EXPLAINING CHANGES IN EXTERNAL FUNDS
Part One: Theory

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FEW 574

Communicated by Prof. Dr. Ir. A. Kapteyn
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Abstract

This paper develops equations that explain changes in stock capital, long debt and short debt (external funds). It is assumed that firms make their external funds decisions so as to minimize the costs of capital that arise from: (i) the need to finance asset changes, and (ii) not having a target capital structure. For the explanation of the target capital structure the several theories of optimal capital structure are used. In part two of the paper the equations are estimated from data for 100 Dutch corporatons for the years 1981–1986. Besides these empirical results, which give reasonable support to the model, there is support from the possibility to embed most previous work in the theoretical equations.
1 Introduction

In order to explain the funds decisions of firms, one might start with a fully dynamic model which minimizes the sum of the discounted yearly expected cost of capital subject to flow of funds restrictions. The optimal first period decision rules for the several funds then would provide the starting point for the positive model. As this is almost certainly not the correct way to model actual funds decisions, a simpler procedure will be proposed. Before elaborating on this, some remarks on previous work will be made.

Previous work, specifying and estimating equations describing capital structure, may conveniently be classified in two categories. One category studies the determinants of long-term (equilibrium) capital structure and another one the short-term (dynamic) behavior. Studies of the equilibrium type are e.g. the ones by Bradley, Jarrell and Kim (1984), Long and Malitz (1985) and Titman and Wessels (1988). Studies of the dynamic type are e.g. the ones by Taggart (1977), Auerbach (1985) and Chowdhury, Green and Miles (1986). In studying this previous work, the question arose whether a common structure for these models could be found, on the basis of a simple static optimizing model. As this common structure may indeed be found, the discussion of this previous work is postponed to Section 4.

In Section 2 the static model will be developed on the basis of the following ideas. Firstly, it is assumed that firms have some (however formed, but quantitative) idea about their optimal capital structure and that this is their target structure; see e.g. Donaldson (1984). Secondly, it is assumed that firms make their actual yearly funds decisions by minimizing the yearly cost of capital of these sources. Thirdly, it is assumed that the behavior of firms concerning their target capital structure may be modelled by incorporating a penalty part, that quantifies the firm’s loss for a chosen, possibly non-target, capital structure, into the cost of capital. Finally, fixed transaction costs are used to explain no-change decisions.

In Section 3 some of the unobserved explanatory variables of the model of Section 2 are modelled. Section 4 discusses previous work and considers how it fits into the framework of Section 2.
2 A model for external funds decisions

2.1 Introduction

For the sake of concreteness the following sources and uses of funds statement is used.

<table>
<thead>
<tr>
<th>Sources of funds</th>
<th>Uses of funds</th>
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<tbody>
<tr>
<td>Depreciation</td>
<td>Asset uses (investment in fixed assets,</td>
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<tr>
<td></td>
<td>changes in inventories, accounts</td>
</tr>
<tr>
<td>Taxes payable</td>
<td>receivable, and liquid assets).</td>
</tr>
<tr>
<td>Dividend payable (D)</td>
<td>Financial uses (payment of dividend and</td>
</tr>
<tr>
<td>Retained profit (ER)</td>
<td>tax, retirements of: allowances (RAL),</td>
</tr>
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<td></td>
<td>stock (RST), long debt (RLD),</td>
</tr>
<tr>
<td>New allowances (NAL)</td>
<td>and short debt (RSD)).</td>
</tr>
<tr>
<td>New stock (NST)</td>
<td></td>
</tr>
<tr>
<td>New long debt (NLD)</td>
<td></td>
</tr>
<tr>
<td>New short debt (NSD)</td>
<td></td>
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</tbody>
</table>

The funds decisions one would like to explain, because these cause changes in capital structure, are: retained profit (ER), new allowances (NAL), new stock (NST), new long debt (NLD), and new short debt (NSD). The model to be developed will explain the variation in the external sources of funds, i.e. NST, NLD, and NSD, together composing the external financial deficit (F), which is defined as total uses of funds minus internal sources (cashflow (CF) plus new allowances).

