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Published in:
Economic Systems Research

Publication date:
1989

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):
Ten Raa, M. H., Chakraborty, D., & Das, T. (1989). Working capital in an input-output model. *Economic Systems Research*, 1(1), 53-68.

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Working Capital in an Input-Output Model

THIJS TEN RAA, DEBESH CHAKRABORTY & TUHIN DAS

ABSTRACT *Working capital is capital in the pipeline of the production process. Little has been done to formalize the concept of working capital in an input-output framework. The objective of the paper is to provide an input-output model of working capital and to assess its empirical value. In section 1 a brief discussion on capital is presented. In section 2 we develop the theoretical framework of the analysis. Section 3 deals with the empirical work, and section 4 concludes the paper.*

1. Capital

The distribution of economic activity over time manifests itself in the form of capital. Such distributions are of great variety and, as a result, capital assumes all sorts of forms. Our discussion of capital will follow Marx, and give a systematic account of the classification in Volume II of *Capital*. The presentation will be geared towards an analysis of economies and temporally distributed input-output relationships.

Capital has different connotations. To the man in the street it is a *store of value*, like money. But economists claim it to be the means of production and as such a *physical good*. Marx used the term in a broad sense which captures both elements. The man in the street's capital corresponds with Marx's capital of circulation (to be distinguished below from circulating capital), while the economist's capital corresponds with Marx's productive capital. We refer to Marx (1974, p. 192) and take up the two kinds in turn.

Capital of circulation consists of commodity-capital and money-capital, denoted C and M, respectively, by Marx (1974, pp. 36 and 26). Capital of circulation is in part the dual of productive capital, in the sense that monetary relations can be obtained by transposition of the physical relations, as has been shown by Brody (1974) and others. For this reason we confine ourselves to a treatment of productive capital. Thus we position ourselves in the classical tradition of attributing no real role to finance. It should be admitted that in this way we do no justice to the peculiarities of the circuits of money-capital and commodity-capital. However, it is possible to treat capital of circulation in the same way as productive capital. This is clear from the symmetry in Foley (1982) and others. Thus our treatment of productive capital may serve as a model for capital of circulation and the complete economic system.

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Marx (1974, p. 218) divides *productive capital* in variable capital and constant capital. Here variable capital adds value in production and consists of labour services and, in Marx's view, nothing but labour services. Constant capital transmits value and consists of all other necessary means of production: flows and stocks of physical goods. The classification is most useful for the theories of value and exploitation. An alternative classification is obtained when the objects are rearranged. Then productive capital is divided into circulating capital and fixed capital following Marx (1974, p. 158). Circulating capital is absorbed in production and consists of flows of goods. Fixed capital must merely be present when production takes place and consists of stocks of goods. This classification is appropriate for our analysis of the production process: circulating and fixed capital are represented by Leontief's input-output flow coefficients matrix, A , and the input-output stock coefficients matrix, B , respectively.

Circulating capital (A) is fluid. However, it can be like water or like syrup. Some circulating capital, such as electricity, is absorbed immediately, but other circulating capital, such as minerals, must be treated during some time. Electricity is (super)fluid capital, while minerals are working capital. (Super)fluid capital, by definition, is processed instantaneously, while working capital is defined to be capital in the pipeline.

The same distinction can be made with regard to fixed capital (B). Some fixed capital, such as a stapler, is ready for immediate use, but other fixed capital, such as a transport container, must be present some time in advance. A stapler is instant capital, the container is advanced capital. Instant capital is fixed capital which can be put to use instantaneously, while advanced capital must be installed in advance, all by definition.

Leontief's A, B -model of circulating and fixed capital is a great advance which facilitates quantitative analysis. His model, however, does not take into account the fact that production takes *time*. In Leontief's world, production is instantaneous. But the time feature is crucial in economic planning, however, as any manager or planner will readily confirm: The faster production takes place, the higher the growth rate. One of Marx's outstanding contributions is that he pays a great deal of attention to the time element in production: a good starting point for the incorporation of the production times in the circulating and fixed capital matrices A and B is Marx (1974, p. 239). For example, if input i 's production time in sector j equals τ_{ij} , then we can write interindustry demand for i at time 0 as

$$\sum_j a_{ij} x_j(\tau_{ij})$$

where $x(t)$ is the output vector at time t . This set-up is still unnecessarily restrictive as it assumes that, for the production of one unit of output, sector j must absorb a_{ij} units of input i exactly τ_{ij} time units prior to the delivery of output and no input at other times, a situation depicted schematically in Figure 1.

In reality input will also be required at other times prior to the delivery of output. The situation can be, for example, as appears in Figure 2.

