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Hollanders, D.A.; Vis, B.

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By David Hollanders, Barbara Vis

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Voters’ commitment problem and welfare-program reforms

David Hollanders a, Barbara Vis b, *

aDepartment of General Economics, University of Tilburg, Postbox 90153, 5000 LE Tilburg, The Netherlands
b Department of Political Science, VU University Amsterdam, de Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
* Corresponding author. Tel.: +31 20 598 6821; fax: +31 20 598 6820.
Email address: b.vis@fsw.vu.nl

Abstract

This paper proposes that reforms by vote-seeking governments and the existence of reform-adverse voters are logically compatible. This results from a commitment problem on the part of voters. Due to economic voting voters cannot credibly commit to reelect a non-reforming government during a recession. The empirical implication of this voter commitment mechanism is that governments only adopt visible welfare-program reforms during economic lows, which is what the empirical political-economic literature has established.

JEL classification: D72; D78; H11; H5; I38; J48
Key words: Commitment; Political Economy; Reform; Welfare-Programs
1. Introduction

Governments often do not adopt reforms that economists consider efficiency enhancing. However, when governments take up such reforms, they typically do it during economic downturns or economic crises. For 21 OECD-countries between 1975-2003, Høj et al. (2006) find that economic crises are associated with higher overall reform activity. Here economic crises were defined as an output gap larger than 4%, while reform activities included both labour market and product market reforms (see also Pitlik and With, 2003).

This paper adds to the comparative welfare state literature and the political economic literature on reform by proposing a mechanism that explains both the occurrence and the timing of reforms in welfare-programs. Whereas there exists much political-economic literature about the commitment problem of politicians, the mechanism here, which we call voters’ commitment problem, instead derives from the commitment problem faced by voters. We present a simple game-theoretical model that formalizes how economic voting makes voters unable to commit to reelect a government that will not reform during economic hardship. This voters’ commitment problem makes that vote-seeking governments are only willing to consider reform when they know they will likely be voted out of office anyway amidst the economic hardship. Consequently, an electorate that opposes reform and a government implementing reform are reconcilable. The central empirical implication is that reforms take place during economic downturns only.2

The argument we present hinges on three underlying assumptions. The first is that reforms are unpopular among most voters. Boeri et al. (2002) indeed find in a survey of the opinions on pension reform in Germany and Italy that citizens oppose reform. Moreover, Brooks and Manza (2006) show that most policy preferences are in favor of welfare-programs (see also Boeri et al., 2001; Blekesaune and Quadagno, 2003; Becker, 2005). Finally, Van Groezen et al. (2009) find that especially the desire to remain at the status quo
induces voters to be weary of pension reforms, even if these might improve their financial position.

A second assumption is the imminence of economic voting; that is, citizens – correctly or not – blame their government for weak economic performance (Tufte, 1978; Hibbs, 1979; Lewis-Beck and Paldam, 2000). There is a widespread consensus in the literature that economic voting is ‘a generalized phenomenon in industrial democracies’ (Pacek and Radeliff, 1995: 44; see Van der Brug et al., 2007). Powell and Whitten (1993) find a mediating effect of the clarity of political responsibility. In majority systems, in which it is clear who is politically responsible, voters are more likely to vote retrospectively (that is economically) than in systems with lower degrees of clarity. Examples of the latter are minority governments or parliamentary ones (see also Whitten and Palmer, 1999).

A final assumption is that government is first and foremost vote-seeking, but can also be rent- or policy-seeking. Both approaches are non-controversial and standard in the literature. Governments make a trade-off between remaining in office by adopting a policy that voters want and adopting a policy that they want themselves for some reason (“rent-seeking”). In particular, when facing sure electoral defeat they turn policy-seeking (or rent-seeking), as winning office is no longer possible.

As both voters and politicians are rational and forward-looking in our model, the argument offers a rationalization of the occurrence and timing of reforms and thereby does not depend on bounded rationality or irrationality of any actor, which is not to deny that this may be relevant.

