Conflict over Arms Accumulation in Market and Command Economies

by
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and
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CONFLICT OVER ARMS ACCUMULATION IN MARKET AND COMMAND ECONOMIES*

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Introduction

Conflict over arms accumulation has in recent years become a more prevalent feature of the relations between West and East. The political aspects of the arms race receive a great deal of attention both in the press and in academic studies (e.g. Richardson 1960; Boulding 1961; McGuire 1965; SIPRI 1982). Much of the theoretical analysis of arms conflict has a game-theoretic nature (e.g. Schelling 1980). The welfare of one country depends on the level of security, which is perceived as an increasing function of its own weapon stock and a decreasing function of the foreign weapon stock. This may be because any imbalance in weapon stocks increases the likelihood of loosing a possible war and it also increases the likelihood that a war might in fact be initiated. Alternatively, a country may simply feel that it gains international prestige from having a more superior army than its rivals. Both of these factors can in principle lead to a balance of terror. Such defence externalities can also be shown to lead to prisoner's dilemma situations. In the absence of cooperation each country builds up a larger weapon stock than with cooperation, because in the absence of commitments no country trusts the other countries to stick to a negotiated level of lower or zero weapon stocks. Other studies concentrate on the technological and strategic aspects of arms and the relationship to the probability that war
breaks out (Saaty 1968; Intriligator 1975; Brito and Intriligator 1976; Intriligator and Brito 1982).

From the point of view of an economist, the purely political analyses of conflict over arms do not pay adequate attention to the 'guns versus butter' dilemma. A higher level of investment in weapons eventually increases security and welfare, but it also means that there are less resources available for private sector consumption and therefore welfare diminishes. A variety of studies employs optimal control and differential game theory to analyse the intertemporal tradeoffs inherent in such 'guns versus butter' dilemmas (e.g. Brito 1972; Deger and Sen 1984). The main problem with these studies is that they consider open-loop Nash equilibrium solutions while feedback Nash equilibrium solutions would in most cases be more appropriate (e.g. Simaan and Cruz 1975). The advantage of the latter type of solution concept is that the resulting equilibrium relies on more realistic information sets, since each country is assumed to be able to monitor the current levels of weapon stocks rather than to be able to observe only the initial weapon stocks. The informational non-uniqueness resulting from closed-loop information sets with memory (Başar and Olsder 1982, Section 6.3) is resolved when the principle of subgame perfection (Selten 1975) is imposed, which has the added advantage that the resulting equilibrium strategies are credible. It is clear that the importance of informational assumptions requires more attention than the literature has given it so far. For example, each country may be able to observe its own weapon stock accurately and to observe the foreign weapon stock not at all or inaccurately.

In an earlier paper (van der Ploeg and de Zeeuw, forthcoming 1989) the conflict between a decentralised market economy (the West) and a command economy (the East) is considered when the western government has the possibility of levying non-distortionary (lump-sum) taxes to finance the investment in arms, when households have quadratic utility functions, and when firms have linear technologies. The open-loop Nash equilibrium solution concept assumes that governments precommit themselves to a given sequence of levels of investment in arms and that governments cannot monitor the weapon stocks of the other countries. The feedback Nash or subgame-perfect equilibrium solution concept assumes that the governments' announcement about investment levels in arms are credible and that governments can monitor the weapon stocks of rival countries at any point in time. The main conclusion of the earlier research is that monitoring
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leads to lower stocks of weapons and to higher consumption of goods and leisure, so that monitoring of each other's weapon stocks is a good thing and is a feasible and desirable form of unilateral disarmament. It can also be argued that the subgame-perfect equilibrium provides a strategic underpinning of the Richardson equations, which give the change in weapon stocks as the sum of a constant, called the 'grievance' or 'hatred' coefficient, a term that depends negatively on the product of the own weapon stock and the 'fatigue' coefficient, and a term that depends positively on the product of the rival weapon stock and the 'defence' coefficient. The desired lead of weapons over the rival country and the relative priority of 'guns' over 'butter' increases the grievance coefficients and thus the steady-state levels of weapon stocks. The discount rate, depreciation rate and the relative priority of 'butter' over 'guns' diminish the defence coefficients and thus the steady-state levels of weapon stocks. The sum of the defence coefficients and physical depreciation rates gives the fatigue coefficients. Hence, with quadratic preferences, linear technologies and non-distortionary taxes, weapon stocks increase proportionately to the level of weapon stocks of the rival nation (defence), decrease proportionately to the economic burden of its own weapon stock (fatigue) and increase due to the desired lead of weapons over the rival nation (grievance or hatred). Furthermore, there is less arms accumulation and higher welfare when monitoring is permitted.

One important shortcoming of the research described above and in van der Ploeg and de Zeeuw (forthcoming 1989) is that it is not really concerned with conflict between a decentralised market economy and a command economy, because the assumption of non-distortionary taxes allows one to invoke the fundamental theorem of welfare economics which says that market economies are efficient and thus equivalent to command economies. The main objective of this paper is to allow for distortionary taxes on labour income, which is an important asymmetry between the West and the East. In addition, the unrealistic assumption of quadratic preferences is replaced by the more realistic assumption of Cobb-Douglas or CES preferences in order to investigate the robustness of previous results. Hence, this paper reconsiders the 'guns versus butter' dilemma, allows for conflict between a decentralised market economy (the West) and a command economy (the East), and contrasts open-loop and subgame-perfect Nash equilibrium solutions with outcomes under coordination. Section 1 formulates a simple general equilibrium model of a market economy and
Section 2 formulates a simple model of a command economy. Each government maximises the discounted utility of the representative household, which depends on consumption, leisure and defence. Defence is a characteristic good which depends positively upon the own weapon stock and negatively upon the foreign weapon stock. The government of the West uses distortionary taxes on labour income to finance the provision of arms, whereas the government of the East commands its constituents directly. Section 3 discusses coordinated decision-making and shows that, under special circumstances, cooperation leads to a gradual running down of weapon stocks via wear and tear. Section 4 analyses decentralised decision-making. The equilibrium for the open-loop information pattern corresponds to a saddlepoint in which the accumulated weapon stock in the East for similar preferences and technologies is larger than the accumulated weapon stock in the West. It is argued that when utilities are separable in home and foreign weapon stocks, as is the case for Cobb-Douglas utility functions, the open-loop Nash equilibrium and the subgame-perfect equilibrium must coincide. Finally, it is shown that closed-loop equilibria which are based on investment strategies with threats can induce a cooperative outcome. Section 5 considers the more general situation in which utility is not separable in defence. This is accomplished with a nested utility function; for example a CES utility function that depends on a composite commodity, given by a Cobb-Douglas sub-utility function that depends on consumption and leisure, and defence. For this more general case, subgame-perfect equilibria lead to less arms accumulation than open-loop Nash equilibria. This suggests that countries should be encouraged to monitor the weapon stocks of their rivals, since this leads to less arms. Section 6 concludes the paper with a summary and some suggestions for further research.

