IMPERFECT INFORMATION, DEMOCRACY, AND POPULISM

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Abstract

The modern world is complex and difficult to understand for voters, who may hold beliefs that are at variance with reality. Politicians face incentives to pander to voters’ beliefs to get reelected. We analyze the welfare effects of this pandering and show that it entails both costs and benefits. Moreover, we explore optimal constitutional design in the presence of imperfect information about how the world works. We compare indirect democracy to direct democracy and to delegation of policy making to independent agents. We find that indirect democracy is often welfare maximizing.

Key words: Imperfect information, beliefs, democracy, populism, accountability, experts.

JEL classification: D72, D78, D83.

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1 Introduction

The best argument against democracy is a five-minute conversation with the average voter.

Winston Churchill

The world is complex and people may not properly understand how the modern society and its economy function. In some cases, voters’ beliefs deviate substantially from the view of experts. Caplan (2001) conducted a survey where he compares the opinions of the general public to the opinions of economists with regard to a number of economic issues. For instance, he asked people whether they think that trade agreements between the United States and other countries have helped create more jobs in the U.S. He coded the answer that trade agreements “have cost the U.S. jobs” as 0, that they “haven’t made much of a difference” as 1, and that they “helped create jobs in the U.S.” as 2. He finds that the mean response among economists is 1.47. In contrast, the mean response among the general audience is only .64. Caplan reports similar discrepancies with respect to other economic issues.

As a result of imperfect knowledge about a complex world, voters may not always be able to judge what policies are truly in their best interest and, as a consequence, hold incorrect beliefs. Politicians who aim to get reelected have an incentive to pander to voters’ beliefs and hence to potentially distort policies.

In this paper, we investigate the welfare consequences of politicians’ pandering to voters’ beliefs in the case of indirect democracy. Second, we explore the conditions under which other forms of government may lead to higher welfare. In particular, we compare indirect democracy to direct democracy and to the case where policy making is delegated to independent agents who are experts in a particular field.

In our model, welfare depends on a unidimensional policy action. In the case of indirect democracy, policy is set by an office-holding politician. In the case of direct democracy, it is determined by voters themselves, whereas in the case of independent agents it is set by the latter.
We capture voters’ imperfect information by assuming that no voter observes which level of the policy action is truly in their own best interest and in the best interest of society. Rather, voters hold prior beliefs about the welfare effects of policy actions.

In the case of indirect democracy, there are two types of politicians in the model. The first type is dubbed competent. This means that he obtains a perfectly revealing signal about the policy action that is best for society from an ex ante point of view. In contrast, the incompetent politician receives a noisy signal about the optimal policy and is, thus, imperfectly informed about the world. The assumption of the existence of incompetent politicians refers to the fact that politicians may have incompetent advisers, or they may have ideological views (as may voters). Politicians’ prime concern is to get reelected.

Voters aim to reelect a politician who appears competent whilst looking at the world through the lenses of their beliefs. This characteristic provides an incentive to politicians to cater for the pivotal voter’s belief in order to maximize their chance of getting reelected.

As it is usual in dynamic games of incomplete information, voters are endowed with a belief about the behavior of politicians. Our analysis allows for (but is not restricted to) a type of beliefs which include an element of bounded rationality. These beliefs are related to the cognitive hierarchy model of Camerer et al. (2004). In a nutshell, we assume that voters may anticipate a politician’s strategic response to their beliefs and, as a result, adapt their beliefs by one additional order not more than \( k \) times. The number \( k \) may be finite or, in the case of unlimited rationality, infinite. We show that, for finite \( k \), the prevailing political equilibrium is a separating equilibrium, in which competent and incompetent politicians implement different policies. For infinite \( k \), the prevailing equilibrium is a pooling equilibrium, in which both politician types implement identical policies. These are exclusively determined by voters’ prior beliefs and do not depend on the incumbent politician’s signal.

Our main positive result is that, under indirect democracy, the policy action is determined as a weighted average of a politician’s signal and the pivotal voter’s prior belief. Thus, policy making is partially populist (except for infinite \( k \), where it is fully populist). Importantly, as
we will show, this populism comes along with both costs and benefits. Our main normative result is that the institution of indirect democracy is often preferable to either that of direct democracy or of delegation of policy making to non-accountable experts, even if the latter are fully benevolent. Our analysis may help explain why indirect democracy is so prevalent around the world.

So far, there has been little attempt to incorporate elements of behavioral economics and bounded rationality into models of voting. Thus, this becomes an important item on the research agenda in political economy. For instance, Besley (2006) writes “going forward it would be interesting to understand better what the differences are between behavioral models of politics and the postulates of strict rationality” (p. 172). By taking into account limitations on voters’ strategic thinking, and by including full rationality as a limit case, we make one step in this direction.

Our analysis is related to a number of existing studies, most notably Maskin and Tirole (2004). These authors also analyze the optimality of the three institutions indirect democracy, direct democracy and independent agents. They consider a binary policy choice where one policy is more popular among voters than the other. Furthermore, they assume that politicians are intrinsically motivated to carry out certain policies and their preferences are either congruent or dissonant to voters. Our analysis differs in three main ways. Technically, we allow policies to lie on the entire real line. This allows us to conceptualize, in a natural way, notions of imperfect knowledge about the world, such as the distance of voters’ beliefs from the truth or the noisiness of a politician’s signal. Second, a key feature of our model is that both voters and politicians are imperfectly informed about optimal policies. In contrast, Maskin and Tirole assume that politicians are perfectly informed about which policy is best. Third, we allow for voters’ beliefs about politicians’ behavioral strategies that are boundedly rational.

Another related study is Canes-Wrone et al. (2001). Their analysis also differs from ours in several respects. Like Maskin and Tirole, they consider a setup with two states of the world and two possible policies. In contrast to our study (and to the one of Maskin and Tirole), Canes-
Wrone et al. do not address the question whether, in the presence of imperfect information, welfare may be improved by forms of government other than indirect democracy. Furthermore, they do not allow for boundedly rational beliefs.

Alesina and Tabellini (2007, 2008) provide an in-depth analysis of the advantages and disadvantages of accountability. In particular, they compare the performance of a politician who aims to get reelected with the performance of a bureaucrat who is concerned about his career perspective. Populism and imperfect knowledge about the world do not play a pivotal role in their analysis. Schultz (2008) analyzes the welfare effects of accountability by focusing on the term length of office periods. In our study, we take this term length as given.

The remainder of this paper is organized as follows. Section 2 introduces our model of indirect democracy. In Section 3, we solve the model. In Section 4, we characterize welfare under indirect democracy. In Section 5, we compare indirect democracy to the case of direct democracy and to the case of delegation of policy making to non-accountable agents. In Section 6, we discuss our findings, and we conclude in Section 7. All proofs are in Appendix A.

2 A Model of Indirect Democracy

2.1 Voters

We consider an economy populated by a unit mass of individuals to which we refer as voters. We consider a setup with two periods, indexed by \( t = 1, 2 \). In each period, voters’ utility is determined as

\[
V_t = -(g_t - x_t^* - \varepsilon_t)^2.
\]

The utility function is identical across voters. The variable \( g_t \in \mathbb{R} \) denotes a policy action. Under indirect democracy, \( g_t \) is set by the office-holding politician. Neglecting \( \varepsilon_t \), the utility maximizing level of \( g_t \) is given by \( x_t^* \in \mathbb{R} \). The crucial assumption in our framework is that \( x_t^* \) is unobserved. We assume that \( x_t^* \) is drawn at the beginning of each period by nature from a
normal distribution with mean \( E x^*_t \) and variance \( \sigma^2_x \). The mean may vary across periods and is unknown.

The variable \( \varepsilon_t \) is a normally distributed random variable with an expected value of zero and a variance of \( \sigma^2_\varepsilon \). We assume that \( \varepsilon_t \) is identically and independently distributed over time and independent of all other random variables in the model. As is the case for \( x^*_t \), \( \varepsilon_t \) is also unobserved. The distribution of \( \varepsilon_t \) is common knowledge.

As will be spelled out in more detail in Subsection 2.3, nature first draws \( x^*_t \), before \( \varepsilon_t \) is realized. The policy action \( g_t \) is to be set after \( x^*_t \) has been determined but before \( \varepsilon_t \) is realized. Thus, \( x^*_t \) determines the _ex ante_ optimal policy in period \( t \). It specifies how, from an ex ante point of view, a choice of \( g_t \) translates into voters’ utility. In contrast, \( \varepsilon_t \) represents a short-term shock to \( x^*_t \) and determines the _ex post_ optimal level of \( g_t \). While \( x^*_t \) and \( \varepsilon_t \) are unobserved in isolation, voters do observe the sum \( x^*_t + \varepsilon_t \) after \( g_t \) has been set. This allows voters to learn, although imperfectly, about \( x^*_t \) (see below).

From an ex-ante perspective, voters’ utility in period \( t \) is given by the expected value of \( V_t \), i.e. by

\[
EV_t = -E \left[ (g_t - x^*_t - \varepsilon_t)^2 \right].
\] (2)

The loss function specification is chosen for tractability. This utility function should be taken as reflecting indirect utility, meaning that optimal values of all other choices that voters may make are already substituted. There are two essential features of (1) or (2). First, \( x^*_t \) determines a unique interior optimum for \( g_t \) from an ex ante point of view. Second, there is risk aversion over the realizations of \( g_t \) if the latter are uncertain. The main theme of our analysis is voters’ imperfect information about \( x^*_t \) (see below). In order to concentrate on this issue, we make the simplifying assumption that \( x^*_t \) and, hence, (1) and (2) are common across voters.\(^1\)

To consider an example, suppose that there is a given budget to be spent for combating

\(^1\)There are standard examples where heterogeneous voters agree about the optimal level of provision of a public good. For instance, this is the case in the presence of income heterogeneity when the utility function is of Cobb-Douglas type with private consumption and a public good as the arguments and with a linear income tax. This is an important benchmark case (see Atkinson and Stiglitz, p. 302).
crime. Suppose that the relevant decision is to determine the share of this budget to spend on preventive measures (schooling, prevention of youth unemployment, quality of neighborhoods etc.) versus the share to spend on punishment (e.g. prison infrastructures). In this example, $x_t^*$ refers to the optimal budget share for preventive measures, given the general current situation in society. This may refer to the degree of income inequality and ethnic heterogeneity, the degree to which people follow certain norms, the general level of youth unemployment etc. The variable $\epsilon_t$ corresponds to a shock to the “threat of crime” and may originate from a sudden rise in youth unemployment, a sudden increase in immigration or the like.\(^2\)

As already stated, $x_t^*$ is not observed and $Ex_t^*$ is unknown. However, voters have prior beliefs about $x_t^*$. Specifically, we make the following assumption.

