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Better Safe than Sorry?

Ex Ante and Ex Post Moral Hazard in Dynamic Insurance Data*

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Abstract

This paper empirically analyzes moral hazard in car insurance using a dynamic theory of an insuree's dynamic risk (ex ante moral hazard) and claim (ex post moral hazard) choices and Dutch longitudinal micro data. We use the theory to characterize the heterogeneous dynamic changes in incentives to avoid claims that are generated by the Dutch experience-rating scheme, and their effects on claim times and sizes under moral hazard. We develop tests that exploit these structural implications of moral hazard and experience rating. Unlike much of the earlier literature, we find evidence of moral hazard.

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1 Introduction

Four decades of theoretical research on asymmetric information have firmly established its importance for insurance relations and competitive insurance markets. The practical relevance of this research, and of the results on the efficiency of insurance markets and the design of optimal contracts that it has produced, depends critically on the empirical relevance of asymmetric information. A substantial and fast growing literature is now assessing this relevance for a variety of markets, using microeconometric methods and micro data on contracts and insurees. For some markets, notably car insurance, evidence is surprisingly mixed and muted, often pointing to a lack of asymmetric information problems. Much of the literature, however, uses static theory and cross-sectional data, which limits both its versatility in dealing with truly dynamic aspects of insurance markets, such as experience rating, and variation in the data that can be turned into robust empirical results. The empirical distinction between moral hazard and selection effects using static methods has turned out to be particularly hard; as argued by Abbring, Chiappori, Heckman, and Pinquet (2003), this is the standard econometric problem of distinguishing causal and selection effects.

This paper instead analyzes moral hazard in car insurance using panel data on contracts and claims provided by a Dutch insurance company. The analysis exploits some remarkable properties of the Dutch experience-rating system. Specifically, we theoretically analyze the Dutch scheme as a repeated contract between an insuree and an insurer, in which each period's interaction involves memory of the relationship's past history. Using control theory, we study the endogenous changes this structure induces in the incentives agents face at each point in time. Under moral hazard, these changes, in turn, generate specific patterns in the time profile of claim occurrences and sizes that we fully characterize; these patterns are specific in the sense that they would not appear under the null of no moral hazard. Finally, we develop structural econometric tests based on this theory and apply them to the Dutch micro data. The tests are flexibly parametric and nonparametric, and valid in the presence of unobserved heterogeneity of a general type. In contrast to much of the earlier literature, we find evidence of moral hazard in car insurance.

We also discuss the empirical distinction between ex ante and ex post moral hazard. Ex ante moral hazard entails that agents respond to changes in incentives by changing the risk of losses. Ex post moral hazard concerns the effects of incentives on claiming actual losses. The distinction between ex ante and ex post moral hazard is important because of their different welfare consequences (e.g. Chiappori, 2001). Because an insurer's administrative data typically only contain data on claims, and not on losses, distinguishing between ex ante and ex post moral hazard requires additional structural assumptions. Under a reasonable set of such assumptions, we find that at least some of the detected moral hazard is due to ex post moral hazard.

Our theoretical model specifies agent's optimal dynamic savings, loss prevention effort, and claim choices under the experience-rating (bonus-malus) scheme in Dutch car insurance. It produces predictions on the joint behavior of the claim occurrence, claim size, and experience-rating processes, for given individual risk and other characteristics. Ex ante moral hazard is captured by the endogenous loss prevention effort; ex post moral hazard by the endogenous claim choice. Endogenous savings allow for self-insurance.

The model provides a characterization of the dynamic heterogeneous incentives to avoid claims inherent to the Dutch experience-rating scheme, and their behavioral consequences under moral hazard. In particular, incentives are defined as the loss in expected discounted utility that would be incurred if a claim would be filed. We show how incentives vary with the current bonus-malus state and contract time, and jump with each claim because of its foreseeable effect on the future bonus-malus state. We present an algorithm for numerically characterizing these effects and provide a quantitative analysis of incentives. We restrict attention to computations under the null of no moral hazard. Because claim rates are constant under the null, these computations are relatively straightforward.

Our tests for moral hazard build on these theoretical computations. We first focus on the timing of claims. Under the null that there is no moral hazard, claim rates do not vary with incentives; under moral hazard, on the other hand, claim rates are lower when incentives are stronger. Our main test exploits the full model structure. It is a score (Lagrange multiplier) test for the dependence of claim rates on incentives in a version of the structural model that allows for flexible heterogeneity in risk. Because it only requires the computation of incentives under the null, it is easy to implement using our algorithm. We find strong evidence that claim rates decrease with incentives, and reject the null of no moral hazard at all conventional levels.

In addition to this structural parametric test for moral hazard, we also present and apply a range of nonparametric tests for state-dependence and contract-time effects on claim rates, controlling for risk heterogeneity. Our theory implies that any such effects must be due to moral hazard and, in this way, identifies the substantial problem of testing for moral hazard with the classical statistical problem of distinguishing state dependence and heterogeneity. This is a hard problem, but one that has been studied at length in statistics and econometrics (Bates and Neyman, 1952, Heckman and Borjas, 1980, Heckman, 1981). Our tests rely on this literature's key insight that, without contract-time and state dependence, the number of claims in a given period is a sufficient statistic for the unobserved heterogeneity in the conditional distribution of claim times in that period. Consequently, any signs of time or state dependence in subsamples with a given number of claims in a period are evidence of moral hazard. Moreover, an implication of the Dutch experience-rating system is that incentives may jump up or down at the time a claim is filed, depending on the current bonus-malus state. Therefore, we not only test for state dependence, but also for appropriate changes in its sign across bonus-malus states. Even though the nonparametric tests have relatively little power with the type of rare events found in insurance data (Abbring and Zavadil, 2008), they corroborate the results from the structural test.

Our theory also attributes a moral-hazard interpretation to state-dependence and contract-time effects on the sizes of claims. Under the assumption that ex ante moral hazard only affects the occurrence, but not the size, of insured losses, the latter are informative on ex post moral hazard. We complement this analysis of ex post moral hazard with data on claim withdrawals. Agents in our data set can withdraw a claim within six months and avoid malus. Under some assumptions, which we spell out in detail, claim withdrawals are observed manifestations of ex post moral hazard.

This paper contributes to a rich literature on asymmetric information in insurance markets. The seminal work on moral hazard and adverse selection by Arrow (1963), Pauly (1974, 1968), and Rothschild and Stiglitz (1976) showed that competitive insurance markets may be inefficient if information is asymmetric. A vast theoretical literature followed up on their key insights. Increasingly, attention has shifted from the development of theory to the empirical analysis of its relevance (see, e.g., Chiappori, 2001, Chiappori and Salanié, 2003, for reviews).[1] Chiappori (2001) forwarded the idea to exploit the rich variation that can be derived from dynamic theory and found in longitudinal data; Abbring, Chiappori, Heckman, and Pinquet (2003) suggested that we base a test for moral hazard on the dynamic variation in individual risk with the idiosyncratic variation in incentives due to experience rating.

The empirical papers most closely related to ours are Abbring, Chiappori, and Pinquet (2003), Dionne et al. (2006), and Pinquet et al. (2007). Our analysis differs from and extends these works in several ways. First, we precisely model the forward-looking behavior of an agent in the actual institutional environment characterizing the insurance market studied. We use this model to define and compute dynamic incentives and construct a structural test that exploits these computations in detail. Secondly, we explicitly

[1] Car-insurance data were studied, among others, by Dionne and Vanasse (1992), Puelz and Snow (1994), Dionne and Doherty (1994), Chiappori and Salanié (1997), Dionne, Gouriéroux, and Vanasse (1999), Richaudeau (1999), Chiappori and Salanié (2000), Dionne, Gouriéroux, and Vanasse (2001), Abbring, Chiappori, and Pinquet (2003), Cohen (2005), Dionne, Dahchour, and Michaud (2006), Chiappori, Jullien, Salanié, and Salanié (2006), and Pinquet, Dionne, Vanasse, and Maurice (2007). Health and life insurance data were analyzed by, for example, Holly, Gardiol, Domenighetti, and Bisig (1998), Chiappori, Durand, and Geoffard (1998), Cardon and Hendel (2001), Hendel and Lizzeri (2003), and Fang, Keane, and Silverman (2006). Finkelstein and Poterba (2002) studied annuities.

distinguish ex ante and ex post moral hazard, which requires a formal analysis of the claim filing behavior. Finally, we model both claim occurrences and claim sizes. Together, this allows us to confront a novel and precise set of dynamic implications for claim occurrences and sizes under moral hazard to longitudinal data.

The remainder of the paper is organized as follows. Section 2 briefly discusses the Dutch car-insurance market, with specific attention for the experience-rating scheme used. It also introduces the data. Section 3 develops the theory. We use the theory to analyze the dynamic incentives inherent to experience rating, and to derive the implications of moral hazard for claim rate and size dynamics. Section 4 develops an econometric framework for testing the effects of moral hazard from data on claim rates and sizes and presents the empirical results. Section 5 concludes. Appendices A and B provide proofs and computational details for Section 3. Appendix C provides additional information on the data.

2 Institutional Background and Data

2.1 Experience Rating in Dutch Car Insurance

In 2006, the 16.3 million inhabitants of the Netherlands were driving 7.2 million private cars.[2] Because liability insurance is mandatory in the Netherlands, this comes with a substantial demand for car insurance. In the same year, 74 insurance companies served this demand.[3] Even though these companies are supervised by the Dutch financial authorities, they are to great extent free to set their premiums and contractual conditions. In doing so, the Dutch insurance companies, united in the Dutch Association of Insurers, have to great extent coordinated their experience-rating systems in car insurance.

Before 1982, car insurers employed a limited experience-rating scheme. This scheme was commonly considered to be inadequate to price observed risk. In the early 1980s, six

[2] Source: Statistics Netherlands (www.cbs.nl).
[3] Source: Dutch Association of Insurers (www.verzekeraars.nl).

of the market's leading firms proposed a much finer *bonus-malus (BM) system*, based on a large actuarial study (de Wit et al., 1982).[4] Early 1982, this system was introduced in Dutch car insurance in a coordinated way. After some early market turbulence, the insurers by and large settled on similar bonus-malus schemes.

This paper uses data from one of the six companies that were leading the introduction of the bonus-malus system in Dutch car insurance. During the data period, January 1, 1995–December 31, 2000, this company used the bonus-malus scheme given in Table 1. The premium discount depends on the insuree's current *bonus-malus class*, which is determined at each annual contract renewal date. Twenty bonus-malus classes are distinguished, from 1 (highest premium) to 20 (lowest premium). Every new insuree starts in class 2 and pays the corresponding premium. We will refer to this premium as the *base premium*. After each claim-free year, an insuree advances one class, up to class 20. Each claim at fault sets an insuree back into a lower class. The worst class is 1, and implies a surcharge to the base premium. This scheme is representative for the bonus-malus schemes used in the Netherlands in this period.[5] Consequently, throughout this paper we assume that the drivers in our data set cannot escape Table 1's bonus-malus system by switching insurers.

The empirical analysis in this paper exploits that the incentives to avoid a claim jump with each claim filed, and vary with contract time and across bonus-malus classes. To gain some first insight in the differences in the "cost" of a claim to an insuree across different

[4] Information on the development of the BM system in Dutch car insurance is scattered throughout the professional literature. de Wit et al. (1982) provides information on the actuarial research underlying the bonus-malus system, and some very early history. Assurantiemagazine (2004) provides more recent historical reflection.

[5]The scheme is similar to the one originally proposed by de Wit et al. (1982), extended with multiple (maximum-bonus) levels that offer good customers some protection against premium increases. Evidence on the development of the bonus-malus system is sketchy— see Footnote 4— but strongly suggests that the sector actively coordinated on similar bonus-malus schemes in the course of the 1980s, before the start of our data period. Moreover, further major innovations to car insurance pricing were only introduced recently, after the end of our data period. We also compared Table 1's scheme to schemes *currently* offered by Dutch insurers and found only minor differences. The maximum discount on premium ranges from 70% to 80% and the maximum surcharge is in the range of 15% to 30%. Some insurance companies offer also collective insurance with more advantageous bonus-malus schemes.

bonus-malus classes and different numbers of claims, we have computed the change in the premium at the next renewal date with each claim in a contract year, for different bonus-malus classes. Table 2 gives the percentage premium change after a claim-free contract year, and the subsequent marginal percentage changes in the premium after each claim in the contract year. For example, after a claim-free year in class 8, an insuree will be upgraded to class 9 and pay 45% instead of 50% of the base premium. This amounts to a 10% reduction in the premium. If she files one claim in the contract year, she will instead be downgraded to class 4 and pay 80% of the base premium. This amounts to a $(80 - 45)/45 = 78\%$ increase relative to the premium that would be paid without the claim. A second claim would take her down further to class 1, and a premium equal to 120% of the base (a 50% increase relative to having one claim). A third claim would have no further effect on the premium.

Clearly, unlike the French scheme studied by Abbring, Chiappori, and Pinquet (2003), this scheme is not proportional. The premium increases after a first claim are largest for those in the intermediate bonus-malus classes, and smallest for those in the top and bottom classes. The marginal premium increases after a second or third claim, however, are increasing nearly monotonically with the bonus-malus class, from 0% in the lowest classes to 100–140% in class 20.[6] This all suggests that incentives to avoid a claim jump down after a first claim for insurees in low classes and jump up after a first claim for insurees in high classes. In Section 3, we formally define incentives in a dynamic theoretical setting and provide some numerical computations to formalize this intuition.

Finally, note that insurees are contractually obliged to claim all their insured losses as soon as possible. However, the contract leaves them the option to withdraw their claims within six months from the loss date. Withdrawn claims do not count as at-fault claims in determining the insuree's bonus-malus class and therefore do not affect the premium.

[6]In the lowest classes, therefore, the bonus-malus scheme itself does not give incentives to avoid a second or third claim. However, the insurance company reserves the right to cancel contracts with three or more claims at fault in a year. Because claims at fault are fairly rare, this is unlikely to affect insurees' decisions a lot. Therefore, we ignore contract cancelations in our theoretical and empirical analysis.

Therefore, throughout most of the paper we treat withdrawn claims like unclaimed losses. That is, we ignore them, together with losses that were not claimed in the first place. Section 4.4 discusses the fact that withdrawals are in fact observed manifestations of ex post moral hazard.

2.2 Data

Our data provide the contract and claim histories of personal car insurance clients of a major Dutch insurer from January 1, 1995 to December 31, 2000. The raw data consist of 1,730,559 records. Each record registers a change in a particular contract (renewal, change of car, etcetera), or a claim. The data include 75 variables, with information on *drivers* (sex, age, occupation, postcode), *cars* (brand, model, production year, price, weight, power, etc.), *contracts* (coverage, bonus-malus class, level of deductible, premium, renewal date), and *claims* (type of claim, damage, etc.).

The raw data contains 163,194 unique contracts. Because they do not contain information on claims in 1995, we excluded this year from the data. We also excluded the contracts that are not covered by the bonus-malus system.[7] This leaves 140,799 unique contracts with a total of 101,074 claims. Of these claims, 34,491 are claims at fault that may lead to a malus.[8] However, in 2,463 of these cases, insurees have avoided a malus by withdrawing their claim.[9] Throughout most of the paper, we treat withdrawn claims as unclaimed losses, and simply exclude them from the analysis. Section 4.4 specifically studies the withdrawal data to learn about moral hazard. Appendix C shows that the empirical results presented in the main text are robust to alternative ways of dealing with withdrawals.

[7] These are the contracts covering companies' fleets of cars. Such contracts have no individual BM coefficients, but general fleet discounts. These discounts are adjusted every year based on the fleets' claim histories.

[8] The data also include so called "nil" claims, which are mostly pro forma claims of amounts below the deductible. These may correspond to an "at fault" event, but typically do not affect the agent's bonus-malus status. Therefore, we treat all nil claims as claims not-at-fault.

[9] We use both direct and indirect information to identify withdrawals. See Appendix C for details.

We restrict our analysis to the claim histories from the contracts' first renewal (or start) date in the sample onwards. In the data, there are 124,021 contracts with observed renewal date. Of these contracts, 6,787 were interrupted for some period of time. In these cases, we only use the contract history from its first observed renewal date to its first interruption.

For each contract, we registered the claim history, with information on the times and sizes of claims at fault that were not withdrawn. We examined the bonus-malus transitions between all observed contract years, corrected some inconsistencies (see Appendix C for details), and registered the initial bonus-malus class (i.e., the bonus-malus class established at the first renewal date). Along the way, we discovered that the data on the bonus-malus class after the 2000 renewal are not reliable.[10] Therefore, we excluded contracts that started in 2000, and the history of ongoing contracts after their 2000 renewal date.

