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Stock and Bond Market Interactions with Level and Asymmetry Dynamics: An Out-of-Sample Application*

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Abstract

We model the dynamic interaction between stock and bond returns using a multivariate model with level effects and asymmetries in conditional volatility. We examine the out-of-sample performance using daily returns on the S&P 500 index and 10 year Treasury bond. We find evidence for significant (cross-) asymmetries in the conditional volatility and level effects in bond returns. The out-of-sample covariance matrix forecasts of the model imply that an investor is willing to pay between 129 and 820 basis points per year for using a dynamic trading strategy instead of a passive strategy.

Keywords: Stock and Bond Market Interaction, Time-Varying Covariances, Asymmetric Volatility, Level Effect.

JEL classification codes: G12, C22.

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1 Introduction

While there exists a large literature on time-varying conditional variances of stock and bond returns, the number of studies on conditional covariances between these returns is rather limited. Moreover, to date, the *empirical* importance of modeling the covariance between stock and bond returns is largely neglected in existing studies. Given their importance in asset pricing, portfolio selection and risk management, it is crucial to obtain accurate estimates and predictions of the conditional covariances between asset returns. Several studies have introduced univariate models that capture the asymmetric volatility effect, for example Nelson (1991), Engle and Ng (1993) and Glosten, Jagannathan and Runkle (1993). Most of these models successfully outperform their symmetric counterparts in practice. Furthermore, De Goeij and Marquering (2004) show that the presence of asymmetric effects in conditional covariances is very likely if there exist asymmetric effects in the conditional variances of asset returns. As a portfolio manager's optimal portfolio depends on the predicted covariance between assets, relaxing the symmetric volatility specification leads to superior investment choices. Moreover, the introduction of asymmetric volatility in financial models could also be useful in other fields in finance, such as risk management and derivative pricing.

Empirical research on asymmetric effects in conditional covariances between asset returns in a multivariate GARCH model has been scarce. Braun, Nelson and Sunier (1995) estimate a bivariate exponential GARCH model with asymmetries in stock return betas for different sectors. However, they do not explicitly consider asymmetries in covariances. In addition, one of the most influential studies on modeling time-varying covariances is the study by Kroner and Ng (1998). They introduce the Asymmetric Dynamic Covariance (ADC) model that proposes asymmetric extensions of the most common multivariate GARCH models. They use data on large and small firms to compare four popular multivariate GARCH models. Their approach does not take into account cross-asymmetric volatilities: the conditional variance and covariance between asset returns can be higher (or lower) after a negative shock in one asset and a positive shock in the other asset, rather than shocks of opposite signs of the same magnitude. In the application of Kroner and Ng (1998) it makes sense not to consider these "cross-asymmetric effects" as these types of shocks are rare between the returns of small and large companies. In contrast, shocks of opposite signs are much more common in stock and bond returns. De Goeij and Marquering (2004) show that these cross-asymmetric effects are statistically significant in a multivariate GJR (Glosten, Jagannathan and Runkle, 1993) framework, by modeling dynamic interactions between stock and bond returns.[1]

Besides the (cross-) asymmetries, another factor which improves the ability to forecast (interest rate) volatility is the level effect. This effect implies that conditional volatility depends on the level of returns in addition to the dependency on innovations. Chan, Karolyi, Longstaff and Sanders (1992) estimate a general non-linear short rate process which nests many of the short rate processes currently assumed in the literature. The level effect in Chan, Karolyi, Longstaff and Sanders (1992) is formulated such that the volatility of the interest rate changes is proportional to the power of the interest rate itself. Their model is able to empirically distinguish between different theoretical term structure models. Brenner, Harjes and Kroner (1996) mix a univariate GARCH process with a model for level effects. They introduce a new class of models for the dynamics of interest rate volatility which allows volatility to depend on both interest rate levels and information shocks. They show that the sensitivity of interest rate volatility to levels is substantially reduced when volatility is a function of both levels and unexpected shocks. More recently, Christiansen (2005) has extended this model in a multivariate framework and includes the level effect in a constant (although time-varying) multivariate correlation model. Her model implies that the time-varying behavior in conditional covariances is caused by time variation in the conditional variances. Consequently, her model does not take into account a direct level effect for the dynamics in the covariances. It is therefore an open question whether the conditional covariance between stock and bond returns contains a level effect as well. Variation in interest rates may induce a positive correlation since the prices of stocks and bonds are negatively related to interest rates.

In this paper we extend the existing literature in three ways. First, we extend the asymmetric multivariate model of De Goeij and Marquering (2004) with the multivariate level effect as in, e.g., Christiansen (2005). Thus the model incorporates level effects and cross-asymmetries in conditional variances and covariances.

[1] Alternatively, Engle (2002) models conditional asymmetric correlations instead of conditional covariances.

To test the appropriateness of this model we examine the asymmetric volatility behavior of stock and bond market returns using daily data. Second, more general models obviously work better in-sample than simpler models. West and Cho (1995), for example, show that in-sample and out-of-sample results could vary substantially because of estimation error. Therefore, in contrast to similar studies, we concentrate on the *out-of-sample* forecasting performance comparison. Finally, recent GARCH literature has been moving towards the direction of examining what is the "best" model using economic loss functions rather than statistical loss functions (see, e.g., Lopez, 2001 and Ferreira and Lopez, 2005). The fact that a model performs better statistically, does not automatically imply that the model performs well in practice. Therefore, we evaluate the out-of-sample performance using an economic framework, taking into account transaction costs, rather than the traditional statistical framework. Overall, we emphasize the *empirical* applicability of our proposed model rather than the theoretical and in-sample properties.

Our empirical results can be summarized as follows. We find the level effect to be statistically significant for bond return volatility which is consistent with findings in the existing literature. However, the level effect is not significant for stock volatility and the covariance between stock and bond returns. We find strong evidence of asymmetric effects in the conditional variances and covariances of stock and bond returns. In addition, we find significant cross-asymmetric effects in the conditional covariances. We show that, after reasonable transaction costs, it would have been economically profitable to have used dynamic volatility timing employing the most general model with level and asymmetric effects in the out-of-sample period January 2003 - September 2005. A mean-variance investor is willing to pay a maximum fee of between 129 and 820 basis points per year to switch from the static strategy to the dynamic strategy. Furthermore, including the asymmetries in the model leads to a higher economic value, out-of-sample. We show that the transaction costs need to be around 17 basis points per trade for the dynamic volatility timing not to be profitable anymore. Finally, our results indicate that the more risk-averse the mean-variance investor is, the more he is willing to pay for using one of the dynamic strategies instead of the passive strategy.

