Abstract: Real wage and price adjustment bring recessions to an end with an upswing of demand and employment. The question is whether the upswing will continue until full employment is achieved. Bottlenecks can lead to rising prices, and balance of payments difficulties, causing the onset of a recession before reaching full employment. A macroeconomic model is constructed in which the market clearing equilibrium is unstable and the system settles into a limit cycle with fluctuating, but persistently high, unemployment. The policy conclusion is to slow the boom, preventing bottlenecks and early recession, allowing full employment to be achieved.

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In many western economies it has proved extraordinarily difficult to return to the low levels of unemployment associated with full employment in the early 1970's. Indeed, a common conclusion has been that the underlying "natural rate" of unemployment has risen (see Phelps (1974)), either due to exogenous microeconomic forces such as the replacement ratio (Minford (1985)) or because of hysteresis effects, such as the deskilling or demotivation of the workforce during a depression (Tobin (1980), Blanchard and Summers (1986)). This paper puts forward an alternative possibility; it is shown that an increase in the variability of the price level can cause a discontinuous increase in the average rate of unemployment.

Inflation can set in below full employment due to supply bottlenecks. If expectations adapt quickly inflation spirals leading to a contraction of real balances, or more likely, the introduction of anti-inflationary macro-economic policy, and a downturn, before full employment has been achieved; this makes expansion difficult and may lead to persistent unemployment. De Long and Summers (1986) show that increased price variability can be destabilising, increasing the variance of employment and output. The basic argument used here is the same as in their model; the expectation of future price changes affects current demand. Excess supply leads to price reductions to clear the market, but the expectation of price reductions tends to encourage money holding rather than investment, exacerbating unemployment during the disequilibrium phase (see Canning (1988)). In this paper increased price variability not only causes employment to be more volatile it also reduces the average level of employment. This is because excess demand and excess supply in the goods market have asymmetric effects on firms' employment decisions; if firms cannot sell their output they tend to reduce employment, but excess demand only has an effect if increased
output and sales raise profits; that is, if the marginal product of labour exceeds the real wage.

Persistently high unemployment in the model depends on the presence of supply bottlenecks. In an economy with high unemployment and insufficient demand real wages and prices will tend to fall. This will increase demand; the economy becomes more competitive internationally and the real balance effect aids domestic consumption and investment. The normal argument seems to be that this increase in demand will allow employment to rise to the full employment level, at which point real wages and prices will begin to rise; there is no problem about achieving full employment, though it is possible to overshoot. This assumes that the only bottleneck at which prices start to rise occurs at full employment.

With adjustment costs employment and output will tend to adjust slowly to demand and profitability. If demand rises too quickly firms will push up prices while adjusting capacity to higher output and wholesalers may turn to imports to meet the excess of demand over current output. Rising prices may set off an inflationary spiral, particularly if inflationary expectations adapt very quickly. The inflation is likely to fuel demand in the short run due to the flight from cash to investment and consumption goods to avoid the implicit inflation tax on money. The emerging balance of payments problem requires either devaluation or higher interest rates, either fuelling the inflation further or putting a brake on economic expansion. The experience of France in 1981-82 and the U.K. in 1988-89 is clear evidence of the problems that can beset rapid expansion.

The model analysed here begins by assuming that the government does not intervene; inflation increases up to the point where the real balance effect is negated, and a recession sets in with falling demand and prices.
In practice it is more likely that the government will engineer a recession before things go too far, increasing interest rates or curbing fiscal policy to reduce inflation and control the current account deficit. There is no reason why the turn around, either endogenous or contrived, should not occur below full employment. The picture is one of continual cycles of boom and depression with unemployment always being high, the turnaround at the peaks being caused by inflationary and balance of payments problems rather than excessive real wages.

The reason for the persistent unemployment in the model lies not in the depression but in the boom. The depression, though bad, always comes to an end, to be followed by an upswing. The problem is that this upswing may come to an end for financial, that is, nominal price, reasons, before the equilibrium position, in real wage and output terms has been attained. This emphasis on the boom was a standard feature of pre-Keynesian cycle theory. Robertson (1922) paints much the same picture of the boom as drawn here, and argues that, in addition, in the early stages of a boom the banking system will increase lending, increasing excess demand (excessive relative to current, not equilibrium, output). Eventually the squeeze on bank reserves due to price rises and an increased demand for cash will lead to high interest rates and recession.

