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Kapteyn, A.J.; Kooreman, P.; van Soest, A.H.O.

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Arie Kapteyn,
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Stockholm School of Economics

Research Coordinators
Eric van Damme
Frank van der Duyn Schouten
Arie Kapteyn
Theo van de Klundert

Address : Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
Phone : +31 13 663050
Telex : 52426 kub nl
Telefax : +31 13 663066
E-mail : center@htikub5.bitnet

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QUANTITY RATIONING AND CONCAVITY IN A FLEXIBLE HOUSEHOLD LABOR SUPPLY MODEL

Arie Kapteyn, Peter Kooreman, and Arthur van Soest*

Abstract—In the first part of this paper we derive explicit expressions for the direct utility function, conditional demand equations, and concavity conditions in both price/income and quantity space for the demand system introduced by Hausman and Ruud (1984). These results are then used in an empirical static family labor supply model, in which kinked budget constraints and unemployment benefits are taken into account for both spouses. Imposition of concavity is necessary for consistent estimation and the concavity constraint appears to be binding.

I. Introduction

The larger part of the recent labor supply literature is devoted to the explanation of female labor supply decisions, thereby addressing the theoretical and econometric problems associated with non-participation, non-linear and non-convex budget sets and stochastic specification (see, for example, Heckman (1974), Hausman (1979, 1980, 1985), Moffitt (1986), Arrufat and Zabalza (1986), Blundell and Meghir (1986) and Blundell, Ham and Meghir (1987)). In these papers, male labor supply decisions usually play a role only through the (by assumption exogenous) explanatory variable "other household income," which includes male labor earnings.

In this paper we adopt the more general approach of modelling male and female labor supply simultaneously. First of all, there is some evidence that the exogeneity assumption of "other household income" in female labor supply models is not always tenable; see Smith and Blundell (1986). More importantly, male and female labor supply decisions within a household are likely to be fundamentally interrelated and a full understanding of a household's labor supply behavior requires taking this interrelationship into account in setting up the empirical model.

The joint modelling of male and female labor supply creates some specific problems in addition to those encountered in modelling individual labor supply. One of the issues is how to represent the household members’ preferences. We will follow the usual approach of assuming that preferences can be represented by a joint household utility function with male leisure, female leisure and total household consumption as arguments.

A second issue that comes up specifically in modelling joint male and female labor supply is that one usually also has to derive conditional supply equations, i.e., equations that give optimal labor supply of a household member, given a fixed number of hours of labor supply by the partner. For example, if the female stops working, the functional form of the male labor supply equation changes from its notional to its conditional form (assuming absence of other quantity constraints). For popular flexible functional forms (in the sense of Diewert (1974)), such as the Almost Ideal Demand Systems and the Indirect Translog, the derivation of conditional commodity demand or labor supply equations is a cumbersome affair, and closed forms can generally not be obtained. See, for example Kooreman and Kapteyn (1986).

It appears that at this moment there exist only two flexible forms suited to deal with conditional equations and unconditional equations in a relatively tractable way. The first one is the direct quadratic utility function, which was used for this kind of problem by Wales and Woodland (1983) and by Ransom (1987a, b). A second one has been introduced by Hausman and Ruud (1984). Since the properties of the Hausman-Ruud system have not been discussed in the literature extensively, we provide a rather elaborate analysis of the system, including the derivation of the conditional labor supply equations, the computation of direct utility, and the imposition of concavity in wages of the expenditure function. The need to compute direct utility in an arbitrary point of the choice set may arise if the budget set is non-convex, in which case different local utility maxima on convex subsets of the budget set have to be compared. Imposition of concavity is sometimes necessary in empirical applications, as the likelihood function of the model may not be well-defined if concavity is not satisfied.

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*Tilburg University.

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The practical importance of these issues will be illustrated in section IV by an empirical example. In section V we make a brief comparison between the direct quadratic and the Hausman-Ruud system. There we also discuss the importance of modeling the labor supply of spouses jointly.

