to an estimated overall increase of 60.2 of the knockout bid in average.

What does not correspond to the prediction is the size of the estimates. The marginal effect of the PV signal is lower than 0.2 in all specifications, significantly lower than in equilibrium. As we point out, this is the main reason for the inefficiency puzzle. Since the efficiency of an auction depends purely on the person of the winner, PVs determine who will be the more efficient of the two human bidders.[79] In other words, efficiency only depends on whether the mechanism induces the right allocation.

While a sizable number of cartels reach an explicit agreement, this does not seem to have an effect on the knockout bids. This is in line with rational bidding behavior and we cannot reject the hypothesis that the bids were not affected by response on "contracting".

Under collusion, the allocation follows the knockout. The random effects panel estimates clearly show that the knockout bids do not correspond to PV signals and the model has a huge unexplained component. Since the hypothesis of consistent beliefs cannot be rejected and beliefs are significantly improved under collusion, we can conclude that efficiency loss is induced by the low effect of the own PV signal on knockout bidding strategy.

[77] The respective dummy variable is defined in a way that it takes value 1 if at least one of the cartel member make a claim on at least one variable or there is an explicit agreement or there is an explicit disagreement.

[78] According to the Hausman test statistics the random effects estimator is consistent for both dependent variable, H_0 cannot be rejected at 5 percent significance level.

[79] While it plays a minor role, the computer wins in less than 1 percent of the cases, both according to prediction and our experimental data.

Table 4.7: Belief updating and its effect

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Belief Other PV		Belief Other CV		Error PV		Error CV	
Claim Other PV	0.736***	0.712***						
	(15.75)	(14.61)						
Treatment		-0.0697***		0.168***	-0.00382	0.0198	0.278***	0.278***
		(-3.62)		(3.41)	(-0.40)	(1.76)	(9.64)	(9.67)
Period		-2.081		-0.494	-0.817	-0.675	-1.067	-1.038
		(-1.32)		(-0.27)	(-0.94)	(-0.80)	(-0.78)	(-0.76)
Claim Other CV			0.738***	0.686***				
			(14.82)	(11.29)				
Collusion					-32.30***	-30.32***	-42.00***	-41.78***
					(-5.83)	(-5.63)	(-5.35)	(-5.33)
Other PV						0.0993***		0.0210
						(3.54)		(0.57)
Constant	105.4***	160.3***	76.43***	-0.243	131.7***	75.31***	13.72	5.166
	(5.63)	(5.39)	(4.62)	(-0.02)	(18.00)	(4.50)	(1.00)	(0.23)
overall R^2	0.6555	0.6753	0.7042	0.7316	0.0304	0.0492	0.1733	0.1737
Observations	439	439	381	381	1260	1260	940	940

note: Random effect panel estimates. Robust standard errors are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.8: Knockout bids

	(1) KO Bid	(2) KO Bid	(3) KO Bid	(4) KO Bid
Own PV	0.195*** (4.41)	0.144* (2.20)	0.160** (3.13)	0.162** (3.25)
Own CV	0.080***(3.32)	0.101**(2.99)	0.066** (2.61)	0.066** (2.64)
Claim Other PV		0.0480 (1.00)		
Claim Other CV		0.0713* (2.12)		
Period		6.544* (2.11)		7.234** (2.93)
Agreement		-0.749 (-0.06)		-15.05 (-1.35)
Belief Other PV			0.0666 (1.36)	0.0592 (1.21)
Belief Other CV			0.0952* (2.53)	0.0938* (2.57)
Constant	161.9*** (8.08)	97.19* (2.56)	126.9*** (4.68)	98.78*** (3.44)
overall R^2	0.0507	0.1042	0.0738	0.0936
Observations	742	368	586	586

note: Random effect panel estimates. Robust standard errors are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

According to our hypothesis, the effect of CV uncertainty on efficiency is negative, with or without a bidding ring. We do reject the former hypothesis, but cannot reject the latter. Our estimates can be seen in Table 4.9. Treatment effect is significant at 10 percent level on the probability of efficient allocation and at 5 percent level on Efficiency Loss. On the other hand, the same can be soundly rejected for auctions with a bidding ring. The latter is consistent with our notions about knockout bids above. Ring members do not take their PV sufficiently into account at that stage.