The remainder of this paper is organized as follows. In the next section we introduce the level asymmetric DVECH model. In Section 3 we present the empirical results and in Section 4 we investigate the quality of the covariance matrix out-of-sample forecasts of the models by determining the economic value of a trading rule exploiting the model forecasts. Finally, Section 5 concludes.

2 The Level Asymmetric Diagonal VECH model

In this section we focus on modeling the asymmetric volatility phenomenon in a multivariate context. First, we describe how the first moments evolve over time in the mean equation. We follow Kroner and Ng (1998) and use a VAR framework to model excess returns. To prevent that asymmetric effects in the volatility equation are due to misspecification of the mean equation, we include extra terms which capture possible asymmetries in the first moments. Some recent studies show that asymmetries are present in the first moments of stock and bond returns; see, e.g. Ang and Chen (2001), Connolly, Stivers and Sun (2005) and Hong, Tu and Zhou (2007). Hence, our mean equation is modeled as:

$$r_{i,t}^{e} = \mu_i + \sum_{j=1}^{N}\sum_{\tau=1}^{L}\left[\alpha_{ij,\tau}^{m} r_{j,t-\tau}^{e} + \beta_{ij,\tau}^{m} r_{j,t-\tau}^{e-}\right] + \varepsilon_{i,t}, \qquad \text{for } i = 1, ..., N, \tag{1}$$

where

$r_{i,t}^{e}$ denotes the excess return of asset i in period t,
$r_{i,t}^{e-} = \min(0, r_{i,t}^{e})$, the *negative* excess return of asset i in period t,
$\varepsilon_{i,t}$ denotes the unexpected excess return of asset i, and
N denotes the number of assets, and L the number of lags.

We assume that $\varepsilon_t|\mathcal{I}_{t-1} \sim N(0, \Sigma_t)$, where $\mathcal{I}_{t-1}$ denotes the information set at time $t-1$, and $\Sigma_t = (\sigma_{ij,t})$, with $\sigma_{ij,t} = Cov_t\{r_{j,t}, r_{i,t}\}$, is the $N \times N$ conditional covariance matrix of the unexpected excess returns; μ_i, $\alpha_{ij,\tau}^{m}$ and $\beta_{ij,\tau}^{m}$ $i, j = 1, ..., N$ are the unknown parameters. Model (1) enables us to examine the importance of the influence of past returns on current levels of returns. Next, we describe how the conditional covariances evolve over time.

We model the time-varying covariances by a multivariate GARCH process.[2]

[2] For an extensive overview of multivariate GARCH models, see Bauwens, Laurent and Rombouts (2006).

More specifically, we employ an asymmetric diagonal VECH model and extend it with a level effect for conditional variances and covariances:

$$\begin{aligned}\sigma_{ij,t+1} &= |r_{i,t}r_{j,t}|^{\gamma_{ij}} \times [\omega_{ij} + \beta_{ij}\sigma_{ij,t} + \alpha_{1ij}\varepsilon_{i,t}\varepsilon_{j,t} + \alpha_{2ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t} \\ &\quad +\alpha_{3ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t} + \alpha_{4ij}(1-I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}],\end{aligned} \qquad (2)$$

$i, j = 1, ..., N$. The extended asymmetric diagonal VECH model in (2) differs in two ways from the standard diagonal VECH specification.

First, generalizing the level specification of Chan, Karolyi, Longstaff and Sanders (1992) and Brenner, Harjes and Kroner (1996), the conditional volatility of the assets depends on the level of the bond and stock returns, more precisely on the γ_{ij}th power of their level. This is known as the level effect. Consequently, the dynamics of the volatility depends on both return levels and information shocks. The larger γ_{ij} is, the more sensitive the (co)variance is to the levels of returns. The univariate level specification of Chan, Karolyi, Longstaff and Sanders (1992), Brenner, Harjes and Kroner (1996) and Christiansen (2005) can be written as $\sigma^2_{t+1} = r_t^{2\gamma} \times f(\sigma^2_t, \varepsilon^2_t; \theta)$, where $f(.)$ is a linear function and θ is a vector of parameters. Christiansen (2005) also specifies a multivariate generalization of the level effect. Her multivariate GARCH model however, assumes constant correlation as it extents Bollerslev's (1990) Constant Conditional Correlation model. In contrast, our multivariate specification allows for a direct level effect in conditional covariance dynamics.

Second, we include several types of asymmetries in the model. In (2), the indicator variable $I_{\varepsilon_{k,t}}$ is equal to 1 if $\varepsilon_{k,t} < 0$ (and zero otherwise), $k = i,\ j$. There are three asymmetry terms in (2), $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$, $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ and $(1-I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$. The first term, $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$, assigns an asymmetric covariance effect on shocks in the same direction (simultaneous positive shocks versus simultaneous negative shocks). The two remaining terms account for a different effect for opposite shocks in asset returns in addition to the existing negative return shock effects. The term $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ is nonzero for negative shocks in asset i and positive in asset j, while $(1-I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$ is nonzero for positive shocks asset i and negative in asset j. These last two terms assigns an asymmetric covariance effect on shocks in the opposite directions. We will refer to these latter effects as *cross-asymmetry effects* or simply *cross effects*. Note that these effects are usually neglected in the literature. However, when modeling the covariance between stock and bond returns these cross effects should be included, as shocks of opposite signs are common and relevant (see De Goeij and Marquering, 2004). The proposed multivariate model provides a generalization of the multivariate GJR model by allowing explicitly for asymmetric conditional covariance terms and for level effects. We will refer this model as the *level asymmetric diagonal VECH* model (ADVECH-L model). The specification in (2) nests several existing models. An overview with restrictions is provided in Table 1. Our proposed specification nests multivariate models such as the (symmetric) diagonal VECH model[3] and the asymmetric diagonal VECH model that was recently introduced by De Goeij and Marquering (2004). In addition, the univariate GARCH(1,1) and GJR models are nested as well.

[Include Table 1 about here]

Diagonalizing the model has the advantage that the number of parameters to be estimated do not become too large. Note that for our application with two assets, $N = 2$, the number of parameters to be estimated is 17. However, by employing a diagonalized model, we constrain the dynamic dependence and may introduce biases in the estimates of the other parameters. For instance, only shocks in asset i can influence the conditional variance of asset i. This assumption is quite restrictive and is obviously a disadvantage of the diagonal VECH model. However we expect the potential biases to be negligible as models allowing for such spillover effects, such as the BEKK model (see Engle and Kroner, 1995) show that these effects are typically small. Finally, to guarantee that the conditional covariance matrix is positive definite, we estimate the model using constrained maximum likelihood.[4] We estimate the ADVECH-L model in (2), along with two other, more restrictive, specifications which we refer to as the DVECH-L and the GJR-L model. The DVECH-L model is the symmetric version of (2). The GJR-L is the multivariate GJR model extended with the level effect, but without cross asymmetries. Again, Table 1 provides an overview of the parameter restrictions

[3]Bollerslev, Engle and Wooldridge (1988) introduced this diagonal VECH model to estimate the trade-off in variance among the returns on a stock index, a bond and a Treasury bill.

