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STRONGLY REGULAR GRAPHS THAT ARE LOCALLY A DISJOINT UNION OF HEXAGONS

René Peeters

FEW 625

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November 8, 1993

Abstract

We prove that there are precisely two srg(64,18,2,6) that are locally a disjoint union of three hexagons and there are no srg(40,12,2,4) that are locally a disjoint union of two hexagons. As a side result we get simple definitions for all eleven srg(64,18,2,6) that are 4-colorable.

1 Introduction

A graph G is said to be *strongly regular* with parameters (v, k, λ, μ) if it has v vertices, is regular of degree k and each pair of distinct vertices has λ or μ common neighbours depending on whether or not the vertices are adjacent. We will refer to such a graph as a srg(v, k, λ, μ). For elementary results of such graphs see [2, 3, 4]. As usual we will denote the two eigenvalues of the adjacency matrix of G, apart from the valency, by r and s and their multiplicities by f and g respectively.

Let G and H be two graphs. We will say that G is locally H if for each vertex x of G its neighbour graph, which will be denoted by G_x, is isomorphic to H. In this paper we concentrate on strongly regular graphs that are locally a disjoint union of hexagons. Table 1 displays the smallest feasible parameter sets of strongly regular graphs that could locally be a disjoint union of hexagons, that means, the smallest parameter sets for which k is divisible by 6, $\lambda = 2$ and f and g satisfy the integrality conditions. The last column tells whether the parameter set satisfies the Krein conditions or not.

There is precisely one srg(13,6,2,3), namely the Paley graph on 13 vertices. Indeed, the neighbour graph of each vertex is the hexagon. There are two srg(16,6,2,2). These are the line graph of $K_{4,4}$ and the Shrikhande graph which are locally $2C_3$ and C_6 respectively. The Paley graph $P(13)$ and the Shrikhande graph are the only strongly regular graphs that are locally the hexagon. In this paper we will deal with the next two parameter sets. We will prove that there are precisely two srg(64,18,2,6) that are locally $3C_6$ and that there exists no srg(40,12,2,4) that is locally $2C_6$. Since there is a unique srg(112,30,2,10): the collinearity graph of the $GQ(3,9)$ which is locally $10C_3$, the smallest parameter set for which it is not yet decided whether or not there exists a strongly regular graph with these parameters that is locally a disjoint union of hexagons is (676,108,2,20).

v	k	λ	μ	r	s	f	g	Krein
13	6	2	3	$\frac{1}{2}(\sqrt{13}-1)$	$-\frac{1}{2}(\sqrt{13}+1)$	6	6	+
16	6	2	2	2	−2	6	9	+
40	12	2	4	2	−4	24	15	+
64	18	2	6	2	−6	45	18	+
112	30	2	10	2	−10	90	21	+
184	48	2	16	2	−16	160	23	−
256	66	2	22	2	−22	231	24	−
400	102	2	34	2	−34	374	25	−
676	108	2	20	4	−22	567	108	+
832	210	2	70	2	−70	805	26	−
1376	150	2	18	6	−22	1075	300	+
1786	84	2	4	8	−10	987	798	+
2146	156	2	12	8	−18	1479	666	+
2370	276	2	36	6	−40	2054	315	+
2704	318	2	42	6	−46	2385	318	+

Table 1: Smallest feasible parameter sets of strongly regular graphs that could locally be a disjoint union of hexagons.

If GQ is a generalized quadrangle of order (s,t) (cf. [9], notation: $GQ(s,t)$) then the collinearity graph of GQ is strongly regular with parameters $((s+1)(st+1), s(t+1), s-1, t+1)$. The parameter sets (40,12,2,4) and (64,18,2,6) correspond to the parameter sets of the collinearity graphs of a $GQ(3,3)$ and a $GQ(3,5)$ respectively. There are precisely two $GQ(3,3)$ and there is a unique $GQ(3,5)$. If a strongly regular graph G with one of the two mentioned parameter sets is locally a disjoint union of triangles, then it follows straightforwardly that the vertices and the 4-cliques of G form the points and lines of a GQ. So G is the collinearity graph of this GQ in this case.

In the next section we will frequently use the next lemma by Haemers:

Lemma 1 *Let G be a strongly regular graph with parameters $(40,12,2,4)$ or $(64,18,2,6)$, then for every vertex x, its neighbour graph G_x consists of cycles of length divisible by 3 and every vertex not adjacent to x is adjacent to precisely $\frac{c}{3}$ vertices of each c-cycle in G_x.*

Proof: See Haemers [6] for graphs with the first parameter set. For graphs with the second parameter set the proof is equivalent. □

So if G is a srg(40,12,2,4) or a srg(64,18,2,6) and locally a disjoint union of hexagons then for each vertex x of G, each non-neighbour of x is adjacent to precisely two vertices of each hexagon of G_x.

2 On srg(64,18,2,6)

In this part we characterize the two srg(64,18,2,6) that are locally $3C_6$. We start with giving alternative constructions for the 11 srg(64,18,2,6) that are 4-colorable and which were found by Mathon [8], one of which is locally $3C_6$. Then we give a definition of the other example and show that no other example exists. Using these results we can also prove that there exists no srg(40,12,2,4) that is locally $2C_6$.

Let Γ be the collinearity graph of $GQ(3,5)$. The first srg(64,18,2,6) that is locally $3C_6$ is obtained from Γ by switching. $GQ(3,5)$ can be described as follows: Consider $AG(3,4)$, the three dimensional affine geometry over $\mathbb{F}_4$. Let S be a set of 6 lines from $AG(3,4)$ passing through one point, such that no three lines lie in one plane. Such a set corresponds to a complete oval in the projective plane $PG(2,4)$. Each plane of $AG(3,4)$ contains two or no lines from S. The points of $GQ(3,5)$ are the points of $AG(3,4)$; its lines are the lines of S, and the lines of $AG(3,4)$ which are parallel to a line of S.