4.4.4. Assessment

Given our estimation results above, we can give the following conclusions to our three hypotheses.

Hypothesis 1. *All bidder types join the ring.*

This hypothesis is rejected. Estimates on Treatments R1, R2 and R3 show that the entry decision depends on type. That is, bidders with low PV and low CV signals enter the ring, while higher types are less likely to do so.

Hypothesis 2. *Forming a bidding ring increases the probability of efficient allocation.*

Table 4.9: The effect of CV uncertainty on efficiency, Treatments R1, R2 and R3

	(1) Efficient	(2) Efficiency Loss	(3) Efficient	(4) Efficiency Loss
Treatment	-0.000352	0.0303*	0.00000709	0.0126
	(-1.82)	(2.11)	(0.04)	(0.76)
Norm. PV	0.000123	-0.00222	0.000316	0.0240
	(0.25)	(-0.08)	(0.78)	(0.57)
Norm. Other PV	0.000121	-0.000399	0.000315	0.0231
	(0.25)	(-0.01)	(0.77)	(0.55)
CompBid	-0.000468	0.0631**	-0.00172***	0.234***
	(-1.15)	(2.77)	(-5.22)	(6.96)
Period	-0.0295	-0.135	-0.00208	-1.613
	(-1.50)	(-0.12)	(-0.13)	(-0.97)
Risk Aversion	-0.00295	1.124	0.00394	3.314
	(-0.10)	(0.46)	(0.12)	(0.97)
Constant	0.551	14.67	0.0900	3.760
	(1.63)	(0.70)	(0.30)	(0.12)
Constant	-11.27		-13.83	
	(-0.68)		(-0.81)	
Collusion	No	No	Yes	Yes
Observations	518	518	742	742

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The hypothesis is rejected. Our estimation strategy addresses this by identifying the effect of endogenous collusion formation as well as the exogenous possibility for collusion, using Treatment C3 precluding collusion. In both cases, the effect is negative and significant on allocative efficiency.

Hypothesis 3. *Common value variance has a negative effect on efficiency in competitive as well as collusive auctions.*

We cannot reject the hypothesis for competitive auctions. This is not particularly surprising, since this is consistent with Goeree and Offerman (2002). On the other hand, we reject the second part. The possible negative effect is counterbalanced by subjects' behavior in the knockout, and this implies no significant difference in efficiency across treatments allowing for collusion (R1, R2 and R3), with different CV support.

4.5. Conclusion

Efficiency in an auction is primarily an allocation problem. Auction theory suggests a strong cartel can paradoxically increase it by allowing for explicit communication and information pooling between bidders. Shared information may improve welfare by reducing information rents. Our laboratory experiment tests the validity of this argument and finds evidence to the contrary. Our estimates show that collusion has a negative effect on efficiency. Colluding subjects tend to truthfully disclose private information and they update their beliefs accordingly. However, they do not implement it in the allocation decision.

This result suggests that efficiency is driven by the allocation mechanism. Collusion provides an environment with pooling non-verifiable information, increasing the stochastic component. We test for the nature of this error. As we find, communication improves beliefs regarding the other ring member's type. The hypothesis of biased beliefs can be rejected, the distribution of beliefs and actual types do not differ significantly on the aggregate level. However, we cannot fully explain why this improvement is implemented in the collusive mechanism. The allocation mechanism might act as a trigger to a different emotional state. This question is relevant and may require additional study.

Three treatments allow for endogenous cartel formation. We can predict that rational, risk neutral bidders always join the bidding ring in equilibrium. However, this only holds for players with lower surplus. Higher types have significantly lower willingness-to-join, and this cannot be attributed to the elicited risk preferences.

Additionally, the distribution of welfare is addressed. Regression estimates confirm the descriptive statistics and show that collusion significantly reduces the seller's revenue and provides additional payoff for colluding bidders. However, the collusive gain of subjects is smaller than the equilibrium prediction and revenue is higher. This occurs for two reasons. Competing bidders tend to bid below equilibrium levels, implying lower level of competition and higher payoffs. Also, inefficiency created by collusion reduces their payoff. A learning effect can be identified. This gap between predictions and data increases in time due to increasing competition.

