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Abstract:

We show that in a non-cooperative transboundary pollution game, a cleaner technology (i.e., a decrease in the emission to output ratio) induces each country to increase its emissions and ultimately can yield a higher level of pollution and reduce social welfare.

JEL classifications: Q55

Keywords: transboundary pollution, technological innovation, differential game

1 Introduction

We analyze a scenario à la Dockner and Long (1993), where two identical countries emit pollutants which accumulate into a stock of pollution over time. Both countries suffer a damage to social welfare from this transboundary stock of pollution. We consider the case where countries set their emissions policies non-cooperatively, and show that the impact of implementing a cleaner technology in the production process on social welfare is ambiguous.

Consider two technologies, one "clean" and one "dirty", and consider the non-cooperative emission strategies under a "dirty" technology that we refer to as the "dirty" equilibrium. Now suppose a cleaner technology is implemented. Clearly the non-cooperative equilibrium strategies under the "dirty" technology are still feasible. However, is this profile of emission strategies still a Nash equilibrium? The answer is no. The adoption of a "clean" technology reduces each country’s damage from pollution along the "dirty" equilibrium and, therefore, gives an incentive to each country to produce more than it would under the "dirty" technology. The negative environmental impact of the increase in production
of both countries can outweigh the positive environmental impact of adopting a "clean" technology. The benefit of the extra consumption from the adoption of the "clean" technology can be outweighed by the loss in welfare due to the increase in pollution. The positive shock of implementing a cleaner technology results in a more "aggressive" and "selfish" behavior of countries that exacerbates the efficiency loss due to the presence of the pollution externality. This phenomenon is similar to the "voracity effect" highlighted, in a different context, in Tornell and Lane (1999), Tornell and Velasco (1992) and Long and Sorger (2006). These papers consider a model of economic growth where multiple powerful interest groups within the economy have open access, via a process of fiscal redistribution, to a common capital stock. They consider a two sector economy with a formal sector where the return on capital is taxable and an informal sector which is nontaxable but yields a lower return. It is shown that if there do not exist institutional barriers to discretionary redistribution, an increase in the raw rate of return of capital reduces growth. This is because the increase in the rate of return of capital in the formal sector induces the "voracity effect" by which each interest group attempts to grab a greater share of national wealth by demanding more transfers. This effect is shown to dominate any direct effect of the positive shock resulting in a negative relationship between the rate of return of capital and growth.

We present the transboundary pollution game model in section 2 and derive a Markov-perfect Nash equilibrium in section 3. In section 4, we determine the impact of the adoption of a cleaner technology. Section 5 presents the concluding remarks.
2 The Model

Consider two countries indexed by $i = 1, 2$. Each country produces a single consumption good, $Q_i$, with a given fixed endowment of factors of production and a given technology. The production of a unit of the consumption good results in the emission of $E_i$ units of pollution where

$$E_i = \theta Q_i.$$  \hfill (1)

The parameter $\theta$ represents the ratio of emissions to output. The development of "cleaner" technologies is reflected by a fall in $\theta$, that is, less emissions generated per unit of output.

The amount of pollution accumulates into a stock, $P(t)$, over time, according to the following transition equation

$$\dot{P}(t) = E_1(t) + E_2(t) - kP(t), \quad k > 0,$$  \hfill (2)

with the initial stock

$$P(0) = P_0$$  \hfill (3)

In (2), $k$ represents the rate of natural purification, that is, the rate at which the stock of pollution naturally decays.

For notational convenience, the argument of time, $t$, is in general omitted throughout the paper although it is understood that all variables may be time dependent.

The instantaneous benefit in country $i$ from consuming a quantity $Q_i$ of the consumption good is given by

$$U_i(Q_i) = A Q_i - \frac{1}{2} Q_i^2, \quad A > 0 \text{ with } i = 1, 2.$$
and the instantaneous welfare loss in terms of pollution damage is given by

\[ C_i(P) = \frac{s}{2}P^2, \quad s > 0 \] with \( i = 1, 2. \)

The instantaneous net benefits of country \( i \) are thus given by

\[ B_i(Q_i, P) = U_i(Q_i) - C_i(P) \] (4)

Using (1), it follows that the objective of country \( i \)'s government is to choose a pollution control strategy \( E_i(t) \) (or equivalently an output strategy) that maximizes the discounted stream of net benefits from consumption:

\[ \max_{E_i} \int_0^\infty e^{-rt} \left[ U_i \left( \frac{E_i(t)}{\theta} \right) - C_i(P(t)) \right] dt \] (5)

subject to the accumulation equation (2) and the initial condition (3). The discount rate, \( r \), is assumed to be constant and identical for both countries. We give below a subgame perfect Nash equilibrium of this two-player differential game.