[4]More details on the constrained maximum likelihood approach can be found in De Goeij and Marquering (2004).

that need to be implemented to obtain these models. The next section proceeds with empirical results.

3 Empirical Results

3.1 Estimation Methodology

The estimation is performed in two steps following Pagan and Schwert (1990), Engle and Ng (1993) and Kroner and Ng (1998). First we estimate mean equation (1) using OLS to obtain the residuals e_t for all $t = 1, ..., T$. In the second step we estimate the conditional covariance matrix parameters using maximum likelihood, treating e_t as observable data. The block diagonality of the information matrix under this setup guarantees that the consistency and efficiency are not lost in such a procedure. The loglikelihood function (for the sample $1, ..., T$) is given by

$$\mathcal{L}(\tilde{\theta}) = -\frac{1}{2}TN\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log(\det \Sigma_t(\tilde{\theta})) - \frac{1}{2}\sum_{t=1}^{T} e_t'\Sigma_t^{-1}(\tilde{\theta})e_t, \tag{3}$$

where $\tilde{\theta}$ denotes the vector of unknown parameters and $\Sigma_t(\tilde{\theta})$ contains the conditional covariance terms, as defined in (2).

If ε_t is not normally distributed, the maximum likelihood procedure may provide consistent estimators for the model parameters, even though the likelihood is incorrectly specified (see Verbeek, 2000 pp.171-172). The reason is that under fairly weak assumptions, the first order conditions of the maximum likelihood procedure are also valid when ε_t is not normally distributed. Bollerslev and Wooldridge (1992) show that the maximum likelihood estimates of $\tilde{\theta}$ are consistent if

$$E_{t-1}\left\{\frac{\varepsilon_{i,t}}{\sqrt{h_{ii,t}}}\right\} = 0 \text{ and } E_{t-1}\left\{\frac{\varepsilon_{i,t}\varepsilon_{j,t}}{h_{ij,t}}\right\} = 1 \text{ for } i, j = 1, ..., N. \tag{4}$$

The resulting estimator for $\tilde{\theta}$ is referred to as the quasi-maximum likelihood or QML estimator. The standard errors are calculated according to Bollerslev and Wooldridge (1992). We report these robust standard errors in the tables below. The estimates are obtained by numerical methods using the Berndt, Hall, Hall and Haussman (1974) (BHHH) optimization algorithm, which approximates the

Hessian with the first derivatives. Note that estimation of multivariate GARCH with many parameters is typically demanding in computer time. In order to improve convergence, a sensible choice of starting values is important. We use starting values based on unconditional sample statistics and preliminary estimates of univariate GARCH models. A range of starting values was used to ensure that the estimation procedure converged to a global maximum. We repeated the estimations with random re-starts of the starting values, conditioned to the range of two times the standard error of the univariate estimates. None of the estimation results indicated any local maximum. The results also seem robust to alternating convergence criteria and optimizing methods. Consequently, we are confident that we have found a global maximum for the loglikelihood function.

3.2 Data Description

In order to examine the asymmetric volatility in the stock and bond market, our data include daily excess returns on the S&P 500 stock market index and the 10 year Treasury bond. We take into account the excess returns on these assets. We follow Jones, Lamont and Lumsdaine (1998) by calculating excess returns as the returns of holding the bond in excess of the risk-free rate, which is approximated by the 3-month Treasury bill rate. We adjust for weekends and holidays (Appendix A provides more details on the calculations). The data cover the period January 4, 1982 - September 30, 2005 (5,930 observations), such that we can examine some volatile periods (1987-1988, 1990 and 1998) and less volatile periods (1991-1995). Table 2 provides a summary of the descriptive statistics at the daily frequency.

[Include Table 2 about here]

The mean excess return on the S&P 500 index is 0.022% per day, which is about 6% per year, while it is around 5% per year for the 10 year bond over the same period. The standard deviation for the S&P 500 return is more than twice as large than for the 10 year Treasury bond. The third row shows the minimum returns, including the large 20% negative return in the S&P 500 index due to the October 1987 crash. Finally, the kurtoses confirm that the distributions of excess returns have quite fat tails, which is common using daily data.

After estimating mean equation (1) the residuals can be divided into four quadrants, based on the signs of the residuals: both residuals have a negative sign: $(-,-)$, S&P 500 residual negative, 10 year bond residual positive: $(-,+)$, etc. About 40 percent of the observations have shocks with opposite signs; i.e. they are in the $(-,+)$ or $(+,-)$ quadrant. This indicates the importance of including the cross-asymmetry effects in (2).

3.3 Estimation Results

In this section we present the estimation results of the dynamic asymmetric interaction between daily U.S. stock and bond returns. Because shocks of the mean equation are the main actors in the multivariate model, it is important that the mean equation is not misspecified. We have estimated VAR models up to ten lags and tested the individual and joint significance of the coefficients.[5] An appropriate model selection criteria in our application is the Schwarz Information Criterion (SIC). The number of lags that minimizes the SIC is obtained for only one lag. Consequently, we use a VAR model with one lag in all calculations below.

Next, we discuss the results of estimating the asymmetric covariance between the S&P 500 index and 10 year Treasury bond. As discussed above, we will estimate the ADVECH-L model (2), along with two other, more restrictive, specifications: the DVECH-L and the GJR-L model. The DVECH-L model is the symmetric version of (2). The GJR-L is the multivariate GJR model extended with the level effect, but without cross asymmetries. The estimates of the conditional second moment parameters for the three different specifications are presented in Table 3.[6] It appears that covariances change substantially over time, as most of the corresponding estimated parameters are statistically significant at the five percent level. Hence, the constant covariance hypothesis can be rejected. This result is consistent with the findings of Bollerslev, Engle and Wooldridge (1988), Harvey (1989) and Bodurtha and Mark (1991), who also document strong evidence in favor of heteroskedastic covariances.

[5] The results can be obtained from the authors upon request.

[6] The sample period contains some major shocks (e.g., the crashes in 1987 and 2000). To check the robustness of our results over time, we re-estimate our models using several subsamples. The subsamples were based on the results in Rapach and Wohar (2006) in which several structural breaks were uncovered, augmented with own research. All the models were re-run using data from January 4, 1982 - November 14, 1988; November 15, 1988 - January 17, 1991; January 18, 1991 - April 14, 2000; and April 15, 2000 - September 3, 2005. Overall, these analyses did not result in qualitative different results. Thus, our results seem robust over time.

