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NOISY COMMUNICATION AND THE FAST EVOLUTION OF COOPERATION

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Abstract

Recent work shows that inefficient outcomes fail to be evolutionary stable in games with pre-play communication. However, this result depends upon drift, and inefficient outcomes may well persist for long periods of time. This paper perturbs the model of cheap-talk, and derives efficiency results which do not rely upon drift. We consider two types of perturbation. If players make mistakes in choosing messages, as in Selten (1975), we obtain efficiency in 2x2 games. Our second perturbation introduces noise in the transmission of messages, so that the message received can be different from the message sent, without the sender being aware of this. In this case we obtain a powerful efficiency result for generic games. Since noise restricts drift, this also allows us to derive existence results which are stronger than in the existing literature.

Key words: equilibrium selection, evolution, cheap talk, perturbed games.

JEL Classification No: C79

*Address for communication: Dept. of Economic Analysis, Universidad de Alicante, Campus San Vicente, 03071 Alicante, Spain. This is a revised version of a 1992 paper with similar title, which contained the efficiency results but not the existence results reported here. This version of the paper has benefited from discussions with Andreas Blume, Sjaak Hurkens, Dilip Mookherjee, Karl Schlag and Eric van Damme. I am also grateful to seminar audiences at CORE, CentER, the Indian Statistical Institute Delhi, the International Economic Association (Moscow), Nuffield College Oxford, and University College London. I am grateful to CentER, Tilburg for providing a for providing support whilst this version of the paper was completed.
1. INTRODUCTION

Consider the unanimity game $G_1$, which is a game of pure coordination, without conflict of interest. The game has two strict Nash equilibria, $(H,H)$ and $(L,L)$, and one in mixed strategies. Till recently, there were not many convincing reasons, from a purely non-cooperative standpoint, for excluding the sub-optimal equilibrium $(L,L)$. It has been argued (see for example, Farrell, 1988) that players could signal their intent to play $H$ by pre-play communication. However, these stories are not fully convincing since it is always a Nash equilibrium for both players to ignore such communication. A second problem was pointed out by Aumann (1990), using the stag hunt example, where a player would like to claim that he intends to "hunt the stag", even if he intends to "hunt the hare".

Recent work has applied evolutionary arguments to obtain positive efficiency results - see Sobel (1993) for an excellent survey. In this setting pairs of players are randomly drawn from a large population. The players exchange costless messages, and then play $G_1$. In this framework, cheap talk de-stabilizes the inefficient outcome, since it allows players to signal their intention to move to the efficient outcome costlessly. The basic idea behind these models is the following. Suppose that all players in the population are playing strategy BAD: choosing message 1 and playing $L$ irrespective of the message. Let a small fraction of the population switch to the strategy REFORMER. REFORMER sends a different message, message 2, and responds to message 1 by choosing $L$, and to message 2 by choosing $H$. When REFORMER is matched against BAD, it gets a payoff of 1, and hence does as well as BAD does against itself. When REFORMER is matched against itself, it gets a payoff of 2, which is strictly greater than the payoff of BAD when matched against REFORMER. Since REFORMER has a higher payoff than BAD in the mixed population, natural selection ensures that REFORMER drives out BAD, so that the population ends up playing the efficient outcome, $(H,H)$.

This argument shows that the strategy BAD is unstable in a very strong sense -
it takes but a single arbitrarily small mutation, introducing REFORMER into this population, and natural selection does the rest of the work in eliminating BAD. BAD fails to be Lyapunov stable or neutrally stable. We can therefore be confident that BAD will be eliminated in a relatively short time. Unfortunately, this is not true in general. As section 2 of this paper makes clear, there are very many inefficient strategies which are neutrally stable, and which cannot therefore be eliminated quickly. An example of such a strategy is DICTATOR, which sends message 1, and plays L in response to message 1, but punishes any deviant message by playing the mixed strategy (2/3 L, 1/3 H). REFORMER gets a payoff on only 2/3 against DICTATOR, and hence cannot invade a population playing DICTATOR. To rule out DICTATOR (and other inefficient strategies) one must rely upon drift. Starting from a population which plays DICTATOR, let a small fraction switch, due to random mutation, to playing BAD. BAD and DICTATOR have equal payoffs in this mixed population. Hence, with many mutations, the proportion of BAD can increase in this population, until DICTATOR is eliminated. At this point REFORMER can enter and displace BAD. Formally, DICTATOR is neutrally stable but does not belong to an Evolutionary Stable set (or asymptotically stable set) of strategies.

This mechanism based on drift is problematic since it can take a long time, due to it reliance upon the accumulation of small random mutations. Inefficient outcomes can therefore persist for very long periods of time before they are eliminated. For example, if in each period a fraction \( \epsilon \) of the population switch randomly from one strategy to another (eg. from DICTATOR to BAD or vice-versa), the expected time before a population playing DICTATOR switches to BAD is of the order \( 1/\epsilon^2 \). This problem is acknowledged by Kim and Sobel (1994, p13): "the arguments in our proofs suggest that convergence (to a stable set) could be slow...the population could remain at an inefficient outcome for a long time." In our view, slow convergence is a serious problem in the human context; the pace of technological and social change is such that the underlying game may itself change
before inefficiency is eliminated.

A second problem with the existing literature is that existence results are very weak. Kim and Sobel (1994) obtain existence results only for games of common interest. Thus ES sets do not exist for games such as the battle of the sexes. Here again, drift plays an important role in destabilizing equilibria.

This paper asks, can we expect the fast evolution of efficiency in games with pre-play communication? We operationalize this notion by not relying upon drift for the purposes of eliminating inefficient outcomes - we seek to get our efficiency results by showing that inefficient outcomes are unstable, i.e. not neutrally stable. (We do not however weaken our existence requirements, since our existence results are obtained using the criterion that equilibrium strategies belong to ES sets). If a strategy fails to be neutrally stable, it only requires a single arbitrarily small mutation which introduces the superior strategy. The unstable strategy will then be eliminated purely due to its lower payoff, without requiring any further mutation. As section 2 of this paper makes clear, neutral stability does not imply efficiency even when the underlying game is as simple as G1. In part the problem arises because behavior at unreached information sets (actions after unused messages in our context) is not subject to any selection pressure. We therefore perturb the cheap talk game, and consider two different types of perturbation. The first perturbation is familiar, and is in the spirit of Selten (1975, 1983) - players make mistakes in choosing messages, so that all messages are chosen with positive probability. This perturbation allows us to get positive efficiency results for 2x2 games - we show that if a strategy is neutrally stable, it must be approximately efficient (i.e. the payoff converges to the efficient payoff as mistakes go to zero). Unfortunately the efficiency result does not generalize when the underlying game has three or more actions per player.

This leads us to consider a second, novel perturbation. We introduce noise in the transmission of messages - when player 1 sends message m, there is a small
probability that player 2 receives a different message, say message $m'$. When this happens, the sender is not aware that player 2 has indeed received message $m'$. This noise creates the possibility of a mis-understanding, since neither the messages sent nor the messages received are mutual knowledge between the players. Noise has powerful effects, and dramatically restricts the set of Nash equilibria of the cheap talk game. This allows us to prove a strong exact efficiency result in general games - in any game satisfying a regularity assumption, inefficient outcomes fail to be neutrally stable. Our formulation of noise is of independent interest, even outside the evolutionary context. Noise has such powerful effects in restricting the equilibrium set essentially because the sequential equilibrium correspondence of the cheap-talk game fails to be lower-hemicontinuous in the level of noise at the point of zero noise.

