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Published in:
Metroeconomica

Publication date:
1991

Citation for published version (APA):
OPTIMAL INTERNATIONAL DEBT AND ENDOGENOUS TIME PREFERENCE IN A DEMOGRAPHICALLY DIVIDED WORLD (*)

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(Received April 20th, 1991; final version December 9th, 1991)

ABSTRACT

The international transmission of shocks in population growth and technology is examined in an interdependent two-country world economy; decisions concerning the intertemporal allocation are made by social planners with an infinite horizon and an endogenous rate of time preference. Steady state shocks in population growth are shown to be negatively transmitted to the rate of time preference. In the steady state, the optimal foreign debt is determinate and it brings about a convergence of time preference. The net creditor and debtor positions in the steady state depend on the discount rate functions, the states of technology and the rates of population growth.

1. INTRODUCTION

Fluctuations in economic activity triggered off by productivity shocks figure prominently in research on closed and open economies (see Kydland and Prescott, 1982; Cantor and Mark, 1987 and Clarida, 1990). A much neglected determinant of economic fluctuations is the change in the demography of a country. Van Imhoff (1989) has shown for a closed economy that a permanent demographic shock can trigger off real

(*) The author gratefully acknowledges comments by Lans Bovenberg, Sukhamoy Chakravarty (who unfortunately passed away in August 1990), Gerbert Hebbink, Evert van Imhoff, an anonymous referee and conference participants at the annual meeting (1991) of the European Economic Association in Cambridge.
business cycles. This finding may be more attractive to the student of (real) business cycles than the results of King et al. (1988), who find that recurrent technology shocks are needed to generate economic fluctuations in a standard neoclassical growth model. Only recently Becker and Barro (1988, 1989) have stressed the need for dynamic general equilibrium models with endogenous population growth. This paper will not go as far as to endogenise fertility choice or migration but instead it will look into the question of how (exogenous) shocks in population growth and technology are transmitted to consumption, time preference, capital and debt accumulation.

A deterministic, perfect foresight model is presented of two interdependent countries, each inhabited by a social planner with an infinite planning horizon and with an endogenous rate of time preference. The interdependence of the two countries is made explicit by international goods and capital markets. A difference in the state of technology and/or population growth rates is assumed throughout the analysis.

The question examined in this paper is the grand question of optimal economic growth: how much should a country consume today and save for the future in order to enjoy the maximum amount of welfare over its planning horizon? The well-known Golden Rule of Capital Accumulation, as derived by Koopmans (1965) and Cass (1965), offers us a guiding principle for closed economies. In a competitive world economy characterised by perfect capital mobility, the central question of the theory of optimal economic growth should be extended to include another question, viz, where should a country invest its savings? The standard one-sector model has, however, its drawbacks in analysing steady state optimal allocations in a world economy characterised by (constant) different population growth rates. The standard optimal growth model is therefore extended with an endogenous rate of time preference. There are three reasons for studying the issue of endogenous time preference. First of all, there is the neglected normative question of how social planners in an interdependent world economy should treat future generations. If planners use the average utility criterion, the problem of optimal economic growth in a world of diverging populations shows some similarities with the case of lending and borrowing between households or agents with differences in impatience. The most patient household will accumulate assets infinitely, while the less patient households are all net debtors living eventually at a subsistence level. In a positive sense there seems some evidence from panel data (see Lawrance, 1991) that the rate of time preference differs significantly among households with different
incomes, race and education. Similar differences among countries is quite likely although empirical evidence is still lacking. The introduction of endogenous time preference tries to establish microeconomic foundations for such a state of affairs, but, as will be shown in the paper, at the same time the theory poses counterintuitive results. Secondly, the rate of time preference is generally treated as a parameter, despite its importance in determining economic activity. For instance, in international and inter-temporal trade, explanatory factors such as technology, endowments and the rules of the game (market structure, trade restrictions) play a dominant role. However, the role of such an obvious factor as taste or preference is generally played down. It is my hunch that one can increase understanding of macroeconomic phenomena, such as the current account imbalances of the U.S. and Japan, by paying attention to differences in time preference. Thirdly, since the concept of endogenous time preference is more and more used in various macroeconomic analyses, it may serve a purpose in pushing this model to its limits and see what it essentially implies for steady state distributions.

The paper is organised as follows. First, as a quick refresher, I will set up the optimal growth model with endogenous time preference in a closed economy (section 2). Section 3 shows how resources are allocated intertemporally in a two-country world economy when the rate of time preference is endogenous in both countries. A normative interpretation is offered and the analysis of comparative statics is performed. In the last section (4) I will sum up the findings.

2. A CLOSED ECONOMY GROWTH MODEL WITH ENDOGENOUS TIME PREFERENCE

The standard problem of optimal economic growth or saving is concerned with the following problem: maximize welfare of the indigenous population over an infinite horizon by appropriate investment and consumption choices. This problem has many variants and I will only consider one particular variation to this theme, viz. the endogeneity of time preference. Decisions are taken by a social planner, who has complete control over the level of consumption per capita, \( c(t) \), and the aggregate capital/labour ratio, \( k(t) \). The planner uses a utility functional \((I)\) with an endogenous rate of time preference, \( \rho(t) \), over an infinite horizon.

