Joint-liability with endogenously asymmetric group loan contracts

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ABSTRACT

Group lending is a common practice that Microfinance Institutions (MFIs) utilize when lending to individuals without collateral. We develop a multi-agent principal-agent model with costly peer monitoring and solve for the optimal group loan contract. The optimal contract exhibits (i) a joint-liability scheme; and, (ii) asymmetric loan terms which can be interpreted as appointing a group leader, who has strong incentives to monitor her peers. Relaxing the joint-liability scheme implies the breakdown of equilibrium monitoring. When the contractual asymmetry is relaxed, the peer-monitoring game exhibits multiple Nash equilibria: a (weak) good equilibrium at which borrowers monitor each other and a (strong) bad equilibrium without monitoring. This key result suggests that profit maximizing MFIs should provide asymmetric group loan contracts - even to a homogeneous group of borrowers - to ensure stability in repayment rates.

1. Introduction

Microfinance institutions utilize group loan contracts when borrowers lack sufficient collateral to support individual lending. Understanding the efficiency of group lending has become more important since Grameen Bank and other financial intermediaries have successfully implemented group loans in alleviating entrepreneurial financial constraints in the developing world. A key feature of group loan contracts is the joint-liability of the group of borrowers. Joint-liability works especially well in rural borrowing environments that have weak formal institutions but exhibit strong social ties. The argument is that by making borrowers liable for each other’s repayments in case of a credit default, joint-liability could mobilize peer-monitoring to overcome financial market imperfections. In this paper we take this argument one step forward and show that in an environment characterized by moral hazard, limited liability and low monitoring costs, sustainable joint-liability mechanisms can require the empowerment of one borrower in the group with strong incentives to monitor peer borrowers. We interpret this feature as the assignment of a group leader, who is set to do the right thing no matter what.

The existing literature on group lending has mostly concentrated on explaining how joint-liability can make use of the locally available knowledge that the group members have about each other.\textsuperscript{1} Another important but not thoroughly explored characteristic term of microfinance lending is the asymmetric treatment of borrowers in a group loan contract. More specifically, microfinance institutions usually request for one agent to serve as a leader of the group loan participants.\textsuperscript{2} The leader has three primary functions: to maintain discipline, to distribute information, and to facilitate repayments. How can the appointment of a leader be rationalized? On what characteristics of group participants does the desirability of within-group asymmetric loan contracts hinge?

A possible explanation put forward in the literature is that loan applicants differ in some exogenous characteristics, so that the appointment of a group leader is motivated by such heterogeneity.\textsuperscript{3} In this paper we provide an alternative explanation and show that group member heterogeneity is not pivotal in making group-leadership desirable. Specifically, we argue that if borrowers’ peer monitoring and

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\textsuperscript{2} The seminal papers in this literature include Stiglitz (1990), Armendáriz de Aghion (1999), Ghatak and Guinnane (1999), Ghatak (2000), LaFont and Rey (2003).
\textsuperscript{3} The Grameen Bank model states that groups are formed through self-selection and the group has to select a leader. This selection is random, which means that in principle any member of the group can be selected as the leader. There are several empirical papers in the literature that we discuss in Section 2, such as Paxton et al. (2000), Hermes et al. (2005) and Hermes et al. (2006), highlighting the group leader’s impact on borrower performance.
\textsuperscript{4} The existing theoretical literature almost ignores the role of a group leader in fostering the performance of the group. To the best of our knowledge, the only two exceptions are Katzur and Lensink (2010) and van Eijkel et al. (2011), where the appointment of a group leader is the consequence of ex-ante heterogeneity in the degree of borrowers risk-aversion or in the profitability of their investment opportunities.
hence social enforcement on each other is not observable by the microfinance institution, joint-liability per se does not guarantee operational efficiency of group lending schemes. Optimality requires as well the involvement of a group leader - even with ex-ante identical group loan participants - who is treated differently: the group leader is provided with strong incentives to produce social enforcement and receives higher expected utility in the optimal contract.