We also obtain stronger existence results than the existing literature. We require that efficient outcomes be asymptotically stable, i.e. elements of Evolutionary Stable sets. These perturbations also allow us to get stronger existence results than usual. In either formulation, if the underlying game has a Nash equilibrium which is efficient (i.e. maximizes the sum of payoffs of the two players), this outcome belongs to an Evolutionary stable set. Thus we ensure existence in games such as the battle of the sexes. Our results imply that inefficient outcomes will be destabilized quickly (by a single mutation), and once the population coordinates on an efficient outcome, it stays there.

The organization of the rest of this paper is as follows. Section 2 sets out the basic model of pre-play communication, in the absence of mistakes or noise. We review the efficiency results obtained by others, and discuss the reasons why inefficient outcomes may be neutral stable. We show that if one does not want to rely on drift, and allow for randomized actions, we need to assume that players have infinitely many messages at their disposal. Section 3 introduces mistakes in the spirit of Selten. Section 4 discusses the effects of noise. Section 5 shows
that our assumption that players have infinitely many messages is important, by
providing an inefficiency result with finite message sets. The final section
concludes.

2. THE BASIC CHEAP TALK GAME

Let the underlying game, $G = \langle \{1, 2\}, A_1, A_2, u_1, u_2 \rangle$ be a finite two-player normal
form game, where 1 and 2 are the two roles in the game, $A_1$ and $A_2$ are finite action
sets, $A = A_1 \times A_2$, and $u_i : A \rightarrow \mathbb{R}$, $i = 1, 2$ are the payoff functions, which are extended
to the set of mixed action profile $\Delta A_1 \times \Delta A_2$, by taking expected payoffs. The
underlying game $G$ defines an asymmetric contest (see Selten, 1980 or van Damme
1991). A behavior strategy in this asymmetric contest is a pair $\alpha = (a_1, a_2) \in
\Delta A_1 \times \Delta A_2$. The payoff of strategy $\alpha$ against strategy $\beta$ is:

$$u(\alpha, \beta) := \frac{1}{2} [u_1(a_1, \beta_2) + u_2(\alpha_2, \beta_1)]$$

(2.1)

An action pair $a^* \in A$ is utilitarian or efficient if $a^*$ is an action pair
which maximizes the ex-ante expected payoff for each of a pair of players, before
the roles are assigned, i.e. if $u(a^*, a^*) \geq u(a, a)$ $\forall a \in A$.

$G$ is compatible if it has a strict Nash equilibrium which is utilitarian i.e.
$\exists a^* = (a_{1}^*, a_{2}^*)$ which is utilitarian, such that $u_{i}(a^*) > u_{i}(a_{j}, a_{j}^*)$, $\forall a_{i} \in A_{i}, i = 1, 2,$
$j = 1, 2$.

$G$ is of common interest if it has a unique weakly Pareto-efficient action
pair, i.e. $\exists a \in A$: $u_{1}(a) > u_{1}(a)$ and $u_{2}(a) > u_{2}(a)$ $\forall a \in A - \{a\}$.

In asymmetric contests, the utilitarian notion of efficiency is clearly more
appropriate than the Paretoian notion, since ex-ante players may occupy either role.
Note that if $G$ is of common interest then it is compatible. The converse is not
true as the game $G_2$ shows. Existing results usually apply to common interest games,
whilst our results will apply to the larger class of compatible games.

The cheap talk game, $G^*$, associated with $G$, is as follows. Players are
randomly drawn from a large (infinite) population, and paired. Each player is
assigned a role, i.e. role 1 or role 2. Each player sends a message from a set of messages, M. The set M may be finite or countably infinite, but contains at least two elements. After these messages are exchanged, the players play G. Payoffs in the cheap talk game, G*, do not depend upon the messages exchanged and depend only upon the actions taken in G.

A local behavior strategy in role 1, \( x_1 \), is a pair \( (\sigma_1, \theta_1) \) where \( \sigma_1 \in \Delta M \), and \( \theta_1: M \times M \rightarrow \Delta A_1 \). A behavior strategy in the cheap talk game, \( \pi \), is a pair \( (x_1, x_2) \). \( x \) can also be written as the pair \( (\sigma, \theta) \), where \( \sigma = (\sigma_1, \sigma_2) \) and \( \theta = (\theta_1, \theta_2) \). Let \( S \) denote the set of behavior strategies in \( G \). The expected payoff of strategy \( x \) against \( x' = (\sigma', \theta') \) is given by:

\[
\begin{align*}
\sum_{\delta_1, \delta_2} \sigma_1 \left( m_1, m_2 \right) \theta_1 \left( m_1, m_2 \right) & \sum_{\delta_2} \theta_2 \left( m_1, m_2 \right) \left( m_1 \right) \delta_2 \left( m_1, m_2 \right) \delta_1 \left( m_1 \right) \\
\sum_{\delta_1, \delta_2} & \sigma_2 \left( m_1, m_2 \right) \theta_2 \left( m_1, m_2 \right) \left( m_1 \right) \delta_2 \left( m_1, m_2 \right) \delta_1 \left( m_1 \right)
\end{align*}
\]

The expected payoff of strategy \( x \) in a mixed population where fraction \( (1-\epsilon) \) play \( x \) and fraction \( \epsilon \) play \( y \) is given by:

\[
u (x; (1-\epsilon)x + \epsilon y) = (1-\epsilon) \nu (x, x) + \epsilon \nu (x, y)
\] (2.3)

Definition 2.1: \( x \in S \) is a neutrally stable strategy (NSS) if and only if there exists \( c', 0 < c' < 1 \), such that \( \forall \epsilon \in [0, c'] \) and \( \forall y \in S \):

\[
u (x; (1-\epsilon)x + \epsilon y) \geq \nu (y; (1-\epsilon)x + \epsilon y)
\] (2.4)

Remark: In games with a finite number of pure strategies (i.e. where \( S \) is finite dimensional), definition 2.1 is equivalent to 2.1':

Definition 2.1': If \( S \) is finite dimensional, \( x \) is a NSS iff \( \forall y \in S \):

\[
u (x, x) \geq \nu (y, x)
\]

\[
u (y, x) = \nu (x, x) \Rightarrow \nu (x, y) \geq \nu (y, y)
\]

van Damme (1991) shows that in finite games, any NSS satisfying definition (2.1') has a uniform invasion barrier. The equivalence of the two definitions does not extend to games where the set of pure strategies is countably or uncountably
infinite. For example, consider an \( x \) such that \( \forall y, u(x,x) > u(y,x) \) so that \( x \) is a strict Nash equilibrium and hence satisfies (2.1'). Nevertheless, there may be a sequence \( \langle y_n \rangle \), such that the limit of \( u(y_n,x) \) equals \( u(x,x) \), while the limit of \( u(y_n,y_n) \) is strictly greater than the limit of \( u(x,y_n) \). In this case there is no uniform invasion barrier \( \epsilon' \). See Bomze and Potscher (1989) and Bhaskar (1992) for further details.