\(^{(1)}\) A similar model is used in Lucas and Stokey (1984) and Buchholtz and Hartwick (1989).
horizon. This objective function has the following form (see for more details Uzawa, 1968):

\[ V_0 = \int_0^\infty U[c(t)] e^{-\rho t} \, dt \]  

(1)

where \( \rho(t) = \left\{ \int_0^t \rho[U(c(s))] \, ds \right\} \)

where \( U \) is a well-defined concave utility function \( (U' > 0, U'' < 0) \).
A number of conditions are needed to guarantee a stable solution. The following Uzawa-conditions should be satisfied for the time preference function \( \rho(.) \):

\[ \rho(U) > 0, \quad \rho'(U) > 0, \quad \rho''(U) > 0 \quad \text{for all} \quad U > 0 \]  

(2)

and

\[ \rho(U) - \rho'(U)U > 0 \]  

(3)

The relationship between the instantaneous rate of time preference and current utility is positive and the first and second derivatives of time preference with respect to utility are also positive. A higher level of consumption at time \( s \) increases the discount factor applied to utility at and after \( s \) (i.e. the second assumption in condition (2)). The condition \( \rho' > 0 \) is not much favoured by economists (e.g., Blanchard and Fischer (1989) warn readers of their book not to use the Uzawa-model for problems in general). Whether impatience to consume should increase or decrease as actual consumption rises is subject to considerable debate. There are two reasons why one could be persuaded to use this assumption. First, as Lucas and Stokey (1984) and Epstein (1987) point out in a setting with many consumers one needs the property of increasing marginal impatience to produce unique and locally stable wealth distributions. One can see their point by considering the alternative \( \rho' < 0 \) for steady state distributions. This last inequality would imply that as consumption rises one would rather consume more at a future date than today. This would seem to be a model with Uncle Scrooge-properties: the rich would get richer and richer as consumption
increases since the weights attached to future income streams increase. In a steady state the equilibrium interest rate would become smaller as consumption increases and hence the capital stock will become larger, and equally so consumption etc., etc. To cut a long story short, the model explodes. Secondly, besides considerations of stability there is an empirical argument. Zin (1987) presents empirical evidence (*) in favour of the assumption of increasing marginal impatience.

Condition (3) ensures that between two stationary consumption paths the one with the higher level of instantaneous utility is preferred. With these conditions in mind, the planner maximizes (1) subject to the following differential equations:

\[ \dot{p} = \rho [U[c(t)]] \]  \hspace{1cm} (4)
\[ \dot{k} = y(t) - (\delta + n)k(t) - c(t) \]  \hspace{1cm} (5)

Equation (4) describes the development of the rate of time preference over time, and differential equation (5) describes the change of the physical capital/labour ratio over time, \(dk/dt\). The capital stock increases if the national product, \(y(t)\), exceeds the level of consumption per capita and the investment necessary to compensate for depreciation and capital dilution, \((\delta + n)k(t)\). The national product is described by the standard production function

\[ y(t) = Zf[k(t)] \]

where \(Z\) denotes the disembodied state of technology and \(f(.)\) represents the functional relationship between production and the capital/labour ratio. The function \(f(.)\) satisfies the standard Inada-conditions.

To simplify the analysis one can solve the optimization problem first by transforming the time variable \(t\) into one in terms of which the rate of time preference becomes a constant. Following Uzawa (1968, p. 491) and Obstfeld (1983), if we take \(p\) as the independent variable instead of \(t\) we can obtain the following maximand:

\[ \int_{0}^{\infty} Ue^{-p} \, dt = \int_{0}^{\infty} \frac{U}{\rho(U)} e^{-p} \, dp \]  \hspace{1cm} (6)

(*) For those persuaded by experimental evidence I should mention a 'reference' in Engel and Kletzer (1989, p. 737). They refer to an unpublished experimental study by Raimond Battalio, who seems to show that low income (underfed) rats tend to have a lower discount rate than high income (well fed) rats. Ratty behaviour is consistent with Uzawa's assumptions.
while the differential equations can be reformulated in a similar fashion:

\[
dk/dp = [Zf[k] - (\delta + n)k - c]/\rho(U)
\]  

(7)

The current-value Hamiltonian as given in (8),

\[
H(c, i, k, \lambda) = U[c(t)] + \lambda(t) \{Zf[k(t)] + c(t) - (\delta + n)k(t)\}
\]  

(8)

can be rewritten in present value terms. The present value of the imputed national income \(H\), to be discounted at the rate of time preference \(\rho[U(\cdot)]\), is given by:

\[
\frac{H}{\rho[U(\cdot)]}e^{-\rho(t)}
\]  

(9)

The optimum consumption level, \(c\), is determined at the level at which the present value (9) of income is maximized. Necessary and sufficient conditions for an interior solution are, in addition to the national resource constraint:

\[
[U''(c) - \lambda] - \frac{\rho'(\cdot) \cdot U'(c)}{\rho[U(c)]} [U(c) + \lambda \dot{k}] = 0
\]  

(10)

\[
\dot{\lambda} = \lambda(t) \{\rho[U(c)] + \delta + n - Zf'(k)\}
\]  

(11)

\[
\lim_{t \to \infty} e^{-\rho(t)} \lambda(t) k(t) = 0
\]  

(12)

To understand condition (10) we can rewrite it as follows:

\[
U'(c) = \lambda + \frac{\rho'(\cdot) \cdot U'(c)}{\rho[U(c)]} H
\]  

(13)

This condition points out that the planner should equate the marginal utility of consumption (i.e. the LHS) to the sum of the imputed value of investment, \(\lambda\), and the marginal increase in the present value of the imputed income due to a marginal decrease in the rate of time preference. If the rate of time preference, \(\rho(\cdot)\), is constant this relation reduces to the condition \(U'(c) = \lambda\) of the standard Ramsey model. The
steady state allocation \((c, k)\) is characterised by the following two conditions:

\[
Zf' = \rho [U(c)] + n + \delta \tag{14}
\]

\[
c = Zf(k) - (\delta + n)k \tag{15}
\]

Equation (14) is the modified golden rule of capital accumulation, i.e. the social planner should accumulate capital up to the point where the marginal productivity of capital equals the sum of the rate of time preference, the population growth rate and the rate of depreciation. The steady-state rate of time preference is determined by the consumption possibilities, as given in equation (15).