The structure of the paper is as follows. First, we discuss the comparative welfare state and political-economic literature on reforms and argue that this literature does not adequately account for the occurrence and, especially, the timing of reforms. Next, we introduce the game-theoretical model. We end with some concluding remarks.
2. Related literature

When does reform occur? That is to say, what is its timing? The answers put forward in the comparative welfare state literature and the political-economic literature on reform do not fully explain the timing of reforms, as we discuss now.

2.1 Comparative welfare state research

A first body of comparative literature on the welfare state argues that the main cause for pressure on the welfare state – and thereby for reform – is socio-economic change and the ensuing problem load (Rodrik, 1997; Garrett and Mitchell, 2001; Huber and Stephens, 2001; Pierson, 2001; Iversen, 2005). Theoretically, this argument makes sense. For example, if population ageing is projected to lead to budgetary problems, it is likely that the government will take measures to try and deal with the issue. However, the socio-economic account provides little theoretical footing as regards when exactly such measures are taken. When do governments pursue cutbacks that may be necessary, but which are also electorally risky?

A second perspective on welfare reform focuses on political struggles, sometimes integrating socio-economic variables too. The argument is that the variation in the degree and type of welfare state reform is influenced by the partisan complexion of the government (e.g. Ross, 2000a; Korpi and Palme, 2003; Allan and Scruggs, 2004) or by the dynamic of party competition (e.g. Kitschelt, 2001; Green-Pedersen, 2002). While offering useful insights into some of the factors that hinder or facilitate reform, this account cannot explain when governments engage in electorally risky activities. Why, for example, have unpopular measures been taken by some right-wing and by some left-wing governments in Germany, Denmark, and the Netherlands, but not by others (see Vis, 2009)?
A third body of comparative welfare state literature focuses on the influence of institutions. The usual argument is that countries with the least institutional hurdles, and therefore the highest degree of power concentration, should display the highest degree of welfare reform. Consequently, reform should be higher in Westminster countries than in political systems with a high level of power fragmentation (like Switzerland and the US). Several empirical studies support this hypothesis (e.g. Bonoli, 2001; Swank, 2001). However, some authors note that the reverse relationship is also plausible (see Ross, 1997). Political systems concentrating political power also concentrate political accountability. As a result, ‘(...) voters know very well who they may blame for unpopular cutbacks’ (Starke, 2006: 109). In political systems where power is fragmented, conversely, avoiding blame for unpopular measures is easier (Weaver, 1986; Pierson, 1994), which may result in more retrenchment. The institutionalist approach has been helpful for explaining the cross-national variation in welfare reform. It cannot explain the when of reform as governments in the same country face the same institutional constraints and opportunities (Armingeon et al., 2005).

A final strand of literature proposes that ideas matter for welfare state change. The argument here is that by invoking a specific discourse or imperative, governments may overcome the hindrances to change and successfully implement reform (Cox, 2001; Schmidt, 2002; Stiller, 2007; see Campbell, 2002; Lieberman, 2002). But when will this happen? According to Ross (2000b: 173), this is most likely when the ‘(...) underlying ideas, frames and policy structures are not wildly incongruous with new initiatives’. Studies focusing on the importance of ideas have added to the knowledge of the process of welfare reform. However, this literature offers little theoretical foothold as regards when ideas matter (see Lieberman, 2002).

Klitgaard (2007) offers a partial solution to the question of when reform occurs by arguing that Social Democratic parties in universal (Social
Democratic) welfare states pursue market-oriented reforms when the party elite considers the policy problems to be a threat to the welfare state’s legitimacy. However, this explanation cannot be generalized to other type of parties or types of welfare regimes, as it premises on the assumption that the universal welfare state is a power resource for Social Democratic parties.

2.2 Political economy of reform

Next we discuss political-economic literature on reforms. Selén and Ståhlberg (2007) posit that the pension reform in Sweden, which gradually transformed the public defined-benefit pension system into a so called notional defined contribution one, could be implemented successfully because the reform would benefit a majority of the voters. Adopting a political-economic perspective, they argue that the winners who would vote in favor of the reform outnumber the losers who would vote against it, accounting for the reform. The underlying assumption that voters know *ex ante*, and with certainty, if they are a winner or loser of the reform is problematic. For most voters, pension systems are complex – to say the least. Calculating the present value of expected pension benefits and expected contributions in the old and the proposed new system is something that surely goes well beyond the capacities of the ‘average’ voter (see Boeri et al. 2002).