[Include Table 3 about here]

The estimates for the coefficients on the level effect are in line with the literature (see, e.g., Chan, Karolyi, Longstaff and Sanders,1992, Brenner, Harjes and Kroner, 1996 and Christiansen, 2005) as far as they are comparable. We find no level effect for the conditional variance of stock returns (the coefficient corresponding to $r_{1,t}^2$ is not statistically significant), but the level effect for the bond returns is statistically significant. Thus, the conditional volatility of the bond returns depends on the level of the bond returns. Looking at the impact of the level effect on the covariance between the stock and bond returns (the coefficient corresponding to $|r_{1,t}r_{2,t}|$), we conclude that there is no direct level effect in the covariance. However, as the level effect in the bond returns is statistically significant for all three specifications, it is important to include the level effects.

The estimates for the coefficients on lagged volatility (i.e. the $\sigma_{ij,t}$'s) are statistically significant and range from 0.913 and 0.956 for the variance of the S&P 500 returns and from 0.939 and 0.942 for the long term bond. These parameter values indicate that the conditional variances for stock and bonds are highly persistent and cluster over time. The lagged volatility for the conditional covariances range from 0.953 to 0.956. Obviously, not only variances, but also covariances tend to cluster over time.

The estimates for the coefficients on the product of the return shocks (i.e. the $\varepsilon_i\varepsilon_j$'s) in the three specifications range from 0.027 to 0.076 for the stock variances, from 0.042 to 0.046 for the bond variances and from 0.020 to 0.038 for the covariances. A positive estimate for the ARCH term in the covariance equation implies that two shocks of the same sign affect the conditional covariance between the corresponding assets positively, while two shocks of opposite signs have a negative effect on the forecasted covariance. Apparently, two negative (or positive) shocks lead to a significant increase in next period's covariance. However, this interpretation only holds if we neglect the asymmetries in covariance. We will see below that the introduction of these asymmetric effects lead to more complex relationships.

Next, we focus on the asymmetric effects in the variances of the stock and bond returns (i.e. $(I_{\varepsilon_{i,t}}\varepsilon_{i,t})^2$, $i = 1, 2$), in other words, the persistency of negative return shocks. The results in Table 3 indicate that these effects are pronounced in the variance of the stock index returns, but not in the variance of bond returns. The estimated coefficient of the variable that captures the negative shocks in the S&P 500 return is equal to 0.076 using the GJR-L specification, and 0.091 using the ADVECH-L specification, which means that negative return shocks in the S&P 500 are followed by a relatively high conditional variance. For the bond returns, there is no asymmetric volatility according to both specifications. Given existing results in the literature, this is not surprising, as most studies only find the asymmetric effect in the variance of the stock (index) returns.

To continue, we address the degree of importance of the asymmetries in covariances. The results in Table 3 show that not only variances, but also covariances exhibit significant asymmetric effects. Both specifications show that the asymmetric effects in the covariance for shocks with the same sign (i.e. $I_{\varepsilon_{1,t}}\varepsilon_{1,t}I_{\varepsilon_{2,t}}\varepsilon_{2,t}$) seem to be important, as the corresponding estimated coefficients are statistically significant. A positive sign of the coefficient indicates that next day's conditional covariance between returns is higher when there are two negative shocks rather than two positive shocks.

Finally, the cross asymmetric effects, i.e. when shocks in the two assets are of opposite signs (i.e. $I_{\varepsilon_{1,t}}\varepsilon_{1,t}(1 - I_{\varepsilon_{2,t}})\varepsilon_{2,t}$ and $I_{\varepsilon_{2,t}}\varepsilon_{2,t}(1 - I_{\varepsilon_{1,t}})\varepsilon_{1,t}$), also appear to be important. The ADVECH-L specification shows a positive and statistically significant influence of the parameter of $I_{\varepsilon_{2,t}}\varepsilon_{2,t}(1 - I_{\varepsilon_{1,t}})\varepsilon_{1,t}$, which indicates that the conditional covariance between returns is lower when there is a negative shock in the stock index return and a positive shock in the long term bond rather than a positive shock in the stock index return and a negative shock in long term bond of the same magnitude. Thus, also cross asymmetries are an important factor when modeling the covariance between stock and bond returns.

The empirical results of the cross asymmetries can be interpreted in the context of flight-to-quality versus contagion. Contagion is present if the covariance between stock and bond returns (strongly) increase after movement of the asset classes in same directions. It implies that during times of increased stock uncertainty the return co-movement between stocks and bonds becomes more positively correlated. A movement in the opposite direction characterized by strongly increasing covariances implies flight-to-quality across asset classes; investors might move from holding stocks to bonds in uncertain times. From our estimates we can infer that the covariance between stock and bond returns is especially high following a negative shock in both the stock and bond market, suggesting that there is a contagion effect across the S&P 500 and the 10 year Treasury bond returns. This finding of a contagion effect between these two assets is consistent with De Goeij and Marquering (2004). A word of caution on the flight-to-quality versus contagion story: note that our models are not specifically designed to distinguish between the flight-to-quality and contagion explanations. Thus while our empirical results are consistent with the contagion story, it should not be seen as convincing empirical evidence.

4 An Out-of-Sample Economic Evaluation

In this section, the performance of the models is evaluated by determining the economic value of a trading rule exploiting the model forecast of the conditional covariance matrix. The estimation results presented in the previous sections do not necessarily imply economically useful implications for forecasting volatility. The basic task in this section is to evaluate the quality of the covariance matrix forecasts, using either the DVECH-L, the GJR-L or the ADVECH-L model. Recently, the GARCH literature has been moving towards the direction of examining what is the "best" model using economic loss functions rather than statistical loss functions. For example, Lopez (2001) and Ferreira and Lopez (2005) examine conditional covariance models within a Value-at-Risk framework. In our context, the most appropriate economic loss function is the maximum fee an investor would be wiling to give up using one volatility model instead of a passive strategy, taking into account transaction costs. In other words, we examine the economic gains of constructing a portfolio using the asymmetric model instead of the restricted (symmetric) portfolio and a passive portfolio.

