A Precursor to Muth: Tinbergen’s 1932 Model of Rational Expectations

by

Hugo A. Keuzenkamp


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Phone  : +31 13 663050
Telex  : 52426 kub nl
Telefax: +31 13 663066
E-mail : "center@htikub5.bitnet"

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MODEL OF RATIONAL EXPECTATIONS*

Hugo A. Keuzenkamp

John Muth's famous 1961 article in *Econometrica* generated a landslide in economics, be it with some delay. Today, we speak of a Rational Expectations Revolution In Macroeconomics (see e.g. Begg, 1982). This development is quite remarkable. Firstly, because of the force with which this Revolution changed macroeconomics in the seventies and eighties. This needs no further discussion. Secondly, because the basic idea is so simple and natural that it is perhaps amazing that this Revolution didn't take place much earlier. Indeed, many economists wonder why rational expectations did not show up in macro-economics directly after John Muth's pathbreaking 1961 paper. These economists will be even more surprised to hear that thirty years earlier, in 1932, a model with explicitly rational expectations was published in a then leading journal. The model was provided by Tinbergen (1932), in an effort to analyse some problems of dynamic economics. In this paper I will describe Tinbergen's model and I will compare it with Muth's famous (1961) paper.

The notion of something familiar to rational expectations had already been around for quite a while. Keynes (1921, 1936) inspired one of his students, Hugh Townshend, to some thoughts about probabilistically based expectations (see Keuzenkamp, 1989). The results did not lead to an explicit model of rational expectations. Similarly, Austrian economists like Morgenstern discussed the idea of predictability in economics. Morgenstern (1928) strongly opposed this. He discussed what Popper (1957) later called the 'Oedipus Effect' of self-fulfilling prophecies.

Later, this attack was directed to the use of so called barometers for prediction (Morgenstern, 1937), or what Koopmans dubbed the 'measurement without theory' approach. Statisticians at various places tried to predict stock market prices using this technique, but also analysed their own (lack of) success (see e.g. Cowles, 1933). The methods of those statisticians did not have much to do with probability theory or regression: in most cases, statistics meant summarising data. Like Morgenstern, Tinbergen was unhappy with the barometer approach. Tinbergen tried to build an alternative by combining economic theory with empirical work. In contrast to many contemporaries, he did make use of modern techniques such as regression analysis.

* This paper elaborates on a paper on the early history of expectations analysis. (Keuzenkamp, 1989). Other contributions to expectations analysis in the inter war years are surveyed in that paper as well. I would like to thank D. Heathfield, R. E. Lucas, N. de Marchi, A. McDonald, F. van der Ploeg, K. Wallis and two anonymous referees for their suggestions and comments. The usual disclaimer applies.

1 In Magnus and Morgan (1987, p. 119-20), Tinbergen's explains that he was dissatisfied with the crude empirical methods underlying these barometers and so-called A-B-C curves, but also with the large distance between theoretical economics and empirical economics: 'my feeling was that there was something lacking in between'.
In the early thirties, Tinbergen was developing the first macro-econometric model. To understand fluctuations in the economy he was very much interested in the study of dynamics. As a physicist, he knew how to deal with difference equations, but he needed an economic interpretation of the lags appearing in these equations. ‘Time to build’ (the fact that investment in capital goods is not an instantaneous process) was one explanation for the appearance of lags, investigated in other studies he made (see e.g. Tinbergen, 1931). Expectations formation was another one and indeed the key element of the 1932 article. Due to his empirical work, Tinbergen also was familiar with statistical theory. He was aware of Morgenstern's critique on economic forecasting (viz. a review, in Dutch, of Morgenstern (1928) in De Economist, 1929), but was not much bothered by his critique, thanks to a rather pragmatic attitude. This set of rare skills led him to invent the first rational expectations model we know of. Tinbergen was the first and, for nearly thirty years the only one who made the link between dynamic economic theory, expectations and uncertainty, and probability theory. In the following I will discuss Tinbergen's work on expectations in more detail.