**Definition 2.2** For \( x, y \in S \), \( x \mathcal{D} y \) if \( u(x,x) - u(y,x) \) and \( u(x,y) = u(y,y) \)

The symmetric binary relation \( \mathcal{D} \) formalizes our notion of drift. Note that if \( x \mathcal{D} y \) and \( z = \lambda x + (1-\lambda)y \) for \( \lambda \in [0,1] \), \( z \mathcal{D} x \). Hence if the population starts at \( x \), and \( y \mathcal{D} x \), the accumulation of small mutations can cause the population to drift to \( y \). Even if \( x \) is a NSS, \( y \) may not be so that the population can move far from \( x \). An Evolutionary Stable set (Thomas, 1985) is a closed set of NSS which is not vulnerable to drift.

**Definition 2.3** A closed set \( L \subseteq B \) is an Evolutionary Stable set (ES set) if

1) \( x \in L \Rightarrow x \) is a NSS

2) if \( x \in L \) and \( y \mathcal{D} x \), then \( y \in L \).

An ES set repels mutants from outside this set. If the population is playing any strategy in this set, and any mutant enters with fraction below \( \epsilon' \), the mutant has strictly lower payoff in the mixed population, and is driven out.

**Remark:** There is a close link between evolutionary stability and dynamic stability under the (mixed-strategy) replicator dynamics. Taylor and Jonker (1978) showed that evolutionary stable strategies are asymptotically stable.\(^3\) Bomze and van Damme (1992) show the converse with mixed strategy dynamics. These results extend naturally to the relation between ES sets and asymptotically stable sets, and the relation between neutrally stable strategies and Lyapunov stability (Thomas, 1985).\(^4\)

We state the following proposition without proof, since it is without claim to originality - it is essentially a translation of results obtained by others (Kim
and Sobel, 1994; Schlag, 1994) to our framework.

**Proposition 0**
1) $G^*$ has an ES set if and only if $G$ is of common interest.
2) If $G$ is of common interest, and $x$ belongs to an ES set of $G^*$, $x$ is utilitarian.

We have two remarks in this context.

**Remark 1:** (1) is extremely restrictive. Consider game $G_2$; since a player may be allotted either role, this has a unique ex-ante efficient outcome, $(T, L)$, which is also a strict Nash equilibrium. Nevertheless, the corresponding game with cheap talk fails to have an ES set. The actions of players after unsent messages is not subject to selection pressure, and therefore drift destabilizes the outcome $(T, L)$. We discuss this game in more detail in the next section.

**Remark 2:** (1i) does not apply if ES set is replaced by neutral stability, i.e. inefficient outcomes can be neutrally stable. This is true even in the simplest case of a $2 \times 2$ unanimity game $G_1$ if we consider mixed strategies. There are two distinct reasons why inefficient outcomes may be neutrally stable even in $G_1^*$.

**No unused messages:** If $M$ is finite with cardinality $|M|$, there may be no unused messages for a mutant to use. Consider the strategy BABBLE; this randomizes with equal probability between all messages and chooses $L$ after every message, with a payoff of 1 against itself. Any mutant which plays $H$ against itself (such as the strategy REFORMER discussed in the introduction) is not a best response to BABBLE since its payoff against BABBLE is $1 - 1/|M|$, which is strictly less than 1. This problem can be solved only if $M$ contains (at least) a countable infinity of messages. In this case, given any incumbent strategy, arrange the messages according to the probability with which they are used. For every $\delta > 0$, there exists $N(\delta)$ such that messages greater than $N(\delta)$ are used with total probability less than $\delta$. We now show that BABBLE cannot be a NSS, since there is no uniform invasion barrier $e'$. Given any $\epsilon' > 0$, let $\delta(\epsilon') = \epsilon'/\left(1 - \epsilon'\right)$, and let
REFORMER use messages greater than \( N(\delta(\varepsilon')) \). It is easy to verify that REFORMER does strictly better than BABBLE in the mixed population.

It is clear that if one wants to get strong efficiency results using neutral stability, one must allow for infinitely many messages, so that there are always messages available which are used with almost no probability. Indeed, given any incumbent strategy, all but a finite number of messages are used with almost no probability, so that in this case "most" messages can be used by mutants to signal to each other. We do not find the assumption that agents have infinite message sets unreasonable, at least in the context of human interaction. Note that the literature which seeks to refine cheap-talk equilibria has to make even stronger assumptions in order to get results - Farrell (1993) for example assumes that message sets are "open-ended" so that there are always neologisms (new messages) which a player can create.

When we perturb the cheap talk game, for example by assuming that players make mistakes, all messages will be sent with positive probability at any strategy profile. The problem of a lack of unused messages arises even if we use stronger criteria such as ES set. In section 5 we show that infinite message sets are essential for efficiency results - with finite message sets we find that even ES sets can be inefficient.

Punishment of deviant messages: The second problem is that inefficient strategies (like regimes) can increase their longevity by punishing potential reformers - specifically by punishing deviant messages. Consider the strategy DICTATOR (which we discussed in the introduction). DICTATOR chooses message 1, and responds to message 1 by playing H, but plays \((2/3L;1/3H)\) after any deviant message. Any strategy which tries to signal using a different message is punished by DICTATOR, and gets a payoff of at most 2/3. DICTATOR thereby ensures that it cannot be invaded by a strategy like REFORMER.

DICTATOR uses a randomized punishment to punish REFORMER. Although this
punishment is not a NSS of the underlying game G, it is not subject to selection pressure, since deviant messages are never sent in a population playing DICTATOR. This suggests that if players make mistakes in sending messages, as in Selten (1975), DICTATOR may no longer be viable. We find that this is the case. Further, in 2x2 games, punishment of deviant messages is not possible since such punishments have necessarily to be via mixed strategies. However, in more general games, deviant messages can be punished by reverting to a strict Nash equilibrium of G. In this case mistakes have no effect, and we have to introduce noise in the transmission of messages in order to rule out punishing strategies.

Jointly-controlled randomization: The third problem is that players may use cheap-talk to generate (an inefficient) jointly controlled lotteries over the Nash equilibria of G. The simplest example (due to Kim and Sobel, 1992) is when the underlying game is G1. Let the set of messages be \{1,2\}, and consider the strategy JCL which sends message 1 with probability one-half and message 2 with probability one-half. If the messages exchanged coincide, the strategy plays L; if they differ, the strategy plays H. The payoff of JCL against itself is 1.5.

It is easy to show that JCL is not neutrally stable in the asymmetric contest if players can condition their strategies upon the role they fill. Consider the mutant which sends message 1 in role 1, and message 2 in role 2, but plays the same actions as JCL after exchanged messages. The payoff of this mutant against JCL is 1.5, but its payoff against itself is 2.6.