In general the i -th input requirement for one unit of sector j -th output is represented by an input profile on the past. We shall now introduce a powerful point of view. Giving up the idea of a_{ij} being some number altogether, we redefine an input-output coefficient as a non-negative and bounded distribution on the non-positive time axis. (The boundedness merely means that the total input requirement, i.e. aggregated over time, is finite.) The whole profile in Figures 1 or 2 is considered to be the input-output coefficient. The width of its support (τ_{ij}) reflects the production

Figure 1.

Figure 2.

time. This set-up obviously applies to capital stock coefficients as well. Then the width of the support of the distribution reflects the investment lead time.

We have modelled the time aspect of productive capital, be it circulating or fixed. The same can be done for capital of circulation. Then a good starting point for the incorporation of the so-called circulation time is Marx (1974, pp. 248–49). If we add up the production time, investment lead time and circulation time, we obtain the turnover time of capital, a centre-piece in the theory of Marx (1974, p. 155). The many forms of capital encountered so far are displayed in Figure 3, where numbers refer to pages in Marx (1974). l , A and B represent Leontief's labour, flows and stock coefficients, respectively. C and M are Marx' own notation for commodity-capital and money-capital which are still open for Leontief like quantitative exploration.

2. The Model

This section models an economy in which any circulating capital in any sector is working capital with a single production time, and any fixed capital in any sector has to be advanced by a single investment lead time. Allowing for technical change, $a_{ij}(t)$

Figure 3.

and $\tau_{ij}(t)$ denote the amount of and the production time for input i entering sector j at time t , and $b_{ij}(t)$ and $\sigma_{ij}(t)$ denote the amount of and the investment lead time for capital good i installed in sector j at time t (see Figure 1). The material balance for good i at time t now becomes

$$x_i(t) - \sum_{j=1}^n [a_{ij}(t)x_j(t + \tau_{ij}(t)) + b_{ij}(t)\dot{x}[t + \sigma_{ij}(t)]] = y_i(t), \quad i = 1, \dots, n$$

where x is the output vector, as before, and y is the final demand vector. Next choose the time unit so small that, first, all $\tau_{ij}(t)$ and $\sigma_{ij}(t)$ become integers and, second $x(t)$ can be approximated by $x(t+1) - x(t)$. Then we may limit attention to integer values of t . For comparison with related studies below, we also neglect the past. Then

$$1x_i(t) - \sum_{j=1}^n [a_{ij}(t)x_j(t + \tau_{ij}(t)) - b_{ij}(t)x_j(t + \sigma_{ij}(t)) + b_{ij}(t)x_j(t + \sigma_{ij}(t) + 1)] = y_i(t),$$

$$i = 1, \dots, n, \quad t = 0, 1, 2, \dots$$

Observe that any coefficient is multiplied by a quantity valued at a non-negative time lag further into the future. All coefficients, including the ones, are sorted out by the respective lags. These lags are contained in the set of $s=0, 1, 2, \dots$. $G_s(t)$ is defined as the $n \times n$ -matrix which contains all $-a_{ij}(t)$ such that $\tau_{ij}(t)=s$, all $b_{ij}(t)$ such that $\sigma_{ij}(t)=s$, and all $-b_{ij}$ such that $\sigma_{ij}(t)+1=s$. The locations of these coefficients are dictated by their subscripts. In addition, the one coefficient of $x_i(t)$ enters the i -th diagonal element of $G_0(t)$. If no coefficient enters a $G_s(t)$ element, then this element is set zero. (Formally $G_s(t)$ is defined as follows. Its (i, j) -th element equals

$$\delta_{ij}\delta_{0,s} - a_{ij}(t)\delta_{\tau_{ij}(t), s} + b_{ij}(t)\delta_{\sigma_{ij}(t), s} - b_{ij}(t)\delta_{\sigma_{ij}(t)+1, s}$$

where δ_{ij} is the Kronecker symbol which is one for $i=j$ and zero otherwise.) The material balances can now be summarized as

$$\sum_{s=0}^{\infty} G_s(t)x(t+s) = y(t), t = 0, 1, 2, \dots$$

which casts it in the mould of ten Raa (1986), where the equation is solved for output levels.

3. Estimation

In principle the classical technique of measuring all plans and stocks at each time is sufficient to determine the flow coefficients $a_{ij}(t)$ and the stock coefficient $b_{ij}(t)$ and to deduce the associated lags $\tau_{ij}(t)$ and $\sigma_{ij}(t)$. However, there are three complications. First, it is hard to know for which output an observed input is required: current output, next period's output, or some future output? In fact, in a stationary economy this is impossible. On the other hand, it is also irrelevant provided that the economy remains stationary. Similar observations hold for intermediately regular economies. The most favourable environment for the determination of lags is a highly irregular economy with a constant structure. Then one has many linearly independent observations for a few lagged coefficients. The second complication is that the structure itself may vary. Observed input-output ratio variation need no longer be the unique manifestation of the lag structure through a varying bill of final goods. It can also reflect technical change, pure and simple. Thirdly, a general complication is that determination of the coefficients and the lags through inference requires massive data collection.