The decisions concerning NAL will be assumed to be autonomous, i.e. they are made independent of the composition of profit after tax (E) and F. In the sequel the composition of F will be considered to be independent of the composition of E. Whether this is a correct assumption, depends on the dividend (D) policy and thus on the way in which ER is determined. With a predetermined dividend policy\(^1\), in the sense of a policy that is independent of the other (than D) decision variables in the two totals, E and F, ER is also predetermined, so that the composition of F is independent of that of E. There are two other possibilities with respect to dividend policy: (i) a purely residual policy, in which case D follows from a policy decision for ER, which is simultaneously determined with the decision variables in F, and (ii) a simultaneous dividend decision,

\(^1\)An example is Lintner's (1956) model, where D depends on D lagged for one year, and on E.
in which case \( D \) is determined simultaneously with the decision variables composing \( F \) (and \( ER \) follows from the dividend decision); see Appendix A for a concise treatment of these two cases. In both of these cases there is interdependence between the decision variables in \( E \) and \( F \). As in this paper only the composition of \( F \) will be modelled, independent of the composition of \( E \), the assumption thus is that neither of these two dividend policies is followed.

2.2 The composition of the external financial deficit without fixed transaction costs

In this subsection, most of the time, there will be abstracted from the above concrete framework: the capital components (funds) composing \( F \) will be denoted by \( v_1, \ldots, v_n \), which thus are the amounts of the external funds. The balance sheet change in fund \( i \), denoted \( \Delta v_i \), is the result of downward changes, called retirements and denoted \( Rv_i \), and of upward changes, called demands and denoted \( Nv_i \): \( \Delta v_i = Nv_i - Rv_i \) \((i = 1, \ldots, n) \). For the firm the \( \Delta v_i \) are of interest; an economist might only be interested in the amounts demanded, the \( Nv_i \).

The balance sheet change \( \Delta v_i \) is the result of positive values of \( Nv_i \) and/or \( Rv_i \). Depending on the particular fund, and, particularly, on the unit of time, one observes \( Nv_i \), or \( Rv_i \), or the net change \( \Delta v_i \). For stock \((i = 1)\) it will be assumed that both demand (stock issue) and retirement (stock repurchase) are observed within the same bookyear, which will be the time unit in the estimation part of the paper. Thus \( \Delta v_1 > 0 \), assumedly, implies pure issue and \( \Delta v_1 < 0 \) pure repurchase. For long debt \((i = 2)\) there are observations for the retirement part of \( \Delta v_2 \), so that the demand for long debt could be calculated. For short debt \((i = 3)\) one only observes \( \Delta v_3 \), which is the result of numerous transactions with respect to new short debt and repayment of short debt within the same time unit. For this reason the balance sheet changes \( \Delta v_i \) \((i = 1, \ldots, n)\), not the demands \( Nv_i \), will be modelled, and the composition of \( F \) will accordingly be adapted. It will be assumed that \( F \) consists of the \( \Delta v_i \):

\[
F_t = \sum_i \Delta v_{it}
\]  

(1)

where the extra index \( t \) denotes time period \( t \).
The approach to modelling the direct costs of the financing decisions, denoted $K_t$, will be to assume that these costs depend only on the $\Delta v_i$ and, specifically, in the following way:

$$K_t = \sum_i c_{it} \Delta v_{it} + \frac{1}{2} \sum_i \sum_j a_{ij} \Delta v_{it} \Delta v_{jt},$$  \hspace{1cm} (2)$$

where the $a_{ij}$ assumedly do not depend on $t$. The linear part of (2) will be called the basic costs; it contains the return related costs of $K_t$, by means of the time dependent $c_{it}$, that will be modelled later; see (7). Because of the assumed linearity, additions (the $N_{v_i}$) have the exactly opposite costs of retirements (the $R_{v_i}$), which is reasonable for return related costs.

The basic costs may also by thought to incorporate variable transaction costs, assuming these to be as in Figure 1, and assuming that $R_{v_i}$ is approximately proportional to $N_{v_i}$ for the non-stock funds. For stock ($i = 1$) there assumedly is either one issue or one repurchase transaction; variable transaction cost can then best be approximated by incorporating them into the quadratic part of (2); see also Figure 1.

Fixed transaction costs, which are more fundamental to the model, are relevant for stock and long debt; they will be introduced into the model in Section 2.3.

\footnote{In a closely related body of literature, regarding the explanation of the (change in) a portfolio of assets, which started with the work of Parkin (1970) on the balance sheet composition of so-called discount houses, this is the usual approach; see e.g. Hunt and Upcher (1979).}
The changes $\Delta v_i$ lead to changes in capital structure and may therefore lead to return related marginal costs that are not satisfactorily described by the $c_{it}$, but need an extra changes dependent component, say $\sum_j a_{ij} \Delta v_{jt}$. This idea is captured in the quadratic part of (2).