We follow Fleming, Kirby and Ostdiek (2001) and Marquering and Verbeek (2004) by evaluating the impact of volatility timing on the economic performance of a dynamic asset allocation strategy. To compare the different volatility timing strategies, we consider an investor who minimizes his portfolio variance subject to a particular target expected rate of return (μ_p). This optimization problem can be written as:

$$\min_{w_{t+1}} \; w'_{t+1}\Sigma^{-1}_{t+1}w_{t+1}, \tag{5}$$

$$\text{s.t. } w'_{t+1}\mu + (1 - w'_{t+1}\iota) r_{f,t+1} = \mu_p,$$

where $\mu = E\{r_{t+1}\}$, ι is a vector of ones and w_{t+1} is the vector of portfolio weights on the risky assets. The proportion invested in the riskfree asset is $w_{0,t+1} = 1 - w'_{t+1}\iota$. Solving (5) for w_{t+1} gives us the optimal weights:

$$w^*_{t+1} = \frac{(\mu_p - r_{f,t+1})\Sigma^{-1}_{t+1}(\mu - r_{f,t+1}\iota)}{(\mu - r_{f,t+1}\iota)'\Sigma^{-1}_{t+1}(\mu - r_{f,t+1}\iota)}. \tag{6}$$

As in Fleming, Kirby and Ostdiek (2001) we assume that there are no short-sale restrictions.

To calculate the portfolio weights of the optimal portfolio, we need the conditional forecasts of the covariance matrix. We employ four different alternative forecasts based on different volatility models: a constant, which we will refer to as the passive strategy, a symmetric time-varying, and two asymmetric time-varying covariance matrices. All models take the level effect into account. The investor determines the optimal mix of three assets: the riskfree asset and the returns on the S&P500 index and 10 year Treasury bond return. In all cases we assume that the expected return is constant over time. The reason for this is threefold. First, we want to concentrate on volatility timing. Second, there is little evidence that (economically significant) predictable patterns in returns exist at the daily level. Third, a long sample period is needed to produce reliable estimates in a forecast regression for first moments (see Merton, 1980).

The vast majority of the multivariate GARCH models are evaluated on the basis of the in-sample performance. This can overstate the performance due to a look-ahead bias. It is more realistic to examine the out-of-sample performance instead. An out-of-sample procedure tries to replicate the way a portfolio manager could have managed its strategic asset portfolio. Such out-of-sample evaluations are very common when modelling first moments, but not for second moments models. Ideally, daily out-of-sample forecasts, generated by the models, are used to evaluate the performance. However, this implies that for each observation (each day) the model has to be re-estimated, which is computational quite demanding. In addition, portfolio managers typically re-estimate volatility models on a weekly or monthly basis instead of every single day. Therefore, to mimic this portfolio manager behavior, we re-estimate the models each calendar month and generate conditional volatility forecasts for each day in the subsequent month. It is an interesting question whether the asymmetries that matter at a daily horizon also make a difference at the monthly horizon. We pursue the following procedure.[7] The models are first estimated using data until December 31, 2002. Using these estimates we generate out-of-sample one-day ahead forecasts for the conditional covariance matrices for every trading day in January 2003. Then we estimate the models again, using data until January 31, 2003 and generate out-of-sample one-day ahead forecast for all the conditional covariance matrices for February 2003, etc. This way we obtain real out-of-sample forecasts for the various specifications for the period January 1, 2003 - September 30, 2005. Figure 1 provides a graphical overview of the procedure.

[Include Figure 1 about here]

Our approach is essentially a recursive scheme where we add most recent observations but do not drop the older observations. Therefore we use all information available to us. Alternatively, one could use a rolling scheme with a fixed number of observations (see also West and Cho, 1995). Note that although the out-of-sample period seems short given the total sample period, we want to test whether the strategy works for a period that corresponds to the length of forecast periods in which typical fund managers are interested.

We compare the out-of-sample performance of the dynamic strategies with the passive strategy, i.e. using a constant covariance matrix. In addition we compare the dynamic strategy which entails the asymmetric effects with the dynamic strategy that only considers the symmetric covariances. If the proposed asymmetric models with level effects have no economic value, then the ex post performance of the two strategies should be indistinguishable. Making this comparison requires a performance measure that captures the trade-off between risk and return. Following Fleming, Kirby and Ostdiek (2001) we use a measure that is based on the close relation between mean-variance analysis and quadratic utility.

Assume that the investor's realized utility in period $t+1$ can be written as:

$$U(W_{t+1}) = W_t r_{t+1}^p - \frac{a}{2}(W_t r_{t+1}^p)^2, \tag{7}$$

[7] We thank an anonymous referee for a suggestion that helped us develop this procedure.

where W_{t+1} is the investor's wealth at period $t+1$, a is his absolute risk aversion, and

$$r^p_{t+1} = w^{*\prime}_{t+1} r_{t+1} + (1 - w^{*\prime}_{t+1} \iota) r_{f,t+1}$$

is the period $t+1$ return on his portfolio p. We hold aW_t constant, which is equivalent to setting the investor's relative risk aversion, $\gamma_t = aW_t/(1 - aW_t)$ equal to some fixed value γ. With relative risk aversion held constant, we can use the average realized utility to consistently estimate the expected utility generated by a given level of initial wealth (normalized to 1). In particular we have

$$\hat{U}_p(\gamma) = \frac{1}{T} \sum_{t=0}^{T-1} \left[r^p_{t+1} - \frac{\gamma}{2(1+\gamma)} (r^p_{t+1})^2 \right]. \tag{8}$$

The above approach enables us to compare alternative investment strategies by calculating the associated average utility levels. We can determine the economic value of volatility timing by calculating the maximum fee an investor would be willing to pay for holding the dynamic portfolio rather than a passive one. A similar approach is applied in Fleming, Kirby and Ostdiek (2001) and Marquering and Verbeek (2004). To find the maximum fee an investor is willing to pay for holding a dynamic portfolio rather than a passive portfolio, Δ, we solve the following equation for Δ:

$$\begin{aligned} & \frac{1}{T} \sum_{t=0}^{T-1} \left[(r_{a,t+1} - \Delta) - \frac{\gamma}{2(1+\gamma)} (r_{a,t+1} - \Delta)^2 \right] \\ = \; & \frac{1}{T} \sum_{t=0}^{T-1} \left[r_{p,t+1} - \frac{\gamma}{2(1+\gamma)} r^2_{p,t+1} \right], \end{aligned} \tag{9}$$

where the indices a and p refer to the active and passive strategies, respectively. The Δ will be reported as annualized fees in percentages using two different risk-aversion levels γ; 1 and 10.

The optimal portfolio weights, based on the out-of-sample forecasts, typically change from day to day. This raises the question whether volatility timing is still profitable after introducing transaction costs. Therefore, additionally we calculate the "break-even" transaction costs, i.e. the maximum transaction costs per trade such that the dynamic strategy is still profitable. In other words, we assume that there is a transaction cost involved with each trade and calculate how much the mean-variance investor would be willing to pay in transaction costs per trade. If the actual transaction costs are higher than the break-even transaction costs, this would imply that the mean-variance investor should not be willing to trade on the basis of the corresponding volatility timing strategy.

