

Predation Under Perfect Information

Argenton, C.

Publication date:
2010

[Link to publication](#)

Citation for published version (APA):

Argenton, C. (2010). *Predation Under Perfect Information*. (CentER Discussion Paper; Vol. 2010-26). Tilburg: Microeconomics.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright, please contact us providing details, and we will remove access to the work immediately and investigate your claim.

No. 2010–26

PREDATION UNDER PERFECT INFORMATION

By Cédric Argenton

March 2010

ISSN 0924-7815

Predation under perfect information*

Cédric Argenton
CentER & TILEC, Tilburg University

March 11, 2010

Abstract

In an oligopoly configuration characterized by high barriers to (re-)entry, a finite horizon, perfect information about demand and costs and the presence of three identical firms, we show that two of them (the predators) can choose to charge an initial price that is so low that the third (the prey) decides to exit immediately, after which the predators can enjoy higher profits, even if they do not raise their price. Predatory prices are thus observed on the equilibrium path and the predators end up earning more than in the best Bertrand (or even, collusive) equilibrium with three firms.

KEYWORDS: predation, predatory pricing, collusion, dynamic game, Bertrand competition

JEL CODES: D43, L13, L41

1 Introduction

Predatory pricing has long been a (contested) fact in search of a theory. Whereas allegations that small entrepreneurs suffered from the predatory practices of large firms prominently featured in the political agitation that led to the enactment of the Sherman Act in the US, in the 1960s and 1970s the influential Chicago critique denied that predation could ever be a profitable business strategy.^{1 2} There is now some evidence that successful predation took place in a number of industries (notably, cement, match, tobacco, telecoms and sugar) in the course of the 20th century.³

Meanwhile, predation has been put on firmer theoretical footing. Modern explanations are all about asymmetric information: the predator tries to exploit the prey's imperfect information (or its creditors') and manipulate its (their) belief about profitability. Absent imperfect information, it is thought that predation cannot occur.⁴

*I thank, without implicating, Michiel Bijlsma, Jan Boone, Eric van Damme and Wieland Müller for very useful comments, as well as seminar audiences at the Netherlands Bureau for Economic Policy Analysis (CPB) and Tilburg University.

¹See e.g. Chandler (1990) for an account of the rapid industrialization process of the US economy at the end of the 19th century and its consequences on "small" producers.

²For a forceful exposition of the Chicago critique, see Bork (1978).

³See the instances and references mentioned by Bolton, Brodley and Riordan (2000, p. 2244-2245).

⁴This viewpoint is well-summarized by Motta (2004, p. 415-416):

In this note, we show that in a Bertrand oligopoly characterized by high barriers to (re-)entry, a possibly finite horizon, perfect information about demand and costs and the presence of three identical firms, two of them (the predators) can choose to charge an initial price that is so low that the third (the prey) decides to exit immediately, after which the predators can enjoy high profits, even if they do not raise their prices above pre-existing levels. Predatory prices are thus observed on the equilibrium path and the predators end up earning higher profits than in the best Bertrand equilibrium (or, even, the best collusive equilibrium). Because costs are assumed to be convex, the disappearance of one producer can harm productive efficiency. The intuition for those results is simple: it is sometimes preferable to share a small pie between a few persons than to share a larger pie with a lot of people! Convex costs allow firms to act on this premise with very simple strategies and under a finite horizon, given the multiplicity of equilibria in the one-shot game.

Note that there is no need for imperfect information (demand and cost functions are perfectly known by all players), that a predator does not have an initial advantage over the prey (all firms are identical), and that recoupment of the initial investment in predatory prices does not necessarily require supra-competitive prices (firms can charge the same price before and after a predatory episode).

In the remainder of this introduction, we review the relevant theoretical literature on predatory pricing. The model we use is laid out in Section 2. Section 3 contains the analysis. Section 4 discusses the relevance of the results for antitrust policy.

The leading theoretical explanations of predation are based on asymmetric information. Kreps and Wilson (1982) proposes a resolution of Selten's chain store paradox that can be interpreted in predatory terms. In a finite-horizon game, a weak incumbent (i.e. endowed with high costs) initially builds a reputation for being strong (i.e. low-cost, which happens with probability ε) by charging low prices. There need not be pricing below cost but there is a sense in which the incumbent initially invests in its reputation in order to deter future entry. Milgrom and Roberts (1982) proposes a limit-pricing model with the same flavor. Observing the price charged by a monopolist, a potential entrant updates its belief about the type of the former (high- or low-cost) and decides about entry. In a pooling equilibrium the high-cost incumbent charges the low-cost monopoly price and the potential entrant chooses to stay out. Sharfstein (1984) develops a model of "test-market predation". A potential entrant is uncertain about the level of demand or profitability in a given market. The incumbent firm knows. In a pooling equilibrium, a high-demand incumbent charges low prices to deter entry. Fudenberg and Tirole (1986) submit a "signal jamming" theory of predation. An entrant is again uncertain about the level of demand. The predator openly cuts prices. In equilibrium, the entrant knows that the price is artificially low due to incumbent's behavior but in the absence of information about what demand would be in normal competitive circumstances, it prefers to exit. Bolton and Scharfstein (1990) rationalize

The manipulation of beliefs can exist only if there is some uncertainty. In a world where all firms (...) knew perfectly what the technologies and financial resources available to each are, their preferences and those of consumers, and their ability to behave rationally, predation would never be observed (...).

the "long-purse" theory of predation. In their principal-agent model, because an entrant's effort is not contractible, the amount of financing it gets depends on the firm's internal assets. By lowering price, the predator reduces the entrant's profitability and thus its retained earnings. Further financing by the principal is thus jeopardized. The prey anticipates at the beginning of the game that it will eventually have to exit.