In a recent political-economic contribution, Kemmerling and Neugart (2009) propose that countries in which financial markets are politically powerful – measured by among other things the degree of assets held by institutional investors as a share of GDP – are more likely to pursue pension reform that increases the private savings component. The reason is that financial markets have an interest in such reforms, as they typically manage defined-contribution schemes. Although this argument is plausible, it fails to account for the large-scale pension reform that included a shift toward defined-contribution in, for example, Sweden (Selén and Ståhlberg, 2007), as
the financial market of that country is comparatively weak (BIS, 2007).

3. A new mechanism: Voters’ commitment problem
We propose a new mechanism, voters’ commitment problem, to account for the timing of welfare-program reforms that is applicable to democratic systems. The trust of the argument is that due to economic voting voters cannot commit to reelect a government that will not reform when the economy is in a poor state. Due to this commitment problem, reforms take place during economic lows only.

This mechanism differs from political-economic explanations that focus on the failure of reform. This literature has often assigned this failure to the ‘nonneutrality’ in the distribution of gains and losses in society. Reform is non-neutral because the winners from the status quo are assumed to be politically strong, whilst the losers are politically weak. Fernandez and Rodrik (1991) expand the argument, stating that it is the uncertainty about the distribution of gains and losses that impedes reform. If some of the winners and losers of the reform cannot be identified ex ante, the status quo is likely to prevail. While this argument offers a convincing account of the conditions under which reform does not occur, our mechanism examines the conditions under which it does.

Elections come with a pre-election commitment problem on the part of politicians, as they cannot commit themselves to actually implement the plans they propagate during elections. When in office, they may use their power to break the election promise with the voters. The crucial aspects of elections, the ability to ‘throw the rascals’ out at the next election, partly solves this commitment problem. There is however a similar commitment problem between elections on the side of the voters. This problem results from economic voting. Voters generally oust a government during an economic recession, because they blame politicians for it. Due to the omnipresence of
economic voting, the promise to do otherwise in the absence of reform is therefore not credible. The pledge of the representative voter before elections to reelect the government if it refrains from a reform is therefore not believable and certainly not enforceable. Therefore a vote-seeking government might reform during a recession, as a reform does not influence the prospect of reelection. The only consideration for the government is whether they intrinsically support the reform in the first place. For several reasons this may be the case. A government may be in favor of a reform because it thinks it is efficiency-enhancing, because it has an ideological preference, or because of rents provided by minorities in favor of the reform. The following game formalizes the argument.

3.1 The game

There are three players in the game: two identical politicians and one representative voter. The two identical politicians both have a time-additive utility function, \( V_\alpha \), with a felicity function \( U(x_i) \) which is concave and positive and where \( x_i \) represents consumption at time \( i \). The discount rate is \( \beta \). The utility-function at time \( t \) is given by:

\[
V_t = \sum_{t=0}^{\infty} \beta^{(i-t)} U(x_i)
\]

At each point of time one and only one politician holds office. If a politician is out of office, he (or she) does not have any decision to make and his utility is normalized to 0. If the politician is in office he receives a positive endowment \( w > 0 \).

For both the politician in office and the voter, the following stage game enrolls:
1. There is a move by nature that determines economic circumstances. With probability \( \lambda \) economic circumstances are good, with probability \( 1-\lambda \) they are
bad.

2. Next, the politician can choose between two actions. The first is to reform, the second is to stick to the status quo with no reform. In the case of reform, the politician receives, next to $w$, a positive amount $r > 0$. This may be interpreted as his personal benefit of reforming, in the form of rents or ideological satisfaction. It may also be interpreted as an efficiency gain, internalized by the politician (and not internalized or not understood by the voters).