**Assumption 1** A voter $i$’s prior belief about $x_t^*$ is given by $x_t^i$ which is a normally distributed random variable with mean $\mu_t^i$ and variance $\sigma_x^2$. The distribution of prior beliefs across voters is common knowledge.

According to Assumption 1, the prior means of $x_t^i$ may be heterogeneous among voters while, for simplicity, we assume that the variance is common across voters.

### 2.2 Politicians

Under indirect democracy, the policy action $g_t$ is chosen and implemented by an incumbent politician. An incumbent politician’s objective in the first office period is to maximize the probability of getting reelected for a second term.\(^3\) Conditional on being (re)elected for office in the second period, a politician’s objective is simply to maximize welfare in this period. The latter assumption is to be understood as a shortcut and does not affect our main conclusions in

\(^2\)We assume that $x_t^*$ is normally distributed because of the high tractability of the normal distribution. For the example of choosing a share of a budget to be spent on preventive measures for combating crime, the policy variable could only take on values between zero and one. This would not be consistent with a normal distribution. However, it is straightforward to find a transformation of the domain of admissible policies such that they may take on any real value. Any function that is bijective and maps $[0, 1]$ onto the entire real line would achieve this.

\(^3\)One interpretation of this is that he derives ego rents from being in office, as in Rogoff (1990) or Besley (2006).
a substantive way.\footnote{In particular, we may allow for rent seeking along the lines of a model discussed in Persson and Tabellini (2000, Ch. 4). See footnote 10 below. We exclude rent seeking here in the interest of transparency.}

A politician knows the distribution of voters’ prior beliefs about $x_t^*$. However, he does not directly observe $x_t^*$. Rather, a politician receives a signal $\xi_t$ that is informative about $x_t^*$. There are two politician types that we dub competent and incompetent, respectively. The prior probability that a politician is competent is denoted by $\alpha$ and is common knowledge. In case of the competent politician, $\xi_t = x_t^*$, i.e. the signal reveals the truth.\footnote{The assumption that the competent politician perfectly observes $x_t^*$ is made for simplicity. The main conclusions from our analysis could also be obtained if the competent type received a more informative, but imperfect, signal than the incompetent type, as, for instance, in the career concerns model of Prat (2005).} An incompetent politician receives a noisy signal. Specifically, in the first period, $\xi_1 = x_1^* + \zeta_1$, where $\zeta_1$ is a random variable with mean zero and variance $\sigma_\zeta^2$. We dub $\zeta_t$ an incompetent politician’s bias. In the second period, $\xi_2 = x_2^* + \zeta_2$ in the case of a challenger winning the election. We assume that $\zeta_t$ is independent of all random variables in the model and that $\zeta_2$ is independent of $\zeta_1$ and identically distributed. Furthermore, the distribution of $\zeta_t$ is common knowledge.

For the case of an incumbent politician we make the following assumption.

**Assumption 2** An incompetent incumbent who gets reelected for a second office period keeps his bias $\zeta_1$, i.e. $\xi_2 = x_2^* + \zeta_1$.

We make Assumption 2 because we find it more plausible than assuming that a politician’s bias is drawn afresh when he gets elected for a second period. In fact, the analysis would be slightly simpler if we assumed that a politician’s bias is determined anew every period.

In principle, it may be natural to allow for $E\zeta_t \neq 0$. One may argue that politicians are drawn from the general population and may thus have systematically biased views about $x_t^*$. We briefly discuss this case in Appendix C but do not consider it in the main model since it complicates the analysis without leading to substantive additional insights.\footnote{Whether politicians should be understood as a “representative sample” drawn from the general population clearly depends on the nature of the political recruitment process and may differ across countries.}

We make two further assumptions about a politician’s information. First, we follow the literature on career concerns by assuming that a politician does not observe his own type (see
Holmström, 1999, and Prat, 2005). This means that he does not observe whether his signal is perfect or noisy. Second, we make the simplifying assumption that a politician treats his signal $x_t^*$ as a best predictor for $x_t^*$. To state this formally, assume that a politician’s belief about the ex ante optimal policy $x_t^*$ is captured by a random variable $x_t^p$. The superscript is an index for politicians. We then state the mentioned assumptions as follows.

**Assumption 3**  A politician does not observe his type. He believes that $E[x_t^p | \xi_t] = \xi_t$. We make the additional simplifying assumption that a politician takes $\xi_t$ as a “point estimate” for $x_t^*$ in the sense of classical statistics and his behavior is only based on this point estimate (rather than on a non-degenerate belief $x_t^p$). We make this assumption only for simplicity and discuss its relaxation in Appendix D. In practical terms, $\xi_t$ should be interpreted as a policy suggestion that a politician gets from his advisers or his party. Thus, a politician’s competence is not only determined by his personal skills but also by the competence of his advisers and party strategists.

**2.3 The Political Game**

Below we indicate the stages of the political game in a more formal manner. We provide a label for each stage of the game. The letters in the labels refer to the players which have their moves at the respective stages. $N$ denotes nature, $P$ denotes the politician, and $V$ denotes voters. The first figure after the letter refers to the office period $t = 1, 2$. The second figure indexes moves within an office period for nature, as nature has two moves within one period.

- **Stage N1.1**: Nature draws $x_1^*$; it determines the type of the incumbent politician and his signal $\xi_1$.
- **Stage P1**: The incumbent politician chooses $g_1$.
- **Stage N1.2**: Nature draws $\varepsilon_1$ and sends the signal $x_1^* + \varepsilon_1$ to voters.
Stage V1: Voters decide whether to reelect or oust the incumbent politician.

Stage N2.1: Nature draws $x^*_2$; if a new politician is in office, nature determines his type and, in case of an incompetent politician, his bias $\zeta_2$; furthermore, nature sends the signal $\xi_2$.

Stage P2: The politician chooses $g_2$.

Stage N2.2: Nature draws $\varepsilon_2$.

Politicians have two moves in the above game, since they set $g_t$ in each period. The politician in period 2 may be different from the politician in office in period 1. Voters have only one move in the entire game, i.e. they decide whether to cast their votes for the incumbent politician or for a challenger.

Our model is comparatively rich. This is due to the fact that, unlike the existing literature, our model incorporates the feature that both voters and politicians are imperfectly informed about what policy is optimal. Furthermore, the problem under study is only of interest if we allow for the possibility that voters have an opportunity to learn about $x^*_t$, but imperfectly so. This motivates the inclusion of the random variable $\varepsilon_t$. There is no other more parsimonious setup where we can still analyze the role of imperfect information in politics on both the voters’ and the politicians’ side in a meaningful way.

3 Analysis of Indirect Democracy

3.1 Overview

The logic of our derivation of the solution of the political game as stated in Section 2.3 is as follows. The two crucial stages of the game are V1 and P1. In an equilibrium, voters’ reelection decisions and an incumbent’s choice of $g_1$ must be mutually best responses, given voters’ beliefs. Furthermore, an equilibrium entails that voters’ beliefs at stage V1 integrate the
information contained in nature’s signal at stage N1.2 and the information contained in $g_1$ about a politician’s type and bias according to Bayes’ law.

For the derivation of the equilibrium it is convenient to start with voters, taking as given a (well-behaved) belief of voters about how an incumbent politician sets $g_1$ as a function of his signal $\xi_1$. Given this belief and voters’ posterior information about the ex ante optimal policy level $x_1^*$ and the incumbent politician’s type and bias, voters determine their expected utility from reelecting the incumbent politician. They compare this level of expected utility to the one resulting from ousting the incumbent politician and electing a challenger. Our analysis focuses on the case where the pivotal voter is the one associated with median of $\mu_1$.\footnote{See Appendix B for conditions that guarantee that the voter associated with the median of $\mu_1$ is pivotal.}

The pivotal voter’s reelection decision implies an incumbent politician’s best response. It is crucial to understand, however, that we will first derive voters’ reelection decisions for given beliefs about how a politician sets $g_1$ as a function of his signal $\xi_1$. An equilibrium requires voters’ beliefs about an incumbent’s behavior and a politician’s actual behavior being mutually related, if not identical. We use a specific equilibrium concept that is based on what we dub beliefs of sophistication of degree $k$. These beliefs contain an element of bounded rationality concerning voters’ strategic thinking. The case of full rationality where voters’ beliefs coincide with a politician’s actual behavior is contained as a limit case and gives rise to a standard sequential equilibrium.

Our main results will be intuitive. However, deriving them in a rigorous manner requires indeed a careful consideration of the role of voters’ beliefs about a politician’s behavior. Thus, the derivation is more intricate than one may infer at first, when simply looking at the results.

### 3.2 Voters’ Beliefs

In this subsection we characterize voters’ sequentially rational beliefs at stage V1 of the game: (i) about the ex ante optimal policy level $x_1^*$; (ii) about the incumbent’s bias $\zeta_1$ conditional on the incumbent being incompetent; and (iii) about the probability that the incumbent is competent.
We start with posterior beliefs about $x_1^*$. These result from observing $x_1^* + \varepsilon_1$ at stage N1.2 of the game. Although the ex post optimal policy level is given by $x_1^* + \varepsilon_1$, rational voters are interested in the ex ante optimal level $x_1^*$ since they want to judge a politician’s competence and are aware that a politician chooses $g_1$ before $\varepsilon_1$ is realized.

Concerning notation, we use a hat for all variables that are associated with voters’ posterior beliefs (i.e. beliefs at stage V1). Variables without a hat refer to prior beliefs. The following lemma states a standard result for normally distributed beliefs.

**Lemma 1 (Posterior beliefs about $x_1^*$)** Voter $i$’s posterior belief $\hat{x}_i^1$ about the ex ante optimal policy level $x_1^*$ is normally distributed with mean $\hat{\mu}_i^1 = (1 - \beta) \mu_i^1 + \beta (x_1^* + \varepsilon_1)$ and variance $\hat{\sigma}_x^2 = \frac{\sigma_x^2 \sigma^2_\varepsilon}{\sigma_x^2 + \sigma^2_\varepsilon}$, where $\beta \equiv \frac{\sigma_x^2}{\sigma_x^2 + \sigma^2_\varepsilon}$.