II. The Hausman-Ruud System

A. The Model

A household is assumed to maximize a utility function with male leisure, female leisure and total household consumption as its arguments. We assume that the expenditure function in real terms corresponding to maximization of the utility function under a linear full income constraint is of the Gorman Polar Form type introduced by Hausman and Ruud (1984):

\[ c(w, u) = u \exp(-\beta'w) - \left\{ \theta + \delta'w + \frac{1}{2}w'Aw \right\}, \]

where \( w = (w_m, w_f)' \), the husband's and wife's (real) after tax wage rate; \( u \) is household utility level; and \( A = \begin{pmatrix} \gamma_m & \alpha \\ \alpha & \gamma_f \end{pmatrix}, \beta = \begin{pmatrix} \beta_m \\ \beta_f \end{pmatrix}, \delta = \begin{pmatrix} \delta_m \\ \delta_f \end{pmatrix}, \) and \( \theta \) are parameters.

The corresponding indirect utility function is given by

\[ v(w, \mu) = \mu^* \exp(\beta'w), \]

\[ \mu^* = \theta + \mu + \delta'w + \frac{1}{2}w'Aw, \] (1)

where \( \mu \) denotes the household's real non-labor income.

Application of Roy's identity yields the following labor supply functions:

\[ h^* = \delta + \mu^* \beta + Aw, \] (2)

where \( h^* = (h_m^*, h_f^*)' \) is the vector of optimal numbers of working hours of husband and wife, respectively.

B. Concavity

The use of the function given by (1) is limited by the usual regularity conditions on expenditure functions. For this specification, only concavity has to be considered, i.e., the matrix of second order partial derivatives of the real expenditure function must be negative definite. Homogeneity of degree zero and monotonicity with respect to \( u \) are satisfied automatically. It is easy to show that concavity is equivalent to

\[ B = \mu^*\beta\beta' - A \text{ is negative definite.} \] (3)

From now on we assume that \( A \) is non-singular. Note that, if \( \beta'A^{-1}\beta \neq 0 \), a necessary condition for concavity is given by

\[ \mu^* < \left( \beta'A^{-1}\beta \right)^{-1}. \] (3')

If \( \beta \neq 0 \) and \( \beta'A^{-1}\beta = 0 \), then \( B \) is negative definite for no value of \( \mu^* \). This case is excluded from now on. In the special case that \( A \) is positive definite, it is easy to prove that (3') is not only necessary but also sufficient for (3). (See, for a proof of a more general result, Bekker (1986).)

The application of duality theory strongly hinges on the concavity condition; without this property, there is no utility maximizing problem behind the labor supply equations. Therefore, (3) must hold for all relevant \((w, \mu)\), including shadow wages and corresponding virtual incomes.

C. The Direct Utility Function

Non-convexity of the budget set makes it necessary to compare the values of the direct utility function in different points. We shall derive the direct utility function by calculating the utility level in some arbitrary point \((h_m, h_f, y)\), where \( y \) is the household's real income:

\[ y = \mu + w_m h_m + w_f h_f. \] (4)

Let \( k \) be the vector \( h - \delta \), where \( h = (h_m, h_f)' \). Given \((h_m, h_f, y)\), we first seek (shadow-)wages \( w \) and corresponding non-labor income \( \mu \) satisfying

\[ k = \mu^* \beta + Aw \] (5)

\[ \mu^* = \mu + \theta + \delta'w + \frac{1}{2}w'Aw \] (6)

\[ \mu = y - h'w. \] (7)

Equations (5) through (7) yield, after substituting (7) into (6):

\[ w - A^{-1}k = -\mu^* A^{-1}\beta \] (8)

\[ \mu^* = \frac{1}{2} (w - A^{-1}k)'A(w - A^{-1}k) - \frac{1}{2}k'A^{-1}k + y + \theta. \] (9)

