THE LENDER OF LAST RESORT: LIQUIDITY PROVISION VERSUS THE POSSIBILITY OF BAIL-OUT

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The lender of last resort: 
liquidity provision versus the possibility of bail-out*

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Abstract

Banking regulation has proven to be inadequate to guard systemic stability in the recent financial crisis. Central banks have provided liquidity and ministries of finance have set up rescue programmes to restore confidence and stability. Using a model of a systemic bank suffering from liquidity shocks, we find that the unregulated bank keeps too much liquidity and takes excessive risk compared to the social optimum. A Lender of Last Resort can alleviate the liquidity problem, but induces moral hazard. Therefore, we introduce a fiscal authority that is able to bail out the bank by injecting capital. This authority faces a trade-off: when it imposes strict bailout conditions, investment increases but moral hazard ensues. Milder bailout conditions reduce excessive risk taking at the expense of investment. This resembles the current situation on financial markets, in which banks take less risk but also provide less credit to the economy.

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1 Introduction

The financial crisis of the last two years has shown that banking regulation has not been adequate to safeguard the stability of the financial system. While prudential regulation such as the Basel II capital requirements has allowed for regulatory arbitrage, the existence of a lender of last resort has been insufficient to deter banks from taking risks that are harmful to the financial system.

As a result, central banks around the world had to provide substantial amounts of liquidity to alleviate liquidity shortages and to prevent the interbank market from breaking down completely. They have provided this liquidity on very generous terms, letting virtually every bank access their facilities. Therefore, although Bagehot (1873) already argued that insolvent banks should not be provided with liquidity, many banks that received assistance were in fact insolvent. However, as they posed a risk for the financial system as a whole, regulators had no choice but to save them. This suggests that the Too-Big-to-Fail problem still exists, although many now call it a Too-Connected-to-Fail problem: the interlinkages between banks are so dense that contagion of bank failures has become inevitable (Nijskens and Wagner, 2008).

Furthermore, to prevent a complete failure of the financial system governments have constructed very large rescue packages additional to central bank liquidity. Halfway through 2009, the amount of resources committed in these packages lay around €5 trillion or 18.8% of GDP for 11 large western countries\(^1\), whereas actual outlays amounted to €2 trillion (Panetta et al., 2009). The rescue measures comprise capital injections into banks, all-out nationalizations, explicit guarantees on bank lending and purchases of troubled assets. This large-scale intervention has turned out to be absolutely necessary to restore confidence and stability.

To provide a correct assessment of crisis management after large systemic shocks, we need to consider thoroughly what has happened. In managing the current crisis, the central bank (as a provider of liquidity) and the fiscal authorities (by providing capital or guarantees) have worked together very closely. It is thus imperative to perform a simultaneous analysis of liquidity provision and solvency regulation. Furthermore, we need to consider large, systemically relevant banks and examine their interaction with both the central bank and the fiscal authorities. Our analytical model will incorporate two principles voiced regarding lender of last resort practices. One is the abovementioned principle of Bagehot, stating that central banks should only provide liquidity to solvent banks. The other is the idea that bailout assistance (e.g. capital injections or loan guarantees) should be costly for banks (Eijffinger, 2008), as they must be punished for threatening financial stability.

The results of our analysis indicate that having solely a Lender of Last Resort (LLR), in the

\(^{1}\)Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Switzerland, the United Kingdom and the United States.
form of a central bank providing liquidity, can decrease excessive liquidity hoarding relative to the case without an LLR. However, having a safety net also leads to engagement in moral hazard by banks: they take excessive risks.

To alleviate this problem the safety net can be extended to comprise also capital provision. We allow this capital provision to be costly to mitigate a possible moral hazard effect. Ultimately we find that the regulators face a trade-off. On the one hand, making capital assistance very costly for the bank increases productive investment, but also increases excessive risk taking. On the other, relatively less costly capital assistance decreases moral hazard at the expense of investment.

This reflects the current situation in the financial world: due to bailout assistance by governments, banks are facing harsher funding requirements and can thus extend less credit for risky investment. However, the risks they are taking are less excessive than before the crisis.

In what follows, we will first provide a short overview of existing literature on LLR and solvency regulation in section 2. Our model will be described in section 3, while section 4 presents the analytical results derived from this model. Section 5 concludes.

2 Related literature on LLR policy and solvency

The academic literature on the Lender of Last Resort (LLR), especially the part also considering solvency (or closure) decisions, is quite modest in size. Interest in this topic has increased since the United States’ Savings and Loans (S&L) crisis in the 1980’s. The literature has since focused on the role of the Central Bank (CB) as an LLR to prevent and manage crises, and on the role of the CB and other institutions in taking care of an orderly solution for bank failures. Of course, the current financial crisis has stimulated research in this area and we are bound to see many more research efforts in the near future.

The result of two decades of research on LLR and closure policy has been nicely documented by Freixas and Parigi (2008). They start by mentioning the classic Bagehot (1873) principle, which has been a starting point of analysis for many authors: the LLR should provide liquidity to illiquid, but solvent banks at a penalty rate and against good collateral. They then note that banking has become much more complex since 1873, causing problems such as the inability to distinguish liquidity from solvency problems (Goodhart, 1987). They also address interbank market imperfections, moral hazard caused by penalty rates, the increasing difficulty in determining the quality of collateral and the change of the banking system in general. The authors end with the recommendation that we should not solely look at the role of the LLR but rather study "what architecture of prudential regulation, risk supervision, monetary policy, deposit insurance and ELA is best to guarantee financial stability". 

The selection of literature we consider has addressed most abovementioned complexities separately. We intend to bring some of them together in our analysis, as described at the end of this section.