To study optimal group loan contracts with non-observable social enforcement, we set up a monitor-then-perform type of moral hazard problem and develop a principal agent model with multiple agents. The principal in the model is a profit maximizing Microfinance Institution (MFI) and the agents are borrowers. Borrowers in the model can behave negligently, reducing the profitability of their projects but earning some private benefits. Intra-group monitoring of peers determine the private benefits from non-diligent behavior. However, since effort needs to be spent to monitor, a borrower chooses to devote time into monitoring her peer only if this is in her best interest. Importantly, the monitoring effort of loan applicants is non-observable by the MFI. As a result, the MFI faces a non-trivial implementation problem, having to consider borrowers’ incentives to monitor their peers when making them jointly liable for each other’s debt. Unobservable monitoring efforts imply that the MFI can punish and reward agents only as a function of project success. A possible risk of this implementation problem is that punishments and rewards could induce multiple equilibria, thus the lender may worry about a coordination failure.

Given this framework we characterize the optimal group lending contract. We show that this optimal contract has two distinctive features. First, it exhibits a joint-liability scheme. Second, it requires the asymmetric treatment of borrowers. This result can be interpreted as the necessity of a group leader, meaning a group member who has strong incentives to monitor her peer borrower and gets compensated in expected terms. Importantly, this key result holds even for a group of borrowers which exhibit homogeneous characteristics in productivity, risk aversion, and monitoring costs. The intuition for our results is related to the implementation problem that the profit maximizing microfinance institution is facing. On the one hand, without a joint-liability scheme peer monitoring cannot be sustained in equilibrium. On the other hand, the absence of a group leader generates the possibility of multiple equilibria which undermines group’s commitment to repay. To the latter end, without a group leader the peer-monitoring game exhibits multiple Nash equilibria: a (weak) good equilibrium at which borrowers monitor each other and a (strong) bad equilibrium without monitoring.

Our theoretical analysis shows that the incorporation of a group leader is a necessary condition for the optimal contract to induce agents to monitor each other and to implement monitoring as the unique Nash equilibrium of the lending game. In this respect, our paper rationalizes the widely applied group leader practice of microfinance programs as the efficient outcome of an optimal lending contract. As highlighted by Bubna and Chowdhry (2010), financial intermediaries all around the world are seeking profitable mechanisms for participating in micro lending. We shed light on particular contractual terms that would help microfinance intermediaries to raise loan repayment performance. In our model, since borrowers are homogeneous, any of them can serve as the group leader. We resolve this problem providing an implementation scheme that assigns the group leadership by the outcome of a lottery whose realization is unknown at the time the contract is signed. When the lottery outcomes are equally likely the group-loan contract generates a fair outcome, where borrowers’ payoffs are ex-ante equal.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents a standard lending problem with moral hazard and Section 4 extends this lending problem into a group lending framework. Section 5 solves for the optimal group loan contract and characterizes the features of the group leader. Section 6 provides a discussion for our key results by showing how the group leader helps to get rid of an undesirable equilibrium multiplicity problem and presents an implementation scheme for the optimal contract. We provide some robustness check in Section 7, where we discuss the existence of the highlighted multiplicity of equilibria problem under alternative specifications of the benchmark model. Section 8 concludes.

2. Related literature

Group lending institutions have been extensively debated in a theoretical literature, which mainly focuses on the efficiency of joint-liability group loan contracts under informational asymmetry.1 We focus on moral hazard and on the efficient design of peer monitoring, while we abstract from adverse selection and peer selection considerations.2 Our paper provides a full-fledged characterization of the solution for a monitor-then-perform type of moral hazard problem in a group lending environment.

The early papers on group lending with moral hazard are Stiglitz (1990), Varian (1990), and Arnott and Stiglitz (1991). The argument of this literature is that facilitating peer monitoring could lead to higher probability of repayment. In particular, the model by Stiglitz (1990) is closely related to ours. Stiglitz (1990) shows that joint-liability can induce peer monitoring and ensure borrowers’ prudence in the use of external funds. However, because of borrowers’ risk aversion, peer monitoring comes with the cost of the higher risk that group participants have to bear. We abstract from risk considerations by assuming that borrowers are risk neutral. More importantly, we relax the assumptions in Stiglitz (1990) that peer monitoring is costless to produce and that loan contracts are exogenously symmetric among borrowers.