Lemma 1 characterizes voters’ posterior beliefs about the ex ante optimal policy level $x_1^*$. The assumption of normally distributed prior beliefs $x_i^1$ implies that the posterior mean $\hat{\mu}_i^1$ is a weighted average of the prior mean and the signal $x_1^* + \varepsilon_1$, observed at stage N1.2 of the game. The degree of updating of prior beliefs depends on the signal-to-noise ratio $\beta = \frac{\sigma_x^2}{\sigma_x^2 + \sigma^2_\varepsilon}$.

While the result in Lemma 1 is highly standard, it is useful to explain its meaning in the context of the current analysis. Consider again the example of what share of a given budget to spend on preventive measures to combat crime. Ex ante, the optimal share is given by $x_1^*$ and voter $i$ believes that expected welfare is maximized by setting $g_1 = \mu_i^1$. Ex post, voters observe the ex post optimal budget share for preventive measures $x_1^* + \varepsilon_1$. The latter depends on the actual ex post threat of crime according to random short-term factors constituting $\varepsilon_1$. If $\sigma^2_\varepsilon$ is very low, then observing $x_1^* + \varepsilon_1$ is very informative about $x_1^*$ and voters will put a high weight on the signal. In contrast, if $\sigma^2_\varepsilon$ is high, voters update their beliefs only in a minor way.

Consider a voter who believes that a high share of the budget to combat crime should be spent on punishment and that the actual share spent on punishment has indeed been high. Suppose that, ex post, the crime rate is high. If $\sigma^2_\varepsilon$ were low, then the voter would infer that his prior beliefs were probably wrong. But if $\sigma^2_\varepsilon$ is high, he will conclude that criminal threat must have been unusually high.
As a special case, voters’ prior beliefs may be understood as ideologies, e.g. about the desirability of punishment for combating crime. An important characteristic of ideologies is that they are persistent, i.e. people are not willing to let their ideologies “erode” (Bénabou, 2008). In light of the above discussion, ideological beliefs have the same effects as assuming that \( \sigma^2_\varepsilon \) is very high. In this case, voters blame \( \varepsilon_1 \) for any observation that is at variance with their beliefs.

We now turn to voters’s posterior beliefs about the realization of the incumbent politician’s bias \( \zeta_1 \), conditional on the incumbent being incompetent. (For the competent type, we have \( \zeta_1 \equiv 0 \).) We denote the posterior belief about the realization of the bias \( \zeta_1 \) by \( \hat{\zeta}_1^i \). As indicated by the notation, this posterior belief is heterogeneous since it depends on heterogeneous beliefs \( x_1^i \) (or \( \hat{x}_1^i \)). The reason why a rational voter wants to updated his beliefs about \( \zeta_1 \) is that an incompetent incumbent who gets reelected for the second office period will keep his bias as stated in Assumption 2. Thus, a voter \( i \) uses \( \hat{\zeta}_1^i \) for determining expected utility in the second period in case of reelection of the incumbent (see Section 3.3 below).

In order to characterize the belief \( \hat{\zeta}_1^i \), we need to introduce a belief of voters about how a politician sets \( g_1 \) as a function of his signal \( \xi_1 \). As outlined in Subsection 3.1, we defer the precise specification of this belief to later (see Section 3.5). Until there, we simply use an abstract notation for this belief and assume that voters believe that \( g_1 = G(\xi_1) \). In technical terms, \( G \) represents voters’ belief about a politician’s behavioral strategy. In order to update beliefs about \( \zeta_1 \), voters’ need to infer a politician’s unobserved signal \( \xi_1 \) from the observed policy choice \( g_1 \). We guess (and verify later) that \( G \) is continuous and strictly increasing. This assures that the inverse function of \( G \), denoted by \( G^{-1} \), exists. Thus, voters simply use this inverse \( G^{-1} \) to infer the unobserved signal \( \xi_1 \) from the observed action \( g_1 \).

Some further comments about the belief \( G \) are in order. First, the fact that \( G \) is a continuous and strictly increasing function implies that a politician’s behavioral strategy is strictly separating across signals. That means that voters believe that if there were two politicians that received different signals, then they would choose different levels of \( g_1 \). Second, we assume that the
belief $G$ is identical across voters. Third, the function $G$ will be made specific in Section 3.5. Under the specific class of beliefs that we consider there, it will turn out that $G$ is linear and strictly increasing. Thus, our assumption that $G$ is continuous and strictly increasing turns out to be consistent. Finally, we do not require that the belief $G$ is a priori correct. Thus, the inferred signal $G^{-1}(g_1)$ may deviate from the actual signal $\xi_1$ that the incumbent politician has received. Our notation indicates this by using a hat for the signal as inferred by voters through their belief $G$. Thus, we have $\hat{\xi}_1 = G^{-1}(g_1)$.

Voter $i$ expects that, conditional on the incumbent politician being incompetent, $\hat{\xi}_1 = \hat{x}_i^1 + \zeta_1$. It follows that $\hat{\zeta}_1$ serves as a signal that a voter $i$ uses to update his beliefs about the realization of the politician’s bias $\zeta_1$, using his posterior belief about $x_1^*$ (see Lemma 1). We then have the following result.

**Lemma 2 (Posterior beliefs about $\zeta_1$)** Voter $i$’s posterior belief $\hat{\zeta}_1$ about the realization of the incumbent’s bias $\zeta_1$ is normally distributed with mean $E\hat{\zeta}_1 = \gamma (\hat{\xi}_1 - \hat{\mu}_1)$ and variance $\hat{\sigma}_\zeta^2 = \frac{\sigma_x^2 \sigma_\zeta^2}{\sigma_x^2 + \sigma_\zeta^2}$, where $\gamma \equiv \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\zeta^2}$.

The proof is almost identical to the one of Lemma 1 and is omitted. Note that in the case where $\hat{\xi}_1 = \hat{\mu}_1$, we have $E\hat{\zeta}_1 = 0$. However, since $\zeta_1$ is normally distributed, $\hat{\xi}_1 = \hat{\mu}_1$ occurs with probability zero. As a result, we generically have $E\hat{\zeta}_1 \neq 0$.

We finally determine a voter’s posterior belief about the probability that the incumbent politician is competent. Conditional on the incumbent being competent, voter $i$ treats $\hat{\xi}_1$ as drawn from the distribution $\hat{x}_i^1$. Similarly, conditional on the incumbent being incompetent, the voter treats $\hat{\xi}_1$ as drawn from $\hat{x}_i^1 + \hat{\zeta}_1$. Denote by $f_i^c$ the density function associated with $\hat{x}_i^1$, and by $f_i^ic$ the density function associated with $\hat{x}_i^1 + \hat{\zeta}_1$. The subscripts $c$ and $ic$ stand for competent and incompetent, respectively. Using this notation, the posterior probability that the incumbent politician is competent is determined as follows.

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8The linearity follows from the fact that $\hat{\mu}_1$ is linear in $x_1^*$ according to Lemma 1.

9Note that it would not be rational to use the prior belief about $x_1^*$ since this would mean neglecting useful information.
Lemma 3 (Posterior $\alpha$) (i): Voter $i$’s posterior belief about the probability that the incumbent is competent is given by

$$\hat{\alpha}_i = Pr[\text{competent} | \hat{\xi}_1] = \frac{\alpha f_c(\hat{\xi}_1)}{\alpha f_c(\hat{\xi}_1) + (1 - \alpha) f_{ic}(\hat{\xi}_1)}.$$  

(ii): $\hat{\alpha}_i$ is a strictly decreasing function of $|\hat{\xi}_1 - \hat{\mu}_i|$. 

To understand the logic of (3), suppose that $\hat{\xi}_1$ would be a discrete random variable. Then, $f_c(\hat{\xi}_1)$ and $f_{ic}(\hat{\xi}_1)$ would denote the probabilities that $\xi_1$ takes on its inferred value in the case of the competent and the incompetent politician, respectively. Thus, (3) would reflect a standard updating formula. Lemma 3 shows that the same logic applies if $f_c$ and $f_{ic}$ refer to continuous random variables, provided they are well-behaved as it is true for normal random variables.

Part (ii) of Lemma 3 will be used for deriving a politician’s best-response choice of $g_1$ below. It shows that the larger the distance between a politician’s signal $\hat{\xi}_1$ (as inferred by the voter) and his posterior belief $\hat{\mu}_i$, the lower the probability that the voter assigns to the event that the incumbent politician is competent. This manifests how a voter judges the competence of an incumbent politician through the lens of his (posterior) belief.

### 3.3 Voters’ Reelection Decision

The politician in office in the second period is either the incumbent from the first period or a newly elected challenger. In either case, he sets $g_2 = \xi_2$ at stage P2 of the game. This follows from our assumption that, conditional on being reelected, a politician’s objective is to maximize welfare$^{10}$ and from Assumption 3.

$^{10}$The aim of this assumption is to simplify the analysis. We could obtain almost identical results if we were allowing for rent extraction in a way similar to Persson and Tabellini (2000, Ch. 4.5). To see this, suppose that there is an upper bound on the amount of rents that a politician can extract. Suppose further that a competent politician makes better use of the remaining government budget by better promoting welfare due to superior information (see Rogoff, 1990). In such a model, rent seeking would not affect a politician’s choice of $g_t$. 

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We consider now a voter \(i\)’s reelection decision at stage \(V_1\) of the game. A voter \(i\) considers his expected utility in case of reelection of the incumbent and compares it to the expected utility obtained in case of the election of a challenger. He casts his vote for the incumbent if and only if expected utility his higher under the incumbent than under the challenger. There is no strategic voting since there is a continuous population of voters.

Expected utility in the case of reelection of the incumbent is determined as follows. From the perspective of voter \(i\), the incumbent is competent with probability \(\hat{\alpha}_i\) (see Lemma 3). In the case of the competent incumbent, \(g_2 = x^*_2\) since \(\xi_2 = x^*_2\). Voter \(i\) does not observe \(x^*_2\) but substitutes his belief \(\hat{x}^*_2\). Using (2), \(EV^i_2 = -E (x^*_2 - \hat{x}^*_2 - \varepsilon_2)^2 = -\sigma^2_\varepsilon\). In case that the incumbent is incompetent, \(g_2 = \xi_2 = x^*_2 + \hat{\zeta}_i\), according to Assumption 2 and Lemma 2. Using (2) again, we have \(EV^i_2 = -E (x^*_2 + \hat{\zeta}_i - x^*_2 - \varepsilon_2)^2 = - \left[ (E\hat{\zeta}_1)^2 + \hat{\sigma}_\zeta^2 + \sigma^2_\varepsilon \right]\). \(^{11}\) Overall, expected utility from reelecting the incumbent is given by

\[
EV^i_2 = -\sigma^2_\varepsilon - (1 - \hat{\alpha}_i) \left[ (E\hat{\zeta}_1)^2 + \hat{\sigma}_\zeta^2 \right].
\] (4)

The logic behind (4) is that the utility loss due to the variance of \(\varepsilon_1\) is realized for both politician types. In contrast, the loss due to the fact that an incompetent politician’s signal is noisy arises only with probability \(1 - \hat{\alpha}_i\) from the perspective of voter \(i\).