In view of these limitations, better results will be obtained by circulation of a questionnaire in which coefficients and lags are asked directly from the production managers. Since these data are technical ones, it may be good to collect them at the source.

Of course it may prove impossible to collect all these data, especially the lags. However, inference can be done with a Canadian time series of input-output tables for the period 1961-71 (Statistics Canada, 1979). We follow the static model, since we have the current transaction tables.

The Canadian tables adhere to the United Nations (1967) Systems of National Accounts which comprise an input or 'use' table $U=(u_{ij})$ of commodities i consumed by industries j and an output or 'make' table $V=(v_{jk})$ of industries j producing commodities k . The aim now is to derive an industry-by-industry transactions matrix. To this end we need an industry-by-industry input-output coefficients matrix and an industry output vector. Multiplication of the two yields the desired industry-by-industry transactions matrix.

The derivation of the industry-by-industry input-output coefficients matrix is standard. One defines a commodity-by-industry coefficients matrix $B=U_{V_e^{-1}}$, where e is the vector with all entries unity, and an industry-by-commodity market share matrix $D=V_{V_e^{-1}}$. The industry-by-industry input-output coefficients matrix is DB . The industry output vector is Ve . It follows that the transactions matrix is $Z=DB_{V_e}=V_{V_e^{-1}}U$.

The static model can be written as

$$x(t) = Z(t)e + y(t)$$

with industry demand Z defined by

$$z_{ij}(t) = a_{ij}x_j(t + \tau_{ij}).$$

In the remainder of this section we fix the transaction, ij , so that the subscripts can be deleted. Our problem is to estimate a and τ simultaneously. We use an empirical regularity. The stability of input-output proportions over time suggests that a geometrical average should be considered with a multiplicative error. In other words, approximate $x(t + \tau)$ by the geometric mean of $x(t)$ and $x(t + 1)$:

$$x(t + \tau) = x(t)^{1-\tau}x(t+1)^\tau.$$

Then we obtain

$$\log z(t) = \log a + (1 - \tau) \log x(t) + \tau \log x(t + 1).$$

This cannot be subjected directly to ordinary least squares estimation, as the coefficients are $(1 - \tau)$ as well as τ . A rewrite does the trick:

$$\log \frac{z(t)}{x(t)} = \log a + \tau \log \frac{x(t+1)}{x(t)}$$

Tacking on a multiplicative lognormally distributed error term to $z(t) = ax(t + \tau)$ can easily be shown to yield a normally distributed error term attached to the present equation. Thus we obtain an ordinary regression of (the log of) the conventional Leontief input-output coefficients on (the log of) the growth rate, with coefficient τ and constant term $(\log) a$. It should be noted that dividing through by $x(t)$ affects neither the deterministic nature of the explanatory variables nor the stochastic nature of the variable to be explained. In other words, the ordinary least squares assumption is maintained.

Note that, statistically a and τ are separated out. Confidence intervals for τ and $\log a$ (hence a) can be obtained immediately, since they coincide with the coefficient and the constant term, respectively, in the regression equation. The estimates, along with 95% confidence intervals, are reported in Tables 1 and 2. Coefficients are multiplied by a factor of 100, so that the unit is cents per dollar. The separation property is a nice by-product of the multiplicative stochastic structure of our input-output model. It does not hold for additive errors which we tried initially, but which seem less appropriate from an input-output perspective. In other words, theoretical purity and statistical convenience go together in this analysis. For those who believe less in beauty, the alternative stochastic specifications can be nested in a model of heteroscedasticity of esteemable degree and thus be tested. See Judge, Hill, Griffiths, Lükepohl and Lee (1982, sections 14.4 and 14.5).