I. TINBERGEN AND THE INTRODUCTION OF RATIONAL EXPECTATIONS

Tinbergen's contribution to the early episode of expectations analysis is truly amazing. In a bold move he introduced by far the most sophisticated analysis of expectations formation of his time. His 1932 paper, *Ein Problem der Dynamik*, is the first that explicitly uses rational expectations, but it disappeared under the dust of history. The fact that it is written in German is one of the probable reasons why no attention was paid to this paper in the anglo-saxon literature. But also the paper in the first volume of *Econometrica* (Tinbergen, 1933), in which some of the ideas in the German paper are repeated, did not shock the world of quantitative economists. The papers were completely forgotten in 1961, when Muth reinvented rational expectations.

Tinbergen's article starts with the formulation of a stochastic dynamic optimisation problem (his doctoral thesis deals with optimising problems in physics and economics). Note that stochastic dynamic optimisation techniques were not yet available. Bellman's Principle of Optimality still had to be invented, certainty equivalence had never been heard of. New concepts or tricks were required, and Tinbergen made an effort to fill the lacuna. Tinbergen did not invent dynamic optimisation techniques, but combining common sense and creativity proved sufficient for solving a simple problem in which rational expectations played a major role.

Tinbergen's article emphasises three concepts that are necessary for studying a dynamic problem: the planning horizon and discount rate, lags, and expectations. The first pair of concepts, time horizon (‘Gesichtsfeld’) and a degree of time preference (Böhm-Bawerk's ‘Perspektivische Verkleinerung’) are not revolutionary. They can be found in other studies of that time as well. Nor was the second concept, lags, revolutionary, although at that time the relationship between lags and dynamics using an economic model was
somewhat novel. Lags appear in Tinbergen's model via inventories, not (at least, in most of his article) in the rules of expectations formation.

These expectations (‘Erwartungen’) are for our goals of course the most interesting concept. Like time preference and lags, expectations were not a new invention, but the way Tinbergen formulated a model of expectations formation certainly was. The step from uncertainty about future variables to expectations and probability theory may seem natural (it certainly is for the ‘post-Muth generation’), but these steps were not yet made, and were not made again until 1961.

As Tinbergen noted, in a dynamic problem there is the inherent difficulty that some future variables are unknown to the economic agents. Therefore, they need to form expectations on, e.g. future prices and harvests. For the sake of simplicity, Tinbergen assumed that these expectations are equal among different individuals. The next assumption is that these expectations are rational (‘vernünftig’):

In my opinion the essential characteristic of ‘expectations’ is not yet eliminated: that is, that they do not have to become reality when new facts that were, and had to be, unknown until that moment, have an influence on these expectations. Therefore, we will go a step further, and assume also, that these expectations are ‘rational’ i.e. are consistent with the economic relationships. (1932, p. 172)

Compare this with how Muth defined rational expectations nearly thirty years later: ‘I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. (...) we call such expectations “rational”’ (Muth, 1961, p. 4). So far the language still may be similar by accident, but the next quote from Tinbergen’s article makes clear that Tinbergen had exactly the same thing in mind as Muth. Tinbergen continues his remarks on expectations by the following:

In certain cases – which will probably be the most fruitful ones for analysis – one can replace these ‘expectations’ by economic-theoretical deductions, certain constants or real variables. For example, in case of a random variable, the rational expectation is the mathematical expectation and therefore a certain constant. Another example is a variable that is the realization of a certain law, to some degree of approximation. This expectation can be replaced by a series, in which the current value of the variable and its derivatives with respect to time are used. (p. 172)

2 This reads in the original: ‘Die nach meiner Ansicht wesentlichste Eigenschaft der “Erwartungen” ist damit keineswegs eliminiert: diese besteht m. E. vielmehr darin, daß sie nicht Wirklichkeit zu werden brauchen, weil neue Tatsachen, die bis dahin unbekannt waren und es auch sein mußten, sie beeinflussen. Wir werden sogar noch einen Schritt weiter gehen und auch annehmen, daß die Erwartungen “vernünftig” sind, d.h. mit den wirtschaftlichen Zusammenhängen übereinstimmen.’

3 This sounds uneasy, but as will be made clear in the next section, simply stems from the fact that Tinbergen only discussed serially uncorrelated Gaussian distributed processes, in which case the ‘constant’ is simply the expected value, i.e. the mean.