If we compare the steady state effect of a population growth decline on the rate of capital accumulation in the case where time preference is exogenous to the above presented case of endogenous time preference, we can derive the following comparative statics results:

\[
\frac{dk}{dn} \bigg|_{\rho} = \frac{1}{Zf''} < 0 \tag{16}
\]

\[
\frac{dk}{dn} \bigg|_{\rho(c)} = \frac{1 - \rho'U'k}{Zf'' - \rho'U'[Zf' - \delta - n]} < 0 \quad \text{if} \quad \rho'U'k < 1 \tag{17}
\]

The additional condition on the product of the marginal rate of change in the rate of time preference times the marginal utility and the capital stock is satisfied since the larger the capital the larger the level of consumption per capita will be. This will give rise to a lower marginal utility of consumption, approaching zero if consumption is extremely high. If we compare (16) to (17) we can see that a population growth decline in the case of exogenous time preference leads to a larger change in capital accumulation than in the case of endogenous time preference. An increase in the rate of time preference 'compensates' part of the population growth decline. This effect of demographic change has been neglected in a large number of practical studies (see, e.g., Cutler et al. 1990). It would seem that a population decline that is accompanied by a shift in time preference is not entirely farfetched.

I will stop the exposition of the closed economy at this point and go on to examine the interaction in a two-country world economy. For a more extensive discussion of the properties of endogenous time
preference one is referred to Uzawa (1968) and for extensions of the basic model one is referred to Obstfeld (1990).

3. A DEMOGRAPHICALLY DIVIDED WORLD ECONOMY

In a neoclassical two-country growth model (\(^3\)), a steady state equilibrium should eventually be characterised by an equality of golden rule parameters. In general this would imply equality of population growth rates plus rates of time preference, whatever the setting may be. This equality would seem at first sight rather improbable. To wit, the past and present world population is non-stable: the population in the developed world grows at a significantly lower rate than the population of the developing world. The 'theory' of demographic transition points out that in the long run population growth rates converge, but given the difficulties developing countries encounter one may well wonder whether this theory is at all times a good description of development problems. There must be more to economic development then imposing the 'transversality' condition of identical population growth rates. This difficulty has led many authors to back away from the problems of divergences in time preference or population growth. Indeed, Ramsey (1928, p. 559) himself conjectured that under conditions of a divergence in rates of time preference:

«... equilibrium would be attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level»

One could, however, well imagine that in the steady state the equality \(\rho_1 + n_1 = \rho_2 + n_2\) holds by some explicit relationship between either time preference and consumption or population growth (\(^4\)) and consumption. A similar assumption has been made by Becker and Barro (1988, p. 14). They assume for a model of endogenous fertility that the rate of (altruism)-time preference depends negatively on the number of children.

Recent work concerning the endogeneity of time preference is found in Epstein and Hynes (1983) and Epstein (1987). They extend and generalise

\(^3\) The limitation and extensions of the standard infinite horizon neoclassical growth model are discussed in Becker and Majumdar (1989) and Van Dalen (1992).

\(^4\) In a two-country overlapping generations model of international lending and borrowing the endogeneity of fertility is analysed by Kondo (1989).
the work by Uzawa (1968) and Becker (1980). The present analysis concentrates on the derivation of a proposition concerning international lending and borrowing by paying attention to differences in population growth or the state of technology. In an international setting Pitchford (1989) used the Uzawa-model to examine the current account dynamics of permanent and transitory (exogenous) income fluctuations for a small open economy. One of the main results was that when discount rates are endogenous one needs to know whether an income shock is permanent or temporary before the economic consequences of income fluctuations can be established. The international trade analyses like Pitchford's (3) are, however, generally of a partial equilibrium nature by assuming a small open economy. In the present setting, the world economy consists of two interdependent countries engaging in international lending and factor price formation is endogenous.

3.1 THE WORLD MODEL

The world economy consists of two countries, each inhabited by a social planner who uses the welfare function (1) in section 2 as the economic policy objective. The differential equations for countries $h$ (for $h = 1, 2$) include one for the development of physical capital, $k_h$, and one for foreign assets, $d_h$, and they can be reformulated in the following fashion:

$$\frac{dk_h}{dp_h} = \frac{[i_h/(\delta_h + n_h) k_h]}{\rho_h(U)}$$  \hspace{1cm} (18)

$$\frac{dd_h}{dp_h} = [Z^{sh}_h[k_h] - c_h - i_h + (r - n_h) d_h]/\rho_h(U)$$  \hspace{1cm} (19)