3 The Markov perfect equilibrium

Firms use Markovian strategies where they condition the level of emissions at a given moment, \( t \), on \( t \) and on the current-state variable, \( P(t) \), that summarizes the latest available information of the dynamic system. The pair \( (\phi_1, \phi_2) \) is a Markov Perfect Nash equilibrium, MPNE, if for each \( i \in \{1, 2\} \), given that \( E_j(t) = \phi_j(P(t), t) \) with \( j \in \{1, 2\}, j \neq i \), an optimal control path, \( E_i(.) \), of the problem (5) exists and is given by the Markovian strategy \( \phi_i \): \( E_i(t) = \phi_i(P(t), t) \).

It is well known that such a game admits a unique linear equilibrium and a continuum of equilibria with non-linear strategies (see Dockner and Long (1993) and Rubio and
Casino (2002)). The linear equilibrium is globally defined and, therefore, qualifies as 
a Markov perfect equilibrium. The non-linear equilibria are typically locally defined, 
i.e. over a subset of the state space. We focus in this analysis on the linear strategies equilibrium. Since our contribution is to highlight an a priori unexpected outcome from the adoption of a "cleaner" technology we wish to make sure that our result is not driven by the fact that countries are using highly "sophisticated" strategies.

**Proposition 1:** The pair

\[ \phi_i(P) = \theta A + \theta^2 (-\alpha P - \beta), \quad i = 1, 2 \]  

constitutes a Markov perfect linear equilibrium and discounted net welfare is given by

\[ W_i(P) = -\frac{1}{2} \alpha P^2 - \beta P - \mu, \quad i = 1, 2 \]  

where

\[ \alpha = \frac{-2k - r + \sqrt{(2k + r)^2 + 12\theta^2}}{6\theta^2} \]

\[ \beta = \frac{2k\alpha\theta}{k + r + 3\alpha\theta^2} \]

\[ \mu = -\frac{(A - 3\beta\theta)(A - \beta\theta)}{2r} \]

The steady state level of pollution

\[ P_{SS}(\theta) = \frac{2A\theta \left( k^2 + 3s\theta^2 - r^2 + (k + r)\sqrt{(2k + r)^2 + 12s\theta^2}\right)}{(k(k + r) + 3s\theta^2) \left( k - r + \sqrt{(2k + r)^2 + 12s\theta^2}\right)} > 0 \]  

is globally asymptotically stable.

**Proof:** We use the undetermined coefficient technique (see Dockner et al (2000) Chapter 4) to derive the linear Markov perfect equilibrium. The details are omitted. (See Proposition 1 of Dockner and Long (1993) for the case where \( \theta = 1 \)).
We note that $E_i > 0$ (and therefore, $Q_i > 0$) iff $P < \bar{P}(\theta) \equiv \frac{1}{\theta\alpha}(A - \theta\beta)$. It is straightforward to show that $\bar{P}(\theta) > P_{SS}(\theta)$ for all $\theta$.

4 Impact of cleaner technology

The development of a cleaner technology is captured by a decrease in the emissions to output ratio, $\theta$. We show that implementing a cleaner technology may end up reducing social welfare, (7), in each country: $W_i(P)$ may be an increasing function of $\theta$. Henceforth, for notational convenience, we explicitly write welfare as a function of $\theta$ as well as $P$: $W_i(P, \theta)$. We note that $W_i(P, \theta)$ is homogeneous of degree zero in $(r, k)$ for all $P$. Therefore, we can normalize one of these parameters without loss of generality: we set $k = 1$. For simplicity, unless otherwise mentioned, we set $A = 100$ and $r = 1$. It can be shown that our main conclusions remain robust to changes in the values of $A$ and $r$.