Our full final sample consists of 123,169 unique contracts with 23,396 claims at fault. Table 3 shows that many of these are observed for the maximum period of 4 years.

We illustrate some of the data's key features using only data on the first fully observed contract years in the sample. Table 4 gives the number of contracts in this subsample by bonus-malus class and by number of claims at fault filed in the contract year. Contracts with one claim and contracts with two or more claims will be important, for different reasons, to our empirical analysis. There are a lot of contracts with one claim in our subsample but, because claims are rare, there are only 278 contracts with at least two claims.

Figure 1 plots the distribution of contracts in our subsample across bonus-malus classes. Classes 1 – 3 have less than 1% of the contracts each; more than 26% of the contracts are in the highest class 20. The majority of contracts (over 57%) is in the high bonus-malus classes 14 – 20, where the premium is just 25% of the base premium.

[10] 40,104 out of 68,515 bonus-malus transitions in 2000 were incorrect.

Figure 1 also plots the shares of contracts in our subsample with at least one and at least two claims at fault, by bonus-malus class. These shares drop substantially with the bonus-malus class. It may be tempting to relate this variation in the number of claims over bonus-malus classes to our discussion of incentives. However, the overall pattern can be well explained by heterogeneity in risk, with high-risk individuals sorted into the lower bonus-malus classes.

3 Model of Claim Rates and Sizes

This section characterizes the dynamic incentives to avoid car insurance claims that are inherent to the Dutch bonus-malus scheme. We do so by analyzing a model of a single agent's risk prevention and claim behavior that combines features of Mossin's (1968) static model of insurance and Merton's (1971) continuous-time analysis of optimal consumption. Our model is related to Briys (1986), but focuses on experience rating and its moral-hazard effects. It is an extension of Abbring, Chiappori, and Pinquet's (2003) model with heterogeneous losses and endogenous claiming, carefully adapted to the Dutch institutional environment. Also, unlike Abbring et al.'s analysis of experience rating in French car insurance, we make the nonstationarity arising from annual premium revision explicit. This is important for our empirical analysis because in the Dutch bonus-malus system, unlike in the French one, both the number of past claims and their distribution across contract years matter for the current bonus-malus status.

3.1 Primitives

We consider the behavior and outcomes of an agent i in continuous time τ with infinite horizon. Time is measured in contract years and has its origin at the moment the agent entered the insurance market.

The wealth of agent i at time τ is denoted by $W_i(\tau)$ and accumulates as follows. At time 0, agent i is endowed with some initial wealth $W_i(0) > 0$. Then, between τ and $\tau + d\tau$, agent i receives a return $\rho W_i(\tau)d\tau$ on her wealth and consumes $c_i(\tau)d\tau$. We ignore any other income, such as labor income.[11]

The agent causes an accident between τ and $\tau + d\tau$ with some probability $p_i(\tau)d\tau$.[12] If so, she incurs some monetary loss. Denote the j-th loss incurred by agent i by L_{ij}. We assume that L_{ij} is drawn independently of the agent's insurance history, including $(L_{i1}, \ldots, L_{i(j-1)})$, from some time-invariant distribution F_i.[13] The losses L_{ij} are covered by an insurance contract involving a fixed deductible D_i and a premium $q_i(\tau)d\tau$ that is paid continuously. The deductible is applied on a claim-by-claim basis, i.e. if a claim for a loss L_{ij} is filed, the insurer pays $L_{ij} - D_i$ to the agent.

The premium $q_i(\tau)$ is determined by agent i's bonus-malus class $K_i(\tau)$ according to Table 1. Thus, we can write $q_i(\tau) = A_i(K_i(\tau))$, where A_i is a mapping from agent i's bonus-malus class into her flow premium. Because the base premium to which the discounts in Table 1 are applied depends on agent i's characteristics, the mapping A_i will be heterogeneous across agents.[14]

Agent i is endowed with an initial bonus-malus class $K_i(0)$. The bonus-malus class is updated at the beginning of each contract year, the renewal date, according to the rule in Table 1. Thus, $K_i(\tau)$ is a right-continuous process, with discrete steps at each renewal date $\tau \in \mathbb{N}$ depending on the past contract year's bonus-malus class and number of claims. Denote by $N_i(\tau)$ the number of claims in the ongoing contract year up to and including time τ. That is, $N_i(\tau)$ is a claim-counting process that is set to zero at the beginning of

[11] For the purpose of our analysis, this is equivalent to assuming that any such income is perfectly foreseen by the agent (Merton, 1971, Section 7).

[12] Accidents that are not caused by the agent are fully covered and have no impact on future premiums. Such accidents can be and are disregarded in our analysis. From now on, by accident or claim we always mean accident or claim *at fault*.

[13] This assumption is violated if agents can influence F_i ex ante by choosing to drive more or less carefully. Then, data on claim sizes do not distinguish between ex ante and ex post moral hazard, but are still informative on the overall presence of moral hazard.

[14] Here, we abstract from time-varying characteristics other than K_i. There is not much harm in treating e.g. age as a time-invariant characteristic, as our empirical analysis will focus on events in only one or a few contract years.

each contract year. Then, at each renewal date $\tau \in \mathbb{N}$,

$$K_i(\tau) = B(K_i(\tau-), N_i(\tau-)), \tag{1}$$

where $K_i(\tau-)$ and $N_i(\tau-)$ are agent i's bonus-malus class and number of claims in the past contract year, respectively, and B represents Table 1's bonus-malus updating rule. Note that this rule is common to all agents. Recall that it moves agents who survive a contract year without claims to a higher bonus-malus class, corresponding to a lower premium, and all other agents to a lower class, with a higher premium.

Insurance claims filed by the agent are potentially affected by ex ante and ex post moral hazard (Chiappori, 2001). Ex ante moral hazard arises if the agent can affect the probability of an accident. We model this by allowing, at each time τ, the agent to choose the intensity $p_i(\tau)$ of having an accident from some bounded interval $[\underline{p}_i, \overline{p}_i]$, at a utility cost $\Gamma_i(p_i(\tau))$. We assume that Γ_i is twice differentiable on $(\underline{p}_i, \overline{p}_i)$, with $\Gamma_i' < 0$, $\Gamma_i'' > 0$. In words, reducing accident rates is costly and returns to prevention are decreasing. For definiteness, we also assume that $\Gamma_i'(\underline{p}_i+) = -\infty$ and $\Gamma_i'(\overline{p}_i-) = 0$. In addition, we allow for ex post moral hazard by allowing the agent to hide a loss she has actually incurred from the insurer. For clarity of exposition, we assume that claiming and hiding losses are costless, but that the agent cannot claim losses that have not actually been incurred.[15]

The agent's instantaneous utility from consuming $c_i(\tau)$ and driving with accident intensity $p_i(\tau)$ at time τ is $u_i(c_i(\tau)) - \Gamma_i(p_i(\tau))$. We assume that u_i is strictly increasing and concave. The agent chooses consumption, prevention and claiming plans that maximize total expected discounted utility[16]

$$\mathbb{E}\left[\int_0^\infty e^{-\rho\tau}\left[u_i(c_i(\tau)) - \Gamma_i(p_i(\tau))\right] d\tau\right],$$

[15]Section 3.3.1 discusses a simple extension of the model in which hiding losses is costly. Such an extension is needed to formalize variation in the degree of ex post moral hazard in general, and the extreme case that agents report all losses (above the deductible) and do not suffer from ex post moral hazard in particular.

[16]For simplicity, we assume that subjective discount rates equal the interest rate.

subject to the intertemporal budget constraint $\lim_{\tau\to\infty} e^{-\rho\tau}W(\tau) = 0$ and given the wealth and premium dynamics described above.

At each time τ, the agent observes her wealth, bonus-malus class and claim histories. As we have implicitly assumed that any labor and other income is perfectly foreseen by the agent, she only has to form expectations on future accidents and their implications.

3.2 Optimal Risk, Claims and Savings

For notational convenience, we now drop the index i. It should be clear, however, that all results are valid at the individual level, irrespective of the distribution of preferences and technologies across agents. In particular, the results hold for any type of unobserved heterogeneity in these primitives of the model.

Because our model is Markovian and, apart from annual contract renewal, time-homogeneous, the optimal consumption, prevention and claim decisions at time τ only depend on the past history through the agent's current wealth $W(\tau)$, bonus-malus class $K(\tau)$, the number of claims at fault $N(\tau)$, and the time $t \equiv \tau - [\tau]$ past in the ongoing contract year.

Let $V(t, W, K, N)$ denote the agent's optimal expected discounted utility at time t in the contract year if her wealth equals W, she is in bonus-malus class K, and has claimed N losses in the ongoing contract year. This value function satisfies the Bellman equation

$$
\begin{aligned}
&V(t, W, K, N) = \\
&\qquad \max_{c,p,\mathcal{X}} \Big\{ u(c)dt - \Gamma(p)dt + e^{-\rho dt}\times \\
&\qquad\qquad \Big[(1 - pdt)V(t + dt, (1 + \rho dt)W - cdt - A(K)dt, K, N) \\
&\qquad\qquad\quad + pdt \int_{\mathcal{X}} V(t + dt, (1 + \rho dt)W - \min\{l, D\} - cdt - A(K)dt, K, N + 1)dF(l) \\
&\qquad\qquad\quad + pdt \int_{\mathcal{X}^c} V(t + dt, (1 + \rho dt)W - l - cdt - A(K)dt, K, N)dF(l)\Big]\Big\},
\end{aligned}
\tag{2}
$$

with

$$V(1,W,K,N) \equiv \lim_{t \uparrow 1} V(t,W,K,N) = V(0,W,B(K,N),0). \tag{3}$$

Equation (2) can be interpreted as follows. Between t and $t+dt$ the agent derives flows of utility from her consumption and disutility from her prevention effort. The value $V(t,W,K,N)$ equals the net value of these utility flows, at the optimal consumption and prevention levels, plus the expected optimal discounted utility at time $t+dt$. With probability $1-pdt$ no accident occurs. Then, the agent's wealth is increased with the interest flow minus consumption and the premium, and the number of claims at fault, N, stays unchanged. If the agent causes an accident, with probability pdt, she will incur an additional wealth loss. The size of this wealth loss is subject to ex post moral hazard. If the damage L caused by the accident lies in the optimal choice of the *claim set* $\mathcal{X}$, she claims for insurance compensation and only looses the minimum of L and the deductible D. Then, the number of claims at fault, N, increases by 1. If L lies in the complement $\mathcal{X}^c$ of the optimal claim set, however, she does not claim and pays the full loss L. Then, the number of claims at fault, N, stays unchanged.

Equation (3) reflects the effects of annual premium renewal. It requires that the value in class K with N claims just before a renewal time equals the value in class $B(K,N)$ with 0 claims just after renewal.

Bellman equation (2) can be rewritten in a more familiar form by rearranging and taking limits $dt \downarrow 0$,

$$\begin{aligned}\rho V(S) = \max_{c,p,\mathcal{X}} \Big\{ & u(c) - \Gamma(p) \\ & + p\Big[\int_{\mathcal{X}} V(t, W-\min\{l,D\}, K, N+1)dF(l) \\ & \qquad + \int_{\mathcal{X}^c} V(t, W-l, K, N)dF(l) - V(S)\Big] \\ & + V_W(S)\left[\rho W - c - A(K)\right] + V_t(S)\Big\},\end{aligned} \tag{4}$$

where V_t and V_W are the partial derivatives of V with respect to, respectively, t and W. The left-hand side of (4) is the flow (or perpetuity) value attached by the agent to state $S \equiv (t, W, K, N)$. It equals the (optimal) instantaneous flow of utility from her consumption net of the disutility from her prevention effort plus three expected value ("capital") gains terms, (i) the expected value gain because of an accident, (ii) the value gain due to net accumulation of wealth, and (iii) the appreciation of the value over time.

Standard arguments guarantee that (4), with (3), has a unique solution V, and that an optimal consumption-prevention-claim plan exists. In Appendix A, we prove that the value function V is strictly increasing in wealth W (Lemma 1) and that it is weakly increasing in the bonus-malus class K and weakly decreasing in the number of claims at fault N (Lemma 2).

One direct implication is that the agent follows a threshold rule for claiming.

Proposition 1. *The optimal claim set in state S is given by $\mathcal{X}^*(S) \equiv (x^*(S), \infty)$, for some claim threshold $x^*(S) \geq D$.*

Thus, if the agent incurs a loss L at time t then, for given wealth W, bonus-malus class K and number of claims at fault N right before t, she claims if and only if $L > x^*(S)$. The threshold is implicitly defined as the loss at which she is indifferent between claiming and not claiming:

$$V(t, W - D, K, N + 1) = V(t, W - x^*(S), K, N). \tag{5}$$

This assumes an internal solution and, in particular, ignores the trivial, and empirically irrelevant, case in which $\mathcal{X} = \emptyset$.

Optimality of the two remaining choices, consumption and prevention, requires that the corresponding first-order conditions are satisfied,

$$\begin{aligned}
u'(c^*(S)) &= V_W(S) \quad \text{and} &(6)\\
-\Gamma'(p^*(S)) &= V(S) - \int_0^{x^*(S)} V(t, W-l, K, N) dF(l) &(7)\\
&\quad - \int_{x^*(S)}^{\infty} V(t, W-D, K, N+1) dF(l),
\end{aligned}$$

where $p^*(S)$ and $c^*(S)$ are, respectively, the optimal accident and consumption intensities in state $S \equiv (t, W, K, N)$. The first equation is the standard Euler condition, which balances the marginal utilities from current and future consumption. The second condition requires equality of the marginal cost of prevention and the marginal cost of an accident.

3.3 Dynamic Incentives from Experience Rating

3.3.1 Measure of Incentives

First, consider ex ante moral hazard. The first-order condition (7) embodies two distinct aspects of ex ante moral hazard, the agent's ability to reduce risk and the incentives she is given to do so. If the marginal cost $-\Gamma'$ of reducing risk quickly increases from 0 to ∞, changes in incentives have little effect on risk and moral hazard is limited. In the limiting case in which $\Gamma(p) = 0$ if $p \geq p_0$ and $\Gamma(p) = \infty$ if $p < p_0$, for some $p_0 > 0$, the agent will choose an accident rate p_0 irrespective of incentives to avoid claims. We will refer to this limiting case as the case of "no (ex ante) moral hazard".

The right-hand side of (7) is the expected discounted utility cost of a claim. This is a measure of the incentives to avoid an accident, for a given prevention technology Γ. In this section, we characterize the variation in these incentives with, in particular, K, N and t.[17] In the next section, we use this characterization to test for moral hazard.

[17] Abbring, Chiappori, and Pinquet (2003) obtain unambiguous theoretical results on the change in incentives after each claim in French car insurance. These results rely on the proportional nature of the French bonus-malus system, and do not carry over to the Dutch system. Moreover, Abbring et al. do not model the nonstationarity arising from annual contract renewal and, therefore, do not provide results on contract-time effects.

We focus on the dynamic incentives inherent to the bonus-malus scheme and set the deductible D to 0. This simplifies the presentation and does not greatly interfere with our objective of learning about *changes* in incentives across states. Section 3.3.2 formalizes this point in the context of a particular model specification.

We will also restrict attention to incentives in the case without moral hazard. This will be sufficient for computing a score test for moral hazard and for interpreting local behavior of econometric tests near the null of no moral hazard. Without moral hazard, the optimal accident rate $p^*(S)$ equals a fixed number $p_0 > 0$ in all states S and all losses are claimed, so the right-hand side of (7) simplifies to

$$V(t, W, K, N) - V(t, W, K, N+1). \tag{8}$$

We will characterize incentives in the case of no moral hazard by characterizing this difference in utility values as a function of the state (t, W, K, N).

Before we move to these computations, note that (8) is also a measure of incentives to avoid a claim given that an accident has occurred. Linearizing (5) as a function of the threshold around the deductible $D = 0$ gives

$$x^*(t, W, K, N) V_W(t, W, K, N) \approx V(t, W, K, N) - V(t, W, K, N+1). \tag{9}$$

The right-hand side of this equation is again the expected discounted utility cost of a claim in (8). The left-hand side is the marginal cost, in expected discounted utility units and at a time a claim decision needs to be taken, of increasing the threshold just above the deductible $D = 0$. Note that this cost, unlike the cost Γ of loss prevention, is not a free parameter of the model. This is a direct consequence of our assumption that claiming and hiding losses are costless, and implies that the model does not have a parameter that indexes the degree of ex post moral hazard. It is straightforward to extend the model with such a parameter. For example, if hiding a loss L leads to a capital loss of γL, for some parameter $\gamma \geq 1$, then the left-hand side of (9) generalizes to $\gamma x^*(t,W,K,N)V_W(t,W,K,N)$. Then, the null of no moral hazard is the limiting case in which $\gamma \to \infty$, and hiding losses is prohibitively expensive. Throughout this paper, it is implicitly understood that the null of no (ex post) moral hazard can be generated this way. For expositional convenience, we will not make this explicit in the notation.