The patterns of international lending and borrowing are restricted by the No-Ponzi-game condition (see for an exposition O'Connell and Zeldes, 1988). This condition requires that the present value of a country's wealth, arbitrarily far in the future, be non-negative. In other words, the country's per capita wealth should not increase asymptotically faster than the interest rate net of population growth, $(r(t) - n_h)$:

$$\lim_{t \to \infty} d_h(t) R_h(t) \geq 0$$  \hspace{1cm} (20)

where the short term discount factor $R_h(t)$ is given by,

$$R_h(t) = \exp \left\{ - \int_0^t [r(s) - n_h] \, ds \right\}.$$

To keep things at an analytical level I will use for the remainder of the analysis identical discount rate functions: $\rho_1(\cdot) = \rho_2(\cdot) = \rho(\cdot)$. The present-value Hamiltonian for this command problem is given by the following formulation:

$$H_h(c, i, k, d, \lambda, \mu) = \frac{e^{-\rho(t)}}{\rho[U(\cdot)]} \left\{ U[c_h(t)] + \lambda_h(t) \left\{ i_h(t) - (\delta_h + n_h) k_h(t) \right\} + \mu_h(t) \left\{ Z_h f_h[k_h(t)] - c_h(t) - i_h(t) + (r - n_h) d_h(t) \right\} \right\}$$

The optimum consumption level, $c_h$, is determined at the level at which the present value (21) of income is maximized. Necessary and sufficient conditions for an interior solution are:

$$[U'_h(c_h(t)) - \lambda_h(t)] - \frac{\rho'(\cdot) \cdot U'_h(c_h(t))}{\rho[U_h(c_h(t))]}[U_h(c_h(t))] + \lambda_h(t) \dot{k}_h + \mu_h(t) \dot{d}_h = 0$$

$$\lambda_h(t) = \mu_h(t)$$

$$\dot{\lambda}_h = \lambda_h \left\{ \rho[U_h(c_h)] + \delta_h + n_h \right\} - \mu_h Z_h f_h$$

$$\dot{\mu}_h = \mu_h \left\{ \rho[U_h(c_h)] + n_h - r \right\}$$

$$\lim_{t \to \infty} e^{-\rho(t)} \mu_h(t) d_h(t) = 0$$

$$\lim_{t \to \infty} e^{-\rho(t)} \lambda_h(t) k_h(t) = 0$$

For the remainder of this paper I will concentrate on steady states, since this is a problem that, until now, has been dealt with in an unsatisfactory
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manner in the economics literature. A steady state allocation in a two-country world (i.e. \( dk_h/dt = dd_h/dt = 0 \) for \( h = 1, 2 \)) can be defined as the allocation that is consistent with the following set of equations:

\[
Z_h f'_h (k^*_h) = \rho [U(c^*_h)] + \delta_h + n_h
\]  
(28)

\[
r^* = Z_1 f'_1 (k^*_1) - \delta_1 = Z_2 f'_2 (k^*_2) - \delta_2
\]  
(29)

\[
c^*_h = Z_h f'_h (k^*_h) - (\delta_h + n_h) k^*_h + (r^* - n_h) d^*_h
\]  
(30)

Equation (28) is the modified golden rule of capital accumulation. Since the stock of foreign assets and the rate of time preference are interdependent one can deduce that under conditions of (i) identical production technologies, but different population growth rates, the low population growth country is a steady state net creditor and the high population growth country a net debtor; (ii) identical population growth rates but different production technologies, the relatively technological advanced country is a steady state net debtor and the less advanced neighbour is a steady state net creditor.

Another comparison must be made with respect to the existing body of literature on borrowing and lending in infinite horizon models. With constant rates of time preference the optimal foreign debt used to be indeterminate. But with the formulation of endogenous time preference the optimal debt in this two-country world economy is determinate. This result arises mainly because in the standard model with a given rate of time preference foreign debt was merely used to smooth consumption streams. With endogenous time preference the foreign debt affects consumption behaviour. The determinacy of wealth distributions is undoubtedly a characteristic of this model that gives it a competitive edge over the standard neoclassical growth model.

One other interesting observation that can be made at this stage is about the steady state consumption and distribution of wealth in a demographically divided world: if all countries have the same discount rate function and production technologies, but different rates of population growth, then all countries have the same steady state capital-labour ratio but the country with the lowest (highest) population growth will have the highest (lowest) steady state consumption level. If the discount functions differ, steady state distributions will be more complex. What is most puzzling about the present formulation of optimal economic growth is that in the standard growth model patience was a virtue in obtaining
wealth and it enabled one to enjoy a higher level of consumption, whereas in the Uzawa-formulation the wealthy are characterised by a high rate of impatience. Perhaps the last interpretation is not quite correct, since 'patience' depends on the level of consumption, hence each individual lives within his/her means. The relatively wealthy are therefore those countries or persons who can shift relatively more resources from the future to the present. In this way one can circumvent the postponed splurge in consumption which arises in case the planner is of a Benthamite persuasion with a finite horizon and the population growth rate exceeds the rate of time preference; a property of the optimal growth model which Koopmans (1967, 1976) found undesirable.