Since inflation can take off below full employment the non-accelerating inflation rate of unemployment (NAIRU), which is approximately the average rate over the cycle, can be below full employment. Contrary to the claims of Friedman (1968) and Lucas (1978) the NAIRU cannot be identified with the equilibrium or natural rate of unemployment based on microeconomic considerations. Indeed, it is easy to show that the NAIRU depends on the speed of adjustment parameters of the model, particularly the rate at which
Our results depend on the somewhat un-Keynesian assumption that employment adjusts slowly, and prices adjust quickly, to excess demand below full employment. This underpins a revival of Robertson's argument that the key to curing unemployment lies in preventing demand and price increases from becoming excessive in the boom. Credit controls can work because they prevent excessive increases in the price level during the temporary disequilibrium phase of the upswing in which demand exceeds output. By slowing adjustment towards equilibrium they can make the equilibrium stable and achievable.

A formal model is presented in order to establish the logical possibility of persistent unemployment when all prices move in response to excess demand and supply. The model has a unique equilibrium given by full employment output and stable prices. The dynamic results are that if expectations adjust slowly this equilibrium is stable while if they adjust quickly the equilibrium is unstable. In the case of instability and non-convergence it is shown that provided the speed of employment and real wage adjustment are slow there exists a globally stable limit cycle in which employment oscillates around a point below full employment, and never achieves full employment, even at its peak. It is shown that a monetary policy involving cutting the money supply in response to inflation and increasing it in response to deflation tends to stabilise prices and so can generate a higher average level of employment. However, if the money supply is endogenous, increasing with inflation and falling when prices fall, the system can become unstable even if expectations are fairly static.

The existence of stable limit cycles in macroeconomic models is a common result. However, these models generate limit cycles around the equilibrium
point. Kaldor (1940), Cheng and Smyth (1971), Goodwin (1967), Rose (1967), and Benassy (1984) all present models of this type. If we accept that the economy can only be in equilibrium at full employment then in order to explain persistent unemployment we require cycles persistently below the equilibrium point. The reason for the absence of persistent unemployment in this literature on cycles are stated in the Poincare-Bendixon and index number theorems (e.g. Hirsch and Smale (1974)). Every path in a two dimensional dynamic model on a bounded domain either converges to equilibrium or cycles around an equilibrium. It follows that in two dimensional models persistent unemployment is possible only if we have an equilibrium with unemployment, or the domain of the system is unbounded.

Varian (1977) presents a model of persistent unemployment with an unemployment equilibrium. Unemployment is due to lack of demand; price reductions are assumed not to increase demand because real balance effects are ruled out. Tobin (1975) has persistent unemployment, but only by allowing prices to fall indefinitely, at an ever increasing rate, so that the real balance effect is always outweighed, even at very low prices, by the depressing effect of the expected future deflation. Neither of these models seems very plausible as an explanation of persistent unemployment. They highlight the problem that, in two dimensional models, persistent unemployment can be explained only in unsatisfactory ways.

In two dimensional models cycles must be around an equilibrium; in three or more dimensions, however, cycles can be constructed far from equilibrium. This suggests that the difficulty in obtaining persistent unemployment in cycle models has been due to the modelling strategy adopted, using the well-understood two dimensional framework, rather than any intrinsic economic forces. This paper combines the demand side of
Tobin's (1975) model with the supply side of Varian's (1977) model into a four dimensional dynamic system on a bounded domain, with cycles below the full employment equilibrium. While more difficult to analyse, higher dimensional models allow a richer range of dynamic behaviour and do not impose, a priori, the stringent restrictions implicit in a two dimensional framework.

I. The Model

The model is a composite of Tobin's (1977) demand system and the supply side of Varian (1977). Aggregate demand, \( D \), measured in real terms, is given by

\[
D = D(Q, m-p, \pi^*)
\]

where \( Q \) is the level of output, \( m \) is the log of the money stock, \( p \) is the log of the price level, and \( \pi^* \) is the expected inflation rate. Demand is taken to be increasing in output, due to consumption out of factor incomes, strictly decreasing in the price level, due to the real balance effect, and rising in the rate of expected inflation, due to the Mundell-Tobin effect.

We further assume that for any fixed \( Q \) and \( \pi^* \) we can find \( p \) small enough so that demand becomes unbounded and \( p \) large enough that \( D \) becomes zero. This assumption ensures that the real balance effect is always large enough, at extreme prices, to clear the goods market and is necessary to guarantee the existence of an equilibrium in the model.

To simplify the analysis when we examine global behaviour it will be assumed that the demand function is separable in output and can be written in the form

\[
D = Z(m-p, \pi^*) Q
\]  

(1')

that is, the propensity to consume out of factor incomes can be written as
a function of real balances and expected inflation.