Let l be a line in $AG(3,4)$ that is not a line of the GQ, then two of the five hyperplanes through l contain no lines of the GQ and the other three contain eight lines each, forming a 4×4-grid on the 16 points of the hyperplane. Since a hyperplane that contains no lines of the GQ together with the 3 hyperplanes parallel to it forms a partition of the vertex set of Γ into 4 cocliques, Γ is 4-colorable. Let $L := \{l_{ij} | i,j \in \{1,2,3,4\}\}$ be the set of 16 lines in the affine geometry parallel to l, such that $l_{i1}, l_{i2}, l_{i3}, l_{i4}$ $(i = 1,2,3,4)$ and $l_{1j}, l_{2j}, l_{3j}, l_{4j}$ $(j = 1,2,3,4)$ form a hyperplane of the affine geometry, containing no lines of the GQ. If $i_1, i_2, j_1, j_2 \in \{1,2,3,4\}$ with $i_1 \neq i_2$ and $j_1 \neq j_2$, then the hyperplane through $l_{i_1j_1}$ and $l_{i_2j_2}$ contains two parallel classes of lines of the GQ forming a 4×4-grid, so in Γ each point of $l_{i_1j_1}$ is adjacent to 2 points of $l_{i_2j_2}$. So if we partition the adjacency matrix A of Γ according to the 16 lines l_{ij} $(i,j \in \{1,2,3,4\})$, then the 4×4-submatrix (block) defined by $l_{i_1j_1}$ and $l_{i_2j_2}$ is the zero-matrix if $i_1 = i_2$ or $j_1 = j_2$ and has rowsums 2 otherwise.

Let L_1 be a subset of L. Let Γ' be the graph obtained from Γ by switching, for each pair of lines $l_{i_1j_1} \in L_1$ and $l_{i_2j_2} \in L \setminus L_1$ such that the hyperplane through these 2 lines contains 2 parallel classes of lines from the GQ (so $i_1 \neq i_2$ and $j_1 \neq j_2$), the adjacency relation between $l_{i_1j_1}$ and $l_{i_2j_2}$ to its complement (that is, edges become non-edges and non-edges become edges). It can be found in [7] (Theorem 3.1), who refers to [5] that Γ' has the same spectrum as Γ and hence is again strongly regular with parameters (64,18,2,6). Since the zero-blocks of A remain zero-blocks, Γ' is again 4-colorable.

If $L_1 = \{l_{i1} | i = 1,2,3,4\}$, then every subgraph of Γ that is defined by a hyperplane containing two parallel classes of the GQ (so the subgraph is isomorphic to $L_2(4)$) is switched into a subgraph that is isomorphic to the Shrikhande graph and hence Γ' is locally $3C_6$. We will call this graph $\mathcal{G}_1$. Mathon [8] determined all 11 strongly regular graphs with parameters (64,18,2,6) that are 4-colorable. It turns out that each of the 10 graphs different from Γ can be obtained from Γ by switching with respect to a suitible subset of L. Table 2 shows which set L_1 can be chosen to obtain each of the 10 graphs. We call a hyperplane of $AG(3,4)$ a zero-plane if it contains no lines of the GQ and a two-plane otherwise. The graphs are ordered as in Mathon's paper. They can be distinguished by the number of 4-cliques they contain and the notion that in G_3 and G_4 each vertex is contained in the same number of 4-cliques (4 and 2 respectively), which

Name	4-cliques	L_1
G_1	96	
G_2	0	4 lines in one zero-plane
G_3	64	2 l. in a zero-pl. + 2 in a par. zero-pl.; other pairs in a two-pl.
G_4	32	4 lines in a two-plane
G_5	72	1 line
G_6	24	3 lines in a zero-plane
G_7	64	2 lines in a two-plane
G_8	32	3 lines in a zero-pl. + 1 line in a zero-pl. with one of these 3
G_9	56	3 not co-planar lines; 2 pairs in a two-pl., 1 pair in a zero-pl.
G_{10}	40	3 lines in a two-plane
G_{11}	48	2 lines in a zero-plane

Table 2: The eleven 4-colorable srg(64,18,2,6)

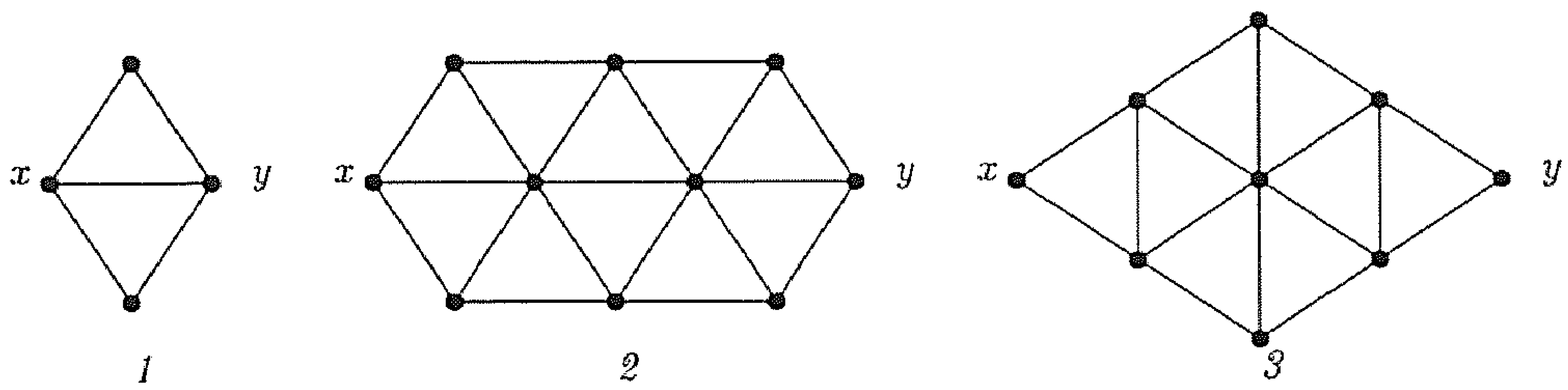

Figure 1: Three types of adjacency.