Substituting (8) into (9) yields a quadratic equa-
tion in $\mu^*$:
\[ \frac{1}{2} \mu^* \beta' A^{-1} \beta - \mu^* - \frac{1}{2} k' A^{-1} k + y + \theta = 0 \]  
(10)
and if $\mu^*$ is known, $w$ can be found from (8):
\[ w = A^{-1}(k - \mu^* \beta). \]  
(11)
Thus $(w, \mu)$ can be determined iff (10) has a real solution, i.e., iff
\[ 1 + \beta' A^{-1} \beta \left( k' A^{-1} k - 2(y + \theta) \right) \geq 0. \]  
(12)
A solution is only feasible if it satisfies concavity condition (3). Obviously, if $\beta = 0$, the solution of (10) and (11) is unique and it satisfies (3) if and only if $A$ is positive definite. If $\beta \neq 0$ and (12) holds, then (10) and (11) yield (at most) two solutions $(w, \mu^*)$ and only the smallest of the two satisfies the necessary condition (3'):
\[ \mu^* = \left( \beta' A^{-1} \beta \right)^{-1} - \left( \beta' A^{-1} \beta \right)^{-1} \left( k' A^{-1} k - 2(y + \theta) \right) \]  
(13a)
\[ w = A^{-1}(k - \mu^* \beta). \]  
(13b)
If this solution satisfies (3), then it is feasible and the utility level follows from (1):
\[ U(h_m, h_f, y) = \mu^* \exp(\beta' w). \]  
(14)
The reader should be aware of the relation between invertibility (i.e., the question whether $(w_m, w_f, \mu)$ can be solved as a function of $(h_m, h_f, y)$) and concavity (i.e., good behavior of the direct or indirect utility function). As usual in dually specified systems, concavity can only be checked in $(h_m, h_f, y)$-space if invertibility is guaranteed, since it involves (shadow-)wages. In the special case that $A$ is positive definite, (3) and (3') are equivalent. This implies that, if $(w_m, w_f, \mu)$ can be found, exactly one solution satisfies the concavity conditions. Thus, in this special case, "invertibility guarantees concavity."

D. Rationed Labor Supply

In this subsection, we derive rationed labor supply functions, i.e., labor supply for one individual if—for some reason—the partner's number of working hours is fixed. This means that the household maximizes utility, taking into account some binding constraint on one of the three goods. Rationed supply functions can be determined using shadow-wages and shadow-income (see Neary and Roberts (1980)). We derive the female's rationed labor supply $h_f$, for given $h_m$, actual wage rates $w_m$ and $w_f$, and non-labor income $\mu$. (The male's rationed labor supply can be derived in the same way.) We search for a shadow wage rate $\bar{w}_m$ and corresponding $\bar{\mu}$, such that
\[ h_m = \beta_m \mu^* + \gamma_m \bar{w}_m + \alpha w_f + \delta_m \]  
(15a)
\[ \bar{\mu} + h_m \bar{w}_m = \mu + h_m w_m \]  
(15b)
\[ \mu^* = \bar{\mu} + \theta + \delta_f w_f + \delta_m \bar{w}_m \]  
(15c)
\[ + \frac{1}{2} \left( \gamma_f w_f^2 + \gamma_m \bar{w}_m^2 \right) + \alpha w_f \bar{w}_m. \]  
(15d)
If a feasible solution $(\bar{w}_m, \bar{\mu})$ (with corresponding $\mu^*$) is found, optimal female labor supply is given by
\[ h_f = \beta_f \mu^* + \gamma_f w_f + \alpha \bar{w}_m + \delta_f. \]  
(16)
System (15) implies
\[ a_2 \bar{w}_m + a_1 \bar{w}_m + a_0 = 0 \]  
(17)
with
\[ a_0 = -h_m + \beta_m (\mu + \theta + h_m w_m + \delta_m w_f + \frac{1}{2} \gamma_m \bar{w}_m^2) + \alpha w_f \bar{w}_m, \]  
(18)
\[ a_1 = \gamma_m + \beta_m (h_m - \delta_m + \alpha w_f), \]  
(19)
\[ a_2 = \frac{1}{2} \beta_m \gamma_m. \]  
(20)
If (18) holds, the solutions for $\bar{w}_m$ are
\[ \bar{w}_m = -\beta_m^{-1} + \left( h_m - \delta_m - \alpha w_f \right) / \gamma_m \]  
(21)
\[ \pm (\beta_m \gamma_m)^{-1} \sqrt{D}. \]  
(22)
The corresponding value of $\mu^*$ is
\[ \mu^* = \beta_m^{-2} \gamma_m \mp \beta_m^{-2} \sqrt{D}. \]  
(23)
Since $\beta - \gamma_m \beta' - A$ is indefinite or semi-definite and $\beta^2$ is positive semi-definite, it is easy to see

2 Rationed supply functions can alternatively be determined using first order conditions for maximization of the direct utility function, which is explicitly derived in section IIC, subject to the budget constraint and the rationing levels.
that only one solution can be feasible:
\[
\bar{w}_m = -\beta_m^{-1} + \frac{(h_m - \delta_m - \alpha w_f)}{\gamma_m} + \frac{(\beta_m \gamma_m)^{-1}}{\sqrt{D}}.
\]
(20)

Note that, even in the special case of a positive definite matrix \( A \), this solution is not necessarily feasible; condition (3) should always be checked. Thus, the relation between “partial invertibility” and concavity is different from the relation between “full invertibility” and concavity, as discussed in Section IIC.