4 In the original: ‘In bestimmten Fällen – und das werden vorläufig wohl die meist erfolgreichen bei der Analyse sein – kann man die “Erwartungen” durch wirtschaftstheoretische Deduktion, durch bestimmte Konstanten oder reelle Größen ersetzen. Handelt es sich z. B. um eine Erwartung bezüglich einer Größe,
And compare Muth again: 'The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about prediction of the theory (or the "objective" probability distribution of outcomes).' (Muth, op. cit. p. 5).

The language used by Tinbergen remarkably resembles that of Muth. Especially if the Wirtschaftstheoretischen Deduktion is translated by 'the relevant economic theory', which is obviously what is meant by Tinbergen.

The problem analysed in Tinbergen's simple model then takes the following form. An individual makes an economic plan for his demands and supplies, given (1) a utility or profit function, (2) the time horizon, and (3), the price expectations. The plans of individuals can be conflicting or mutually inconsistent. This results in excess demand vectors, $X(p)$. The equilibrium condition for period $t$ is defined as $X_t(p_t) = 0$, i.e. excess demands are zero. Therefore, equilibrium realisations are mutually consistent. Now the setup of Tinbergen's model is given, we may have a closer look at the model itself, in order to see if Tinbergen's rational expectations do indeed match our current view of what constitutes such rational expectations.

II. Tinbergen's Model

The setup of Tinbergen's model very closely resembles Muth's model of Rational Expectations: the way 'rational expectations' is defined is identical, using the mathematical expectation with reference to 'the relevant economic model' for the expected value of economic variables. The next step of course is to find out whether Tinbergen and Muth use their theory of expectations in the same way. If not, the mutual use of the word rational (or 'vernünftig') would be purely accidental. The comparison is made easy because, coincidentally, both brilliant economists apply their idea to a simple model of demand and supply, and even better, both analyse inventories. The divergence between the two articles quickly arises as Muth concentrates on autocorrelated processes whilst Tinbergen only discusses the Gaussian or Normal case with serially independent disturbances (NID, in short). It is clear that Tinbergen's model is much simpler than Muth's, but this may be expected of a thirty-year precursor in a rapidly advancing area.

As Muth remarks, solving for prices is not very interesting in a model with uncorrelated residuals: the mathematical expectation is the mean. Indeed, Tinbergen must have had the same insight, hence he is more interested in solving for quantities (allocation of harvests and inventories over time).

Tinbergen starts with a number of assumptions in addition to the rational expectations hypothesis; these extra assumptions are relaxed in later sections of his article. The assumptions are as follows.
1. There is just one market, where producers and consumers meet (traders are introduced in Tinbergen, section 9).
2. Demand is static.
3. Demand depends on current prices, not on past prices: $\epsilon_t = \mathcal{f}(p_t)$

Section 10 discusses the case where demand also depends on last year's prices,

$$\epsilon_t = \alpha - \beta p_t + \epsilon(p_t - p_{t-1})$$

and section 11 introduces a time trend:

$$\epsilon_t = \mathcal{f}(p_t) + \alpha t$$

4. Supply is dynamic. Producers maximise profits, calculated over a planning horizon of two years ($\tau = 2$) (a longer horizon is discussed in section 12).
5. There is perfect competition (monopolistic competition is briefly discussed in section 13).
6. The amount of farmland is fixed (i.e. production capacity is constant, expected harvest size as well) (section 14 discusses the case where the amount of farmland depends on price expectations).
7. Inventory costs are negligible (section 15 drops this assumption).

A final assumption, which holds throughout the paper, is that good and fast information is available, as well as perfect insight of the producers in the functioning of the market, which implies that the 'Rational Expectations' of the different individuals are the same.

Tinbergen then proceeds with his basic problem: given the demand function

$$\epsilon_t = \mathcal{f}(p_t)$$

and given inventories and current harvests, how big will current consumption (and, therefore, the new inventory size) be? The solution is fairly straightforward: let $P$ denote production (i.e. harvests), $I$ inventories, $Q^s$ gross supply, $Q^d$ gross demand. Then:

$$Q^s_t = P_t + I_{t-1}$$
$$Q^d_t = \epsilon_t + I_t$$
$$Q^s = Q^d$$ (equilibrium condition).