[Include Table 4 about here]

Table 4 presents the performance measures of the passive and dynamic portfolios for the different model specifications for different levels of target returns. The Sharpe ratio for the passive portfolio is lower than that for each of the dynamic portfolio, indicating that the dynamic strategy is superior than the passive strategy. The Sharpe ratios for the dynamic strategies are very similar and it is not possible to discriminate one strategy against another. However, it is important to realize that the Sharpe ratio does not appropriately take into account time-varying volatility. The risk of the dynamic strategies is typically overestimated by the sample standard deviation, particularly in the presence of volatility timing, because the ex post (unconditional) standard deviation is an inappropriate measure for the (conditional) risk an investor was facing at each point in time (see Kirby, Fleming and Ostdiek, 2001 and Marquering and Verbeek, 2004). This indicates a potentially severe disadvantage of the use of Sharpe ratios to evaluate dynamic strategies. Consequently, to compare the dynamic strategies we should not look at the Sharpe ratios, but at the economic gains of constructing a portfolio using the asymmetric model instead of one of the restricted portfolios.

First, Table 4 shows the maximum fee the mean-variance investor is willing to pay for holding the dynamic portfolio. When using the DVECH-L model, the investor is willing to pay more than 98 basis points per year for using that dynamic strategy instead of the passive strategy. Using the same target return level, the same investor would be wiling to pay between 129 and 830 basis points per year for using the ADVECH-L model instead of the passive strategy. Furthermore, the economic value of using the asymmetric model instead of the symmetric model is between 31 and 403 basis points, and the value from switching from a model without cross asymmetries to a model with these asymmetries is between 33 and 283 basis points return.[8] Second, Table 4 shows that the mean-variance investor

[8]We have also compared the models with an exponentially-weighted sample covariance, which assigns more weights to recent observations than a simple moving average method. When we

would be willing to pay a break-even transaction cost of around 17 basis points per trade to use the ADVECH-L model instead of the passive strategy to calculate its optimal weights. The break-even transaction costs are lower for the GJR-L and DVECH-L models.

Obviously, the covariance asymmetries that matter at the daily horizon also make a difference at the out-of-sample monthly horizon. Note that these results are conservative because of two reasons. First, transaction costs were not taken into account in the optimization problem. Second, while the horizon for the portfolio manager is monthly, we assume that the portfolio manager changes his portfolio weights daily. In practice, a portfolio manager will not trade so frequently, thereby incurring fewer transactions costs. To conclude, the empirical results of Table 3 also show that the more risk-averse the mean-variance investor is, the more he is willing to pay for using one of the dynamic strategies instead of the passive strategy.

5 Concluding Remarks

In this paper we develop a multivariate model to forecast the conditional volatility of stock and bond returns and their interaction, taking into account level effects as well as asymmetric volatility. Our approach contributes to the literature across three dimensions. First, we extend the asymmetric multivariate model of De Goeij and Marquering (2004) by mixing the model with a multivariate level effect. Our model incorporates level effects and cross-asymmetries in conditional variances and covariances. Second, in contrast to comparable studies, we concentrate on the out-of-sample forecasting performance comparison. Finally, we evaluate the out-of-sample performance using an economic framework, taking into account transaction costs, rather than the traditional statistical framework. This approach is important in the light of recent GARCH literature that has been moving towards the direction of examining what is the "best" model using economic loss functions rather than statistical loss functions.

In order to examine the asymmetric volatility in the stock and bond market, we

use this exponentially-weighted sample covariance with a commonly used (see, e.g., Lopez, 2001) decay rate of 0.94 as comparable performance, we find that all the models outperform this benchmark, using different measurements. This, together with the fact that in our set-up a real passive portfolio is a more natural benchmark, led us to leave out these results in this article.

use daily excess returns on a stock market index and a long term Treasury bond. The empirical results indicate that taking into account asymmetries is important for forecasting the conditional covariance between stock and bond returns. Our main findings can be summarized as follows. The level effect is statistically significant for bond returns, thus the conditional volatility of the bond returns depends on the level of the bond returns. However, there exists no direct level effect in the covariance between the stock and bond returns. We find strong evidence of asymmetric effects in the conditional variances and covariances of stock and bond returns. In addition, we find significant cross-asymmetric effects in the conditional covariances. From the estimates we can infer that the covariance between stock and bond returns is especially high following a negative shock in both the stock and bond market, suggesting that there is a contagion effect across the S&P 500 and the 10 year Treasury bond returns.

We show that in the out-of-sample period January 2003 - September 2005, it would have been economically profitable, after taking into account reasonable transactions costs, to have used a dynamic volatility timing strategy that is implied by the most general volatility model with level and asymmetric effects. A mean-variance investor is willing to pay between 129 and 820 basis points per year for using this model instead of the passive strategy. This is the maximum fee the investor would be willing to pay to switch from a static strategy to the dynamic strategy. Furthermore, including the asymmetries in the model leads to a higher economic value out-of-sample. We show that the transaction costs need to be around 17 basis points per trade for the dynamic volatility timing not to be profitable anymore. The asymmetries that matter at the daily horizon also contribute to the economic value using estimates at the monthly horizon. Finally, our results show that the more risk-averse the mean-variance investor is, the more he is willing to pay for using one of the dynamic strategies instead of the passive strategy.

To conclude, our proposed model has practical value for portfolio managers to anticipate volatility. The model provides accurate out-of-sample predictions of the conditional covariances between asset returns, which are crucial inputs in asset pricing, portfolio selection and risk management.

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Appendix A: Calculation of the Returns

We obtained the "daily constant maturity interest rate series" from the federal reserve bank in Chicago. We have followed the method in Jones, Lamont and Lumsdaine (1998) to calculate the bond returns.[9] The U.S. Treasury bonds have semi-annual coupon payments, and the coupon on the hypothetical bonds is half the stated coupon yield. Hence, the price of the bond at the beginning of the holding period is equal to its face value. We have calculated an end-of-period price on this bond using the next day's yield augmented with the accrued interest rate:

$$P_{n-\#hd,t+1} = \sum_{i=1}^{2n-1} \frac{\frac{1}{2}y_{nt}}{(1+\frac{1}{2}y_{n,t+1})^i} + \frac{1+\frac{1}{2}y_{nt}}{(1+\frac{1}{2}y_{n,t+1})^{2n}} + \frac{\#\text{ holding days}}{365} y_{nt}, \tag{10}$$

where $P_{n-\#hd,t+1}$ is the end-of-period price of the bond, n is the number of years the bond is referring to, t is the time and y_{nt} is the yield of an n-period bond at time t. The $\#hd-$return, is calculated as

$$r_{t+1} = P_{n-\#hd,t+1} - 1. \tag{11}$$

Finally, the excess returns are calculated using the 3-month interest rate as the risk free rate that accrues over the holding period, which varies from one to five days due to weekends and holidays.

$$r^e_{t+1} = r_{t+1} - \frac{\#\text{ holding days}}{365} y_{3mo,t}.$$

The returns on the S&P 500 index, obtained from Datastream, are calculated as

$$r_{index,t+1} = \frac{P_{index,t+1} - P_{index,t}}{P_{index,t}}. \tag{12}$$

Excess returns are calculated by substracting the risk free rate that accrues over the holding period

$$r^e_{index,t+1} = r_{index,t+1} - \frac{\#\text{ holding days}}{365} y_{3mo,t}. \tag{13}$$

[9] We thank Charles Jones, Owen Lamont and Charlotte Christiansen for their suggestions to write a program that generates the data used in the paper.