In this section we derived the conditional female labor supply function \( h_f(w_f, h_m, \mu + w_m h_m) \) corresponding to household preferences given by (14). The result is a closed form expression for \( h_f \). Lundberg (1988) follows a different strategy: She starts with conditional demand functions in some convenient form and does not discuss the issue whether it is possible to find a household utility function corresponding to these equations. Our approach has the advantage that a closed form expression of the indirect utility function is available. As a consequence, it is easy to check whether the underlying system of preferences satisfies regularity properties (e.g., concavity) and it is possible to use non-convex budget sets.

III. Applications

The rationed labor supply functions derived in section IID can be applied in several situations. The most common example is the nonnegativity constraint for females. If this restriction is binding, the husband's labor supply function should be replaced by a rationed labor supply function, as described in section IID.

A similar situation arises if individual budget sets are piecewise linear and convex (see, e.g., Blomquist (1983) and Hausman (1979)), as in the case where spouses file separately and the tax system is progressive and piecewise linear. If, for example, the optimal number of the husband's working hours is at a kink, then female labor supply is not given by (2) but by the conditional labor supply function given in section IID. If the budget set is non-convex, comparison of values of the direct utility function, derived in section IIC, is necessary to determine the optimum.

Apart from constraints arising from the shape of the budget set, restrictions may stem from demand side factors or institutional constraints on the labor market. Particularly in The Netherlands, actual hours are not only determined by labor supply decisions of the household, but also strongly depend on institutional constraints and demand side factors. Possibilities to work a non-standard number of hours are rare. It therefore seems unrealistic to treat actual hours as if they were chosen freely by the members of the family. This is one of the reasons why several recent Dutch labor market surveys do not only contain information on actual hours worked, but also on preferred hours, i.e., the number of hours someone would like to work at a given wage rate.3 Preferred hours are provided by respondents in a ceteris paribus context, i.e., it is assumed that the partner does not change his or her actual number of working hours. This way of questioning implies that preferred hours in the data set are to be interpreted as optimal hours, conditional on the fact that the actual number of hours worked by the partner is fixed. Thus, a conditional labor supply equation is needed to explain preferred hours.

Some further explanation may be useful at this point. Of course, preferred hours are not very interesting by themselves from an economist's point of view; it is actual hours that we want to study eventually. But, due to institutional constraints and demand side factors, preferred hours appear to be a better reflection of the household's preferences than actual hours. Thus, certainly in The Netherlands, preferred hours should be used to reveal preferences. In a later stage, information on family preferences can be used in a labor market model, in which actual hours are linked to preferences as well as institutional constraints and demand side factors.

IV. An Empirical Example

In this section, we present an application of the model studied in section II. A similar model, estimated for a different data set, can be found in Kapeyn and Woittiez (1988). In that paper, some of the results derived here have been used. For the rest, the Kapteyn and Woittiez paper concentrates

3 A typical wording of the survey question asking for preferred hours is: "How many hours a week would you like to work if you could choose freely and if your average hourly wage rate remains as it is now? Assume that other family members do not change their number of working hours."
on different issues, particularly habit formation and preference interdependence. In our model preferred hours of husband and wife are the endogenous variables, for reasons discussed in section III.

A. Specification of the Model

Since each individual provides his or her preferred number of working hours taking the partner's actual labor supply as given, only conditional labor supply functions are relevant. Thus, from the individual's point of view the household budget set is only two-dimensional. In the absence of non-convexities and with a piece-wise linear budget constraint, the optimal number of hours for each spouse, given the number of hours worked by the other spouse, can be found by computing conditional labor supply for each of the linear segments, as described by, for example, Hausman (1979). If a spouse is unemployed and receives a benefit, the budget set is non-convex and the optimum is found by explicit utility comparison of various points.