1. Optimal Dynamic Taxation and the Provision of Arms in a Market-oriented Economy

Consider a decentralised market economy with a representative household, a representative firm and a government. There are no domestic or foreign financial assets and the economy does not engage in international trade. There is no private sector capital accumulation, although the government invests in weapon stocks. There is only one domestically produced
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Commodity, which is like 'jelly' as it can be used for both consumption and investment purposes. The government demands goods for investment purposes, the household supplies labour and demands goods for consumption purposes, and the firm demands labour and supplies goods. The real wage adjusts in order to ensure labour market equilibrium. The government finances the provision of public goods, i.e., weapon stocks, by means of distortionary taxes on labour income and maximises the utility of the representative household. The problem of optimal taxation is that the household values spending on public goods, but that it is not prepared to pay for it.

The representative household maximises its utility subject to its budget constraint. The main analysis in this paper is based upon a Cobb-Douglas utility function, although the expressions for general utility functions are also given. This utility function is increasing in consumption of goods, leisure and defence and, what will be important, it is separable in defence. Section 5 extends the analysis for a CES utility function, which allows for non-separability between goods and leisure on the one hand and defence on the other hand. Both of these utility functions lead to linear Engel curves, so that aggregation across households is possible and therefore the assumption of a representative household is justified. Utility is assumed to be homogeneous of degree one in consumption and leisure. Defence is a characteristic (cf. Lancaster 1966), which is an increasing function of the own weapon stock and a decreasing function of the foreign weapon stock. It is assumed to be homogeneous of degree one in the respective weapon stocks. Consumption cannot exceed disposable income, which consists of after-tax wages and dividends. The household's problem is therefore:

Maximise $U(c, l, l, d(a, a^*)) = \alpha_1 \ln(c) + \alpha_2 \ln(l-l) + \alpha_3 \ln(a/a^*)$,  
subject to the household's budget constraint

$$0 \leq c \leq w(l-\tau) + \pi = y.$$
where

\[ c : \text{real consumption of goods} \]
\[ \ell : \text{supply of labour} \]
\[ \ell : \text{total amount of time available to the household} \]
\[ d : \text{level of defence or security} \]
\[ w : \text{real wage rate} \]
\[ \tau : \text{rate of taxation on labour income} \]
\[ \pi : \text{real profits or dividends} \]
\[ y : \text{real disposable income} \]
\[ a : \text{weapon stock of the decentralised market economy} \]
\[ a^* : \text{weapon stock of the command economy.} \]

For an interior solution, the marginal rate of substitution between leisure, \( \ell - \ell \), and consumption equals the real opportunity cost of leisure:

\[ \frac{\partial U}{\partial C} w(1-\tau). \]  

Because utility is homogeneous of degree one, this yields consumption

\[ c = (\ell - \ell) h(w(1-\tau)) = \rho(w(1-\tau))y_0, \quad \rho' \leq 0, \quad h' > 0 \]  

(1.4)

and the supply of labour

\[ \ell = \ell - \rho(w(1-\tau))y_0/h(w(1-\tau)). \]  

(1.5)

where \( h^{-1}(c/(\ell - \ell)) = \frac{\partial U}{\partial C}, y_0 = w(1-\tau)\ell + \pi \) is full-employment income

An increase in profits (or time available to the household) increases income, so that consumption increases and labour supply falls as both consumption and leisure are normal goods given the assumption of homogeneous utility functions. For Cobb-Douglas utility functions,
h(w(1-τ)) = α_1 w(1-τ)/α_2 and \( p(w(1-τ)) = α_1 \) so that an increase in the after-tax wage increases consumption and the supply of labour.

The representative firm (with linear technology) chooses the demand for labour to maximise profits:

\[
\text{Maximise } \pi = f(l) - w l = β l - w l, \quad f' > 0, \quad f'' < 0,
\]

so that \( f'(l) = β = w \) and \( π = 0 \).

Labour market equilibrium gives \( w = W(τ) = β \), employment

\[
l = L(τ) = α_1 \ l,
\]

and consumption

\[
c = C(τ) = α_1 β(1-τ) l.
\]

For Cobb-Douglas utility functions and constant-returns-to-scale production functions, a tax cut increases the opportunity cost of leisure so that the household substitutes away from leisure towards consumption. Labour supply is unaffected, since the substitution effect is exactly offset by the income effect.

Upon substitution of (1.7) and (1.8) into (1.1), one obtains the indirect utility function:

\[
U(τ, a, a*) = U(C(τ), L(τ), d(a, a*)) = α_1 ln(1-τ) + α_2 ln(a/a*) + α_0.
\]

\[
\frac{∂}{∂τ} > 0, \quad \frac{∂}{∂a} < 0, \quad \frac{∂}{∂a*} < 0,
\]

where \( α_0 = α_1 ln(α_1 β) + α_2 ln α_2 + ln l. \) The role of the government is to provide the public good, defence, and finance it with taxes on labour income. Government investment, \( g \), leads to the accumulation of weapon stocks,

\[
\dot{a} = g - δa, \quad a(0) = a_0.
\]
where \( b \) is the depreciation rate, and needs to be financed by distortionary taxation,

\[
g = \dot{W}(\tau) - L(\tau) = \tau \beta \alpha l.
\]

(1.11)

Note, that summing of the household budget constraint, (1.2), and the government budget constraint, (1.11), yields the familiar national accounting identity:

\[
c + g = w l + \pi = f(l).
\]

(1.12)

The government of the market economy chooses the tax rate in order to maximise the discounted utility of the representative household,

\[
\text{Maximise } \int_0^\infty \exp(-rt) \dot{U}(\tau, a, a') dt.
\]

(1.13)

where \( r \) is the pure rate of time preference, subject to (1.10) and (1.11). The dilemma of 'guns versus butter' for a market economy is that high tax rates are required to ensure a large build-up of weapons, but that this necessarily implies less private sector consumption.