4. Conclusion

Table 1 displays an interesting picture of the times material inputs spend in the production process, that is of working capital. The confidence intervals are disturbingly wide, though. Basically, there is too little information in the data. In the extreme case of no departure from balanced growth, the lags cannot be identified at all, as is easily illustrated. The last equation reduces to

$$\log \frac{z(t)}{x(t)} = \log a + \tau \log \frac{e^g x(t)}{x(t)} = \log a e^{g\tau}$$

where g is the growth rate. One can estimate $a e^{g\tau}$, but a and τ cannot be identified. The

Table 1. Estimated values of τ

	Agriculture	Forestry	Fishing and hunting	Mines and quarries
Agriculture	0.55 (-0.29, 1.40)	2.80 (-3.13, 8.73)	-2.77 (-8.11, 2.58)	-5.99 (-22.1, 10.1)
Forestry	-0.36 (-2.73, 2.00)	0.83 (-2.11, 3.76)	0.53 (-1.00, 2.05)	8.62 (-9.62, 26.9)
Fishing and hunting	1.39 (-0.35, 3.13)	-0.14 (-5.56, 5.29)	2.07 (-0.97, 5.10)	-1.45 (-6.35, 3.44)
Mines and quarries	-0.83 (-3.34, 1.67)	-1.66 (-3.60, 0.28)	0.31 (-3.50, 4.12)	-0.67 (-5.67, 4.12)
Manufacturing	0.36 (-0.48, 1.19)	0.34 (-1.58, 2.26)	0.22 (-0.86, 1.30)	0.22 (-2.76, 3.20)
Construction	-0.48 (-14.4, 13.4)	-0.12 (-17.6, 17.4)	-1.09 (-16.0, 13.8)	-4.99 (-46.7, 36.7)
Transport and storage	0.46 (-0.33, 1.24)	-1.12 (-3.58, 1.33)	0.47 (0.16, 0.79)	0.44 (-2.41, 3.28)
Communication	0.67 (-0.01, 1.34)	0.45 (-0.49, 1.39)	-0.22 (-1.61, 1.17)	1.11 (-0.10, 2.32)
Utilities	-0.21 (-1.13, 0.71)	-2.25 (-10.2, 5.73)	0.19 (-1.47, 1.85)	0.62 (-0.69, 1.92)
Wholesale trade	0.49 (-0.29, 1.27)	-0.04 (-0.57, 0.48)	-0.07 (-1.12, 0.98)	0.65 (-0.94, 2.24)
Retail trade	0.56 (-0.11, 1.23)	-0.36 (-1.19, 0.47)	-1.71 (-8.27, 4.85)	0.11 (-2.19, 2.41)
Finance	-0.28 (-0.92, 0.36)	-0.04 (-1.15, 1.08)	0.13 (-1.19, 1.44)	0.46 (-1.01, 1.94)
Services	0.46 (-0.09, 1.01)	-0.45 (-1.00, 0.10)	0.99 (-0.50, 2.48)	0.48 (-0.94, 1.91)
Transport margins	0.09 (-0.95, 1.12)	0.52 (-0.35, 1.39)	0.03 (-0.75, 0.81)	-0.18 (-1.87, 1.50)
Office supplies	-0.57 (-1.95, 0.81)	-0.03 (-1.41, 1.36)	0.93 (-0.71, 2.56)	-0.94 (-4.73, 2.84)
Travel and advtg		0.01 (0.00, 0.01)		1.03 (-0.87, 2.93)

Table 1 (continued).

	Manufacturing	Construction	Transport and storage	Communication
Agriculture	2.54 (0.84, 4.34)	-0.16 (-3.33, 3.00)	-4.29 (-8.40, -0.18)	-17.2 (-61.0, 26.5)
Forestry	1.89 (-0.10, 3.89)	-0.22 (-5.62, 5.19)	12.9 (1.96, 23.8)	43.7 (-3.74, 91.0)
Fishing and hunting	1.24 (-0.47, 2.95)	0.10 (-6.85, 7.06)	-12.8 (-21.8, -3.84)	-21.0 (-32.4, -9.62)
Mines and quarries	1.41 (0.64, 2.19)	-2.09 (-4.75, 0.57)	-2.01 (-6.35, 2.33)	11.8 (1.90, 21.7)
Manufacturing	-0.84 (-1.44, -0.25)	0.06 (-0.22, 0.35)	2.05 (-2.03, 6.13)	5.14 (-17.0, 27.3)
Construction	-9.05 (-46.9, 28.8)	4.20 (-15.8, 24.2)	-22.6 (-74.1, 28.9)	-21.9 (-175, 131)
Transport and storage	-0.44 (-1.88, 1.00)	-0.40 (-4.56, 3.75)	-2.40 (-5.17, 0.37)	7.29 (-6.20, 20.8)
Communication	-0.96 (-2.34, 0.43)	-0.12 (-3.96, 3.72)	-1.62 (-3.42, 0.19)	0.24 (-4.14, 4.63)
Utilities	-0.43 (-1.15, 0.29)	0.10 (-0.77, 0.97)	-0.19 (-1.19, 0.81)	-11.7 (-23.3, -0.14)
Wholesale trade	-0.86 (-1.76, 0.03)	0.65 (0.17, 1.13)	1.00 (1.39, 3.39)	2.62 (-5.67, 10.9)
Retail trade	1.40 (-0.38, 3.18)	0.37 (0.09, 0.64)	-0.93 (-1.82, -0.04)	-15.8 (-44.9, 15.4)
Finance	0.04 (-0.93, 1.01)	0.69 (-2.31, 3.68)	-3.19 (-5.27, -1.11)	8.80 (0.80, 16.8)
Services	-2.02 (-3.79, -0.26)	-1.30 (-4.47, 1.88)	-3.54 (-5.94, -1.14)	-1.03 (-6.05, 3.98)
Transport margins	0.49 (0.06, 0.93)	-0.05 (-0.78, 0.67)	-0.17 (-1.47, 1.13)	-1.57 (-6.37, 3.23)
Office supplies	-0.47 (1.66, 0.72)	1.04 (-2.20, 4.28)	2.19 (0.07, 4.46)	11.3 (-4.86, 27.5)
Travel and advtg.	1.38 (-0.04, 2.79)	0.40 (-0.93, 1.72)	-0.07 (-1.37, 1.22)	7.19 (2.50, 11.9)