One striking comment on Bagehot’s LLR view is the idea that the CB should be ambiguous about liquidity provision, meaning that the bank cannot always expect the CB to provide liquidity when it is needed. Instead, the bank will face some uncertainty about whether it will receive liquidity, and will face requirements on funding. This so-called "creative ambiguity" doctrine is analyzed by among others Freixas (1999), Goodhart and Huang (1999), Repullo (2005) and Cordella and Yeyati (2003), with contrasting results. While Freixas (1999) finds that ambiguity may have its merits in some cases (by reducing moral hazard), he also provides a rationale for a Too-Big-to-Fail (TBTF) policy. When banks are large, the social cost of their bankruptcy is too high and the CB will always assist them with liquidity; a practice that is detrimental to welfare if the bank is insolvent. Essentially the same result is found by Goodhart and Huang (1999). They find a justification for ambiguity, although the optimal degree of ambiguity decreases strongly in bank size. This ultimately leads to a TBTF policy motivated by contagion concerns.

Repullo (2005) finds a similar result regarding excessive risk taking: certainty about liquidity provision does not increase moral hazard, although banks hold too little liquidity in this case. The introduction of penalty rates, however, does cause moral hazard. This is motivated by arguing that penalty rates decrease the expected return in the illiquid state. The bank tries to offset this by taking a higher risk, thereby increasing its return in the good state.

Cordella and Yeyati (2003) also conclude that moral hazard is not sufficient to justify criticism on standard LLR policies. They argue that the possible moral hazard effect of having an LLR is compensated by an increase in charter value. This increase is generated by the CB’s commitment to an unambiguous LLR policy, conditional on an aggregate macroeconomic shock and some "good practice" conditions on funding.

Rochet (2004) explores this further by letting the bank choose its riskiness under an optimal regime of prudential regulation. Here, riskiness is measured by the exposure to aggregate macroeconomic shocks. He finds that banks with an exposure above a certain threshold are perceived as too risky and should not receive liquidity assistance. However, he also finds a time inconsistency in providing liquidity assistance, leading to ex post regulatory forbearance.

Regulatory forbearance occurs after a macroeconomic shock reminds of the regulatory response to the current financial crisis, and the concepts used to justify this: systemic risk and contagion. Although we do not explicitly model these phenomena, systemic risk is implied in our analysis and we thus review shortly the literature on this topic. Freixas et al. (2000) have been among the
first to model systemic risk, in the interbank market. The interbank market can reduce the cost of holding liquid reserves, which banks need to satisfy uncertain consumption demand. However, coordination failures in this market can lead to a gridlock, which may lead to inefficient closure of solvent banks. Moreover, money center banks may not be allowed to fail as their failure might lead to contagion. This resembles the "too-connected-to-fail" problem that has also manifested in the current crisis.

Recently, Acharya and Yorulmazer (2007, 2008) have explicitly considered interlinkages between banks that invest in similar projects. If the correlation between these projects is low, it is possible that only a few banks fail when some projects yield a low return. These may be taken over by the surviving ones, who may receive CB liquidity for this purpose. However, if correlation is high and many banks fail, there are not enough surviving banks that can purchase assets. The CB thus has to choose whether to bail them out or liquidate them, which can be very costly because of asset specificity. This leads again to a time inconsistency problem, as the CB cannot credibly commit ex ante to not bail out these banks. The authors call this a "too-many-to-fail" problem, related to the correlation between banks’ investments in the current financial system.

Although the models discussed above can explain several phenomena present in the financial system, they also suffer from one deficiency: they focus mainly on the central bank as a lender of last resort, which provides liquidity assistance up to a certain threshold. However, the bank may still be solvent, although the central bank does not have enough information to judge this; banks may be inefficiently closed. Furthermore, as described above and noted by e.g. Boot and Thakor (1993) and Rochet (2004), the inability to discriminate between liquidity and solvency problems can lead to regulatory forbearance: insolvent banks are allowed to stay open.

Freixas et al. (2004) thoroughly examine indistinguishable liquidity and solvency problems. They explicitly model liquidity and solvency shocks separately, thereby assuming that the CB cannot determine ex ante whether the bank is only illiquid or also insolvent. Their main finding is that a CB providing LLR support is optimal when insolvent banks are not detected, it is costly for banks to screen borrowers and interbank market spreads are high. This resembles crisis episodes (such as the recent one) with inefficient supervision.

In Rochet and Vives (2004), the same problem does not stem from failing supervision, but is caused by coordination failures between market participants. In their case, participants in the interbank market cannot distinguish between illiquid and insolvent banks. Below a certain threshold for bank fundamentals, participants in the interbank market are not willing to lend to the bank anymore. Although the bank may still be solvent in this case, the interbank market will see it as insolvent. This suggests a role for LLR policy by the CB, which should be complemented with prompt corrective action to implement the incentive-efficient solution.
To incorporate this notion, some authors have considered explicitly the co-existence of multiple regulators (with different responsibilities regarding liquidity and solvency) in the banking system. Repullo (2000), for instance, suggests a model of a bank suffering liquidity shocks that requires LLR borrowing. His model is based on the literature of incomplete contracts. It contains two regulatory agencies with different preferences, namely the central bank and the deposit insurance fund (DIF), that may provide this liquidity using nonverifiable information. Under the assumption that the allocation of liquidity can be made contingent on the size of the shock, he finds that in case of small shocks the CB should be the LLR, and the DIF should fulfil this role in case of large shocks.

Kahn and Santos (2005) extend this model significantly by considering closure authority in addition to liquidity decisions. This allows for a distinction between illiquidity and insolvency and for examining how regulator’s information incentives are influenced by the institutional allocation. They find that having only one regulator deciding on both LLR and closure leads to excessive forbearance and suboptimal bank investment. Like Repullo (2000), they also find that multiple regulators may improve the situation, especially when supervision is allocated to the DIF. However, at low levels of liquidity shortage the forbearance problem may be exacerbated.

Additional to having two regulatory bodies, it is argued that capital provision may complement liquidity provision and help to solve the problem of inefficient closure. Diamond and Rajan (2005), for instance, have set up a general equilibrium model in which an endogenous liquidity problem occurs as entrepreneurs need to refinance their projects (Holmstrom and Tirole, 1998). This leads to an aggregate liquidity shortage, which the CB can partly alleviate. Furthermore, they find that a capital injection may make insolvent banks solvent again, improving their ability to raise liquidity. They also note that this intervention does not work when the bank is fundamentally insolvent, in line with Bagehot’s principle: genuinely insolvent banks should not receive liquidity.