The steady state equilibrium pollution stock is then given by

$$P_{SS}(\theta) = F(\theta) \equiv \frac{200\theta(3s\theta^2 + 2\sqrt{9 + 12s\theta^2})}{(2 + 3s\theta^2)\sqrt{9 + 12s\theta^2}}. \quad (9)$$

**Proposition 2:** There exists $\bar{\theta} \simeq \frac{126}{\sqrt{s}}$ such that for all $\theta > \bar{\theta}$ we have

$$\frac{\partial P_{SS}}{\partial \theta} < 0$$

A decrease in the emissions to output ratio results in a larger stock of pollution at the steady state.

**Proof:** See Appendix A.

Following the adoption of a cleaner technology each country increases its production and the resulting increase in emissions outweighs the positive shock of a decrease in the
emissions to output ratio.

The impact of a decrease of $\theta$ on social welfare $W_i(P, \theta)$ depends on the level of pollution $P$ at the moment the change in $\theta$ occurs. To economize on space we shall focus on one particular value of $P$ which is $P_{SS}$.\(^1\) More precisely, suppose that the emissions to output ratio drops from $\theta_0$ to $\theta < \theta_0$ then from (9), we obtain $P_{SS}(\theta_0)$ and $P_{SS}(\theta)$, that is, the steady state pollution stocks when the emissions to output ratio are given by $\theta_0$ and $\theta$ respectively. We shall compare $W_i(P_{SS}(\theta_0), \theta_0)$ to $W_i(P_{SS}(\theta_0), \theta)$. Note that our analysis, is, thus, not a simple analysis of steady states levels of welfare, we take into account the impact on welfare during the transition from $P_{SS}(\theta_0)$ to the new steady state pollution stock $P_{SS}(\theta)$.

Given the cumbersome expression of $W_i(P_{SS}(\theta_0), \theta)$, we illustrate our main finding with the analysis of a marginal reduction in $\theta$, i.e. in the neighborhood of $\theta_0$. We provide in Figure 1 the plot of $\frac{\partial W_i(P_{SS}(\theta_0), \theta)}{\partial \theta}$ as a function of $\theta_0$. From Figure 1, we can state that there exists $\theta_0 \approx 3.14$ such that $\frac{\partial W_i(P_{SS}(\theta_0), \theta)}{\partial \theta} |_{\theta=\theta_0} > 0$ for all $\theta > \theta_0$: that is, a decrease in $\theta$, the emissions to output ratio, reduces social welfare.

\(^1\)We have also considered an initial pollution stock of zero. This does not add any new insights.
Figure 1: The effect of a marginal change in $\theta$ on welfare

We also consider a non-marginal reduction in $\theta$: we set $\theta_0 = 5$ and plot, in Figure 2, $W_i(P_{SS}(5), \theta)$. Figure 2 shows that the value of $\theta$ should decrease to levels smaller than 2.7 (a decrease by more than 46%) before the reduction in the emissions to output ratio results in an increase in welfare.\(^2\)

\(^2\)We note that $\max\{P_{SS}(5), P_{SS}(2.7)\} < \min\{\tilde{P}|_{\theta=5}, \tilde{P}|_{\theta=2.7}\}$. 

Figure 2: The effect of non-marginal changes in $\theta$ on welfare
In Figures 1 and 2, we set \( s = 1 \). It can be shown that our main conclusions remain qualitatively robust to changes in \( s \).

The non-cooperative equilibrium strategies under the "dirty" technology are feasible under the "clean" technology. However, this profile of emission strategies is not a Nash equilibrium. The adoption of a "clean" technology reduces each country’s damage from pollution along the "dirty" equilibrium, thereby giving each country an incentive to produce more than it would produce under the "dirty" technology. The negative environmental impact of the increase in the production of both countries can outweigh the positive environmental impact of adopting a "clean" technology. The positive shock of a cleaner technology results in a more "aggressive" and "selfish" behavior of countries that exacerbates the efficiency loss due to the presence of the pollution externality. This phenomenon is similar to the "voracity effect" highlighted in Tornell and Lane (1999), Tornell and Velasco (1992) and Long and Sorger (2006).