Expected utility from a challenger is determined very similarly. There \(\hat{\alpha}_i\) has to be replaced by \(\alpha\) and \(\hat{\zeta}_1\) by \(\zeta_2\). Since \(E\zeta_2 = 0\), we obtain, in analogy to (4),

\[
EV^i_2 = -\sigma^2_\varepsilon - (1 - \alpha) \sigma^2_\zeta.
\] (5)

A voter reelects the incumbent if and only if expected utility as given by (4) exceeds expected utility as given by (5). Rearranging directly leads to the condition stated in the below lemma.

\(^{11}\)Here we have used the fact that, for any random variable \(z\), \(E (z^2) = (Ez)^2 + Var(z)\).
Lemma 4 (Reelection Decision) Voter $i$ reelects the incumbent if and only if

$$\frac{1 - \hat{\alpha}_i}{1 - \alpha} \left( E\hat{\zeta}_1 \right)^2 + \hat{\sigma}_\zeta^2 \leq 1.$$ (6)

To understand condition (6), consider first the limit case in which a voter would not learn anything about the incumbent’s bias $\zeta_1$ from observing the incumbent’s action at stage P1. This would be the case if the function $G$ were constant for all levels of the signal $\xi_1$. In this case, $E\hat{\zeta}_1 = E\zeta_1 = 0$ and $\hat{\sigma}_\zeta^2 = \sigma_\zeta^2$. Thus, condition (6) simplifies to $\hat{\alpha}_i \geq \alpha$. This means that a voter casts his ballot for the incumbent if and only if it is more likely that the incumbent is competent than that a challenger is competent.

If voters update their beliefs about an incumbent’s bias $\zeta_1$ by observing $g_1$ at stage P1, reelection of the incumbent is compatible with $\hat{\alpha}_i < \alpha$. Thus, it is possible that a voter prefers to reelect an incumbent politician even if he believes that the probability that the incumbent is competent is lower than the probability that a challenger would be competent. Lemma 4 shows that this requires that $\left( E\hat{\zeta}_1 \right)^2 + \hat{\sigma}_\zeta^2$ is sufficiently smaller than $\sigma_\zeta^2$. This is the case if $|\hat{\xi}_1 - \hat{\mu}_i|$ is small and $g_1$ provides a relatively sharp signal about the incumbent’s bias $\zeta_1$, such that $\hat{\sigma}_\zeta^2$ is small (see Lemma 2). In this case, reelecting the incumbent, a voter expects a relatively small variance associated with $g_2$ relative to the variance associated with $g_2$ when being set by a challenger. This comes at a benefit since voters are risk averse over $g_2$.

3.4 Politician Behavior

A politician’s behavior at stage P2 has already been discussed at the beginning of last subsection. At stage P1, a politician chooses $g_1$ such that he maximizes the probability of getting reelected. This entails maximizing the probability that (6) holds for the median voter, i.e. the voter associated with the median of $\mu_1$, denoted by $\mu_1^m$. (See Appendix B for a discussion of sufficient conditions for the voter associated with $\mu_1^m$ being pivotal.)

From Lemma 3(ii), $\hat{\alpha}_m$ strictly decreases in $|\hat{\xi}_1 - \hat{\mu}_1|$. Furthermore, from Lemma 2,
\( (E\hat{\zeta}_1^m)^2 \) strictly increases in \( |\hat{\xi}_1 - \hat{\mu}_1^m| \). It follows that a politician maximizes the probability that (6) holds for the median voter by setting \( g_1 \) such that \( \hat{\xi}_1 = E [\hat{\mu}_1^m | \xi_1] \), since this maximizes \( \hat{\alpha}_m \) and minimizes \( E (\hat{\zeta}_1^m)^2 \). A politician determines \( \hat{\xi}_1 \) via his choice of \( g_1 \) since \( \hat{\xi}_1 = G^{-1} (g_1) \). Using this, it follows that the condition \( \hat{\xi}_1 = E [\hat{\mu}_1^m | \xi_1] \) is equivalent to \( g_1 = G (E [\hat{\mu}_1^m | \xi_1]) \). Substituting for \( E [\hat{\mu}_1^m | \xi_1] \) from Lemma 1 and using Assumption 3, we state this as follows.

**Lemma 5 (Politician’s Behavior)** The incumbent politician chooses \( g_1 = G ((1 - \beta) \mu_1^m + \beta \xi_1) \).

This lemma establishes that, given voters’ belief \( G \) about a politician’s behavioral strategy, setting \( g_1 = G ((1 - \beta) \mu_1^m + \beta \xi_1) \) is a best response to this belief and to voters’ reelection decision as characterized by (6).

### 3.5 Strategic Beliefs of Sophistication of Degree \( k \)

Lemmas 4 and 5 characterize mutually best responses for voter’s election decisions at stage V1 of the game and a politician’s choice of \( g_1 \) at stage P1. These mutually best responses have been derived under the assumption that voters’ have a belief \( G \) about how a politician sets \( g_1 \) as a function of his signal \( \xi_1 \). We have assumed that \( G \) is a strictly increasing function of \( \xi_1 \) and is common across voters.

A full solution of the game requires determining \( G \) such that it relates to a politician’s actual behavioral strategy as characterized in Lemma 5. We concentrate on what we dub beliefs of strategic sophistication of level \( k \), or sophistication-\( k \) beliefs, for short. Sophistication-\( k \) beliefs are related to the cognitive hierarchy model of Camerer et al. (2004). They contain an element of bounded rationality, but full rationality is obtained as a limit case. In the case of bounded rationality, voters’ beliefs about a politician’s behavioral strategy \( G \) and a politician’s actual behavioral strategy are mutually consistent up to one order. The precise meaning of this statement will become clear below.
From a technical point of view, we develop sophistication-\(k\) beliefs recursively. We start with a baseline belief about how a politician sets \(g_1\) as a function of \(\xi_1\). This baseline belief corresponds, by definition, to a belief of strategic sophistication of degree zero. A natural choice of the baseline belief is the one where voters believe that a politician maximizes expected welfare, given his signal \(\xi_1\). Denoting this baseline belief by \(G_0(\xi_1)\), it entails that \(G_0(\xi_1) = \xi_1\). This follows from (2) and Assumption 3.

Consider now a politician’s best response to the belief \(G_0\). Using Lemma 5, it follows that a politician maximizes the probability of getting reelected by choosing \(g_1 = G_0 [E [\hat{\mu}_1^m | \xi_1]] = (1 - \beta) \mu_1^m + \beta \xi_1\). It follows that a politician’s best response deviates from voters’ belief.

The intuition for this fact is as follows. Suppose that the median voter believes that a politician chooses \(g_1 = \xi_1\). This implies that, at stage V1, the median voter judges an incumbent competent if \(g_1\) comes close to \(\hat{\mu}_1^m\). In other words, the median voter judges competence through the lens of his posterior belief about \(x_1^*\), i.e. \(\hat{\mu}_1^m\). A politician anticipating this has an incentive to set \(g_1\) equal to his expectation of \(\hat{\mu}_1^m\), i.e. \(E [\hat{\mu}_1^m | \xi_1]\), rather than equal to \(\xi_1\). Using Lemma 1 and Assumption 3, if follows that \(E [\hat{\mu}_1^m | \xi_1] = (1 - \beta) \mu_1^m + \beta \xi_1\).

By definition, a belief of strategic sophistication of degree 1 entails that voters anticipate a politician’s incentive to deviate from the sophistication-0 belief \(G_0\). Thus, \(G_1(\xi_1) := G_0((1 - \beta) \mu_1^m + \beta \xi_1) = (1 - \beta) \mu_1^m + \beta \xi_1\) (since \(G_0(\xi_1) = \xi_1\)). A politician’s best response to this belief is again determined by Lemma 5 and we obtain \(g_1 = G_1 [E [\hat{\mu}_1^m | \xi_1]] = (1 - \beta^2) \mu_1^m + \beta^2 \xi_1 \neq G_1(\xi_1)\). Thus, a politician also has an incentive to deviate from sophistication-1 beliefs. Proceeding with this recursion, we define sophistication-\(k\) beliefs as follows.

**Definition 1** Beliefs of strategic sophistication of degree \(k\) are defined by the recursion

\[
G_k(\xi_1) = G_{k-1}((1 - \beta) \mu_1^m + \beta \xi_1),
\]

where \(G_0(\xi_1) = \xi_1\).

The following Lemma provides a direct analytical expression for \(G_k\) and states a politician’s
best response to this belief. The proof is straightforward and is omitted.

Lemma 6 Under beliefs of sophistication of degree \( k \), voter \( i \)'s belief about a politician's behavioral strategy is given by \( G_k(\xi_1) = (1 - \beta^k)\mu_1^m + \beta^k\xi_1 \). A politician's best response to this belief is given by \( g_1 = (1 - \beta^{k+1})\mu_1^m + \beta^{k+1}\xi_1 \).

Confirming our earlier claim, \( G_k \) is indeed linear. Furthermore, it is strictly increasing in \( \xi_1 \) for finite \( k \). Hence, \( \hat{\xi}_1 = G^{-1}(g_1) \) is well-defined for finite \( k \). The solution of the political game for infinite \( k \) is obtained as a limit case.

Under sophistication-\( k \) beliefs, voters' beliefs about a politician's behavioral strategy and a politician's actual behavioral strategy are mutually consistent up to one order. Both converge when \( k \) approaches infinity. We do not restrict the analysis to this limit case since, in our view, it is not particularly realistic. Rather, our approach is to take \( k \) as exogenously given and identical across voters. We take it as a constraint on the sophistication of voters' strategic thinking. Alternatively, it may also be interpreted as a belief of voters about the strategic sophistication of the incumbent politician. The evidence discussed in Camerer et al. (2004) suggests that experimental subjects are able to foresee about one or two rounds of strategic reactions. This stands in sharp contrast to the requirement of unlimited rationality of anticipating strategic reactions for infinitely many rounds. Thinking about the chess game makes it salient how difficult it is in practice to anticipate higher-order strategic reactions of other players.

