ON THE IDENTIFIABILITY OF HOUSEHOLD PRODUCTION FUNCTIONS WITH JOINT PRODUCTS: A COMMENT

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1. Introduction

Since the appearance of the classical paper by Becker (1965) on household production theory, several attempts have been made to estimate household production functions. To disentangle empirically the effects of the household's preferences and its production technology, either the household products have to be observable, which is rarely the case,\(^1\) or one has to assume that the household products can also be bought on the market. In the latter case preferences hardly play a role, because the perfect substitutability of the household products output for market goods generates efficiency conditions that relate the number of hours spent on homework by each spouse to their market wages, at least if they work in a paid job. Having data on time spent on homework, wages and non-labor income, (some) production function parameters can then be estimated. Examples are the papers by Gronau (1977, 1980) and a recent paper in this Review by Graham and Green (1984) (G & G in the sequel).

A particular feature of the G & G model is that it allows for joint production, meaning that the spouses may enjoy homework and hence count part of the time spent on it as pure leisure. After choosing specific functional forms G & G derive an equation for female homework which they estimate by OLS using cross-section data on 921 two-earner households. Since their model is underidentified different sets of additional restrictions on structural parameters are imposed. These different sets of restrictions yield dramatically different estimates of the value of home production.

The ostensible underidentification in G & G's model emanates from ignoring the information contained in the corresponding male homework equation. In Section 2 we show that if the equations for female homework and for male homework are considered jointly, the model is in fact over-identified.

However, the possibility to identify the parameters of the household production function and the parameters describing the extent of joint production does not only require the assumption that the output could also

\[^1\) An exception is the paper by Rosenzweig and Schultz (1983) who use birth weight as an indicator of the output of the household health production function.\]
be bought on the market. It also requires the use of specific, restrictive functional forms as the ones employed by G & G. In Section 3 we show that in general a model with joint production is fundamentally distinguishable from a model without joint production. So, in G & G’s model conclusions on joint production will entirely rest upon assumptions about functional form.

2. Identification in the G & G model

The model formulated by Graham and Green consists of the following six relations

\[ U = U(C, M_{h}, M_{w}) \]  \hspace{1cm} (1)
\[ C = X_{m} + Z \]  \hspace{1cm} (2)
\[ Z = Z(X_{z}, M_{h}^{h}, M_{w}^{h}) \]  \hspace{1cm} (3)
\[ X_{m} + X_{z} = W_{h}N_{h} + W_{w}N_{w} + v \]  \hspace{1cm} (4)
\[ \lambda_{i} + H_{i} + N_{i} = T; i = h, w \]  \hspace{1cm} (5)
\[ L_{i} = \lambda_{i} + g_{i}(H_{i}); i = h, w \]  \hspace{1cm} (6)

Here \( U(.) \) is the household utility function. \( C \) represents goods which are either obtained in the market \( (X_{m}) \) or produced at home \( (Z) \). Eq. (3) describes the production of \( Z \) within the household, where \( X_{z} \) equals market purchased inputs used in production and \( M_{h}^{a} \) and \( M_{w}^{b} \) are the "effective" time inputs of husband and wife, respectively. Eq. (4) is the household budget constraint where \( v \) is non-labor income, \( W_{h} \) and \( W_{w} \) are hourly wages, and \( N_{h} \) and \( N_{w} \) are hours of work in paid jobs by the husband and wife, respectively. The function \( g_{i}(H_{i}) \) is introduced to account for joint production and denotes the fraction of time spent on home production which is valued as pure leisure.

For empirical implementation, G & G choose the following particular functional forms for \( Z(X_{z}, M_{h}^{a}, M_{w}^{b}) \) and \( g_{i}(H_{i}) \):
Next, they show that in order to maximize (1) s.t. (2)-(6), $H_h$, $H_w$ and $X_z$ should satisfy the following first order conditions:

$$Z = A(M^a_{h\cdot h})^{\gamma_h} (M^b_{w\cdot w})^{\gamma_w} X^\beta_z$$

$$g_i(H_i) = H_i - T_i (1+\delta_i)^{-1} H_i^{-1}, i = h, w.$$  \hspace{1cm} (8)

Note that the utility function is left unspecified and does not play a role in the system (9)-(11); due to the assumption of perfect substitutability between home products and market goods, $H_m$, $H_g$ and $X_z$ are completely determined by the conditions for efficient production of $Z$, at least for exogenous $W_m$ and $W_w$.

Solving the system simultaneously for $H_w$ and $H_h$ and expressing the solution in logs yields

$$\log H_w = c_w + q_w^{-1} \log A + q_w^{-1} \left[ \frac{\gamma_h}{1+\delta_h} + \beta - 1 \right] \log W_w$$

$$-q_w^{-1} \left[ \frac{\gamma_h}{1+\delta_h} \right] \log W_h + q_w^{-1} a\gamma_h \log M_h + q_w^{-1} b\gamma_w \log M_w$$

and

$$\log H_h = c_h + q_h^{-1} \log A + q_h^{-1} \left[ \frac{\gamma_w}{1+\delta_w} + \beta - 1 \right] \log W_h$$

$$-q_h^{-1} \left[ \frac{\gamma_w}{1+\delta_w} \right] \log W_w + q_h^{-1} a\gamma_h \log M_h + q_h^{-1} b\gamma_w \log M_w$$