To our knowledge, only two papers studied predation in the context of perfect information and none delivered predatory prices on the equilibrium path under a finite horizon. Harrington (1989) studies whether cartel members can sustain cooperation over time under the threat of free entry in an infinitely-repeated game and shows that firms can deter entry by credibly threatening to meet any entry with an episode of below-cost pricing. Roth (1996) shows that predatory pricing is rationalizable in an infinite-horizon, perfect-information war-of-attrition model and stresses the role of strategic uncertainty.

2 Model

We consider a dynamic game of perfect information. The demand for a homogenous good is given in each period by demand function $D(p)$, which is twice continuously differentiable, strictly decreasing (in the range where it is positive), concave and cuts both axes. (In particular, $D(a) = 0$ for some $a > 0$.) It emanates from a representative consumer who buys from the firm(s) offering the lowest price. In case this price is charged by several firms, sales are equally split.

There are three identical firms labelled 1, 2, and 3 present on the market. The fact that the number of firms is fixed can be attributed to the existence of barriers to entry. For instance, potential entrants may have to pay a fixed set-up cost in order to access the market. Unless the profit that an entrant can make is high enough, a potential entrant (called firm 4 in what follows) will stay out.

In any period, firms incur a fixed cost $F \geq 0$ for remaining active and face a strictly increasing marginal cost of production. So,

$$C(q_i) = F + \int_0^{q_i} MC(x)dx,$$

where $MC(x)$ is a continuously differentiable, strictly increasing function.⁵ (We denote by AVC , AFC , and AC the average variable cost function, average fixed cost function, and average total cost function, respectively.) Firms strive to maximize the present value of the flow of profits. They discount the future at a common rate $\delta \in (0, 1]$.

The market operates over an sequence of periods $t = 1, 2, 3, \dots, T$, where T can be any natural number greater than 2 or, abusing notation, ∞ .⁶ In every period, active firms choose an action in $S \equiv [0, a] \cup \{E\}$. E stands for "exit" while an element in $[0, a]$ is the price charged by a firm choosing to remain active. If, in a given period, a

⁵This is a simple instance of the U-shaped cost curves used to teach students in intermediate microeconomics courses.

⁶We need $\delta < 1$ if $n = \infty$.

firm has chosen to exit, then it becomes inactive for the remainder of the game. That is, exit is irreversible. We denote the set of existing firms at the beginning of period t by $N^t \subset \{1, 2, 3\}$. This set then follows the following law of motion:

$$N^{t+1} = N^t \setminus \{i \in N^t : s_i^t = E\}.$$

Notationwise, we write:

- $\pi^{(n)}(p)$ for the per-period, per-firm profit when n firms charge the same price p ;
- $\hat{\pi}_i(p, N)$ for the highest profit⁷ to firm i in a given period when all firms in N charge p and the strategy set of firm i is restricted to $[0, a]$.

We are interested in constructing certain subgame-perfect equilibria.

Abstracting from the exit decision, the results from the classical, static Bertrand competition model with convex costs form the first building block of our model. An important institutional feature of Bertrand competition is that firms are committed to serve any demand addressed to them at the posted price; they cannot turn customers down or ration demand. As Vives (1999) indicates, apart from cases in which continuous provision is legally mandated, this is a reasonable assumption in industries in which customers have an on-going relationship (subscription, repeat purchases, etc) with suppliers or the costs of restricting output in real time are high. Those probably include water supply, electricity, or (to some extent) telecommunications. In any case, because of this feature, under convex costs there is a continuum of Nash equilibria including average-cost pricing and marginal-cost pricing. See the seminal paper by Dastidar (1995) and, for the case where the monopoly profit function is concave (as in this paper), Weibull (2006). We summarize the main results in

Proposition 1 *Suppose that demand is twice continuously differentiable with $D'(\cdot) < 0$ in the range in which it is positive, and the n identical firms in the market have a strictly increasing, twice continuously differentiable, strictly convex cost function, $C(\cdot)$. Let $\check{p}^{(n)}$ be such that $\pi^{(1)}(\check{p}^{(n)}) = \pi^{(n)}(\check{p}^{(n)})$, and $\dot{p}^{(n)}$ such that $\pi^{(n)}(\dot{p}^{(n)}) = -C(0)$. Then, $\dot{p}^{(n)} < \check{p}^{(n)}$ and all firms charging a price in the interval $[\dot{p}^{(n)}, \check{p}^{(n)}]$ is a Bertrand equilibrium.*

The intuition for those results is as follows. When costs are strictly convex, undercutting competitors is very costly because the unit margin decreases a lot (or even turns negative) as output goes up, as a result of the rise in marginal cost. As a consequence, a whole range of prices can be sustained in equilibrium, for price undercutting doesn't typically benefit a potential deviator.

Now, in our game, because E belongs to the strategy space, the minimum payoff that a player can achieve in the one-shot game is not $-C(0)$ but 0. (It is as if the fixed

⁷Or, to be precise, the supremum over $[0, a]$ of all possible profits. As we work with Bertrand competition, the best reply to a set of prices charged by other firms is not defined in many cases: by "marginally undercutting" the lowest price, it is possible to come arbitrarily close to serving the whole market at this lowest price.

cost were avoidable and the cost function discontinuous at 0.) The definition of $\dot{p}^{(n)}$ must be accordingly adjusted to $\pi^{(n)}(\dot{p}^{(n)}) = 0$. It is easy to extend Dastidar's (1995; lemma 7) results to show that the maximum and the minimum static Nash equilibrium prices are still decreasing in n :

$$\begin{aligned}\ddot{p}^{(n)} &< \ddot{p}^{(n-1)}, \\ \dot{p}^{(n)} &< \dot{p}^{(n-1)}.\end{aligned}$$

Although the dynamic game we consider is not a standard repeated game (because exit is irreversible), the theory of repeated games will constitute the second building block of our model. In particular, since the one-shot Bertrand game allows for multiple equilibria, it is well-known from Benoît and Krishna (1985) that it is possible to give firms incentives to depart from those equilibria over time, even under a finite number of repetition.