Table 1 (continued).

	Utilities	Wholesale trade	Retail trade	Finance
Agriculture	-7.10 (-24.2, 10.0)	1.18 (-2.40, 4.75)	-0.13 (-10.4, 10.1)	-1.42 (-37.3, 34.5)
Forestry	3.66 (-4.68, 12.0)	8.52 (-3.75, 20.8)	2.42 (-3.44, 8.28)	-0.13 (-6.60, 6.34)
Fishing and hunting	-0.35 (-18.2, 17.5)	-0.21 (-6.12, 5.69)	-7.16 (-19.1, 4.83)	3.48 (-19.8, 26.8)
Mines and quarries	-2.15 (-24.3, 20.0)	-0.53 (-3.56, 2.49)	0.63 (-3.57, 4.83)	1.16 (-4.31, 6.64)
Manufacturing	20.4 (-1.42, 42.2)	0.88 (-0.52, 2.29)	1.73 (-1.43, 4.89)	29.4 (-26.1, 84.8)
Construction	-76.8 (-136, -17.5)	-1.09 (-20.2, 18.0)	-13.5 (-61.7, 34.7)	-51.1 (-159, 57.1)
Transport and storage	-2.97 (-15.3, 9.39)	1.14 (-1.01, 3.29)	0.33 (-1.86, 2.52)	3.90 (-8.91, 16.7)
Communication	0.12 (-4.12, 4.35)	0.21 (-0.48, 0.90)	2.08 (-2.18, 6.35)	1.42 (-8.47, 11.3)
Utilities	2.08 (-3.32, 7.47)	0.67 (-0.77, 2.11)	1.55 (-2.71, 5.81)	2.03 (-10.6, 14.7)
Wholesale trade	-0.46 (-11.0, 10.1)	1.13 (-0.81, 3.07)	1.80 (-4.92, 8.51)	2.89 (-4.46, 10.2)
Retail trade	0.15 (-6.54, 6.84)	1.51 (-1.46, 4.47)	0.46 (-3.69, 4.61)	1.64 (-2.35, 5.63)
Finance	0.43 (-10.2, 11.0)	1.23 (-0.20, 2.26)	0.60 (-0.68, 1.88)	0.02 (-2.42, 2.47)
Services	0.65 (-4.81, 0.11)	0.26 (-1.25, 1.76)	-0.13 (-2.13, 1.87)	0.57 (-5.72, 6.86)
Transport margins	-1.67 (-24.2, 20.6)	0.64 (-0.12, 1.40)	0.18 (-2.19, 2.56)	0.66 (-4.12, 5.44)
Office supplies	-3.39 (-16.0, 9.26)	0.73 (-1.54, 3.01)	3.50 (-1.66, 8.66)	1.57 (-9.75, 12.9)
Travel and advt.	0.87 (-7.73, 9.46)	0.84 (-1.52, 3.21)	0.01 (-1.90, 1.92)	0.79 (-0.85, 2.44)

Table 1 (continued).