\hspace{1cm} (12)\hspace{1cm} (13)
with

\[ q_w = (1-\beta)(1+\delta_w) - \gamma_w - \frac{\gamma_h(1+\delta_w)}{1+\delta_h} \]  

and

\[ q_h = (1-\beta)(1+\delta_h) - \gamma_h - \frac{\gamma_w(1+\delta_h)}{1+\delta_w} \]

Now consider the following reduced form regressions

\[ \log H_w = C_w + k_w \log A + \ell_w \log W_w + \mu_w \log W_h + \nu_w \log M_w + \omega_w \log M_h \]  

(16)

\[ \log H_h = C_h + k_h \log A + \ell_h \log W_w + \mu_h \log W_h + \nu_h \log M_w + \omega_h \log M_h \]  

(17)

The estimated coefficients in (16) and (17) can be used to obtain estimates for the structural parameters in (12) and (13).

For some unknown reason, G & G ignore the equations for male homework, although their data set seems to contain the information required for the estimation of (17).

Since the unknown parameters \( \gamma_h, \delta_h, \delta_w, \alpha, \) and \( \beta \) cannot be retrieved from the estimated coefficients in (16) only, G & G have to impose additional restrictions.

However, if one exploits the information from both (16) and (17), the structural parameters can be identified as follows. \( \beta \) can be estimated by either \( (k_w + \ell_w + m_w)k_w^{-1} \) or \( (k_h + \ell_h + m_h)k_h^{-1} \). Given an estimate for \( \beta \), estimates for \( \gamma_w, \gamma_h, \delta_w, \) and \( \delta_h \) can be obtained from

\[ \frac{\gamma_h}{1+\delta_h} = -\frac{m_w}{k_w}, \quad k_w^{-1} = (1-\beta)(1+\delta_w) - \gamma_w - \frac{\gamma_h(1+\delta_w)}{1+\delta_h} \]

\[ \frac{\gamma_w}{1+\gamma_w} = -\frac{m_h}{k_h}, \quad k_h^{-1} = (1-\beta)(1+\delta_h) - \gamma_h - \frac{\gamma_w(1+\delta_h)}{1+\delta_w} \]
The solution is given by

\[ \hat{\delta}_w = k_w^{-1}(1-\beta + m_w^{-1} + m_h^{-1})^{-1} - 1 \]

\[ \hat{\delta}_h = k_h^{-1}(1-\beta + m_w^{-1} + m_h^{-1})^{-1} - 1 \]

\[ \gamma_w = -m_h k_h^{-1}(1+\delta_w) \]

\[ \gamma_h = -m_w k_w^{-1}(1+\delta_h) \]

Finally, given the estimates for \( \gamma_h \) and \( \gamma_w \), \( a \) can be estimated by \( o_w k_w^{-1} \cdot \gamma_h \) or by \( o_h k_h^{-1} \cdot \gamma_h \), whereas \( b \) can be estimated by \( n_w k_w^{-1} \cdot \gamma_w \) or by \( n_h k_h^{-1} \cdot \gamma_w \).

3. Non-identifiability of joint production in the general model

Reconsider the general formulation (1)-(6) and let us ignore the productivity variables \( M_h \) and \( M_w \), i.e. we take \( M_h = M_w = 1 \). This does not affect the main argument. The assumptions made by G & G regarding the function \( g_i \) are

(i) \( 0 \leq g'_i(H_i) \leq 1 \)

(ii) \( g''_i(H_i) < 0 \)

(iii) \( \lim_{H_i \to 0} g'_i(H_i) = 1 \)

(iv) \( \lim_{H_i \to T} g'_i(H_i) = 0, \ i = h, w \)

It is easy to show that the first order conditions determining \( X_z, H_h \) and \( H_w \) are (cf. Kooreman and Kapteyn, 1987, Appendix A):

\[ \frac{\partial Z}{\partial X_z} = 1 \] (18)
\[ \frac{\partial Z}{\partial H_i} = \theta_i[1 - g'_i(H_i)], \quad i = h, w \]  

(19)

Now define

\[ H_i^* = H_i - g_i(H_i) = k_i(H_i), \]  

(20)

as the "pure household production time" (i.e. \( H_i \) minus the part that is considered to be leisure by the individual). By construction the function \( k(.) \) is monotonically increasing. Let \( F \) be the production function that is obtained by substituting \( H_i = \frac{1}{\partial}^{-1}(H_i^*) \) into (3), i.e.

\[ Z = F(X, H_h, H_w) = Z(X, H_h, H_w) \]  

(21)

We then have

\[ \frac{\partial Z}{\partial H_i} = \frac{\partial F}{\partial H_i} \cdot \frac{\partial H_i^*}{\partial H_i} = \frac{\partial F}{\partial H_i} [1 - g'_i(H_i)], \quad i = h, w \]  

(22)

and we can write (5) as

\[ L_i + H_i^* + N_i = T, \quad i = h, w \]  

(23)

The model consisting of the equations (1), (2), (21), (4), (23) is equivalent to the model consisting of (1)-(6). Both models generate identical first order conditions and it is also easy to show that if \( Z \) is a quasi-concave twice differentiable function, then so is \( F \). Since (1), (2), (21), (4), (23) is a model without joint production and (1)-(6) a model with joint production, it is clear that in the G & G framework it is fundamentally impossible to identify jointness in household production.
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