3 Analysis

Obviously, there are infinitely many subgame-perfect equilibria in the game that we consider. Any sequence of prices in $[\dot{p}^{(3)}, \ddot{p}^{(3)}]$ is an equilibrium path, for instance. We are looking at a particular candidate equilibrium: in period 1 firms 1 and 2 charge a price p_1 that induces a loss to firm 3 in case it stays on the market; thereafter, they charge a profitable price p . We want firm 3 not to be able to recoup its initial loss if it decides to stay. In contrast, we want 1 and 2 to be able to recoup their initial loss in case 3 withdraws and they charge p thereafter. We have to specify what happens in the case where, in period 1, firm 3 deviates by staying on the market and best-responding to p_1 . In that case, firms 1 and 2 will min-max firm 3 by subsequently charging $\dot{p}^{(3)}$ until it (ever) exits. Thus, upon deviating, firm 3 is assured that due to the ensuing price war, it will make zero profit in the continuation equilibrium (either by staying and charging $\dot{p}^{(3)}$ until the end or by exiting). In case firm 1 or firm 2 deviates in the first period, then the equilibrium will prescribe that both of them charge $\dot{p}^{(2)}$ in all subsequent periods. Thus, our candidate equilibrium prescribes firms to 'coordinate' on zero profit in all the periods that follow a deviation.

Our claim is that this equilibrium is predatory, in the sense that the predators (firm 1 and 2) 'invest' in low prices and accept losses in the short term only because the exclusion of the prey (firm 3) allows them to recoup their investment in subsequent periods.

In order to avoid profitable deviations, we need:

$$\begin{aligned}\text{(i)} \quad & \hat{\pi}_3(p_1, \{1, 2\}) \leq 0; \\ \text{(ii)} \quad & \pi^{(2)}(p_1) + \delta \frac{1 - \delta^{T-1}}{1 - \delta} \pi^{(2)}(p) \geq 0; \\ \text{(iii)} \quad & \hat{\pi}_1(p_1, \{1, 2\}) \leq \pi^{(2)}(p_1) + \delta \frac{1 - \delta^{T-1}}{1 - \delta} \pi^{(2)}(p).\end{aligned}$$

The first condition specifies that the best possible deviation for firm 3 (best-responding to p_1 and then, say, exiting) is not profitable. The second condition specifies that firms

1 and 2 are willing to charge first p_1 in the first period, then p for $(T - 1)$ periods rather than exit immediately. The third condition specifies that, conditional on staying the game, firm 1 (and firm 2, by symmetry) is willing to post p_1 in the first period rather than let firm 2 (firm 1) take all the losses and be subsequently min-maxed.

We introduce the residual demand addressed to each of n firms charging the same price p :

$$D^{(n)}(p) \equiv \frac{D(p)}{n}. \quad (1)$$

By definition,

$$\pi^{(n)}(p) = pD^{(n)}(p) - C [D^{(n)}(p)]. \quad (2)$$

Because of convex costs, for a given p , gross industry profits (i.e. $n\pi^{(n)}(p) + nF$) are an increasing function of n . (Once fixed costs are taken into account, there is a trade-off between the level of marginal cost and the level of fixed costs, delivering an optimal number of producers.)

For the time being, we want our candidate equilibrium to yield predators more profit than the best Bertrand equilibrium. Given the initial investment in low prices, this is possible only if the predators make more profit post exit than in the best Bertrand equilibrium. The first question to address, then, is whether there is a price p such that

$$\pi^{(3)}(\ddot{p}^{(3)}) < \pi^{(2)}(p), \quad (3)$$

which happens to be true in all circumstances.

Lemma 2 $\pi^{(3)}(\ddot{p}^{(3)}) < \pi^{(2)}(\ddot{p}^{(3)})$.

Proof. Recall that by definition,

$$\pi^{(3)}(\ddot{p}^{(3)}) = \pi^{(1)}(\ddot{p}^{(3)}),$$

or

$$\frac{D(\ddot{p}^{(3)})}{3} \int_{\frac{D(\ddot{p}^{(3)})}{3}}^{D(\ddot{p}^{(3)})} MC(x)dx = \frac{2}{3}\ddot{p}^{(3)} D(\ddot{p}^{(3)}).$$

Now, if the summation is taken on the first fourth of the integration interval, because MC is strictly increasing,

$$\frac{D(\ddot{p}^{(3)})}{3} \int_{\frac{D(\ddot{p}^{(3)})}{3}}^{\frac{2}{3}D(\ddot{p}^{(3)})} MC(x)dx < \frac{1}{4} \int_{\frac{D(\ddot{p}^{(3)})}{3}}^{D(\ddot{p}^{(3)})} MC(x)dx = \frac{1}{6}\ddot{p}^{(3)} D(\ddot{p}^{(3)}).$$

Therefore,

$$\frac{D(\ddot{p}^{(3)})}{3} \int_{\frac{D(\ddot{p}^{(3)})}{3}}^{\frac{2}{3}D(\ddot{p}^{(3)})} MC(x)dx < \frac{\ddot{p}^{(3)} D(\ddot{p}^{(3)})}{6},$$

which is equivalent to

$$\pi^{(3)}(\ddot{p}^{(3)}) < \pi^{(2)}(\ddot{p}^{(3)}).$$

■

So, at $\ddot{p}^{(3)}$ the benefit of sharing demand among fewer firms is always higher than the cost associated with less efficient production. In other words, by having one more firm active in the industry, the unit margin of the firms that are already present goes up (because of cost savings) but not sufficiently to compensate for the decrease in volume. For analogous reasons, $\ddot{p}^{(3)}$ is a static Nash equilibrium with two firms.