Prices are set by firms. Firms attempt to move their relative price down
if they have excess supply, and move it up if they have excess demand.
Letting $d$ be the log of demand and $q$ the log of output this gives
\[ \dot{p} = \pi^* + \alpha (d - q) \]  
(2)
Firms adjust prices in line with expected inflation so as to keep their
relative price steady. If there is excess demand in the product market
firms will attempt to raise their price relative to expected inflation,
while if there is excess supply they will attempt to lower their relative
price.

This price adjustment equation is called Walrasian by Tobin; prices move
to try to clear the goods market. He contrasts it with a Keynesian model in
which firms raise relative prices only after full employment has been
achieved; a theory which can be based on a Phillips' curve relationship
between wages and unemployment, with prices set as a mark up over labour
costs. Letting price adjustment depend on the difference between demand and
current output allows a role for bottlenecks; prices can start to rise
below full employment if output is slow to respond to demand pressure.

Expectations are taken to be adaptive
\[ \pi^* = \beta (\dot{p} - \pi^*) \]  
(3)
Setting $\beta$ equal to zero gives static expectations while, as $\beta$ becomes
large, we approximate rational expectations. Tobin completes his model by
adding an equation for output adjustment as a response to excess demand.
Instead we follow Varian (1977) and model the labour market explicitly.
Output is given by
\[ Q = f(E) \]  
(4)
where $E$ is the level of employment and the production function, $f$, is
increasing but with decreasing marginal productivity; that is, we assume
\( f'(E) > 0, f''(E) < 0 \). Labour supply is given by \( L \) where
\[
L = L(w)
\]  
and \( w \) is the log of the real wage rate. We assume \( L'(w) > 0 \) so that labour
supply is increasing in the real wage. This labour supply equation could be
much more complex; the wealth effect of changes in real balances,
intertemporal substitution effects, the effects of rationing in the goods
and labour market, both current and anticipated, could all influence labour
supply. Equation (5) is adopted merely for simplicity.

The log of money wages, \( \hat{w} \), changes in response to inflation and excess
demand in the labour market:
\[
\hat{w} = \dot{p} + \gamma (E - L)
\]  
Money wages change in line with actual inflation rather than expected
inflation. This implies that labour market condition determine the rate of
change of the real wage. If we were to use the same expected inflation rate
in the wage equation as was used in the price change equation real wages
would depend on conditions in the goods market as well as the labour
market. While this seems realistic, and agrees with Keynes (1936) who
argues that workers can determine money wages, but not real wages, it
complicates the model. Equation (6) can be justified if wages agreements
are fully indexed to the nominal price level, so agreements determine real
wages, that is:
\[
\dot{\hat{w}} = \gamma (E - L)
\]  
To close the model we require an equation determining employment. A
common assumption is that labour demand is given by the minimum of the
profit maximising level and the level which satisfies current demand.
Setting employment equal to labour demand assumes firms can adjust
employment instantaneously to their desired level. Here we use a dynamic version of this approach, assuming that employment adjusts in response to profit opportunities and demand constraints:

$$\dot{E} = \sigma (g(E) - w) + \delta \min (0, (d - q)^2)$$

where $g(E) = \log f'(E)$. If $(d - q)$ is positive, so firms are not demand constrained, employment moves so as to maximise profits at the current real wage; employment rises if the marginal product of labour is higher than the real wage and falls if it is lower. If firms are demand constrained, so that $(d - q) \leq 0$, and cannot sell their current output, they take this into account, reducing employment and output. The effect of demand constraints is taken to be non-linear, a little oversupply has negligible effects but a large oversupply leads to a rapid contraction.

Equation (8) assumes that employment adjusts slowly in response to changes in profitability and demand constraints; it implies labour hoarding when recessions begin but vacancies, and slow expansion of output, when demand pressure builds up. Given the costs associated with hiring, firing and training, including redundancy payments to laid off workers and the lowering of entry standards if the firm increases the rate at which it hires workers, it seems reasonable to take employment adjustment to be smooth, rather than instantaneously jumping to the desired level.

We assume that employment is determined by labour demand, though with a lag rather than instantaneously. Labour supply constraints do not affect employment directly; workers continue to work even if their real wage is lower than the disutility of working, instead of quitting they push up the real wage rate.