Building on this body of literature, we acknowledge that there are indeed often two different authorities responsible for financial crisis management. These authorities have a division of tasks, where the central bank is generally responsible for liquidity provision and a fiscal authority (Treasury or Ministry of Finance) has to decide whether the bank receives a capital injection or not. We will argue that the existence of a fiscal authority beside a central bank may reduce excessive risk taking. The model with which we show this slightly resembles that of Repullo (2005). We take a similar game theoretic approach, but we introduce an additional regulatory authority into the model. Our model will be explained in the next section.
3 The Model

Let us consider an economy with risk-neutral agents and three dates: \( t = 0, 1, 2 \). In this economy, there is one (systemically important) bank that collects deposits and has some equity capital; it operates under limited liability. The economy also contains two regulatory agencies: a central bank (CB) fulfilling the role of Lender of Last Resort (LLR) and a fiscal authority that, in case of a bank failure, has to decide on the failure resolution procedure.

The bank’s size is equal to one\(^2\), and its balance sheet looks as follows:

\[
I + M = E + D \tag{1}
\]

Investments \( I \) provide a random gross return\(^3\) \( \tilde{R} \) per unit of investment in period 2:

\[
\tilde{R} = \begin{cases} 
R_H > 1 & \text{with probability } p \\
R_L < 1 & \text{with probability } 1 - p
\end{cases}
\]

\( p \in [0, 1] \) is the success probability of investment, increasing in the efforts of the bank to monitor this investment. Expected return \( E(\tilde{R}) \) is greater than 1, and investments are illiquid in the sense that they cannot be sold before \( t = 2 \). The other item on the asset side is \( M \), holdings of liquid assets. These are called "liquid" since they represent investment in a storage technology, which provides a riskless return of \( R_M = 1 \) per unit of \( M \). This implies that the riskless interest rate in our model is equal to 0.

On the liability side we first equity and deposits. Equity capital \( E \) comes from the bank owner, who operates under limited liability. Deposits \( D \) are fully insured, and yield a return \( R_D = 1 \) at \( t = 2 \) since they are riskless. To abstract completely from deposit insurance issues, we assume that the bank pays no deposit insurance premium.

We will further assume that \( I > E \), to give the bank owner the opportunity to work with leverage. This assumption reflects that holding liquidity may be too costly as it foregoes potential returns on \( I \) (Rochet and Vives, 2004). It is thus profitable for the bank owner to invest the bank’s funds in the risky asset (as \( E(\tilde{R}) \geq 1 \)). These profits will be even amplified when \( I > E \).

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\(^2\)Since we have assumed that there is only one bank and thus bank failure is costly for society, we may abstract from letting bank size determine bank closure policy.

\(^3\)Note that all returns in our model are gross returns.
Finally, to simplify our analysis, we will introduce the following assumptions on $p$ and $R$:

\begin{align*}
R_L &= 0, \quad R_H = R(p), \quad (2) \\
R'(p) &< 0, \quad R''(p) \leq 0, \quad (3) \\
R(1) &\geq 1, \quad R(1) + R'(1) < 0, \quad (4) \\
E(\bar{R}) &\geq 1 \quad (5)
\end{align*}

This return function is also used by Repullo (2005), Allen and Gale (2000) and Cordella and Yeyati (2003), and implies that $E(\bar{R}) = pR(p)$ will be maximized at $\hat{p} \in (0, 1)$ where $R(\hat{p}) + \hat{p}R'(\hat{p}) = 0$. The assumptions on the function thus suffice for an interior maximum\(^4\). Given the above assumptions we can write down bank value at $t = 2$ as follows:

$$V_2 = p[R(p)I + M - D] \quad (6)$$

This expression will be modified in the next section.

### 3.1 A liquidity shock

In its operation, the bank is subject to liquidity shocks. A liquidity shock, consisting of depositors withdrawing a fraction $x$ of their deposits, occurs at date $t = 1^5$. $x$ is uniformly distributed on the interval $(0, 1)$ with cumulative density $F(x) = x$ and probability density $f(x) = 1$. The size of the shock is public information when it occurs at $t = 1$. Taking into account that we have two regulatory agencies, we can distinguish three cases (similar to Rochet and Vives (2004)):

1. $x \leq \frac{M}{D} = \bar{x}$, with probability $\bar{x}$: the liquidity shock can be resolved using liquid reserves.
2. $\bar{x} < x \leq \bar{x}$ with probability $\bar{x} - x$: the bank is illiquid and will apply for emergency lending at the LLR. $\bar{x}$ will be determined below.
3. $x < \bar{x}$ with probability $1 - x$: the solvency of the bank is insufficient to warrant LLR borrowing and the fiscal authority will have to take a closure/continuation decision.

In case 1, the shock is small and the bank can repay the withdrawn deposits using its liquid reserves $M$. Note that we assume there is no interbank market; the bank’s only liquidity comes

\(^4\)Note that, for $p = 0$, \(\frac{dpR(p)}{dp} = R(0) > 0\) and, for $p = 1$, $R(1) + R'(1) < 0$. The second order condition for a maximum is $\frac{d^2pR(p)}{dp^2} = 2R'(p) + pR''(p) < 0$ for all $p > 0$. This suffices for an interior maximum at $\hat{p}$.

\(^5\)Taking the credit crisis as a reference point, this kind of liquidity shock is very similar to investors in asset-backed securities selling their claims back to the bank. Banks were obliged to return the money, which led to severe liquidity problems. We can see this as analogous to deposit withdrawals.
from the amount of liquid reserves it has kept at $t = 0^6$.

In case 2, when $x < x < \bar{x}$, the bank cannot finance the liquidity shortage by itself, so it has to apply for emergency liquidity from the LLR at an amount of $xD - M$. The LLR will ask a repayment rate equal to 1 (there is no penalty rate) at $t = 2$ and will only lend to solvent banks.