	Services	Transport margins	Office supplies	Travel and advtg.
Agriculture	-1.59 (-3.80, 0.62)		2.36 (0.37, 4.34)	
Forestry	-1.33 (-2.88, 0.22)	10.3 (-11.9, 32.5)	2.89 (1.22, 4.56)	6.23 (-1.32, 13.8)
Fishing and hunting	0.12 (-4.01, 4.25)		1.48 (-1.18, 4.14)	-2.95 (-10.9, 5.00)
Mines and quarries	3.62 (-3.27, 10.5)		-4.39 (-9.22, 0.45)	-2.66 (-8.18, 2.86)
Manufacturing	-0.80 (-1.87, 0.28)	-13.0 (-28.5, 2.51)	-0.10 (-0.51, 0.30)	0.28 (-0.40, 0.97)
Construction	6.83 (-16.0, 29.7)		-8.15 (-20.0, 3.73)	-1.28 (-6.57, 4.01)
Transport and storage	0.29 (-1.75, 2.34)	-0.01 (-0.05, 0.02)	0.67 (-1.11, 2.45)	-2.60 (-4.42, -0.79)
Communication	1.27 (-0.30, 2.84)		-0.91 (-2.81, 0.98)	0.86 (-1.16, 2.88)
Utilities	1.11 (-3.54, 5.77)		-0.77 (-3.19, 1.66)	2.04 (0.42, 3.65)
Wholesale trade	0.46 (-0.40, 1.31)		0.12 (-0.54, 0.77)	-0.31 (-1.95, 1.21)
Retail trade	-0.09 (-1.15, 0.97)	2.59 (-4.40, 9.58)	0.34 (0.02, 0.64)	1.03 (0.08, 1.98)
Finance	0.69 (-0.73, 2.11)		-3.63 (-6.31, -0.97)	-4.35 (-12.3, 3.56)
Services	-0.39 (-2.07, 1.28)		0.71 (-0.15, 1.57)	1.03 (-0.05, 2.11)
Transport margins	-0.02 (-1.53, 1.49)		-0.42 (-1.30, 0.46)	-0.66 (-2.29, 0.97)
Office supplies	0.02 (-2.51, 2.55)			
Travel and advtg.	0.82 (-1.05, 2.70)			

Table 2. Estimated values of A (times 100)

	Agriculture	Forestry	Fishing and hunting	Mines and quarries
Agriculture	4.19 (3.88, 4.50)	0.31 (0.22, 0.40)	0.00 (0.00, 0.00)	0.00 (-0.00, 0.01)
Forestry	0.01 (0.01, 0.02)	6.91 (5.89, 7.94)	0.01 (0.01, 0.01)	0.00 (-0.00, 0.01)
Fishing and hunting	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)	1.25 (0.96, 1.54)	0.00 (0.00, 0.00)
Mines and quarries	0.36 (0.28, 0.44)	0.10 (0.09, 0.11)	0.56 (0.40, 0.72)	3.93 (2.69, 5.18)
Manufacturing	20.3 (18.8, 21.8)	4.95 (4.47, 5.43)	22.6 (20.7, 24.4)	5.67 (4.55, 6.78)
Construction	1.27 (-0.28, 2.82)	1.22 (0.14, 2.30)	0.28 (-0.04, 0.60)	1.66 (-2.91, 6.23)
Transport and storage	0.35 (0.33, 0.38)	10.2 (8.98, 11.5)	1.29 (1.26, 1.32)	1.02 (0.83, 1.21)
Communication	0.30 (0.28, 0.32)	0.33 (0.32, 0.35)	0.31 (0.27, 0.34)	0.31 (0.28, 0.33)
Utilities	0.95 (0.89, 1.04)	0.16 (0.09, 0.22)	0.13 (0.11, 0.15)	2.01 (1.84, 2.19)
Wholesale trade	2.24 (2.08, 2.39)	0.86 (0.84, 0.88)	2.06 (1.89, 2.22)	0.80 (0.72, 0.89)
Retail trade	0.67 (0.63, 0.71)	0.52 (0.50, 0.54)	0.71 (0.35, 1.06)	0.21 (0.18, 0.24)
Finance	3.78 (3.57, 4.00)	8.04 (7.59, 8.49)	1.13 (1.02, 1.24)	7.38 (6.66, 8.10)
Services	0.25 (0.24, 0.26)	2.33 (2.26, 2.39)	0.76 (0.67, 0.84)	2.34 (2.12, 2.57)
Transport margins	1.78 (1.61, 1.94)	0.25 (0.24, 0.26)	1.05 (0.99, 1.11)	0.46 (0.41, 0.51)
Office supplies	4.43 (3.89, 4.97)	12.6 (11.7, 13.5)	1.47 (1.28, 1.65)	7.78 (5.83, 9.73)
Travel and advtg.		0.57 (0.48, 0.67)		0.53 (0.47, 0.60)

Table 2 (continued).