5 Concluding remarks

An alternative that has been proposed to the Kyoto agreement (which is currently riddled with free-riding issues) is a set of treaties: one promoting cooperative R&D and the other encouraging the collective adoption of new cleaner technologies arising from this R&D. For example, Barrett (2006) sets up a model in which if all countries adopt the "breakthrough" technology, this leads to a discrete drop in emissions, leaving the countries collectively better off. In this paper, we developed a model where the adoption of a cleaner technology may leave all countries worse off.

We analyzed a scenario where two identical countries emit pollutants which accumu-
late into a stock of pollution over time. Both countries suffer a damage to social welfare from this stock of pollution. Within this context, we showed that the implementation of a cleaner technology in the production process can have an ambiguous effect on welfare if the countries behave non-cooperatively. If the countries were to cooperatively set their individual emission levels to maximize joint welfare, then welfare would unambiguously increase in response to the implementation of cleaner technologies. However, in the non-cooperative equilibrium, each country’s response to the cleaner technology is to increase its own output without internalizing the negative impact of the resultant increase in the stock of pollution on the other country’s welfare. We identified the range of emission to output ratio for which this causes welfare in each country to fall as cleaner technologies are implemented.

There seems to be a general consensus amongst governments, international organizations and academics that a significant effort in the creation of clean technologies is needed. The main implication of our finding, however, is that the need for international cooperation to overcome the inefficiency due to the presence of transboundary negative externalities does not necessarily diminish with the development of cleaner technologies.

Appendix A: Proof of Proposition 2.

We have
\[
\frac{\partial F(\theta)}{\partial \theta} = \frac{200 \left( (4 - 6s^2\theta^2) (4s\theta^2 + 3)^{\frac{3}{2}} + 18\sqrt{3}s\theta^2 + 25\sqrt{3}s^2\theta^4 \right)}{(3s\theta^2 + 2)^2 (4s\theta^2 + 3)\frac{3}{2}}
\] (10)

Let
\[
X \equiv 4s\theta^2 + 3
\] (11)

Note that \( s\theta^2 = \frac{X - 3}{4} \) and for all \( s > 0 \) and \( \theta > 0 \) we have \( X > 3 \). Also, let
\[
Z(X) \equiv \left( 4 - 6\frac{X - 3}{4} \right) X^{\frac{3}{2}} + 18\sqrt{3}\frac{X - 3}{4} + 25\sqrt{3}\left( \frac{X - 3}{4} \right)^2
\] (12)
Together, (10), (11) and (12) imply that

\[
\frac{\partial F(\theta)}{\partial \theta} = \frac{2Z(X)}{(3s\theta^2 + 2)^2 (4s\theta^2 + 3)^{\frac{3}{2}}}
\]  

(13)

We have

\[
\frac{\partial Z(X)}{\partial X} = \frac{5}{8} \left( \frac{102}{5} X^\frac{1}{2} - 6X^\frac{3}{2} - \frac{39}{5} \sqrt{3} + 5X \sqrt{3} \right)
\]

(14)

with

\[
\frac{\partial^2 Z(X)}{\partial X^2} = \frac{5}{8X^\frac{1}{2}} \left( 5X^\frac{1}{2} \sqrt{3} - 9X + \frac{51}{5} \right)
\]

(15)

and

\[
\frac{\partial^3 Z(X)}{\partial X^3} = \left( -\frac{3}{16X^2} \right) \left( 17X^\frac{1}{2} + 15X^\frac{3}{2} \right) < 0.
\]

(16)

From (16), we have that \(\frac{\partial^2 Z(X)}{\partial X^2} < \frac{\partial^2 Z(X)}{\partial X^2} \big|_{X=3} = -0.64952 < 0\) for all \(X > 3\). Thus \(Z(X)\) is continuously differentiable and strictly concave in \(X\) for all \(X > 3\) with \(Z(3) = 20.79 > 0\) and \(\lim_{X \to \infty} Z(X) = -\infty\). Therefore, \(Z(X)\) has one and only one root over \(X \in [3, \infty)\), given by \(\bar{X} \simeq 9.38\), and we have \(\frac{\partial F(\theta)}{\partial \theta} < 0\) for all \(s\theta^2 > \frac{\bar{X} - 3}{4} \simeq 1.59\) or \(\theta > \bar{\theta} \simeq \frac{1.26}{\sqrt{s}}\).

**References**