This observation is easily generalized. Suppose the G is a game of common interest over equilibria, i.e. a game where the Nash equilibria of G can be be Pareto ordered. Jointly controlled randomization over the Nash equilibria of G can never be a neutrally stable in G*. To see this, let the mutant simply choose the message pair which induces the best Nash equilibrium of G, and choose actions as JCL. Since all messages used by JCL must have the same payoff against JCL, the mutant is a best response to JCL which does strictly better against itself.
Consider the game G3, which is a game of common interest, but without common interest over equilibria - players have conflicting interests over the two inefficient equilibria (L,M) and (M,L), but these are Pareto-dominated by (H,H). Consider the strategy JCR which randomizes with equal probability between messages 1 and 2. If the realized messages coincide, JCR plays (L,M); if they are (1,2) or (2,1) JCR plays (M,L). If either player receives a message different from 1 or 2 (say message 3), the player plays M. The payoffs to various message combinations are shown in Fig. 4. If a mutant sends a message different from 1 or 2 in order to recognize each other and coordinate on (H,H), this mutant is punished by JCR. This can be verified from Fig. 4 which shows that the expected payoff to message 1 or 2 when playing against JCR is 1.5, whereas the expected payoff to message 3 is 1. Hence such a mutant cannot invade JCR.

JCR is a NSS of the cheap talk game. Furthermore, as we show in section 4, since JCR plays a jointly controlled lottery, it continues to be a NSS even in the game with noise. Nevertheless, the game G3 is exceptional, and JCR will not be a NSS of G3* if we perturb the payoffs in G3 slightly. The critical feature of this example is that the sum of payoffs to the two roles at (L,M) equals the sum at (M,L). Hence we can show that if the underlying game is generic, jointly controlled randomization will not be neutrally stable in the cheap talk game.

A strategy which performs a jointly controlled lottery, as in the example above, can be viewed as playing a mixed strategy over messages, where the payoffs are constructed using the Nash equilibria of G. Neutral stability of this strategy is therefore closely related to the neutral stability of mixed strategy Nash equilibria in asymmetric contests. Although mixed strategies are never evolutionary stable in asymmetric contests, (Selten, 1980), there exist examples of mixed strategies that are neutrally stable - for example, mixed strategy Nash equilibria of a constant-sum game. In a companion paper (Bhaskar, 1994, Theorem 2), I show that such examples are rare - if G satisfies a regularity condition, which is
generically satisfied, a mixed strategy equilibrium of $G$ cannot be neutrally stable. The same condition also rules out jointly controlled randomization in the cheap talk game.

Define the following:

If $a \in A$, let $u(a) := u_1(a) + u_2(a)$.

$\text{supp}(\sigma) := \{m \in M \times M: \sigma(m) > 0\}$

**REGULARITY ASSUMPTION** : The payoffs in the underlying game $G$ satisfy:

R1 *No payoff ties in equilibrium* : If $a = (a_1, a_2)$ is a pure strategy Nash equilibrium, $a$ is a strict equilibrium.

R2 Let $a, a', a'', a''' \in A$:

If $u(a) + u(a') = u(a'') + u(a''')$, then $a = a' = a'' = a'''$.

**Lemma 2.1** Let $G$ satisfy the regularity assumption:

1) any NSS of $G$ is in pure strategies.

11) If $x = (\sigma, \theta)$ is a NSS of $G^*$, where $\theta: M \times M \rightarrow A$, then $\theta$ is a constant function on $\text{supp}(\sigma)$.

**Proof** Part (i) of the lemma is proved in a companion paper (Bhaskar, 1994, lemma 3), where it is shown that if a mixed strategy $p = (p_1, p_2)$, $i.e.$ a NSS of the asymmetric contest only if payoffs in $G$ satisfy the following condition: consider the *restricted game* associated with $p$, i.e. the game where each player 1 has pure strategy set $\text{supp}(p_1)$, the set of pure strategies in the support of $p_1$. In the restricted game, consider any pair of pure strategies for player 1, $h$ and $i$, and any pair of pure strategies for player 2, $j$ and $k$. Payoffs in the restricted game must satisfy:

$$u(h, j) + u(i, k) = u(h, k) + u(i, j)$$

(2.5)

where $u(h, j)$ is the sum of the payoffs of the two players at the strategy combination $(h, j)$, and so on. Hence part (i) of the lemma follows.

In the case of jointly controlled randomization, if player 1 uses messages $h$ and $i$ with positive probability and player 2 uses messages $j$ and $k$, the payoffs to
these message combinations must satisfy (2.5). By the hypothesis of the lemma, these payoffs must be constructed using the pure action combinations of \( G \) so that there must be a combination which violates \( R_2 \).

Note that Lemma 2.1 does not restrict players actions after unsent messages, and hence does not rule out strategies which punish deviant messages. Hence perturbing the cheap talk game is essential if we are to rule out such punishments.

3. MISTAKES

We now allow for the possibility that players may make mistakes in sending messages. This ensures that all messages are sent with positive probability, so that there are no unsent messages. This formulation, of the "trembling-hand", was introduced by Selten (1975), in order to refine Nash equilibria. Selten (1983) extended the idea to the evolutionary context, suggesting the notion of "limit ESS" in order to alleviate the existence problems for ESS in extensive form games. In our paper mistakes will play a dual role: on the one hand, they help refine the set of neutrally stable strategies, by introducing selection pressure at information sets which are otherwise unreached. On the other hand, mistakes also restricts drift, thereby allowing ES sets to exist in a larger class of games.

Let \( M \), the set of messages be the set of natural numbers.

Let \( \mu = \left< \mu_n \right> \) be a probability measure on \( M \), with full support.

Let \( \eta > 0 \) be a small positive number.

The cheap talk game with mistakes, \( G_M(\eta) \), is as follows:

1. Players are randomly matched and assigned roles. Each player chooses a message from \( M \).

2. For each player, the chosen message is sent with probability \((1-\eta)\). With probability \( \eta \), the player makes a "mistake". Mistakes are independent across players. If the player makes a mistake, a message is selected from \( M \) by the probability measure \( \mu \), and the player sends this message.
3. Players observe each others messages, and choose randomized actions from $\Delta A_1$. Payoffs depend only on the actions chosen.

Note that $G^M(\eta)$ and $G^*$ have the same strategy sets, $S$, and $G^M(0) = G^*$. The payoff of strategy $x = (\sigma, \theta)$ against $x'=(\sigma', \theta')$ in $G^M(\eta)$ is:

$$\begin{align*}
u(x,x',\eta) &= \frac{1}{2} \sum_{m_1,m_2} \left[ \theta_1(m_1,m_2) \sigma_1(m_1) + \eta \mu(m_1) \right] \left[ (1-\eta) \sigma_2(m_2) + \eta \mu(m_2) \right] \\
&+ \frac{1}{2} \sum_{m_1,m_2} \left[ \sigma_2(m_1,m_2) \theta_1(m) \right] \left[ (1-\eta) \sigma_1(m_1) + \eta \mu(m_1) \right]
\end{align*}$$

(3.1)

The payoff of $x$ in the mixed population $(1-\epsilon)x + \epsilon y$ is:

$$\begin{align*}
u(x;(1-\epsilon)x+\epsilon y;\eta) &= (1-\epsilon) \nu(x,x,\eta) + \epsilon \nu(x,y,\eta)
\end{align*}$$

(3.2)

We want to define $x$ to be neutrally stable in $G^M$ if the payoff of $x$ is weakly greater than the payoff of any other strategy $y$, provided that $\eta$ and $\epsilon$ are both "small". Since we have two "perturbations", mistakes and mutations, the order in which we take limits could potentially be important. However, the results all our efficiency and existence results, in this section and the next, do not depend upon the relative importance of mistakes versus mutations. Accordingly, we provide two alternative definitions of both our solution concepts (NSS and ES sets), and prove our efficiency results using the weak definition of NSS, and our existence result with the strong definition of ES set.