Lemma 3 $\dot{p}^{(3)} \in [\dot{p}^{(2)}, \ddot{p}^{(2)}]$.

Proof. By definition,

$$\pi^{(3)}(\ddot{p}^{(3)}) = \pi^{(1)}(\ddot{p}^{(3)}),$$

or

$$\int_{\frac{D(\ddot{p}^{(3)})}{3}}^{D(\ddot{p}^{(3)})} MC(x)dx = \frac{2}{3}\ddot{p}^{(3)}D(\ddot{p}^{(3)}).$$

Now, because MC is strictly increasing,

$$\int_{\frac{D(\ddot{p}^{(3)})}{2}}^{D(\ddot{p}^{(3)})} MC(x)dx > \frac{3}{4} \int_{\frac{D(\ddot{p}^{(3)})}{3}}^{D(\ddot{p}^{(3)})} MC(x)dx.$$

Therefore,

$$\int_{\frac{D(\ddot{p}^{(3)})}{2}}^{D(\ddot{p}^{(3)})} MC(x)dx > \frac{\ddot{p}^{(3)}D(\ddot{p}^{(3)})}{2},$$

which is equivalent to

$$\pi^{(2)}(\ddot{p}^{(3)}) < \pi^{(1)}(\ddot{p}^{(3)}).$$

In addition, by the previous lemma, $\pi^{(3)}(\dot{p}^{(3)}) < \pi^{(2)}(\dot{p}^{(3)})$ and $\pi^{(3)}(\dot{p}^{(3)}) > 0$.

Thus, $\dot{p}^{(3)}$ is a static Nash equilibrium with two firms and $\dot{p}^{(2)} \leq \dot{p}^{(3)} \leq \ddot{p}^{(2)}$. ■

Our candidate equilibrium involves firms 1 and 2 pricing so low as to force a loss on firm 3 in case it decided to stay. So, one needs a price at or below $\dot{p}^{(3)}$. Let's consider pricing right at $\dot{p}^{(3)}$. In case firm 3 decided to stay, it would make zero profit in that period and zero profit in all subsequent ones (given the candidate equilibrium strategies). If it exited, it would make zero profit (by assumption), while 1 and 2 would make large losses (because of convex costs):

$$\pi^{(2)}(\dot{p}^{(3)}) < 0. \tag{4}$$

Conditional on staying in the game, each predator may or may not have the incentive to price itself out of the market (for instance, by posting price a) and let the other one take the losses. For the moment, we have taken care of that incentive problem by specifying reversion to zero-profit static Nash equilibrium forever upon observing a deviation in period 1. This incentive will be obviously sufficient if the profit to firm 1 and firm 2 is positive along the candidate equilibrium path, for by deviating a firm would then be guaranteed zero profit. In an appendix, we show that charging $\dot{p}^{(3)}$ can even be a short-term best response for the predators (in the sense that the revenues covers variable costs), conditional on staying in the game.

We are now in the position to state our first possibility result.

Proposition 4 *Provided δ and T are large enough, there exists a subgame-perfect equilibrium in which firms 1 and 2 charge $\dot{p}^{(3)}$ in the first period and $\ddot{p}^{(3)}$ thereafter. In this equilibrium they make more profit than in the equilibrium in which all three firms charge $\ddot{p}^{(3)}$ in all periods.*

Proof. The equilibrium strategies are as follows. Firms 1 and 2 play $\dot{p}^{(3)}$ and firm 3 plays E in period 1. Upon observing the exit of firm 3, firms 1 and 2 charge $\ddot{p}^{(3)}$ in all subsequent periods. If exit does not occur in period 1, all firms charge $\dot{p}^{(3)}$ until exit is observed. If in period 1, firm 3 exits but either firm 1 or firm 2 deviates, then they both charge $\dot{p}^{(2)}$ in all subsequent periods.

By the previous lemma,

$$0 < \pi^{(3)}(\ddot{p}^{(3)}) < \pi^{(2)}(\ddot{p}^{(3)}).$$

Thus,

$$\lim_{\delta \rightarrow 1, T \rightarrow \infty} \delta \frac{1 - \delta^{T-1}}{1 - \delta} [\pi^{(2)}(\ddot{p}^{(3)}) - \pi^{(3)}(\ddot{p}^{(3)})] > \pi^{(3)}(\ddot{p}^{(3)}) - \pi^{(2)}(\dot{p}^{(3)}).$$

■

Therefore, for T sufficiently large (but finite) and δ sufficiently close to 1, firm 1 and firm 2 not only make positive profit but make more than in the best Bertrand outcome with three firms. It is all the truer if we prescribe that the predators charge a price p such that $\ddot{p}^{(3)} < p \leq \ddot{p}^{(2)}$ after the exit. They could even collude. Observe, however, that by increasing their price following exit, the predators would invite ‘hit-and-run’ entry by any firm whose entry cost is below $\pi^{(1)}(p)$. By contrast, in our candidate equilibrium, a potential, equally efficient entrant, firm 4, would face the same incentives for entry before and after the exit of the prey. In our notation, this corresponds to the following statement.