Our design allows for unrestricted communication between cartel members. Actual cases show that bidders do not necessarily share detailed information with each other and some of them apply fairly sophisticated methods. The extent of inefficiency might be different in such schemes and it is theoretically possible they can increase efficiency. However, our data does not support this hypothesis. Antitrust literature gathered vast evidence against bid rigging and price fixing. Reasons for the implementation of antitrust policies include the information rents of colluding parties, loss of the seller and inefficiency. Our paper supports the argument that antitrust efforts are necessary.

Appendices

Appendix 4.A Instructions

The following session is an experiment in decision-making. The instructions are written here and read out loud before we start. If you follow them and depending on your decisions, you can earn a considerable amount of money, which will be transferred to you after the end of the experiment. The amount of payment you receive depends on your decisions, on the decisions of others, and on chance. Your ID number on the experiment is the computer ID you can find next to you. The entire experiment is anonymous, so we will use this ID to call you when the payments are paid out at the end.

The currency used in the experiment is *Coin*. All amounts will be expressed in terms of Coin. The cash payment at the end of the experiment will be given to you in Euro. The conversion rate is 100 Coins to 1 Euro. Once the experiment has started, no one is allowed to talk to anybody other than the experimenter. If you have a question please raise your hand and we will go to your place. During the entire experiment, please remain seated.

The experiment consists of two parts. Your final payment will be the sum of your payments of these two parts. In the following you will learn the details of the first part.

Auction experiment

The experiment will consist of a sequence of 11 rounds in which you participate in an auction where you can bid for an object X. Of this 11 rounds, 1 will be practice and 10 counts towards your payment. At the beginning of each round, you will begin with an endowment of 800 Coins. A round's outcome is independent from other rounds. So, your payment in any given round will not be affected by your or others' decisions in other rounds.

In each round, you are randomly paired with an anonymous other participant in the room. That is, we will never tell you the identity of the participant you are paired with. Neither will we tell the other participant your identity. When a new round starts, new pairs are formed, and everyone in the room has equal chance to be paired up with you.

Before the auction takes place, you are allowed to communicate with the other bidder in your group and to make an agreement. The following list summarizes the timing of each round; you can read the details below.

1. *Redemption value*: In the first stage you learn values A_{you} and B_{you} affecting your *redemption value*. If you acquire a unit of X, you can redeem it for Coin. The amount of Coins that a unit of X gives you is called your *redemption value*.

2. *Decision*: You are asked about your willingness to cooperate with the other bidder in your group. If you both reply 'Yes', you continue with the *Chat stage*. In other words, no agreement is made unless both of you wants to make one.

3. *Chat*: If you and the other bidder decide to cooperate, you can chat and make an agreement.

4. *Guessing game*: You are asked to guess the redemption value of the other bidder in the group.

5. *Agreement*: If you and the other bidder decides to cooperate, and makes an agreement, you can read the details of this agreement.

6. *Auction*: The auction takes place.

7. *Results*: You can read the final results of the round.

Redemption value

In the auction you are bidding for a good called X with the other bidder and a computerized bidder.

If you acquire the good in the auction, you redeem it for your *redemption value*. The *redemption value* can be different for bidders in the same auction. Your redemption value will be denoted by R_{you}. Now we explain how is it calculated.

You and the other human bidder are given two random numbers privately, denoted by A and B. A is between 200 and 600 Coins, B is between 0 and 400 Coins. Your and the other bidder's numbers are denoted by A_{you}, A_{other}, B_{you} and B_{other}. These numbers are integers, and all possible numbers have equal chance of being drawn. All numbers are independently drawn.

Your *redemption value* is calculated from these numbers. Before the auction, you only know A_{you} and B_{you} but you do not know A_{other} and B_{other}. Similarly, the other player knows A_{other} and B_{other}, but does not know $A_{you}$and B_{you}.