Expression (2) may also be rationalized by considering it as a quadratic approximation of a cost of capital function in the point $(v_{1,t-1}, \ldots, v_{n,t-1})$. As this is a somewhat mechanistic point of view, the above interpretation has been offered.

As mentioned in Section 1, firms assumedly have a target capital structure, which will be restricted to the external funds, $v_1, \ldots, v_n$, as the explanation of retained profit and changes in allowances has been excluded in Section 2.1. Capital structure then is specified by fractions $f_{it}^*$ ($i = 1, \ldots, n$), which are the desired proportions of (external) funds $i = 1, \ldots, n$ to total (external) funds in year $t$. These fractions are translated into target values, $v_{it}^*$ ($i = 1, \ldots, n$), which should, for consistency of desires, sum to the actual values. By definition of the $f_{it}^*$ this is indeed the case.

The firm takes these target values into account when choosing values for the $\Delta v_{it}$: the actual values, $\Delta v_{it}$, which the firm chooses, may assumedly be described as the outcome of the minimization of the direct cost $K_t$ plus a quantity which describes the undesirability of not being on target. This penalty quantity is denoted by $P_t$ and it supposedly is a function of the differences $v_{it} - v_{it}^*$ ($i = 1, \ldots, n$).

If the $v_{it}^*$ would be optimal values in a purely static problem, the cost of making suboptimal decisions $v_{it}$ could be approximated by a quadratic form in the $v_{it} - v_{it}^*$; see Theil (1964). Although the present problem is not a static one, and the $v_{it}^*$ are not the optimal values for period $t$, but the long-term target (equilibrium) values, it could analogously be assumed that the above mentioned penalty cost $P_t$ may be approximated as follows:

$$P_t = \frac{1}{2} \sum_i \sum_j b_{ij} (v_{it} - v_{it}^*) (v_{jt} - v_{jt}^*),$$

(3)

where it has been assumed that the $b_{ij}$ do not depend on time. For an interpretation of the $b_{ij}$ see e.g. Hunt and Upcher (1979).

Alternatively one might add a linear term to (3). This would give room for the possibility that values above $v_{it}^*$ are treated differently from those below $v_{it}^*$; see also Theil (1964) for a linear part added to a function of type (3). As this does not alter the equations to be derived no linear term will be added to (3).
Total cost in year $t$, $K_t + P_t$, is a quadratic function of $v_{1t}, \ldots, v_{nt}$. For this part of the paper, which is of a theoretical nature, a much less specific function of $v_{1t}, \ldots, v_{nt}$ could have been used. The above quadratic specification is used mainly for the following two reasons: (i) to discuss previous specifications of equations for the sources of funds, which are all linear, and (ii) to prepare the way for the application part of the paper.

The firm's financing behavior in year $t$ with respect to the components of $F_t$ assumedly follows from the minimization of its total cost of capital in year $t$, $K_t + P_t$, with respect to its net sources of funds$^3$, the $\Delta v_{it} = v_{it} - v_{i,t-1}$ $(i = 1, \ldots, n)$, taking into account equality (1). Strictly speaking one should also impose the restrictions $v_{it} \geq 0$ (or $\Delta v_{it} \geq -v_{i,t-1}$), so that there would be a programming problem. As it will be assumed that $v_{it} > 0$, it suffices to impose equality (1) as the only restriction.

The Lagrangian function is:

$$L_t = K_t + P_t - \lambda_t \left\{ \sum_{i=1}^{n} (v_{it} - v_{i,t-1}) - F_t \right\}.$$  

(4)

The (necessary) conditions for a minimum are:

$$\frac{\partial L_t}{\partial v_{it}} = \frac{\partial K_t}{\partial v_{it}} + \frac{\partial P_t}{\partial v_{it}} - \lambda_t = 0 \ (i = 1, \ldots, n),$$

(5a)

$$\frac{\partial L_t}{\partial \lambda} = \sum_{i} (v_{it} - v_{i,t-1}) - F_t = 0.$$  

(5b)

In the minimum the marginal cost in year $t$ is the same for each component. Condition (5a) implies, for each component, the same trade-off in the minimum between an increase in the direct cost, $\partial K_t/\partial v_{it}$, and a decrease in the penalty cost, $-\partial P_t/\partial v_{it}$. The increase in $K_t$ caused by $\partial v_{it} = 1$ evidently consists of two parts: a part ($\lambda_t$) caused by the (net) sources of funds restriction ($\partial v_{it} = 1$ causes an increase of $F_t$ by 1, with shadowcost $\lambda_t$) and a part ($-\partial P_t/\partial v_{it}$) caused by the decrease in penalty ($-\partial P_t/\partial v_{it}$ is positive (negative) if $v_{it}$ moves towards (away from) $v_{it}^*$).