is not the case for G_7 and G_8. Note that G_2 is the same graph as $\mathcal{G}_1$.

The other strongly regular graph with parameters (64,18,2,6) that is locally $3C_6$ can best be defined by its embedding in the triangular grid. Identify each point of the grid with the point eight positions further on along one of the three lines through the point. This induces 64 equivalence classes of points, which are the vertices of the graph. Two vertices, x and y say, are adjacent if two of the corresponding points are in one of the three positions as in Figure 1. We will call this graph $\mathcal{G}_2$.

Theorem 2 *There are precisely two strongly regular graphs with parameters* $(64, 18, 2, 6)$ *that are locally* $3C_6$, *namely* $\mathcal{G}_1$ *and* $\mathcal{G}_2$.

Proof: Let G be a strongly regular graph with parameters (64,18,2,6) that is locally $3C_6$. Now each vertex x and a 6-cycle C_x from its neighbour graph define an embedding of a subgraph of G, G' say, in the triangular grid in a canonical way. So points that are adjacent in the grid correspond to adjacent vertices of G and the two common neighbours of two adjacent points in the grid correspond to the two vertices that are adjacent to the vertices corresponding to these two points.

CLAIM 1

G' is isomorphic to either the Shrikhande graph or the graph $\mathcal{G}_2$.

Again, the vertices of G' are represented by equivalence classes of points of the grid.