Remark 5 $\hat{\pi}_4(\ddot{p}^{(3)}, \{1, 2\}) = \hat{\pi}_4(\ddot{p}^{(3)}, \{1, 2, 3\})$

Proof. $\ddot{p}^{(3)}$ is not a Bertrand equilibrium with four identical firms (for $\dot{p}^{(n)} < \ddot{p}^{(n-1)}$ for any n). So, if firm 4 considered entry when $\ddot{p}^{(3)}$ is charged by three firms, his static best-response would be to marginally undercut $\ddot{p}^{(3)}$ and

$$\hat{\pi}(\ddot{p}^{(3)}, \{1, 2, 3\}) = \pi^{(1)}(\ddot{p}^{(3)}).$$

Now, $\ddot{p}^{(3)}$ is a Bertrand equilibrium with three firms. So, by definition,

$$\hat{\pi}(\ddot{p}^{(3)}, \{1, 2\}) = \pi^{(3)}(\ddot{p}^{(3)}).$$

Thus, by definition of $\ddot{p}^{(3)}$:

$$\hat{\pi}_4(\ddot{p}^{(3)}, \{1, 2\}) = \hat{\pi}_4(\ddot{p}^{(3)}, \{1, 2, 3\}).$$

■

So far, we have shown that providing the “shadow of the future” is important enough, predation on the part of two firms is a natural outcome delivering more profit to those firms than the best non-collusive outcome without predation. It is also possible to compare our predatory outcome to perfect collusion. Before proceeding, we show that the eviction of the prey is more profitable to the predators, the higher the price charged. Let p^{JPn} is the price that maximizes the static joint profit of n firms.

Lemma 6 *For any p such that $\ddot{p}^{(3)} \leq p \leq p^{JP1}$, one has $\pi^{(2)}(p) > \pi^{(3)}(p)$. Moreover,*

$$\pi^{(2)}(p) - \pi^{(3)}(p) > \pi^{(2)}(\ddot{p}^{(3)}) - \pi^{(3)}(\ddot{p}^{(3)}).$$

Proof. Recall that $\pi^{(3)}(\ddot{p}^{(3)}) < \pi^{(2)}(\ddot{p}^{(3)})$ by Lemma 1. So,

$$\frac{\ddot{p}^{(3)} D(\ddot{p}^{(3)})}{3} - C \left[\frac{D(\ddot{p}^{(3)})}{3} \right] < \frac{\ddot{p}^{(3)} D(\ddot{p}^{(3)})}{2} - C \left[\frac{D(\ddot{p}^{(3)})}{2} \right],$$

or

$$\frac{\ddot{p}^{(3)} D(\ddot{p}^{(3)})}{6} > C \left[\frac{D(\ddot{p}^{(3)})}{2} \right] - C \left[\frac{D(\ddot{p}^{(3)})}{3} \right].$$

For $\ddot{p}^{(3)} \leq p \leq p^{JP1}$, one has

$$pD(p) > \ddot{p}^{(3)} D(\ddot{p}^{(3)})$$

by strict concavity of the monopolist’s programme. In addition,

$$D(\ddot{p}^{(3)}) > D(p)$$

and

$$C \left[\frac{D(\ddot{p}^{(3)})}{2} \right] - C \left[\frac{D(\ddot{p}^{(3)})}{3} \right] > C \left[\frac{D(p)}{2} \right] - C \left[\frac{D(p)}{3} \right]$$

since MC is a strictly increasing function of quantity.

Therefore,

$$\begin{aligned} & \frac{pD(p)}{6} - \left\{ C \left[\frac{D(p)}{2} \right] - C \left[\frac{D(p)}{3} \right] \right\} \\ & > \frac{\ddot{p}^{(3)} D(\ddot{p}^{(3)})}{6} - \left\{ C \left[\frac{D(\ddot{p}^{(3)})}{2} \right] - C \left[\frac{D(\ddot{p}^{(3)})}{3} \right] \right\}, \end{aligned}$$

which is equivalent to

$$\pi^{(2)}(p) - \pi^{(3)}(p) > \pi^{(2)}(\ddot{p}^{(3)}) - \pi^{(3)}(\ddot{p}^{(3)}).$$

■

Going back to the possibility of collusion, one realizes that there are in fact two issues at hand. The first one is to compare the predators' payoff in the predatory pricing equilibrium we have developed above to the best collusion payoff. The predators prefer predation to perfect collusion whenever

$$\pi^{(2)}(\ddot{p}^{(3)}) + \delta \frac{1 - \delta^{T-1}}{1 - \delta} \pi^{(2)}(p) > \frac{1 - \delta^{T-1}}{1 - \delta} \pi^{(3)}(p^{JP3}).$$

As a matter of fact, the right-hand side is an upper bound to the best collusive profit, for in a finite-horizon game it will not be possible to sustain $\pi^{(3)}(p^{JP3})$ in every period. Obviously, if

$$\pi^{(2)}(\ddot{p}^{(3)}) > \pi^{(3)}(p^{JP3}),$$

then it may be possible to construct an equilibrium in which, on the equilibrium path, predators make more profit by sharing the market between the two of them over sufficiently many periods than by colluding all along. When is this inequality satisfied? A simple sufficient condition can be worked out.

Lemma 7 *There exists $\delta > 0$ such that if $p^{JP3} \in [\ddot{p}^{(3)}, \ddot{p}^{(3)} + \delta)$, then $\pi^{(2)}(\ddot{p}^{(3)}) > \pi^{(3)}(p^{JP3})$.*

Proof. Suppose that $p^{JP3} \in [\ddot{p}^{(3)}, \ddot{p}^{(3)} + \delta)$, then by the previous lemma:

$$\pi^{(3)}(p^{JP3}) < \pi^{(2)}(p^{JP3}),$$

so that by continuity of $\pi^{(2)}$:

$$\pi^{(3)}(p^{JP3}) < \pi^{(2)}(\ddot{p}^{(3)}),$$

for δ sufficiently small. ■

It is easy to produce simple examples displaying the relevant feature.⁸ However, if one thinks that collusion could be sustained with three firms on the basis of the classical threat and reward possibilities allowed by repetition, then there is no reason not to explore the option for the predators to collude *after* the exit of the prey. After all, a large body of theoretical and experimental evidence supports the idea that collusion is easier when the number of firms is smaller.⁹ Thus, one should allow the predators to collude on a price between $\ddot{p}^{(3)}$ and p^{JP2} post exit. The next result indicates that any collusive path followed by three firms in equilibrium can be also followed by two firms.