Your redemption value is calculated from A_{you}, B_{you} and B_{other} as:

$$R_{you} = A_{you} + \frac{B_{you} + B_{other}}{2}$$

Similarly redemption value of the other human bidder is:

$$R_{other} = A_{other} + \frac{B_{you} + B_{other}}{2}$$

That is, the first component is different for you and the other human bidder. The second one, $\frac{B_{you}+B_{other}}{2}$ is the same, and determined by B_{you}, which only you can see, and B_{other}, which only the other bidder can see.

Decision

Before the *auction* takes place, you are allowed to chat with the other human player, and you can make an agreement. Participation at the *decision* stage depends on your and the other player‘s decision. A question appears in each round and your answer only affects that particular round.

The following question appears on your screen. 'Do you wish to cooperate with the other bidder in your group?' If you both reply 'Yes', you continue with the *chat* stage. If at least one of you replies 'No', you proceed to the *guessing game* and to the *auction*. In other words, no agreement is made unless both of you wants to make one.

Chat

If you both replied 'Yes' at the *decision* stage, a chat box and a proposal box appear. In the chat box you send a message by pressing the Enter button on your keyboard. You have 60 seconds available for chat. During that time, you are allowed to talk about the numbers you observed (A_{you}, A_{other}) and you can agree on a strategy for the remainder of the round. The other player in your group is able to see the message what you send, but nothing more. So he or she cannot see on the screen whether you write the truth.

You are free to discuss whatever you like except: Do not use any words or phrase which helps to identify you. Only communicate in English. In case you violate these rules, you must leave the experiment and you receive no payment.

You leave the chat by making a proposal to other player in your group next to the text 'Please, make your proposal!'. By clicking OK, you leave the chat. The purpose of this part of the round is to decide who will participate in the auction and how much compensation the other participant should get for not participating.

In the bracket 'Make your proposal', you are allowed to type a number, which is not bigger than your endowment. The one making the higher proposal has to pay *the lower of the two amounts* (in other words, the second highest proposal) to the other player, which is deducted from her endowment. If the numbers are equal, one of you is chosen with equal chance to do so. If you receive a payment, you do not participate in the auction, your bid will be automatically 0. The payoff of this round is:

$$\text{payoff} = \text{endowment} + \text{smaller proposal}$$

The other player participates in the auction with the computerized bidder with:

$$\text{balance} = \text{endowment} - \text{smaller proposal}$$

The proposal part can make two things possible for your group. First, you can share information during chat. Second, you can decide who should participate in the auction. If an agreement is made, one of you who pays can have only one opposing bidder, the computer.

Guessing game

The next stage is a guessing game on which everyone participates, even the pairs who decided not to cooperate. On your screen you can see two questions and two brackets in which you are asked to write numbers. These is 'How much do you think A_{other} is?' and 'How much do you think B_{other} is?'

In the brackets, you are asked to guess the numbers of the other player in your group. The available information to you is what you can have read in the instructions and what the other player writes to you during the chat. There is an extra payment for guessing right. The closer your guesses are to the actual number, the more you earn. This amount is not added to your balance in this round, but it is counted at the end of the experiment. The maximum you can earn with guessing is 200 Coins, with a perfect guess.

For example, if $A_{other} = 550$, $B_{other} = 200$ and your guess is 550 and 200, your payoff is 200 Coins for the guessing game. Your payoff is determined using the difference between A_{other}, B_{other} and your guesses. Suppose your guesses are 630 and 280 it means your guess was more far away from A_{other}, you get 72 Coins. The least you can earn is 0 Coin.

Agreement

Before the auction, a new stage appears, which provides you information about the outcome of the proposals. This can only be seen by pairs who decided to cooperate. You can see your new balance and whether you are allowed to participate in the auction.

Auction

The auction has two or three participants, depending on whether you and the other player in your group had an agreement. The auction has the same rules as the proposal part. That is, you are required to make your bids at the same time. The winner of the auction is the one making the highest bid. In case of a tie, the winner is chosen randomly among the ones with the highest bids. The amount paid is the *second highest bid* submitted. In the auction stage, you are allowed to submit an integer number in the bracket 'Please, make your bid', that is at least 0.

The computer submits a number, that is between 0 and 500 Coins.