3. After observing the state of the economy and the action of the politician, the representative voter has the option to either reelect the politician or not. If the politician is not reelected, the other politician is automatically elected. The graph depicts the sequence of the stage game where the (re)installment of the new government at $t=4$ closes the stage game.

=Graph 1 here=

The preferences of the voter are such that he prefers no reform to a reform under any circumstance.

The action space of the politicians consists of two actions, reform and no reform. The action space of the voter also consists of two actions, reelection or no reelection.

We restrict the strategies and the equilibria of the players in several ways. First, we only consider pure strategies. Second, we restrict attention to Markov equilibria. In Markov equilibria, actions of players are a function of the current, pay-off relevant state. Here the state is defined as the state of the economy (either good or bad). This rules out that agents condition their actions on the entire economic history or the history of others agents’ actions. Third, as the two politicians are identical we only consider symmetric equilibria where both politicians have the same strategy.
For the politician a strategy maps the state of the economy into the action reform or no reform. A strategy thus consists of a pair that prescribes the action when the economy is in a bad and good state respectively. The voter has a strategy that maps the economic condition and the action of the politician into the action reelection or no reelection. Therefore the strategy of the voter has to prescribe an action in four circumstances, conditional on the state of the economy (either good or bad) and on the action of the politician.

3.2 Equilibria with perfect conditioning

First, the situation is considered where the voter can condition reelection perfectly on the occurrence of reform. So, economic circumstances are not relevant in this case. Then, the voter has the optimal strategy to reelect a politician that does not reform and does not reelect a politician that reforms. Subsequently, there are two potential pure strategy equilibria, one with both politicians always reforming and one with both politicians never reforming.

The equilibrium with both politicians always reforming may occur if the following condition holds:4

\[ U(w + r) - U(w) > \frac{\beta}{1 - \beta} U(w) - \frac{\beta^2}{(1 - \beta)^2} U(w + r) \]  

The left-hand side gives the immediate gain of reforming compared to not reforming. The right-hand side gives the difference of the remaining lifetime utility of never reforming and the lifetime utility of always reforming (given that the other politician always reforms). In the latter case both politicians are in office every second period and reform when they are. Under condition i, given that the other politician always reforms, it is best to do likewise. As the two politicians are similar, this constitutes a Nash-equilibrium. Note that if \( \beta = 0 \), the condition is always met, as \( r > 0 \). In that case, future income is not considered at all and reforming is more attractive.

Another possible equilibrium is one with both politicians not reforming.
A necessary condition for such an equilibrium is:

\[ U(w + r) - U(w) < \frac{\beta}{1 - \beta} U(w) \]  

(ii)

Note that if \( \beta = 0 \), the condition is never met. This condition states that - given that the other politician never reforms- never reforming and hence holding office forever after, leads to higher life-time utility than reforming once and never being (re)elected again.

Summarizing, there are three possibilities.

1. \( U(w + r) - U(w) \) is small and condition ii is met and condition i is not met. This means that lifetime utility of always holding office is large. Reforming is not attractive, even if the other politician does likewise. The equilibrium with both politicians never reforming occurs.

2. \( U(w + r) - U(w) \) is large and condition i is met and condition ii is not. Utility of even a one time reform is large and there will always be reform. In that case there is no way for the voter to discipline the government by not reelecting him.

3. \( U(w + r) - U(w) \) has an intermediate value and both condition i and ii are met; then both equilibria are possible. Which one will occur depends on the ability of the two politicians to coordinate the equilibrium of both of them reforming. That equilibrium will provide both politicians with a higher lifetime utility than the equilibrium where both never reform. If the politicians indeed succeed in coordination, a further strategy of the voter could be to never reelect one of the two politicians once he reformed and to always reelect the other one, irrespective of him reforming or not. With such a strategy of the voter, the politician the strategy is aimed at will not reform.

Note that it is not possible that both conditions are not met, as the right-hand side of condition ii is larger than the right-hand side of condition i.