3.3.2 Theoretical Characterization of Incentives

We compute the value function and the incentives for the constant absolute risk aversion (CARA) class of utility functions, which is given by

$$u(c) = \frac{1 - e^{-\alpha c}}{\alpha},$$

with $\alpha > 0$ the coefficient of absolute risk aversion, $-u''(c)/u'(c)$. Linear utility, $u(c) = c$, arises as a limiting case if we let $\alpha \downarrow 0$. The CARA class brings analytical and computational simplifications that we believe outweigh, for the purpose of this paper at least, its disadvantages (see e.g. Caballero, 1990, for some discussion).

Merton's (1971) results that, with CARA utility, the value and utility functions have the same functional forms and consumption is linear in wealth, provide intuition for

Proposition 2. *In the case of no moral hazard with accident rate p_0, $D = 0$, and CARA utility,*

$$c^*(S) = \rho\,[W - Q(t,K,N)] \quad \textit{and} \quad V(S) = \frac{1 - e^{-\alpha\rho[W - Q(t,K,N)]}}{\alpha\rho},$$

with $S \equiv (t,W,K,N)$ and Q the unique solution to the system of differential equations

$$\begin{aligned} \rho Q(t,K,N) &= \pi(K) + p_0 \frac{e^{\alpha\rho[Q(t,K,N+1) - Q(t,K,N)]} - 1}{\alpha\rho} + Q_t(t,K,N) && (10) \\ Q(1,K,N) &= Q(0,B(K,N),0). && (11) \end{aligned}$$

Here, $Q_t(t, K, N)$ is the partial derivative of $Q(t, K, N)$ with respect to t.

Proposition 2 is proved in Appendix A. It provides a characterization of the value function V that can be used to compute incentives under the null of no moral hazard.

To gain some insight in Proposition 2's characterization of optimal consumption and the value function, first note that equation (10) reduces to

$$\rho Q(t, K, N) = \pi(K) + p_0 \left[Q(t, K, N + 1) - Q(t, K, N)\right] + Q_t(t, K, N)$$

if we let $\alpha \downarrow 0$. Thus, in the limiting case of linear utility— that is, a risk-neutral agent— $Q(t, K, N)$ reduces to the expected discounted flow of future premia. The agent simply consumes the flow value $\rho[W - Q(t, K, N)]$ of her *net* wealth, which produces a value $V(S) = W - Q(t, K, N)$. The expected discounted utility cost of a claim in state S is given by

$$V(S) - V(t, W, K, N + 1) = Q(t, K, N + 1) - Q(t, K, N).$$

Conveniently, incentives are independent of the level of wealth in this case.

With a risk-averse agent— that is, for fixed $\alpha > 0$— the right-hand side of equation (10) involves an additional term,

$$p_0 \left\{ \frac{e^{\alpha\rho[Q(t,K,N+1)-Q(t,K,N)]} - 1}{\alpha\rho} - \left[Q(t, K, N + 1) - Q(t, K, N)\right] \right\},$$

which is strictly positive for all (t, K, N). As a consequence, $Q(t, K, N)$ strictly exceeds the expected discounted flow of premia, and optimal consumption is lower than with linear utility. This reflects precautionary savings. Incentives in state S are now given by $V(S) - V(t, W, K, N + 1) = (\alpha\rho)^{-1} e^{-\alpha\rho W} \left[e^{\alpha\rho Q(t,K,N+1)} - e^{\alpha\rho Q(t,K,N)}\right]$, so that a wealth-invariant measure of incentives is given by

$$\Delta V(t,K,N+1) \equiv \frac{V(S)-V(t,W,K,N+1)}{e^{-\alpha\rho W}} = \frac{e^{\alpha\rho Q(t,K,N+1)}-e^{\alpha\rho Q(t,K,N)}}{\alpha\rho}.$$

Note that this measure again reduces to $Q(t,K,N+1)-Q(t,K,N)$ if we let $\alpha \downarrow 0$.

Before we move to a numerical characterization of incentives, briefly consider the case of a general but state-invariant deductible D. In this case, with linear utility, incentives in state S are the sum of the increase $Q(t,K,N+1)-Q(t,K,N)$ in the expected discounted premium flow and the deductible D. Because the deductible is not state dependent, changes in incentives across states are not affected. Consequently, tests that focus on changes in incentives across states within agents are robust to an extension to general deductibles (see Section 4.2).

3.3.3 Numerical Characterization of Incentives

In the remainder of this section, we will numerically characterize incentives by computing $\Delta V(t,K,N+1)$ for various values of (t,K,N), α, and p_0. An algorithm for computing the underlying function Q is presented in Appendix B. We set $\rho = \ln(1.04)$ to be consistent with a 4% annual interest and discount rate. In our baseline computations we take $p_0 = 0.053$, which corresponds to a 94.8% probability of having no claim in the contract year. This equals the share $\frac{105{,}650}{111{,}394}$ of contracts without claims in our subsample of single contract years (see Table 4). We measure the premium $\pi(K)$ in multiples of the base premium. That is, $\pi(K)$ is set equal to the premium reported in Table 1 and, in particular, $\pi(2) = 1$.

Figure 2 plots the (wealth-invariant measures of the) present discounted utility costs of a first ($\Delta V(1,K,1)$), a second ($\Delta V(1,K,2)$) and a third ($\Delta V(1,K,3)$) claim just before contract renewal, as a function of the bonus-malus class K. The bold graphs correspond to the linear-utility case $\alpha = 0$ and give the expected discounted premium cost of a claim in multiples of the base premium. The other graphs correspond to $\alpha = 0.1, 0.2, \ldots, 0.5$, in that order and with the graphs corresponding to $\alpha = 0.1$ closest to the bold graph. At a consumption level equal to 20 times the base premium, $\alpha = 0, 0.1, \ldots, 0.5$ correspond to coefficients of *relative* risk aversion equal to $0, 2, \ldots, 10$, respectively. This is roughly the range considered, with some empirical support, by Caballero (1990).

Incentives near the null of no moral hazard are considerable. In the linear case, total wealth drops by more than the annual base premium. Recall that the base premium is four times the premium in class 20 paid by most insurees in our sample. The cases with risk aversion are very similar. Incentives also vary a lot between bonus-malus classes. The incentives to avoid a first claim are small in the lowest classes, where the premium paid is already high. They then increase substantially, and again fall to a lower level in the highest classes. Robustly across the values of α, these incentives are larger than the incentives to avoid a second or a third claim in low classes K, and smaller in high classes K. Thus, for agents in high bonus-malus classes, the Dutch bonus-malus system has implications that are similar to those of the French proportional experience-rating scheme studied by Abbring, Chiappori, and Pinquet (2003): The first and also the second claim in a contract year lead to jump up in incentives, and therefore jump down in claim rates under moral hazard. However, the Dutch system allows us to contrast this implication with the effects of low bonus-malus classes, where incentives jump *down* after a first claim.

Figure 3 plots the change $\Delta V(1, K, N+1) - \Delta V(1, K, N)$ for $N = 1$ (resp. $N = 2$) in incentives to avoid a claim when a first (resp. second) claim is filed just before contract renewal, again for different degrees of risk aversion. This graph shows that incentives after a first claim jump down for low K and up for high K. The incentives after a second claim do not change for low K (they are already equal to zero), but jump down for middle K and up for high K. These effects, and those in Figure 2, are computed at a specific point, $t = 1$, in time, but are robust to considering alternative times.

To illustrate this, Figure 3 also plots the changes over the course of a contract year in incentives to avoid, respectively, a first, a second and a third claim. Because $\Delta V(t, K, N) -$

$\Delta V(0, K, N)$ is for all $N = 1, 2, 3$ close to linear as a function of time t, these graphs of $\Delta V(1, K, N) - \Delta V(0, K, N)$ summarize well the time patterns in incentives, and the variation in these time patterns between bonus malus classes. The changes in incentives over time are small relative to the jumps in incentives when a claim is filed just before contract renewal. The *differences* between $\Delta V(1, K, N+1) - \Delta V(0, K, N+1)$ and $\Delta V(1, K, N) - \Delta V(0, K, N)$ are even smaller for $N = 1$ and almost the same for $N = 2$. Because these differences give the change in $\Delta V(t, K, N+1) - \Delta V(t, K, N)$ over the contract year, this implies that the graphs of $\Delta V(1, K, N+1) - \Delta V(1, K, N)$ for both, $N = 1$ and 2, indeed characterize well the jump in incentives after a first and a second claim at all times across the contract year.

Even if the time-variation in incentives is small relative to the jumps in incentives at the times of a claim, it may still affect some of our empirical procedures that focus on the latter. After all, the time-variation in incentives affects all contracts, but only some contracts experience jumps in incentives. We will return to this in Section 4 in the specific context of an econometric model.

Finally, we explore the robustness of these results to changes in the accident intensity under the null, p_0. Figure 4 again plots the jumps in incentives after a first and a second claim and changes in incentives over the course of a contract year for different levels of risk aversion, but for $p_0 = 0$. The graphs are qualitatively similar to those in Figure 3 for the average-risk case. Time effects are smaller in the zero-risk case because agents do not expect any accident during the contract year; time preference is the only source of nonstationarity in this extreme case.

Figure 5 plots the same graphs for $p_0 = 0.232$, which is the average risk level consistent with the share of contracts without a claim in the worst bonus-malus class, $K = 1$. At this risk level, incentives at the time of a first claim only increase in the highest bonus-malus classes, and decrease at low and intermediate bonus-malus classes. The jumps in incentives at the time of a second claim have similar features as before. Time effects are now substantial. Because agents are very likely to experience an accident during the contract year anyhow, incentives do not jump much early in the year even if they would jump a lot close to renewal.

In sum, the qualitative conclusions for our baseline case with average risk continue to hold as long as p_0 is average or low, but change for very large p_0. Nevertheless, we can robustly conclude that incentives at the time of a first claim drop in all low classes, and increase in very high classes. The results on jumps in incentives at the time of a second claim are more robust to changes in accident intensity; these incentives do not change in low classes, drop in middle classes and increase in high classes.

4 Empirical Analysis

The empirical analysis uses the data set introduced in Section 2.2. We formalize this using Section 3's notation, with an appropriate change of the time scale's origin.

The full sample consists of n contracts. Time is measured in years. For each contract $i \in \{1, \ldots, n\}$, we let time τ have its origin at the start of the contract's first contract year included in the sample. Then, $K_i(0)$ is the initial bonus-malus class in the *sample*. Contract i is observed until some random "attrition" time C_i.

Let $N_i(\tau)$ count the number of claims at fault on contract i in the ongoing contract year up to and including time τ. Note that the total number of claims incurred on contract i until time C_i is $\bar{N}_i(C_i)$, with

$$\bar{N}_i(\tau) \equiv N_i(\tau) + \sum_{u=1}^{[\tau]} N_i(u-).$$

Denote the time and size of the jth claim on contract i with T_{ij} and L_{ij}, respectively. Then, for each contract i, we observe C_i, the contract's initial bonus-malus state $K_i(0)$, and its claim history $H_i[0, C_i)$ up to time C_i, where

$$H_i[0, \tau) \equiv (\{N_i(u); 0 \leq u < \tau\}; L_{i1}, \ldots, L_{i\bar{N}_i(\tau)}).$$

Note that the bonus-malus history up to time C_i, $\{K_i(\tau); 0 \leq \tau < C_i\}$, does not vary within contract years, and can be constructed from $K_i(0)$ and $\{N_i(u); 0 \leq u < C_i\}$.

The full unbalanced sample is $\{C_i, K_i(0), H_i[0, C_i); i = 1, \ldots, n\}$. We assume that it is a random sample from the distribution of its population counterpart $\{C, K(0), H[0, C)\}$. The claim history $H \equiv H[0, \infty)$, and its relation to the bonus-malus class $K(0)$ initially occupied by the agent, are the focus of our empirical analysis.

4.1 Econometric Model

At the core of our econometric model is the intensity θ_l of claims of size $l \in \mathbb{R}_+$ or up at time τ, conditional on the claim history $H[0, \tau)$ up to time τ, the initial bonus-malus class $K(0)$, and a nonnegative individual-specific effect λ. We specify the following model:

$$\theta_l\left(\tau|\lambda, H[0,\tau), K(0)\right) = \vartheta\left(t|\lambda, N(\tau-), K(\tau)\right) \cdot \overline{F}\left(\max\{l, x^*(t, \lambda, N(\tau-), K(\tau))\}|\lambda\right), \quad (12)$$

where $t \equiv \tau - [\tau]$ is time elapsed in the contract year and $\vartheta\left(t|\lambda, N(\tau-), K(\tau)\right)$ is the rate at which losses are incurred at time τ by an agent with characteristics λ who has been in class $K(\tau)$ and has claimed $N(\tau-)$ times in the year up to time τ. Recall that the bonus malus class $K(\tau)$ is fully determined by the initial class $K(0)$ and the claim history $H[0, \tau)$. The second factor, $\overline{F}\left(\max\{l, x^*\left(t, \lambda, N(\tau-), K(\tau)\right)\}|\lambda\right)$, is the conditional probability that the loss is of size $L \geq l$ and that it is claimed, i.e. $L \geq x^*$. This specification incorporates Section 3's assumption that losses are drawn from an exogenous and time-invariant distribution $F(\cdot|\lambda) = 1 - \overline{F}(\cdot|\lambda)$ that may differ between agents. It also reflects the result that agents follow a threshold rule for claiming (Proposition 1).

Without further loss of generality, and to facilitate a discussion of theory's implications for (12), we write

$$\vartheta\left(t|\lambda, N(\tau-), K(\tau)\right) = \lambda \cdot \psi(t) \cdot \beta\left(t|\lambda, N(\tau-), K(\tau)\right), \tag{13}$$

with ψ a continuous and positive function representing external contract time effects, and β an almost surely bounded and positive function. We frequently use the notation $\Psi(t) \equiv \int_0^t \psi(u)du$ and normalize $\Psi(1) = 1$. We assume that λ has distribution G_K conditional on $K(0) = K$.

Together with equation (12), this fully specifies the distribution of $H|K(0)$. We assume independent censoring, that is $C \perp\!\!\!\perp H|K(0)$. This is a standard assumption in event-history analysis (e.g. Andersen, Borgan, Gill, and Keiding, 1993). It ensures that $\theta_l\left(\tau|H[0,\tau), K(0)\right)$ can be identified with the claim rate among surviving contracts, $\theta_l\left(\tau|H[0,\tau), K(0), C > \tau\right)$.

We are now ready to define the tests' hypotheses within the context of the econometric model. First, consider the simplification of (12) that is implied by the absence of moral hazard. In the empirical analysis, we will refer to this case as the *null of no moral hazard*.

Prediction 1. The claims process under the null of no moral hazard. *Without moral hazard, $\beta \equiv 1$ and $x^* \equiv 0$, so that*

$$\theta_l\left(\tau|\lambda, H[0,\tau), K(0)\right) = \lambda\psi(t)\overline{F}(l|\lambda).$$

Given λ, claim rates and sizes do not depend on the past number of claims $N(\tau-)$ or the bonus-malus class $K(\tau)$; they only depend on contract time through the function ψ. That is, there is no state dependence in the claims process.

Taken literally, Section 3's theory implies that $\psi \equiv 1$, so that $\theta_l\left(\tau|\lambda, H[0,\tau), K(0)\right) = \lambda\overline{F}(l|\lambda)$ is time-invariant, with $\lambda = p_0$. Thus, ψ captures contract-time effects that are *external* to the model, that is, that are independent of the claim history and the bonus-malus class. We entertain the possibility of such effects because, if they are there for some reason, they are likely to confound our analysis of state dependence.[18] We will present both tests that assume $\psi \equiv 1$ ("stationarity") and tests that allow for nonparametric ψ. The proportional specification of (13) will then capture the first-order effects of any external contract-time effects. Note that in addition we explicitly allow, through β and x^*, for contract-time effects that arise *internally* because of the fact that contracts are renewed at discrete times. These internal time effects will in general enter the claim rate nonproportionally.