The expected harvest for next year is $P^*_{t+1}$, which due to rational expectations together with the NID assumption can be substituted by its mathematical expectation, the mean: $\mathbb{E}$.

$$P^*_{t+1} = \mathbb{E}(P_{t+1}) = \bar{P}.$$ 

Total expected supply for the two year period over which expected profits are maximised is $P_t + I_{t-1} + \bar{P}$. Now three cases are distinguished: current production is below, at or above average.

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5 The German notation is adjusted to the one used by Muth.

6 Gaussian, serially uncorrelated disturbances are implicitly assumed by Tinbergen until two paragraphs further where this assumption is made explicit. The mathematical expectation is conditional on the information set of the agents, i.e. their insight in the economic model and the disturbance generating process.
If current supply is below average then $p_t > p_{t+1}^*$. In this case, total available supply will go to the market to be consumed in order to maximise profits, and inventories are zero.

If the current supply is equal to the average harvest size, expected prices for next and current year will be equal: $p_t = p_{t+1}^*$. Supply and consumption will now be half of total expected supply because equal current and next year's expected prices are only rational if expected consumption will be equal in these periods:

$$C_t = \frac{1}{2}(P_t + \bar{P} + I_{t-1}) = \bar{P} + \frac{1}{2}(P_t - \bar{P} + I_{t-1}).$$

The last term between brackets is called 'overproduction', half of it is supplied to consumers.

A third case would be $p_t < p_{t+1}^*$, but then suppliers will withhold production until at least the expected next year's price will be equal to the current one. Case two will result. This is consistent with Tinbergen's observations that (expected) future prices are nearly always lower than current prices (see the figures on page 173 of his article, where the thin connected line denotes current prices, thick lines are future prices; upper figure: corn, lower figure: wheat). Note: storage and interest costs are nil.

Case one and two are thus the relevant ones. Only if $P_t + I_{t-1} < \bar{P}$, will case one occur. Inventories are nonnegative, therefore $P_t \leq \bar{P}$. If harvests are normally distributed this will happen less than half of the time.

The general conclusions are: variation in consumption is less than variation in production, the proportionality-factor lies between $0.5$ and $1$. Inventories are a positive function of the difference between expected and current prices. (In a footnote (p. 178, fn. 3) Tinbergen computes the relations between variation in production consumption, and inventories, but his results ($\sigma_R^2 = 3\sigma_C^2 = 3\sigma_I^2$) are not entirely correct.)

Now compare with Muth, section 3 where he discusses optimal speculation and market adjustment. His equation 28, or 29, attains the same kind of inventory function (Muth defines his variables as deviations from equilibrium, Tinbergen uses levels, but the difference is not important)

$$I_t = \alpha(p_{t+1}^* - p_t)$$

(Muth eq. 29)

with $\alpha$ between zero and one. In Muth, $\alpha$ depends on the variance of price forecasts, whilst Tinbergen does not specify this.

Muth's equations 30 to 31 describe basically the same model as Tinbergen (Table 1).

The difference between Tinbergen and Muth is that Muth proceeds with serially correlated disturbances, whereas Tinbergen assumes NID throughout. However, as I mentioned before, Tinbergen does make some extensions. An interesting one is his section 14, where he drops his assumption that capacity is fixed. The new assumption is: planned cropping for next year depends on present prices. Farmers choose capacity in such a way, that expected production will be:

$$P_{t+1}^* = \bar{P} + \epsilon(p_t - \bar{P}), \text{ where } \epsilon > 0$$

(eq. C, Tinbergen p. 183)
Demand is implicitly assumed to take the form:

\[ C_t = -\beta p_t. \]

Tinbergen derives supply for consumption as:

\[ C_t = \bar{p} + \frac{\beta}{2\beta + \epsilon} (p_t + I_{t-1} - \bar{p}). \]

(eq. D, Tinbergen p. 183)