⁸For instance, take $D(p) = 1 - p$, $C(q) = q^2$, $F = 0$, $\delta = 1$ and $T = 10$.

⁹For an early statement of the relationship between the "critical discount rate" required for collusion and the number of firms, see Friedman (1971). For a recent survey of the experimental evidence, see Haan, Schoonbeek and Winkel (2009).

Lemma 8 *If the sequence (p_1, p_2, \dots, p_T) , such that $p^{JP1} \geq p_t \geq \dot{p}^{(3)}$ for every $t \in \{1, 2, \dots, T\}$, is a subgame-perfect equilibrium outcome when three firms are active throughout the game, then it is also a subgame-perfect equilibrium outcome when two firms are active.*

Proof. Because the sequence (p_1, p_2, \dots, p_T) is a subgame-perfect equilibrium outcome when three firms are active, it must be the case that in every period t ,

$$\pi^{(3)}(p_t) + \pi^{(3)}(p_{t+1}) + \dots + \pi^{(3)}(p_T) > \hat{\pi}_i(p_t, \{1, 2, 3\} \setminus i).$$

The left-hand side is the continuation equilibrium payoff, while the right-hand side stands for the sum of the best deviation profit and the worst subgame-perfect punishment (i.e. minmax) payoff (which is zero, since a player can always exit the market in the period following a deviation and all firms playing $\dot{p}^{(3)}$ in all periods following a deviation is obviously subgame-perfect).

Now, by the previous lemma, since $\pi^{(3)}(\dot{p}^{(3)}) < \pi^{(2)}(\dot{p}^{(3)})$, we have $\pi^{(3)}(p_t) < \pi^{(2)}(p_t)$ for every t . Therefore, in every period t ,

$$\pi^{(2)}(p_t) + \pi^{(2)}(p_{t+1}) + \dots + \pi^{(2)}(p_T) > \hat{\pi}_i(p_t, \{1, 2, 3\} \setminus i).$$

Again, the left-hand side is the continuation equilibrium payoff, with two firms while the right-hand side stands for the sum of the best deviation profit and the worst subgame-perfect punishment payoff (which is again zero, since a player can always exit the market in the period following a deviation and playing $\dot{p}^{(2)}$ in all periods following a deviation is obviously subgame-perfect). So, (p_1, p_2, \dots, p_T) is a subgame-perfect equilibrium outcome with two firms. ■

As a corollary, we have that the best equilibrium outcome for three firms can be profitably replicated with two firms only. This puts us in the position of stating our second main result.

Proposition 9 *Consider the best symmetric subgame-perfect equilibrium outcome with three firms, $(p_1, \bar{p}_2, \dots, \bar{p}_T)$. Provided δ and T are large enough, then there exists a subgame-perfect equilibrium in which firms 1 and 2 charge $\dot{p}^{(3)}$ in the first period, firm 3 exits and firms 1 and 2 charge $(\bar{p}_2, \bar{p}_3, \dots, \bar{p}_T)$ thereafter. In this second equilibrium, firms 1 and 2 make more profit than in the first.*

Proof. The strategies are as follows. In period 1, firms 1 and 2 charge $\dot{p}^{(3)}$ and firm 3 chooses E . Thereafter, firms 1 and 2 play $(\bar{p}_2, \bar{p}_3, \dots, \bar{p}_T)$. If firm 3 fails to exit in period 1, then all three firms charge $\dot{p}^{(3)}$ until exit is observed. If either firm 1 or firm 2 fails to play in accordance with the equilibrium in any period, then they play $\dot{p}^{(2)}$ in all subsequent periods.

Because $(p_1, \bar{p}_2, \dots, \bar{p}_T)$ is a subgame-perfect equilibrium outcome with three firms, $(\bar{p}_2, \bar{p}_3, \dots, \bar{p}_T)$ is also a subgame-perfect equilibrium outcome with three firms. By the previous lemma, it is then also a subgame-perfect equilibrium outcome with two firms.

So, the only incentive problem is whether firms want to deviate in period 1. Firm 3 is indifferent between exiting and best-responding to $\dot{p}^{(3)}$. Firm 1 (or firm 2) makes the following profit on the equilibrium path:

$$\pi^{(2)}(\dot{p}^{(3)}) + \delta\pi^{(2)}(\bar{p}_2) + \dots + \delta^{T-1}\pi^{(2)}(\bar{p}_T).$$

By deviating in period 1, firm 1 can avoid the loss associated with charging $\dot{p}^{(3)}$ and achieve zero profit, but then it will also make zero profit in all subsequent periods. Thus, if the equilibrium profit is positive, then there is no incentive to deviate.

Now, by a previous lemma, for any p such that $\dot{p}^{(3)} \leq p \leq p^{JP1}$, one has

$$\pi^{(2)}(p) - \pi^{(3)}(p) > \pi^{(2)}(\dot{p}^{(3)}) - \pi^{(3)}(\dot{p}^{(3)}) \equiv K.$$

In the best, symmetric, subgame-perfect equilibrium outcome with three firms, $p^t \geq \dot{p}^{(3)}$ for any t . Thus, the equilibrium profit is such that

$$\pi^{(2)}(\dot{p}^{(3)}) + \delta\pi^{(2)}(\bar{p}_2) + \dots + \delta^{T-1}\pi^{(2)}(\bar{p}_T) > \pi^{(2)}(\dot{p}^{(3)}) + \delta\frac{1 - \delta^{T-1}}{1 - \delta}K.$$

Since one has

$$\lim_{\delta \rightarrow 1, T \rightarrow \infty} \delta\frac{1 - \delta^{T-1}}{1 - \delta}K = +\infty,$$

then for δ and T large enough firms 1 and 2 make more profit in the constructed equilibrium than in the original one. ■

Thus, provided the “shadow of the future” is sufficiently extended, two firms *always* have an incentive to induce the exit of the third in order to share demand between the two of them only. As before, this can be achieved without giving any extra incentive to (re-)entry.