The highest bidder receives the unit of X being sold and earns his or her *redemption value* for it. If you win, your redemption value equals R_{you}. Other participants, who do not win any X, do not pay any Coin and do not receive any X, so their earnings for the period equal their balance. Thus, if you win, your payoff is:

$$\text{payoff} = \text{balance} - \text{second highest bid} + R_{you} =$$
$$= \text{balance} - \text{second highest bid} + A_{you} + \frac{B_{you} + B_{other}}{2}$$

Let us see an example. Suppose you have cooperated with the other player in your group and you have given 150 Coins. Your new balance is $800 - 150 = 650$ Coins. Suppose your values are $A_{you} = 500$ and $B_{you} = 100$. You decide to bid 650. The computer bids 200. The other bidder does not participate, so the second highest bid is 200 Coins. You win the auction, pay 200 Coins and receive your *redemption value.* In order to calculate your redemption value, you also need to know B_{other}. The only way you can learn it before the auction is by participating in the Chat round. Even in that case, you can only see what the other human bidder writes, we do not show you B_{other}.

Suppose $B_{other} = 300$. So, you redemption value is $R_{you} = 500 + \frac{300+100}{2} = 500 + 200 = 700$. So, your payoff of this round is $650 + 700 - 200 = 1150$ Coins. If someone does not win the auction, the resulting payoff is payoff = balance. So, the other player receives $800 + 150 = 950$ Coins, the sum of the endowment and the amount he or she has received by the agreement.

Results

After all bidders have submitted their bids, the outcome of the auction is announced. You can read whether you have won, all the submitted bids and your earnings for the round. You can see this screen even if you did not participate in the auction. The process is repeated 11 times, from which 1 round is only practice, whereas one of the remaining 10 count towards your payment. At the beginning of the first round which counts, we ask you type your seat number. This number you can find on the separating wall, and it is between 1-20. We need this in order to make the payments at the end of the session.

Your payoff and end of session

After the second experiment, you will receive your payments in cash. The amount for the first experiment is your payoff from a random round. Your payment for the auction experiment is the sum of that and the payment for the guessing game. After the auction game you will participate in a second part, which result will be added to your final payment.

After the two parts, payments are made anonymously and individually. Please remain seated until we call you.

Appendix 4.B Control questions

Please make a choice for each question by encircling (a), (b) or (c).

1. In a round you learn that your numbers are $A_{you} = 500$ and $B_{you} = 200$. In the previous round, your number was $A_{you} = 400$ and $B_{you} = 100$. In this round

   (a) My *redemption value* is 700, it does not depend on the previous round.

   (b) My *redemption value* is 500.

   (c) I do not know my own *redemption value* with certainty.

2. Before the chat part, to the question 'Do you wish to cooperate with the other player in the auction?', you reply 'Yes' and he other player replies 'No'. What does happen next?

   (a) We proceed to the auction, since we both need to reply 'yes' in order to start the chat and make an agreement.

   (b) We proceed to the chat, since I replied 'yes'.

3. At the proposal for making an agreement, you offer 150 Coins whereas the other player offers 100 Coins. What does happen next?

   (a) We can both participate in the auction and we keep our endowments.

   (b) I can participate in the auction, my pair cannot. I pay 100 Coins to my pair, so my new balance is 700 Coins, my endowment minus what I paid out.

   (c) I can participate in the auction, my pair cannot. I pay 150 Coins to my pair, so my new balance is 650 Coins, my endowment minus what I paid out.

4. In the auction

   (a) there are always three bidders.

   (b) there are two or three bidders, but I always participate.

   (c) there are two or three bidders, and I do not participate if we cooperated and I received an amount.

5. You are the winner of the auction and you are required to pay 300 Coins. Previously, you have made an agreement, and paid 100 Coins to your pair. Your numbers are $A_{you} = 500$ and $B_{you} = 200$. Your pay-off of this round

   (a) is $800 - 100 + 500 + 200 - 300 = 1100$ Coins.

(b) is $800 - 100 - 300 = 400$ Coins.

(c) you do not know with certainty.