Under the alternative of moral hazard, Prediction 1 generally fails. In that case, $\theta_l\left(\tau|\lambda, H[0,\tau), K(0)\right)$ depends negatively on incentives, which in turn vary with t, λ, and, in particular, $N(\tau-)$ and $K(\tau)$. In Subsection 4.2, we impose the full structure of Section 3's theory on the econometric model, including $\psi \equiv 1$. We present and apply a score test that can be interpreted as a Lagrange multiplier test for moral hazard in the structural model. In Section 4.3, we use more general tests for state dependence in claim times and sizes. There, we only rely on qualitative predictions of the effect of incentives on claim rates and sizes, without directly using incentive computations.

Before we present these tests, we briefly reflect on the possibility that they pick up alternative sources of state dependence in the claims process, such as learning, fear, or cautionary responses to accidents, that are unrelated to financial incentives and moral hazard. In Section 3, we have assumed these away by specifying the prevention technology, as represented by the cost function Γ, to be independent of the accident history. There are two reasons not be overly concerned about this. First, many of these alternative sources of

[18] The theoretical and econometric models only recognize contract time and do not explicitly consider the effects of calendar time (or duration since last event for that matter). In our sample, different contracts have different renewal dates, so that contract time and calendar time do not coincide. If renewal dates are evenly distributed over calendar time, seasonal calendar-time effects are not likely to matter much to the empirical analysis. However, we observe in our sample that the share of contracts starting in January is 13.3%, which is more than twice as much as at the end of the calendar year (6.1% contracts start in November and 6.2% in December). This variation could be explained by the fact that it is more advantageous to buy a new car at the beginning of a calendar year because the ageing of a car in years depreciates its value much faster than the ageing in months. On the other hand, the shares of contracts starting in other (middle) months of the year are almost equal; they range from 7.1% (August) to 9.4% (April).

state dependence are expected to work in one direction, unlike the financial incentives in the Dutch bonus-malus system. Learning from accidents, for example, is likely to reduce the accident rate in all states, irrespective of financial incentives. Therefore, it is unlikely that, for example, learning exactly replicates the implications of moral hazard. Second, learning effects are likely to be small for older drivers. We have confirmed the robustness of the empirical conclusions that follow by repeating our analysis on a subsample of insurees of 28 years old and up.[19]

4.2 Structural Test on the Full Sample of Claim Times

We first focus on the timing of claim and ignore information on claim sizes. Section 3 proves that ex ante and ex post moral hazard work in the same direction (see also Section 4.3). Thus, we can view tests based on claim times as overall tests for moral hazard.

Assume that there are no external time effects, $\psi \equiv 1$, so that all nonstationarity arises from behavioral responses to variation in incentives over time. In addition, suppose that there are R risk-types $\lambda_1, \ldots, \lambda_R$ of agents. Consider the following auxiliary model of claim rates,

$$\theta(\tau|\lambda, H[0,\tau), K(0)) = \lambda \cdot \exp\left[-\beta \Delta V(t, K(\tau), N(\tau-) + 1|\lambda)\right] \tag{14}$$

with $\Pr(\lambda = \lambda_r | K(0) = K) = \xi_r(K)$, $r = 1, \ldots, R$, and $\sum_{r=1}^{R} \xi_r(K) = 1$, for $K = 1, \ldots, 20$, and $\Delta V(\cdot|\lambda)$ equal to $\Delta V(\cdot)$ evaluated at $p_0 = \lambda$. The distributions of $\lambda|K(0) = K$ have the same supports $\{\lambda_1, \ldots, \lambda_R\}$ across K, but with different probability masses at each support point because of sorting into classes K.

Under the null of no moral hazard, $\beta = 0$ and claim rates are time-invariant. Under moral hazard, we expect to find evidence that $\beta > 0$. We will now argue that a score test for $\beta = 0$ in (14) can be interpreted as a structural test for moral hazard.

The auxiliary model's specification corresponds exactly to theory under the null; in

[19] Details are available from the authors upon request.

that case $\theta(\tau|\lambda, H[0,\tau), K(0)) = \lambda = \theta_0(\tau|\lambda, H[0,\tau), K(0))$. It can be seen as an approximation to the theoretical model under local alternatives to the null and a specific functional form of Γ. Suppose that an agent with characteristics λ chooses p from $(0,\lambda]$, with cost function $\Gamma_\lambda(p) = (p/\tilde{\beta})\left[\ln(p/\lambda) - 1\right] + \lambda/\tilde{\beta}$, so that $\Gamma'_\lambda(p) = \tilde{\beta}^{-1}\ln(p/\lambda)$. Substituting in the first-order condition (7) and assuming that there is no ex post moral hazard gives

$$\begin{aligned} p^*(S) &= \lambda \cdot \exp\left(-\tilde{\beta}\left[V(t,W,K,N|\lambda) - V(t,W,K,N+1|\lambda)\right]\right) \\ &\approx \lambda \cdot \exp\left[-\tilde{\beta}e^{-\alpha\rho W}\Delta V(t,K,N+1|\lambda)\right], \end{aligned} \tag{15}$$

where the approximation in the second line holds near the null of no moral hazard. Thus, the auxiliary model (14) is a good approximation to the optimal claiming hazard near the null, that is, for small $\tilde{\beta}$, with $\beta = \tilde{\beta}e^{-\alpha\rho W}$. Note that $\beta = \tilde{\beta}$ is homogeneous in the population, as in the auxiliary model, in the limiting case of a risk-neutral agent $(\alpha = 0)$. In this case, the derivative of $p^*(S)$ with respect to β at $\beta = 0$ exactly equals the corresponding derivative of the auxiliary model's claim rate in (14). Consequently, a score test for $\beta = 0$ in the auxiliary model exactly equals a Lagrange multiplier test for moral hazard in the structural model.

The score test for moral hazard has so far been narrowly developed for the case without ex post moral hazard, a specific functional form of the cost function Γ, a zero deductible, and linear utility. However, the intuition for a test based on the auxiliary model (14) does not rest on this example's specific assumptions, and we expect such a test to have power against moral hazard more generally. For example, with a general but state-invariant deductible D, the approximation in (15) becomes

$$p^*(S) \approx \tilde{\lambda} \cdot \exp\left[-\tilde{\beta}\Delta V(t,K,N+1|\lambda)\right],$$

with $\tilde{\lambda} = \lambda \exp\left(-\tilde{\beta}D\right)$. Clearly, a score test for $\beta = 0$ in the auxiliary model continues to be a test for moral hazard in this extension.

We estimate both restricted ($\beta = 0$) and unrestricted versions of the auxiliary model with parametric maximum likelihood, using the full unbalanced sample and computing ΔV using the linear specification ($\alpha = 0$). We compute the likelihood using a discrete (daily) approximation, building on

$$\Pr\left(N\left(\tau + \frac{1}{365}-\right) - N\left(\tau-\right) = 1 \,\middle|\, \lambda, N\left(\tau-\right), K(\tau)\right) \approx \frac{\theta(\tau|\lambda, H[0,\tau), K(0))}{365},$$

$\tau \in \left\{\frac{k}{365}; k \in \mathbb{Z}_+\right\}$. Each likelihood computation for the unrestricted model, and the computation of the score test statistic, embed the algorithm in Appendix B to compute $\Delta V(\cdot|\lambda_r)$ (that is, $\Delta V(\cdot)$ at $p_0 = \lambda_r$), $r = 1, \ldots, R$, at daily times. In addition to the score test, we also compute Wald and likelihood-ratio statistics to test for $\beta = 0$ against the alternative that $\beta \neq 0$. Because the latter two tests involve estimates of the auxiliary model under the alternative of moral hazard, where it only approximates the structural model, their interpretation as structural tests is less clear cut. However, because the approximation holds near the null, we expect them to have good power against, at least, local moral-hazard alternatives.

We estimated the unrestricted model with various numbers of support points for the distribution of λ, $R = 2, 3, 4, 5$, and obtained stable estimates of β. Moreover, between $R = 4$ and $R = 5$, the maximum log likelihood only increased by 5.83 points, even though 21 parameters were added.[20] Table 5 gives the estimates of β and the λs in the unrestricted model, with their estimated standard errors, for the specification of the model with 3, 4 and 5 support points. It also presents the score, Wald and likelihood-ratio test statistics for the hypothesis that there is no moral hazard: $\beta = 0$. The estimate of β is significantly positive. All three tests reject the null of no moral hazard at all conventional levels.

Figure 6 plots the probability masses $\xi_r(K)$ of the unrestricted model with 3 support

[20] Computation time also became an issue: Estimating the model with five support points took almost a week on a standard PC.

points for each class K. For expositional convenience, the estimates of λs in Table 5 are in ascending order, i.e. $\hat{\lambda}_1 < \hat{\lambda}_2 < \hat{\lambda}_3$. With this in mind, it is easy to see that the probability masses are slowly moving from the highest risk ($\hat{\lambda}_3$) in bonus-malus class 1 to the lowest risk ($\hat{\lambda}_1$) in bonus-malus class 20. This pattern is consistent with dynamic sorting of agents across bonus-malus classes.

4.3 Tests for State Dependence in Claim Times and Sizes

The previous section presents a tightly structured test for state dependence. It is tightly structured in the sense that it concentrates on local alternatives in which all state dependence is channeled through the dynamic incentives computed using Section 3's theory. In this section, we explore the application of more universal, nonparametric tests for state dependence from the literature. The interpretation and, in a few cases, construction of these tests rely on the theory's *qualitative* predictions on the claims process for given λ under moral hazard. We first develop and present these predictions.

4.3.1 Theoretical Implications for the Claims Process

The theoretical analysis of Section 3 can now be applied to predict the properties of the claims process for given λ under local moral-hazard alternatives. First note that the theory implies that incentives to avoid claims vary between initial bonus-malus classes K. However, in data the resulting moral-hazard effects on claims are confounded with sorting of agents with different characteristics λ into different classes K. The problem of empirically separating these selection effects from the causal effects of incentives is the standard problem of causal inference from cross-sectional data. This is a notoriously hard problem that we avoid here. Instead, we exploit that there is idiosyncratic variation in incentives over time.

Prediction 2. Dependence of claims on $N(\tau-)$, by class $K(\tau)$ under moral hazard. *Conditional on λ, loss rates jump down ($\beta(t|\lambda, 0, K) > \beta(t|\lambda, 1, K) > \beta(t|\lambda, 2, K)$)*

and claim sizes increase $(x^*(t,\lambda,0,K) < x^*(t,\lambda,1,K) < x^*(t,\lambda,2,K))$ *at the times of the first and the second claims in high classes* K*. In contrast, in low classes* K *loss rates jump up* $(\beta(t|\lambda,0,K) < \beta(t|\lambda,1,K) \le \beta(t|\lambda,2,K))$ *and claim sizes decrease* $(x^*(t,\lambda,0,K) > x^*(t,\lambda,1,K) \ge x^*(t,\lambda,2,K))$ *after the first and the second claims. There is no change in loss rates and claim sizes after the second claim in classes* $K \le 5$*. Because the state-dependence effects on loss rates and claim probabilities work in the same direction, the results for the loss rates carry over to claim rates.*

Next, for expositional convenience, suppose that there are no external time effects, $\psi \equiv 1$. Then, we have

Prediction 3. Dependence of claims on time t**, by class** $K(\tau)$ **under moral hazard.** *Conditional on* λ*, loss rates of an agent with* 0 *claims, resp.* 1 *claim (or, more particularly,* $\beta(t|\lambda,0,K)$*, resp.* $\beta(t|\lambda,1,K)$*) weakly decrease with* t *in most classes* K*, but may increase in the highest classes. Loss rates of an agent with 2 claims* $(\beta(t|\lambda,2,K))$ *are time-invariant in classes* $K \le 9$ *and strictly decrease with* t *in classes* $K > 9$*. The opposite results hold for claim thresholds* x^**, so that the effects on loss rates carry over to claim rates. All these time effects are small compared to the jumps at the time of a claim (Prediction 2), except for very high loss rates.*

If there are external contract-time effects, that is if ψ is nontrivial, then Prediction 3 holds relative to these external effects.

Predictions 1-3 are all conditional on λ; they are predictions at the level of an individual contract. Because λ is not observed, tests based on contrasting the predicted behavior under the null (Prediction 1) with the predicted behavior under the moral-hazard alternative (Predictions 2 and 3) are not feasible. The econometric challenge is to develop tests that use these predictions without requiring data on λ.

Our tests exploit the dynamics of claims implied by Predictions 2 and 3. Rather than studying cross-sectional variation in incentives, and trying to separate these from selection effects, we exploit variation in incentives over time. The problem of separating the corresponding dynamic moral hazard effects from dynamic selection is the classic problem of distinguishing state dependence and heterogeneity. Like the problem of distinguishing causal effects and selection effects in a static setting, this is a hard problem. However, it is a richer problem that has been well-studied in the statistics and econometrics literature. A key result from this literature implies that, under the null, the total number of claims in the contract year is a sufficient statistic for the unobserved heterogeneity in the loss intensities. We use this result to control for unobserved heterogeneity in the loss rates. We build on Abbring, Chiappori, and Pinquet's (2003) adaptations and extensions of the tests developed in the seminal work by Bates and Neyman (1952), Heckman and Borjas (1980) and Heckman (1981).

We first study time effects in claim rates. Prediction 1 implies that, under the null and after controlling for heterogeneity, time effects should be identical between classes K. Moreover, there should be no time effects at all under the theory's assumption of stationarity ($\psi \equiv 1$). On the other hand, both Predictions 2 and 3 imply that there will be time effects under moral hazard.

Time effects in claim rates are likely to be small and tests for moral hazard based on observed time effects are not likely to be very powerful. More importantly, they may be confounded by external time effects (ψ). Therefore, we quickly move to comparing (distributions of) first and second claim times and sizes. Here, Prediction 2 takes center stage. Because the jumps in incentives at the time of a claim are much larger than the time-variation in incentives, Prediction 2's "structural occurrence dependence" (Heckman and Borjas, 1980) effects dominate Prediction 3's time effects. Therefore, we can test for moral hazard by testing the implications of Prediction 2 for the relation between first and second claim times and sizes, across classes K and controlling for heterogeneity and, possibly, external time effects.

For the state-dependence tests, we use the balanced subsample consisting of the first fully observed contract years, presented in the Table 4. We will only use data on contracts with one claim and contracts with (exactly or at least) two claims in the contract year. We will use the same notation as before, i.e. $K_i(0)$ will denote the initial bonus-malus class (which is the bonus-malus class in the first observed contract year); T_{ij} and L_{ij} will refer to the time and size of the jth claim (in the first contract year).

4.3.2 Distribution of First Claim Time

Consider the distribution of the first claim time T_1 in the subpopulation with exactly one claim in the contract year and in one of the bonus-malus classes in $\mathcal{K}$,

$$H_1(t|\mathcal{K}) = \Pr(T_1 \leq t | N(1-) = 1, K(0) \in \mathcal{K}),$$

and its empirical counterpart

$$\hat{H}_{1,n}(t|\mathcal{K}) = \frac{1}{M_{1,\mathcal{K},n}} \sum_{i=1}^{n} I(T_{i1} \leq t, N_i(1-) = 1, K_i(0) \in \mathcal{K}),$$

where n is the total number of contracts in the sample and $M_{k,\mathcal{K},n} \equiv \sum_{i=1}^{n} I(N_i(1-) = k, K_i(0) \in \mathcal{K})$ is the number of contracts in the sample of contracts in a class in $\mathcal{K}$ with exactly k claims.

Under the null of no moral hazard, $H_1(\cdot|\mathcal{K}) = \Psi(\cdot)$ (Prediction 1 and Abbring, Chiappori, and Pinquet, 2003). Under the moral hazard alternative, $H_1(\cdot|\mathcal{K})$ will typically depend on the choice of $\mathcal{K}$ and differ from $\Psi(\cdot)$. This variation is caused by both changes in incentives at the time of a claim (Prediction 2) and changes in incentives over time (Prediction 3). We tested the null that $H_1(\cdot|K)$ is equal for all $K \in \{1, 2, \ldots, 20\}$ using the Kruskal-Wallis test and do not reject the null at conventional levels (see Table 6).

Figure 7 plots $\hat{H}_{1,n}(t|K(0) \in \mathcal{K})$ for low BM classes $\mathcal{K} = \{1, \ldots, 10\}$ and high BM classes $\mathcal{K} = \{11, \ldots, 20\}$. The difference between these two empirical distributions is not significant: The p-values of Wilcoxon rank-sum and Kolmogorov-Smirnov tests (given in first lines of the Table 6) are above conventional levels.