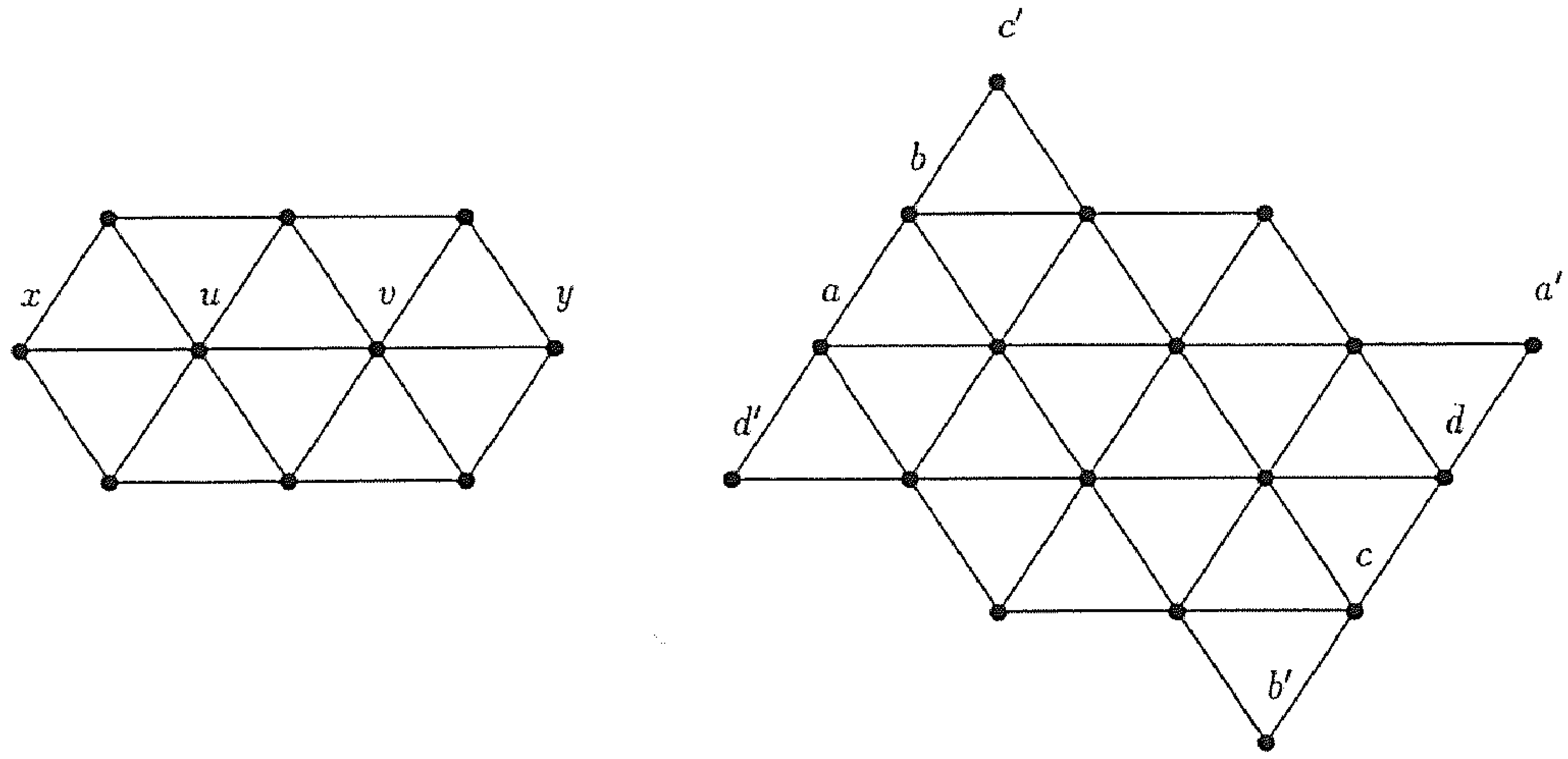

Figure 2: Two parts of the embedding of G' in the triangular grid

Consider two points x and y as in Figure 2. Since y is not adjacent to u, it is (by Lemma 1) adjacent to two vertices of each 6-cycle of G_u, so except to v, y is also adjacent to x. So each point of the grid is also adjacent to the point that is three positions further on along one of the three lines through this point. Now consider four points a, b, c, d as in Figure 2. One can check a and b are adjacent to precisely one of c and d and vica versa, so either a is adjacent to c and b to d or a to d and b to c. If a is adjacent to d and b to c then the points a and a' correspond to the same vertex and the same holds for b and b', c and c' and d and d'. It follows straightforward that each point must represent the same vertex as the point 4 positions further on, so G' is isomorphic to the Shrikhande graph. In the other case two points are adjacent if they are in one of the three positions as in Figure 1.

Now consider vertex (point) x of Figure 3. This vertex is adjacent to the points at the left side of the thick line that are represented with the big dots and is not adjacent to those represented by the small dots. So x must also be adjacent to vertex 1 and in fact each point of the grid is adjacent to the point 5 positions further on. Now x must be adjacent to either 2 or 3. If x is adjacent to 2, then 4 and x represent the same vertex of G, so x is adjacent to a, b and c. However a, b and c form a triangle which was not allowed. So two points that are 6 positions apart do not represent the same vertex and x is adjacent to 3 and hence also to all points represented by the big dots and not to the points represented by the small dots. It follows that x is also adjacent to 5, so each point of the grid is also adjacent to the point 7 positions further on. Now x must be adjacent to either 6 or 7. Suppose that x is adjacent to 6. Then it is also adjacent to the vertex l. Now the vertices k, l, m, n induce a subgraph of the neighbour graph of x isomorphic to $K_{1,3}$, a contradiction. So x is adjacent to 7 and hence x and x' represent the same vertex. So each point of the grid represents the same vertex as the point 8 positions further on, so G' is isomorphic to $\mathcal{G}_2$.