The ability of the present model to generate time paths with a positive level of consumption at some point in the future in both countries, despite a divergence in population growth rates, must not be underestimated. The early literature on optimal capital accumulation takes the view (e.g., Inada, 1968, p. 324) that it is irrelevant to treat countries with different growth rates. For instance, Inada examines the consequences of different population growth rates and asserts that identical population growth rates are not a severe limitation to international growth models because «[i]f the population growth rate in one country is greater than in the other country, after a sufficiently long period the former economy becomes overwhelmingly large compared with the latter economy...» and «[t]he capital accumulation process in [the former economy] under free trade is described approximately by the same model as in the no-trade situation». In the long run this assertion is certainly true, but in a medium term context the divergence of population growth rates must be of influence on international capital flows and consumption behaviour. Dixit (1981, p. 283) draws attention to the difficulties involved in the treatment of different growth rates in the trading countries. E.g., «during the course of the supposed steady state, trading prices must change from the autarky prices of one country to those of the other». These consequences of different growth rates are «inconsistent with the logic of a steady state». The model seems to embody the feeling that is inherent in intertemporal planning and expressed vividly by Solow (1974, p. 9): «We have actually done quite well at the hands of our ancestors. Given how poor they were and how rich we are, they might properly have saved less and consumed more». However, the model seems to have some implications that run counter to plain observation. The rich (i.e. the country with the superior production technology) turn out to be net debtors in the steady state and the poor are net creditors in the long-run.
At present it would seem like a fair description to say that the developed world can be seen as a net creditor and the developing world as the net debtor. The implicit assumption in this paper was that countries have identical preferences and, of course, countries can have different utility and time preference functions which give rise to more real-to-life descriptions. But then one has to study the relationship between how preferences develop in response to technology and endowments.

The present model is quite flexible when it comes to describing the transition of an interdependent two-country world economy to a world economy in which one country, the high population growth country, acts in the long run as if it is a closed economy and the low population growth country has become so small that it faces a world interest rate, determined by the high population growth country. A question that springs to mind is: can the low population growth country have a non-negligible influence on the world capital market, even in the long-run? Different discount rate functions could certainly give rise to this situation. The per capita debt, corrected for the relative population sizes, must then tend to some finite number. For every identical steady state consumption allocation, $c$, one has to assume that $\rho_1[U(c)] > \rho_2[U(c)]$, for the case $n_1 < n_2$, so that $c_1 > c_2$. The low population growth country is under those circumstances the net creditor and the high population growth country the net debtor. The trouble with this solution is that one tends to fall back on arbitrary differences in time preference functions that cannot be justified in any normative sense. Hence we are left with the unsatisfactory state of affairs that by removing an arbitrary element from the optimal growth model – constant rate of time preference – one has to face a new arbitrary element, viz. the functional form of the rate of time preference.

3.2 NORMATIVE INTERPRETATION

The normative significance of the present section is of some considerable importance. The way in which we value living standards and especially population developments in the world of today can fall into one of the two extreme welfare criteria: the total utility criterion $L \cdot U[C/L]$, as proposed by Meade (1955), or the average utility criterion, $U[C/L]$. The total utility (or Benthamite) criterion could imply a Vatican stance in questions of population policy, viz. the more the merrier whereas the average utility criterion leads to a Malthusian attitude: the less, the merrier (see Nerlove et al., 1987). As recognised by Ng (1989),
when numbers differ both principles are unsatisfactory policy devices when one has to make a judgment over the value of life (as represented by living standards). Ng (1989) 'solves' the problem by introducing an ad hoc compromise: maximization of number-dampened total utility, $q(L) \cdot U[C/L]$ where $q' > 0$, $q'' < 0$ and $q(\cdot)$ is bounded from above, i.e. the function $q$ never reaches infinity, even if $L$ goes to infinity. This function which establishes a compromise has no economic or ethical content at all. The value attached to the first person exceeds all values attached to other persons in the economy.

Another solution is offered by Michel (1990) who opts for the ethical stance in which the social rate of time preference equals the growth rate of population. This would indeed solve the problem of a demographically divided world, but it remains ad hoc in that it is merely a translation of the undiscounted average utility case to the conditions of undiscounted total utility. It embodies the rather unhappy implication that the planners of relatively fast growing populations may use a higher discount rate than the planners representing slower growing populations. The welfare analysis with the Uzawa-utility function avoids the ad hocrity (6) of Ng (1989) and Michel (1990) and, in fact, endogenises the care for future generations. Endogenous time preference formation establishes a compromise between the average utility criterion and the total utility criterion. Besides that, the country with the relatively superior production technology will consume less than it would have done in autarky and raise consumption abroad by borrowing resources. In case production technologies are identical but population growth rates differ one will arrive at the conclusion that, in an interdependent world economy, the social planner of the country with the fast(er) growing population should care relatively more about future generations than the planner of the slow(er) growing population. Each social planner has to take into account the domestic and foreign rate of population growth ($n_d$) and the state of technology ($Z_t$, $\delta_t$).

In order to discern how the allocation changes as shocks in technology or population growth occurs. I will perform the analysis of comparative

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(6) A referee pointed out that the model of endogenous time preference is equally ad hoc in that one has to impose certain restriction to ensure stability. This is indeed true, but then again every model is ad hoc in that one needs additional assumptions to ensure stability. The model is however not ad hoc in that it comes close to a microeconomic foundation of a crucial growth parameter, whereas North-South theorists and the analyses by Michel (1990) and Ng (1989) do not provide such a link.
statics for an interdependent world economy with diverging population developments in the next section.

3.3 COMPARATIVE STATICS

If we totally differentiate the set of equations (28)-(30) we have six equations explaining the variables $c_h$, $r$, $d$ and $k_h$ (for $h = 1, 2$). In the appendix to this paper the full derivation is given of the comparative statics. At this point I will restrict myself to discussing the qualitative outcomes of the comparative statics.