$^3$The term 'source of fund' will, from here on, be used synonymous with 'net source of fund', which is the change $\Delta v$. 


Substituting into (5) expressions (2) and (3) for $K_t$ and $P_t$, respectively, and solving for the optimal levels $\tilde{v}_{it}$ of the funds $i = 1, \ldots, n$ gives, of course, the following linear decision rule:

$$\tilde{v}_{it} = \alpha_i F_t + \sum_{j=1}^{n} \beta_{ij} v_{j,t-1} + \sum_{j=1}^{n} \gamma_{ij} v_{j,t}^* + \sum_{j=1}^{n} \delta_{ij} c_{jt}. \quad (6)$$

Subtracting $v_{i,t-1}$ from both sides of (6) gives the optimal sources of funds, the $\Delta \tilde{v}_{it}$. As $\sum_i \tilde{v}_{it} = \sum_i v_{i,t-1} + F_t$, the following restrictions across the equations hold: $\sum_i \alpha_i = 1$, $\sum_i \beta_{ij} = 1$, $\sum_i \gamma_{ij} = 0$ and $\sum_i \delta_{ij} = 0$.

Details on the derivation of (6) may be found in Appendix B, where also the restrictions on the coefficients are proved.

### 2.3 The composition of the external financial deficit with fixed transaction costs

In the data used more than 60 per cent of the $\Delta v_1$ (change in stock) are zero’s. In this subsection it will be shown, that considerable fixed transaction costs may explain this phenomenon. (Remember that variable transaction costs were already included in the model of Section 2.2.)

In general the optimal change in stock of the previous section is not zero, i.e. $\Delta \tilde{v}_{1t} \neq 0$. If $\Delta \tilde{v}_{1t} > 0$, the firm considers a stock issue, and if $\Delta \tilde{v}_{1t} < 0$ a stock repurchase. Whether the firm actually makes the issue or the repurchase, depends on the amount of fixed transaction costs. These are added to $\tilde{K}_t + \tilde{P}_t$, the costs in the point $(\tilde{v}_{1t}, \ldots, \tilde{v}_{nt})$, resulting in the total costs, including fixed transaction costs, which total costs are compared to the total costs of having $\Delta v_{1t} = 0$ and optimal values for the other funds, with costs say $\bar{K}_t + \bar{P}_t$.

Figure 2 illustrates this procedure for the case $n = 2$, so that $\Delta v_{2t} = F_t - \Delta v_{1t}$, with fixed transaction costs $A_t$, and assuming $\Delta \tilde{v}_{1t} > 0$. Illustrated is the case that $\Delta v_{1t} = 0$ is optimal. An analogous illustration may be given for the case $\Delta \tilde{v}_{1t} < 0$. 
If one assumes that the fixed transaction costs for both cases are about the same, say \( A_t \), then (in the terminology of the extended Tobit model) the selection equation, which determines whether \( \Delta v_{1t} \) is zero or not, is:

\[
y_t = \tilde{K}_t + \tilde{P}_t - \tilde{K}_t - \tilde{P}_t - A_t,
\]

with the dummy \( d \) (which is 1 if \( \Delta v_{1t} \neq 0 \) and 0 otherwise) as follows: \( d = 1 \) if \( y_t > 0 \) and \( d = 0 \) if \( y_t \leq 0 \).

There are two sets of (linear) decision rules, one for each of the two cases, \( \Delta v_{1t} = 0 \) and \( \Delta v_{1t} \neq 0 \), assuming approximately the same fixed transaction costs for both cases. (Otherwise there would be three regimes.) The set of decision rules for the case \( \Delta v_{1t} \neq 0 \) has been given in Section 2.2. For the case \( \Delta v_{1t} = 0 \) there is a different set of decision rules, as \( F_t \) is now distributed over \( n - 1 \) funds, instead of \( n \); these decision rules have the same variables as for the case \( \Delta v_{1t} \neq 0 \), but they have different coefficients. The regression equation for \( \Delta v_{1t} \), for the case \( \Delta v_{1t} \neq 0 \), comes with selection equation (7), which is a linear function of the determinants of the \( \tilde{v}_{it} \) and of their squares and cross products, as a quadratic cost function has been assumed.
Actually, things are somewhat more complicated than one fund having fixed transaction costs, as, in addition to the change in stock, with 62 per cent zero's, there is the change in long debt (say \( \Delta v_2 \)) with 6.5 per cent zero's.