In fulfilling its role of LLR, the central bank (CB) will want to minimize the social cost of a bank’s risk taking. This is reflected in the bankruptcy cost $C$, which will be realized if the bank fails. The CB will therefore provide liquidity up until a certain threshold$^7$. This follows from the generally accepted principle stated by Bagehot (1873): central banks should not lend to banks that are both illiquid and insolvent. In determining this so-called solvency threshold, the CB takes into account an expected cost of $(1 - p)[\alpha C + (xD - M)]$ when it supports the bank with emergency liquidity. When it does not support the bank, the CB incurs the certain loss $\alpha C$. In these expressions, $\alpha$ is the weight the regulator attaches to the bankruptcy cost. This can be interpreted as the political or reputational cost to the central bank and is assumed to be greater than zero$^8$.

Comparing the two above expressions we can deduce the solvency threshold for the CB at $t = 1$, denoted by $\bar{x}$:

$$x \leq \bar{x} \equiv \frac{p}{1 - p} \frac{\alpha C}{D} + \frac{M}{D}$$

(7)

Otherwise stated: the bank will apply for an amount of $xD - M$ and the CB will only provide liquidity when (7) holds. This means that the certain cost of a bank failure at $t = 1$ is greater than the expected cost of failure at $t = 2$. In this case the bank is considered to be solvent ($x \leq \bar{x}$), but illiquid.

Therefore, when $x > \bar{x}$, the bank cannot borrow from the LLR (case 3). A bank failure will occur, and the bank will enter into a prompt corrective action programme by the fiscal authority (FA). A bailout from the FA is needed to continue the bank’s business, and the FA will require a certain repayment that is potentially costly for the bank owner.

In a bailout, the FA has to provide an amount of funds equal to $xD - M$ to make the bank solvent; we will call these funds "capital". The FA then decides upon the conditions on which this $^6$This assumption can be justified since we are focusing on crisis management. In the current financial crisis the interbank market nearly broke down (Allen et al., 2009; Diamond and Rajan, 2009). Massive intervention by central banks seemed to be the only way to get it going again.

$^7$Depositors get $D$ back in case of insolvency, but this is dealt with by the DIF (a separate authority). We assume that the deposit insurance is not part of the loss functions.

$^8$In Kahn and Santos (2005), but Repullo (2000) assumes $\alpha < 1$ and Repullo (2005) assumes $\alpha = 1$. We will not yet make any assumptions other than $\alpha > 0$. The same holds for $\beta$ in the case of the fiscal authority.
capital will be provided. These are meant to discipline the bank owner for taking too much risk and letting the bank become insolvent, and will consist of the regulator determining the amount of gains it appropriates from the bank. The FA will require a share $\gamma$ of bank value at $t = 2$ in case of success, and will incur the bankruptcy cost $\beta C$ in case of failure. However, when the it does not provide assistance, it will incur the cost $\beta C$ with certainty.

The FA will then choose the repayment $\gamma$ such that it at least breaks even in expectation:

$$
\gamma \geq \frac{xD - M - p\beta C}{p((R(p) - 1)I + E)}
$$

where $\beta$ is the weight the FA attaches to the cost of bankruptcy in the same vein as the CB’s $\alpha$. Note, however, that these weights may differ for the CB and the FA. This reflects the political relation between the CB and the FA; they may have different responsibilities regarding financial stability.

This possibility of bailout, with a required period 2 return of $\gamma$, is a stylized representation of the situation in which a bank is nationalized, recapitalized or provided with guarantees on its borrowing. These measures have been used extensively in crisis management during the last 2 years. Of course, these measures have not been free for banks: regulators have set a premium on the rates to be paid for access to these facilities, as the government has taken over part of the risk from the bank. This is epitomized by the $\gamma$ in our model, which may contain the abovementioned risk premium. Bailout assistance thus comes at a cost for the bank owner.

### 3.2 The bank’s objective

Taking the liquidity shock and the regulatory system into account, the bank owner will maximize total bank value at $t = 2$. The bank operates with an exogenously given capital structure (following Wagner (2007)), consisting of positive amounts of both equity and deposits. The choice variables for the bank owner are the effort put into monitoring, embodied in the probability of success $p$, and the amount of investment $I$. This investment is productive, meaning that it is desirable for the economy to allocate funds to it (since $E(\tilde{R}) > 1$). The probability of success, which increases with monitoring effort at $t = 0$, is the inverse of the amount of risk taken.

Using the properties of the liquidity shock and the aforementioned conditions $\pi$ and $\gamma$, set by the regulatory authorities, we can refine the bank’s objective function. We assume that there is no time discounting. Let us first write down the bank owner’s $t = 2$ payoff, denoted by $\tilde{V}_2$, in the
different scenarios:

\[
\tilde{V}_2 = \begin{cases} 
V^L_2(p) = p[R(p)I + M - D] & \text{w.p. } \bar{x} \\
V^M_2(p) = p[R(p)(1 - x)D - (xD - M)] & \text{w.p. } \bar{x} - \bar{\bar{x}} \\
V^H_2(p) = (1 - \gamma)p[R(p)I + M - D] & \text{w.p. } 1 - \bar{\bar{x}}
\end{cases}
\]

where "w.p." means "with probability" and the subscripts denote the magnitude of the shock: low, medium or high. We can thus write the bank’s objective function at \( t = 0 \) as follows:

\[
\max_{p,I} \left\{ E(\tilde{V}(p,I)) = p[R(p)I + M - D][1 - \gamma(1 - \bar{\bar{x}})] \right\} \quad (9)
\]

The fact that expected bank value is not only varying with \( p \), but also with \( \bar{x} \) and \( \gamma \), indicates that it depends on the choices made by the bank owner as well as those made by the regulators. In the next section we will characterize this interdependence.

4 Liquidity or liquidation

To summarize the previous sections, we can systematically go through the sequence of events. Following Repullo (2005), we let the bank simultaneously choose its risk \( p \) (determined by its monitoring effort) and its portfolio of risky investments \( I \) at \( t = 0 \), taking into account the possibility of liquidity shocks at \( t = 1 \) and responses by the CB and the FA.