Appendix B: Tables

Table 1: **Model Specification Comparisons and Parameters Restrictions**

This table presents an overview of the parameter restrictions of several univariate and multivariate conditional volatility models using volatility equation (2) as benchmark.

Model		Number of Assets	Parameters				
			γ	α_1	α_2	α_3	α_4
		Panel A: Univariate Models					
GARCH(1,1)	Bollerslev (1986)	$N = 1$	$\gamma = 0$	unrestricted	$\alpha_2 = 0$		
GJR(1,1)	Glosten, Jagannathan and Runkle (1993)	$N = 1$	$\gamma = 0$	unrestricted	unrestricted		
GARCH(1,1)-L	Brenner, Harjes and Kroner (1996)	$N = 1$	unrestricted	unrestricted	$\alpha_2 = 0$		
		Panel B: Multivariate Models					
DVECH	Bollerslev, Engle and Wooldridge (1988)	$N > 1$	$\gamma = 0$	unrestricted	$\alpha_2 = 0$	$\alpha_3 = 0$	$\alpha_4 = 0$
ADVECH	De Goeij and Marquering (2004)	$N > 1$	$\gamma = 0$	unrestricted	unrestricted	unrestricted	unrestricted
DVECH-L	This paper	$N > 1$	unrestricted	unrestricted	$\alpha_2 = 0$	$\alpha_3 = 0$	$\alpha_4 = 0$
GJR-L	This paper	$N > 1$	unrestricted	unrestricted	unrestricted	$\alpha_3 = 0$	$\alpha_4 = 0$
ADVECH-L	This paper	$N > 1$	unrestricted	unrestricted	unrestricted	unrestricted	unrestricted

Table 2: **Descriptive Statistics for Stock and Bond Excess Returns**

The table presents descriptive statistics for the excess return on the S&P 500 index and the 10 year Treasury bond for the period January 4, 1982 - September 30, 2005. All returns are daily returns in percentages calculated according to the details in Appendix A.

	S&P 500	10 yr bond
Mean	0.0225	0.0189
Std. Dev.	1.0570	0.4815
Minimum	-20.473	-2.7120
Maximum	9.0828	4.0802
Skewness	-1.2356	0.0651
Kurtosis	31.261	6.9028
Jarque-Bera	198,825	3,767.2
Observations	5,929	5,929

Table 3: **Estimation Results**

This table reports the maximum likelihood estimation results of model (2) using data from January 4, 1982 to September 30, 2005 ($T = 5,930$). Index $i = 1$ refers to the S&P 500 index and $i = 2$ to the long term bond. Robust Bollerslev Wooldridge standard errors are reported in parentheses, while '*' denotes statistical significance at the 5% level.

Explanatory	**DVECH-L**		**GJR-L**		**ADVECH-L**	
Variables	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
$r_{1,t}^2$	0.0026	(0.0061)	−0.0045	(0.0063)	−0.0031	(0.0063)
$\lvert r_{1,t} r_{2,t}\rvert$	0.0151	(0.0123)	0.0122	(0.0121)	0.0139	(0.0127)
$r_{2,t}^2$	0.0115*	0.0055)	0.0113*	(0.0055)	0.0115*	(0.0055)
Constant_{11} #	10.5526*	(1.3246)	13.5974*	(1.3788)	15.0623*	(1.4349)
Constant_{12}	1.0439*	(0.2295)	0.8266*	(0.3101)	2.2054*	(0.4072)
Constant_{22}	3.7717*	(0.5139)	3.7539*	(0.5210)	4.3181*	(0.5797)
$\sigma_{1,t}^2$	0.9178*	(0.0035)	0.9134*	(0.0040)	0.9138*	(0.0040)
$\sigma_{12,t}$	0.9556*	(0.0030)	0.9535*	(0.0031)	0.9531*	(0.0034)
$\sigma_{2,t}^2$	0.9420*	(0.0045)	0.9413*	(0.0046)	0.9391*	(0.0049)
$\varepsilon_{1,t}^2$	0.0758*	(0.0026)	0.0370*	(0.0044)	0.0274*	(0.0046)
$\varepsilon_{1,t}\varepsilon_{2,t}$	0.0383*	(0.0028)	0.0357*	(0.0029)	0.0203*	(0.0037)
$\varepsilon_{2,t}^2$	0.0438*	(0.0031)	0.0455*	(0.0041)	0.0419*	(0.0041)
$(I_{\varepsilon_{1,t}}\varepsilon_{1,t})^2$	.	.	0.0759*	(0.0053)	0.0908*	(0.0059)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t} I_{\varepsilon_{2,t}}\varepsilon_{2,t}$	.	.	0.0074*	(0.0037)	0.0283*	(0.0054)
$(I_{\varepsilon_{2,t}}\varepsilon_{2,t})^2$	.	.	−0.0018	(0.0035)	0.0047	(0.0037)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t}(1 - I_{\varepsilon_{2,t}})\varepsilon_{2,t}$	.	.	.	.	0.0043	(0.0055)
$I_{\varepsilon_{2,t}}\varepsilon_{2,t}(1 - I_{\varepsilon_{1,t}})\varepsilon_{1,t}$	.	.	.	.	0.0387*	(0.0053)
Log Likelihood	−11241.4		−11194.9		−11175.3	

"Constant" refers to estimate for the constant term ω_{ij} of the volatility equation (2).

Table 4: **Economic Evaluation**

This table shows the results of the economic evaluation exercise. Mean and Std. Dev. denote the mean return and the standard deviation of the return on the corresponding strategy in percentage a year, respectively. Sharpe denotes the Sharpe ratio, which is equal to the average excess return of the strategy divided by the sample standard deviation. The maximum fee, Δ, an investor is willing to pay for holding one of the dynamic portfolios rather than the passive portfolio is represented in percentages per year. TC represents the (break-even) transaction cost in percentages per trade for which the dynamic strategy would have the same utility as the passive strategy.