It could have been argued that the arms accumulation game should be modelled as an insurance where the level of defence decreases the probability of being attacked and therefore increases the probability that nobody survives and the utility of the current and all future generations is zero (see Shepherd 1988). This argument suggests that, if an attack only affects the utility of the current generation, an appropriate utility function might be \( P(a-a') \dot{U}(\tau) \), where \( P(.) \) denotes the instantaneous probability of not being attacked, \( P' > 0 \), and \( \dot{U}(\tau) \) denotes the indirect utility function. Taking logarithms yields (1.9) with \( \dot{U}(\tau, a, a') = \ln[P(a-a')] + \ln(\dot{U}(\tau)) \). A proper analysis of the probabilities of survival, when an attack destroys the current and all future generations, requires an intertemporal and stochastic framework, but this leads to a differential game formulation which is extremely difficult to solve. In any case such an intergenerational analysis is more appropriate for a nuclear than for a conventional arms build-up. However, if the analysis allows for nuclear attacks where all future generations can be destroyed, then the only credible, non-cooperative equilibrium is for neither country...
to accumulate missiles. When the build-up of nuclear weapons leads to a finite probability of an attack which is too horrendous to contemplate and when there is a zero probability of attack, there is no incentive to accumulate arms. In other words, deterrence requires the probability of a commitment to investments which in the future may imply launching missiles and blowing up the world, and which is therefore not rational to be carried out. This seems to exclude perfect equilibrium as an appropriate solution concept for deterrence games.

2. The 'Guns versus Butter' Dilemma in a Command Economy

The previous section discussed a stylised model of the market-oriented Western economies (e.g. the USA). The objective of this paper is to analyse conflict over arms accumulation between the Western economies and the Eastern economies (e.g. the USSR). It is probably more realistic to describe the Eastern bloc by a command or centrally planned economy than by a decentralised market economy. The effects of such asymmetries in economic organisation on the arms race have not been discussed previously in the literature. For simplicity, it is assumed that the two economies have identical tastes, technologies and population sizes. The variables and expressions describing the Eastern bloc will be distinguished from the ones describing the West by an asterisk.

The government of the centrally planned economy does not levy taxes, but commands the household how much to consume and how much to work and commands the firm how many workers to hire and how much to produce. The optimal plan follows from maximising the utility of the representative household, $U^*(c^*, \tilde{l} - \tilde{l}^*, d^*)$, subject to the material balance condition, $f(\tilde{l}^*) = c^* + g^*$. Hence, the marginal rate of substitution between leisure and consumption equals the marginal productivity of labour, so that

$$c^* = (\tilde{l} - \tilde{l}^*) h^*(f'(\tilde{l}^*)), \quad h^* > 0 \quad (2.1)$$

holds. With the material balance condition this yields

$$\tilde{l}^* = L^*(g^*) = \alpha_1 \tilde{l} + \alpha_2 g^*/\beta \quad (2.2)$$

and
where \( L_{g^*}^* = \{ f'(l^*) + h^* - (l-l^*) h''(l^*)\}^{-1} > 0 \) and
\[ C_{g^*}^* = -\{ h^* - (l-l^*) h''(l^*)\} L_{g^*}^* < 0. \]

Consumption and employment do not depend directly on the level of security, since it has been assumed that utility is separable in defence. When the central planning authority allocates more resources to investment in weapon stocks, there are less resources available for consumption purposes and, consequently, the people consume less and work more hours. This one-to-one crowding out captures the 'guns versus butter' dilemma for the Eastern bloc. Note that for identical levels of investment in weapons (and identical tastes and Technologies), the Eastern bloc employs more labour and consumes more than the West:

\[ c^* = C^*(g^*) = \alpha_1(\beta l - g^*), \quad (2.3) \]

when \( g(=\alpha_1 \tau) = g^* \). The reason for this result is that the Western economy levies distortionary taxes on labour income, which reduces the real opportunity cost of leisure and therefore Western households supply less labour and consume less. Obviously, if the West has a more productive technology (say, \( \beta > \beta^* \)), it is possible for the West to be more affluent than the Eastern bloc (\( c > c^* \) yet \( l < l^* \)).

Using (2.2) and (2.3), one obtains the indirect utility function:

\[ \hat{U}(g^*, a^*, a) = U(C^*(g^*), l-L^*(g^*), d^*(a^*, a)) \]
\[ = \ln(1-g^*/\beta l) + \alpha_3 \ln(a^*/a) + \alpha_0, \]
\[ \hat{U}_{a^*} > 0, \quad \hat{U}_{a^*} < 0, \quad \hat{U}_{g^*} < 0. \quad (2.5) \]

The optimal defence strategy of the central planning authority of the Eastern bloc follows from maximising the discounted utility of the representative household,
Maximise \[ \int_0^\infty \exp(-rt) \hat{U}^*(g^*, a^*, a) \, dt, \] subject to
\[ \dot{a}^* = g^* - \delta a^*, \quad a^*(0) = a_0^*. \] (2.7)

3. Cooperative Behaviour

Before the problems of conflict over weapons accumulation are discussed, it seems appropriate to consider briefly the coordination of arms accumulation. Pareto-efficient outcomes for the Western and Eastern blocs may be found from choosing \( \tau \) and \( g^* \) to maximise joint welfare,

\[ \int_0^\infty \exp(-rt) \{ \theta \, \hat{U}(\tau, a, a^*) + (1-\theta) \, \hat{U}^*(g^*, a^*, a) \} \, dt, \] subject to \( \tau, g^* \geq 0, \) (1.9) - (1.11), (2.5) and (2.7). This yields

\[ \begin{align*}
\theta \, \hat{U}_{\tau} + \lambda \{ W(\tau) \, L(\tau) + \tau \, W_{\tau}(\tau) \, L(\tau) + \tau \, W(\tau) \, L_{\tau}(\tau) \} & \leq 0 \\
\tau & \geq 0 \\
1-\theta \, \hat{U}_{g^*} + \lambda^* & \leq 0 \\
g^* & \geq 0
\end{align*} \] (3.1 - 3.3)

\[ \lambda = (r+\delta) \lambda - \theta \, \hat{U}_a(\tau, a, a^*) - (1-\theta) \, \hat{U}_{a^*}(g^*, a^*, a), \] (3.4)