	Manufacturing	Construction	Transport and storage	Communication
Agriculture	4.98 (4.41, 5.55)	0.06 (0.05, 0.07)	0.02 (0.02, 0.03)	0.00 (-0.01, 0.01)
Forestry	1.98 (1.72, 2.24)	0.07 (0.05, 0.10)	0.01 (0.00, 0.02)	0.00 (-0.00, 0.00)
Fishing and hunting	0.31 (0.27, 0.34)	0.00 (0.00, 0.00)	0.00 (0.00, 0.01)	0.00 (0.00, 0.00)
Mines and quarries	5.51 (5.23, 5.78)	1.54 (1.31, 1.77)	0.29 (0.21, 0.38)	0.01 (0.00, 0.01)
Manufacturing	34.5 (32.3, 36.7)	41.1 (40.5, 41.8)	7.16 (5.19, 9.13)	2.47 (-1.59, 6.52)
Construction	0.60 (-0.89, 2.08)	0.06 (-0.01, 0.13)	8.20 (-20.3, 36.7)	8.98 (-92.9, 111)
Transport and storage	0.85 (0.77, 0.93)	0.86 (0.66, 1.06)	9.19 (7.47, 10.9)	2.73 (-0.00, 5.47)
Communication	0.62 (0.57, 0.68)	0.36 (0.28, 0.43)	1.33 (1.17, 1.50)	2.44 (1.65, 3.23)
Utilities	1.06 (1.01, 1.11)	0.06 (0.06, 0.06)	0.48 (0.45, 0.52)	0.60 (0.09, 1.11)
Wholesale trade	2.23 (2.10, 2.36)	4.34 (4.22, 4.45)	1.35 (1.13, 1.57)	0.21 (0.08, 0.34)
Retail trade	0.24 (0.21, 0.26)	0.90 (0.88, 0.91)	0.98 (0.92, 1.04)	2.89 (-3.35, 9.13)
Finance	1.25 (1.17, 1.33)	1.15 (0.96, 1.35)	2.62 (2.25, 2.99)	0.90 (0.37, 1.43)
Services	1.10 (0.97, 1.22)	3.49 (2.88, 4.11)	2.13 (1.78, 2.47)	2.56 (1.61, 3.51)
Transport margins	2.08 (2.02, 2.14)	2.36 (2.26, 2.45)	0.40 (0.36, 0.43)	0.08 (0.05, 0.11)
Office supplies	3.01 (2.77, 3.24)	1.79 (1.47, 2.11)	4.00 (3.39, 4.62)	0.99 (-0.20, 2.17)
Travel and advtg.	2.26 (2.05, 2.47)	0.41 (0.38, 0.43)	0.98 (0.90, 1.07)	0.53 (0.35, 0.72)

Table 2 (continued).

	Utilities	Wholesale trade	Retail trade	Finance
Agriculture	0.00 (-0.00, 0.00)	0.10 (0.08, 0.14)	3.94 (2.06, 5.23)	0.00 (-0.00, 0.01)
Forestry	0.00 (0.00, 0.00)	0.01 (0.00, 0.01)	0.01 (0.01, 0.01)	0.00 (0.00, 0.00)
Fishing and hunting	0.00 (-0.00, 0.00)	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)
Mines and quarries	3.26 (-1.78, 8.30)	0.10 (0.08, 0.12)	0.20 (0.16, 0.23)	0.08 (0.06, 0.09)
Manufacturing	0.35 (-0.18, 0.88)	4.26 (3.84, 4.68)	3.47 (2.96, 3.98)	0.18 (-0.28, 0.64)
Construction	4.41 (-13.85, 22.67)	0.19 (-0.01, 0.45)	0.89 (-1.10, 2.87)	46.1 (-184, 276)
Transport and storage	0.34 (0.05, 0.63)	3.26 (2.76, 3.75)	1.09 (0.98, 1.20)	0.07 (0.03, 0.11)
Communication	0.35 (0.25, 0.46)	2.08 (1.98, 2.18)	2.07 (1.66, 2.49)	1.31 (0.71, 1.90)
Utilities	1.63 (1.02, 2.24)	0.62 (0.56, 0.69)	1.72 (1.37, 2.05)	0.41 (0.17, 0.66)
Wholesale trade	0.37 (0.10, 0.64)	0.96 (0.83, 1.10)	0.42 (0.29, 0.56)	0.07 (0.05, 0.10)
Retail trade	0.15 (0.08, 0.23)	0.39 (0.31, 0.47)	0.30 (0.24, 0.36)	0.10 (0.08, 0.12)
Finance	2.00 (0.52, 3.48)	4.47 (4.14, 4.79)	6.85 (6.44, 7.26)	6.03 (5.35, 6.71)
Services	0.74 (0.46, 1.03)	2.24 (2.00, 2.47)	1.15 (1.04, 1.25)	1.61 (1.14, 2.08)
Transport margins	0.53 (-0.30, 1.37)	0.22 (0.21, 0.23)	0.21 (0.19, 0.23)	0.01 (0.01, 0.02)
Office supplies	1.59 (0.19, 2.99)	4.18 (3.51, 4.85)	4.35 (3.30, 5.39)	2.77 (1.32, 4.22)
Travel and advtg.	0.39 (0.16, 0.64)	5.44 (4.53, 6.35)	3.12 (2.85, 3.40)	1.93 (1.79, 2.08)

Table 2 (continued).