Passive Portfolio

Target Return	Mean	Std. Dev	Sharpe
6	9.237	7.340	0.065
7	10.492	8.562	0.065
8	11.746	9.785	0.065
9	13.001	11.007	0.065
10	14.256	12.230	0.065
11	15.510	13.452	0.065
12	16.765	14.675	0.065
13	18.020	15.897	0.065
14	19.274	17.120	0.065

Dynamic Portfolio (DVECH-L)

				$\gamma = 1$		$\gamma = 10$	
Target Return	Mean	Std. Dev	Sharpe	Δ	TC	Δ	TC
6	9.975	7.273	0.072	0.984	0.124	1.184	0.132
7	11.353	8.484	0.072	1.196	0.126	1.470	0.134
8	12.731	9.695	0.072	1.423	0.128	1.781	0.136
9	14.108	10.906	0.072	1.663	0.130	2.118	0.137
10	15.486	12.117	0.072	1.917	0.131	2.479	0.138
11	16.864	13.328	0.072	2.185	0.133	2.866	0.139
12	18.241	14.539	0.072	2.467	0.134	3.278	0.140
13	19.619	15.750	0.072	2.763	0.135	3.716	0.141
14	20.997	16.961	0.072	3.073	0.136	4.179	0.142

Dynamic Portfolio (GJR-L)

				$\gamma = 1$		$\gamma = 10$	
Target Return	Mean	Std. Dev	Sharpe	Δ	TC	Δ	TC
6	9.786	7.227	0.070	0.961	0.132	1.297	0.142
7	11.132	8.430	0.070	1.202	0.135	1.661	0.146
8	12.478	9.634	0.070	1.466	0.138	2.066	0.148
9	13.824	10.837	0.070	1.753	0.140	2.514	0.150
10	15.171	12.040	0.070	2.063	0.142	3.003	0.151
11	16.517	13.244	0.070	2.397	0.144	3.535	0.152
12	17.863	14.447	0.070	2.754	0.146	4.108	0.153
13	19.209	15.651	0.070	3.133	0.146	4.724	0.154
14	20.555	16.854	0.070	3.536	0.148	5.382	0.155

Table 4: **Economic Evaluation** *(Continued)*

This table shows the results of the economic evaluation exercise. Mean and Std. Dev. denote the mean return and the standard deviation of the return on the corresponding strategy in percentage a year, respectively. Sharpe denotes the Sharpe ratio, which is equal to the average excess return of the strategy divided by the sample standard deviation. The maximum fee, Δ, an investor is willing to pay for holding one of the dynamic portfolios rather than the passive portfolio is represented in percentages per year. TC represents the (break-even) transaction cost in percentages per trade for which the dynamic strategy would have the same utility as the passive strategy.

Dynamic Portfolio (ADVECH-L)							
				$\gamma = 1$		$\gamma = 10$	
Target Return	Mean	Std. Dev	Sharpe	Δ	TC	Δ	TC
6	9.849	7.151	0.072	1.294	0.159	1.853	0.169
7	11.206	8.342	0.072	1.644	0.162	2.404	0.171
8	12.562	9.533	0.072	2.031	0.165	3.025	0.173
9	13.919	10.724	0.072	2.457	0.167	3.716	0.174
10	15.276	11.915	0.072	2.920	0.168	4.475	0.176
11	16.632	13.106	0.072	3.422	0.170	5.304	0.177
12	17.989	14.296	0.072	3.962	0.171	6.203	0.178
13	19.346	15.487	0.072	4.541	0.172	7.171	0.179
14	20.702	16.678	0.072	5.157	0.174	8.208	0.179

Appendix C: Figures

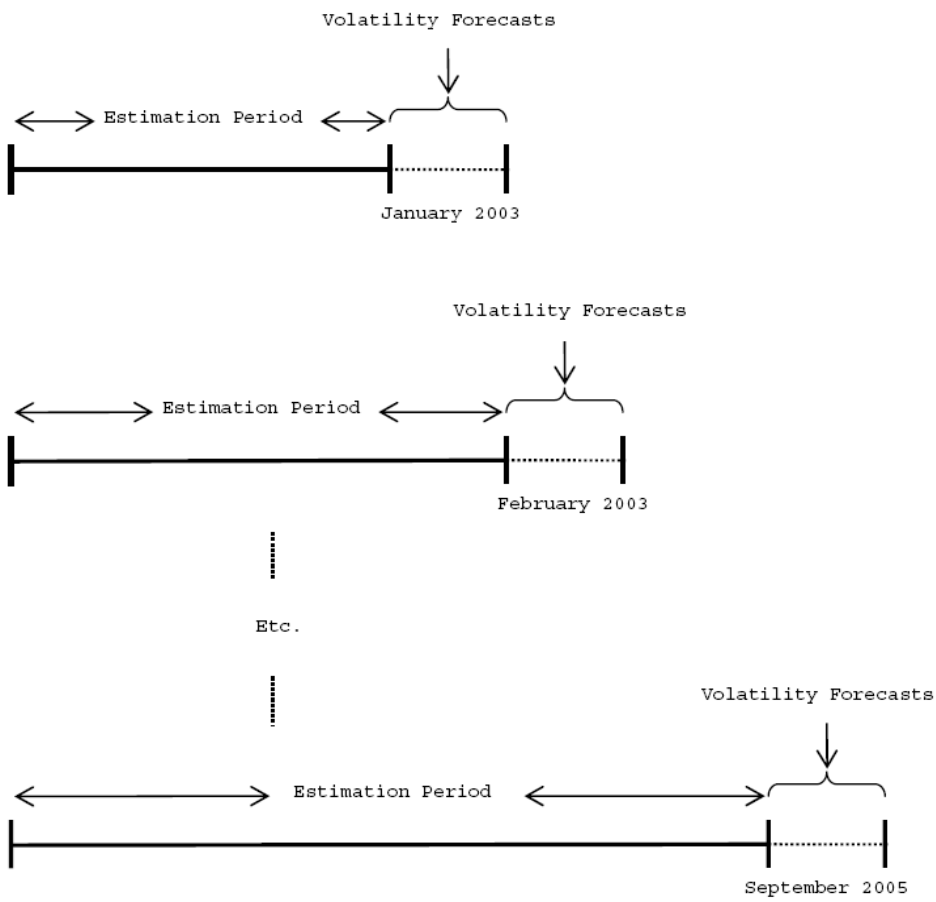

Figure 1: **Out-of-Sample procedure**