The same approach may be used as for the case of one fund having fixed transaction costs. This leads to four different sets of decision rules (regimes), which are for the cases (0,0), (0,1), (1,0) and (1,1), with \([\delta(\Delta v_1), \delta(\Delta v_2)]\) indicating whether \( \Delta v_1 \) and \( \Delta v_2 \) are zero or not. \( \delta \) is 0 if \( \Delta v_i = 0 \) \((i = 1, 2)\) and 1 otherwise. The firm compares the costs of the four regimes and chooses the one with least cost. There is then not one 'simple' selection equation.

Assuming \( n = 3 \) (with \( v_1 = ST, v_2 = LD, \) and \( v_3 = SD \)), for regime (0,0) \( F \) is 'distributed' only over \( \Delta SD \), i.e. the 'equations' are: \( \Delta ST = 0, \Delta LD = 0, \) and \( \Delta SD = F \). For regime (1,1) \( F \) is distributed over \( \Delta ST, \Delta LD, \) and \( \Delta SD \) and there are three complete equations as in (6). For regime (0,1) \( F \) is distributed over \( \Delta LD \) and \( \Delta SD \); there are two complete equations for \( \Delta LD \) and \( \Delta SD \), with the same variables but with other coefficients than in the corresponding equations for regime (1,1). An analogous statement holds for regime (0,1).

3 Modelling the unobserved variables \( v_{jt}^* \) and \( c_{jt} \)

In (6) there are several unobserved variables, the \( v_{jt}^* \) and the \( c_{jt} \) \((j = 1, \ldots, n)\). As to the \( c_{jt} \) direct calculation from data on dividend, interest, etc. is possible. Alternatively one may use the following model, which is easier to implement and which is more fundamental in that there is a more explicit link between the firm and the aggregate economy. According to the CAPM the basic marginal cost of capital, the \( c_{jt} \), can be modelled as follows:

\[
c_{jt} = RF_t + \{E_s(R_{jt}) - RF_t\}g_j \quad (j = 1, \ldots, n), \tag{8a}
\]

where \( RF \) is the riskfree interest rate, \( E_s(R_{jt}) \) the short-term expectation of the market rate of funds of type \( j \) and \( g_j \) the (corporation characteristic) beta factor of funds of type \( j \), which is specified as:
where \( z \) is a vector of corporation characteristic variables. Note that (8a) does not use a tax rate; this will be introduced later.

The \( v^*_t \) in relations (6) are also unobservable. They are modelled as follows:

\[
v^*_t = f^*_j T C_t \quad (j = 1, \ldots, n),
\]

(9a)

where \( T C_t \) is total (external) capital at the end of year \( t \) and the \( f^*_j \) (specifying the optimal capital structure) are assumed to depend on the same firm characteristic variables as the ones determining the beta factors in (8b) and moreover on the long-term expectations of some relevant market rates, assembled in the vector \( E_t(R_t) \):

\[
f^*_j = f^*_j(z_t, E_t(R_t)).
\]

(9b)

The motivation for using these determinants of the \( f^*_j \) is that the \( f^*_j \) have been determined, e.g. in the long-term financial plan of the firm, on the basis of the long-term expectations of the various (marginal) costs of capital, for which the same type of model as in (8) is assumed to hold; see Appendix C for a more formal approach.

Further corroboration for using the same determinants, \( z_t \), in (8) and (9), may be found in two bodies of literature, one concerning so called 'fundamental' betas and an other concerning optimal capital structure. For the former see e.g. Elton and Gruber (1991) and for the latter Harris and Raviv (1991).

Specification (9) ensures that the target values \( v^*_j \) are indeed given for the decision problem of determining the \( \Delta v^*_t \): \( F_t \) is given, so that \( T C_t = T C_{t-1} + F_t \) is also given; and the target ratio's \( f^*_j \) are given by assumption.
4 Previous work

As mentioned in the Introduction, previous work on empirical capital structure may be classified in two categories: one on long-term (equilibrium) and another on short-term capital structure.