Note that equation (8) rules out the underemployment equilibrium found in Varian (1977). If the marginal product of labour exceeds the real wage
firms will expand, even if \((d - q) = 0\). Varian argues that such expansion is irrational since firms cannot sell the extra output. However, it can be justified on two grounds. The excess supply produced will tend to lower prices and increase demand for the firm due to the real balance effect; more importantly, from the individual firm's point of view extra output may lead to a larger market share through a lower relative price, even if the price level is fixed or the real balance effect weak.

Combining equations (1) - (8) we have a dynamic model of the economy which can be written in terms of four variables, \(p, \pi^*, w\) and \(E\):

\[
\begin{align*}
\dot{p} &= \pi^* + \alpha(d(f(E), m-p, \pi^*) - q(E)) \\
\dot{\pi}^* &= \beta(p - \pi^*) \\
\dot{\omega} &= \gamma(E - L(\omega)) \\
\dot{E} &= \sigma(\log f'(E) - \omega) + \delta \min\{0, (d(f(E), m-p, \pi^*) - q(E))\}^2
\end{align*}
\]

where \(d = \log D\) and \(q = \log f\). We assume that all adjustment parameters, \(\alpha, \beta, \gamma, \delta,\) and \(\sigma\) are positive. The equilibrium conditions for the model are:

\[
\begin{align*}
E^* &= L(\omega^*) \\
\log f'(E^*) &= \omega^* \\
\pi^{**} &= 0 \\
D(f(E^*), p^*, \pi^{**}) &= f(E^*)
\end{align*}
\]

The first two conditions imply we are on both the Walrasian demand curve for, and the supply curve of, labour; since one is upward sloping and the other downward sloping this determines \(E^*\) and \(\omega^*\) uniquely. Expected inflation must be zero in equilibrium, and the demand equals supply condition in the goods market determines the equilibrium price level. By assumption the real balance effect is large so we can always find \(p\) to solve for demand equals supply and, since demand is strictly decreasing in the price level, this equilibrium is unique.
We assume that the functions $D$ and $L$ are twice differentiable and that $f$ is three times differentiable (this is necessary to ensure that $f'$ is twice differentiable). Note that the function $\min \{0, (d - q)^3\}$ is twice differentiable; the only problem occurs at $(d - q) = 0$ and by taking the cube we have ensured that the first two derivatives are well defined (and equal to zero) at this point. It follows that we can linearize the system about the equilibrium point and investigate its local stability. This avoids the problem of change of regime when excess demand switches sign; here the system remains differentiable. Eckalbar (1980) discusses the problems associated with analysing models with change of regime.

Let $d_*$ and $d_{**}$ be the partial derivatives of the log of demand, at equilibrium, with respect to expected inflation and real balances.

**Lemma 1.** The equilibrium is locally asymptotically stable if $\beta d_* < d_{**}$ and unstable if $\beta d_* > d_{**}$.

**Proof.** Linearizing the system around the equilibrium point gives

$$
\begin{bmatrix}
\dot{p} \\
\dot{\pi} \\
\dot{w} \\
\dot{E}
\end{bmatrix} = 
\begin{bmatrix}
-\alpha d_* & 1+ \alpha d_* & 0 & \alpha q'(d_{**}-1) \\
-\alpha \beta d_* & \alpha \beta d_* & 0 & \alpha \beta q'(d_{**}-1) \\
0 & 0 & -\gamma L' & \gamma \\
0 & 0 & -\sigma & \sigma f'/f
\end{bmatrix}
\begin{bmatrix}
p \\
\pi \\
w \\
E
\end{bmatrix}
$$

where '$' denotes the derivative and a subscript a partial derivative and all functions are evaluated at equilibrium. The linearized system has four eigenvalues given by the solutions of
\[ \lambda_{1} + (\gamma L' - \sigma f'/f) \lambda + \gamma \sigma (1 - L'f'/f) = 0 \]

and

\[ \lambda_{2} + \alpha (d_{-} - \beta d_{+}) \lambda + \alpha \beta d_{+} = 0 \]

It is easy to check that the roots of the first equation have negative real parts. The second equation has roots with negative real parts if \( \beta d_{+} < d_{-} \) and has roots with positive real parts if \( \beta d_{+} > d_{-} \). It follows that if \( \beta d_{+} < d_{-} \) the system is locally asymptotically stable but if \( \beta d_{+} > d_{-} \) it is unstable.

Q.E.D.