Appendix 4.C Instruction HL protocol

In this part of the experiment you will be making choices between two lotteries, such as those represented as ”Option A” and “Option B” below. The money prizes are determined by the computer equivalent of throwing a ten-sided die. Each outcome, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, is equally likely. If you choose Option A in the decision shown below, you will have a 1 in 10 chance of earning 500 Coins and a 9 in 10 chance of earning 400 Coins. Similarly, Option B offers a 1 in 10 chance of earning 960 Coins and a 9 in 10 chance of earning 25 Coins.

Decision 1:

Option A: 500 Coins if the die is 1 and 400 Coins if the die is 2 - 10.

Option B: 960 Coins if the die is 1 and 25 Coins if the die is 2 - 10.

Each box of the decision table contains a pair of choices between Option A and Option B. You make your choice by clicking on the “A” or “B“ buttons on the bottom. Only one option in each box can be selected, and you may change your decision as you wish before you submit it.

Even though you will make ten decisions, only one of these will end up being used. The selection of the one to be used depends on the “throw of the die” that is the determined by the computer's random number generator. No decision is any more likely to be used than any other, and you will not know in advance which one will be selected, so please think about each one carefully.

For example, suppose that you make all ten decisions and the roll of the die is 9, then your choice, A or B, for decision 9 would be used and the other decisions would not be used.

After the random die throw determines the decision box that will be used, a second random number is drawn that determines the earnings for the option you chose for that box. In Decision 9 below, for example, a throw of 1, 2, 3, 4, 5, 6, 7, 8, or 9 will result in the higher payoff for the option you chose, and a throw of 10 will result in the lower payoff.

Decision 9:

Option A: 500 Coins if the die is 1-9 and 400 Coins if the die is 10

Option B: 960 Coins if the die is 1-9 and 25 Coins if the die is 10

For decision 10, the random die throw will not be needed, since the choice is between amounts of money that are fixed: 500 Coins for Option A and 960 Coins for Option B.

Your earnings in this part of the experiment will be added to your final payoff.

Appendix 4.D Instructions chat coding

The following task is part of an experiment in decision-making. The instructions are written here. If you follow them, you can earn a considerable amount of money, which will be transferred to you after delivery. The amount of payment is fixed, 70 EUR. The content of all attached files is confidential. You are not allowed to share it with anybody other than the experimenter. If you have a question please contact us.

The instructions for the coding follow. Attached you can find a MS Excel file with two spreadsheets, *chat* and *data*. Your task is to read the content of *chat* and fill the content of *data*. You can do this job anytime before the deadline of December 17, 17:00. At that point, the spreadsheet must be complete. After delivering the completed file, you will receive your payment by bank transfer, which will be sent to your account.

Chat

Spreadsheet *chat* contains observations from an anonymous economic experiment. The entries are chat messages and identification values. The meaning of each columns is explained below. This database records the chat messages of participants in experimental sessions.

- Period: This refers to the round in which the chat has taken place. There were 11 rounds, numbered between 0 and 10. Note, that not every participant had chat in every round.

- Text: This is a chat message.

- Group: Code number of the group. A group consists of two participants in a *Period*.

- Time: The time the message was sent.

- ID: identification number of the participant used (only) in the experiment.

In spreadsheet *chat*, the entries are ordered such that you can read a conversation between two participants easily. For example, rows 2, 3 and 4 contain a conversation between subjects 102 and 105 in round 0, with chat messages in chronological order. Note, that not all participants participated in chat for all periods. In all chats there were exactly two participants, but it is possible that only one of them has chat entries.

Data

The second spreadsheet must be filled in using the *Chat* spreadsheet. Each row corresponds to a participant with identification *ID* in a certain *Period*. These two variables are already filled in, please do not modify them nor change their order.

There are three cases. A participant with *ID* in a *Period*

- has no chat entry in a period. In this case, leave that row empty. Never delete entries for *ID* and *Period*.
- has chat entry, but he or she was the only one communicating. Please write 1 in the row *onesided*. In any other case, leave this variable empty.
- has participated in a chat with two-sided communication. That is, both participants of the group have sent at least one line.

If a participant has chat entries (case 2 and 3), you need to fill in the remaining cells. The conversation is about a proposal. After this chat, both participants in the group made a proposal, which was always a number. They can refer to this as *offer*, *proposal*, *payment* or *bid*. This is a decision made by the participants *after* the chat.