In order to evaluate the specification of studies of the equilibrium type, the following equation for the observed fraction of funds of type $i$ ($i = 1, \ldots, n$) to total funds, derived from (6), is presented:

$$\frac{v_{it}}{TC_t} = \alpha_i \frac{F_t}{TC_t} + \sum_j \beta_{ij} \frac{v_{jt-1}}{TC_t} + \sum_j \gamma_{ij} f_j^*(z_t, \hat{E}_t(R_t)) + \sum_j \delta_{ij} \frac{c_{jt}}{TC_t},$$

where the 'deflator' $TC_t$ is a predetermined variable (see Section 3) and where the $v^*_j$ are seen as equilibrium values (see Appendix C). The $f_j^*$ have been defined as follows:

$$\frac{v^*_j}{TC_t} = f_j^*(z_t, \hat{E}_t(R_t)).$$

Of course, these are not observed ratio's, which obey relation (10). Nevertheless, relations of type (11) have implicitly been used in studies of the determinants of long-term capital structure. These studies use cross-sectional data so that variables that are only time dependent, like $\hat{E}_t(R_t)$ in (11), are constant. (Strictly speaking, $\hat{E}_t(R_t)$, being an expectation, may be different across firms.) The studies of Bradley, Jarrell and Kim (1984), Long and Malitz (1985) and Titman and Wessels (1988) are in this mode. As the variables used in the first two studies have also been used by Titman and Wessels (1988), only this last study will now be discussed.

Titman and Wessels (1988) explain the debt-equity ratio, as follows. Denote debt by $v_1$ and equity by $v_2$. They postulate:

$$\frac{v_{1t}}{v_{2t}} = h(z_t),$$

see Appendix C.
with \( h(\cdot) \) a linear function. Note that \( v_{1t}/v_{2t} \) is the ratio of two endogenous variables (in the model of Section 2) and cannot be described as simple as (12)\(^5\). Another objection to studies of this type is the neglect of many variables that also influence observed capital structure.

In order to evaluate the studies of the determinants of the dynamic behavior of capital structure, one may use (6). The following studies will be reviewed: Taggart (1977), Auerbach (1985) and Chowdhurry, Green and Miles (1986).

Taggart (1977) takes the financial deficit \((F)\) as exogenous and develops equations for the following five sources: new long debt \((\Delta LDBT)\), stock issues, stock retirements, change in liquid assets \((\Delta LIQ)\), and change in short debt \((\Delta SDBT)\). For more detailed comments Taggart’s equations for \(\Delta LDBT\) and \(\Delta SDBT\) (for quarter \(t\)) will be used. They are:

\[
\Delta LDBT_t = \alpha_1(LDBT^*_t - LDBT_{t-1}) + \alpha_2(PCB^*_t - PCB_{t-1} - RE_t) + \alpha_3STOCKT_t + \alpha_4RT_t,
\]

\[
\Delta SDBT_t = \Delta LIQ^*_t + \lambda_2(TC^*_t - TC_{t-1}) + \lambda_3F_t + \lambda_4RT_t.
\]

Here \(PCB\) is the book value of permanent capital (long-term debt plus equity), \(RE\) is retained earnings, \(STOCKT\) and \(RT\) are two timing-variables, and \(TC\) is temporary capital (short-term debt minus liquid assets). The target values are modelled as linear functions of known variables.

Essentially Taggart (1977) postulates some form of stock-adjustment, first for \(\Delta PCB\), and then for the five sources of funds. In a footnote he says that "the stock-adjustment mechanisms can be rationalized as an attempt to balance costs of adjustment against the costs of being out of equilibrium". This is, among other things, what has been done in Section 2, so that Taggart’s equations may very well be compared with equations (6).

\(^5\)Titman and Wessels (1988) mention in a footnote that they have also used, correctly, debt to total assets (which is comparable to \(TC\)).

\(^6\)The symbols of the authors discussed will be used. Starred symbols denote target values.
Consider then Taggart’s specification for new long debt, $\Delta LDBT$. As target values $LDBT^*$ and $PCB^*$ occur, and implicitly thus $PCB^* - LDBT^*$, the equity target, so that the other targets, $SDBT^*$ and $LIQ^*$ are lacking. The lagged capital components $LDBT_{-1}$ and $PCB_{-1}$ occur, but $SDBT_{-1}$ and $LIQ_{-1}$ are lacking. The marginal cost of capital for period $t$ does not occur explicitly ($LDBT^*$ depends on a long-term interest rate). The financing deficit $F_t$ occurs partly $(-RE_i)$.

For Taggart’s specification for the change in short debt, $\Delta SDBT$, the reader may verify analogous remarks (except for $F_t$, which does occur). For the other sources of funds analogous remarks may be made.

Taggart’s equations may be characterized as a special case of equations (6) with quite a number of essential variables lacking in each equation.