3.3 Equilibria with economic voting

We now turn to the case where the voter can only condition reeelection
imperfectly on the action of the politician in office. As an extreme case of economic voting, the politician is never reelected when the economy is slowing down, irrespective of whether he reformed. This constitutes the commitment problem of the voter who cannot credibly commit to reelecting a government that does not reform. Consequently, the politician will always reform during a recession. For the politician reforming does not alter the prospects of being reelected while there is a positive pay-off \( r>0 \). During booms, a politician is still never reelected after a reform, as before.

Again, two equilibria are possible. The appendix shows the following necessary condition for an equilibrium where both politicians will not reform in good times. This is the analogy of condition ii:

\[
U(w + r) - U(w) < \frac{\beta (1 - \lambda \beta)[\hat{\beta} U(w) + (1 - \hat{\beta})U(w + r)]}{1 - 2\beta \lambda - \beta^2 + 2\beta^2 \hat{\lambda}} - \frac{\beta^2 (1 - \lambda)U(w + r)}{1 - \beta \lambda - (1 - \lambda)\beta^2} \tag{iii}
\]

Note that condition iii reduces to condition ii if \( \lambda = 1 \). In that case economic circumstances are always positive and the voter can perfectly condition reelection on the actions of the politicians. Note also that if \( \beta = 0 \), the condition never holds; in that case the future is not taken into account by both politicians and they will therefore always reform.

For the equilibrium with both politicians reforming the necessary condition reads:

\[
U(w + r) - U(w) > \frac{\beta}{1 - \beta \lambda - (1 - \lambda)\beta^2} [\hat{\beta} U(w) + (1 - \hat{\beta})U(w + r)] - \frac{\beta^2}{1 - \beta^2} U(w + r) \tag{iv}
\]

Note that, as before, condition iv reduces to condition i if \( \lambda = 1 \). Note also that if \( \beta = 0 \), the condition always holds.

The appendix shows that the right-hand side of condition iii is larger than the right-hand side of condition iv. Therefore it is not possible that both conditions are not met and there is always at least one equilibrium.
Generally, there are again three possibilities. A unique equilibrium with both politicians always reforming, a unique equilibrium with both never reforming during booms or the possibility that there are two equilibria. Which one occurs in the latter case, depends on which one the two politicians coordinates. The equilibrium where both reform has higher life-time utility than the one where neither reforms. This follows from the observation that condition iii is met and it is then better not to reform than to reform, given that the other politician does not reform. Condition iv is also met, implying that it is better to reform than not to reform, given that the other politician reforms. Generally it holds that not reforming when the other reforms gives a higher lifetime utility than not reforming when the other does not reform. In both cases, the politician has the same income when in office and is only out of office after bad economic circumstances. In the latter case however the probability of coming back into office is smaller, as the other politician does not reform during booms. Combining these observations, it holds that in the case of multiple equilibria, the two politicians have higher lifetime utility in the equilibrium of both reforming than of both not reforming. For the voter the opposite holds; the equilibrium with both not reforming provides higher lifetime utility.

3.4 Comparative statics

We investigate the comparative statics to assess how the willingness to reform and the ability of voters to discipline politicians is influenced by the four different parameters in the model.

=Table 1 here=

It can be shown that, ceteris paribus and for all \( w \), condition iii will more likely be met when the base wage \( w \) increases, that is the right-hand side
increases more than the left-hand side. If the wage increases, reform is less likely to occur. This follows as reform leads to the loss of the base salary \( w \) in the next period and possibly subsequent periods. The higher this loss is, the less likely a government is to reform. This implies that higher income for government members decreases the probability of reforms during prosperous economic times. The opposite holds for condition iv; the higher is \( w \), the less likely the condition is met and the less likely is an equilibrium with both reforming.

Furthermore, ceteris paribus and for all \( r \), condition iii will less likely be met when rents \( r \) increase; then the right-hand side decreases more than the left-hand side. The higher is \( r \), the more likely reform will be. This formalizes that higher rents of reform make its undertaking more attractive. The opposite holds for condition iv; the higher is \( r \), the more likely the condition is met and the more likely an equilibrium with both reforming is.