At \( t = 1 \), the liquidity shock realizes and it is observable. If \( x \leq \bar{x} \), the bank pays depositors out of its liquidity reserves. If \( \bar{x} < x \leq \bar{\bar{x}} \), the bank applies for liquidity and the CB will provide it. Finally, if \( x > \bar{\bar{x}} \), the CB is not willing to provide liquidity and the FA will take action. This will lead to a required repayment \( \gamma \), which depends in turn on the amounts of investment and monitoring chosen by the bank at \( t = 0 \).

4.1 First Best

As a benchmark, we first analyze the socially efficient solution to the problem of choosing optimal investment and risk taking. In the first best case, there is a central planner who chooses risk, investment and the regulatory instruments such that the social value of bank investments is maximized. The gains to society are the total value of bank investments at \( t = 2 \) minus the value of investments.
The central planner’s problem is:

$$\max_{p,I} \quad p[(R(p) - 1)I + M] + (1 - p)[M - I]$$

(10)

As we have assumed $E(\tilde{R}) = pR(p) > 1$, this function is strictly increasing in $I$ and it is optimal to set $I$ equal to 1. Furthermore, the derivative of (10) w.r.t. $p$ is $[R(p) - pR'(p)]I$. The optimal quantities of $p$ and $I$ are thus given by:

$$R(p^{fb}) + p^{fb}R'(p^{fb}) = 0$$

(11)

$$I^{fb} = 1$$

(12)

which means that $p^{fb}$ is such that equation (11) is satisfied. It is optimal to set $M = 0$ and invest all funds into the risky asset $I$; with this knowledge, monitoring effort (and thus $p$) is chosen to maximize the expected return on these investments.

Of course the central planner/regulator takes into account the full social value when setting a solvency threshold for liquidity provision. The threshold rule will thus be determined by comparing the expected cost of providing liquidity, which is $(1 - p)(C + (xD - M)) - p(R(p)I - (xD - M))$ with the cost of failure $C$ (from society’s point of view). This leads to the following solvency threshold:

$$\bar{x}^{fb} = \frac{p^{fb}[R(p^{fb})I^{fb} + C]}{D}$$

(13)

which says that the bank only fails when its expected return on investment at $t = 2$ plus the possible bankruptcy costs is less than the liquidity shock. As $x \in (0, 1)$, $xD \leq I^{fb}$ and $p^{fb}R(p^{fb}) \geq 1$, we see that this threshold is larger than 1 and thus not binding. The bank will always get a liquidity injection from the government in the first best scenario: socially optimal risk taking and investment justify unconditional liquidity assistance.

### 4.2 Bank optimization without regulation

Let us now consider the case of a private bank choosing an optimal portfolio, and analyze whether it reaches the first best allocation.

We assume that there are no regulatory authorities, such as a Lender of Last Resort or a fiscal authority, which may provide assistance. There is also no interbank market, as mentioned above. The bank thus has to cope with liquidity shocks on its own, which means that the bank fails if
\( x > \bar{x} \equiv \frac{M}{D} \), i.e. when the sudden demand for liquidity is larger than the bank’s liquid assets. In case of failure, the returns at \( t = 2 \) are zero, since effectively \( \gamma = 1 \) when there is no FA. The bank thus maximizes the following expected bank value of (where we have replaced \( M - D \) by \( E - I \)):

\[
E(V(p, I)_2) = p((R(p) - 1)I + E)[\bar{x}]
\]

(14)

The bank simultaneously chooses optimal values \( p = p^n \) and \( I = I^n \). We can analyze the decision process by first letting the bank choose \( p^n \), assuming \( I \) is already at its optimum \( I^n \); subsequently, the bank chooses \( I^n \) taking \( p^n \) as given. The choice of \( p^n \) is given by the following first order condition (FOC), replacing \( M \) by \( 1 - I \):

\[
R(p^n) + p^n R'(p^n) = 1 - \frac{E}{I^n}
\]

(15)

which holds since \( I < 1 \): if \( I = 1 \), \( \bar{x} = 0 \) and the bank would always fail. The bank would thus choose \( I^n < 1 \) to receive a positive payoff at \( t = 2 \). The optimal \( p^n \) also satisfies the second order condition (SOC) for a maximum:

\[
\frac{\partial^2 V(p^n, I^n)_2}{\partial p^n \partial I^n} = \frac{1 - I^n}{D} I^n [2R'(p^n) + p^n R''(p^n)] < 0
\]

(16)

Next, taking \( p^n \) as given, we can analyze the bank’s choice of \( I^n \). The following FOC holds:

\[
I^n = \frac{1}{2} \left[ 1 - \frac{E}{R(p^n) - 1} \right]
\]

(17)

where we have used \( \frac{\partial p}{\partial I} = -\frac{1}{D} \). This FOC also fulfills the SOC:

\[
\frac{\partial^2 V(p^n, I^n)_2}{\partial I^n \partial I^n} = p^n \left\{ 2(R(p^n) - 1)(-\frac{1}{D}) \right\} < 0
\]

(18)

where we have used \( \frac{\partial^2 p}{\partial I \partial I} = 0 \). We can deduce from equations (15) and (17) that the bank takes more risk than is desirable from a social perspective. This follows from our assumption that the bank invests with leverage (i.e. \( D > M > 0 \)), which means \( I^n > E \) and thus \( R(p^n) + p^n R'(p^n) > 0 \). As \( \frac{E}{x} < 1 \) and \( R(p^n) + p^n R'(p^n) \) is decreasing in \( p \), we see that \( p^n < p^{fb} \).

Furthermore, we can state that \( I^n < I^{fb} \), which follows from assuming that \( E > 0 \) and \( R(p^n) > 1 \) (otherwise it would not be profitable to invest in the risky asset):

\[
I^n - I^{fb} = \frac{1}{2} \left[ 1 - \frac{E}{R(p^n) - 1} \right] - 1 < 0
\]

(19)

The bank owner thus generates too little productive (but risky) investment compared to the first
best case, and takes too much risk while doing so. The investment decision follows from the assumption that there is no safety net in the form of a central bank able to provide emergency liquidity; the bank has to reserve part of its funds to cope with liquidity shocks. As it has to keep more liquidity on its balance sheet, the bank tries to make up for the foregone investment returns by taking more risk. This means the bank owner "gambles" for a higher return in the case of success, which is harmful to social welfare.