Definition 3.1 (Weak Definition) $x \in S$ is a weak NSS of $G^M$ if $\exists \eta^* > 0 : \forall \eta \in (0, \eta^*)$, $\exists \epsilon'(\eta) : \forall \epsilon \in [0, \epsilon'(\eta)]$:

$$\nu(x;(1-\epsilon)x+\epsilon y;\eta) \geq \nu(y;(1-\epsilon)x+\epsilon y;\eta)$$

Definition 3.2 (Strong Definition) $x \in S$ is a strong NSS of $G^M$ if $\exists \epsilon' > 0$, $\eta^* > 0 : \forall \eta \in (0, \eta^*)$, $\forall \epsilon \in [0, \epsilon']$:

$$\nu(x;(1-\epsilon)x+\epsilon y;\eta) \geq \nu(y;(1-\epsilon)x+\epsilon y;\eta)$$

Observe that if $x$ is a strong NSS of $G^M$, it is a weak NSS. For our efficiency result, we shall prove that inefficient outcomes are not weak NSS.
Definition 3.3 \( D(\eta) \) y if and only if
\[ u(x,x,\eta) = u(y,x,\eta) \] and
\[ u(x,y,\eta) = u(y,y,\eta) \]

Definition 3.4 \( L \subseteq S \) is an weak ES set of \( G^M \) if
i) \( x \in L \Rightarrow x \) is a weak NSS of \( G^M \)
ii) if \( x \in L \) and \( y D(\eta) x \) for \( \eta \in (\eta',0) \), then \( y \in L \).

Definition 3.5 \( L \subseteq S \) is an strong ES set of \( G^M \) if
i) \( x \in L \Rightarrow x \) is a strong NSS of \( G^M \)
ii) if \( x \in L \) and \( y D(\eta) x \) for \( \eta \in (\eta',0) \), then \( y \in L \).

Observe that if \( L \) is a strong ES set of \( G^M \), it is a weak ES set. We shall use the strong version of ES set for our existence results.

Given \( a \in A \), we define the subset of \( S \), \( \Omega(a) \) as follows:
\[ \Omega(a) := \{ (\sigma, \theta) \in S : \theta(m) = a \forall m \in M \times M \} \]

**EXISTENCE**

We begin by proving our existence result, in proposition 1.

**PROPOSITION 1 (EXISTENCE)** If \( G \) is a compatible game, \( G^M \) has a strong ES set which induces the utilitarian outcome.

Proof Let \( a^* \) be the efficient strict Nash equilibrium of \( G \). We show that \( \Omega(a^*) \) is an ES set in \( G^M \) by the strong definition. Let \( x \in \Omega \) and \( \eta > 0 \). Let \( y \not\in \Omega \), implying that for some \( m \), \( y(m) \neq a^* \). Since \( a^* \) is a strict Nash equilibrium, \( u(y,x) < u(x,x) \), implying not \( y D(\eta) x \) for any \( \eta > 0 \).

We now show that any \( x \in \Omega(a^*) \), is neutrally stable since there exists a \( \epsilon^* \) which is a uniform invasion barrier against any \( y \). This follows since \( a^* \) is a strict Nash equilibrium, and \( a^* \) is efficient. Let \( a' \neq a^* \) so that \( u(a',a^*) < u(a^*,a^*) \). Then there exists \( \epsilon'(a') \) such that \( (3.3) \) is positive for all \( \epsilon \leq \epsilon'(a') \).

\[ (1-\epsilon)[u(a^*,a^*)-u(a'a')] + \epsilon[u(a^*,a') - u(a^*,a^*)] \quad (3.3) \]

Let \( \epsilon^* = \min \{ \epsilon'(a') : a' \in A - \{a^*\} \} \)

Let \( x \in \Omega \), and given any \( \eta \), let \( y \) play \( a^* \) against \( x \) with probability \( \pi \), and play action \( a' \) against \( x \) with probability \( (1-\pi) \). The difference in payoffs between
x and y in the mixed population is
\[ u(x; (1-\varepsilon)x + \varepsilon y, \eta) - u(y; (1-\varepsilon)x + \varepsilon y, \eta) = \varepsilon \pi [u(a^*, a^*) - u(y, y, \eta)] 
\]
\[ + \{(1-\varepsilon)(1-\pi)[u(a^*, a^*) - u(a', a')] + \varepsilon(1-\pi)[u(a^*, a^*) - u(y, y, \eta)]\} \]

If \( \varepsilon \leq \varepsilon^* \), the term in curly brackets {} is positive. Since \( a^* \) is efficient, \( u(a^*, a^*) \geq u(y, y, \eta) \). Hence for any \( \eta \), \( \varepsilon^* \) is an invasion barrier for any \( x \in \Omega(a^*) \) against any \( y \in \Omega(a^*) \).

Contrast Propositions 0 and 1; the latter generalizes the existence result from the class of common interest games to the class of compatible games. This distinction is illustrated by compatible game G2, which has an efficient Nash equilibrium (T,L). The game with cheap talk, but without noise, fails to have an ES set. Consider the strategy \( x \) which sends message 1 in both roles, and plays (T,L) after every message pair. Message 2 is never sent by \( x \), so that the response to message 2 can change via drift arbitrarily. Hence \( x \) can be replaced by the strategy \( y \) which sends 1 in both roles, plays (T,L) in after (1,1), but plays (B,L) after any other message pair. Consider strategy \( z \), which sends message 2 in either role, and plays which always plays T irrespective of the message in role 1, and plays L in role 2 after (2,2) and R after (1,2). \( u(y, y) = 2 \) whereas \( u(z, y) = 2.5 \), so that \( y \) is driven out by \( z \).

With mistakes, the population cannot drift from \( x \) to \( y \), since \( y \) is not a best response to \( x \) in the perturbed game. Without drift, \( z \) cannot enter since (B,R) is inefficient as compared to (T,L), and (T,L) is a strict Nash equilibrium.

**EFFICIENCY**

We now turn to efficiency, to showing that in 2x2 games, inefficient outcomes are unstable. However, the efficiency result is approximate, as the following definition makes clear.

Definition 3.6 \( x \) is approximately efficient if \( u(x, \eta) \) converges to the efficient payoff \( u(a^*, a^*) \) as \( \eta \) tends to zero.

**Proposition 2 (Efficiency)** If \( G \) is a 2x2 game satisfying the regularity
assumption R, and \( x \) is a weak NSS of \( G^+ \), \( x \) is approximately efficient.

The basic idea of the proof is to show that an inefficient strategy cannot punish any invading mutant. Since all messages are received with positive probability, the response after any message must be a NSS of \( G \). In 2x2 games, there are at most two NSS, which restricts punishment possibilities. This rules out strategies like DICTATOR which punish deviant signals. A mutant can therefore invade this inefficient strategy and coordinate on the efficient outcome. This basic argument is somewhat complicated by the possibility of jointly controlled randomization, which we discuss later.