\[ \lim_{t \to \infty} e^{-rt} \lambda(t) \, a(t) = 0, \] (3.5)
(1.10) and (2.7), where \( \lambda \) and \( \lambda^* \) are the marginal values of the weapon stocks of the Western and Eastern blocs, respectively. Under the assumption that the 'world peace authority' attaches equal weight to the Western and Eastern blocs, i.e. \( \theta = \frac{1}{2} \), and using the specific functional forms for the utility functions adopted in the previous sections, it follows that \( \lambda(t) = \lambda^*(t) = 0 \), for all \( t \). This result holds whenever \( \hat{u}_a = -\hat{u}_a^* \) and \( \hat{u}_a^* = -\hat{u}_a^* \), that is whenever the game between the two economies is zero-sum at the margin with respect to \( a \) and \( a^* \). It implies that \( \tau(t) = g^*(t) = 0 \), for all \( t \), so that the cooperative outcome for both economies is to stop investing in arms and to run down weapon stocks (via the natural process of wear and tear) until they have fallen to zero. This outcome is probably close to one's intuition, although it should be noted that it pertains only under rather special conditions. Due to the asymmetry in economic organisation of the West and the East, it may well be that the Nash bargaining solution does not coincide with \( \theta = \frac{1}{2} \). Furthermore, the utility functions may not be separable or homogeneous of degree one in \( a \) and \( a^* \) and then the cooperative outcome need not necessarily lead to a moratorium on investment in weapons. The cooperative outcome is not sustainable, since each country has an incentive to deviate from it and increase its security at the expense of its rival by investing more in weapons.

4. Competitive Behaviour and the Arms Race

Consider the situation in which the Western and the Eastern blocs do not cooperate when they accumulate weapon stocks. Since there is a unilateral incentive to deviate from the cooperative outcome, there are serious problems with implementing and sustaining the cooperative outcome and therefore non-cooperative outcomes may be more relevant. It seems reasonable to consider Nash equilibria, since neither the Western nor the Eastern bloc is dominant in the arms race. This implies that there should be no unilateral incentive to deviate from the equilibrium. Typically, there are an infinite number of Nash equilibrium solutions in differential games. Sometimes uniqueness can be obtained for special information sets. For example, with open-loop information sets, i.e. the information set of each country at any point of time is assumed to be \( \{a_0, a^*_0\} \), the unique open-loop Nash equilibrium solution (OLNES) can be obtained (e.g. Starr...
and Ho 1969a). In general multiple Nash equilibrium solutions exist. For example, with closed-loop (memory) information sets, i.e. the information set of each country at time $t$ is assumed to be $\{t, a(s), a^*(s), 0 \leq s \leq t\}$, an infinite number of closed-loop Nash equilibrium solutions (CLNES) exist (Başar and Olsder 1982). However, if one restricts the class of CLNES to subgame-perfect equilibria (Selten 1975), one can obtain uniqueness within the class of CLNES. The resulting outcome will be called the subgame-perfect Nash equilibrium solution (SPNES). Starr and Ho (1969b) and Simaan and Cruz (1975) refer to the SPNES as the feedback (stagewise) Nash equilibrium solution.

4.1 Nash Equilibrium with Open-loop Information Sets

The OLNES implies that each country conditions its optimal investment strategy on the initial stocks of weapons and therefore commits itself to a path of levels of investment in weapons. The expected investment strategy of the rival country only depends on initial weapon stocks and not on past or current weapon stocks or on past or current investment levels of the country under consideration. It follows that Pontryagin's Maximum Principle can be used, hence

$$\dot{U}_t + \lambda \{\omega l + \tau(W_t + W_L_t)\} = \alpha_1 \{\beta \lambda - (1-\tau)^{-1}\} \leq 0$$

$$\tau > 0 \quad \text{c.s.,} \quad (4.1)$$

$$\dot{g}^* + \lambda^* = \lambda^* - (\beta \lambda - g^*)^{-1} \leq 0$$

$$g^* \geq 0 \quad \text{c.s.,} \quad (4.2)$$

$$\dot{\lambda} = (r+\delta) \lambda - \dot{U}_a = (r+\delta) \lambda - \alpha_3/a,$$

$$\lim_{t \to \infty} e^{-rt} \lambda(t) a(t) = 0, \quad (4.3)$$

$$\dot{\lambda}^* = (r+\delta) \lambda^* - \dot{U}^*_a = (r+\delta) \lambda^* - \alpha_3/a^*,$$

$$\lim_{t \to \infty} e^{-rt} \lambda^*(t) a^*(t) = 0, \quad (4.4)$$
where $\lambda$ ($\lambda^*$) is the marginal value of the Western (Eastern) weapon stock to the Western (Eastern) bloc, are necessary conditions. The interpretation of (4.3) and (4.4) is that the marginal utility of weapons of each country has to equal the rate of time preference plus the depreciation charge minus the rate of capital gains in the marginal value of its weapons. Equation (4.1) says that no taxes will be levied in the West when the marginal indirect disutility of the tax rate $-\frac{\partial T}{\partial \tau}$, arising from lower consumption, exceeds the marginal value of the increase in investment in arms made possible by a marginal increase in the tax rate. Otherwise, the marginal indirect disutility equals the marginal shadow value of the tax rate which gives the tax rate as an increasing function of the marginal value of Western weapon stocks:

$$\tau = T(\lambda) = 1 - \left(\beta \lambda^*\right)^{-1}, \quad T_{\lambda} > 0. \quad (4.5)$$

It follows that private sector consumption is a decreasing function of the marginal value of weapons, $c = \alpha_1/\lambda$, which is of course quite intuitive. Upon substitution of (4.5) into (1.11), one obtains

$$g = G(\lambda) = T(\lambda)W(T(\lambda))L(T(\lambda)) = \alpha_1 \left(\beta \lambda^*\right)^{-1}. \quad (4.6)$$

Equation (4.2) says that the Eastern bloc does not invest in arms when the marginal disutility of arms expenditure, in terms of foregone consumption, exceeds the marginal value of weapon stocks. In general, $-\frac{\partial g^*}{\partial \lambda^*} = \lambda^*$ and consequently Eastern bloc investment in arms is an increasing function of the marginal value of weapon stocks:

$$g^* = G^*(\lambda^*) = \beta \lambda^* - \lambda^*-1, \quad G^*_{\lambda^*} > 0. \quad (4.7)$$

It turns out that the OLNES can be easily characterised, since the policy instrument, weapon stock and shadow price of each country are independent of the corresponding foreign variables. This is a direct consequence of the assumption that the utility function is separable in $a$ and $a^*$. The game is in a sense degenerate. The OLNES for the West follows from
\[
\dot{a} = G(\lambda) - \delta a = \alpha_1 (\beta l - \lambda)^{-1} - \delta a, \quad a(0) = a_0 
\] (4.8)

and (4.3). The \(\dot{a} = 0\)-locus is upward-sloping and the \(\dot{\lambda} = 0\)-locus is downward-sloping. The full phase-diagram is given in Figure 1.