4.3 Introducing a Lender of Last Resort

It may be possible to improve the situation, by setting up a central bank (CB) that can provide temporary liquidity to an illiquid bank. The bank owner then chooses risk-taking and the amount of investment in this new situation by setting \( p \) and \( I \), with equilibrium values \( p^l \) and \( I^l \) (where \( l \) denotes that we are dealing with the possibility of liquidity provision). As in Repullo (2005) and Kahn and Santos (2005), bank and CB play a simultaneous Nash game of incomplete information in the determination of \( p \) and \( \pi \). In this game, the CB can only observe the choice of \( I \) when it has to make a liquidity provision decision at \( t = 1 \); this observation of \( I \) is not verifiable. The CB does not know the choice of \( p \) at this moment. However, the CB can form a belief about \( p^l \) through its knowledge of \( I \) and \( x \). Expressing \( M \) as \( 1 - I \), the threshold can be written as follows:

\[
\pi = \frac{p^l}{1 - p^l} \frac{\alpha C}{D} + \frac{1 - I}{D} \tag{20}
\]

with equilibrium value \( \pi^l = \pi(p^l, I^l) \). This threshold shows that the CB only faces downside risk; the bank gets the upside. We can also see that the threshold depends only on the bank’s actual choice of \( I \); it doesn’t change directly with the actual choice of \( p \). Instead, it is determined by \( p^l \), the equilibrium value of \( p \).

Furthermore, if \( x > \pi \) the bank finds itself in a crisis situation and it will be taken over completely by the fiscal authority. The depositors will be compensated by the DIF, and the remaining parts of the bank will be sold by the FA at \( t = 2 \): \( \gamma = 1^9 \). The bank owner will thus get a zero return in this case; we will relax this assumption in the next section.

At \( t = 0 \), the bank will take all this into account while choosing \( p \) and \( I \). It maximizes the new objective function

\[
E(V(p, I)_{2}) = p[(R(p) - 1)I + E][\pi^l] \tag{21}
\]

taking into account the equilibrium decision by the CB. The corresponding FOC w.r.t. \( p \) and \( I \)

\footnote{Note that this is equivalent to taking over, recapitalizing and selling the bank.}
are:

\[ R(p^l) + p^l R'(p^l) = 1 - \frac{E}{I^l} \]  

(22)

\[ I^l = \frac{1}{2} \left[ \frac{p^l}{1-p} \alpha C + 1 - \frac{E}{R(p^l) - 1} \right] \]  

(23)

where we can see that \( p^l \) and \( I^l \) are determined in a similar way as \( p^n \) and \( I^n \).

However, we also see that \( I^n \neq I^l \) when \( \alpha > 0 \) and \( C > 0 \), which means that \( p^n \neq p \). To determine the relative size of \( I^n \) and \( I^l \), we note that a decrease in \( \alpha \) means that the Central Bank cares very little about bank failure. This leads to very little liquidity injections: the equilibrium threshold \( p^l \) will fall towards \( x \). Analogously, when \( \alpha = 0 \) the CB will never intervene as it will not incur any political cost from failure. This is equivalent to the earlier situation without a Lender of Last Resort. It is thus straightforward to perform comparative statics regarding \( \alpha \) by taking the derivative of \( I^l \) w.r.t. \( \alpha \):

\[ \frac{dI^l}{d\alpha} = \frac{C}{2} \frac{p^l}{1-p} > 0 \]  

(24)

This expression indicates that \( I^l \) decreases when \( \alpha \) decreases: \( I^l \rightarrow I^n \) when \( \alpha \rightarrow 0 \). This means that \( I^l > I^n \), or that an introduction of a Lender of Last Resort leads to an increase in productive investment.

However, regulation is also established to mitigate risk taking. Let us therefore analyze whether the riskiness of the bank has improved, by comparing \( p^l \) with \( p^n \). To this end, we can totally differentiate equation (22) and perform comparative statics:

\[ \frac{dp^l}{dI^l} = \frac{E}{2R(p^l) + p^l R''(p^l)} < 0 \]  

(25)

where the inequality holds because of the assumptions on \( R(p) \). As we have found that \( I^l > I^n \), we must also conclude that \( p^l < p^n \) because of equation (25).

The bank, in equilibrium, thus invests more in risky assets than in the situation without a liquidity provider, which is a positive development. However, it also takes more risks when doing so, which is worse from a social point of view. This may reflect a moral hazard effect caused by the introduction of a safety net: since there is a Lender of Last Resort that comes to the rescue when an intermediate liquidity shock occurs, the bank takes more risk.

To illustrate this phenomenon, we have calibrated our model using reasonable parameter values.
We have specified the returns as a concave decreasing function of $p$, namely $R(p) = 3 - 2p^2$ (satisfying the assumptions from section 3.2), and the cost of bankruptcy is set to 0.10 or 10% of the bank’s balance sheet (Repullo, 2005). $\alpha$ is set to 1 (Cordella and Yeyati, 2003) and the capital ratio $E$ is assumed to be at the minimum Basel II requirement, which is 8% of risk weighted assets. We assume that the risky asset $I$ gets a 100% weight.

Figure 1 shows that investment and the solvency threshold are indeed negatively related, as an increase in investment means a decrease in liquidity buffers. We also see that the probability of success and the solvency threshold are positively related. This means that an increase in investment should be met with an increase in its success probability to keep the threshold at the same level. The bank will thus face a trade-off between investment and risk-taking if it wants to induce the CB to set the optimal solvency threshold. In the end, this leads to a threshold $x_l > x_c$ with $I_l > I_n$ but $p_l < p_n$: there is more productive investment, but this goes with increased risk taking.