Also in this literature, Besley and Coate (1995) identify two opposite effects of joint liability on loan repayment: a positive effect associated with the successful borrower covering the repayment of an unsuccessful one whenever her project returns are high enough; and, a negative effect where the successful borrower defaults on the repayment if her project return is not that high, even though she would have repaid if she were in an individual liability contract. In this respect, the framework of Besley and Coate (1995) generates the foundations where joint liability agreements can lead to bad as well as good outcomes from the perspective of a lending institution. Therefore, they discuss the existence of an equilibrium multiplicity similar to what we study in our framework. In a different context, our paper also uncovers an equilibrium multiplicity situation. We argue that the lender can overcome the multiplicity problem by offering a group loan contract with asymmetric terms. Ghatak and Guillen (1999) analyze an ex-ante moral hazard model that is similar to ours. The authors show that borrowers’ effort choices are strategic complements when loan contracts exhibit joint-liability. Peer monitoring and social sanctions can improve welfare and overcome coordination failures, provided that social sanctions are effective and monitoring costs are low. Using a similar moral-hazard set-up, we uncover an equilibrium multiplicity situation that can undermine the profitability of lending. In this context incentivizing peer monitoring per se cannot rule out borrowers’ coordination to a bad equilibrium. Profitable lending requires also asymmetric treatment of borrowers which we interpret as the involvement of a leader-borrower with strong monitoring incentives.3

Our results relate to the literature that questions the efficiency of joint-liability. Gangopadhyay et al. (2005) claim that joint-liability lending contracts might violate an ex post incentive-compatibility constraint which says that the amount of joint-liability cannot exceed the amount of individual-liability. Ahlin and Townsend (2007) show

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1 Murdoch (1999, JEL) and Ghatak and Guillen (1999) provide detailed reviews of the key models in this literature.
2 The seminal papers on group lending with adverse selection are Ghatak (2000) and LaFont and N’Guesen (2000).
3 Other important papers on moral hazard and group lending are Conning (1991), Armendariz de Aghion (1999), Wydick (1999), LaFont and Rey (2003), and Chowdhry (2005).

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that, for a given base interest rate, joint liability within the group is a negative predictor of repayments, whereas the strength of social sanctions positively predicts repayment. Rai and Sjöström (2004) show that it is the ability to side-contract and cross-report that is crucial for the efficiency of group loan contracts and not necessarily the joint-liability scheme by itself. Our paper contributes to this debate by providing a characterization of optimal group lending contracts with costly and unobservable peer monitoring and showing that the optimality of joint-liability requires asymmetric loan contracts.

A number of empirical studies highlighted the role of group leaders in enhancing the performance of joint liability contracts. For instance, Paxton et al. (2000) use a data of 140 group borrowers from a micro-lending institution in Burkina Faso. They find that the quality of the group leader in running the group transactions is positively related to the repayment performance. Hermes et al. (2005) investigate the role of the group leader in reducing moral hazard. The authors use a dataset of 102 groups from Eritrean group lending institutions. They find evidence that the monitoring effort of the group leader reduces moral hazard behavior of group members. To the best of our knowledge, the only two theoretical papers that aimed to understand the role of a group leader in fostering the group performance are van Eijkel et al. (2011) and Katzen and Lensink (2010). The study of van Eijkel et al. (2011) shows that the entrepreneur with the highest future profits is motivated to put the highest effort to peer monitor and would desire to become the group leader. Katzen and Lensink (2010) find that riskier borrowers are motivated to act as group leaders. The key difference between our model and these two studies is that we do not assume any ex-ante heterogeneity in borrower characteristics, such as heterogeneity in risk attitudes and profitability.