The reason for the emergence of instability is the same as in Tobin (1975); if agents have inflationary expectations the Mundell-Tobin effect leads to increased demand, which tends to push up prices. These price increases lead to further inflationary expectations, giving higher demand and so on. The system is unstable if the effect of higher prices on inflationary expectations and consequent demand increases is sufficient to outweigh the depressing effect of higher prices on demand through the fall in real balances. Since the equilibrium of the system may be unstable it is necessary to discuss global behaviour, and the non-linear terms in the equations become important.
3. Behaviour far from Equilibrium

In order to perform a global analysis of the system we assume that the non-linear terms are such that all trajectories starting near equilibrium are restricted to a bounded set. This seems economically plausible, we assume that prices, employment and the real wage do not become arbitrarily high or low. It is possible to derive such bounded behaviour from assumptions on the underlying equations, but the assumption of bounded behaviour seems more acceptable in itself than technical conditions on the equations which would ensure that trajectories point "inwards" far from equilibrium.

Replacing (1) with (1') the system can be written

\[ \dot{p} = \pi^* + \alpha z(m-p, \pi^*) \]  \hspace{1cm} (9)
\[ \dot{\pi}^* = \beta (\dot{p} - \pi^*) \]  \hspace{1cm} (10)
\[ \dot{w} = \gamma (E - L(w)) \]  \hspace{1cm} (11)
\[ \dot{E} = \sigma (\log f'(E) - w) + \delta \min \left(0, (z(m-p, \pi^*))^3\right) \]  \hspace{1cm} (12)

where \( z(m-p, \pi^*) \) is \( \log Z(m-p, \pi^*) \). The important point about this model is that the pair of equations (9) and (10) form an independent subsystem which can be solved without reference to the other equations. The solution to this pair of equations can then be used in solving (11) and (12).

**Proposition 1.** If \( B_z < z \), the equilibrium of the system is locally asymptotically stable. If \( B_z > z \), the equilibrium is unstable, and, if \( \sigma, \delta \) and \( \gamma \) are small enough, all trajectories (except for those starting at the equilibrium point itself) converge to a stable periodic orbit in which employment oscillates around a level \( E_0 < E^* \). Further, for \( \sigma, \delta \) and \( \gamma \) small
the fluctuations around $E_0$ are small.

proof in appendix.

The solution of the system proceeds in two stages. First we solve equations 9 and 10. These form a two dimensional system on a bounded domain with a unique equilibrium. If $Bz_0 < z_0$ the equilibrium is stable, while if $Bz_0 > z_0$ prices and expected inflation are unstable and must, by the Poincare Bendixson theorem, tend to a stable limit cycle. Now note that equations 11 and 12 can be written, for the appropriate choice of $h$, as

$$\dot{w} = \gamma(E - L(w))$$

$$\dot{E} = \sigma(\log f'(E) - w) + h(p, \pi^*)$$

If prices and expected inflation are stable and converge to their equilibrium values the function $h$ eventually generates a constant term. The dynamics of the real wage and employment levels can then be shown to be stable, converging to the Walrasian equilibrium.

While real wages and employment are stable in the absence of the function $h$, cycles in prices and expected inflation generate a periodic forcing term on employment which may cause instability to emerge. The cycles in prices and expected inflation, and so in aggregate demand, generate cycles in employment. The key to the underemployment result in the theorem is the asymmetric effects of excess demand and excess supply on employment; excess demand causes firms to increase their workforce only if it is profitable so to do, excess supply always has a downward effect on employment.

The economic system described by the model consists of two interacting cycles. There is a cycle, of the type described by Goodwin (1967) in his growth model, in which the real wage and level of employment oscillate, though in the model described here this part of the system, if taken in
isolation, is stable, the real variables tending to their equilibrium levels. In addition there is a "financial" cycle involving the price level and expected inflation rate. This financial cycle does not converge if expectations adapt too quickly; a limit cycle in the nominal variables appears. The reason for this is that price rises lead to the expectation of further rises which are then reflected in price setting; small perturbations may set off a price spiral, though such a spiral eventually ends due to the real balance effect.

The interesting point is the interaction of the two cycles. Unemployment can occur for one of two reasons; excessive real wages or lack of demand for produced goods. Neither of these states can persist indefinitely; real wages will fall, as will prices, and while falling prices may depress demand at a price level sufficiently low the real balance effect will dominate. Once the system has turned around a boom occurs with rising prices and employment.

The boom comes to an end either because real wages become excessive or prices rise so high that the lack of real cash balances causes a financial crisis and a fall in demand. This lack of demand may come about due to the high interest rate associated with low real cash balances, the Keynes effect, or, ultimately, the lack of an adequate means of exchange, a negative Pigou effect.