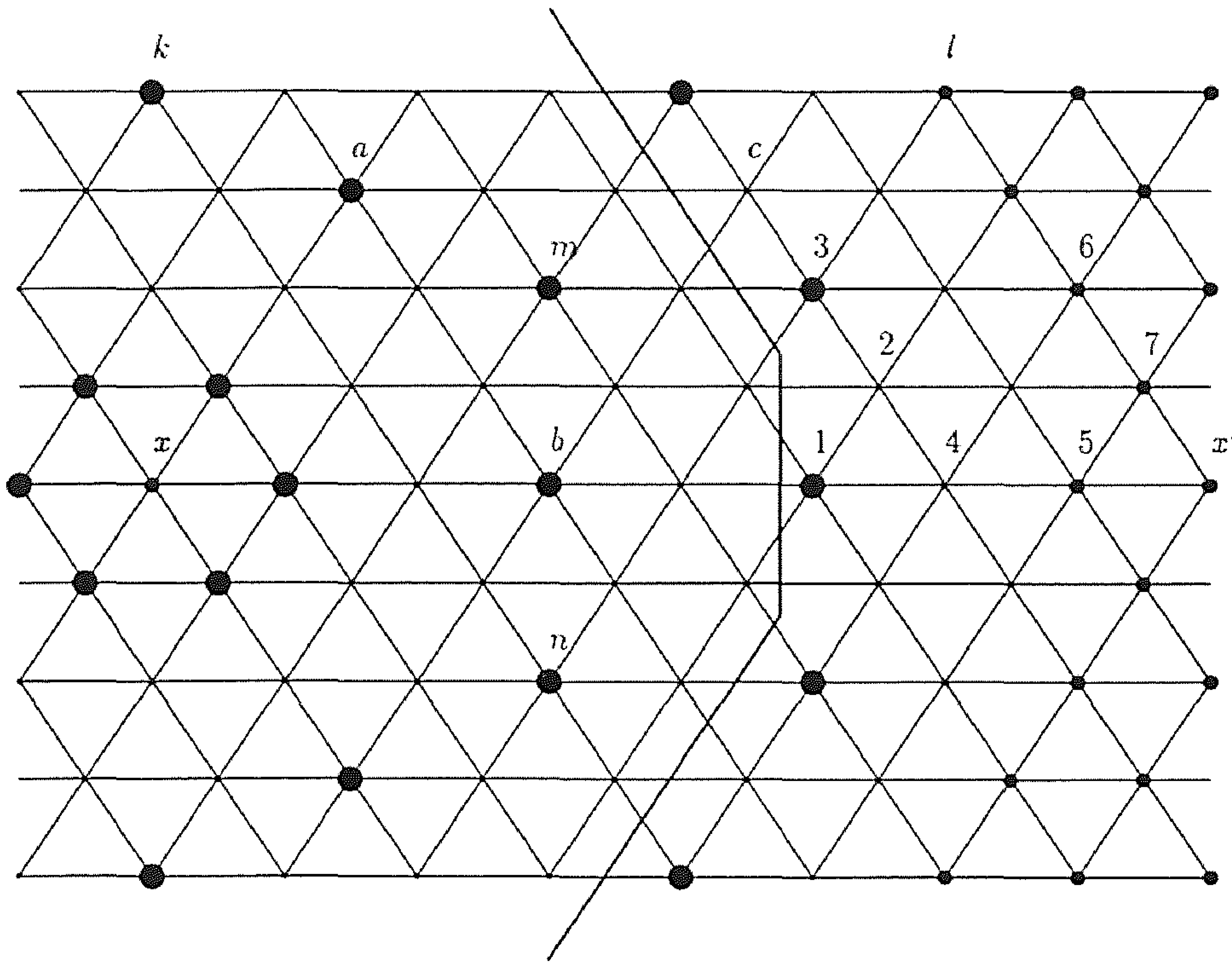

Figure 3: The embedding of G' in the triangular grid

If G is isomorphic to $\mathcal{G}_2$, then for an adjacency (edge) of type *1* or *2*, G' is isomorphic to $\mathcal{G}_2$, and for an adjacency of type *3* G' is isomorphic to the Shrikhande graph. So the vertices of $\mathcal{G}_2$ can be partitioned into four Srikhande subgraphs.

If G is not isomorphic to $\mathcal{G}_2$, then each vertex x of G and each 6-cycle C_x of its neighbour graph G_x define an embedding of a subgraph of G in the triangular grid that is isomorphic to the Shrikhande graph. In fact each edge of G defines a unique Shrikhande subgraph of G containing that edge. Counting edges we find that G contains precisely 12 Shrikhande subgraphs. Each vertex is contained in three of these.

Let G' be a subgraph of G that is isomorphic to the Shrikhande graph. Since $\lambda = 2$ for the Shrikhande graph the neighbours of a vertex from $G \setminus G'$ in G' form a coclique. The largest size of a coclique in the Shrikhande graph is four, so a vertex from $G \setminus G'$ has at most four neighbours in G'. But the average number of neighbours in G' over all vertices in $G \setminus G'$ is also four, hence the neighbours in G' of a vertex from $G \setminus G'$ form a maximal coclique in G'. Mention that the complement of the Shrikhande graph is the latin square graph of the cyclic latin square of order 4, or, equivalently, the collinearity graph of the cyclic 3-net of order 4. Each vertex from the Shrikhande graph is contained in four cocliques of size four: three corresponding to the three lines of the 3-net through each point (we will call these the maximal cocliques of type I), and one (type II) corresponding to the unique sub-3-net of order 2 (or affine plane of order 2) containing the corresponding point. Number the vertices of G' as in Figure 4. The four cocliques of size four containing vertex 1 are $\{1,3,9,11\}$ of type II and $\{1,3,10,12\}$, $\{1,9,7,15\}$ and $\{1,11,8,14\}$