Suppose now that $\psi \equiv 1$. Then, under the null of no moral hazard, H_1 should be a uniform distribution. In the Figure 7, both empirical distributions of H_1 (for low and high BM classes) lie below the diagonal which suggests that agents file claims later in the year in all bonus-malus classes. This is consistent with the theory's Prediction 2 under moral hazard for low bonus-malus classes, but violates this prediction for high classes. Moreover, this apparent anomaly is significant since the p-value of the Kolmogorov-Smirnov test for uniformity of H_1 is 0.015 for high classes. For low classes, the p-value is 0.083; and for all classes it is 0.002 (see Table 7).

These results should, however, be interpreted with considerable care, because Predictions 2 and 3 correspond to only small effects of K on H_1 and, moreover, work in opposite directions. Therefore, even small external time effects in ψ can explain the anomaly and, together with moral hazard, generate the pattern observed in Figure 7. To see this, note that the estimate of H_1 for high bonus-malus classes lies above that for low classes. Thus, consistently with Prediction 2, agents in high classes claim *earlier* in the year *relative to agents in low classes.*

By comparing across bonus-malus classes, we have controlled for external time effects. Another way to control for such effects is to compare first and second claim times.

4.3.3 Marginal Distributions of First and Second Claim Times

Consider the distribution of the second claim time T_2 in the subpopulation with exactly two claims in the contract year and in one of the bonus-malus classes in $\mathcal{K}'$,

$$H_2(t|\mathcal{K}') = \Pr(T_2 \leq t|N(1-) = 2, K(0) \in \mathcal{K}'),$$

and its empirical counterpart,

$$\hat{H}_{2,n}(t|\mathcal{K}') = \frac{1}{M_{2,\mathcal{K}',n}} \sum_{i=1}^{n} I(T_{i2} \leq t, N_i(1-) = 2, K_i(0) \in \mathcal{K}').$$

Abbring, Chiappori, and Pinquet's (2003) analysis implies that, under the null of no moral hazard, $H_2(t|\mathcal{K}') = H_1(t|\mathcal{K})^2$, for all ψ and $\mathcal{K}, \mathcal{K}'$. They also show that this equality breaks down under moral hazard, and is likely to do so in one direction. The immediate implication of this result is that under no moral hazard, $H_2(t|\mathcal{K}')$ will not depend on the choice of $\mathcal{K}'$. A Kruskal-Wallis test for the null that $H_1(\cdot|K)$ is equal for all $K \in \{1, 2, \ldots, 20\}$ gives a p-value of 0.271. The result changes if we group the BM classes into low $(1-10)$ and high $(11-20)$. Then, both Wilcoxon and Kolmogorov-Smirnov tests reject the null at conventional levels; see Table 6.

Another test of moral hazard compares $\hat{H}_{2,n}(\cdot|\mathcal{K}')$ and $\hat{H}_{1,n}(\cdot|\mathcal{K})^2$ for appropriate choices of $\mathcal{K}$ and $\mathcal{K}'$. Figure 8 plots $\hat{H}_{1,n}(\cdot|\mathcal{K})$, $\hat{H}_{1,n}(\cdot|\mathcal{K})^2$ and $\hat{H}_{2,n}(\cdot|\mathcal{K}')$ for $\mathcal{K} = \mathcal{K}' = \{1, \ldots, 10\}$ (low BM classes) and for $\mathcal{K} = \mathcal{K}' = \{11, \ldots, 20\}$ (high BM classes). We find some evidence that $H_2 > H_1^2$ in low classes, and that $H_2 < H_1^2$ in high classes. From Abbring, Chiappori, and Pinquet's (2003) analysis and Prediction 2, we may expect the opposite rankings under moral hazard.[21] However, none of the Kolmogorov-Smirnov tests for $H_2 = H_1^2$ with different choices of $\mathcal{K}$ and $\mathcal{K}'$ rejects the null; see Table 7. This is consistent with Abbring and Zavadil's (2008) finding that nonparametric state-dependence tests, unlike Section 4.2's structural test, have little power with data on rare events.

4.3.4 Joint Distribution of First and Second Claim Durations

So far, we have only compared marginal distributions of first and second claim times. Intuitively, much can be gained by comparing first and second claim times within contracts, that is, by studying the *joint* distribution of first and second claim times. Thus, we compare the time of the first claim T_1 and the time between the first and the second claim $T_2 - T_1$ in the subpopulation with exactly two claims in the contract year.

[21] Abbring, Chiappori, and Pinquet (2003) focus on local behavior near the null. Abbring and Zavadil (2008) show that the global implications are less clear-cut. This may also explain some of this result.

Assuming stationarity ($\psi \equiv 1$) and under the null of no moral hazard, we have that

$$\Pr(T_1 \geq T_2 - T_1 | N(1-) = 2, K(0) \in \mathcal{K}) = \frac{1}{2}$$

for all $\mathcal{K}$. Under moral hazard, on the other hand, we would expect this probability to be larger than 1/2 in low classes, where incentives jump down after the first claim, and smaller than 1/2 in high classes, where incentives jump up. Note that here we again use that these jumps in incentives dominate the changes in incentives over time.

Thus, under stationarity ($\psi \equiv 1$), a test for moral hazard can be based on the share of contracts in classes in $\mathcal{K}$ with two claims for which the time to the first claim is larger than the time between the first and the second,

$$\hat{\pi}_n(\mathcal{K}) = \frac{1}{M_{2,\mathcal{K},n}} \sum_{i=1}^{n} I(T_{i1} \geq T_{i2} - T_{i1}, N_i(1-) = 2, K_i(0) \in \mathcal{K}).$$

Under the null of no moral hazard, $\hat{\pi}_n(\mathcal{K})$ is asymptotically normal with mean 1/2 and variance $1/(4n_{\mathcal{K}}P_{2,\mathcal{K}})$, where $n_{\mathcal{K}}$ is the total number of contracts in all classes from $\mathcal{K}$, and $P_{k,\mathcal{K}}$ is more generally the probability that a contract in a class in $\mathcal{K}$ has k claims in the contract year. The variance of $\hat{\pi}_n(\mathcal{K})$ can be consistently estimated by $1/(4M_{2,\mathcal{K},n})$.

Another test for moral hazard under stationarity can be based on

$$\widehat{\ln \beta_n}(\mathcal{K}) = \frac{1}{M_{2,\mathcal{K},n}} \sum_{i=1}^{n} \ln\left(\frac{T_{i1}}{T_{i2} - T_{i1}}\right) I(N_i(1-) = 2, K_i(0) \in \mathcal{K})$$

which is asymptotically normal under the null of no moral hazard, with expectation 0 and variance $\pi^2/(3n_{\mathcal{K}}P_{2,\mathcal{K}})$. The variance can be consistently estimated by $\pi^2/(3M_{2,\mathcal{K},n})$.

The first two columns of Table 8 give $\hat{\pi}_n(\mathcal{K})$ and $\widehat{\ln \beta_n}(\mathcal{K})$ with their estimated standard errors for various choices of $\mathcal{K}$. The two statistics' values, and their variation with classes, are consistent with moral hazard. However, the null of no moral hazard is not rejected at a 5% level, because the small numbers of observations imply low precision. This is consistent with Abbring and Zavadil's (2008) result that these tests have limited power with data on rare events.

Precision can be increased by pooling classes at both ends of the bonus-malus scheme, but reversing the comparison for the high classes. For example, we can use

$$\begin{aligned}\hat{\pi}_n(\mathcal{K}_L, \mathcal{K}_H) = & \frac{1}{M_{2,\mathcal{K}_L \cup \mathcal{K}_H, n}} \sum_{i=1}^{n} [I(T_{i1} \geq T_{i2} - T_{i1}, N_i(1-) = 2, K_i(0) \in \mathcal{K}_L) + \\ & I(T_{i1} \leq T_{i2} - T_{i1}, N_i(1-) = 2, K_i(0) \in \mathcal{K}_H)],\end{aligned}$$

with $\mathcal{K}_L$ and $\mathcal{K}_H$ disjoint sets of low and high bonus-malus classes, respectively. Under moral hazard, we would expect this share to be larger than 1/2. Therefore, we can use one-sided test.

The first two columns of Table 9 give the values of $\hat{\pi}_n(\mathcal{K}_L, \mathcal{K}_H)$ and a similar variant $\widehat{\ln \beta_n}(\mathcal{K}_L, \mathcal{K}_H)$ of $\widehat{\ln \beta_n}(\mathcal{K})$. We expect the latter to be positive under moral hazard. The results are again consistent with moral hazard, now with some rejections of the null at a 5% level in very high and very low BM classes.

Abbring, Chiappori, and Pinquet (2003) develop a variant $\hat{\pi}_n^*(\mathcal{K}|\mathcal{K}')$ of the statistic $\hat{\pi}_n(\mathcal{K})$ that allows for general external time effects ψ. Adapted to our setting, it compares the transformed durations $H_1(T_1|\mathcal{K}')$ and $H_1(T_2|\mathcal{K}') - H_1(T_1|\mathcal{K}')$ in the subsample with classes in $\mathcal{K}$. As before, $\mathcal{K}$ and $\mathcal{K}'$ can be wisely chosen to maximize power.

Proposition 7 in Abbring, Chiappori, and Pinquet implies that, under the null of no moral hazard, $\hat{\pi}_n^*(\mathcal{K}|\mathcal{K}')$ is asymptotically normal with expectation 1/2 and variance $1/(4n_{\mathcal{K}} P_{2,\mathcal{K}}) + 1/(6n_{\mathcal{K}'} P_{1,\mathcal{K}'})$, which can be consistently estimated by $1/(4M_{2,\mathcal{K},n}) + 1/(6M_{1,\mathcal{K}',n})$. The last two columns of Tables 8 and 9 plot the values of $\hat{\pi}_n^*$ with the estimated standard errors for different bonus-malus classes. First we estimated H_1 using all bonus-malus classes (taking $\mathcal{K}' = \{1, \ldots, 20\}$) and then using only the tested (current) bonus-malus classes (taking $\mathcal{K}' = \mathcal{K}$). The values of $\hat{\pi}_n^*(\mathcal{K}|\mathcal{K}')$ statistic and their variation with classes, are again consistent with moral hazard. However, the null of no moral hazard is not rejected at a 5% level, because the π_n^* test has lower power than the π_n test

(Abbring and Zavadil, 2008).

4.3.5 Claim Sizes

As discussed in Section 4.3.1, the jumps in incentives at the time of a claim dominate the variation in incentives over time. Therefore, in comparing claim sizes within a contract year, we can focus on Prediction 2's occurrence-dependence effects, and ignore Prediction 3's time effects.

This facilitates a test for ex post moral hazard based on a comparison of the sizes of agents' first and second claims in a contract year, even though these occur at different times. Under ex post moral hazard the size L_2 of a second claim in a contract year is stochastically larger than the size L_1 of a first claim in high classes where incentives jump up after the first claim. On the other hand, first claim sizes are stochastically larger than second claim sizes in low classes.

Under the null of no moral hazard, first and second claim sizes share the same distribution $F(\cdot|\lambda)$. Table 10 reports p-values of Wilcoxon and sign tests for this hypothesis against one-sided and two-sided alternatives, using subsamples of contracts with two or more claims in different bonus-malus classes. They suggest that L_1 and L_2 are not identically distributed. In particular, the second claim is stochastically larger in the subpopulation in higher classes. This is consistent with ex post moral hazard: Agents in high classes K increase their claiming thresholds x^* after experiencing a jump up in their incentives at the time of their first claim.

4.4 Claim Withdrawals

So far, we have ignored withdrawn claims. We will now argue that withdrawals are directly informative on moral hazard, and present some evidence.

Suppose that it takes time for loss amounts to be assessed, so that agents have to file a claim before the loss amount is fully known. Furthermore, suppose that there are no costs – administrative or informational – of filing and withdrawing claims. Then, agents will report all losses to the insurer to secure an option on compensation, and typically withdraw those claims for losses that fall below the threshold. Our data on claims and withdrawals are thus directly informative on ex post moral hazard (withdrawals), and ex ante moral hazard (initial claims). If we relax our assumptions, some ex post moral hazard will end up reducing initial claims. In any case, the mere fact that some claims are withdrawn in the sample points to the evidence of ex post moral hazard.

Under the null of no ex-post moral hazard, agents will claim all accidents to the insurer and withdraw only those which damage falls below the level of deductible. The agent's decision whether to withdraw a claim or not will therefore depend only on the size of a damage and not on the bonus-malus class. Consequently, the shares of withdrawn claims should be roughly the same among all BM classes.

Figure 9 plots the shares of withdrawn claims for each bonus-malus class. We observe that the shares are small for low and high bonus-malus classes and big for the bonus-malus classes in between. This is consistent with the incentives to avoid a first $(\Delta V(1, K, 1))$ and a second $(\Delta V(1, K, 2))$ claim that we presented in the Figure 2.

5 Conclusion

Putting novel theoretical insights into the dynamic incentives implied by experience rating to empirical use, we find evidence of moral hazard in Dutch car insurance. The earlier literature often fails to find such evidence.

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Appendices

A Proofs of Results in Section 3

Lemma 1. *The value function V is strictly increasing in wealth W.*

Proof. Consider a state (t, W, K, N) and denote the (stochastic) optimal consumption-prevention-claim plan following this state by $(c^*, p^*, \mathcal{X}^*)$. Then, in state (t, W', K, N) with $W' > W$, the agent can attain an expected discounted utility equal to $V(t, W, K, N)$ by following the same plan $(c^*, p^*, \mathcal{X}^*)$. Because consuming $c^* + \rho(W' - W) > c^*$ is feasible and instantaneous utility u is strictly increasing, $V(t, W', K, N) > V(t, W, K, N)$. □

Lemma 2. *The value function V is weakly increasing in the bonus-malus class K and weakly decreasing in the number of claims at fault N.*

Proof. Consider a state (t, W, K, N) and denote the (stochastic) optimal consumption-prevention-claim plan following this state by $(c^*, p^*, \mathcal{X}^*)$. Then, in state (t, W, K', N') with $K' \geq K$ and $N' \leq N$, the agent can attain an expected discounted utility equal to $V(t, W, K, N)$ by following the same plan $(c^*, p^*, \mathcal{X}^*)$. In this case future insurance premia are weakly smaller, in the sense of stochastic dominance, than under optimal behavior in state (t, W, K, N), because $B(K, N)$ is weakly increasing in K and weakly decreasing in N, premia are weakly decreasing in K, and the agent faces the same distribution of future claims. Therefore, choosing $(c^*, p^*, \mathcal{X}^*)$ in state (t, W, K, N) is feasible and, indeed, $V(t, W, K', N') \geq V(t, W, K, N)$. □

Proof of Proposition 2. First, note that the proposition's specifications of the consumption rule and value function satisfy the Euler equation:

$$u'(c^*(S)) = e^{-\alpha\rho[W - Q(t,K,N)]} = V_W(S)$$

Second, note that $\rho V(S) = u(c^*(S))$, so that Bellman equation (4) is satisfied if

$$0 = p_0\Big[V(t, W, K, N+1) - V(S)\Big] + V_W(S)\left[\rho W - c^*(S) - A(K)\right] + V_t(S).$$

Because

$$V(t, W, K, N+1) - V(S) = e^{-\alpha\rho[W - Q(t,K,N)]}\left(\frac{1 - e^{\alpha\rho[Q(t,K,N+1) - Q(t,K,N)]}}{\alpha\rho}\right),$$

$$V_W(S)\left[\rho W - c^*(S) - A(K)\right] = e^{-\alpha\rho[W - Q(t,K,N)]}\left[\rho Q(t, K, N) - A(K)\right], \text{ and}$$

$$V_t(S) = -e^{-\alpha\rho[W - Q(t,K,N)]} Q_t(t, K, N),$$

this is guaranteed by equation (10). Third, the Bellman equation's premium renewal conditions (3) are satisfied by equation (11):

$$\begin{aligned} V(1, W, K, N) &= \frac{1 - e^{-\alpha\rho[W - Q(1,K,N)]}}{\alpha\rho} \\ &= \frac{1 - e^{-\alpha\rho[W - Q(0,B(K,N),0)]}}{\alpha\rho} = V(0, W, B(K, N), 0). \end{aligned}$$

Finally, using standard methods it can be proved that there exists a unique solution Q to the system (10)–(11). □

B Computation of Proposition 2's Function Q

Let A and B be given by Table 1 and attach some values to the parameters ρ, α, and p_0.