![Phase-diagram for the open-loop Nash equilibrium](image)

**Figure 1: Phase-diagram for the open-loop Nash equilibrium - Increase in \(\alpha_3\)**

The equilibrium is a saddlepoint, which follows from \(\det(J) = -6(r + \delta) - \alpha_1 \alpha_3 (\lambda a)^{-2} < 0\), where \(J\) is the Jacobian of (4.8) and (4.3) evaluated at the equilibrium. The equilibrium is given by \(\lambda(\omega) = \alpha_3 / ((r + \delta) a(\omega))\), where

\[
a(\omega) = \alpha_1 \alpha_3 \beta l / ((\alpha_1 + \alpha_3) \delta + \alpha_1 r). 
\] (4.9)

Hence, the steady-state weapon stock of the West is an increasing function of productivity \(\beta\) and the weights attached to security \(\alpha_3\) and private sector consumption \(\alpha_1\), but a decreasing function of the weight attached
to leisure ($\alpha_2$), the rate of depreciation ($\delta$) and the rate of impatience ($r$).

The OLNE for the East follows from

$$\dot{a}^* = G^*(\lambda^*) - \delta a^* = (\beta \lambda - \lambda^{-1}) - \delta a^*, \quad a^*(0) = a_0^* \quad (4.10)$$

and $(4.4)$. This yields a similar phase-diagram, where $\text{det}(J^*) = -\delta(r+\delta) - \alpha_3(\lambda^*a^*)^{-2} < 0$. The steady-state weapon stock of the East is given by

$$a^*(\omega) = \alpha_3 \beta l / [(1+\alpha_3)\delta + r] > a(\omega). \quad (4.11)$$

Since $\lambda^*(\omega) = \alpha_3 / (r+\delta) a^*(\omega)$, $g(\omega) = \delta a(\omega)$ and $g^*(\omega) = \delta a^*(\omega)$, it follows that $\lambda^*(\omega) < \lambda(\omega)$ and $g^*(\omega) > g(\omega)$. Also,

$$l^*(\omega) = \frac{(\alpha_1 + \alpha_3)\delta + \alpha_1 r}{(1+\alpha_3)\delta + r} \quad l > l(\omega) = \alpha_1 \frac{l}{(4.12)}$$

and

$$c^*(\omega) = \frac{(r+\delta)\alpha_1 \beta l}{(1+\alpha_3)\delta + r} > c(\omega) = \frac{(r+\delta)\alpha_1^2 \beta l}{(1+\alpha_3)\delta + \alpha_1 r} \quad (4.13)$$

An unanticipated increase in the weight attached to security ($\alpha_3$) leads in both countries to an immediate increase in the marginal value of weapon stocks from $E$ to $A$ (see Figure 1), which causes an immediate increase in investment in arms and associated reduction in consumption. Afterwards, the marginal value of weapons falls gradually until the weapon stocks have reached their new higher equilibrium values at $E'$ (see Figure 1).

Since the West has to levy distortionary taxes to finance the provision of arms, employment and therefore output, consumption, investment in arms and the eventual weapon stock are clearly below the corresponding levels attained in the East. However, the West may have a more productive technology ($\beta > \beta^*$ rather than $\beta = \beta^*$) in which case consumption, leisure and arms in the West may exceed the levels attained
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in the East. It is obvious that competitive arms accumulation leads to excessive investment in weapons relative to the cooperative outcomes of Section 3, since competitive arms accumulation does not lead to a moratorium.

4.2 The Subgame-perfect Nash Equilibrium

The CLNES implies that each country conditions its optimal investment strategy on the current and, possibly, past stocks of weapons. This type of information structure admits, among other things, threat strategies which are briefly considered in Section 4.3. The solution set under CLNES is typically non-unique. However, if the principle of subgame perfection is applied, uniqueness might result. An equilibrium solution is subgame perfect, if the relevant part of this solution is also a Nash equilibrium for each subgame. A subgame in this respect is a game over a remainder of the planning period, say over \([\tilde{t}, \infty)\) rather than over \([0, \infty)\). The subgame-perfect equilibrium must be a Nash equilibrium for any \(\tilde{t} \in [0, \infty)\) and for any \((a(\tilde{t}), a^*(\tilde{t}))\). Considered this way Selten's (1975) principle of subgame perfection is very similar to Bellman's principle of optimality, which is now assumed and not concluded. The solution technique follows dynamic programming. This implies that each country uses information on the current weapon stocks and does not commit itself to a fixed investment strategy from the beginning. In fact the principle of subgame perfection rules out threat equilibria which rely on information patterns with memory, and rules out equilibria which cease to be equilibria when something unexpected occurs. Under closed-loop information patterns there might be different sets of equilibrium strategies which yield the same equilibrium path of arms accumulation. This is a problem of non-uniqueness in representation of the optimal policies. The only set of equilibrium strategies, which remains an equilibrium solution when mistakes or disturbances cause a deviation from the expected equilibrium path, is the subgame-perfect Nash equilibrium solution (SPNES).

Although Pontryagin's Maximum Principle is primarily constructed for problems with an open-loop information pattern, an extended version can be used to find equilibrium solutions for closed-loop (no-memory) information patterns. To the right-hand sides of equations (4.3) and (4.4) the extra terms \((-g_a^* \lambda^e_a\) and \((-\tau_a^* \lambda^w_a\) have to be added respectively, where \(\lambda_a \) (\(\lambda^e_a\)) is the marginal value of the Eastern (Western) weapon stock to the Western
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(Eastern) bloc and where \( \lambda^* \) and \( \lambda^{**} \) have to satisfy the additional necessary conditions

\[
\dot{\lambda}^* = (r+\delta) \lambda^* + \alpha^*_3 \frac{a^*}{a^*} - g^*_a \lambda^*, \quad \lim_{t \to \infty} e^{-rt} \lambda^*_a(t) a^*(t) = 0. \tag{4.14}
\]

\[
\dot{\lambda}^{**} = (r+\delta) \lambda^{**} + \alpha^{**}_3 \frac{a^{**}}{a^{**}} - \tau^{**} \lambda^{**}, \quad \lim_{t \to \infty} e^{-rt} \lambda^{**}_a(t) a(t) = 0. \tag{4.15}
\]

Note that the user cost of defence, that is the rental plus depreciation charges minus capital gains \((r+\delta-\lambda/\lambda)\lambda\), now has to match the marginal benefit of weapons \((\alpha^*_3/a)\) plus an extra term to allow for the closed-loop nature of the information patterns \((g^*_a \lambda^*)\). Since one normally expects the rival country to step up its investment in arms when the home country's weapon stock increases \((g^*_a>0)\), and the marginal value of the Eastern weapon stock to the West to be negative \((\lambda^*<0)\), one expects this extra term to reduce the marginal benefit of weapons. Hence, one expects less weapon stocks and therefore greater welfare when countries can monitor their rival's weapon stock. Indeed, van der Ploeg and de Zeeuw (forthcoming 1989) prove this for a symmetric model with lump-sum taxation and quadratic preferences. However, these insights only hold when defence is not separable in home and foreign weapon stocks.