In practice, \( k \) may be influenced by many factors. One factor may be the structure of the media market, e.g. to what degree the market for TV news is dominated by a public provider (see Prat and Strömberg, 2006). One may speculate that private news provider have a greater incentive to present "stories" where politicians are portrayed as strategic actors that do everything only to get reelected. This may increase voters' level of \( k \), in turn.

3.6 The Political Equilibrium

We start the discussion of the equilibrium with a definition of populism.
Definition 2 (Populism) A politician’s choice is populist if it does not only depend on his signal $\xi_t$ but also on the prior belief of the median voter $\mu_t^m$.

Our main positive result, which characterizes the outcomes in an indirect democracy in the first office period, is the following.

Proposition 1 (Equilibrium First Period) Suppose voters hold beliefs of degree of sophistication $k$. (i): If $k$ is finite, then there exists a unique equilibrium in which

$$g_1 = (1 - \beta^{k+1}) \mu_1^m + \beta^{k+1} x^*_1$$

in case of the competent politician and

$$g_1 = (1 - \beta^{k+1}) \mu_1^m + \beta^{k+1} (x^*_1 + \zeta_1)$$

in case of the incompetent politician. (ii): If $k$ is infinite, there exists a unique equilibrium that is obtained as a limit case for $k \to \infty$. Then both politician types set $g_1 = \mu_1^m$.

Part (i) of Proposition 1 shows that $g_1$ is equal to a weighted average of the politician’s signal about $x^*_1$ and the median voter’s prior belief about $x^*_1$. Remember that the signal of the competent politician is equal to $x^*_1$ while the signal of the incompetent politician is equal to $x^*_1 + \zeta_1$. Any equilibrium involves pandering to the median voter’s belief and thus a populist policy choice. The degree to which policy making is populist is the higher, the lower $\beta^k$. In the limit case where $k = \infty$, (part (ii) of Proposition 1), policy making is perfectly populist and neither politician type makes use of his signal. Thus, $\beta^k$ can be understood as indicating the susceptibility to populism. Since $0 < \beta < 1$, a higher $k$ implies a higher susceptibility to populism. We discuss the intuition for this result below.

In the case of a finite $k$, the prevailing equilibrium is separating. This means that: (i) a politician’s choice depends on his signal $\xi_1$ and different values of the signal lead to different policy choices; (ii) both politician types choose different levels of $g_1$ with probability one.
The difference in policy choices shrinks with a higher level of $k$. For the limit case of full-rationality, i.e. an infinite $k$, the prevailing equilibrium is a pooling equilibrium. In particular, both politician types choose an identical policy action that does not depend on the signal $\xi_1$.

A crucial determinant of $\beta$ is $\sigma^2$ (see Lemma 1). If $\sigma^2$ is low, $\beta$ is close to one and populism vanishes. To understand this, recall that voters receive the signal $x_1^* + \varepsilon_1$ before making their election decision. They use this signal to judge an incumbent politician’s competence. If $\sigma^2$ is very low, voters observe the ex ante optimal policy level, i.e. $x_1^*$, almost perfectly, and they know it. As a result of getting a very precise signal, the median voter’s prior $\mu^m_1$ has very little influence on his posterior belief about $x_1^*$. A politician’s aim is to be judged competent through the lens of the median voter’s posterior belief. If this posterior belief depends only very little on the prior $\mu^m_1$, a politician’s incentive to pander to the median voter’s prior belief is low. As a result, the policy is almost exclusively determined by the politician’s signal that he uses to predict the median voter’s posterior belief.

In the opposite case, where $\sigma^2$ is large, $\beta$ is low. Thus, voters’ beliefs are highly persistent and $\mu^m_i$ has a high weight in influencing policy. A high $\sigma^2$ has the same effect as ideological beliefs about $x_1^*$, which voters may be motivated to keep intact (Bénabou, 2008).

The result that a higher $k$ leads to more populism seems rather surprising at first. Its logic is best understood from a formal point of view. Consider the limit case where $k$ is infinite. Then, it is in fact a logical impossibility that $g_1$ can depend on $\xi_1$ in equilibrium. To see this, suppose that $g_1$ would indeed depend on $\xi_1$. Then voters would be aware of this and would also understand that $\xi_1 = x_1^*$ in the case of the competent politician. However, they do not observe $x_1^*$ and voter $i$ substitutes $\hat{\mu}_1^*$ for $x_1^*$. Thus, a politician who wants to appear competent to the median voter will not actually want to let his policy depend on $\xi_1$ but rather on $E[\hat{\mu}_1^m | \xi_1] = (1 - \beta) \mu^m_1 + \beta \xi_1$ (see Lemma 1 and Assumption 3). Here $\xi_1$ enters only with a weight $\beta$, which lies between zero and one. But now $g_1$ would still depend on $\xi_1$, hence the same argument can be repeated and we would find that $g_1$ could in fact only depend on $(1 - \beta^2) \mu^m_1 + \beta^2 \xi_1$. This argument can be iterated an infinite number of time. Since $0 < \beta < 1$, $\xi_1$ must necessarily vanish and $g_1$ cannot
depend on $\xi_1$. If $k$ is finite, then this argument can be repeated only a finite number of times that increases with $k$. With each iteration, $g_1$ depends less on $\xi_1$ and more on $\mu_1^m$.

We conclude this section by summarizing the equilibrium outcome in the second period. The result follows directly from the discussion at the beginning of Section 3.3.

**Proposition 2 (Equilibrium Second Period)** In the second office period, we have $g_2 = x_2^*$ in case of a competent politician. Furthermore, $g_2 = x_2^* + \zeta_1$ in case of a re-elected incompetent politician and $g_2 = x_2^* + \zeta_2$ in case of an incompetent challenger.

## 4 The Costs and Benefits of Populist Policies

Using Proposition 1 and 2, it is straightforward to characterize welfare under indirect democracy. We do so by using the concept of a loss function defined as $L_t = EV_{t}^{FB} - EV_{t}^{EQ}$ for period $t$. $L_t$ is defined as the difference between expected utility as achieved when $g_t$ is set to its ex ante welfare-maximizing level $x_t^*$ and expected utility as achieved in the equilibrium of the political game. The first-best utility value ex ante results if $g_t = x_t^*$ (see (2)), which leads to $EV_t^{FB} = -\sigma_\xi^2$. We obtain:

**Proposition 3 (Welfare Indirect Democracy)** Under indirect democracy, welfare is characterized by

$$L_{1D} = (1 - \beta^{k+1})^2 (x_1^* - \mu_1^m)^2 + \beta^{2(k+1)} (1 - \alpha) \sigma_\xi^2,$$

$$L_{2D} = [1 - \alpha - \Delta_\alpha] \sigma_\xi^2,$$

where $\Delta_\alpha \geq 0$.

Consider the first period. The welfare loss from indirect democracy is equal to a weighted average of the distortion associated with the median voter’s belief and the variance of the incompetent politician’s bias. The first term arises from pandering. The second term arises from
the fact that no equilibrium for finite $k$ entails full pandering. Politicians will always partially base their policy choice upon their signal $\xi_1$. This follows from the fact that voters observe $x^*_1 + \varepsilon_1$ before they make their reelection decision. Anticipating this induces the incumbent politician to not fully ignore his signal. Since the signal of the incompetent politician is noisy, the fact that $g_1$ depends on this signal increases the variance of $g_1$. This comes at a cost to risk averse voters. Note that the weights $(1 - \beta^{k+1})^2$ and $\beta^{2(k+1)}$ do not add to one. We will come back to this in the next section.

It follows from Proposition 3 that there are both costs and benefits to populist policies. The costs relate to the fact that the competent politician partially ignores his signal. Since the signal reveals the ex ante optimal policy level, this means wasting useful information. On the other hand, populism also leads the incompetent politician to partially ignore his signal. This comes at a benefit to risk averse voters and prevents policy making from being too erratic. The key insight from this result is that populism can have beneficial consequences in a world where both voters and policy makers are imperfectly informed. This is a crucial insight for judging the benefits of indirect democracy.

In the second period, populism does not arise since no politician has an incentive to manipulate voters’ perception of his competence. As a result, only the noise term $\sigma^2_{\zeta}$ contributes to the welfare loss. It can be checked that $\Delta_\alpha \geq 0$ follows from the fact that the probability that a competent politician holds office in the second period exceeds $\alpha$ (see the proof of Proposition 3). Thus, the probability that a competent politician holds office in the second period is higher than that a competent politician holds office in the first period. This means that an election leads to a screening of competent politicians, even though voters are imperfectly informed about the optimal policy. The reason is that a competent politician is better at predicting the median voter’s posterior belief about the optimal policy. As a result, the coefficient of $\sigma^2_{\zeta}$ in (11) is smaller than $1 - \alpha$, which appears in the coefficient of $\sigma^2_{\zeta}$ in (10).\footnote{Of course, it must be the case that $1 - \alpha - \Delta_\alpha \geq 0$. This requires, in particular, that $\Delta_\alpha = 0$ if $\alpha = 1$. It can be inferred from the proof of Proposition 3 that this is indeed the case.}

\textsuperscript{12}
5 Comparing Constitutional Regimes

Now we turn to the question, which constitutional regime is optimal if voters are imperfectly informed about the world and politicians and other agents may be so, too. We analyze this question according to the degree of delegation of decision making from voters to their agents in three constitutional regimes. Decision making can either be delegated to completely independent agents such as experts, or to politicians who want to get reelected and, thus, are only partially independent (indirect democracy), or it may not be delegated at all (direct democracy). The question we address now is which of these regimes leads to higher welfare under what circumstances?

We first consider direct democracy. We follow Maskin and Tirole (2004) by modeling direct democracy as a political institution where $g_t = \mu_t^m$, i.e. it is the median voter who directly chooses $g_t$. The idea is that in a direct democracy voters have the right to ask for referenda and that this would lead to a strong link between policy making and the beliefs of the median voter.

In this simple benchmark model of direct democracy the median voter is the only relevant actor and there are no strategic elements involved. The following proposition follows directly from inserting $g_t$ into (2) and taking expectations.

Proposition 4 (Welfare under Direct Democracy) Under direct democracy, $L_{DD}^t = (x_t^* - \mu_t^m)^2$. The loss function is again defined as the deviation of expected utility from its first-best level.

Before we compare this to the case of indirect democracy, we introduce the third institution that we consider here: delegation of policy making to independent agents. In the following, we will dub these agents experts, as this reflects more accurately what we have in mind.

