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Sustainability and investment rules

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Abstract

We develop conditions under which an economy aiming at sustainability has to keep the value of the total stocks of capital constant over time.

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1. Introduction

There exists an abundant literature on the subject of sustainability, which is commonly defined as constant per capita well-being over time.[1] We refer to Asheim (1986, 1994b, 1995), Dasgupta and Mäler (1993), Dixit et al. (1980), Hartwick (1977, 1978, 1994a, b, 1995a, b), Hartwick and Van Long (1995), Mäler (1991), Pezzey (1994), Sefton and Weale (1994), Solow (1974, 1986, 1993), Vellinga and Withagen (1996), and Weitzman (1976, 1995). This literature aims at finding rules for sustainability. A common feature of such rules seems to be that the value of capital is constant over time (beginning with the well-known Hartwick rule). These results were obtained for special cases and particular models. We will show in this paper that a considerable generalization is possible in a control-theoretic framework. The central idea can be sketched as follows.

The state in which an economy finds itself at a particular instant of time can be characterized by its stocks. Examples are the stocks of natural exhaustible resources, renewable natural resources, man-made physical capital, human capital, international financial wealth, the stock of labor and the stocks of pollutants. Some of these stocks influence

[1] We are aware of the fact that this anthropocentric interpretation and operationalization of sustainability is subject to some criticism, but for our purpose this is irrelevant. See also Toman et al. (1995) and Asheim (1994a).

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instantaneous well-being directly, such as those pollutants that have negative health effects. Others have an indirect impact, because well-being is derived from using their services, which is the case when raw materials are extracted from the exhaustible resources and when human capital is used as a factor to produce consumer commodities. It is usually assumed that the motion of the stocks in the economy can be described by a set of differential equations. Moreover, technological and other constraints can be modelled in another set of equations. With the particular concept of sustainability given above in mind, maximal constant instantaneous per capita well-being can be found as the outcome of an optimal control problem with a (generally) non-constant rate of discount. This is the case in most of the papers mentioned above. The next step is then to show that the current value of the corresponding Hamiltonian just equals the constant value of well-being, corrected for non-autonomous elements in the constraints. If such elements are absent, then the current value Hamiltonian is constant, which implies that the total value of net investments in the stocks involved should be zero. It is to be understood that the stocks are evaluated at their shadow prices originating from the optimal control problem. These prices need not prevail on the actual markets, especially not when negative externalities are present. With non-autonomous differential equations, caused for example by exogenous technical change, varying world market prices or non-constant exogenous interest rates, the situation becomes more complicated in the sense that no easy economic interpretation can be given. However, it is clear from a mathematical point of view what corrections should be made to achieve sustainability. This is particularly useful when we are looking for a national income concept that incorporates sustainability. See Sefton and Weale (1994), Vellinga and Withagen (1996) and Johanson and Löfgren (1996).

2. The main result

We consider an optimal control problem with n state variables, r control variables and m constraints. There are given an open set $X \subset \mathbb{R}^n$, a set $U \subset \mathbb{R}^r$ and $T := \mathbb{R}_+$. We shall only be interested in state variables $x: T \to X$ having piece-wise continuous derivatives and control variables, $u: T \to U$ being piece-wise continuous. Also given are the functions $\rho: T \to \mathbb{R}_+$, $f_0: X \times U \to \mathbb{R}$, $f := (f_1, f_2, \ldots, f_n): X \times U \times T \to \mathbb{R}^n$ and $g := (g_1, g_2, \ldots, g_m): X \times U \times T \to \mathbb{R}^m$. We assume that f_0, f and g are continuously differentiable. We shall also assume that

$$\pi(t) := \exp\left(-\int_0^t \rho(\tau)\,\mathrm{d}\tau\right) \to 0 \text{ as } t \to \infty .$$

Now suppose that $(\hat{x}, \hat{u})$ solves the following problem:

$$\max \int_0^\infty \pi(t) f_0(x(t), u(t))\,\mathrm{d}t \tag{1}$$

subject to

$$\dot{x}(t) = f(x(t), u(t), t) \quad (t \in T), x(0) = x_0 \text{ given}, \tag{2}$$

$$g(x(t), u(t), t) \geq 0 \quad (t \in T). \tag{3}$$

It is well known that in order to apply the usual necessary conditions, some kind of constraint qualification must be satisfied. It is assumed here that this is indeed the case. We define the current value Hamiltonian and the current value Lagrangian as follows:

$$\mathcal{H}(x, u, t, \lambda) := f_0(x, u) + \lambda \cdot f(x, u, t),$$
$$\mathcal{L}(x, u, t, \lambda, \mu) := \mathcal{H}(x, u, t, \lambda) + \mu \cdot g(x, u, t).$$

Here dots between variables refer to the inner product. Now the following holds: there exist $\lambda: T \to \mathbb{R}^n$ and $\mu: T \to \mathbb{R}^m_+$ with λ continuous and piece-wise continuously differentiable and μ piece-wise continuous, such that

$$\partial\mathcal{L}/\partial u_j = 0 \quad (j = 1, 2, \ldots, m; t \in T), \tag{4}$$

$$\mu(t) \cdot g(\hat{x}(t), \hat{u}(t), t) = 0 \quad (t \in T), \tag{5}$$

$$-\dot{\lambda}_i(t) = \partial\mathcal{L}/\partial x_i - \rho(t)\lambda_i(t) \quad (i = 1, 2, \ldots, n; t \in T). \tag{6}$$

Here all the partial derivatives are evaluated in the optimum.