In the limiting case $\alpha \downarrow 0$, the corresponding initial-value problem (10)–(11) has an explicit analytical solution Q. In particular, the initial values $Q(0, \cdot, 0)$ can be computed directly using

$$\begin{pmatrix} Q(0,1,0) \\ \vdots \\ Q(0,20,0) \end{pmatrix} = \frac{1-e^{-\rho}}{\rho}(I - e^{-\rho}T)^{-1} \begin{pmatrix} \pi(1) \\ \vdots \\ \pi(20) \end{pmatrix}. \tag{16}$$

Here, I is the 20×20 identity matrix and T is the annual transition probability matrix among bonus-malus classes implied by p_0 and B. The solution Q then satisfies the recursive system

$$\begin{aligned}
Q(t,K,N) &= \frac{\pi(K)}{\rho} + e^{-\rho(1-t)}\left[Q(0,1,0) - \frac{\pi(K)}{\rho}\right] \quad \text{for } N \geq 3, \\
Q(t,K,2) &= Q(t,K,3) + e^{-(p_0+\rho)(1-t)}[Q(0,B(K,2),0) - Q(0,1,0)], \\
Q(t,K,1) &= Q(t,K,2) + e^{-(p_0+\rho)(1-t)}\{Q(0,B(K,1),0) - Q(0,B(K,2),0) \\
&\quad + p_0(1-t)[Q(0,B(K,2),0) - Q(0,1,0)]\}, \text{ and} \\
Q(t,K,0) &= Q(t,K,1) + e^{-(p_0+\rho)(1-t)}\{Q(0,B(K,0),0) - Q(0,B(K,1),0) \\
&\quad + p_0(1-t)[Q(0,B(K,1),0) - Q(0,B(K,2),0)] \\
&\quad + \frac{1}{2}p_0^2(1-t)^2[Q(0,B(K,2),0) - Q(0,1,0)]\}.
\end{aligned}$$

In the general case, the function Q can be computed iteratively using

Algorithm 1. *Give starting values to $Q(0,K,0)$, $K = 1,\ldots,20$, and repeat*

1. *set $Q^*(0,K,0) = Q(0,K,0)$, $K = 1,\ldots,20$;*

2. *for $K = 1,\ldots,20$,*

   (a) *for $N \geq 3$, set*

   $$Q(t,K,N) = \frac{\pi(K)}{\rho} + e^{-\rho(1-t)}\left[Q(0,1,0) - \frac{\pi(K)}{\rho}\right];$$

   (b) *for $N = 2,1,0$, set $Q(\cdot,K,N)$ to the numerical solution of the corresponding* *single-equation initial-value problem in (10)–(11);*

until $\max_K |Q(0, K, 0) - Q^*(0, K, 0)| \leq \varepsilon$*, for some small* $\varepsilon > 0$*.*

The values $Q(0, \cdot, 0)$ for the linear-utility case, those that satisfy (16), can be used as starting values in Algorithm 1. Note that in the linear-utility case itself, this produces Q in one iteration; in cases with $\alpha > 0$, more iterations are typically needed. If we have to compute Q for multiple values of α, we can use the linear-utility values of $Q(0, \cdot, 0)$ as starting values for the computations with the lowest value of α, the resulting $Q(0, \cdot, 0)$ as starting values for the computations with the second-lowest value of α, etcetera.

C Data

Recall from Section 2.2 that the data provide contract and claim histories of personal car insurance clients of a major Dutch insurer from January 1, 1995 to December 31, 2000. All data, except information on claim withdrawals, came in a single file with 1,730,559 records for 163,194 unique contracts. A second file provided information on the withdrawal of claims by agents after they were filed. Recall from Section 2 that agents had the option to avoid a malus after filing a claim by timely withdrawing it.

We excluded information on the year 1995, because it lacked information on claims. From the remaining 142,175 contracts, we deleted 1,376 contracts that were not subject to the bonus-malus system. We also deleted 16,778 contracts with unobserved renewal date. Most of these contracts started in 1995 and did not renew in 1996. Many were also short-term contracts covering only a couple of weeks or months. This left a sample of 124,021 contracts.

We matched the second file's withdrawal information to the main data set based on contract and claim identifiers, but this matching was not complete. This is important, because consistency of the claim and BM information is crucial to this paper's empirical analysis. The remainder of this section discusses the ways we enforced such consistency by correcting the claim withdrawal and BM information, and checked for our empirical work's robustness to these corrections.

First, few BM transitions in 2000 were recorded correctly. Therefore, we truncated all contract histories that were renewed in 2000 at the 2000 renewal date. This cut another 219 contracts that were first observed in 2000 from the sample, leaving 123,802 unique contracts.

Of these 123,802 contracts, 103,930 are observed for more than one year. For each such contract we observe the sequence of BM classes in consecutive contract years, with the number of claims at fault that were not withdrawn in each contract year. For 14,206 contracts, we observed one or more deviations from Table 1's BM updating rule.

Many of these deviations can be explained by unobserved withdrawals, which may exist because observed withdrawals could not be perfectly matched to the main data file. For example, some contracts were awarded a bonus after a contract year with a claim. This is only consistent with the BM system if the claim was withdrawn. Therefore, we decided to treat those claims as (unobserved) withdrawals. Consequently, we excluded them from the sample. All in all, we found 1,355 unobserved withdrawals in the sample constructed so far.

Even after excluding unobserved withdrawals, the sample still contained incorrect BM transitions. We corrected these anomalies by constructing the most appropriate BM class for the first contract year— that is, the class that minimized changes to the raw data— and deriving the BM classes in all consecutive contract years from this initial BM class and claims using the BM updating rule.

Of the 14,325 contracts observed for two years only, 1,269 have an incorrect BM transition. In these 1,269 cases, we simply set the BM class in the second year to be consistent with the BM class and the number of claims observed in the first year.

For most contracts with inconsistencies observed for more than 2 years, a BM sequence based on the first year's BM class delivered the best fit to the observed BM sequence. A single inconsistency in the middle of a BM sequence, sandwiched between consistent BM classes, also often occurred. Then, we simply corrected this single BM class using the previous year's BM class and claim information.

In some cases with more than two years of data, the first year's BM class was inconsistent with the BM classes in all later years. Then, we forced the first year's BM class to be consistent with the first BM class later in the sequence that was consistent with later BM classes and claims. Because the BM updating rule in Table 1 is not a one-to-one mapping, there were often more consistent choices of a first BM class. For example, an agent who was downgraded to BM class 1 in the second contract year after having one claim in the first, could have been in any of the BM classes 1 – 5 in the first year. In these cases, we chose the highest consistent BM class. In a very few cases we were not able to correct the first year's BM class this way. For example, no choice of a first year's BM class is consistent with a claim in the first year and a BM class 15 or higher in the second year. We deleted 12 such contracts, observed for 4 years, from the sample.

Finally, we deleted 621 contracts that were observed for more than 2 years and had only inconsistent BM transitions. This leaves a final sample with 123,169 unique contracts and 23,396 claims at fault that were not withdrawn. All empirical results reported in this paper are based on this sample.

We checked the robustness of these results with respect to the ways we have selected the sample and corrected the BM classes and claim withdrawal information by recomputing all results on different samples employing different ways of correcting for inconsistencies. First, we used an alternative sample that included only observations with consistent raw data on claims and BM classes. No corrections were applied to these data. Second, we used a sample that was alternatively corrected for inconsistent BM information by deriving all BM sequences from the initial BM classes observed, using the BM updating rule. Third, we used the main, corrected sample, but included all withdrawn claims as claim-at-fault events. We find that the results reported in this paper are robust. Tables and figures are available from the authors upon request.

Figure 1: Distribution of Contracts Observed for At Least One Full Contract Year Across Bonus-Malus Classes; and Shares of Those Contracts with At Least One and At Least Two Claims at Fault in the First Contract Year, by Bonus-Malus Class

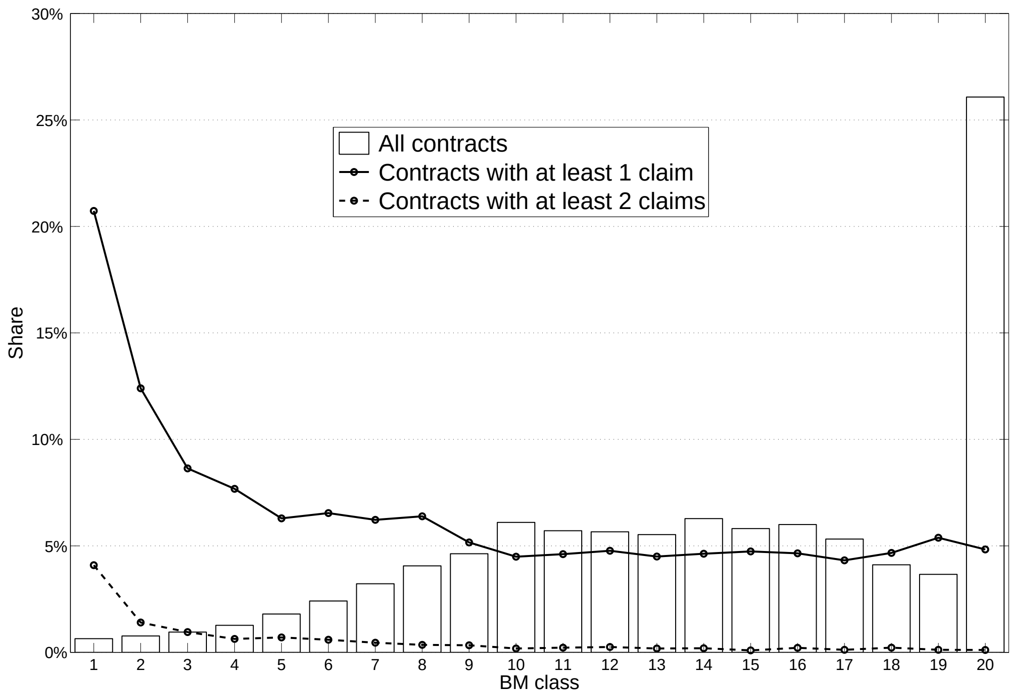
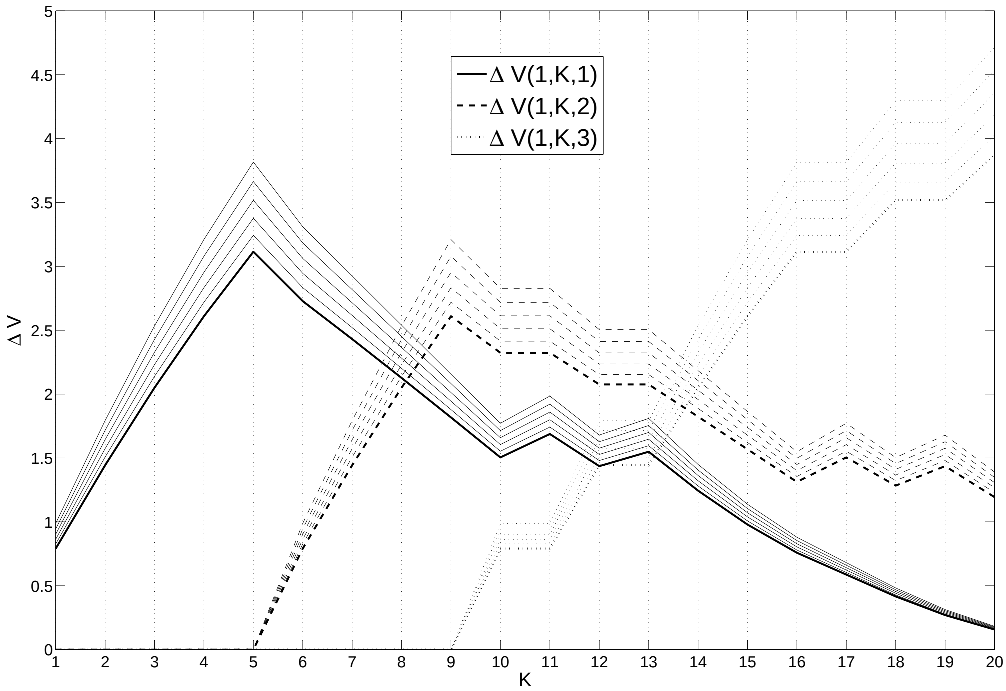
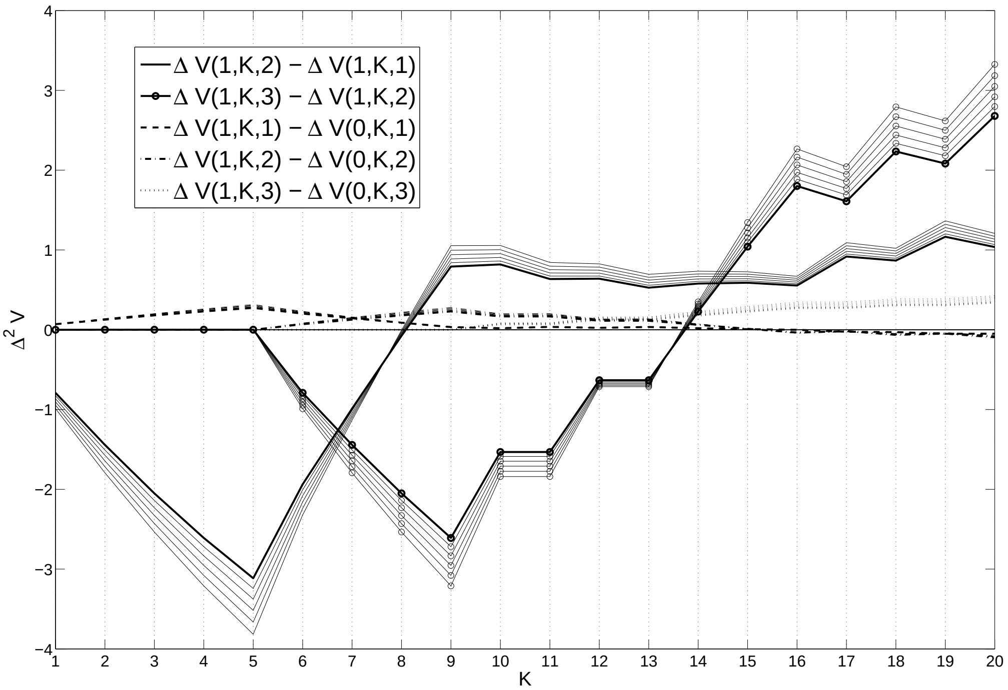


Figure 2: Incentives to Avoid First, Second and Third Claim; at an Average Risk Level



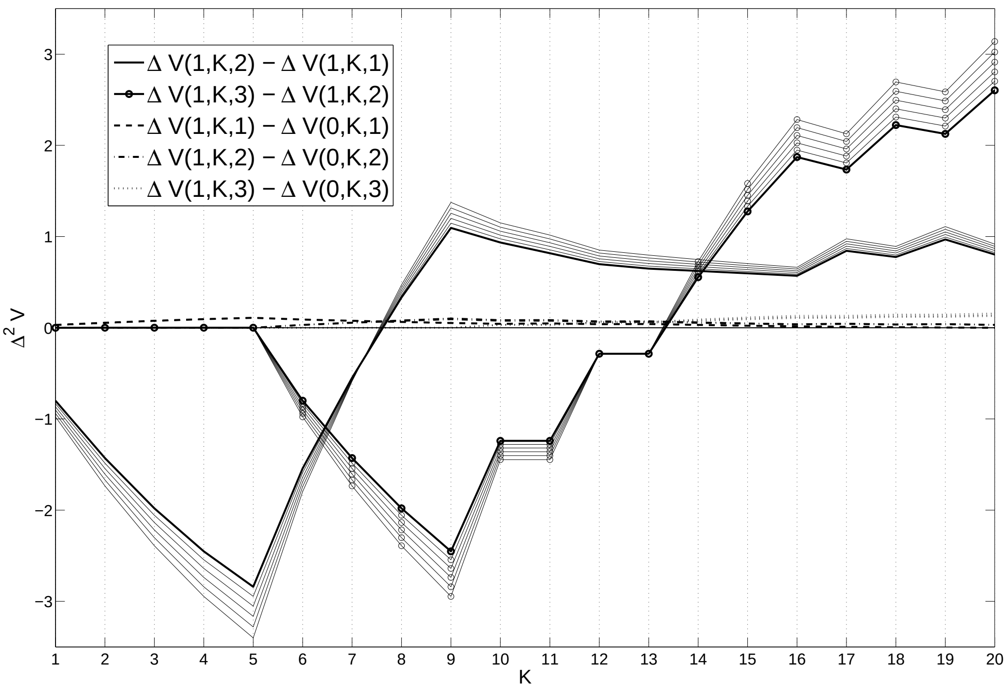
Note: This figure plots $\Delta V(1, K, N)$ for $N = 1, 2, 3$ as functions of K for the CARA case without moral hazard, for $p_0 = 0.053$ and different values of the coefficient of absolute risk aversion α. The premium $\pi(K)$ is measured in multiples of the base premium, as in Table 1. The bold graphs correspond to the linear-utility case $\alpha = 0$ and give the expected discounted premium cost of a claim in terms of the base premium. The other graphs correspond to $\alpha = 0.1, 0.2, \ldots, 0.5$, in that order and with the graphs corresponding to $\alpha = 0.1$ closest to the bold graph. At a consumption level equal to 20 times the base premium, $\alpha = 0, 0.1, \ldots, 0.5$ correspond to coefficients of *relative* risk aversion equal to $0, 2, \ldots, 10$, respectively.