In our framework asymmetric terms of repayment imply that borrowers of a group loan get rewarded differently in expectation. Optimality of differential rewards and discrimination have been also proposed in a line of organization literature on team leadership. For instance, Winter (2004, 2009) argue that discrimination in organizations might be unavoidable - even with identical agents - when the mechanism aims to induce everybody to exert effort. Goege and Kube (2010) provide experimental evidence to show that unequal rewards can potentially increase firm productivity by facilitating coordination. Chen (2012) identifies an all-or-nothing type of compensation heterogeneity in organizations with within-complementarities. Finally, Lopez-Pintado and Moreno-Ternero (2014) study the optimal management of teams in which agents’ effort decisions are mapped (via a production technology) into the probability of the team's success. Optimal wage schemes in such a context are largely discriminatory, but the authors also show that the extent of the discrimination crucially depends on the existence of moral hazard.

3. The lending problem

We consider a fairly general lending problem with moral hazard, which is similar to the framework studied in Holmstrom and Tirole (1997). There are three agents: a risk-neutral Microfinance Institution (MFI or bank) and two identical risk-neutral borrowers. The borrowers are denoted as 1 and 2. Borrowers need 100% external bank finance to run investment projects. Each borrower has access to a set of mutually exclusive projects. Project choice is private information, which implies that in the absence of monitoring or right incentives borrowers would choose opportunistically and reduce the probability of project success in order to enjoy a private benefit.

Formally, each borrower can privately choose one of the two project options that we illustrate in Table 1. We assume that the probability of success of the good project (p) is greater than the success probability of the bad project (p_B), i.e. p > p_B. However, the bad project yields a private benefit to the borrower worth of B > 0, which is non-predictive to the bank. As in Holmstrom and Tirole (1997), we interpret the private benefit that accrues to the borrower as the opportunity cost of managing the project diligently. Each project requires 1 unit of cash investment and generates a verifiable financial return equaling either 0 (failure) or θ (success). Project outcomes are publicly observable and verifiable. The opportunity cost of the unit cash is 1 and in autarky the borrower-payoffs are normalized to zero. Borrowers and the MFI are protected by a limited liability clause, which implies that the repayments to the bank cannot exceed the project output and they cannot be negative at any contingency.

We impose the following parameter restrictions, which make the problem that we investigate non-trivial from the perspective of the bank. First, the rate of return on invested capital satisfies:

A1. \( pθ > 1 > p_Bθ \)

Assumption A1 implies that the good project has positive net present value whereas the bad project’s pledgeable portion has negative net present value and hence it is undesirable for the bank. Second, the bank can incur \( ψ_{B\text{-bank}} \) units of disutility (effort) - called as monitoring - and eliminate the Bad-project from the borrower’s feasible set of actions. By construction we assume that it is too costly for the bank to rule out the Bad-project by its own monitoring effort. Formally:

A2. \( ψ_{B\text{-bank}} > \theta \)

We also make the following assumption:

A3. \( \frac{p}{p_B} > B > pθ - 1 \)

Assumption A3 ensures the unprofitability of incentivizing the diligent behavior at a group loan contract without “peer monitoring” to which we turn next.

4. Group lending with peer monitoring

We incorporate peer monitoring into the framework that we presented in Section 3 by assuming that the borrower i could monitor her peer, incurring a disutility of ψ. Peer monitoring eliminates the private benefit action from a borrower –i’s set of feasible investment projects, where we use the convention that –i refers to borrower i’s peer. We assume ψ to be sufficiently small - with explicit conditions on ψ to be derived at the end of our analysis - so that the bank might find it optimal to mobilize peer monitoring when maximizing the profits from extending a group loan. Importantly, peer monitoring is non-observable and non-verifiable by the bank. Since peer monitoring is non-contractible, delegation of monitoring to the borrowers of a group loan becomes non-trivial.

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7 In a related paper the same authors show that the role of the group leader is highly relevant for improving the repayment performance of the group (Hermes et al., 2006).

8 Following the standard group-lending framework in the literature we work with an environment, where the group of borrowers consist of two individuals.