<table>
<thead>
<tr>
<th>Shocks in: Population growth rate</th>
<th>Depreciation rate</th>
<th>Technology</th>
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<tbody>
<tr>
<td></td>
<td>$n_1$</td>
<td>$n_2$</td>
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<tr>
<td>$k_1$</td>
<td>$-a$</td>
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<tr>
<td>$k_2$</td>
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<td>$r$</td>
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<td>$d$</td>
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<tr>
<td>$c_2$</td>
<td>$+a$</td>
<td>$-$</td>
</tr>
</tbody>
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a) if $\rho'U'a_1 < 1$, where $a_1 = k_1 + d$.
b) if $\rho'U'a_2 < 1$, where $a_2 = k_2 - d \cdot L_1/L_2$.
c) if $\rho'U'k_h < 1$ for $h = 1, 2$.

First of all, it must be said that some of the standard results of neoclassical capital theory still stand. What does differ from standard theory are the international spillover effects. For instance, the effect of a population growth decline on consumption per capita is always positive for the country where the population shock occurs but the effect abroad depends on the sign of $(\rho'U'a_h - 1)$. In effect, the influence of a population growth shock on capital accumulation, foreign assets, the world interest rate and consumption abroad depend on the sign of $(\rho'U'a_h - 1)$. In Table I I have presentend the effects with the restriction that $\rho'U'a_h < 1$ (for $h = 1, 2$). As we saw in section 2 this is a rather likely restriction: with a very large asset position one’s consumption is bound to be large and therefore the marginal utility as well as the marginal increase in time preference are small.
To proceed with the comparative statics, the effects of technology \((\delta_h, Z_h)\) are to a large extent unambiguous. Positive technology shocks, \(Z_h\), give rise to an increase in consumption and thereby the rate of time preference, irrespective of the country where the shock occurred. A shock in the state of technology does, however, have an ambiguous effect on the rate of capital accumulation at home (i.e. \(dK_h/dZ_h \leq 0\)). E.g., a positive shock in the state of technology occurring in country 1 leads to a rise in the world interest rate as a consequence of the rise in the rate of time preference \((\rho[U_1])\). The increase in the world interest rate leads to a fall in capital accumulation in country 2, but the effect on \(k_1\) is ambiguous because, on the one hand, the need to accumulate more capital has increased (see es. 28) and on the other hand the planner feels the need to consume part of this newly acquired wealth and increase the rate of time preference.

There is however a difficulty when differences in population growth arise. As long as a difference in population growth rates exists (and discount rate functions are identical), the country with the relatively high rate of population growth (i.e. country 2) will have a negligible amount of interest (corrected for the rate of population growth) to pay since:

\[
\lim_{t \to \infty} (r - n_2) d \cdot L_1/L_2 = 0
\] (31)

In the steady state, the high population growth country accumulates assets as if it is a closed economy. The low population growth country becomes under those circumstances a small open economy and the social planner receives interest payments on its foreign assets.

Summarizing, the interdependent nature of the world economy of this section is apparent. Whether a change in endowments or technology occurs at home or abroad, it always affects consumption, investment and factor prices in both countries.

4. SUMMARY

The central aim of this paper was to explore the question how shocks in the population growth rate and the state of technology of a country affect international lending and borrowing, time preference, consumption and investment in an interdependent world economy. Empirically, investment fluctuations appear to be an important determinant of current account fluctuations (Sachs, 1981). However, many authors (Sachs, 1982
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and Obstfeld, 1983) abstract from investment by the use of the partial equilibrium framework of a small open economy. Interdependent economies with endogenous investment turn out to be extremely complicated as Bovenberg (1989) and Cantor and Mark (1987) show, but still it brings to the fore how actions and endowments abroad affect actions at home. However, a simple one-sector model of optimal growth in a two-country world economy allows only a limited number of population growth paths to be studied. In an infinite horizon model convergence of population growth rates is a necessary condition for steady state patterns of international lending and borrowing to be a viable solution, without ending up with a repugnant conclusion. When time preference is endogenous and depends on the utility of current consumption streams, a divergence in population growth rates will be offset by the rate of time preference. A low population growth country will therefore have a high(er) rate of time preference and the opposite proposition applies to the high population growth country. The model is more flexible since it can trace the time path of asset accumulation in a permanently demographically divided world economy. Another important aspect of the steady state equilibrium is that the wealth distribution in an interdependent world economy is determinate, as opposed to the standard infinite horizon model where this distribution remains indeterminate. The long-run consequences of a divergence of population growth rates are however trivial: the high population growth country accumulates capital as if it is a closed economy and the low population growth country accumulates assets as a small open economy facing a perfectly competitive international capital market, represented by the economy of the high population growth country.