In order to facilitate the comparison to indirect democracy, our assumptions about experts parallel our assumptions about politicians. In particular, we also assume that there are two types

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13 Relating to Maskin and Tirole (2004), the higher the degree of delegation of decision making the lower the accountability of decision makers in a constitutional regime.

14 In New Zealand, Switzerland, and some U.S. states, a referendum can be initiated by voters by means of a citizen petition.
of experts, competent and incompetent. Exactly as in the case of politicians, we assume that the experts receive a signal, denoted $\xi_e^t$, about $x^*_t$. For the competent expert we have $\xi_e^t = x^*_t$ while, for the incompetent expert, we have $\xi_e^t = x^*_t + \nu_t$. The random variable $\nu_t$ reflects a noise term with an expected value of zero and a variance $\sigma^2_{\nu}$. The probability that an expert is competent is $\pi$. By definition, experts as non-accountable agents conduct policy for both periods and cannot be fired after the first period.

The term “incompetent expert” may sound rather odd at first. What we have in mind is that if experts disagree, at most one expert opinion can be right. Thus, experts may be wrong even if they are highly trained. Combating crime provides one salient example where experts disagree substantially (see Levitt, 1998, and Buscaglia, 2008), climate change provides another one (see McKibbin and Wilcoxen, 2002; Weitzman, 2007; Stern 2008).

In one important aspect, our assumptions about experts deviate from the assumptions made about politicians. We assume that experts are fully benevolent. Thus, they set $g_t = \xi_e^t$. We make this assumption since the case of benevolent experts is often considered as an ideal, if unfeasible, benchmark for government. Here we are interested in the question under which conditions this ideal benchmark would actually be desirable in a world of imperfect knowledge.

The welfare loss under the expert system is given in the following proposition. The proof is very similar to the one of Proposition 3 and is omitted.

Proposition 5 (Welfare Experts) In the case of non-accountable experts, $L_{t}^{EXP} = (1 - \pi) \sigma^2_{\nu}$.

We are now in a position to compare the three political institutions. We start the discussion by noting that, concerning the first period, indirect democracy can be understood as a mix of direct democracy and governance by non-accountable experts. To see this, denote $\xi_p^t$ the signal of a politician and assume that $\xi_e^t = \xi_p^t \equiv \xi$. Furthermore, assume that the likelihood that an expert or a politician is incompetent is equal, i.e. $\alpha = \pi$. Then $g_t^{EXP} = \xi$, where $EXP$ stands for experts. Furthermore, $g_t^{DD} = \mu^m_t$, where $DD$ refers to direct democracy. From Proposition 1 it follows that $g_t^{ID} = (1 - \beta^{k+1}) g_t^{DD} + \beta^{k+1} g_t^{EXP}$, where $ID$ refers to indirect democracy.
This weighted-average nature of indirect democracy makes it attractive to risk averse voters in the sense that \( L_1 (g^1_{ID}) < (1 - \beta^{k+1}) L_1 (g^1_{DD}) + \beta^{k+1} L_1 (g^1_{EXP}) \). This follows from the fact that \( L_1 \) is strictly convex. The fact that the loss associated with \( g^1_{ID} \) is lower than a weighted average of the losses associated with either \( g^1_{DD} \) or \( g^1_{EXP} \) is also the reason why the weights associated with the two terms in \( L^1_{ID} \) in Proposition 3, namely \((1 - \beta^{k+1})^2 \) and \( \beta^{2(k+1)} \), add to less than one for finite \( k \). We summarize this finding in a corollary.

**Corollary 1 (Comparative Advantage of Indirect Democracy)** Suppose that \( \alpha = \pi \) and \( \xi^e_t = \xi^p_t \). Then \( g^1_{ID} = (1 - \beta^{k+1}) g^1_{DD} + \beta^{k+1} g^1_{EXP} \) and \( L_1 (g^1_{ID}) < (1 - \beta^{k+1}) L_1 (g^1_{DD}) + \beta^{k+1} L_1 (g^1_{EXP}) \).

Comparing the loss functions in Propositions 3, 4, and 5, the elements that crucially affect which constitutional regime is optimal are: The distortion associated with the median voter’s belief, i.e. \(|\mu^m_t - x^*_t|\); the variance of the incompetent politician’s bias, i.e. \( \sigma^2_\zeta \); and the corresponding variance of the incompetent expert’s bias, i.e. \( \sigma^2_\nu \). We discuss the implications of our analysis in the next section. In the following corollary, we simply point out the comparative statics.

**Corollary 2 (Constitutional Comparison)** (i): Non-accountable experts are optimal if \( \sigma^2_\nu \) is small relative to \(|\mu^m_t - x^*_t|\) and \( \sigma^2_\zeta \). (ii): Direct democracy is optimal if \(|\mu^m_t - x^*_t|\) is small relative to \( \sigma^2_\zeta \) and \( \sigma^2_\nu \). (iii): In case that neither of these conditions applies, the weighted-average nature of indirect democracy may often make it optimal.

### 6 Discussion

We first provide arguments why the variance of the noise of experts’ signal \( \sigma^2_\nu \) may not be small in many cases. This leads us to the conclusion that delegation to experts may not be optimal in many domains. We proceed by discussing the relative merits of direct and indirect democracy. Our conclusion is that the latter may be optimal in many instances.
The noise of experts’ signal $\sigma^2_\nu$ reflects the degree to which experts can draw on reliable knowledge that has been derived from a large amount of high-quality data. An example of a piece of knowledge associated with a very low $\sigma^2_\nu$ would be “HIV causes AIDS.”

The desirability of many policies depends on behavioral reactions to certain interventions, e.g. the reactions of criminal activities to more severe punishment, or the reactions of labor supply to a five-percent increase in the income tax. Unfortunately, there are few examples where social scientists can draw upon high quality data in order to pin down the consequences to a policy intervention with a high degree of precision. This is a consequence of the fact that it is rarely possible to conduct large-scale randomized experiments that would allow for collecting high quality data.\(^\text{15}\) Second, many policy interventions have a unique element in that they are carried out for the first time or for the first time under certain circumstances.

In contrast, $\sigma^2_\nu$ is low and hence delegation to experts desirable in domains where choosing the right policy is simply a matter of applying technical knowledge. Here, we come to the same conclusions as Maskin and Tirole (2004), and Alesina and Tabellini (2007, 2008). As an example, we mention the calculation of an annuity conversion factor for a public pension system as a function of life expectancies. Central banking may be another important case.

Overall, $\sigma^2_\nu$ is unlikely to be small in many important cases. In line with this observation, there are many examples where experts widely disagree (e.g. combating crime or climate change; see references above). In these cases, it is conceivable that the median voter’s prior belief lies somewhere in between the “signals” of various experts.

Now we turn to the comparison of direct and indirect democracy. Our analysis implies that direct democracy is desirable if the distortion of the median voter’s prior $|\mu^{m}_t - x^*_t|$ is small. This term may indeed be small in some circumstances and large in others. It is expected to be small when judging the effect of a policy does not so much require formal knowledge but rather a good “feeling” about what would be an appropriate policy action and what the likely behavioral reactions are. Second, in some cases, voters may actually have more information\(^\text{15}\) See Banerjee and Duflo (2008) for a discussion.
than politicians (and experts). An example for the former case may be a judgment about how much free choice of savings and investment strategies there should be in a pension system. Ordinary voters may be better able to judge how easy or difficult it may be for them to make such choices themselves than politicians (and experts). An example for the latter case may be combating crime in local neighborhoods where residents of such neighborhoods may have superior information. In both cases, $|\mu_t^m - x^*_t|$ may be small relative to $\sigma^2_\zeta$ and $\sigma^2_\nu$.

In other instances, judging the desirability of a policy may require more formal knowledge. An important example may be globalization, although even here ordinary workers may have some additional information about how they are affected by globalization in the short-run that is not readily available to politicians and experts. To the degree that formal knowledge is important for judging policies, politicians have greater incentives to acquire this knowledge than voters (Maskin and Tirole, 2004). The reason is that the latter may anticipate that they are not pivotal and thus not invest in the acquisition of this formal knowledge. This makes the case for delegating policy making to politicians. On the other hand, there is the danger that politicians are incompetent or may lack important informal information as discussed above. This is reflected in the parameter $\sigma^2_\zeta$. This is where the positive effects of “populism” come into play: politicians are induced not to rely exclusively on their signal but to incorporate voters’ beliefs.

Our main positive result in this paper is that policies under indirect democracy are determined as a weighted average of voters’ priors and politicians’ signals. This weighted average nature makes indirect democracy a very balanced and well-diversified institution, which is desirable from a normative point of view. Our analysis may help to explain why it is so prevalent around the world.

7 Conclusion

In this paper, we have analyzed policy making under indirect democracy. The key innovation of our setup is that we take into account that both voters and politicians may be imperfectly
informed about what policies are desirable. Our main positive result is that policies under indirect democracy are a weighted average of voters’ prior beliefs and politicians’ signals. This implies our main normative result that, due to its balanced nature, indirect democracy should often be expected to be preferable to either direct democracy or delegation to non-accountable agents.

These results crucially depend on the two effects of populism, a generic characteristic of indirect democracy. Just as suggested by the entry quote of Winston Churchill, populism comes at a cost because the reelection goal of competent politicians lets them partly neglect useful information about optimal policies. In contrast, the same objective restrains incompetent politicians from implementing their erratic ideas one by one. This good side of populism has not been identified in the literature before and is due to our assumption that politicians (and even experts) can err, too.

Although our findings are intuitive, their derivation requires a careful consideration of voters’ beliefs about how politicians make use of their information when they choose a policy. Working out the role of these beliefs is another main contribution of our analysis.

There is a range of issues to be addressed in future research. First, one may generalize the model to an indefinite time horizon, where each politician may serve in office for two or more consecutive periods. Second, it would be interesting to consider the case where a policy affects welfare of different subgroups of the population in different ways. Third, it would also be important to explore which forms of indirect democracy may be most desirable, either presidential or parliamentary, majoritarian or proportional. Fourth, it would be interesting to consider the role of the media in shaping voters’ beliefs. Finally, the setup of this paper may also apply to decision making in corporations.
Proof of Lemma 1

Voters observe $x^*_i + \varepsilon_1$. From the point of view of voter $i$, $x^*_i + \varepsilon_1$ is a realization of the random variable $x^*_i + \varepsilon_1$. The voter aims to update his belief about $x^*_i$. The random variables $x^*_i$ and $x^*_i + \varepsilon_1$ are jointly normally distributed with $E[x^*_i] = \mu^*_i$, $Var[x^*_i] = \sigma^2_x$, $E[x^*_i + \varepsilon_1] = \mu^*_i$, $Var[x^*_i + \varepsilon_1] = \sigma^2_x + \sigma^2_\varepsilon$. Furthermore, $Cov[x^*_i, x^*_i + \varepsilon_1] = \sigma^2_x$. Inserting this in the formulas for conditional expectations and variances for jointly-normal random variables (see e.g. Hogg and Craig, 1995, p. 148) yields the result.