Auerbach (1985) postulates a stock-adjustment model to explain changes in the ratio’s of long-term debt to assets ($\ell$) and of short-term debt to assets ($s$), as follows:

$$
\Delta \ell_t = \lambda_1(\ell_t^* - \ell_{t-1}^*) + \varphi_1(s_t^* - s_{t-1}^*) + \gamma_1 f_t,
$$

$$
\Delta s_t = \lambda_2(\ell_t^* - \ell_{t-1}^*) + \varphi_2(s_t^* - s_{t-1}^*) + \gamma_2 f_t,
$$

where $f_t$ is the ratio of the (adjusted) external deficit to assets, and $\ell_t^*$ and $s_t^*$ have a common set of explanatory variables that characterize the firm in period $t$ (the $z_t$ of Section 2). Besides the ad hoc use of ratio’s, this model is an improvement on Taggart (1977)’s. However, there are no cost of capital variables (the $c_t$ of Section 2) in the specification and no long-term expectations of market rates (the $E_t(R_t)$ of Section 2). The rather rigid stock-adjustment restriction on the coefficients of the lagged debt variables is still present.

The most recent study discussed is Chowdhurry, Green and Miles (1986). They explain the four components of the following balance: short-term bank borrowing plus trade credit received minus trade credit given minus the change in liquid assets, as follows:

$$
y_i = \alpha_i + \sum_k \beta_{ik} m_k + \sum_j \gamma_{ij} Y_{j,t-1} + \sum_{\ell} \delta_{\ell i} M_{\ell,t-1} + \sum_h \varphi_{ih} Z_h \quad (i = 1, \ldots, 4),
$$
where the $m_k$’s are the other sources and uses of funds (other than the $y_i$’s), the $Y_{j,t-1}$’s the starting levels corresponding to the $y_j$’s, the $M_{t,1-1}$’s the starting levels corresponding to the other sources and uses and the $Z_h$’s other variables.

This specification comes closer to the ‘correct’ specification (6) than Taggart (1977)'s on two counts: Chowdhurry c.s. include all lagged capital components, and they include (without mentioning them as such) most possible determinants of this paper’s $v_{it}$ among their $Z_h$’s. They also correctly use only short-term interest rates; see Section 5. The role of the financing deficit, $F_t$, is played by the separate sources and uses other than the one’s they explain, while it should be played by one short-term total; see also Section 5 (last paragraph).

5 Discussion

The purpose of embedding previous models on empirical capital structure, especially short-run models, is achieved by the static decision model of Section 2. In addition the many no-change decisions for stock and long debt are explained, by means of fixed transaction costs.

This same type of model could be used to explain (changes in) portfolio composition. Especially the zero holdings of many assets, may, besides by risk averse behavior, be explained by fixed transaction cost. (Transaction costs are defined to include all sorts of costs besides the price, including costs of information.)

Possibly the firm takes into account (expected) inflation in year $t$ in its decision problem concerning the $\Delta v_{it}$’s. Appendix D shows how equations (6) should be modified in that case.

Assuming that firms determine aggregate fund changes first, like the change in total short debt ($\Delta SD$), the above line of reasoning may be applied to the parts of the aggregate (external) funds. Taking as an example total short debt and its parts, the role of $F$ in equations (6) is played by $\Delta SD$, $TC$ is replaced by $SD$, and the market rates $R$ are replaced by several types of short-term rates corresponding to these parts, etc.
Appendix A  Two dividend policies causing dependence between the decision variables in $F$ and $E$

1. Dividend determined residually
Retained profit ($ER$) is a decision variable such that

$$ER + \Delta ST + \Delta LD + \Delta SD = F + E,$$

and, of course, $0 \leq ER \leq E$. Total cost of funds (one of the funds now is cumulative retained profits) is minimized subject to these restrictions. Dividend $D = E - ER$ (which, by the way, defines a regression equation for $D$). If $0 < D < E$, then there is an internal optimum for $ER$. In the decision rule of the main text one may then replace $F$ by $F + E$.

As $ER$ and $\Delta ST$ are very close substitutes, they entail approximately the same costs and thus could be taken together as one variable, $ER + \Delta ST$.