Remark 10 *The second equilibrium outcome in the previous proposition is such that*

$$\hat{\pi}_4(p_t, \{1, 2\}) \leq \hat{\pi}_4(\bar{p}_t, \{1, 2, 3\})$$

for any $t \in \{1, 2, \dots, T\}$.

Proof. In period 1, since $\dot{p}^{(3)} < p_1$, we have

$$\hat{\pi}_4(\dot{p}^{(3)}, \{1, 2\}) = \pi^{(3)}(\dot{p}^{(3)}) \leq \pi^{(3)}(\ddot{p}^{(3)}) \leq \hat{\pi}_4(\bar{p}_t, \{1, 2, 3\}).$$

In any period $t \geq 2$, we have $p_t \geq \ddot{p}^{(3)}$, so that

$$\hat{\pi}_4(p_t, \{1, 2\}) = \hat{\pi}_4(p_t, \{1, 2, 3\}) = \pi^{(1)}(p_t).$$

■

4 Discussion

Our claim is that the constructed equilibria are predatory. The prey does not actually incur losses (this is impossible under perfect information, since the value of exiting is set at zero) but there is a sense in which it is forced to exit not to incur them and period 1 price is clearly predatory in the sense that it leads to losses that are borne by the predators only in expectation of higher future profits. As a matter of fact, in the specific equilibrium we construct, the prey is indifferent between staying in the

market and sustaining the price war or exiting. The equilibrium can be made strict by requesting the predators to charge $\dot{p}^{(3)} - \varepsilon$ in the first period. Similarly, predatory pricing need not start in period 1. It is easy to construct equilibria in which all firms make profit for a while before predation starts. The model can also be extended to include more than two predators and more than one prey.

According to the standard interpretation of the repeated-game literature, predators can be said tacitly to collude on decreasing prices because their strategy in period 2 depends on what happened in period 1 (play $\dot{p}^{(3)}$ if firm 3 has not exited). However, they fix prices *downward* and they do not necessarily collude after firm 3's exit; instead, they can play a static Nash equilibrium! That is: they do not necessarily prey in order to collude but they collude so as to prey!

This result shows that joint predation can arise in equilibrium under perfect information, so long as the prey expects a bad outcome (low price) to follow her decision to resist the predation attempts. As usual, equilibria necessitate that firms' beliefs be mutually consistent, an achievement that may be difficult to obtain without explicit communication. Nevertheless, observe that joint predation is in a sense easier than single-firm predation. In the model, a single predator could not help with charging the monopoly price in all periods where it is the only active firm. Because it could not commit not to raise its price in period 1, it would never induce the exit of the prey. Even if it succeeded, following exit, it would give potential entrants incentives to 'hit and run'.

Notice that our theory of joint predation also generates the prediction that predation should be easier towards "weaker" rivals. It is indeed easily seen that if the prey is "small" or "weak" (in the sense of facing more convex costs than the predators), then predators can achieve exit while charging a higher price in period 1.

In equilibrium, consumers benefit from predatory prices: consumer surplus goes up in period 1. From period 2 on, the price is the same as when three firms charge the highest Bertrand (or collusive) price. In terms of total surplus, the inefficiency stems not only from overconsumption in period 1 but also from the inefficient production following exclusion. (Productive efficiency can decrease since convex costs call for multiple firms to share production.) This last feature is what makes predation not only anticompetitive but also antisocial. In a model where variable costs were linear, for any nonzero level of fixed costs total surplus maximization would call for only one firm to produce, so that a benevolent social planner (or competition authority) would in some sense be happy to see predation take place.

Joint predation is not a theoretical curiosity. Sea shipping has historically been cartelized and the sea shipping conferences (i.e. the cartels) have often fought entrants with so-called fighting ships that are scheduled to depart on, or close to, the day rival ships are to depart at a rate that equals or beats the one of the entrant. (See Yamey, 1972 and Scott Morton, 1997 for historical evidence.) This practice was at the heart of recent European case *Compagnie maritime belge*¹⁰. Similarly, in the US, the two

¹⁰European Court of First Instance, Cases C-395 & 396/96 *Compagnie Maritime Belge Transport NV v. Commission*.

leading cases, *Matsushita*¹¹ and *Brooke*¹² allegedly involved many predators.

Antitrust laws would likely treat any concerted attempt at evicting a rival as a hard-core cartel practice. When this outcome is achieved by tacit coordination, the case law is notoriously difficult and, overall, favorable to the predators. In particular, the US Supreme Court has shown extreme skepticism toward the possibility of joint predation.¹³ One reason, which is valid and also applies to tacit collusion in general, is the difficulty for firms to align their expectations and conducts without an explicit agreement. Another one, which we show to be non valid, is the absence of a convincing economic theory of joint predation.

If the abuse is treated as a predation case, of particular concern is the requirement that the initial investment in predatory prices be recouped through so-called “supra-competitive prices” after the exit of the prey. Proposition 4 shows that such higher prices are not needed. In addition, the possibility for defendants to argue that their pricing pattern has a legitimate business purpose or to invoke a meet-competitor’s-price defense is especially problematic in the case of joint predation. By definition, joint predators do exactly what their competitors do and we can show that charging $\dot{p}^{(3)}$ can even qualify as a static best-response for two firms which envision staying in the industry. (See appendix.) Therefore, it is fair to say that the existing laws and judicial interpretations governing predatory pricing are likely to leave a number of anticompetitive practices unchecked.