For both conditions, the comparative statics of \( \lambda \) and \( \beta \) are not straightforward. The partial derivative of the bound can be both positive and negative. The sign depends on the particular values of the parameters and the functional form of the utility function, making general predictions of the effect impossible.

4. Conclusion
This paper argues that the coexistence of vote-seeking governments pursuing unpopular welfare-program reforms and reform-averse voters are reconcilable, a finding that helps solve a theoretical puzzle in the literature on such reforms. In line with for example Høj et al. (2006), the empirical implication of our theoretical model is that reforms, if at all, are initiated during recessions. Our contribution lies in presenting the theoretical mechanism, which is that the occurrence and timing of reforms springs from an intrinsic commitment problem of voters in times of economic recession.
Due to economic voting, there is a high chance that the incumbent party or parties will not be re-elected, irrespective of their particular policy. Subsequently, other considerations to reform come to forefront for governments, which may then undertake a reform. These other factors include rents, ideology or a genuine wish to implement efficient policies.

References


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The dynamics of public opinion toward unemployment and inflation. 


Graph 1

Move by nature: good or bad conditions
Incumbent decides whether to reform or not
Elections
Incumbent or opposition (re)installed
Table 1

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<thead>
<tr>
<th>Comparative statics</th>
<th>Condition iii</th>
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Notes

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2 Note that our argument focuses on visible welfare reforms. The argument does not necessarily apply to a series of silent and/or small cutbacks that combined might have a large impact.

3 This section draws heavily on Vis (2009).

4 Here and in the remainder of the paper, the familiar convergence result of a geometric series is used.
The Appendix

This appendix derives some results given in the main text.

Derivation of equation iii

Equation iii gives the condition for the equilibrium where both politicians do not reform during booms:

\[ U(w + r) - U(w) < \frac{\beta(1 - \lambda \beta)\{\lambda U(w) + (1 - \lambda)U(w + r)\}}{1 - 2\beta\lambda - \beta^2 + 2\beta^2\lambda} - \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} U(w + r) \]

To derive this condition, assume the first of the two politicians does not reform. It is best for the second politician to do likewise if, given the first politicians’ strategy, the life-time utility of no reform is at least as high as that of always reforming.

If the second politician reforms during good times, he has utility \( U(w + r) \) when in office and is then voted out. When out of office he will at one point be back in office, he also has a positive life-time utility at the beginning of the next period when still out of office, denoted here \( U^{\text{out}} \). \( U^{\text{out}} \) can be determined in a recursive manner:

\[ U^{\text{out}} = (1 - \lambda)[\beta U(w + r) + \beta^2 U^{\text{out}}] + \beta\lambda U^{\text{out}} \]

With probability 1- \( \lambda \) economic circumstances will be bad, and the other party will be voted out. Then the politician will be back in office within one period. Otherwise, he remains out of office which provides lifetime utility of \( U^{\text{out}} \) the next period. Solving this equation:

\[ U^{\text{out}} = \frac{(1 - \lambda)\beta}{1 - \beta\lambda - (1 - \lambda)\beta^2} U(w + r) \]

This gives life-time utility of reforming of:

\[ U(w + r) + \beta U^{\text{out}} = U(w + r) + \beta \frac{(1 - \lambda)\beta}{1 - \beta\lambda - (1 - \lambda)\beta^2} U(w + r) \]
If the second politician does not reform during good times, he has utility $U(w)$ and he stays in office. This gives lifetime utility

$U(w) + \beta U^\text{in}$

It remains to determine $U^\text{in}$. This can be determined with the following two equations:

$U^\text{in} = \lambda[U(w) + \beta U^\text{in}] + (1 - \lambda)[U(r + w) + \beta U^\text{out}]$

$U^\text{out} = \lambda \beta U^\text{out} + (1 - \lambda) \beta U^\text{in}$

Here $U^\text{in}$ and $U^\text{out}$ are the lifetime utility of entering the stage game while being in and out of office respectively. When a politician is currently in office, he faces a probability $\lambda$ that economic times will be good. If so, he receives both his wage $w$ and he remains in office, which offers again the prospect of $U^\text{in}$ the next period, discounted by $\beta$. With a probability $1 - \lambda$ economic times will be gloomy, in which case he will reform and thus receive $w + r$. In the next period he will be out of office, and has the prospect of $U^\text{out}$, discounted.