While real wages are unlikely to become excessive before full employment is achieved it may well be the case, particularly if prices adjust quickly, that the financial crisis and downturn in demand occur below full employment. The system in this case is characterized by a limit cycle in which unemployment is perpetually oscillating; however, even at its peak, the volume of employment is always below the full employment level.
The result requires the somewhat un-Keynesian assumption that employment adjust slowly, but prices adjust quickly, to excess demand. While it has become something of a dogma that Keynesian economics concerns only the short period in which wages and prices are relatively stable, and employment moves to meet demand, this is not the only story told in the General Theory. Keynes makes two points; if wages and prices adjust quickly there is no guarantee that full employment results, but if money wages are stable then demand determines employment, justifying demand management. While the emphasis has been placed on stable money wages, with the consequent decline of Keynesian economics due to the obvious non-stickiness of money wages in modern times, the question of the ability of speedy wage and price adjustment to clear the labour market remains.

The central reason for the persistent unemployment in the paper lies not in the depression but in the boom. The depression, though bad, always comes to an end, to be followed by an upswing. The problem is that this upswing may come to an end for financial, that is nominal price, reasons, before the equilibrium position, in real wage and output terms, has been attained.

Despite the fact that employment is always below the equilibrium level the real wage and inflation rate move procyclically. The usual argument that the macroeconomic NAIRU and the natural rate of unemployment, based on microeconomic considerations, coincide is based on the assumption that below full employment money wages are rising less fast than inflation and that future inflation is determined by a mark up over labour costs. In our model inflation can accelerate below full employment if demand rises too sharply. Inflation rises if the difference between demand for goods and current output widens, independently of whether or not current output is the full employment level. It is easy to show that by increasing the rate
at which inflationary expectations adapt the average level of unemployment, and the NAIRU rate calculated from the cycle generated by the model, can both be raised.

If we consider policy the following simple rule suggests itself

\[ m = m^* + k(p^* - p) \]

The money stock moves in the opposite direction to price changes; alternatively we can write the dynamics of the money stock as

\[ \dot{m} = -k \dot{p} \]

An increase in prices now has a double effect on demand, reducing the real value of the money stock and the money stock itself. The full employment equilibrium is stable providing \( \delta > 0, t > d \), \( (1+k) \). By making \( k \) large enough the monetary authorities can stabilise the price level, and hence employment. The monetary authorities should reduce the money supply during the boom to slow inflation and increase in the depression to prevent price reductions going too far.

Alternatively, the banking system may be such that the money supply is endogenous, and, near equilibrium, a rise in prices brings forth an increased supply of money; in this case the system tends to be unstable. For \( k \) large and negative instability can occur even if expectations are static. This lends support to Robertson's argument that the trade cycle may be due, in part, to the credit creation policy followed by Banks. Indeed in the Federal Reserve Bank Report (1923) the monetary policy of the U.S.A. was openly stated as being one of "Productive Credit", the role of money being seen as financing circulating capital or inventories (fixed capital being left to long term financing). Productive Credit amounts to the doctrine that Banks are prepared to lend against the value of goods in circulation; a rise in prices, and the value of inventories, gives rise to
an expansion of credit. This amounts to a negative k, with a value of approximately minus one, in the money supply rule with the consequent generation of instability. There seems little doubt that the failure of central banks to counteract the reductions in the price level during the Great Depression of 1929-39, and indeed to allow a large reduction in the broad money stock due to Bank failures was at least a contributory factor to the duration and depth of the recession (see Friedman and Schwartz (1956)). The policies of the Federal Reserve Bank can be seen as not only prolonging the recession but as the root cause. Keynes (1930) argues:

"the high market rate of interest which, prior to the collapse, the Federal Reserve system, in their effort to control the enthusiasm of the speculative crowd, caused to be enforced in the United States - and, as a result of sympathetic self-protective action, in the rest of the world - played an essential part in bringing about the rapid collapse"

While willing to finance productive activities the authorities were unwilling to finance lending to speculators in the stock market (which rose 98% between 1926 and 1929).