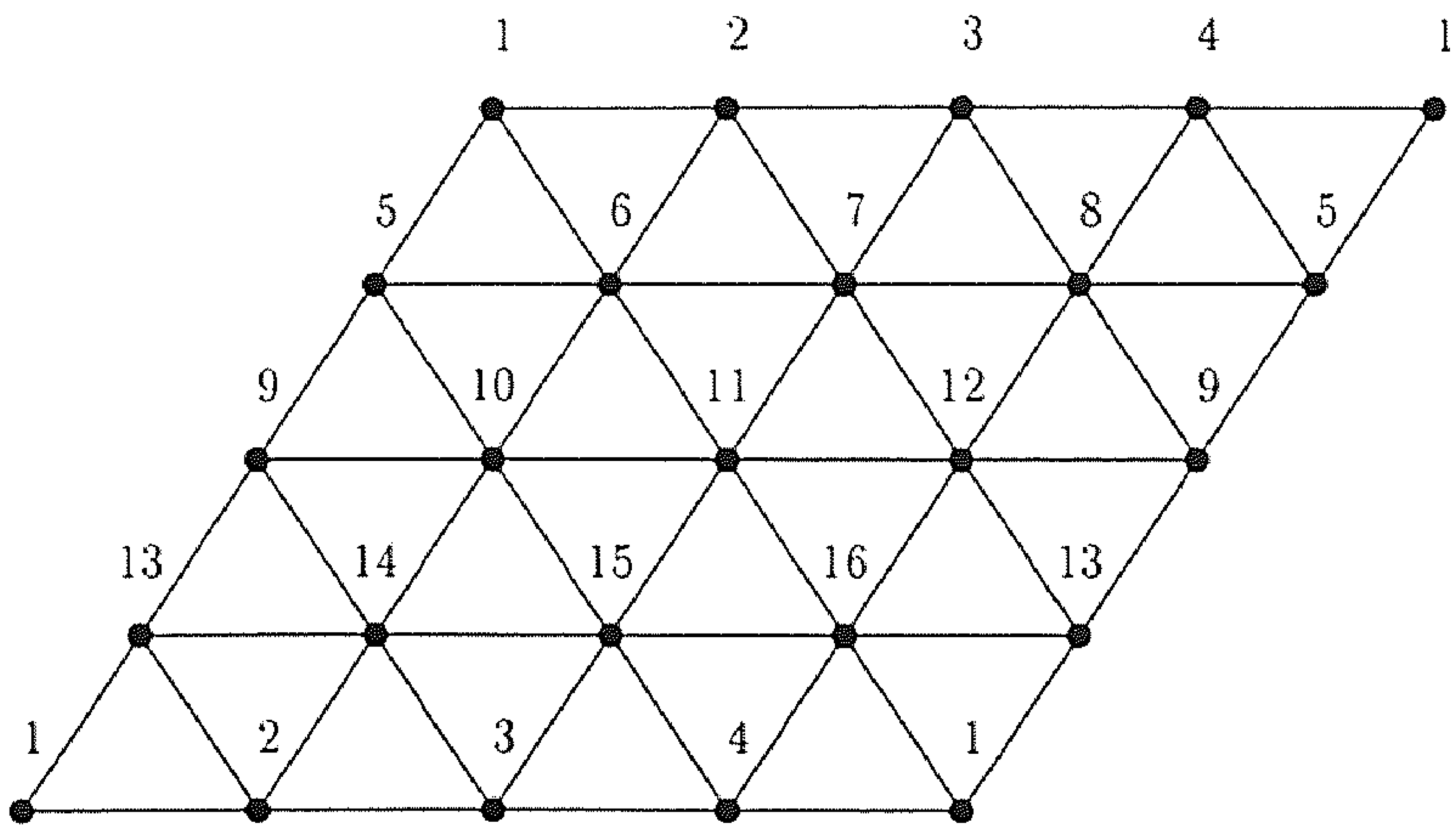

Figure 4: The Shrikhande subgraph G'

of type I. Since e.g. vertices 1 and 7 are not adjacent and $\mu = 2$ for the Shrikhande graph, there must be four vertices from $G \setminus G'$ that are adjacent to both 1 and 7 and hence to the 4-tuple $\{1, 9, 7, 15\}$. Similarly there are four vertices from $G \setminus G'$ that are adjacent to $\{1, 3, 10, 12\}$ and four that are adjacent to $\{1, 11, 8, 14\}$. Together these are the 12 vertices from $G \setminus G'$ that are adjacent to vertex 1 of G'. In particular each vertex of $G \setminus G'$ is adjacent to four vertices from G' that form a maximal coclique of type I in G'.

CLAIM 2

Two different Shrikhande subgraphs of G are either disjoint or intersect in a maximal coclique of type II.

Let $\mathbf{G}'$ be a Shrikhande-subgraph of G that is defined by vertex 10 of G' and a 6-cycle from its neighbour graph different from (5,6,11,15,14,9). Number the vertices of $\mathbf{G}'$ similarly as those of G' with $\mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \mathbf{16}$ such that $10 = \mathbf{10}$. We prove that without loss of generality: $2 = \mathbf{2}$, $4 = \mathbf{4}$, $10 = \mathbf{10}$ and $12 = \mathbf{12}$.

Each of the vertices of $C_{10} = \{5, 6, 11, 15, 14, 9\}$ is adjacent to one of the following three 4-tuples: $\{10, 2, 16, 8\}$, $\{10, 12, 1, 3\}$ and $\{10, 13, 7, 4\}$. Since each vertex not adjacent to 10 is adjacent to two vertices of C_{10}, each 4-tuple occurs twice as a set of neighbours of a vertex from C_{10}. In addition, two adjacent vertices from C_{10} are adjacent to different 4-tuples, since the have only 2 common neighbours. So w.l.o.g. we are in one of the following two cases:

1. **6** and **14** are adjacent to $\{10, 2, 16, 8\}$, **5** and **11** are adjacent to $\{10, 12, 1, 3\}$, and **9** and **15** are adjacent to $\{10, 13, 7, 4\}$.

2. **6** and **14** are adjacent to $\{10, 2, 16, 8\}$, **5** and **15** are adjacent to $\{10, 13, 7, 4\}$, and **9** and **11** are adjacent to $\{10, 12, 1, 3\}$.

Consider we are in the first case. The vertices 12,1,3 are adjacent to both **5** and **11**, but are not the same as the vertices **10** or **6**, so 12,1 and 3 are not vertices of $\mathbf{G}'$, hence they are adjacent to the 4-tuple $\{\mathbf{5},\mathbf{11},\mathbf{13},\mathbf{3}\}$. Since **5,11,13** and **3** are adjacent to 12,1 and 3 they are not vertices of G'. Similarly 13,7 and 4 are not vertices of $\mathbf{G}'$ and are adjacent to the 4-tuple $\{\mathbf{9},\mathbf{15},\mathbf{1},\mathbf{7}\}$ which are not vertices of G'. Let x be one of the vertices from $\{5,6,9,11,14,15\}$, then x is adjacent to one of the following three 4-tuples: $\{\mathbf{10},\mathbf{2},\mathbf{16},\mathbf{8}\}$, $\{\mathbf{10},\mathbf{12},\mathbf{1},\mathbf{3}\}$ and $\{\mathbf{10},\mathbf{13},\mathbf{7},\mathbf{4}\}$. Since x is adjacent to one of the vertices 12,1,3 and to one of the vertices 13,7,4, x cannot be adjacent to **5,11,13,3,9,15,1** or **7**. So x must be adjacent to the 4-tuple $\{\mathbf{10},\mathbf{2},\mathbf{16},\mathbf{8}\}$, but not all six vertices (5,6,9,11,14,15) can be adjacent to the same 4-tuple. A contradiction.

So we are in the second case. Now either all the vertices 2,16,8 are adjacent to (**6,14,4,12**), or two are and one is the same as vertex **2**. Similarly either all the vertices 13,7 and 4 are adjacent to (**9,11,2,4**) or two are and one is the same as vertex **12** and either all the vertices 12,1 and 3 are adjacent to (**5,15,2,12**) or two are and one is the same as vertex **4**. So G' and $\mathbf{G}'$ have at most 4 common vertices, The only candidates from $\mathbf{G}'$ are **10,2,4** and **12**. Because of symmetry the only candidates from G' are 10,2,4 and 12, so the only possible identities are $10 = \mathbf{10}$ (by definition), $2 = \mathbf{2}$, $4 = \mathbf{4}$ and $12 = \mathbf{12}$. Suppose $2 \neq \mathbf{2}$; then **2** is not a vertex of G' and should have four neighbours in G'. But **2** is adjacent to two 'opposite' neighbours of 10 in G' plus to at least four of the vertices 13,7,4,12,1 and 3, so $2 = \mathbf{2}$ and similarly $4 = \mathbf{4}$ and $12 = \mathbf{12}$.