The SPNES is expressed in terms of the value functions \(V(t,a,a^*)\) and \(V^*(t,a^*,a)\) which are the equilibrium values for the indirect intertemporal utility functions when the game would start in time period \(t\) with weapon stocks \(a\) and \(a^*\). Note that \(V_a = \lambda, V_{a^*} = \lambda^*\), \(V^*_a = \lambda^{**}\) and \(V^*_{a^*} = \lambda^{**}\) establishes the relationship between the SPNES and the extended Maximum Principle above. The value functions \(V\) and \(V^*\) have to satisfy the Hamilton-Jacobi-Bellman equations

\[
V_t - rV + \max_{\tau} \left\{ V_a \left[ \tau \left( W(\tau) L(\tau) - \delta a \right) + V_{a^*} \left[ g^* - \delta a^* \right] + \hat{U}(\tau,a,a^*) \right] \right\} = 0, \quad \tau
\]

\[
V^*_t - rV^* + \max_{g^*} \left\{ V^*_a \left[ \tau w L - \delta a \right] + V^*_{a^*} \left[ g^* - \delta a^* \right] + \hat{U}^*(g^*,a^*,a) \right\} = 0, \quad \tag{4.17}
\]

yielding
Even for the stationary situation with interior solutions the explicit functional forms of the value functions \( V \) and \( V^* \) are very difficult to find. However, on the basis of two specific aspects of the present problem it can be concluded that the equilibrium paths for the weapon stocks and the equilibrium values for the investments in arms of the SPNES and the OLNES must coincide. These specific aspects are the separability of the indirect utility functions in the weapon stocks \( a \) and \( a^* \) and the absence of a direct influence of each investment and stock on the accumulation of arms in the rival country. These aspects imply a decoupling of the dynamic game, which restores the one-player property that subgame-perfect control and open-loop control yield the same control values. The decoupling can also be recognised in the analysis of the stationary OLNES in Section 4.1. In Section 5 the separability is relaxed by considering the more general CES utility function instead of the Cobb-Douglas utility function.

4.3 Threats as an Inducement of Cooperative Behaviour

Suppose each country aims at the results of cooperative behaviour but cannot rely on a cooperative agreement with the rival country, because the rival country then has an incentive to deviate by accumulating more weapons than the level agreed upon in the cooperative outcome. In other words, the cooperative outcome is in general not sustainable under competitive arms accumulation. However, if each country can employ memory strategies, threats can be formulated in such a way that the cooperative outcome can be sustained. Each country announces that, if the rival country deviates from the cooperative equilibrium, it will invest so heavily in arms accumulation, that the rival country is deterred from deviating. The structure of such a threat equilibrium is given by:

\[
\begin{align*}
\hat{V} + V_a \{w, l + r(W, l + w, l, r)\} &= \alpha_1 \{\beta v_a - (1 - \tau)^{-1}\} \leq 0 \\
\tau &\geq 0 \\
\hat{V}^* + V^{*a} &= V^{*a} - (\beta l - g^*)^{-1} \leq 0 \\
g^* &\geq 0
\end{align*}
\]
There are two possibilities. Either each country submits to the threat and the cooperative equilibrium results or a country does not submit to the threat (because it is not credible and thus not optimal to execute it when called upon to do so) and then anything can happen.

5. Consequences of Non-separable Preferences

In Section 4.2 it was argued that the separability of the Cobb-Douglas utility function is one of the reasons why the OLNES and the SPNES coincide. This section therefore considers a more general nested CES utility function:

\[
U(c, l-l, d(a, a^*)) = \left[ \frac{1}{1+\alpha_3} \left[ c^{1/(1-\ell)} \left( \frac{\alpha_1}{\alpha_2} c^{\alpha_2} + \frac{\alpha_3}{1+\alpha_3} \left( \frac{a}{a^*} \right)^{-\alpha_4} \right) \right]^{\alpha_4} \right]^{-1},
\]

\[
\alpha_1, \alpha_2, \alpha_3 \geq 0, \quad \alpha_4 \geq -1, \quad \alpha_1 + \alpha_2 = 1, \quad (5.1)
\]

where \( \sigma = 1/(1+\alpha_4) \) is the elasticity of substitution between private goods, a composite of consumption and leisure, and defence. The special case of a unit elasticity of substitution (\( \alpha_4 = 0 \)) yields the Cobb-Douglas utility function employed so far. This nested approach has the advantage that the logarithm of the sub-utility function is still separable, so that the choices for consumption and leisure are unaffected. Upon substitution into (5.1), one obtains the indirect utility functions

\[
\hat{U}(\tau, a, a^*) = \left[ \frac{1}{1+\alpha_3} \left[ \frac{\alpha_1}{\alpha_2} \left( \frac{a}{a^*} \right)^{\alpha_2} + \frac{\alpha_3}{1+\alpha_3} \left( \frac{a}{a^*} \right)^{-\alpha_4} \right]^{-1} \right]^{\alpha_4},
\]

\[
(5.2)
\]
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\[ \hat{u}^*(g^*,a^*,a) = \left[ \frac{1}{1+\alpha_3} \left[ \alpha_0 (1-\alpha_1 g^*) \right]^{1-\alpha_4} + \frac{\alpha_3}{1+\alpha_3} \left[ a^* / a \right]^{-\alpha_4} \right]^\frac{-1}{\alpha_4}, \]  

(5.3)

where \( \alpha_0 = \exp(\alpha_0) \). As before, the OLNES is found with the aid of Pontryagin's Maximum Principle and the SPNES with the aid of dynamic programming. Since an analytical treatment becomes very cumbersome, a numerical example will be discussed in such a way that the length of the working day in the West will be eight hours and the normalisation \( \alpha_1 \beta \ell = 1 \) is used.