Figure 3: Change in Incentives to Avoid a Claim after a First and a Second Claim, and Changes in Incentives to Avoid a First, a Second and a Third Claim over the Course of a Contract Year; at an Average Risk Level



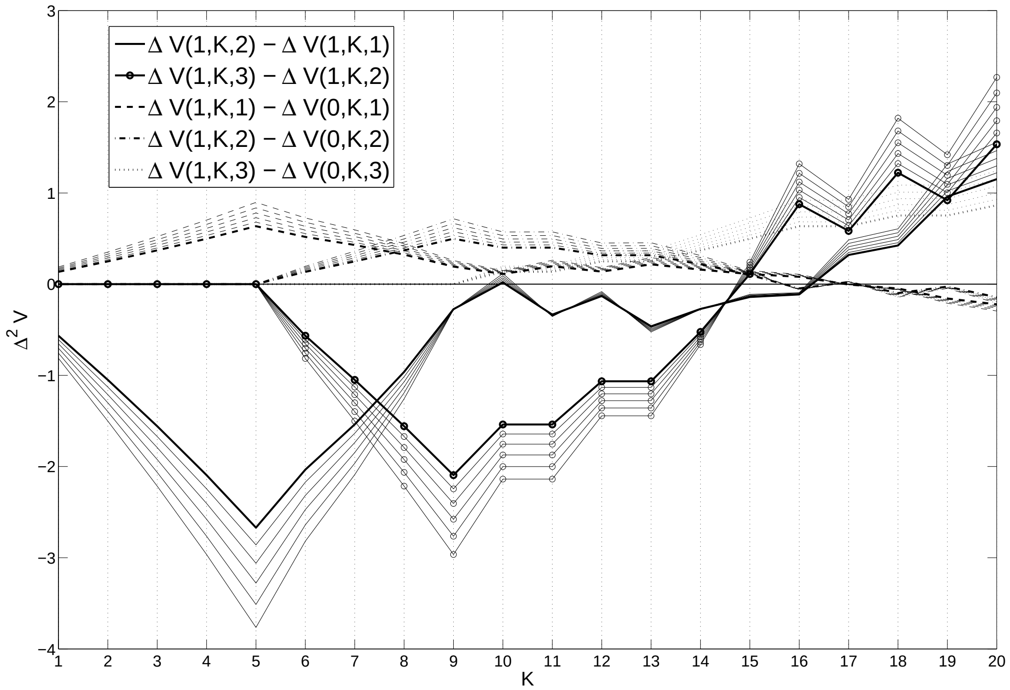
Note: This figure plots $\Delta V(1,K,N+1) - \Delta V(1,K,N)$ for $N = 1,2$, and $\Delta V(1,K,N) - \Delta V(0,K,N)$ for $N = 1,2,3$ as functions of K for the CARA case without moral hazard, for $p_0 = 0.053$ and different values of the coefficient of absolute risk aversion α. The premium $\pi(K)$ is measured in multiples of the base premium, as in Table 1. The bold graphs correspond to the linear-utility case $\alpha = 0$ and give the expected discounted premium cost of a claim in terms of the base premium. The other graphs correspond to $\alpha = 0.1, 0.2, \ldots, 0.5$, in that order and with the graphs corresponding to $\alpha = 0.1$ closest to the bold graph. At a consumption level equal to 20 times the base premium, $\alpha = 0, 0.1, \ldots, 0.5$ correspond to coefficients of *relative* risk aversion equal to $0, 2, \ldots, 10$, respectively.

Figure 4: Change in Incentives to Avoid a Claim after a First and a Second Claim, and Changes in Incentives to Avoid a First, a Second and a Third Claim over the Course of a Contract Year; at a Zero Risk Level



Note: This figure plots $\Delta V(1, K, N+1) - \Delta V(1, K, N)$ for $N = 1, 2$, and $\Delta V(1, K, N) - \Delta V(0, K, N)$ for $N = 1, 2, 3$ as functions of K for the CARA case without moral hazard, for $p_0 = 0$ and different values of the coefficient of absolute risk aversion α. The premium $\pi(K)$ is measured in multiples of the base premium, as in Table 1. The bold graphs correspond to the linear-utility case $\alpha = 0$ and give the expected discounted premium cost of a claim in terms of the base premium. The other graphs correspond to $\alpha = 0.1, 0.2, \ldots, 0.5$, in that order and with the graphs corresponding to $\alpha = 0.1$ closest to the bold graph. At a consumption level equal to 20 times the base premium, $\alpha = 0, 0.1, \ldots, 0.5$ correspond to coefficients of *relative* risk aversion equal to $0, 2, \ldots, 10$, respectively.

Figure 5: Change in Incentives to Avoid a Claim after a First and a Second Claim, and Changes in Incentives to Avoid a First, a Second and a Third Claim over the Course of a Contract Year; at a High Risk Level



Note: This figure plots $\Delta V(1, K, N+1) - \Delta V(1, K, N)$ for $N = 1, 2$, and $\Delta V(1, K, N) - \Delta V(0, K, N)$ for $N = 1, 2, 3$ as functions of K for the CARA case without moral hazard, for $p_0 = 0.232$ and different values of the coefficient of absolute risk aversion α. The premium $\pi(K)$ is measured in multiples of the base premium, as in Table 1. The bold graphs correspond to the linear-utility case $\alpha = 0$ and give the expected discounted premium cost of a claim in terms of the base premium. The other graphs correspond to $\alpha = 0.1, 0.2, \ldots, 0.5$, in that order and with the graphs corresponding to $\alpha = 0.1$ closest to the bold graph. At a consumption level equal to 20 times the base premium, $\alpha = 0, 0.1, \ldots, 0.5$ correspond to coefficients of *relative* risk aversion equal to $0, 2, \ldots, 10$, respectively.

Figure 6: Estimated Probability Masses $\xi_r(K)$ of the Auxiliary Model (14) with Three Mass Points

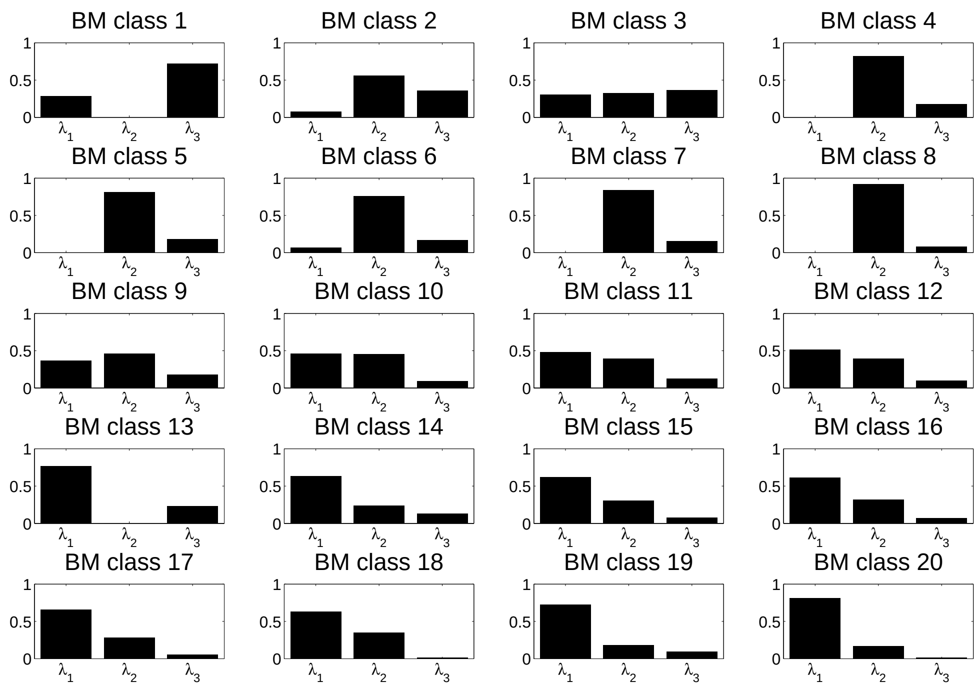


Figure 7: Comparison of $\hat{H}_1$ with the Uniform Distribution for Low and High Bonus-Malus Classes

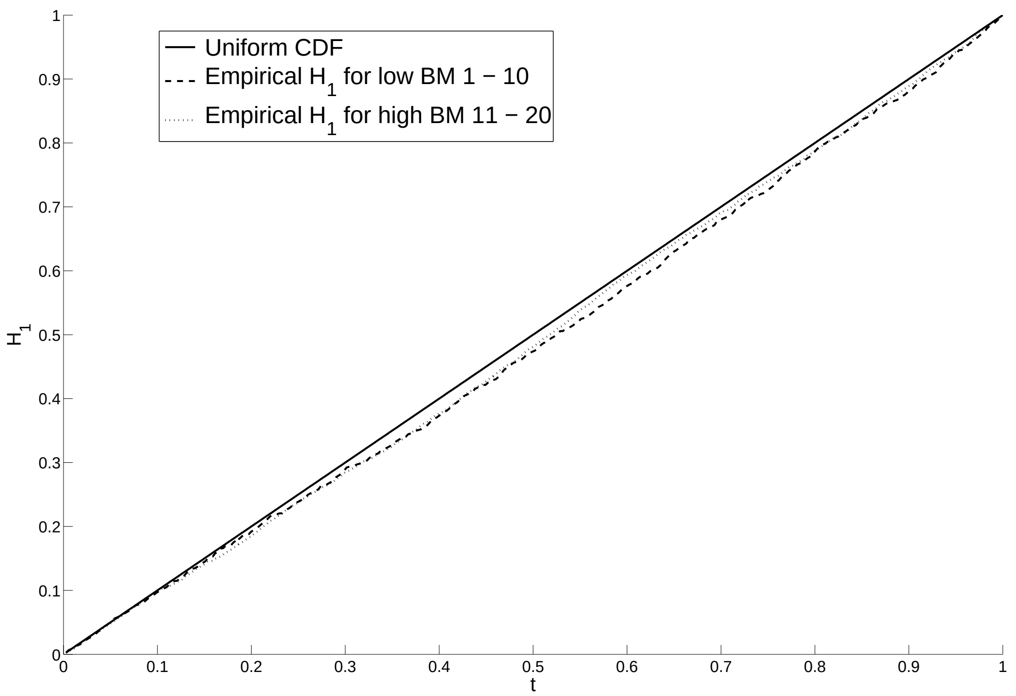


Figure 8: Comparison of $\hat{H}_1$ with the Uniform Distribution and of $\hat{H}_1^2$ with $\hat{H}_2$ for Low and High Bonus-Malus Classes, with $\hat{H}_1$ and $\hat{H}_2$ Estimated on the Same Classes

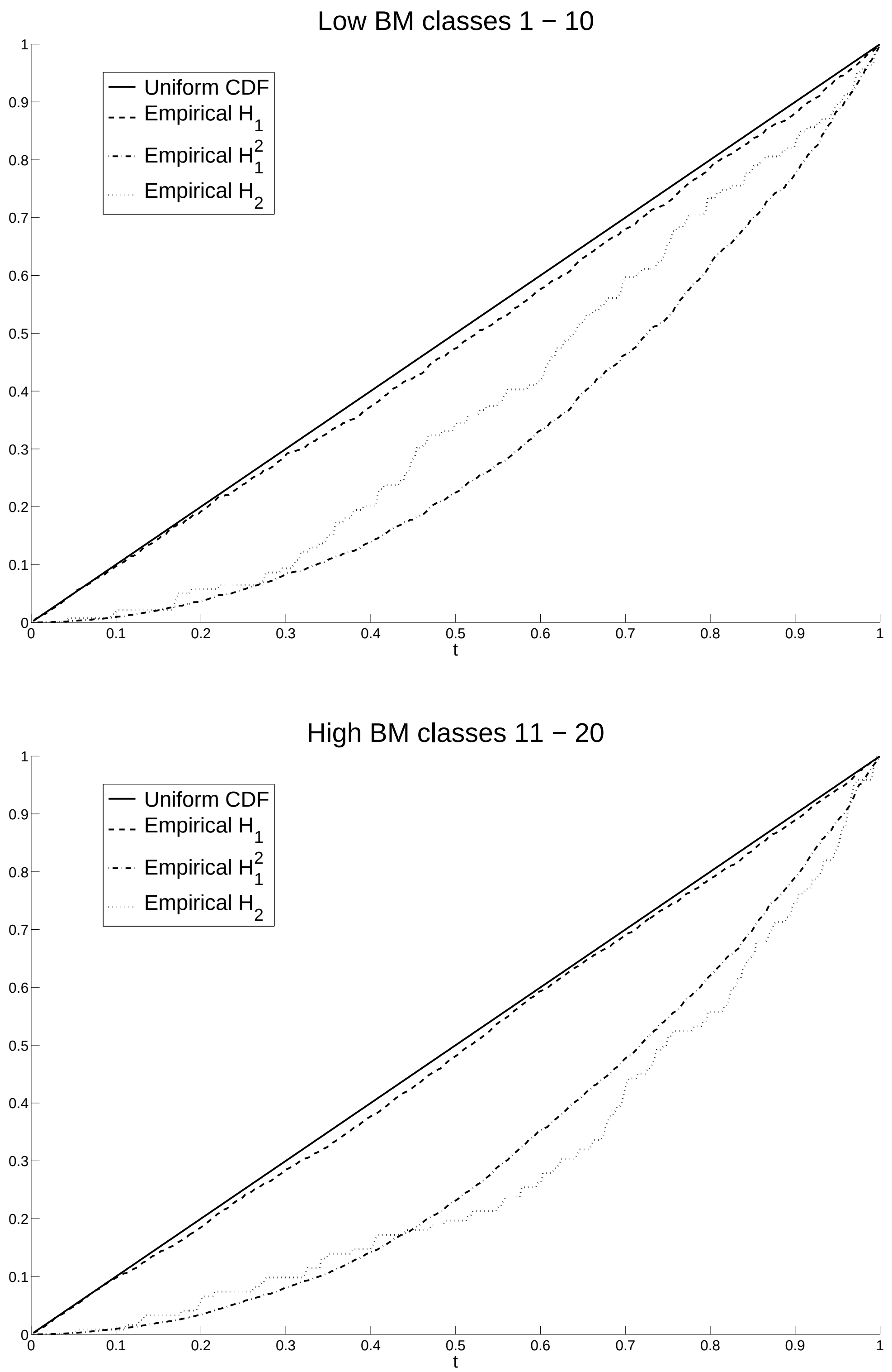
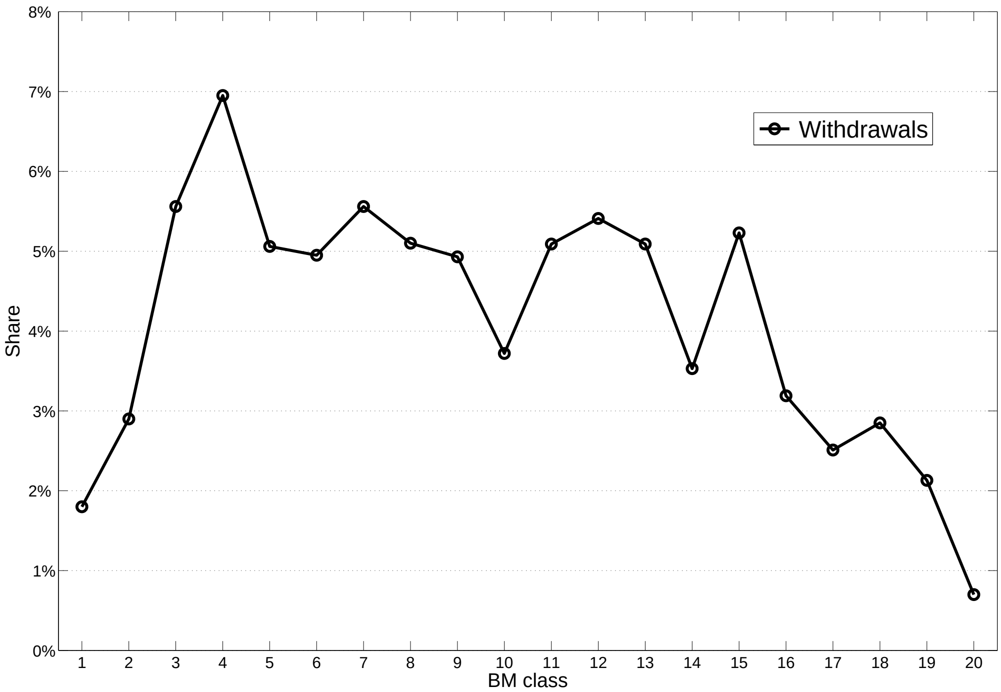


Figure 9: Share of Withdrawn Claims per Bonus-Malus Class



Note: This graph only includes withdrawals that are directly observed. See Appendix C.