	Services	Transport margins	Office supplies	Travel and advtg.
Agriculture	0.95 (0.83, 1.08)		0.67 (0.57, 0.77)	
Forestry	0.01 (0.01, 0.01)	0.07 (-0.04, 0.19)	0.05 (0.04, 0.06)	0.08 (0.05, 0.11)
Fishing and hunting	0.11 (0.09, 0.14)		0.02 (0.02, 0.03)	0.00 (0.00, 0.00)
Mines and quarries	0.09 (0.05, 0.13)		0.33 (0.21, 0.45)	0.08 (0.06, 0.11)
Manufacturing	13.0 (12.2, 13.9)	0.09 (-0.01, 0.19)	60.5 (58.7, 62.4)	33.2 (32.0, 34.3)
Construction	0.21 (-0.08, 0.50)		0.23 (0.02, 0.44)	0.19 (0.14, 0.24)
Transport and storage	0.32 (0.28, 0.36)	99.6 (99.4, 99.9)	0.14 (0.12, 0.16)	16.3 (14.8, 17.9)
Communication	1.57 (1.43, 1.72)		0.16 (0.14, 0.18)	7.58 (6.78, 8.37)
Utilities	0.63 (0.45, 0.80)		0.06 (0.05, 0.07)	0.07 (0.06, 0.07)
Wholesale trade	1.04 (0.99, 1.10)		8.99 (8.54, 9.44)	2.43 (2.23, 2.64)
Retail trade	1.23 (1.10, 1.35)	0.16 (0.08, 0.24)	5.89 (5.75, 6.03)	4.45 (4.23, 4.67)
Finance	4.61 (4.22, 5.00)		0.09 (0.07, 0.11)	0.14 (0.08, 0.20)
Services	3.33 (3.00, 3.67)		10.2 (9.58, 10.9)	23.9 (22.5, 25.2)
Transport margins	0.44 (0.40, 0.48)		2.80 (2.61, 2.99)	0.63 (0.57, 0.68)
Office supplies	6.16 (5.24, 7.09)			
Travel and advtg.	2.33 (2.07, 2.59)			

standard procedure is, of course, to fix $\tau=0$ and to estimate a by contemporary division of flows. This is observationally equivalent to a sluggish economy with smaller coefficients. In our data set, the situation is not this bad, but a scatter diagram exposes the problem. In Figure 4 we have plotted

$$\log \frac{z(t)}{x(t)}$$

against the growth rate of agriculture for the agricultural and manufacturing inputs, respectively. The slopes estimate the lags. It is clear that the slopes are poorly determined.

We conclude that our input-output model lends itself to sound statistical analysis of working capital, but that the data contain too little information. We need either

Figure 4.

observations of more irregular economies, or direct information about time spent in the pipeline of production. This conclusion holds *a fortiori* for more complicated production functions and, therefore, casts some doubt on the precision of results on time to build studies such as Kydland and Prescott (1982).

References

- Brody, A. (1974) *Proportions, Prices and Planning* (Budapest, Akademiai Kiado, and Amsterdam-London, North Holland Publishing Company).
- Foley, D.K. (1982) 'Accumulation, realization, and crisis', *Journal of Economic Theory* 30, 2, pp. 300-19.
- Judge, G.G., Hill, R.C., Griffiths, W., Lütkepohl, H. & Lee, T.-C. (1982) *Introduction to the Theory and Practice of Econometrics* (New York, John Wiley & Sons).
- Kydland, F.E. & Prescott, E.C. (1982) 'Time to build and aggregate fluctuations', *Econometrica* 50, 6, pp. 1345-70.

- Leontief, W. (1970) 'The Dynamic Inverse', in: *A.P. Carter & A. Brody, eds., Contributions to Input-Output Analysis*, (Amsterdam-London, North Holland Publishing Company) pp. 17-46.
- Marx, K. (1974) *Capital II: The Process of Circulation of Capital* (New York, International Publishers).
- Nikaido, H. (1962) 'Some dynamic phenomena in the Leontief model of reversely lagged type', *Review of Economic Studies* 29, pp. 313-23.
- Statistics Canada (1979) *The Input-Output Structure of the Canadian Economy in Constant Prices, 1961-74*.
- ten Raa, Th. (1986) 'Applied dynamic input-output with distributed activities', *European Economic Review* 30, 4, pp. 805-31.
- UN Statistical Commission (1967) *Proposals for the Revision of SNA, 1952* (Document E/CN.3/356).