**Example.**

\[ \ell = 24, \quad \alpha_1 = 1/3, \quad \beta = 1/8, \quad \delta = 0.1, \quad r = 0.1. \]

The indirect instantaneous utility functions for this example are

\[ \hat{u}(\tau,a,a^*) = \left[ \frac{1}{1+\alpha_3} \left[ (1-\tau)^{1/3} 16^{2/3} \right]^{1-\alpha_4} + \frac{\alpha_3}{1+\alpha_3} \left[ a^* / a \right]^{-\alpha_4} \right]^\frac{-1}{\alpha_4}, \]

and

\[ \hat{u}^*(g^*,a^*,a) = \left[ \frac{1}{1+\alpha_3} \left[ (1-g^*/3)^{16^{2/3}} \right]^{1-\alpha_4} + \frac{\alpha_3}{1+\alpha_3} \left[ a^* / a \right]^{-\alpha_4} \right]^\frac{-1}{\alpha_4}. \]

### 5.1 Nash Equilibrium with Open-loop Information Sets

The first-order conditions describing the OLNES are (1.10) - (1.11).

(2.7), \( H_\tau = 0, H_{g^*} = 0, r\lambda - \dot{\lambda} = H_a, r\lambda - \dot{\lambda} = H_{a^*}, r\lambda - \dot{\lambda} = H_{a^*} \), and \( r\lambda - \dot{\lambda} = H_a \), where the Hamiltonians for the Western and Eastern blocs are given by \( H = \hat{u} + \lambda(\tau-\delta a) + \lambda_a(g^*-\delta a^*) \) and \( H^* = \hat{u}^* + \lambda^*(g^*-\delta a^*) + \lambda_a^*(\tau-\delta a) \), respectively. Again, the equations describing the dynamic behaviour of \( \lambda \) and \( \lambda \) are decoupled from the other equations. The OLNES
for the example above is a saddlepoint and its steady state can be shown to satisfy the following set of equations:

\[
\frac{1}{3} (r+\delta) 16 - \frac{2}{3} \alpha_4 (1-6a) - \frac{1}{3} \alpha_4^{-1} = \alpha_3 a^* - \alpha_4^{-1} \tag{5.4}
\]

and

\[
\frac{1}{3} (r+\delta) 16 - \frac{2}{3} \alpha_4 (1-6a^*/3) - \alpha_4^{-1} = \alpha_3 a^* - \alpha_4^{-1} \tag{5.5}
\]

The values of the weapon stocks and utilities in equilibrium for \( \alpha_3 = 0.2 \) and for different values of the parameter \( \alpha_4 \) have been calculated and are:

<table>
<thead>
<tr>
<th>( \alpha_4 )</th>
<th>( \sigma )</th>
<th>( a^*(\cdot) )</th>
<th>( a^*(\cdot) )</th>
<th>( \hat{\theta} )</th>
<th>( \hat{\theta}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>2.00</td>
<td>0.514</td>
<td>2.526</td>
<td>4.651</td>
<td>5.660</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>2.308</td>
<td>2.727</td>
<td>4.219</td>
<td>4.431</td>
</tr>
<tr>
<td>0.5</td>
<td>0.67</td>
<td>4.303</td>
<td>5.183</td>
<td>3.353</td>
<td>3.764</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>6.337</td>
<td>8.042</td>
<td>2.532</td>
<td>3.219</td>
</tr>
</tbody>
</table>

The values for \( \alpha_4 = 0 \) correspond to the results (4.9) and (4.11) which were obtained for Cobb-Douglas utility functions. An increase in \( \alpha_4 \) or, alternatively, a decrease in the elasticity of substitution between, on the one hand, consumption and leisure and, on the other hand, defence leads to larger weapon stocks in equilibrium. The reason for this is that less substitution possibilities between private goods and defence makes each country more vulnerable to increases in foreign weapon stocks, so that each country escalates its accumulation of arms. As before, for any given value of the elasticity of substitution, except for \( \sigma \rightarrow \infty \), the East is more efficient and therefore has more weapons and a higher level of welfare.
5.2 The Subgame-perfect Nash Equilibrium

The SPNES is considerably more difficult to calculate. In fact, it has proven impossible to find the asymptotic solution to the coupled system of partial differential equations describing the SPNES for the example discussed in this section. One possibility is to solve a linear-quadratic approximation of the problem, since then the value functions associated with the SPNES can be found. It is not clear that this is a very satisfactory approach, hence some progress has been made in the development of a computer algorithm for solving discretised finite-horizon dynamic games.

Each country is given a limited set of investment possibilities and for a finite number of stages a 'game-tree' is built in which the branches represent the investment choices with the corresponding utility levels and the nodes represent the resulting weapon stocks. The algorithm solves backwards recursively for each node the remaining subgame starting at the end of the tree. In this discretised setting these subgames take the form of bimatrix games. The horizon is set at 7 stages and the security parameter $\alpha_3$ is given a value of at least 0.2 in order to evoke positive investments in arms. Suitable discrete investment possibilities were experimentally established at 0.1 and 0.3 and the initial values of the weapon stocks, $a_0$ and $a_0^*$, are set at the equilibrium values of the corresponding OLNES.

The purpose of this section is to compare, for non-separable preferences, the SPNES, in which the countries are able to observe each other's weapon stocks, and the OLNES, in which the countries precommit themselves to a sequence of investment choices and cannot monitor foreign weapon stocks. Therefore, the OLNES is also calculated for the same discretised finite-horizon dynamic game which implies solving one large bimatrix game for the Nash equilibrium. Thus the OLNES solves one 128 x 128 bimatrix game and the SPNES solves 5461 (i.e., $(4^7-1)/3$) 2 x 2 bimatrix games. Even for this simple example, the computational costs of calculating the OLNES and the SPNES are considerable. When the horizon or the number of investment possibilities goes up, the computational cost goes up exponentially.