**Lemma 3.1** Let \( x = (\sigma; \theta) \) be a weak NSS of any cheap talk game (i.e. \( G^+ \) or \( G^M(\eta) \)). If \( m = (m_1, m_2) \) is a message sent with positive probability, then \( \theta(m) \) is an NSS of \( G \).

**Proof:** Let \( \alpha = \theta(m^*) \) not be an NSS of \( G \) so that there exists \( \beta \in \Delta A_1 \times \Delta A_2 \), which can invade \( \alpha \) in the asymmetric contest defined by \( G \). Let \( y = (\sigma, \theta') \) where \( \theta'(m) = \theta(m) \forall m \neq m^* \), and \( \theta(m^*) = \beta \).

\[
u(y; (1-c)x+ey) - u(x; (1-c)x+ey) = \text{prob}(m)[u(\beta; (1-c)x+c\beta) - u(\alpha; (1-c)x+c\beta)]
\]  

Since \( \text{prob}(m) \) is positive, and \( \beta \) can invade \( \alpha \), \( x \) is not a NSS. \( \Box \)

We are now in a position to prove proposition 2.

**Proof of Proposition 2:** By lemma 2.1 and the regularity assumption, \( G \) has at most two NSS, which are strict equilibria. Label these \( a \) and \( a' \). Let \( x = (\sigma, \theta) \) be a NSS of \( G^M \). By lemma 3.1 let \( \theta(m) = a \) or \( \theta(m) = a' \) for any \( m \in \text{supp}(\sigma) \). Further, by lemma 2.1 \( \theta \) is constant on \( \text{supp}(\sigma) \), so let \( \theta(m) = a \forall m \in \text{supp}(\sigma) \).

We now show that if \( \theta(m) = a \forall m \in M \times M \), then \( a \) is efficient. Let \( m \in \text{supp}(\sigma) \) such that \( \theta(m) \neq a \). If \( \eta > 0 \), by lemma 3.1 \( \theta(m) \) must equal \( a' \). We claim that:

\[
u_1(a) > u_1(a'), \quad i = 1, 2.
\]

If not, consider the strategy \( y \), which sends \( m_1 \) (the \( i \)-th component of \( m \)) instead of \( \sigma_1 \), but is otherwise the same as \( x \). If (3.6) does not apply, the reverse inequality must hold (by the regularity assumption there are no payoff ties at pure
Nash equilibria of G). Hence $u(y, x) > u(x, x)$ so that x is not a NSS.

(3.6) and the fact that $a$ and $a'$ are strict Nash equilibria implies the inequality:

$$u_i(g) > u_i(a') > u_i(a_i, a_j') \text{ for } i=1,2, j=3-i. \ldots \ldots (3.7)$$

Hence a Pareto dominates all other outcomes and is hence utilitarian. We have therefore proved that if a is inefficient, $\theta$ is constant on $M \times M$.

If a is inefficient, let $a^*$ be the efficient action pair. We now show that x is not a weak NSS. The probability with which message $n$ is sent by x is $(1-n)\sigma_n + \eta \mu_n$. Let $\delta > 0$ be given. Consider the following sets:

$$\{n \in M: \sigma_n < \delta/2\}$$
$$\{n \in M: \mu_n < \delta/2\}$$

Since $\sigma_n$ and $\mu_n$ are convergent series, both the above sets contain all but a finite number of elements of M. Let the mutant strategy select a message in the intersection of these two sets. For any $\eta$, this message is sent by x with probability less than $\delta$. Hence the mutant can use such messages to signal and switch to the efficient outcome. Since the mutant plays as x does after messages which arise with high probability, the mutant's payoff loss when matched against x can be made arbitrarily small. Hence x is not a weak NSS. \(\square\)

Remark 1. The efficiency result is approximate, since the strategy can choose actions which are inefficient after messages which only sent by mistake. Consider the unanimity game $G_1$, and let x send message 1, play $(H, H)$ after the message pair $(1,1)$, and play $(L, L)$ after any other message. x is a NSS which is approximately efficient, and $u(x, x) = (1-\eta)^2 + [1-(1-\eta)^2]1$, which converges to 2 as $\eta$ tends to zero.

The efficiency result does not generalize when the underlying game, G, is 3x3 or more, since in this case G may have more than two pure strategy Nash equilibria. Consider the 3x3 unanimity game, $G_5$, which has three strict Nash equilibria. Consider the strategy PERFECT DICTATOR, which sends message 1, and
plays M after the message pair (1,1), and plays L after any other message. Since
PERFECT DICTATOR plays a strict Nash equilibrium after every message pair, it
satisfies the condition of lemma 3.1. Any mutant sending a deviant message is
punished, and gets a payoff of 1, and is hence not (even approximately) a best
response to PERFECT DICTATOR. Hence PERFECT DICTATOR is an inefficient NSS of the
cheap talk game with mistakes. This example is fully robust. We therefore consider
a different perturbation of the cheap talk game, in the following section.

4. NOISY COMMUNICATION

We now consider a different perturbation of the cheap talk game. Rather than
assuming that a player makes a mistake, we assume that there is some noise in the
transmission of messages. Suppose player 1 sends message $m$ to player 2. With high
probability player 2 receives message $m$; however, with a small probability, player
2 receives a different message, say message $m'$. When this happens, player 1 is not
aware that this has indeed occurred. In other words, player 1 knows the message he
sent, and the message that he received, but does not know the message actually
received by player 2, nor the message sent by player 2. This type of noise in the
transmission of messages can lead to a misunderstanding. The possibility of a
misunderstanding seems ever present whenever there is communication between agents,
and it is this that we take into account. The cheap talk game with noise, $G^N(\eta)$, is
as follows:

1. Players are randomly matched and assigned roles. The player in role 1
chooses a message from $M$.

2. Nature moves, independently for the two players. With probability $(1-\eta)$,
player 2 receives the message sent by 1. With probability $\eta$, 2 receives a
different message; in this case the message received by 2 is selected from $M$ by the
probability measure $\mu$.

3. Player 1 only observes the message he receives; he does not observe the
message that \( j \) sent, nor the message that \( j \) receives. Players choose randomized actions from \( \Delta \mathbb{A}_1 \). Payoffs depend only on the actions chosen.