Regarding the stochastic specification, it is important to distinguish between different sources of random errors, i.e., measurement errors, optimization errors and random preferences. Preference variation across households in our model could be incorporated by allowing, e.g., the parameters $\delta_m$ and $\delta_f$ to depend upon household characteristics:

$$\delta_i = \sum_{j=1}^{K} x_{ij} \delta_{ij} + \epsilon_i \quad (i = m, f)$$

where $x_{ij}$ $(j = 1, \ldots, K)$ are observed characteristics (including a constant term) and $\epsilon_i$ is a random variable representing unobserved sources of preference variation. This corresponds to translating; see McElroy (1987). Random $\delta$'s however, lead to random shadow wages and a complicated likelihood function. Moreover, the lack of global convexity, as discussed in section IIB, implies that it is necessary to truncate the distribution of the $\epsilon$'s in some rather intricate way. It is easy to see that conditions like (3') or (12) imply that the $\epsilon$'s have to lie in a polyhedron and it is hard to find a tractable distribution which allows for such a kind of truncation. Although we do recognize the importance of a stochastic specification that allows for random preference variation, the ensuing complications make this an issue beyond the scope of this paper.

Our stochastic specification is "ad hoc" in the sense that it only allows for optimization (or measurement) errors. We add normally distributed error terms to the conditional labor supply functions. Thus, for a female not receiving an unemployment compensation, we have

$$h_f^p = \max \{0, h_f + \epsilon_f\}$$

where $h_f^p$ is the observed preferred number of working hours and $h_f^p$ is the optimal choice given the budget constraint.4

If a female does receive an unemployment compensation, we only know whether she is seriously looking for a job or not. The optimization error $\eta_f$ is incorporated as an error in the "regime choice":

$$v = u_1 - u_0 + \eta_f$$

where $u_1$ and $u_0$ are the utilities of working and not working, respectively. If $v > 0$, the female wants to work; if $v < 0$, she is not seriously looking for a job. Male preferred labor supply is treated in the same way.

The vector of error terms $(\epsilon_m, \epsilon_f, \eta_m, \eta_f)'$ is assumed to follow a normal distribution with mean zero and covariance matrix

$$
\begin{pmatrix}
\sigma_m^2 & \rho \sigma_m \sigma_f & 0 & 0 \\
\rho \sigma_m \sigma_f & \sigma_f^2 & \cdot & \cdot \\
0 & \cdot & \sigma_v^2 & \cdot \\
0 & \cdot & \cdot & \sigma_o^2
\end{pmatrix}
$$

An asterisk indicates that the variance does not appear in the likelihood function, so that it cannot be estimated. Because of the small number of people in the sample receiving an unemployment benefit, we impose $\text{cov}(\epsilon_m, \eta_f) = \text{cov}(\epsilon_f, \eta_m) = 0$, and $\text{var}(\eta_m) = \text{var}(\eta_f)$.

B. Data and Estimation Results

The data used stem from a labor mobility survey conducted in The Netherlands in 1982 by the Institute of Social Research of Tilburg University jointly with the Netherlands Central Bureau of Statistics. The data set has been used by various researchers in The Netherlands for studies on

4 For individuals who work less than 15 hours a week, it is only known whether preferred hours exceed actual hours or not. It is straightforward to take this into account, considering $h_f$ as a latent variable.
labor supply, labor mobility, and income distribution. The survey was held among a random sample of Dutch households with at least one member between 16 and 65 years of age. In each household, all members between 16 and 65 years have been interviewed. The information collected pertains to incomes, hours worked per week, desired working hours per week, search behavior, demographics, etc. Non-response is equal to 35.7%.

Comparison with population characteristics shows that the survey is fairly representative of the population from which it was drawn, although students and unemployed people appear to be somewhat underrepresented. Altogether the survey comprises 2677 persons in 1299 households. The analysis here is restricted to families with at least two adults. Also, self-employed, students, and disabled people are omitted from the sample. As a result, data on 520 households were used in the estimation.

For non-participating individuals before tax wage rates are predicted using a wage equation with log(age), log(age)-squared and education as predictors. For males and females separate wage equations were estimated, using Heckman's two-stage procedure (see Heckman (1979)). Apart from an intercept \( \delta_{10} \) (i = m, f), the variables on the right hand side of (21) are log(family size) (with coefficient \( d_{t} \)) and a dummy for the presence of children younger than six (coefficient \( d_{s} \)).