2. Dividend determined simultaneously
As in the above case, total direct cost, $K_1$, now is defined with also $ER$ among the sources of funds. There now is a target value for dividend, $D^*$, and $D - D^*$ occurs in the penalty function $P_t$. This sees to it that $D > 0$; otherwise cheap retained profit could use up all of profit after taxes ($E$). The restrictions are:

$$D + ER = E$$
$$\Delta ST + \Delta LD + \Delta SD = F,$$
$$D \geq 0, \ ER \geq 0.$$  

Interaction between the decision variables in the two equality restrictions occurs because of $D - D^*$ being in the penalty function. Assuming non-negativity of $D$ and $ER$, the decision rules (which are as in the main text) now include separate terms for $E$ and $F$, also those for $D$ and $ER$. 
Appendix B  Derivation of equations (6)

The total cost of funds, $K_t + P_t$, with $K_t$ in (2) and $P_t$ in (3), has to be minimized with respect to the funds $v_1, \ldots, v_n$ under restriction (1). The cost of funds is written in matrix notation as follows:

$$K_t + P_t = c_t^T(v_t - v_{t-1}) + \frac{1}{2}(v_t - v_{t-1})^T A(v_t - v_{t-1}) + \frac{1}{2}(v_t - v_t^*)^T B(v_t - v_t^*),$$

(B.1)

where $c_t^T = (c_{1t}, \ldots, c_{nt})$, $v_t^T = (v_{1t}, \ldots, v_{nt})$, $A = [a_{ij}]$, and $B = [b_{ij}]$, with $A$ and $B$ symmetric positive-definite matrices. Restriction (1) is in matrix notation:

$$i^T(v_t - v_{t-1}) = F_t,$$

(B.2)

where $i^T = (1, \ldots, 1)$.

From the Lagrangian function,

$$K_t + P_t - \lambda_t \{i^T(v_t - v_{t-1}) - F_t\},$$

(B.3)

the following first-order conditions follow:

$$\begin{bmatrix} A + B & -i \\ - & - \\ i^T & 0 \end{bmatrix} \begin{bmatrix} v_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} Bu_t^* + Av_{t-1} - c_t \\ - \\ i^T v_{t-1} + F_t \end{bmatrix},$$

(B.4)

The inverse of the first partitioned matrix is:
with $C = A + B$ and $b = \mathbf{1}^T C^{-1} \mathbf{1}$. More compactly (B.5) is written as:

\[
\begin{bmatrix}
G & -d \\
\end{bmatrix}
\]

with obvious definitions of $G$ and $d$. The solution of (B.4) for $v_t$ thus is:

\[
v_t = GBv_t^* + (GA - dt^T)v_{t-1} - Gc_t - dF_t,
\]

which is equation (6).

Restrictions on the coefficients of the equations may be deduced from (B.2) and (B.7), which imply:

\[
t^T v_{t-1} + F_t = t^T GBv_t^* + (t^T GA - t^T dt^T)v_{t-1} - t^T Gc_t - t^T dF_t.
\]

So one expects the following restrictions to hold:

\[
t^T GB = 0, \quad t^T GA - t^T dt^T = t^T, \quad t^T G = 0, \quad -t^T d = 1.
\]

These equalities may be checked by substituting $G$ and $d$ from (B.6).
Appendix C  The target ratio’s $f^*_{jt}$

It will be shown that the target values $v^*_{jt}$ may be chosen such that they specify a capital structure which equals the equilibrium capital structure, viz. by choosing the $f^*_{jt}$ according to the equilibrium capital structure.

In equilibrium the target values in equations (6) are the equilibrium values, to be denoted by $v^e_j$. As, moreover, in equilibrium $F = \sum \Delta v_j = 0$, equation (6) collapses to:

$$v^e_i = \sum_j \beta_{ij}^e v^e_j + \sum_j \gamma_{ij} v^e_j + \sum_j \delta_{ij} c_j \quad (i = 1, \ldots, n),$$

with the $c_j$ the long-run marginal costs of capital, which depend on $z$ and $E_t(R)$. In equilibrium $\sum_j \delta_{ij} c_j$ is a constant, so one may solve these equations for the $v^e_j$. Evidently this solution depends on $z$ and $E_t(R)$:

$$v^e_j = v^e_j(z, E_t(R)).$$

If the functions $f^*_{j}$ of (9b) are defined as:

$$f^*_{j} = v^e_j / \sum_j v^e_j,$$

then the target values in period $t$, the $v^*_{jt}$ of (6), have the equilibrium capital structure.

Appendix D  Accounting for inflation

In the decision problem concerning the $v_t$, the $v_{t-1}$ also occur. The purchasing power, in terms of assets, for $v_{t-1}$ and the other (unlagged) quantities may differ. By using (expected) inflation in year $t$, e.g. by using the percentage price change ($p_t$) of (composite) assets, $v_{t-1}$ may be made comparable to the other quantities: replace $v_{t-1}$ by $(1 + p_t)v_{t-1}$ in the decision problem, and thus in equations (6).
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