Let \( m_1 \) denote the message sent by player 1 and let \( r_j \in M \) denote the message received by player \( j \). Note that \( r_j \) does not necessarily equal \( m_1 \). Let \( \nu(r_j/m_1) \) denote the conditional probability that the message received by \( j \) is \( r_j \) given that \( m_1 \) was sent by 1.

\[
\nu(r_j/m_1) = \begin{cases} 
(1-\eta) + \eta \mu(r_j) & \text{if } r_j = m_1 \\
\eta \mu(r_j) & \text{if } r_j \neq m_1
\end{cases} \tag{4.1}
\]

The action taken by \( i \), \( \theta_i \), is now a function of \( m_1 \) and \( r_i \). The payoff of strategy \( x \) against \( x' \) is given by:

\[
u(r_j/m_1) = (1-\eta) + \eta \mu(r_j) \quad \text{if } r_j = m_1 \\
\eta \mu(r_j) \quad \text{if } r_j \neq m_1
\]

\[
\theta_i(m_1, r_i) = \begin{cases} 
\sigma_1(m_1) & \text{if } r_i = m_1 \\
\sigma'_1(m_1) & \text{if } r_i \neq m_1
\end{cases}
\]

\[
\sigma_2(m_2) \quad \text{if } r_2 = m_2 \\
\sigma'_2(m_2) \quad \text{if } r_2 \neq m_2
\]

\[
\sigma_1(m_1) \quad \text{if } r_1 = m_1 \\
\sigma'_1(m_1) \quad \text{if } r_1 \neq m_1
\]

\[
u(r_j/m_1) = (1-\eta) + \eta \mu(r_j) \quad \text{if } r_j = m_1 \\
\eta \mu(r_j) \quad \text{if } r_j \neq m_1
\]

\[
\theta_2(m_2, r_2) = \begin{cases} 
\sigma_2(m_2) & \text{if } r_2 = m_2 \\
\sigma'_2(m_2) & \text{if } r_2 \neq m_2
\end{cases}
\]

\[
u(r_j/m_1) = (1-\eta) + \eta \mu(r_j) \quad \text{if } r_j = m_1 \\
\eta \mu(r_j) \quad \text{if } r_j \neq m_1
\]

Note that if \( m_1 \) is a message which is not sent by 1, \( \theta_i(m_1, r_j) \) can be completely arbitrary since it does not affect payoffs.

The definitions of neutral stability and ES sets in the noisy cheap talk game are exactly as in definitions 3.1-3.5, with the \( G^N \) replacing \( G^M \) in these definitions.

The existence result is the same as in section 3. The proof is omitted since it is the same as the proof of Proposition 1.

**PROPOSITION 3 (EXISTENCE)** If \( G \) is a compatible game, \( G^M \) has a strong ES set which induces the utilitarian outcome.

Noise allows us to get a significantly stronger efficiency result provided that \( G \) satisfies the regularity assumption.

**PROPOSITION 4 (EFFICIENCY)** Let \( G \) be a normal form game satisfying the
regularity assumption. If $x$ is a weak NSS of $G^N$, $x$ is efficient.

**COROLLARY** If $G$ satisfies the regularity assumption, $x$ is a weak NSS of $G^M$ if and only if $x$ belongs to a strong ES set.

They key to the proof of Proposition 4 is the observation that strategies that punish deviant messages, such as the strategy PERFECT DICTATOR in $G_5^*$ (or DICTATOR in $G_1^*$) are sub-optimal in the presence of noise - although they are subgame perfect equilibria in the game without noise, they are not even Nash equilibria of the perturbed game. Recall that PERFECT DICTATOR (abbreviated to PD henceforth) is an inefficient NSS in the game with mistakes. PD sent message 1, and chose $M$ if the other player had sent message 1, but chose $L$ after any other message. PD is not a Nash equilibrium with noisy communication. The payoff of PD against itself is:

$$u(PD,PD) = (1-\eta)^2 + 2\eta(1-\eta)x0 + \eta^2 \cdot 1$$

(4.3)

The payoff of BAD2, the strategy which sends message 1 and chooses $M$ irrespective of the message, against PERFECT DICTATOR is:

$$u(BAD2,PD) = [(1-\eta)^2 + \eta(1-\eta)]^2 + [\eta(1-\eta) + \eta^2] \cdot 0$$

(4.4)

The difference in payoffs is:

$$u(BAD2,PD) - u(PD,PD) = \eta(1-\eta)2 - \eta^2 \cdot 1$$

(4.5)

which is strictly positive for $\eta$ sufficiently small.

The above example shows that the sequential equilibrium correspondence of the cheap talk game fails to be lower-hemicontinuous in $\eta$ at the point $\eta = 0$. This is shown more generally in lemma 4.1. Recall from lemma 2.1 that if $G$ satisfies the regularity assumption, $\theta_i$ is constant on $\text{supp}(\sigma)$. In this case player $i$ does not condition his action on the message he sent. Hence, $\theta_i$ may be, without loss of generality be taken to be a function from $M_j$ to $A_1$.

**Lemma 4.1** Let $x = (\sigma, \theta)$, let $\theta_i(m_j, r_j) = a_j \forall r_j \in \text{supp}(\sigma_i)$, where $a = (a_1, a_j)$ is a pure strategy Nash equilibrium of $G$. If $x$ is a Nash equilibrium of $G^M(\eta)$ for $\eta > 0$ and sufficiently small, $\theta_i(m) = a_i \forall m \in \text{supp}(\sigma_i) M_j$, $i=1,2$, $j=3-i$. 
Proof Since $\theta_j(\text{supp}(\sigma)) = a_j$, player $j$ plays $a_j$ with probability greater than $(1-\eta)$. This probability is independent of the message received by $i$, since the noise is independent. Since $a_i$ is a strict best response to $a_j$, it is also strict best response to $\theta_j$ for $\eta$ sufficiently small. Hence if $\eta > 0$, player $i$ must play $a_i$ after any $m_j$.

Proof of Proposition 4. If $G$ satisfies the regularity condition and $x=(\sigma, \theta)$ is a NSS of $G^N$, lemmata 2.1 and 4.1 imply that $\theta_i(m) = a_i$ (a constant action) $\forall m \in \text{supp}(\sigma_i) \times M_j$, for $i=1,2$, $j=3,1$. The rest of the proof is along the lines of the proof of proposition 2, and is hence abbreviated. If $\sigma$ is not efficient, let the mutant choose messages which occur with arbitrarily small probability under $x$, and play the efficient action after receiving such a message, and play $a$ otherwise.

Remark 1. In contrast with proposition 2 the efficiency result here is exact – approximately efficient strategies are not Nash equilibria by lemma 4.1.

We now return to the example of game $G3$ to show that if the regularity assumption is not satisfied, jointly controlled randomization may allow an inefficient outcome to persist even in the presence of noise. Recall that the strategy JCR sent messages 1 and 2 with equal probability, and played $(L,M)$ if the messages exchanged coincided, and $(M,L)$ if they differed. If the other player sent a different message (say message 3), JCR plays $M$. The payoffs to messages are shown in Fig. 4. Although JCR punishes deviant messages, this is not ruled out with noisy messages. In fact it is optimal to play $M$ after message 3 in a population of JCR players. If I receive message 3 in such a population, this must be due to noise. With probability one-half my opponent has sent message 1 and with probability one-half he has sent message 2. Hence, independently of my message, he is equally likely to play actions $L$ and $M$, making $M$ a strict best response. In other words, the punishment of deviant messages by playing $M$ is optimal in the presence of noise.
Although JCR is neutrally stable, it does not belong to an ES set, i.e. the corollary to proposition 4 also fails without the regularity assumption. Consider the strategy SIMPLE which sends message 1 in both roles, and takes the same actions as JCR. It is clear that SIMPLE ∉ JCR, so that SIMPLE can be replaced by JCR via drift. However, SIMPLE is not a Nash equilibrium of the game with noise (by lemma 4.1), and can hence be replaced by the strategy which plays (L,M) irrespective of the message, which can then be invaded by a mutant which coordinates on (H,H).