Three typical situations occur for the following three sets of parameter values: \{\alpha_3 = 0.2, \alpha_4 = 1.0\}, \{\alpha_3 = 0.2, \alpha_4 = 0.5\} and \{\alpha_3 = 0.3, \alpha_4 = 0.5\}, labelled I, II and III, respectively.
The results are:

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th></th>
<th></th>
<th>Case II</th>
<th></th>
<th></th>
<th>Case III</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPNES</td>
<td>OLNES</td>
<td>SPNES</td>
<td>OLNES</td>
<td>SPNES</td>
<td>OLNES</td>
<td>SPNES</td>
<td>OLNES</td>
</tr>
<tr>
<td>$\tau(1)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(1)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(2)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(2)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(3)$</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(3)$</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(4)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(4)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(5)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(5)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(6)$</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$g^*(6)$</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau(7)$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$g^*(7)$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$a(7)$</td>
<td>4.396</td>
<td>4.396</td>
<td>2.832</td>
<td>2.935</td>
<td>3.618</td>
<td>3.724</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a^*(7)$</td>
<td>4.794</td>
<td>5.032</td>
<td>3.250</td>
<td>3.356</td>
<td>4.279</td>
<td>4.279</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{q}$</td>
<td>14.326</td>
<td>14.033</td>
<td>15.015</td>
<td>14.906</td>
<td>10.469</td>
<td>10.520</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{q}^*$</td>
<td>17.423</td>
<td>17.527</td>
<td>16.770</td>
<td>16.599</td>
<td>12.720</td>
<td>12.577</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{q} + \hat{q}^*$</td>
<td>31.749</td>
<td>31.560</td>
<td>31.785</td>
<td>31.505</td>
<td>23.189</td>
<td>23.097</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The most remarkable conclusion is that the SPNES leads to less arms accumulation in both the West and the East than the OLNES and is therefore better for the world as a whole. This is exactly what one expected from Section 4.2, since closed-loop information effectively reduces the marginal benefit of weapons as each country now takes account of the fact that, when it invests in an additional weapon, the rival will react and escalate its arms accumulation. It is clear that a unilateral arms treaty should give each country the opportunity to monitor the rival's weapon stocks. The familiar result that the East has more weapons, greater security and therefore greater welfare than the West is also illustrated in this discretised version of the dynamic game. The general pattern in each case is that heavy investments in arms take place in the early periods of the game and that more consumption takes place later on. This is not unreasonable in view of the fact that the marginal value of weapons after the game is zero. In situation II both countries invest less in guns
in the SPNES and thus consume more butter than in the OLNES and both countries achieve a higher level of welfare. In situation I the East invests less in guns in the SPNES and thus consumes more butter. However, because the West invests the same in weapons in the SPNES as in the OLNES, the East feels less secure and the tradeoff between more butter and less security results in a lower level of welfare for the East. The West consumes the same amount of butter but feels more secure, so that the West achieves a higher level of welfare in the SPNES than in the OLNES. In situation III it is just the other way around. Situations I and III are peculiar to asymmetric economies and discretised games and cast doubt on the robustness of the result obtained in Situation I.

6. Concluding Remarks

This paper tries to contribute to the literature on the arms race in two ways. Firstly, the arms race is modelled from an economic point of view so that adequate attention is given to the 'guns versus butter' dilemma and to the way in which investment in arms is realised. A decentralised market economy where distortionary taxes finance the investment in arms is contrasted with a centrally planned economy. The analysis is based on a utility function, which depends on consumption, leisure and defence. Secondly, the impact of information on the investment strategies is investigated where especially information on both the own and the foreign weapon stock over time is considered. For Cobb-Douglas utilities the separability ensures that there are no differences in the outcomes under precommitment with static information patterns and in the subgame-perfect outcomes with dynamic information patterns. However, more general CES utilities lead to different equilibrium solutions. It has been argued that the subgame-perfect Nash equilibrium leads to lower stocks of weapons than the open-loop (or precommitment) Nash equilibrium. This implies that the previous literature (e.g. Brito 1976) has tended to overestimate the extent of the accumulation of arms. As far as policy is concerned, this suggests that countries should be given the opportunity to monitor each other's weapon stocks as this will lead to less arms accumulation and therefore to a safer world. It has also been shown that a decrease in the elasticity of substitution between private goods, consumption and leisure on the one hand, and defence on the other, leads to larger weapon stocks.
In this paper it has been argued that monitoring of each other's weapon stocks may be a good thing, because it leads to smaller weapon stocks, higher consumption levels and greater welfare. However, monitoring is not necessarily the same thing as verification. Historically, verification has been endogeneous in the sense that once countries agree on less arms accumulation they agree on verification by formalising how the results of monitoring procedures should be interpreted. Verification was a major issue prior to SALT, but once the limits had been agreed both sides accepted that existing monitoring methods, National Technical Means (satellites), were adequate for verification which is something they could have accepted before. Likewise scientists apparently think that a Comprehensive Test Ban Treaty can be monitored, but governments refuse this and use verification as a major argument against an agreement. Hence, verification should really be considered a bargaining issue rather than the provision of public information. Another problem is that monitoring itself, i.e. counting weapons, is not so much of a problem, but agreeing on what the numbers mean (given different capabilities of the weapon stocks) is almost always a problem. Such problems may make feedback more difficult.

There are at least four directions for future research. The first is to extend the framework to allow for international trade between the West and the East. This would make the welfare of each country depend on the government policies of the rival country. For example, an increase in foreign government spending or taxes might lead to a reduction in foreign consumption and the resulting balance of payments deficit might be choked off by a depreciation of the real exchange rate and therefore home welfare might fall. Such trade wars introduce flow externalities over and above the stock externalities caused by conflict over arms accumulation, but they do not change the qualitative character of the results described in this paper. The second direction is to think more carefully about the asymmetries between the West and the East. It seems more reasonable to assume that the West is in a regime of Keynesian unemployment and the East is in a regime of repressed inflation (e.g. Malinvaud 1977). Since the West has an excess supply of goods and labour, Western arms now have benefits not only in terms of higher security but also in terms of Keynesian employment-generating effects. However, the East has an excess demand for goods and labour, so that expenditure on arms in the East increases security but leads to longer queues and inflation. This seems to
suggest that the West may have more of an incentive to invest in arms than the East. The third direction is to consider extensions that allow the private sector to accumulate assets and the government to issue debt. This may introduce problems of time inconsistency and credibility of each government vis-à-vis its private sectors. For example, the government might announce to levy low capital taxes and high labour taxes in order to encourage investment. However, once the machines have been accumulated, the government has an incentive to renege and levy higher capital taxes than promised. Another example is a surprise inflation tax. It may well be that, when there is international trade and governments cannot credibly precommit themselves, international cooperation is counter-productive as multilateral reneging does not induce a depreciation of the exchange rate and therefore inflation costs (see Rogoff 1985; van der Ploeg, 1988). It would be exciting to investigate whether such counter-intuitive results can be obtained in multi-country models of arms conflict. It would also be interesting to analyse reputational equilibria within the context of such models. The final direction of future research extends the economic framework of competitive arms accumulation from two to more than two countries. It may well be that, if the analysis is restricted to two countries, important adverse responses from third countries are overlooked. Cooperation may no longer lead to multilateral disarmament, i.e. a moratorium on investment in weapons, as this might provoke an arms build-up or even an attack of third countries. The economic analogue of this result for international monetary coordination between three countries is provided by Canzoneri and Henderson (forthcoming).

Note

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