Furthermore, they mention *values*, which they refer to as A or B. If in a conversation they mention only one value without explicitly saying A or B, you should assume they talk about value A. All participants knew their own values *before* the chat.

In the chat, participants mainly talked about these numbers. It is possible, that a chat contains no mention of some or all of them. You need to fill in the followings. As a general rule, do not write anything in a cell if the respective information cannot be found in the chat. In each case, prefix *own* refers to chat messages of the participant *ID*, this is the information the participant provided. Prefix *other* always refers to the other participant in the same group. This is the information the participant received. As a general rule, if there are multiple numbers for one entry, please type in only the last one.

- *ownproposal*: If *ID* in *Period* mentions a *offer*, *proposal*, *payment* or *bid* with a specific number, please write that number here.
- *otherproposal*: If the participant receives an offer, please write it here. Similarly, if there are multiple numbers, write the last one here.
- *ownproposaltoother*: If the participant suggests a number what the other participant should propose, please write it here.
- *otherproposaltoown*: If the participant is suggested a number, please write it here.

- *ownavalue*: Claim about value A.
- *otheravalue*: Received claim about A.
- *ownbvalue*: Claim about value B.
- *otherbvalue*: Received claim about B.
- *agreement*: Please fill in 1, if an agreement has been reached in the conversation.
- *disagreement*: Please fill in 1, if there was a disagreement at the end of the conversation, 0 otherwise. Do not fill it in if there is no chat entry.
- *noresponse*: Please fill in 1, if there was an attempt to make an agreement, but the conversation ended without explicit agreement or disagreement.

It is possible that there was only a hint about a certain value, but no precise number. In that case, write down your best guess in that cell, i.e. the mid of an interval, do not leave it empty.

Good luck with finishing the task. If you have any questions, please write an email to g.seres@tilburguniversity.edu. It is important that you only send inquiries to this email address, *do not discuss* any part of this task with other participants.

Appendix 4.E Variables

Table 4.10: Variables

Own PV	PV of the subject.
Own CV	CV signal of the subject.
Other PV	PV of the other subject in the group.
Other CV	CV signal of the other subject in the group.
Collusion	Dummy, takes 1 if bidding ring is formed.
Collusion Attempt	Dummy, takes 1 if subjects agrees to form a ring.
Treatment	Width of the CV support, takes 0, 400 or 800.
Collusion Allowed	Dummy, takes 1 in R1, R2 and R3.
KO Bid	Knockout bid of the subject.
Bid	Auction bid of the subject.
OthersBid	Auction bid of the other subject in the group.
CompBid	Auction bid of the computerized bidder.
Period	Number of the current period.
Belief Other PV	Belief about the PV of the other subject in the group.
Belief Other CV	Belief about the CV signal of the other subject in the group.
Auction Payoff	Payoff of the auction game, without belief elicitation.
Risk Aversion	Holt-Laury measure of risk aversion, takes values $0, 1, \ldots, 10$.
Claim Own PV	Chat: What a subject claims as PV.
Claim Own CV	Chat: What a subject claims as CV signal.
PV Misreport	Chat: Difference between claimed and actual PV.
CV Misreport	Chat: Difference between claimed and actual CV signal.
Claim Other PV	Chat: What the other subject claims as PV.
Claim Other CV	Chat: What the other subject claims as CV signal.
Norm. PV	Normalized PV, takes $0 - 400$.
Norm. Other PV	Normalized Other PV.
Efficient	Dummy, takes 1 if the auction winner is the efficient bidder.
Efficiency Loss	Absolute difference between the efficient and the actual welfare.
Surplus	Surplus $x_i + \frac{y_i}{2}$.
Revenue	Revenue of the seller.
Absolute Difference PV	Absolute difference between PV Other and the corresponding belief.
Absolute Difference CV	Absolute difference between CV Other and the corresponding belief.
Collusion Percentage	The ratio of preceding Periods the subject was in a ring.
Payoff Average	Average Auction Payoff of the preceding Periods of the subject.
_L1	Lagged variable.

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