Table 1: Bonus-Malus Scheme

Present BM class (K)	**Premium paid ($\mathbf{q = A(K)}$)**	**Future BM class ($\mathbf{B(K, N)}$) after a contract year with no claim ($\mathbf{N = 0}$)**	**1 claim ($\mathbf{N = 1}$)**	**2 claims ($\mathbf{N = 2}$)**	**3 or more claims ($\mathbf{N \geq 3}$)**
20	25%	20	14	8	1
19	25%	20	13	7	1
18	25%	19	12	7	1
17	25%	18	11	6	1
16	25%	17	10	6	1
15	25%	16	9	5	1
14	25%	15	8	4	1
13	30%	14	7	3	1
12	35%	13	7	3	1
11	37.5%	12	6	2	1
10	40%	11	6	2	1
9	45%	10	5	1	1
8	50%	9	4	1	1
7	55%	8	3	1	1
6	60%	7	2	1	1
5	70%	6	1	1	1
4	80%	5	1	1	1
3	90%	4	1	1	1
2	100%	3	1	1	1
1	120%	2	1	1	1

Note: The notation in parentheses is taken from Section 3's model.

Table 2: Percentage Premium Change after a Claim-Free Contract Year and Marginal Percentage Changes in the Premium after each Claim, by Bonus-Malus Class

Present BM class (K)	**Premium change if no claim (N = 0)**	**Increase in premium after 1st claim (N = 1)**	**2nd claim (N = 2)**	**3rd claim (N = 3)**
20	0%	0%	100%	140%
19	0%	20%	83%	118%
18	0%	40%	57%	118%
17	0%	50%	60%	100%
16	0%	60%	50%	100%
15	0%	80%	56%	71%
14	0%	100%	60%	50%
13	-17%	120%	64%	33%
12	-14%	83%	64%	33%
11	-7%	71%	67%	20%
10	-6%	60%	67%	20%
9	-11%	75%	71%	0%
8	-10%	78%	50%	0%
7	-9%	80%	33%	0%
6	-8%	82%	20%	0%
5	-14%	100%	0%	0%
4	-13%	71%	0%	0%
3	-11%	50%	0%	0%
2	-10%	33%	0%	0%
1	-17%	20%	0%	0%

Note: The notation in parentheses and below is taken from Section 3's model. The second column reports

$$\frac{\text{New premium after claim-free year} - \text{Old premium}}{\text{Old premium}} = \frac{A\left[B(K,0)\right] - A(K)}{A(K)}$$

for each bonus-malus class K. The third, fourth and fifth columns report

$$\frac{A\left[B(K,N)\right] - A\left[B(K,N-1)\right]}{A\left[B(K,N-1)\right]}$$

for respectively $N = 1, 2, 3$, for all bonus-malus classes K. Here, $A\left[B(K,N)\right]$ is the new premium after a year in class K with N claims.

Table 3: Contract Exposure Durations in the Sample

Number of years	**Number of contracts observed**		
Y	**exactly Y years**	**between $Y-1$ and Y years**	**Total**
1	8,097	11,775	19,872
2	4,709	9,616	14,325
3	6,262	7,387	13,649
4	68,820	6,503	75,323
Total	87,888	35,281	**123,169**

Table 4: Number of Contracts Observed for At Least One Full Contract Year, by Bonus-Malus Class and Number of Claims in the First Contract Year

BM	**Number of contracts with**					
class	**no claim**	**1 claim**	**2 claims**	**3 claims**	**4 claims**	**Total**
1	562	118	24	4	1	709
2	749	94	11	1		855
3	962	81	10			1,053
4	1,311	100	9			1,420
5	1,876	112	13	1		2,002
6	2,514	160	14	2		2,690
7	3,363	207	16			3,586
8	4,232	273	16			4,521
9	4,889	249	15	2		5,155
10	6,490	293	11		1	6,795
11	6,063	279	12	2		6,356
12	6,004	285	16			6,305
13	5,879	266	11			6,156
14	6,669	311	13			6,993
15	6,165	301	6			6,472
16	6,377	297	13	1		6,688
17	5,671	249	7			5,927
18	4,367	204	10			4,581
19	3,855	214	5			4,074
20	27,652	1,373	29	2		29,056
Total	105,650	5,466	261	15	2	111,394

Note: Nil and withdrawn claims were excluded from the sample.

Table 5: Maximum-Likelihood Estimation of the Auxiliary Model (14) with Three, Four, and Five Support Points

Three Support Points

Parameter	Estimate	Std. Error
β	0.4229	0.0428
λ_1	0.0427	0.0050
λ_2	0.1770	0.0254
λ_3	0.3514	0.0234

Tests of $\beta = 0$

- LM test: 26.14, p-value = 0.00
- LR test: 89.74, p-value = 0.00
- Wald test: 97.54, p-value = 0.00

Four Support Points

Parameter	Estimate	Std. Error
β	0.3810	0.0337
λ_1	0.0000	0.0000
λ_2	0.0552	0.0074
λ_3	0.2211	0.0125
λ_4	0.8715	0.1260

Tests of $\beta = 0$

- LM test: 72.73, p-value = 0.00
- LR test: 94.96, p-value = 0.00
- Wald test: 127.90, p-value = 0.00

Five Support Points

Parameter	Estimate	Std. Error
β	0.4017	0.0390
λ_1	0.0135	0.0118
λ_2	0.0531	0.0117
λ_3	0.1889	0.0164
λ_4	0.2629	0.0171
λ_5	0.8896	0.1219

Tests of $\beta = 0$

- LM test: 76.56, p-value = 0.00
- LR test: 106.39, p-value = 0.00
- Wald test: 106.09, p-value = 0.00

Note: The left side of each panel presents maximum-likelihood estimates of the relevant parameters in the unrestricted auxiliary model (14). The right side of each panel presents Lagrange multiplier (LM), likelihood-ratio (LR) and Wald tests for moral hazard.

Table 6: Nonparametric Tests Based on Comparison of H_1 and H_2 for Different Bonus-Malus Classes

Kruskal - Wallis test	p-value
$H_1(K)$ equal for all K	0.592
$H_2(K)$ equal for all K	0.271

Wilcoxon test	p-value
$H_1(\text{low } K) \sim H_1(\text{high } K)$	0.629
$H_2(\text{low } K) \sim H_2(\text{high } K)$	**0.006**

Kolmogorov - Smirnov test	p-value
$H_1(\text{low } K) \sim H_1(\text{high } K)$	0.676
$H_2(\text{low } K) \sim H_2(\text{high } K)$	**0.004**

Note: This table is computed using a subsample of first fully-observed contract years from Table 4's sample. The values in **bold** imply rejection of the null of no moral hazard at a 5% level. Low classes are BM classes 1 – 10 and high classes are BM classes 11 – 20.

Table 7: Kolmogorov-Smirnov Test Comparing H_1 with the Uniform Distribution and H_2 with H_1^2 for Different Bonus-Malus Classes

Kolmogorov - Smirnov test			p-value
H_1(all K)	$\sim$	$Uniform$	**0.002**
H_1(low K)	$\sim$	$Uniform$	0.083
H_1(high K)	$\sim$	$Uniform$	**0.015**
H_2(all K)	$\sim$	H_1^2(all K)	0.524
H_2(low K)	$\sim$	H_1^2(low K)	0.065
H_2(high K)	$\sim$	H_1^2(high K)	0.344
H_2(low K)	$\sim$	H_1^2(high K)	0.149
H_2(high K)	$\sim$	H_1^2(low K)	0.541

Note: This table is computed using a subsample of first fully-observed contract years from Table 4's sample. The values in **bold** imply rejection of the null of no moral hazard at a 5% level. Low classes are BM classes $1-10$ and high classes are BM classes $11-20$.

Table 8: Tests Based on Comparison of First and Second Claim Durations, for Different Bonus-Malus Classes

BM	**Test statistics (std. error)**			
classes	$\hat{\pi}_n$	$\widehat{\ln \beta_n}$	$\hat{\pi}^*_n$(all)	$\hat{\pi}^*_n$(current)
1	62.5% (10.2%)	0.392 (0.370)	54.2% (10.2%)	54.2% (10.9%)
1 – 2	65.7% (8.5%)	0.570 (0.307)	60.0% (8.5%)	60.0% (8.9%)
1 – 3	62.2% (7.5%)	0.384 (0.270)	57.8% (7.5%)	57.8% (7.8%)
1 – 4	55.6% (6.8%)	0.211 (0.247)	51.9% (6.8%)	53.7% (7.1%)
1 – 5	55.2% (6.1%)	0.180 (0.222)	52.2% (6.1%)	55.2% (6.4%)
1 – 6	54.3% (5.6%)	0.046 (0.202)	50.6% (5.6%)	54.3% (5.8%)
1 – 7	53.6% (5.1%)	0.086 (0.184)	50.5% (5.1%)	53.6% (5.3%)
1 – 8	53.1% (4.7%)	0.085 (0.171)	50.4% (4.7%)	53.1% (4.9%)
1 – 9	52.3% (4.4%)	0.083 (0.160)	49.2% (4.5%)	50.8% (4.6%)
1 – 10	53.2% (4.2%)	0.106 (0.154)	50.4% (4.3%)	51.1% (4.4%)
All	52.9% (3.1%)	0.107 (0.112)	50.6% (3.1%)	50.6% (3.1%)
11 – 20	52.5% (4.5%)	0.109 (0.164)	50.8% (4.6%)	50.8% (4.6%)
12 – 20	51.8% (4.8%)	0.035 (0.173)	50.0% (4.8%)	50.0% (4.8%)
13 – 20	51.1% (5.2%)	0.070 (0.187)	50.0% (5.2%)	50.0% (5.2%)
14 – 20	53.0% (5.5%)	0.132 (0.199)	51.8% (5.5%)	51.8% (5.5%)
15 – 20	51.4% (6.0%)	0.101 (0.217)	50.0% (6.0%)	50.0% (6.0%)
16 – 20	48.4% (6.3%)	-0.040 (0.227)	46.9% (6.3%)	46.9% (6.3%)
17 – 20	45.1% (7.0%)	-0.188 (0.254)	43.1% (7.0%)	43.1% (7.1%)
18 – 20	47.7% (7.5%)	-0.105 (0.273)	45.5% (7.6%)	47.7% (7.6%)
19 – 20	44.1% (8.6%)	-0.057 (0.311)	41.2% (8.6%)	44.1% (8.6%)
20	41.4% (9.3%)	-0.112 (0.337)	41.4% (9.3%)	44.8% (9.3%)

Note: This table is computed using a subsample of first fully-observed contract years from Table 4's sample. The values in **bold** imply rejection of the null of no moral hazard at a 5% level (two-sided test). In the computation of $\hat{\pi}^*_n$, we first used all bonus-malus classes to estimate H_1, and then only the tested (current) bonus-malus classes listed in the first column.

Table 9: Tests Based on Comparison of First and Second Claim Durations that Pool Low and High Bonus-Malus Classes

BM classes		**Test statistics (std. error)**			
low	**high**	$\hat{\pi}_n$	$\widehat{\ln \beta_n}$	$\hat{\pi}_n^*$(all)	$\hat{\pi}_n^*$(current)
1	20	60.4% (6.9%)	0.239 (0.249)	56.6% (6.9%)	56.6% (6.9%)
1 − 2	20	**62.5%** (6.3%)	0.363 (0.227)	59.4% (6.3%)	59.4% (6.3%)
1 − 2	19 − 20	**60.9%** (6.0%)	0.317 (0.218)	59.4% (6.0%)	59.4% (6.1%)
1 − 2	18 − 20	58.2% (5.6%)	0.311 (0.204)	57.0% (5.7%)	58.2% (5.7%)
1 − 2	17 − 20	**59.3%** (5.4%)	**0.343** (0.196)	58.1% (5.4%)	58.1% (5.5%)
1 − 3	19 − 20	**59.5%** (5.6%)	0.243 (0.204)	58.2% (5.7%)	58.2% (5.7%)
1 − 3	18 − 20	57.3% (5.3%)	0.246 (0.192)	56.2% (5.3%)	57.3% (5.4%)
1 − 4	17 − 20	55.2% (4.9%)	0.200 (0.177)	54.3% (4.9%)	54.3% (4.9%)
1 − 5	16 − 20	53.4% (4.4%)	0.112 (0.158)	52.7% (4.4%)	52.7% (4.4%)
1 − 6	15 − 20	51.7% (4.1%)	-0.022 (0.148)	50.3% (4.1%)	50.3% (4.1%)
1 − 7	14 − 20	50.6% (3.7%)	-0.014 (0.135)	49.4% (3.8%)	49.4% (3.8%)
1 − 8	13 − 20	51.2% (3.5%)	0.015 (0.126)	50.2% (3.5%)	50.2% (3.5%)
1 − 9	12 − 20	50.4% (3.2%)	0.028 (0.118)	49.6% (3.3%)	49.6% (3.3%)
1 − 10	11 − 20	50.6% (3.1%)	0.006 (0.112)	49.8% (3.1%)	49.8% (3.1%)

Note: This table is computed using a subsample of first fully-observed contract years from Table 4's sample. The values in **bold** imply rejection of the null of no moral hazard at a 5% level (one-sided test). In the computation of $\hat{\pi}_n^*$, we first used all bonus-malus classes to estimate H_1, and then only the tested (current) bonus-malus classes listed in the first column.

Table 10: Comparison of First and Second Claim Sizes for Various Bonus-Malus Classes

BM classes	**# obs.**	**Wilcoxon test** p-value	**Sign test** $L_1 \sim L_2$ **against** $L_1 \succ L_2$	$L_1 \prec L_2$	$L_1 \not\sim L_2$
1	29	0.336	0.068	0.969	0.136
1 – 2	41	0.791	0.378	0.734	0.755
1 – 3	51	0.633	0.500	0.610	1.000
1 – 4	60	0.802	0.449	0.651	0.897
1 – 5	74	0.942	0.546	0.546	1.000
1 – 6	90	0.755	0.701	0.376	0.752
1 – 7	106	0.458	0.809	0.248	0.497
1 – 8	122	0.219	0.913	0.120	0.239
1 – 9	139	0.173	0.913	0.117	0.235
1 – 10	151	0.240	0.873	0.164	0.329
All	278	**0.028**	0.993	**0.010**	**0.019**
11 – 20	127	0.061	0.994	**0.010**	**0.021**
12 – 20	113	**0.043**	0.996	**0.007**	**0.014**
13 – 20	97	**0.035**	0.993	**0.012**	**0.025**
14 – 20	86	**0.029**	0.994	**0.011**	**0.023**
15 – 20	73	0.055	0.995	**0.009**	**0.019**
16 – 20	67	**0.038**	0.998	**0.003**	**0.007**
17 – 20	53	0.090	0.986	**0.027**	0.053
18 – 20	46	0.127	0.973	0.052	0.104
19 – 20	36	0.388	0.879	0.203	0.405
20	31	0.493	0.763	0.360	0.720

Note: This table is computed using a subsample of contracts with at least two claims in the first fully-observed contract year from the Table 4's sample. The values in **bold** imply rejection of the null of no moral hazard at a 5% level.