The model was estimated by means of maximum likelihood. To impose concavity of the cost function in wages in a relevant region of the \((h_{m}, h_{f}, y)\)-space, the parameter \( \theta \) has been restricted, i.e., an upper bound in terms of other parameters in the model has been set to \( \theta \), such that concavity is guaranteed in all data points; it turns out that this restriction is binding. It should be noted that testing of the restriction is impossible, since the likelihood is not well-defined under the alternative. We have discussed this more fully in Van Soest, Kooreman, and Kapteyn (1988).

Table 1 presents the parameter estimates. \( \beta_{m} \) ("the male non-labor income effect") is significantly negative and \( \gamma_{f} \) (representing the largest part of the female own wage effect) is significantly positive, whereas \( \beta_{f}, \gamma_{m}, \alpha \) and \( \gamma_{m} \) do not differ significantly from zero. \( \beta_{m} \) and \( \beta_{f} \) have the expected sign, indicating that leisure is a normal good. The variables concerning family composition play a significant role in the female hours equation but not in the male hours equation. A direct economic interpretation for the parameters other than \( \beta_{m} \) and \( \beta_{f} \) is hard to give. The economic meaning of the estimates is brought out more clearly by graphs and elasticities.

In figures 1a through 1d family labor supply functions are drawn for a family without children as a function of before-tax wage rates. In each case the remaining variables are set at their sample means. We distinguish between "short run" (the partner is rationed at a certain number of hours) and "long run" (the partner is not rationed) labor supply functions. In each figure two short-run labor supply functions are drawn: one for the case that actual hours worked by the partner equal the sample mean (\( h_{f} = 22.6 \) or \( h_{m} = 42.3 \)) and one for the case that the partner does not work. Figure 1a shows a backward bending male labor supply function implying that the negative income effect
HOUSEHOLD LABOR SUPPLY WITH RATIONING

Figure 1. Preferred Hours as a Function of Before Tax Hourly Wage Rates for a Couple Without Children

The optimal number of male working hours. In figure 2b, where the wife's indifference curves are drawn if the husband works $h_m = 42.2$ hours a week, the (own) wage impact is much larger (note the difference in scale between both figures).

V. Conclusions

Modelling household labor supply under different regimes (i.e., taking account of kinks and corners) requires the use of shadow prices if one wants to work with specifications that are given in dual form. Unfortunately, most of the known flexible forms have the undesirable property that shadow prices cannot be found in closed form, except for some special cases. The only known exceptions are the direct quadratic utility function and the Hausman-Ruud specification. Of course, knowing shadow prices at some point amounts to knowing the value of the direct utility function at that point. Indeed, the first thing accomplished in this paper is the derivation of the direct utility function corresponding to the Hausman-Ruud specification. Secondly, the application of rationing theory requires that the system considered satisfies the Slutsky conditions in all data points. Hence, we have imposed concavity conditions for all data points in the empirical example considered.

A drawback of the Hausman-Ruud specification might seem to be that it is difficult to allow for random preferences in a utility consistent way. At first sight the direct quadratic utility function does not suffer from such a problem. Ransom (1987b) presents a specification with random errors and provides conditions under which the ensuing model is coherent. The conditions are easy to impose and estimation of the model is relatively
straightforward. It turns out, however, that for certain values of the random preferences the bliss point of the direct quadratic utility function is inside the budget constraint, and in such a case the demand equations do not represent a utility maximum. We have shown elsewhere (Van Soest, Kooreman, and Kapteyn (1988)), that the restrictions on random preferences required to prevent this from happening are quite similar to the restrictions which have to be imposed on the random preferences in the Hausman-Ruud system to guarantee a well-behaved system. Therefore, there are no compelling a priori reasons to prefer one system or the other. Thus we have two reasonably tractable flexible systems available which can be used for the analysis of household labor supply in the presence of kinks and corners, and the choice between them in each case should be based on the data at hand.

REFERENCES


No. 5  Th. ten Raa and F. van der Ploeg, A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, vol. 6, no. 1, 1989, pp. 2 - 19.


No. 8  Th. van de Klundert and F. van der Ploeg, Wage rigidity and capital mobility in an optimizing model of a small open economy, *De Economist*, vol. 137, nr. 1, 1989, pp. 47 - 75.


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