Proof of Lemma 3

Proof of (i). We omit the time subscript as well as the superscript $i$ when there is no danger of confusion. Let, for clarity, $\xi$ denote the random variable whose realization is $\hat{\xi}$. The idea of the proof is to derive the posterior probability $\hat{\alpha}$ for the case that the random variable $\xi$ falls into the (small) interval $I_\delta(\hat{\xi}) := [\hat{\xi} - \delta, \hat{\xi} + \delta]$ and to consider the limit $\delta \to 0$.

Denote by $C$ the event that a politician is competent and by $IC$ the complementary event. Using the definition of conditional probabilities, it follows that

$$P(C|\xi \in I_\delta) = \frac{\alpha P(\xi \in I_\delta|C)}{\alpha P(\xi \in I_\delta|C) + (1 - \alpha) P(\xi \in I_\delta|IC)}.$$  \hfill (A.1)

Note that the denominator is equal to $P(\xi \in I_\delta)$. In order to consider the limit of (A.1) for the case where $\delta \to 0$, it is useful to rewrite it as

$$P(C|\xi \in I_\delta) = \left[1 + \frac{1 - \alpha \int_{\hat{\xi} - \delta}^{\hat{\xi} + \delta} f_C(\xi) d\xi}{\int_{\hat{\xi} - \delta}^{\hat{\xi} + \delta} f_C(\xi) d\xi}\right]^{-1}.$$  \hfill (A.2)
where the two normal densities $f_c$ and $f_{ic}$ are defined as in the main text preceding Lemma 3. Since normal densities are well-behaved, it follows from standard arguments using the definition of the Riemann integral that $\lim_{\delta \to 0} \frac{\int_{\xi - \delta}^{\xi + \delta} f_c(\xi)d\xi}{\int_{\xi - \delta}^{\xi + \delta} f_c(\xi)d\xi} = f_{ic}(\xi)$. Substituting this into (A.2) for $\delta \to 0$ and rearranging yields (3).

Proof of (ii). We consider the case that $\hat{\xi} - \hat{\mu}_1 \geq 0$. (Similar arguments apply to the symmetric case $\hat{\xi} - \hat{\mu}_1 < 0$.) Write $\hat{\alpha} = \left[1 + \frac{1}{\alpha} \frac{f_c(\hat{\xi})}{f_c(\hat{\xi})}\right]^{-1}$. The posterior $\hat{\mu}_1$ is taken as given here. Therefore, it is sufficient to show that $\frac{f_c(\hat{\xi})}{f_c(\hat{\xi})}$ increases with $\hat{\xi}$. $f_c$ is the normal density describing the distribution of $\xi_c \equiv \hat{x}_1$, while $f_{ic}$ is the normal density associated with $\xi_{ic} \equiv \hat{x}_1 + \hat{\zeta}_1$. Using the formula for the normal density, we have $f_c(\xi) = \frac{1}{\sqrt{2\pi \text{Var}(\xi_c)}} \exp\left[-\frac{(\xi - E\xi_c)^2}{2\text{Var}(\xi_c)}\right]$ and $f_{ic}(\xi) = \frac{1}{\sqrt{2\pi \text{Var}(\xi_{ic})}} \exp\left[-\frac{(\xi - E\xi_{ic})^2}{2\text{Var}(\xi_{ic})}\right]$. It then follows that

$$d \left[ \frac{f_{ic}(\xi)}{f_c(\xi)} \right] /d\xi = \frac{\sqrt{\text{Var}(\xi_{ic})}}{\sqrt{\text{Var}(\xi_c)}} \left[ \xi - E\xi_{ic} / \text{Var}(\xi_{ic}) - \xi - E\xi_c / \text{Var}(\xi_c) \right] \exp\left[\frac{(\xi - E\xi_c)^2}{2\text{Var}(\xi_c)} - \frac{(\xi - E\xi_{ic})^2}{2\text{Var}(\xi_{ic})}\right].$$

We are interested in the case that $\xi = \hat{\xi} \geq \hat{\mu}_1$. Lemma 2 implies then that $\hat{\xi} \geq E\xi_{ic} \geq E\xi_c = \hat{\mu}_1$ (note that $E\xi_{ic} = (1 - \gamma) \hat{\mu}_1 + \gamma \hat{\xi}_1$). Hence, $\hat{\xi} - E\xi_c \geq \hat{\xi} - E\xi_{ic}$. Furthermore, $\text{Var}(\xi_{ic}) > \text{Var}(\xi_c)$. Thus $\left[ \frac{\xi - E\xi_{ic}}{\text{Var}(\xi_{ic})} - \frac{\xi - E\xi_c}{\text{Var}(\xi_c)} \right] > 0$ and hence $d \left[ \frac{f_{ic}(\xi)}{f_c(\xi)} \right] /d\xi > 0$.

The case where $\hat{\xi} < \hat{\mu}_1$ is symmetric and analyzed by following the same steps.

**Proof of Proposition 1**

Proof of part (i). This follows directly from Lemma 5 and 6.

Proof of part (ii). Clearly, part (i) implies that there is a unique limit for $k \to \infty$. This limit is indeed an equilibrium for appropriate off-equilibrium beliefs about a politician’s type if he deviates from setting $g_1 = \mu_1^m$. For instance, consider the belief, identical across voters, that a politician setting $g_1 \neq \mu_1^m$ is incompetent with probability one and that $E\hat{\zeta}_1$ is sufficiently high. Given this off-equilibrium belief, setting $g_1 = \mu_1^m$ maximizes a politician’s probability of getting reelected from Lemma 4. Thus, a politician does indeed not want to deviate from $g_1 = \mu_1^m$. Furthermore, voters do not have any incentive deviate from (6).
Proof of Proposition 3

Denote by \( g_{t,c} \) the level of \( g \) set in period \( t \) by the competent politician and let \( g_{t,ic} \) refer to the incompetent politician. Let \( \lambda_t \) denote the probability that a politician is competent in period \( t \). Then

\[
EV_t = - \left[ \lambda_t E \left[ \left( g_{t,c} - x^*_t - \varepsilon_t \right)^2 \right] \right] + (1 - \lambda_t) E \left[ \left( g_{t,ic} - x^*_t - \varepsilon_t \right)^2 \right].
\]  

(A.3)

Consider the first period. Clearly, \( \lambda_1 = \alpha \). Using this an inserting for \( g_1, c \), \( g_1, ic \) from Proposition 1 into (A.3), we obtain

\[
EV_1^{EQ} = - \left( 1 - \beta^{k+1} \right)^2 \left( \mu_1^m - x_1^* \right)^2 - (1 - \alpha) \beta^{2(k+1)} \sigma_\zeta^2 - \sigma_\varepsilon^2.
\]

\( EV_1 \) is maximized for \( g_1 = x_1^* \) which yields \( EV_1^{FB} = -\sigma_\varepsilon^2 \). Inserting this and the above expression into the definition of \( L_t \) yields (10).

We turn next to the second period. We show first that it is more likely that a competent politician gets reelected than that an incompetent politician gets reelected. To establish this, we show that the probability that (6) holds for the median voter is lower for an incompetent incumbent than for a competent incumbent. We first prove this for finite \( k \). By Lemma 3, \( \hat{\alpha}_m \) is a strictly decreasing function of \( |\hat{\zeta}_1 - \hat{\mu}_1^m| \). By Lemma 6, the median voter’s belief is that \( g_1 = G_k(\xi_1) = (1 - \beta^k) \mu_1^m + \beta^k \xi_1 \). Hence, \( \hat{\zeta}_1 = \frac{g_1}{\beta} - \frac{1 - \beta^k}{\beta} \mu_1^m \). By Proposition 1, \( g_1 = (1 - \beta^{k+1}) \mu_1^m + \beta^{k+1} \xi_1 \). In the case of the competent politician, \( \xi_1 = x_1^* \). Inserting this into the expression for \( g_1 \), then inserting \( g_1 \) into the expression for \( \hat{\zeta}_1 \) and using Lemma 1 yields that \( \hat{\zeta}_1 - \hat{\mu}_1^m = -\beta \varepsilon_1 \equiv \phi_c \). Similarly, it follows for the incompetent politician that \( \hat{\zeta}_1 - \hat{\mu}_1^m = \beta (\zeta_1 - \varepsilon_1) \equiv \phi_{ic} \). The variance of \( \phi_{ic} \) is equal to \( \beta^2 (\sigma_\zeta^2 + \sigma_\varepsilon^2) \), whereas the variance of \( \phi_c \) is equal to \( \beta^2 \sigma_\zeta^2 \) and thus strictly smaller than the variance of \( \phi_{ic} \). It follows that the probability that \( |\hat{\zeta}_1 - \hat{\mu}_1^m| \geq A \), for any \( A \in (0, \infty) \), is strictly greater for the incompetent than for the competent politician. Hence, the probability that \( \hat{\alpha}_m \leq B \), for any \( B \in (0, 1) \), is strictly greater for the incompetent than for the competent politician.

From Lemma 2, \( E\hat{\zeta}_1^m = \gamma \left( \hat{\zeta}_1 - \hat{\mu}_1^m \right) \). Thus, the above arguments also imply that the prob-
ability that \( (E \hat{\xi}_1^m)^2 \geq C \), for any \( C \in (0, \infty) \), is strictly greater for the incompetent incumbent than for the competent incumbent. Since \( \sigma^2_\xi \) does not differ across types, this establishes that the probability that (6) holds for the median voter is strictly smaller in case of an incompetent incumbent than in case of a competent incumbent for finite \( k \).

If \( k \) is infinite, \( \hat{\xi}_1 \) cannot be inferred and no information about the incumbent’s type is observed since we have a pooling equilibrium. Hence, the probability of getting reelected must be equal for both types. Overall, we have shown that it is more likely that a competent politician gets reelected than that an incompetent incumbent gets reelected.