The intuition for this result is as follows. Assume this year has relatively high production, which results in low current prices. Producers expect that next year's production will be lower. Therefore, depending on the size of \( \epsilon \), part of the 'overproduction' will be withheld (added to inventories) in order to spread out the amount of production available for consumption. The argument can be compared with the cobweb tale, discussed at large by Muth (and, as an aside, Tinbergen mentions that one often sees a term \( p_{t-1} \) in equation C). At first sight, this extension seems indeed to be a simple cobweb. But there is a difference: producers are rational, be it in a somewhat peculiar way. A high \( \epsilon \) for example does not seem to be very rational. However, intertemporal inventory allocation remains consistent with the Rational Expectations hypothesis, and helps to dampen changes in net supply (and, therefore, in consumption). The larger \( \epsilon \), the more stable consumption, and therefore prices, will be. The net effect on production of a high \( \epsilon \) and convergence of \( p_t \) and \( \bar{p} \) is not analysed.

III. EPILOGUE

One may wonder why Tinbergen's paper remained neglected. A weak explanation, already mentioned, is that it is written in German, while the language of quantitative economics became English in the 1930s. A second explanation for the neglect of Tinbergen's rational expectations article may be, that the initial problem, how lags in economic models relate to expectations, was solved (or illustrated) by his model. Hence, he may have felt warranted to continue the research programme of dynamic economic modelling. Furthermore, enough other stirring developments were going on: one revolution (the econometric) was maybe good enough.
It seems that even Tinbergen himself forgot about his Rational Expectations model. But it may be argued that in a sense his later work is not completely at odds with the 1932-article, be it at a practical level. In the 1932 article, an empirical illustration presented in his introduction, may have warranted Tinbergen's feeling that current realisations are fairly good forecasts of future realised prices. This conclusion stems from comparing future prices with realised prices. In Tinbergen (1933), the model of 1932 is very briefly repeated, but instead of 'rational', Tinbergen translated 'vernunftig' with the word 'reasonable' and he does not dwell on what he meant by that (no mention was made of 'mathematical expectation', for example). A more or less unrelated section in this article deals with the relation between dividend paid and the 'worth' of stocks. The question that Tinbergen posed, was on what factors dividend expectations depend. The conclusion is, that 'The chief determinant factor (...) is the last dividend paid' (p. 261). Tinbergen concludes, that there is no 'forecasting quality' of stock price. The results are close to the modern random walk hypothesis of stock prices (see e.g. Granger and Morgenstern, 1970).

In his later work in building dynamic macroeconomic models Tinbergen never referred to his theory of rational expectations. Expected profit was an important factor in his empirical explanation for investment, but no use was made of a rational expectations theory to defend the use of profit and profit lagged as proxies for expected profit (see Tinbergen 1937, p. 25: 'It could be asked whether profit expectations rather than past profits should be considered as determinants of investment. In principle this is no doubt correct, but it seems to me that the chief factors in expectations are the actual profits that have been made.'). Other sources for dynamics in his models, such as natural and technological production lags (pig breeding, ship building) were analysed extensively in different contributions of Tinbergen. The sum of these studies in dynamic economics gave Tinbergen more confidence in using lag structures for his macroeconometric models.

The fact that rational expectations had a much wider area for application than Tinbergen, and later Muth, imagined, became apparent only after the research of Lucas and his collaborators. It is interesting to note that Tinbergen belongs to the founding fathers of interventionist macro-economic policy, based upon economic models, but also anticipated the stick by which this research programme was hit most severely much later. It is unlikely that Tinbergen ever realised this unpleasant implication of a seemingly harmless assumption of rationality.

Tilburg University

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REFERENCES


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<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
<th>Journal/Book Details</th>
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<tr>
<td>5</td>
<td>Th. ten Raa and F. van der Ploeg</td>
<td>A statistical approach to the problem of negatives in input-output analysis</td>
<td>Economic Modelling, vol. 6, no. 1, 1989, pp. 2 - 19.</td>
</tr>
<tr>
<td>8</td>
<td>Th. van de Klundert and F. van der Ploeg</td>
<td>Wage rigidity and capital mobility in an optimizing model of a small open economy</td>
<td>De Economist 137, nr. 1, 1989, pp. 47 - 75.</td>
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