Given a vertex of G, the three Shrikhande subgraphs of G containing this vertex have a maximal coclique of type II in common so the other three vertices of this coclique are contained in the same three Shrikhande subgraphs. The 64 vertices of G can be partitioned into 16 maximal 4-cocliques of type II, such that the four vertices of each coclique are contained in the same three Shrikhande subgraphs. Each Shrikhande subgraph contains precisely four of these 16 cocliques.

Consider a Shrikhande subgraph S and one of the twelve disjoint 4-cocliques C. Then the three Shrikhande subgraphs containing C are either disjoint from S or have a 4-coclique of type II with it in common. If one has a 4-coclique with S in common, then each vertex of C is adjacent to two vertices of this 4-coclique. Since each vertex of G not in S has four neighbours in S, two of the three Shrikhande subgraphs containing C have a 4-coclique with S in common and the other one is disjoint from S. So if we consider the 16 cocliques of type II as points ($\mathcal{P}$) and the 12 Shrikhande subgraphs of G as lines ($\mathcal{L}$) we get a partial geometry with 16 points, 12 lines of size four, three lines through each point and if a point is not incident with a line it is incident with two points of that line. Hence the geometry $(\mathcal{P}, \mathcal{L})$ has the structure of a 3-net of order 4. There are precisely two 3-nets of order 4: the cyclic 3-net of order 4 and the 3-subnet of the affine plane of order 4.

CLAIM 3
The partial geometry $(\mathcal{P}, \mathcal{L})$ is isomorphic to the 3-subnet of the affine plane of order 4.

We prove that every two collinear points of $\mathcal{P}$ are in a sub-affine plane of order 2. Let C_1 and C_2 be two points of the line (Shrikhande subgraph) S. Let T_1 and U_1 be the other two Shrikhande subgraphs containing C_1 and T_2 and U_2 the other two Shrikhande subgraphs containing C_2, with T_1 and U_1 parallel to T_2 and U_2 respectively. Let C_3 be the common point of T_1 and U_2 and C_4 the common point of T_2 and U_1; see Figure 5.

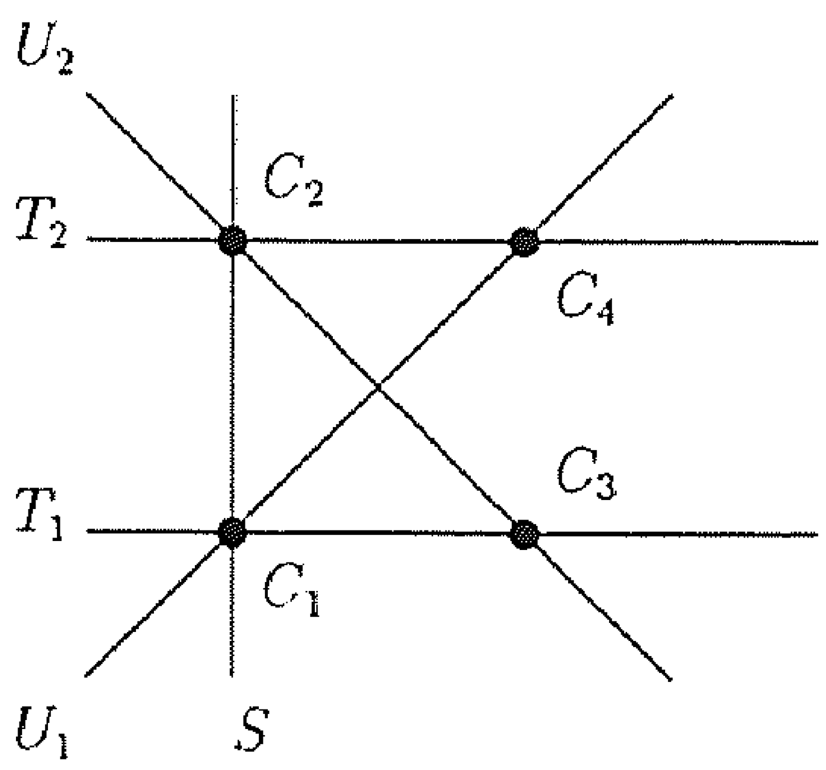

Figure 5: A substructure of $(\mathcal{P}, \mathcal{L})$

Number the vertices of T_1 with $1, 2, 3, \ldots, 16$ and those of T_2 with $17, 18, 19, \ldots, 32$ in the same way as G' in Figure 4. Now w.l.o.g. $C_1 = \{1, 3, 9, 11\}$, $C_3 = \{2, 4, 10, 12\}$, $C_2 = \{17, 19, 25, 27\}$ and $C_4 = \{18, 20, 26, 28\}$. So the vertices of C_2 are adjacent to the 4-tuple $\{1, 3, 10, 12\}$ or $\{2, 4, 9, 11\}$ and the vertices of C_1 are adjacent to the 4-tuple $\{17, 19, 26, 28\}$ or $\{18, 20, 25, 27\}$. So w.l.o.g. 1 and 3 are adjacent to $\{17, 19, 26, 28\}$ and 9 and 11 to $\{18, 20, 25, 27\}$, and hence 17 and 19 are adjacent to $\{1, 3, 10, 12\}$ and 25 and 27 to $\{2, 4, 9, 11\}$. So 10 and 12 are adjacent to $\{17, 19, 26, 28\}$, 9 and 11 to $\{18, 20, 25, 27\}$, 25 and 27 to $\{2, 4, 9, 11\}$ and 26 and 28 to $\{1, 3, 10, 12\}$. So each vertex of C_3 is adjacent to two vertices of C_4 and vica versa, so there is a Shrikhande subgraph containing both C_3 and C_4. This means for the geometry $(\mathcal{P}, \mathcal{L})$ that there is a line through C_3 and C_4 and hence C_1, C_2, C_3 and C_4 form an affine (sub-)plane of order 2.