There are three events in which the politician in the second period is competent: (1) A competent incumbent gets reelected; (2) a competent incumbent gets ousted and replaced by a competent politician; (3) an incompetent incumbent gets ousted and replaced by a competent politician. Denote the probability that a competent politician gets reelected by \( \rho_c \) and the probability that an incompetent politician gets reelected as \( \rho_{ic} \). Denote the event that the second period politician is competent by \( C_2 \). We have then

\[
Pr [C_2] = \alpha \rho_c + \alpha^2 (1 - \rho_c) + \alpha (1 - \alpha) (1 - \rho_{ic})
\]

\[
= \alpha [1 + (1 - \alpha)(\rho_c - \rho_{ic})] \geq \alpha. \tag{A.4}
\]

The last inequality follows from the fact that it is more likely that a competent politician gets reelected than that an incompetent politician gets reelected, as shown above. Overall, we have now established that \( \lambda_2 = \alpha + \Delta_\alpha \) for some \( \Delta_\alpha \geq 0 \) (see (A.3)).

Inserting the expressions for \( g_{2,c}, g_{2,ic} \) given in Proposition 2 into (A.3) yields

\[
EV_2 = - (1 - \alpha - \Delta_\alpha) \sigma^2_\xi - \sigma^2_\varepsilon.
\]

Again, \( EV_{2 FB} = -\sigma^2_\varepsilon \). Inserting this and the above expression into the definition of \( L_t \) yields (11).
B  Sufficient Conditions for the Median Voter Theorem to Apply

A voter \( i \) is pivotal in our model if and only if the following two conditions hold: (i) If the pivotal voter casts his ballot for the incumbent then at least half of voters prefer the incumbent; (ii) if the the pivotal voter casts his ballot for the challenger, then at least 50 percent of voters prefer the challenger. If the two conditions hold for the \( i \) associated with the median of \( \mu_1 \) (i.e. \( \mu_1^m \)), we say that the median voter theorem applies.

We first show that a voter \( i \) reelects the incumbent if and only if \( \hat{\xi}_1 \) falls within a finite interval. The reelection decision is determined by (6). The posterior probability \( \hat{\alpha}_i \) is a strictly decreasing function of \( |\hat{\xi}_1 - \hat{\mu}_1| \) as stated in Lemma 3(ii). Furthermore, \( (E\hat{\zeta}_1)^2 \) is a strictly increasing function of \( |\hat{\xi}_1 - \hat{\mu}_1| \) from Lemma 2. Since \( \hat{\alpha}_i \) enters negatively on the left-hand side of (6) while \( (E\hat{\zeta}_1)^2 \) enters positively, it follows that (6) holds if and only if \( |\hat{\xi}_1 - \hat{\mu}_1| \) is sufficiently small. Using Lemma 1, we can state that voter \( i \) reelects the incumbent if and only if

\[
\hat{\xi}_1 \in [(1 - \beta) \mu_1^i + \beta (x_1^* + \varepsilon_1) \pm \delta_{\text{crit}}] \equiv I_{re}^i, \tag{B.1}
\]

where \( \delta_{\text{crit}} \) is a strictly positive real number that is common across voters and is a function of \( \sigma_x^2, \sigma_x^2, \sigma_{\varepsilon}^2 \). We dub \( I_{re}^i \) voter \( i \)'s reelection interval.

The median voter theorem applies if (i) \( \hat{\xi}_1 \in I_{re}^m \) implies that \( \hat{\xi}_1 \in I_{re}^i \) for at least half of voters and, conversely, (ii) \( \hat{\xi}_1 \notin I_{re}^m \) implies that \( \hat{\xi}_1 \notin I_{re}^i \) for at least half of voters. The following lemma states two sufficient conditions for this to hold.

**Lemma 7 (Median Voter)** The median voter is pivotal if either of the following conditions are fulfilled:

(i) \( \mu_1^i = \mu_1^m \) for at least half of voters;

(ii) \( \mu_1^{\text{max}} - \mu_1^{\text{min}} \leq 2\delta_{\text{crit}} / (1 - \beta) \)

where \( \mu_1^{\text{min}} \equiv \min \{\mu_1^i\} \) and \( \mu_1^{\text{max}} \equiv \max \{\mu_1^i\} \).
Proof. Proof of (i). If $\mu_i^m = \mu_i^m$ for at least half of voters then it follows immediately that $\hat{\xi}_1 \in I_{re}^m$ implies that $\hat{\xi}_1 \in I_{re}^i$ for at least half of voters. Conversely, $\hat{\xi}_1 \not\in I_{re}^m$ implies that $\hat{\xi}_1 \not\in I_{re}^i$ for at least half of voters.

Proof of (ii). Using Lemma 1 and (B.1), it can be checked that the condition $\mu_i^{\text{max}} - \mu_i^{\text{min}} \leq 2\delta_{\text{crit}} / (1 - \beta)$ is equivalent to $\max I_{re}^{\text{min}} \geq \min I_{re}^{\text{max}}$, where the superscripts $\text{min}$ and $\text{max}$ refer to the $i$ with the lowest and highest $\mu_i^1$, respectively. If this holds then $\hat{\xi}_1 \in I_{re}^m$ implies that either $\hat{\xi}_1 \in I_{re}^{\text{max}}$ or $\hat{\xi}_1 \in I_{re}^{\text{min}}$. The reason is that every point in $I_{re}^m$ is contained in $I_{re}^{\text{min}}$ or $I_{re}^{\text{max}}$ since $I_{re}^m$ is situated between the latter two and the latter two overlap in at least one point. Furthermore, whenever $g \in I_{re}^m$ and $g \in I_{re}^{\text{min}}$, then also $g \in I_{re}^i$ for all $i$ with $\mu_i^{\text{min}} \leq \mu_i^1 \leq \mu_i^m$. Similarly, whenever $g \in I_{re}^m$ and $g \in I_{re}^{\text{max}}$, then also $g \in I_{re}^i$ for all $i$ with $\mu_i^{\text{min}} \leq \mu_i^1 \leq \mu_i^{\text{max}}$. Hence, in either case, if $\hat{\xi}_1 \in I_{re}^m$ then $\hat{\xi}_1$ belongs to the reelection interval of at least half of voters.

Consider now the case that $\hat{\xi}_1 \not\in I_{re}^m$. It follows that either $\hat{\xi}_1 \not\in I_{re}^i$ for all $i$ with $\mu_i^{\text{min}} \leq \mu_i^1 \leq \mu_i^m$ or for all $i$ with $\mu_i^{\text{min}} \leq \mu_i^1 \leq \mu_i^{\text{max}}$. In either case, $\hat{\xi}_1 \not\in I_{re}^i$ holds for at least 50 percent of voters. Overall, this establishes that $m$ is pivotal.

Obviously, according to part (i), the voter associated with $\mu_i^m$ is pivotal if a majority of voters share his belief. The second condition in Lemma 7 limits the range of $\mu_i^1$. In particular, it requires, that the upper limit of the reelection interval for the voter with the lowest $\mu_i^1$ is at least as great as the lower limit of the reelection interval for the voter with the highest $\mu_i^1$. It is possible to derive further sufficient conditions for the median voter theorem to apply.

C Indirect Democracy for Biased $\zeta_1$

As mentioned in the main text, politicians may be understood as a representative sample of the general population if the political selection process is not biased in favor of the elite or any other particular group. If politicians are representative for the general population, then we would expect that the incompetent politician’s signal is related to the distribution of voters’
beliefs. (For the competent politician, this does not apply, since he observes the truth.) A simple way of capturing this is assuming that $E\zeta_t$ is related to $\mu_t^m - x_t^*.$

In the following, we consider the limit case where, from an objective point of view, $E\zeta_t = \mu_t^m - x_t^*.$ By objective we mean from the point of view of the economic theorist analyzing the problem. In contrast, we need to assume that a politician believes that $E\zeta_t = 0$ for himself. Otherwise, he could make use of the information about $E\zeta_t$ to unbias his belief about $x_t^*.$ Second, we also assume that voters believe that $E\zeta_t = 0.$ More precisely, we assume here that a majority of voters hold beliefs that are identical to the beliefs of the median voter. In this case, it is indeed appropriate to assume that a majority of voters believe that $E\zeta_t = 0.$ Otherwise, their beliefs about $E\zeta_t$ would be inconsistent with their own beliefs about $x_t^*.$

In this case, all positive results in Section 3 continue to hold. However, the welfare expressions in Proposition 3 are modified. In particular, we obtain

\begin{align*}
L_1 &= \alpha (1 - \beta^{k+1})^2 + 1 - \alpha \right) (x_1^* - \mu_1^m)^2 + \beta^{2(k+1)}(1 - \alpha) \sigma_\zeta^2. \quad (C.1) \\
L_2 &= [1 - \alpha (1 + \Delta_\alpha)] \left[ (x_2^* - \mu_2^m)^2 + \sigma_\zeta^2 \right], \quad (C.2)
\end{align*}

As to be expected, the terms $(x_t^* - \mu_t^m)^2$ have a stronger influence on the welfare loss than in the baseline case. In particular, in the case of Proposition 3, the coefficient for $(x_1^* - \mu_1^m)^2$ is smaller than in the case of (C.1). The expression $(x_2^* - \mu_2^m)^2$ does not appear at all in Proposition 3. The overall conclusion is that if the incompetent politician’s signal is biased towards the beliefs that are prevalent among voters, indirect democracy becomes more similar to direct democracy as defined in Section 5.
D Indirect Democracy in the Case of Non-Degenerate Beliefs of Politicians

Our analysis is based on the assumption that politicians receive a signal $\xi_t$ which they use as a “point estimate” for $x^*_t$ in the sense of classical statistics. This introduces an asymmetry between voters and politicians since the former are Bayesian and hold non-degenerate prior beliefs $x^i_t$.

It is straightforward to turn a politician into a Bayesian in our framework by assuming that his prior belief about $x^*_t$ is given by $x^p_t \sim N(\xi_t, \sigma^2_x)$, where the superscript $p$ indexes a politician. This would affect the analysis insofar as an incumbent politician would be able to update the probability that he is competent. More important, an incumbent would partially learn about his bias $\zeta_1$ (under the hypothesis that he is incompetent). An incumbent politician’s updating would parallel Lemma 2 and 3.

If an incumbent politician learns about his bias $\zeta_1$, Assumption 2 implies that he can partially “unbias” his signal $\xi^*_2$ in period 2. This leads to a further incumbent advantage from the perspective of voters since this reduces the expected level of bias in the second period arising from an incompetent politician if this politician is a reelected incumbent. Since voters would take this into account, the reelection condition (6) would get somewhat more complicated. Loosely speaking, the fact that an incumbent can partially unbias his signal reduces the second term on the left hand side of (6). In spite of this modification, the logic of our main results remains entirely valid.
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