In this section, we have abstracted from penalty rates for emergency liquidity. Although this is one of Bagehot’s key arguments, there are some issues with this view. One problem is that it may be rather difficult for a CB to commit to these penalty rates, especially in times of crisis. This is exemplified in the recent financial crisis, where the ECB, Fed and other central banks have provided liquidity far below market rates. Another problem is that banks may internalize this penalty rate by taking higher risk ex ante; the higher expected return in the good state will then compensate for higher expected LLR payments in the bad state (Repullo, 2005).

Furthermore, the concept of "creative ambiguity" has been introduced (by i.e. Freixas (1999)) as a possible solution to mitigate moral hazard. However, this concept also suffers from commitment problems, as Freixas (1999) already noted himself: it may cause a Too-Big-to-Fail problem. Cordella and Yeyati (2003) have also shown that committing explicitly has a possible "[charter] value" effect that may outweigh the moral hazard effect. Acharya and Yorulmazer (2007, 2008) find similar results for systemically important banks.

We thus abstract from penalty rates and the "creative ambiguity" principle, and instead focus on a situation in which the regulator will bail out the bank by injecting capital (as the bank is a systemic one). At the same time, the regulator can determine what cost will be attached to this assistance. We will analyze this situation in the next section.

### 4.4 The possibility of bailout

After analyzing the case where a bank goes simply bankrupt when a crisis occurs ($x > x$), we will now introduce the possibility of the FA injecting capital into the bank to possibly mitigate excessive risk taking. When doing so, the FA will stipulate its required share in the equity returns
at \( t = 2 \). This is denoted by \( \gamma \). While in the previous section this \( \gamma \) was set equal to 1, it is now possible for the FA to let the bank keep a share of its profits when a crisis occurs. However, the FA can discipline the bank for taking too much risk by setting a high \( \gamma \).

It is assumed that the fiscal authority gets supervisory information from the central bank. Therefore, the bank and the FA, just as the bank and the CB, play a simultaneous Nash game with incomplete information. We will assume additionally that the CB and the FA observe each other’s actions, but take them for granted; there is no interaction between the CB and the FA.

The bank again chooses risk-taking and the amount of investment in this new situation by setting \( p \) and \( I \), with equilibrium values \( p^c \) and \( I^c \). The \( c \) indicates that we have added the possibility of capital provision. Furthermore, the fiscal authority chooses the repayment fraction such that it breaks even in expectation. This means that it chooses \( \gamma = \gamma^c \), with equilibrium value \( \gamma^c \). The \( \gamma \) is determined by the following equation, where we can see it depends on the bank’s actual choice of \( I \), but only on its equilibrium choice of \( p \), which is \( p^c \):

\[
\gamma \equiv \frac{xD - 1 + I^c - p^c \beta C}{p^c([R(p^c) - 1]I + E)}
\]  

For the bank, this \( \gamma \) will be a function of the expectation of \( x \), conditional on \( x > \pi \): \( E(x) = \frac{1}{2}(\pi + 1) \), where \( \pi \) is determined as in section 4.3. We thus find an expected minimum repayment fraction of:

\[
E(\gamma) = \frac{1}{2}(\pi + 1)D - 1 + I^c - p^c \beta C}{p^c([R(p^c) - 1]I + E)}
\]  

The bank’s objective function in the case of bailout possibility is thus as follows:

\[
\max_{p,I} E(V(p, I)) = p([R(p) - 1]I + E)[1 - E(\gamma)[1 - \pi]]
\]  

which is optimized according to the following FOCs:

\[
R(p^c) + p^c R'(p^c) = 1 - \frac{E}{I^c}
\]  

\[
p^c \left\{ [R(p^c) - 1][1 - \gamma^c(1 - \pi^c)] - [R(p^c) - 1]I^c + E \left[ \frac{\partial \gamma^c}{\partial I^c} (1 - \pi^c) + \gamma^c \left( \frac{1}{I^c} \right) \right] \right\} = 0
\]

where \( \pi^c = \pi(p^c, I^c) \) and \( \gamma^c = \gamma(p^c, I^c) \). It is not straightforward to write an explicit solution for both \( p^c \) and \( I^c \) from these conditions. However, we can see that the FOC for \( p^c \) is similar to that of \( p^l \); the only difference is that \( I^c \) may differ from \( I^l \).

To gauge the effect of having the possibility of bailout on \( I \) and \( p \), let us again perform

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comparative statics. Since the introduction of a bailout possibility means that $\gamma < 1$ (as opposed to $\gamma = 1$), our analysis should focus the effect of this change. As in section 4.3, we perform comparative statics by totally differentiating equations (30) and (29):

$$\frac{dI}{d\gamma} = \frac{p^*_r[(R(p^*_f) - 1)I^*_1 + E]}{\frac{1}{2} - \gamma^c p^*_r(R(p^*_f) - 1)}$$  \hspace{1cm} (31)

$$\frac{dp^*_r}{dI^c} = \frac{-[R(p^c) + p^c R'(p^c) - 1]}{I^c [2R'(p^*_f) + p^c R''(p^c)]} < 0$$ \hspace{1cm} (32)

Inequality (32) holds because of equation (29), $I \geq 0$ and the assumptions on $R(p)$. However, the sign of equation (31) is not unambiguous; it depends on the sign of the denominator (since the numerator, representing a part of expected bank value, is positive). We see that this sign depends on the relative size of $p^*_r(R(p^*_f) - 1)$ and the equilibrium $\gamma^c$. To assess the effect of a possible bailout, we thus need to consider two situations.

First, when $\gamma^c$ is relatively large, $p^*_r(R(p^*_f) - 1)\gamma^c > \frac{1}{2}$ and we find a negative effect of an increase in $\gamma$ on investment. The $\gamma^c$ is large when $\beta$ is small, meaning that the regulator cares little about bankruptcy. This leads to a strict FA, which will to discipline banks fiercely when in a crisis by making capital assistance very costly.

The bank owner takes into account that he will thus lose a large share in period 2 profits. An increase in this expected repayment will induce him to invest less in risky assets and keep more reserves to fend off liquidity shocks. As $\gamma \to 1$ and we move towards the case without an FA, we thus see that investment decreases. This means that the introduction of an FA with bailout capabilities (and $\gamma < 1$) can stimulate productive investment: $I^c > I^l$.