Now consider the latter equation:

$U^\text{out} = \lambda \beta U^\text{out} + (1 - \lambda) \beta U^\text{in}$

$U^\text{out} = \frac{(1 - \lambda) \beta}{1 - \lambda \beta} U^\text{in}$

This gives an expression for $U^\text{in}$ in terms of $U^\text{out}$. Using this:
\[ U^{in} = \lambda[U(w) + \beta U^{in}] + (1 - \lambda)[U(r + w) + \beta U^{out}] \Rightarrow \]
\[ U^{in} = \lambda[U(w) + \beta U^{in}] + (1 - \lambda)[U(r + w) + \frac{\beta^2(1 - \lambda)}{1 - \beta \lambda} U^{in}] \Rightarrow \]
\[ U^{in} - \beta \lambda U^{in} = \frac{\beta^2(1 - \lambda)^2}{1 - \beta \lambda} U^{in} = \lambda U(w) + (1 - \lambda)U(r + w) \Rightarrow \]
\[ U^{in}[1 - \beta \lambda - \frac{\beta^2(1 - \lambda)^2}{1 - \beta \lambda}] = \lambda U(w) + (1 - \lambda)U(r + w) \Rightarrow \]
\[ U^{in}\left[\frac{(1 - \beta \lambda)^2 - \beta^2(1 - \lambda)^2}{1 - \beta \lambda}\right] = \lambda U(w) + (1 - \lambda)U(r + w) \]

Now it remains to working out the brackets:
\[ U^{in}\left[\frac{(1 - \beta \lambda)^2 - \beta^2(1 - \lambda)^2}{1 - \beta \lambda}\right] = \lambda U(w) + (1 - \lambda)U(r + w) \Rightarrow \]
\[ U^{in}\left[\frac{1 - 2 \beta \lambda - \beta^2 + 2 \beta^2 \lambda}{1 - \beta \lambda}\right] = \lambda U(w) + (1 - \lambda)U(r + w) \Rightarrow \]
\[ U^{in} = \left[\frac{1 - \beta \lambda}{1 - 2 \beta \lambda - \beta^2 + 2 \beta^2 \lambda}\right][\lambda U(w) + (1 - \lambda)U(r + w)] \]

Lifetime utility of no reform is:
\[ U(w) + \beta U^{in} = U(w) + \beta \frac{1 - \beta \lambda}{1 - 2 \beta \lambda - \beta^2 + 2 \beta^2 \lambda} [\lambda U(w) + (1 - \lambda)[U(r + w)] \]

The politician will not reform if:
\[ U(w) + \frac{\beta(1 - \beta \lambda)[\lambda U(w) + (1 - \lambda)[U(r + w)]}{1 - 2 \beta \lambda - \beta^2 + 2 \beta^2 \lambda} > U(w + r) + \frac{(1 - \lambda)\beta^2 U(w + r)}{1 - \beta \lambda - (1 - \lambda)\beta^2} \]

Derivation of condition iv
An equilibrium with both reforming may arise if:
\[ U(w + r) - U(w) > \frac{\beta}{1 - \beta \lambda - (1 - \lambda)\beta^2} [\lambda U(w) + (1 - \lambda)U(w + r)] - \frac{\beta^2}{1 - \beta^2} U(w + r) \]

Given that the other politician reforms, it is best to do likewise during a boom if the life-time utility of reform is at least as high as that of not reforming
during booms. If the politician also reforms, he has \( U(w+r) \) immediately and every second period. This leads to life-time utility of:

\[
\frac{1}{1-\beta^2}U(w+r) = U(w+r) + \frac{\beta^2}{1-\beta^2}U(w+r)
\]

When the politician does not reform he receives utility \( U(w) \) and stays in office. Denote the lifetime utility of being in office \( U^{in} \) and of being out of office \( U^{out} \). These can be determined by solving the following two equations that recursively define both:

\[
U^{in} = \lambda[U(w) + \beta U^{in}] + (1-\lambda)[U(r+w) + \beta U^{out}]
\]

\[
U^{out} = \beta U^{in}
\]