Since the 3-subnet of the affine plane of order 4 can be completed into the affine plane of order 4 by adding two parallel classes of four lines each, we can partition the 16 4-cocliques of G in two different ways into four sets of four 4-cocliques such that no two vertices of two cocliques belonging to the same set are adjacent. So these two partitions correspond to two 4-colorings of the vertices of G. Now by Mathons paper [8] G must be isomorphic to $\mathcal{G}_1$, since $\mathcal{G}_1$ is the only 4-colorable strongly regular graph with parameters (64,18,2,6) that is locally $3C_6$. On the other hand the two 4-colorings of G partition the adjacency matrix of G into 256 blocks that are either half full or contain the zero matrix. If we swich the adjacencies in the half full blocks corresponding to a color class (so the blocks that contain edges with one end point in the color class), then the 12 Shrikhande subgraphs are switched into lattice graphs $L_2(4)$. So we get a srg(64,18,2,6) that is locally $6C_3$ which must be the collinearity graph of the unique GQ(3,5). □

Remark

Concerning the 2-rank of the adjacency matrices of $\mathcal{G}_1$ and $\mathcal{G}_2$ (cf. [1]) we find by computer that $r_2(A) = r_2(A + J) = 16$ for the adjacency matrix A of $\mathcal{G}_1$ and $r_2(A) = r_2(A + J) = 18$ if A is the adjacency matrix of $\mathcal{G}_2$.

Theorem 3 *There exists no srg(40,12,2,4) which is locally* $2C_6$.

Proof: Suppose G is a srg(40,12,2,4) which is locally $2C_6$. Then Claim 1 and 2 of the previous proof still hold for G, so G contains 5 Shrikhande subgraphs which intersect pairwise in a coclique of size 4. The vertices of G can be partitioned into 10 cocliques of size 4 each of which is defined by the intersection of two Shrikhande subgraphs. Again, a vertex not in some Shrikhande subgraph of G is adjacent to four vertices of this Shrikhande subgraph forming a coclique of type I. If you try to reconstruct G from its five Shrikhande subgraphs by identifying pairs of points it follows straightforwardly but not immediatly that such a graph can not exist. A shorter completion of the proof, which unfortunately relies on computer results, is as follows: Let A be the adjacency matrix of G and partition A into 100 blocks according to the partitioning of G into 10 4-cocliques. Then each block of A is either half full (has row sums 2) or is a zero-matrix. Let C be one of the 4-cocliques partitioning G. Now, if we swich the adjacencies between vertices of C and vertices of $G \setminus C$ that are in one of the two Shrikhande subgraphs containing C we get a srg(40,12,2,4), G' say, for which the graphs $4C_3, 2C_3 + C_6$ and $2C_6$ occur 4,24 and 12 times respectively as the neighbour graph of a vertex. Spence [11] determined all (23) srg(40,12,2,4) which contain a vertex for which the neighbour graph is $4C_3$ but none of these has a distribution of the neighbour graphs of the 40 vertices as G' has, so G' and hence G does not exist. □

In fact no srg(40,12,2,4) are known without a 4-clique and maybe none exist (see [11]).

References

[1] A.E. Brouwer, and C.A. van Eijl, On the p-Rank of the Adjacency Matrices of Strongly Regular Graphs, *Journal of Alg. Comb.* **1** (1992), 329-346.

[2] A.E. Brouwer and J.H. van Lint, Strongly regular graphs and partial geometries, pp. 85-122 in *Enumeration and Design*-Proc. Silver Jubilee Conf. on Combinatorics, Waterloo, 1982 (D.M. Jackson and S.A Vanstone, eds.), Academic Press, Toronto, 1984.

[3] P.J. Cameron, Strongly regular graphs, in: *Selected topics in graph theory*, L.W. Bieneke and R.J. Wilson(eds.), Academic Press, London 1978, 337-360.

[4] P.J. Cameron and J.H. van Lint, *Designs, Graphs, Codes and their Links*, London Mathematical Society Student Texts 22, Cambridge 1991.

[5] C.D. Godsil and B.D. McKay, Constructing cospectral graphs, *Aequationes Math.* **25** (1982), 257-268.

[6] W.H. Haemers, Eigenvalue Techniques in Design and Graph Theory, Math. Centre Tracts 121, Mathematisch Centrum, Amsterdam 1980.

[7] W.H. Haemers, Distance-Regularity and the Spectrum of Graphs, Tilburg University, Research Memorandum FEW 582, 1992.

[8] R. Mathon, The systems of linked (16, 6, 2) designs, *Ars Combinatoria*, Vol. **11** (1981), 131-148.

[9] S.E. Payne and J.A. Thas, *Finite Generalized Quadrangles*, Pitman, New York 1985.

[10] J.J. Seidel, Strongly regular graphs, in *Surveys in Combinatorics*, B. Bollobas(ed.), Proc. 7th British Comb. Conf., London Math. Soc. Lecture Notes Series, No. 38, Cambridge (1979), 157-180.

[11] E. Spence, $(40,13,4)$-designs derived from strongly regular graphs, University of Glasgow, Department of Mathematics preprint series No. 90/50 1990.

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