There is also a downside to having a regulator that can provide bailout assistance. Because of equation (32) we also see that the banker takes more risk when investment increases: the $p^c$ decreases with $I$. This is the negative effect of the introduction of a strict FA. It is similar to the moral hazard effect that may ensue when penalty rates on liquidity are introduced: the banker will compensate the higher expected repayment with higher risk taking, to increase the return when investment is successful.

However, the effect is reversed when $\gamma^c$ is relatively small $(p^*_r(R(p^*_f) - 1)\gamma^c < \frac{1}{2})$. In this case, $\frac{dI}{d\gamma} > 0$ which means that investment decreases when an FA is introduced (decreasing $\gamma$ to $\gamma^c \ll 1$). A reason for this small $\gamma^c$ may be a large $\beta$, which means that the FA’s political cost of a bank failure is large. This will thus lead to a small required repayment, which ex ante provides the banker with a relatively high expected return.

Counterintuitively, a small $\gamma^c$ leads to a lower level of investment: the bank’s optimal payoff is reached at $I^c < I^l$. Investing more than $I^c$ in the risky asset is considered as "gambling" by
the bank. Similarly, we can see that the decrease in investment leads to a positive effect on risk taking (since $\frac{dp}{d\gamma} < 0$). As $\gamma^c$ is quite small, investments have a higher expected return. Less risk taking is thus necessary to achieve the optimal bank value at $t = 2$.  

Table 1: The effect of having an FA on $I$ and $p$

<table>
<thead>
<tr>
<th></th>
<th>Low $\beta$</th>
<th>High $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Success Probability</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4.4 summarizes the different situations described above. An FA that cares little about bankruptcy (low $\beta$) demands a large repayment for banks in a crisis, leading to an increase in investment, but also an increase in risk taking. On the other hand, an FA that cares a lot about bankruptcy (high $\beta$) demands a small repayment from crisis banks, thereby decreasing productive investment but also mitigating risk taking.

Figure 2 shows the case of an FA that is much concerned about bankruptcy. This is probably the most realistic case, especially since we consider the bank to be systemic. In this situation $\beta$ is relatively high; in our set of parameter assumptions we have set $\beta = 2$. This means that the weight the FA attaches to bankruptcy is twice as large as that of the CB$^{10}$. It also means that $\gamma^c$ is relatively low$^{11}$. We can clearly see that keeping $\gamma^c$ low can lead to a low $I$, but a higher $p$ and thus less risk taking. It is especially interesting to see that an increase (as well as a decrease) in risk taking is met with a punishment for the banker by increasing $\gamma^c$.

We can conclude that a high $\beta$ can mitigate risk taking, but also causes a decrease in investment; a low $\beta$ induces more risk taking, but has a positive effect on investment. Regulatory authorities thus face a trade-off when establishing regulation in the form of a safety net. They have to decide whether they attach more value to an increase in investment, or to a decrease in risk taking. This seems to be realistic: the current nationalization, bailout and guarantee efforts by government have led banks to mitigate their risk taking, while at the same time they have cut back on (risky) lending to entrepreneurs.

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$^{10}$The exact size of this number is not very important; with $\beta = 1$ we get a similar result, but it is much more pronounced for $\beta = 2$.

$^{11}$Only at the top left we see the situation where $dI/d\gamma < 0$, caused by a very high $\gamma^c$. Since we only consider cases with too low $p$ and $p^{1/\nu} = 0.71$ with our parameter values, this extreme range is not relevant.
5 Conclusion

The ongoing financial crisis has provoked governments and central banks to supply unusually large amounts of capital and liquidity to banks. Regard for systemic stability is the main motivation with which this support to the financial system has been provided. However, the risk for financial stability (ultimately leading to the financial crisis) has arisen because of excessive risk taking by individual institutions that were central to the system. Since they thus posed a risk for the financial system as a whole, regulators had no choice but to prevent them from failing.

Because of the enormous costs that are associated with financial system failure, but also with its prevention, it is necessary to thoroughly assess the management of crises by regulatory authorities. In our analytical model, we have thus simultaneously allowed for liquidity provision (by a central bank) and capital assistance (by a fiscal authority) to examine how they interact with a bank facing a crisis.

We have assessed this interaction for a systemic bank suffering from liquidity shocks, with which it can only cope by keeping liquid reserves. There is no interbank market in our model, reflecting a crisis situation in which the interbank market does not function well. We find that being in this situation without any regulation leads a bank to hoard too much liquid assets and take too much risk, compared to the first best situation.

The introduction of a liquidity provider in the form of a central bank (CB) should alleviate this problem. This CB has no information other than the bank’s investment level. It cannot observe the bank’s choice of risk ex ante and can thus not condition its Lender of Last Resort (LLR) policy upon this information. We find that this measure indeed induces a higher investment level. However, the introduction of a safety net also increases moral hazard as found by Freixas (1999).

To improve the situation, we set up a second regulator in the form of a fiscal authority (FA) that is responsible for the bank closure decision. However, it can also decide to give the bank a capital injection if it deems the bank solvent. This FA has the same information as the CB. We find that this set-up leads to a trade-off between mitigating risk and promoting investment. When the FA is mild in its bailout conditions (demanding a low repayment) it can, counterintuitively, reduce moral hazard at the expense of investment. A strict FA achieves the opposite result: the investment level is higher, but there is an increase in moral hazard.

We must conclude that an additional regulatory authority with responsibility for solvency is not a completely satisfactory solution for curbing excessive risk taking. This result is in line with the current situation: although banks take less risk, they provide less credit to the economy partly due to the terms of their rescue packages. Furthermore, relative effects of CB and government policies are also likely to play a role: central banks continue to provide liquidity to stimulate lending, while banks are hoarding liquid reserves as the government induces them to reduce risk.
References


A Figures

Figure 1: The optimal solvency threshold $\pi$
Figure 2: The optimal required return $\gamma^c$