4. Conclusions

The restriction of dynamic analysis to models in two dimensions rules out many types of behaviour. In particular, persistent unemployment is impossible unless we postulate an unemployment equilibrium or there are no bounds on the values variables can achieve. A four dimensional model has been presented, in which wages and prices move to clear their respective markets, expectations are adaptive and employment change depends on the ability to sell output as well as the real wage. It has been shown in this example that persistent disequilibrium unemployment is possible; employment can oscillate indefinitely around a low level.
The model presented here is very simple and may be criticised on many grounds, not least the absence of a representative agent, microeconomic foundation. A crucial question is the robustness of the results presented. The argument for robustness lies in considering equations 13 and 14. The real variables in the model adjust slowly, but are stable if taken alone. However, aggregate demand constraints, which depend on nominal variables, can affect employment. Any model which allows aggregate demand to affect employment, and in which nominal variables are unstable, will give similar results. The central question is whether there is a plausible route through which nominal variables can affect real outcomes. The classical dichotomy between real and nominal variables applies only to market clearing equilibrium levels; the real equilibrium is not changed by changing the nominal variables. However, in disequilibrium dynamics the nominal variables can affect real outcomes; the propositions that Walrasian excess demands are homogeneous of degree zero in the price level, and that money is neutral, though often confused, are not identical.
APPENDIX

PROOF OF PROPOSITION 1

Proof. If $\beta z < z_m$ the locally stability of the system follows directly from Lemma 1 since the system is a special case of the one examined there.

Now consider the case where $\beta z_m > z_m$. It is clear that in this case the equilibrium is unstable, both eigenvalues have positive real parts and it is a source.

The following theorem, originally due to Poincare and Bendixon, may be found in Hirsh and Smale (1974).

Theorem. (Poincare-Bendixon.) A nonempty compact limit set of a $C^1$ planar dynamical system, which contains no equilibrium point, is a closed orbit.

The system (9)-(10) is certainly planar (two dimensional) and by assumption is twice differentiable, so is certainly $C^1$. It follows that, since the equilibrium point is a source, and by assumption the trajectories of the system are restricted to a compact set, we have that every trajectory which starts from a point other than the equilibrium converges to a closed orbit, that is, a non-trivial periodic solution. It can be shown (e.g. see Hirsh and Smale (1974)) that this periodic solution encircles the equilibrium point.

It follows that for almost all initial conditions the price level and the expected inflation rate will converge to a limit cycle. In general there may be many limit cycles around the equilibrium, being alternatively stable and unstable. Which of these limit cycles the system converges to depends on the initial conditions.

I now assume that the trajectory $(p(t), \pi^e(t))$ has settled down and actually lies on the limit cycle, as we shall see later this can be relaxed to cover the case of asymptotic convergence.

We now turn to the analysis of the system given by equations (7) and (8).
These can now be written as
\[ \dot{y} = \beta(y - L(x)) \]
\[ \dot{x} = \xi \sigma(g(y) - x) + \xi \int_{0}^{S} h(t) \, dt \]
\[ \dot{y} = \xi \sigma(g(y) - x) + \xi \int_{0}^{S} h(t) \, dt \]
where \( h(t) = \min \{0, z(p(t), \pi^e(t))\} \) and \( p(t) \) and \( \pi^e(t) \) are the solution of the system (9) and (10) for given initial conditions. By theorem 2 of chapter 15 of Hirsch and Smale the flow of a system of differential equations is differentiable to the same degree as system from which it arises. So
\[ h''(t) = 6 \min \{0, z(p(t), \pi^e(t))\} \left( \frac{d}{dt} (z(p(t), \pi^e(t))) \right)^2 \]
\[ + 3 \left( \min \{0, z(p(t), \pi^e(t))\} \right)^2 \frac{d^2}{dt^2} (z(p(t), \pi^e(t))) \]
is well defined, since both \( z \) and the flows \( p(t), \pi^e(t) \) are twice differentiable. Hence \( h \) is twice differentiable. Since \( p(t) \) and \( \pi^e(t) \) are periodic it is clear that \( h(t) \) is periodic.

Suppose that \( h \) is of period \( S \), then let
\[ h_0 = \frac{1}{S} \int_{0}^{S} h(t) \, dt \]
b be the average value of \( h(t) \) over its cycle. By construction we have that \( h(t) \) is always non-positive. If the financial cycle is stable we have that \( h_0 = 0 \) while if it unstable, and cycles we have \( h_0 < 0 \).