We now state and prove the main result of the paper.

Theorem. Suppose that $f_0(\hat{x}(t), \hat{u}(t)) = \alpha$, a constant $(t \in T)$, and that $\pi(t)\mathcal{H}(\hat{x}(t), \hat{u}(t), t, \lambda(t)) \to 0$ as $t \to \infty$. Then

$$\mathcal{H}(\hat{x}(t), \hat{u}(t), t, \lambda(t)) = \alpha - \frac{1}{\pi(t)} \int_t^\infty \pi(s)\left[\lambda(s) \cdot \frac{\partial f(\hat{x}(s), \hat{u}(s), s)}{\partial s} + \mu(s) \cdot \frac{\partial g(\hat{x}(s), \hat{u}(s), s)}{\partial s}\right] \mathrm{d}s.$$

Proof. Let f_t and g_t denote the partial derivatives of f and g with respect to time. It follows from Seierstad and Sydsaeter (1987, p. 277, note 2) that

$$\frac{\mathrm{d}\mathcal{L}^*}{\mathrm{d}t} = \frac{\partial\mathcal{L}^*}{\partial t}$$

at all continuity points of u, where $\mathcal{L}^* := \pi\mathcal{L}$. Hence,

$$\dot{\mathcal{L}} = \rho[\mathcal{L} - \alpha] + \lambda \cdot f_t + \mu \cdot g_t$$

at all continuity points of u. Therefore, in shorthand:

$$\mathcal{H}(t) = \mathcal{L}(t) = \frac{1}{\pi(t)} \int_t^\infty \pi(s)[\rho(s)\alpha - \lambda(s) \cdot f_s - \mu(s) \cdot g_s] \, \mathrm{d}s,$$

as is easily verified. Moreover,

$$\frac{1}{\pi(t)} \int_t^\infty \pi(s)\rho(s)\alpha \, \mathrm{d}s = \alpha. \quad \square$$

The following is now immediate:

Corollary. Suppose that f and g are autonomous. Then $\lambda(t)\cdot\dot{\hat{x}}(t)=0$ *(all* $t\in T$*).*

Proof.

$$\mathcal{H}(t)=\alpha+\lambda(t)\cdot f(\hat{x},\hat{u},t)=\alpha+\lambda(t)\cdot\dot{\hat{x}}(t)=\alpha\,. \quad \square$$

The condition that the present value Hamiltonian converges to zero as time goes to infinity is satisfied if f and g are autonomous. However, the condition is indeed required when f and g are non-autonomous.

3. Discussion

The result presented above strongly resembles the findings of Dixit et al. (1980). The major difference is that our result is obtained in an optimal control framework, whereas they work in the context of a competitive equilibrium.

It would go too far to verify for each model employed in this line of research whether or not it satisfies the assumptions we made. We shall therefore restrict ourselves to some important examples.

In the Solow (1974) model there are two stocks: a stock of an exhaustible natural resource and a stock of man-made capital. Capital and the raw material are used in a Cobb–Douglas production process that yields a consumer commodity. In the case at hand, the maximum rate of consumption can be calculated explicitly and it is easily seen that this is also the outcome of an optimal control problem with a decreasing rate of time preference.

Also, the recent work of Hartwick and Van Long (1995) fits into our general framework. They consider constant consumption, C, in an economy which derives instantaneous profits, P, from for example selling raw material from exhaustible resources on world market. The optimal profits are an exogenous function of time. Then the accumulation of financial capital, W, is given by

$$\dot{W}(t)=P(t)+r(t)W(t)-C(t)\,,$$

where r is the exogenous world market interest rate. The maximand is

$$\int_0^\infty \pi(t)C(t)\,\mathrm{d}t\,.$$

it is easy to see that along an optimum the rate of time preference must equal the interest rate. Moreover, it follows immediately from our theorem that with a constant rate of consumption

$$\dot{W}(t)=-\frac{1}{\pi(t)}\int_t^\infty \pi(s)[\dot{r}(s)W(s)+\dot{P}(s)]\,\mathrm{d}s\,.$$

If we define V as the current value at time t of all future profits:

$$V(t) := \frac{1}{\pi(t)} \int_t^\infty \pi(s) P(s) \, ds ,$$

then it readily follows that

$$\dot{W}(t) = -\dot{V}(t) - \frac{1}{\pi(t)} \int_t^\infty \pi(s)[\dot{r}(s) W(s) + V(s)] \, ds$$

which is exactly the investment rule derived by Hartwick and Long.

4. Conclusion

In this paper we have derived a general investment rule that must be satisfied in a situation of sustainable economic development. To the best of our knowledge it can be applied to all the models used in the literature on this subject. As was already mentioned in the Introduction, the investments should be weighted by the optimal shadow prices, which do not necessarily prevail in reality. This implies that even if the value of net investments measured in actual prices equals zero at all times, this does not guarantee that the economy finds itself in sustainable development. Another caveat applies to the interpretation of the Hamiltonian. It is tempting to identify the (linearized) Hamiltonian with national income and hence to see national income (after correction, for example, for the extraction of exhaustible resources) as an indicator for sustainability. However, in view of the above this might lead to conclusions that are at variance with reality.

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