Solving these two equations give:

\[
U^{in} = \frac{\lambda U(w) + (1-\lambda)U(w+r)}{1-\beta\lambda - (1-\lambda)\beta^2}
\]

This gives the condition for both reforming:

\[
U(w+r) + \frac{\beta^2}{1-\beta^2} U(w+r) > U(w) + \beta \lambda U(w) + (1-\lambda)U(w+r)
\]

Right-hand side of condition iii larger than that of condition iv

Condition iii and iv are respectively:

\[
U(w+r) - U(w) < \frac{\beta(1-\lambda\beta)}{1-2\beta\lambda - \beta^2 + 2\beta^2\lambda}[\lambda U(w) + (1-\lambda)U(w+r)] - \frac{\beta^2(1-\lambda)U(w+r)}{1-\beta\lambda - (1-\lambda)\beta^2}
\]

\[
U(w+r) - U(w) > \frac{\beta}{1-\beta\lambda - (1-\lambda)\beta^2}[\lambda U(w) + (1-\lambda)U(w+r)] - \frac{\beta^2}{1-\beta^2}U(w+r)
\]

To show:

\[
\frac{\beta(1-\lambda\beta)}{1-2\beta\lambda - \beta^2 + 2\beta^2\lambda}[\lambda U(w) + (1-\lambda)U(w+r)] - \frac{\beta^2(1-\lambda)}{1-\beta\lambda - (1-\lambda)\beta^2}U(w+r) > \frac{\beta}{1-\beta\lambda - (1-\lambda)\beta^2}[\lambda U(w) + (1-\lambda)U(w+r)] - \frac{\beta^2}{1-\beta^2}U(w+r)
\]

First note that:
\[ \frac{\beta^2}{1 - \beta^2} > \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \] 
This follows from:

\[ \frac{\beta^2}{1 - \beta^2} > \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \iff \frac{1}{1 - \beta^2} > \frac{(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \iff \\
1 - \beta\lambda - \beta^2 + \lambda\beta^2 > (1 - \lambda)(1 - \beta^2) \iff 1 - \beta\lambda - \beta^2 + \lambda\beta^2 > 1 - \lambda - \beta^2 + \lambda\beta^2 \iff \\
- \beta\lambda > -\lambda \iff \beta < 1 \]

Second note that:

\[ \frac{\beta(1 - \lambda\beta)}{1 - 2\beta\lambda - \beta^2 + 2\beta^2\lambda} > \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \]
This follows from:

\[ \frac{\beta(1 - \lambda\beta)}{1 - 2\beta\lambda - \beta^2 + 2\beta^2\lambda} > \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \iff \frac{\beta}{1 - \beta^2} + \frac{(1 - \lambda\beta)}{1 - 2\beta\lambda - \beta^2 + 2\beta^2\lambda} > \frac{\beta(1 - \lambda) + 1}{1 - \beta\lambda - (1 - \lambda)\beta^2} \]

For \( \lambda = 0 \), there is equality with both terms equaling \( \frac{1 + \beta^2}{1 - \beta^2} \). The derivative of right-hand side with respect to \( \lambda \) equals zero, whereas the derivative of the left-hand side is proportional to \( \beta(1 - \beta)^2 > 0 \). Therefore the left-hand side is larger than the right-hand side for all \( 0 < \lambda < 1 \). As this holds for all \( 0 < \beta < 1 \), the inequality follows.

From this inequality the original condition would follow if \( r = 0 \). When \( r > 0 \), it holds that \( U(w + r) > \lambda U(w) + (1 - \lambda) U(w + r) \). From \( \frac{\beta^2}{1 - \beta^2} > \frac{\beta^2(1 - \lambda)}{1 - \beta\lambda - (1 - \lambda)\beta^2} \) it follows that the right-hand side decreases faster in \( r \) than the left-hand side. Therefore the condition also holds for any \( r > 0 \).