The present paper has addressed a question that numerous authors have assumed away (') or considered unimportant. The introduction of diverging population growth developments does however add a piece of reality to the Ramsey-model that would have gone unnoticed if we conveniently assumed population growth rates constant and equal to one another. With an increasing use of the Koopmans-Cass model of capital accumulation in normative and positive analyses (such as the real business

') E.g., Bardhan (1965), Oniki and Uzawa (1965) and Inada (1968) all play down the difficulties of using different growth rates in models of international lending and borrowing. The only exception to this rule is Deardorff (1985, 1987) who considers the economic consequences of a world of diverging populations in a model that resembles the outcomes of Pasinetti's two-class growth model.
cycle strand of literature) it seems like a worthwhile exercise to explore and elaborate the use of this model in an international setting. The extension of endogenous time preference proved a more flexible edifice for questions of international lending and borrowing in a demographically divided world. It is shown that international debt in an interdependent world economy can bring about a convergence of time preference; a 'fact' which may, just as well as the spillover effect of human capital (see Tamura, 1991), explain income convergence. However, one should not get carried away by these remarks. Endogeneity of time preference begs the question about the form of time preference functions. Here, as elsewhere in the economics literature stability assumption were imposed. Perhaps unstable paths may not be so elegant but on the other hand they may be more realistic. Future empirical work will have to show whether the theory of endogenous time preference was a worthwhile exercise. Despite these warnings, the normative question of optimal economic growth posed in this paper may be of some importance for studying development problems of LDCs and ageing problems of developed countries hand in hand, since it is one of the stylised facts and forecasts that the present day world economy is and will be demographically divided (see, e.g., Brown and Jacobson, 1986).

REFERENCES


APPENDIX: COMPARATIVE STATICS AND ENDOGENOUS TIME PREFERENCE

The following set of equations is sufficient to derive the steady state general equilibrium effects of changes in parameters (for \( h = 1, 2 \)):

\[
Z_h f'_h = \rho [U(c_h)] + n_h + \delta_h \tag{A.1}
\]

\[
r = Z_h f'_h - \delta_h \tag{A.2}
\]

\[
c_1 = Z_1 f(k_1) - (\delta_1 + n_1) k_1 + (r - n_1) d \tag{A.3}
\]

\[
c_2 = Z_2 f(k_2) - (\delta_2 + n_2) k_2 - (r - n_2) d \cdot L_1/L_2 \tag{A.4}
\]

If \( n_1 < n_2 \) one can use the limit property of the long-run population size ratio:

\[
\lim_{t \to \infty} L_1(t)/L_2(t) = 0 \tag{A.5}
\]

Under those circumstances the comparative statics apply to the case where the high population growth country accumulates capital as if it is a closed economy, while the low population growth country is a small
open economy with the world interest rate determined by the high population growth country. Totally differentiating these equations we obtain in matrix notation:

\[
\begin{bmatrix}
    \frac{dk_1}{dr} \\
    \frac{dk_2}{dr} \\
    \frac{dr}{dr} \\
    \frac{dd}{dr} \\
    \frac{dc_1}{dr} \\
    \frac{dc_2}{dr}
\end{bmatrix}
= A^{-1} \cdot B \cdot
\begin{bmatrix}
    \frac{dn_1}{dr} \\
    \frac{dn_2}{dr} \\
    \frac{d\delta_1}{dr} \\
    \frac{d\delta_2}{dr} \\
    \frac{dZ_1}{dr} \\
    \frac{dZ_2}{dr}
\end{bmatrix}
\]  

(A.6)

where

\[
B = \begin{bmatrix}
1 & 0 & 1 & 0 & -f'_1 & 0 \\
0 & 1 & 0 & 1 & 0 & -f'_2 \\
0 & 0 & -1 & 0 & f'_1 & 0 \\
0 & 0 & 0 & -1 & 0 & f'_2 \\
-a_1 & 0 & -k_1 & 0 & f'_1(k_1) & 0 \\
0 & -a_2 & 0 & -k_2 & 0 & f'_2(k_2)
\end{bmatrix}
\]

where \( a_1 = k_1 + d \) and \( a_2 = k_2 - d \cdot L_1/L_2 \), and (6)

\[
A^{-1} = \frac{1}{\text{Det}}
\begin{bmatrix}
    A_{11} & A_{21} & A_{31} & A_{41} & A_{51} & A_{61} \\
    A_{12} & A_{22} & A_{32} & A_{42} & A_{52} & A_{62} \\
    A_{13} & A_{23} & A_{33} & A_{43} & A_{53} & A_{63} \\
    A_{14} & A_{24} & A_{34} & A_{44} & A_{54} & A_{64} \\
    A_{15} & A_{25} & A_{35} & A_{45} & A_{55} & A_{65} \\
    A_{16} & A_{26} & A_{36} & A_{46} & A_{56} & A_{66}
\end{bmatrix}
\]

(6) To shorten notation and derivatives, production \( y_h = Z_h f_h(\cdot) \) and \( \rho' \) denotes \( (\partial \rho / \partial U) \cdot (\partial U / \partial c) \).
\[ \text{Det} = y''y_2^2 \rho_2 (r - n_2) \frac{L_1}{L_2} - \rho_1 \left\{ y_1''y_2'' \left\{ - (r - n_1) + \rho_2 d(n_1 - n_2) \frac{L_1}{L_2} \right\} + 
 + y_2'' \rho_2 (r - n_2) \frac{L_1}{L_2} \rho_1 + y_1'' \rho_2 (r - n_1) \rho_2 \right\} > 0 \]

\[ A_{11} = \rho_2 y_2'' (r - n_2) L_1/L_2 < 0 \]

\[ A_{12} = \rho_2 y_1'' (r - n_2) L_1/L_2 < 0 \]

\[ A_{13} = y_1''y_2'' \rho_2 (r - n_2) L_1/L_2 > 0 \]

\[ A_{14} = -y_1''y_2'' - \rho_2 [y_2''y_2'dL_1/L_2 - y_1'' \rho_2] < 0 \]