To summarize, we have proved existence and efficiency results for the class of compatible games satisfying the regularity condition. This implies that if the underlying game is generic, one can expect cooperation to evolve quickly, and to be stable.

5. FINITELY MANY MESSAGES

One assumption which is relatively non-standard is our assumption that players have infinitely many messages. This assumption ensures that there are always messages which are (almost) unused, so that a mutant can use such messages to coordinate upon the efficient outcome. We note that Kim and Sobel (1994) have to make a similar assumption - they consider a finite population, restrict players to use pure strategies and assume that there are more messages than players. We note at this point that with the introduction of noise, we do require infinitely many messages, since otherwise we have an inefficiency result, with either formulation of noise. More precisely, we have inefficiency unless we can make an assumption about the relative importance of mutations versus noise.

The following example may clarify matters. Consider the game G1 and let M = \{1,2\}. If a player sends any message, say message 1, with probability (1-\(\eta\)) message 1 is received by the other player, and with probability \(\eta\) the other message, message 2 is received. Consider the stability of the set of strategies \(\Omega(L,L)\), which sends arbitrary messages and plays L irrespective of the message. It
is clear that if we to are show that Q(L,L) is not an ES set, we must show that one of the pure strategies in it is not a NSS. BAD is such a pure strategy; this sends message 1 and plays L irrespective of the message. Consider the invading strategy GOOD, which sends message 2, and plays L if it receives 1, and H if it receives message 2. The payoffs when GOOD and BAD play each other are:

\[
\begin{align*}
    u(\text{BAD, BAD}) &= 1 \\
    u(\text{GOOD, BAD}) &= 1 - \eta \\
    u(\text{BAD, GOOD}) &= 1 - \eta \\
    u(\text{GOOD, GOOD}) &= 2(1 - \eta)^2 + \eta^2
\end{align*}
\]

The expected payoff of GOOD in the mixed population is greater if its share \( \epsilon \) is greater than the critical value, \( \epsilon'(\eta) \), which is given by:

\[
\epsilon'(\eta) = \eta(1 - \eta)/[1 - \eta(1 - \eta)]
\]

\( \epsilon'(\eta) \) is of the same order as \( \eta \), and in this example is larger than \( \eta \). Hence mutations must be sufficiently important as compared to the noise for BAD to be invaded by GOOD. To put things differently, for every \( \eta \), there exists a \( \epsilon'(\eta) \) such that if the size of mutations is below this, the inefficient outcome belongs to an ES set. This is formalized in the following proposition, which adopts the weak definition of an ES set:

**Proposition 5** If \( \varphi \) is a strict Nash equilibrium of \( G \), there exists a weak ES set of \( G^\varphi \) (resp. \( G^N \)) which induces \( \varphi \).

**Proof** We show \( \Omega(\varphi) \) is a weak ES set, i.e. given \( \eta > 0 \) \exists \( \epsilon'(\eta) \) such that any \( x \) in \( \Omega(\varphi) \) cannot be invaded by any \( y \) from outside. Fix \( \eta \). Since \( M \) is finite, \( G^M(\eta) \) defines a finite game. Since \( \varphi \) is a strict Nash equilibrium, \( u(y, x) < u(x, x) \), \( \forall y \notin \Omega(\varphi) \) and hence \( \exists \) an invasion barrier \( \epsilon'(\eta) \).

**Remark** \( \Omega(\varphi) \) is not a strong ES set, i.e. there does not exist \( \epsilon' \) such that \( \Omega(\varphi) \) is an ES set of \( G^M(\eta) \) with \( \eta \) arbitrarily small. To put it differently, to get even asymptotic efficiency in the presence of noise one has to assume that mutations are more important than noise. This may not be a reasonable assumption;
noise can be seen as a random phenomenon, whereas mutations represent coordinated deviations by a positive measure of a large population. In our view, the assumption of infinitely many messages is more palatable.

6 CONCLUDING COMMENTS

There is a large literature on how evolution ensures efficiency when mutants are able to use actions to signal their identities, and thereby coordinate a deviation to the efficient outcome. Such possibilities arise in the context of pre-play communication as well as in repeated games. In the former context, we must mention the work of Fudenberg and Maskin (1991), Kim and Sobel (1994), Matsui (1991), Schlag (1994a, 1994b) and Warneryd (1991). In the repeated game context, similar ideas have been used by Binmore and Samuelson (1992) and Fudenberg and Maskin (1990). Since there already exists an excellent review of this literature (Sobel, 1993), we have felt free to focus on the novel features of our approach.

The essence of our approach has been to obtain efficiency results which do not rely upon drift. We do this by perturbing the game with pre-play communication. In this respect, we differ from most of the preceding literature, with the exception of Fudenberg and Maskin (1991). They also consider a model of pre-play communication, where players make mistakes. This model is very similar to their paper on repeated games (Fudenberg and Maskin, 1990). There are major differences between our approach and theirs'. First, they have to allow for infinitely many rounds of communication. Second, they also have to assume that mistakes occur with arbitrarily higher probability than mutations - in fact they assume that for any $n$, $n$ mistakes occur with higher order of probability than a single mutation. In contrast, our model does requires only a single round of communication, and needs make no assumption about the relative probabilities of noise versus mutations. Finally, our general efficiency result requires the device of noise rather than mistakes.
We conclude with the observation that the specification of noise introduced in this paper may be of independent interest, outside the evolutionary context. The introduction of noise dramatically reduces the set of sequential equilibria of the game. This is due to the fact that messages are not mutual knowledge between players in a model with noise, even though they are arbitrarily close to being so. Thus this formulation of noise may have wider application, even in contexts where players are assumed to be perfectly rational.
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1 This mechanism may also not be robust; the strategy profile can drift from x to y only if there are two exact equalities among the four possible payoffs which arise when x and y are matched. These equalities may not be robust to perturbations in the game.

2 The importance of considering the speed of convergence to equilibrium has been highlighted in the evolutionary context by Ellison (1993).

3 $x \in S$ is Lyapunov stable if for every small neighborhood $N(x) \subseteq S$, there is another neighborhood $N'(x)$ such that every trajectory in $N'(x)$ does not leave $N(x)$. $x$ is asymptotically stable if $x$ is Lyapunov stable and if $\exists N''(x)$ such that every trajectory in $N''(x)$ converges to $x$.

4 These equivalences hold for finite games, but the methods of proof suggest that the arguments should extend to bimatrix games with countably infinite strategy sets.

5 Warneryd (1991) obtains an efficiency result for $G1$ using neutral stability, but considers only pure strategies.

6 With a finite number of messages, the mixed strategy is in fact evolutionary stable if players cannot condition their strategies upon the role they fill, as Kim and Sobel (1992) and Schlag (1994) point out. However, with infinitely many messages, this strategy fails to be neutrally stable even if players cannot condition upon roles.

7 See for example Samuelson (1994) who finds that iteratively weakly dominated strategies can be eliminated only if mistakes occur with higher order of probability than mutations.
It is interesting that the same example was independently discussed by Kim and Sobel (1992). This example posed a problem for them for quite different reasons - the problem arises, in their framework, since players could use up all the messages in performing the randomization, leaving no unused messages. In our case the problem is due to the "credibility" of punishments even in the presence of noise, a problem which does not arise when one can rely on drift.
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