A Legitimate business response

Lemma 11 *If the fixed cost F is high in relation to the variable costs, then, conditional on not exiting, charging $\dot{p}^{(3)}$ in period 1 is a static best-response to firm 3’s exit for firms 1 and 2.*

Proof. Given the candidate equilibrium strategies, conditional on staying in the game, $\dot{p}^{(3)}$ is a static best-response for firms 1 and 2 iff $\dot{p}^{(3)}$ is at least as large as the corresponding average variable cost. Now, by definition:

$$\dot{p}^{(3)} = AC \left(\frac{D(\dot{p}^{(3)})}{3} \right).$$

¹¹475 U.S. 574 (1986).

¹²509 U.S. 209 (1993).

¹³In *Brooke*, the Court wrote

In *Matsushita*, we remarked upon the general implausibility of predatory pricing. *Matsushita* observed that such schemes are even more improbable when they require coordinated action among several firms. *Matsushita* involved an allegation of an express conspiracy to engage in predatory pricing. The Court noted that in addition to the usual difficulties that face a single firm attempting to recoup predatory losses, other problems render a conspiracy ‘incalculably more difficult to execute’. In order to succeed, the conspirators must agree on how to allocate present losses and future gains among the firms involved, and each firm must resist powerful incentives to cheat on whatever agreement is reached.

So, $\dot{p}^{(3)}$ is a static best-response for firms 1 and 2 iff

$$AC\left(\frac{D(\dot{p}^{(3)})}{3}\right) \geq AVC\left(\frac{D(\dot{p}^{(3)})}{2}\right),$$

or

$$AFC\left(\frac{D(\dot{p}^{(3)})}{3}\right) \geq AVC\left(\frac{D(\dot{p}^{(3)})}{2}\right) - AVC\left(\frac{D(\dot{p}^{(3)})}{3}\right)$$

or

$$F \geq \frac{2}{3} \int_{\frac{D(\dot{p}^{(3)})}{3}}^{\frac{D(\dot{p}^{(3)})}{2}} MC(x) dx - \frac{1}{3} \int_0^{\frac{D(\dot{p}^{(3)})}{3}} MC(x) dx$$

When F increases, $\dot{p}^{(3)}$ increases and $D(\dot{p}^{(3)})$ decreases. Observe that

$$\frac{d}{dy} \left[\frac{2}{3} \int_{\frac{y}{3}}^{\frac{y}{2}} MC(x) dx - \frac{1}{3} \int_0^{\frac{y}{3}} MC(x) dx \right] = \frac{1}{3} \left[MC\left(\frac{y}{2}\right) - MC\left(\frac{y}{3}\right) \right],$$

which is positive since MC is strictly increasing. Since $\dot{p}^{(3)}$ increases with F , if F is sufficiently large, then the inequality is verified. ■

References

- [1] Benoît, Jean-Pierre and Vijay Krishna (1985). "Finitely-Repeated Games." *Econometrica*, 53(4), 905-922.
- [2] Bolton, Patrick, Joseph F. Brodley, and Michael H. Riordan (2000). "Predatory pricing: strategic theory and legal policy." *Georgetown Law Journal*, 88, 2239-2330.
- [3] Bolton, Patrick and David S. Scharfstein (1990). "A theory of predation based on agency problems in financial contracting." *American Economic Review*, 80, 93-106.
- [4] Bork, Robert (1978). *The antitrust paradox: a policy at war with itself*. New York: Basic Books.
- [5] Chandler, A.D. (1990). *Scale and scope: the dynamics of industrial capitalism*. Cambridge, Mass.: Harvard University Press.
- [6] Dastidar, Krishnendu Gosh (1995). "On the existence of pure strategy Bertrand equilibrium." *Economic Theory*, 5, 19-32.
- [7] Friedman, James (1971). "Non-cooperative equilibrium for supergames." *Review of Economic Studies*, 38, 1-12.

- [8] Fudenberg, Drew and Jean Tirole (1986). "A "signal-jamming" theory of predation." *Rand Journal of Economics*, 17, 366-376.
- [9] Haan, M.A., L. Schoonbeek, and B.M. Winkel (2009): Experimental results on collusion; in Jeroen Hinloppen and Hans-Theo Normann, *Experiments and competition policy*, New York: Cambridge University Press, 9-33.
- [10] Harrington, Joseph E. (1989). "Collusion and predation under (almost) free entry." *International Journal of Industrial Organization*, 7, 381-401.
- [11] Kreps, D.M. and R.J. Wilson (1982). "Reputation and imperfect information." *Journal of Economic Theory*, 27, 253-279.
- [12] Milgrom, P.R. and J. Roberts (1982). "Predation, reputation and entry deterrence." *Journal of Economic Theory*, 27, 280-312.
- [13] Motta, Massimo (2004). *Competition policy. Theory and practice*. Cambridge: Cambridge University Press.
- [14] Roth, David (1996). "Rationalizable predatory pricing." *Journal of Economic Theory*, 68, 380-396.
- [15] Scharfstein, David (1984). "A policy to prevent rational test-market predation." *Rand Journal of Economics*, 15, 229-243.
- [16] Scott Morton, Fiona (1997). "Entry and predation: British shipping cartels 1879-1929." *Journal of Economics & Management Strategy*, 6, 679-724.
- [17] Vives, Xavier (1999). *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, Mass.: MIT Press.
- [18] Yamey, B.S. (1972). "Predatory price cutting: notes and comments." *Journal of Law and Economics*, 15(1), 129-142.
- [19] Weibull Jörgen W. (2006): "Price competition and convex costs", SSE/EFI Working Paper Series in Economics and Finance No 622.