\[ A_{15} = -y_1''y_2'' \rho_4 - \rho_2 [y_2''y_2'd(n_2 - n_1) L_1/L_2 - y_2'' (r - n_1) \rho_2] + 
 + \rho_2 y_2'' \rho_1 (r - n_1) L_1/L_2 < 0 \]

\[ A_{16} = y_1''y_2'' (r - n_2) L_1/L_2 > 0 \]

\[ A_{21} = \rho_1 y_2'' (r - n_1) < 0 \]

\[ A_{22} = \rho_1 y_1'' (r - n_1) < 0 \]

\[ A_{23} = \rho_1 y_1''y_2'' (r - n_1) > 0 \]

\[ A_{24} = y_1''y_2'' - \rho_1 [y_2''y_2'd + y_2'' \rho_1] ? \]

\[ A_{25} = y_1''y_2'' (r - n_1) > 0 \]

\[ A_{26} = -y_1''y_2'' (r - n_2) L_1/L_2 - \rho_1 [y_1''y_2''d(n_2 - n_1) L_1/L_2 - 
 - y_1'' \rho_2 - y_2'' \rho_1 \rho_2 L_1/L_2] < 0 \]

\[ A_{31} = \rho_1 [-y_2'' (r - n_1) - \rho_2 y_2''d(n_2 - n_1) L_1/L_2 + \rho_2 \rho_1 \rho_2] > 0 \]

\[ A_{32} = y_1'' \rho_2 (r - n_2) L_1/L_2 - \rho_1 \rho_2 \rho_1 \rho_2 L_1/L_2 < 0 \]

\[ A_{33} = \rho_2 y_2'' \rho_2 L_1/L_2 [y_1'' - \rho_1 \rho_1] > 0 \]

\[ A_{34} = -y_1''y_2'' - \rho_2 y_2'' [y_2''dL_1/L_2 - \rho_2] + \rho_1 y_2'' \rho_1 + 
 + \rho_1 \rho_2 \rho_1 \rho_2 \rho_2 L_1/L_2 - \rho_1 \rho_2 \rho_1 \rho_2 < 0 \]

\[ A_{35} = -y_1''y_2'' \rho_1 - y_1''y_2'' \rho_2 d(n_2 - n_1) L_1/L_2 + y_1'' \rho_2 \rho_1 \rho_2 < 0 \]

\[ A_{36} = y_1'' \rho_2 L_1/L_2 [y_1'' - \rho_1 \rho_1] > 0 \]

\[ A_{41} = \rho_1 \rho_1 [y_1'' - \rho_2 \rho_2] < 0 \]

\[ A_{42} = -y_1'' \rho_2 \rho_2 L_1/L_2 + \rho_2 \rho_2 [y_1''d(n_1 - n_2) L_1/L_2 + \rho_1 \rho_2 L_1/L_2] > 0 \]

\[ A_{43} = \rho_1 \rho_1 y_1'' [y_1'' - \rho_2 \rho_2] > 0 \]
\[ A_{44} = y_1^2 y_2^2 - y_1^2 \rho_2 \rho_2 - \rho_1 y_2^2 \left[ y_1^2 \rho_1 + \rho_1 \rho_1 \rho_2 \rho_2 \right] + \rho_1 \rho_1 \rho_2 \rho_2 \left[ y_1^2 \rho_1 + \rho_1 \right] \]

\[ A_{45} = y_1^2 y_2^2 \rho_1 - \rho_2 \rho_1 \rho_2 y_1^2 > 0 \]

\[ A_{46} = -y_1^2 y_2^2 \rho_1 \rho_2 L_1/L_2 + \rho_1 y_2^2 \left[ y_1^2 \rho_1 \rho_2 (n_1 - n_2) L_1/L_2 + \rho_1 \rho_2 L_1/L_2 \right] < 0 \]

\[ A_{51} = \rho_1 \rho_2 \rho_2 y_1^2 L_1/L_2 < 0 \]

\[ A_{52} = \rho_1 \rho_2 \rho_2 y_1^2 L_1/L_2 < 0 \]

\[ A_{53} = \rho_1 \rho_2 \rho_2 y_1^2 y_2^2 L_1/L_2 > 0 \]

\[ A_{54} = -y_1^2 y_2^2 \rho_1 + \rho_1 \rho_2 y_1^2 \left[ \rho_2 - y_2^2 \rho_1 \rho_2 L_1/L_2 \right] < 0 \]

\[ A_{55} = y_1^2 y_2^2 \rho_2 \rho_2 L_1/L_2 > 0 \]

\[ A_{56} = y_1^2 y_2^2 \rho_1 \rho_2 L_1/L_2 > 0 \]

\[ A_{61} = \rho_1 \rho_2 \rho_2 y_2^2 < 0 \]

\[ A_{62} = \rho_1 \rho_2 \rho_2 y_1^2 < 0 \]

\[ A_{63} = \rho_1 \rho_2 \rho_2 y_1^2 y_2^2 > 0 \]

\[ A_{64} = y_2^2 \left[ y_1^2 \rho_2 - \rho_1 \rho_2 \left[ y_1^2 \rho_1 + \rho_1 \right] \right] \]

\[ A_{65} = y_2^2 \rho_2 \rho_2 \rho_1 > 0 \]

\[ A_{66} = y_2^2 y_2^2 \rho_1 \rho_1 > 0. \]