Now consider the family of averaged systems of equations

\[ \dot{x} = \xi \sigma(y - L(x)) \]

for \( \xi > 0 \), where the effect of \( h(t) \) is averaged over its cycle. This system has a unique equilibrium point defined by
\[ y_0 = L(x_0), \quad x_0 = g(y_0) + \frac{S}{\sigma} h_0 \]
Consider the function

\[ V(x, y) = \varepsilon^2 (y - L(x))^2 + \varepsilon \sigma (g(y) - x - \frac{\delta}{\varepsilon} h_0)^2 \]

It is easy to check that \( V \) is a strict Liapunov function for the averaged system over the entire phase space. That is

(i) \( V(x_0, y_0) = 0, \ V(x, y) > 0 \) for all \( (x, y) \neq (x_0, y_0) \)

(ii) \( \dot{V}(x, y) < 0 \) for all \( (x, y) \neq (x_0, y_0) \)

(i) is obvious and (ii) follows from the fact that

\[ \dot{V}(x, y) = 2( \varepsilon^2 \frac{f''(y)}{f'(y)} - \varepsilon^2 L'(y)) < 0 \]

since the marginal product of labour is positive but strictly decreasing while the supply schedule of labour is increasing. It follows that the averaged system is globally asymptotically stable, that is, for any initial conditions the system converges to \( (x_0, y_0) \).

By the Averaging Theorem of Guckenheimer and Holmes (1983), for a twice differentiable dynamical system on a bounded domain, if the averaged system has a hyperbolic fixed point then there exists \( \varepsilon_0 > 0 \) such that the original system written as

\[ \dot{w} = \varepsilon \sigma (E - L(w)) \]
\[ \dot{E} = \varepsilon \sigma (g(E) - w) \]

possesses a unique hyperbolic periodic orbit of the same stability type for all \( 0 < \varepsilon < \varepsilon_0 \). Further, if \( (x_0, y_0) \) is the equilibrium point of the averaged system the periodic orbit of the original system can be written as

\( (w(t), E(t)) = (x_0, y_0) + O(\varepsilon) \) where \( O(\varepsilon) \) denotes terms which are \( \varepsilon \) the same order as \( \varepsilon \).

Clearly, the critical value \( \varepsilon_0 \) increases as \( \sigma, \delta \) and \( \varsigma \) fall. By making these small enough we can ensure that \( \varepsilon_0 > 1 \) and the system is stable for \( \varepsilon = 1 \). It follows that, if we choose \( \sigma, \delta \) and \( \varsigma \) sufficiently small, that is to say, the speed of employment adjustment and real wage adjustment are slow
enough, the system (7) - (8') has a unique stable periodic orbit which is
close to the point defined by $E_0$ and $\omega_0$ where

$$E_0 = L(\omega_0), \quad \omega_0 = g(E_0) + \frac{\delta}{\partial_\omega} h_0$$

For $h_0 < 0$ we have that employment is on the labour supply curve but the
marginal product of labour exceeds the real wage. It follows that $E_0$ is
below the equilibrium full employment level since the demand curve for
labour is downward sloping while the supply curve is upward sloping.

If the financial cycle is stable the real wage and employment converge to
their equilibrium (full employment) levels, since we have $h_0 = 0$; the
periodic orbit is trivial, it is a fixed point. If the financial system is
unstable, and becomes a limit cycle, the real variables cannot converge but
themselves cycle. However, these cycles are not around the equilibrium point
but are in the neighbourhood of a level of employment which is below the
equilibrium full employment level.

The result depends on $h(t)$ being periodic. However, if the price-
expected inflation system produces a trajectory which converges to a limit
cycle, but never actually reaches it, we can apply the generalized averaging
theorem for almost periodic functions from Hale (1969).

The result shows that for $\sigma, J$ and $\delta$ small the system (7) - (8) has a
periodic orbit near $\omega_0, E_0$ which is locally stable. To prove global
stability consider the Poincaré map of the original flow; this is generated
by taking a section across the flow and noting the point at which the flow
returns to this section as a mapping from its previous intersection.

We have shown that the averaged system converges globally to the fixed
point, which is asymptotically stable (and hence hyperbolic). It follows
that the entire space is the stable manifold of the fixed point of the
averaged system, the unstable manifold is the empty set and so the
intersection of these two manifolds is trivially transverse. It follows, by theorem 4.4.1 of Guckenheimer and Holmes (1983) that the Poincare map of the original system is topologically equivalent to the flow of the averaged system for $\epsilon$ small. Since the Poincare map of the original flow now has a globally stable fixed point the original flow has a globally stable periodic orbit.

For each possible periodic orbit of $p$ and $\pi^e$ the real wage $w$ and employment level $E$ converge globally to a periodic orbit of their own. Clearly, precisely which periodic orbit the system converges to depends on the initial values of $p$ and $\pi^e$. The larger the periodic orbit of $p$ and $\pi^e$ the smaller the values $E_0$ and $w_0$ of the point around which the real variables oscillate. Q.E.D.
References


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