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<table>
<thead>
<tr>
<th>Table 1 Investment projects.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Good Project</strong></td>
</tr>
<tr>
<td>Private Benefit</td>
</tr>
<tr>
<td>Prob. of Success</td>
</tr>
</tbody>
</table>

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9 In our model borrowers’ project outcomes are observable by the bank whereas project choices and peer monitoring are not. We think of this set-up as a natural representation of a rural production environment. In rural areas, due to lack of sufficient resources a formal lender (such as an MFI) might not be able to observe a borrower’s day-to-day decisions and activities (which includes project choices and peer monitoring efforts). However, since in rural areas production is primarily based on farmland, the final output can be easily observed by many - including a lender - almost costlessly: a person who walks by an agriculture-field can observe the amount of crops during a harvest season. Therefore, we believe that our framework is a good representation of rural borrowing environments, for which microfinance group lending is vital.
As we describe in Fig. 1, the timing of actions can be represented as a dynamic game consisting of two-stages. In the first-stage, the bank offers a group lending contract to borrowers 1 and 2 who decide whether to accept the contract (A) or not (R). Afterwards, the two borrowers simultaneously choose the monitoring effort. Formally, they decide on whether to monitor (m) the peer and incur the monitoring disutility ψ conditional on having obtained finance. The first-stage game ends with the monitoring effort of each borrower being revealed to her peer, but not to the bank. In the second-stage of the game borrowers choose investment projects. Following that, the project returns are realized and borrowers repay back to the bank. The individual specific repayment rates - as determined by the contractual terms - can be conditioned on the realization of one’s own project success as well as her peer’s project success. Finally, at the end of the second-stage, borrowers and the bank consume.

Since the only observable (and hence contractible) variables for the bank are borrower 1 and borrower 2 project realizations, the terms of the group loan will be conditioned only on project outcomes. Letting $s_i \in \{H, L\}$ be the realization of borrower $i$’s project - where $H$ denotes the high-state with θ-return and $L$ denotes the low-state with zero-return - a group loan contract is a collection of repayments, contingent on the realizations of the two projects. Formally, we define a group loan contract as follows.

**Definition.** A group loan contract is a collection of repayment functions $(R_{ii}^{n, s})_{i=1,2}$. For instance, $R_{ii}^{n, s}$ is borrower $i$’s repayment when her project return is in high state, $s_i = H$, while borrower 2 project return is in the low state, $s_2 = L$.

Stage 1

<table>
<thead>
<tr>
<th>Bank offers</th>
<th>Borrower $i$ chooses $e_i \in A_i^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>group loan contract ${R_{ii}^{n, s}}_{i=1,2}$</td>
<td>$A/R$</td>
</tr>
</tbody>
</table>

Stage 2

<table>
<thead>
<tr>
<th>Borrower $i$ chooses $e_i \in A_i^s$</th>
<th>State $(s_1, s_2)$ realises</th>
<th>The contract is executed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrowers and bank consume</td>
<td>$\sum_{h \in {H, L}} \pi_h \delta e \mathbb{1}_{s_2}^R$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Timing.

[Stage 2 diagram]

5. Optimal group loan contract

In this section we define an optimal group loan contract restricting our attention to contracts that implement a unique Nash equilibrium in each possible second-stage game, as well as a unique equilibrium in the first-stage game. In Section 6 we will show that without the restriction on equilibrium uniqueness in the first-stage, a group loan design would give rise to a game with multiple Nash equilibria: a weak good equilibrium, in which borrowers monitor each other and it is profitable for the bank to extend credit, and a strong bad equilibrium, in which borrowers take the private benefit action and the MFI incurs a loss in expected terms. The following definitions are essential for our analysis.

**Definition.** A group lending contract $(R_{ii}^{n, s})_{i=1,2}$ is incentive compatible for the strategy profiles $(\delta_i^*, \{e_i^h\}_{i=1}^{12})$ if

- (i) $(e_i^h, e_{-i}^h)$ is a Nash equilibrium of the second-stage game that starts at history $h_k$, and for any other feasible strategy $(e_i^\theta, e_{-i}^\theta) \in \mathcal{A}_i^s \times \mathcal{A}_{-i}^s$, it is true that for all $\epsilon > 0$, either $u_i(e_i^h, e_{-i}^h) + \epsilon > u_i(e_i^\theta, e_{-i}^\theta) + \epsilon > u_i(e_i^\theta, e_{-i}^h)$.

- (ii) In the first-stage, $(\delta_1^*, \delta_2^*)$ is a Nash equilibrium of the supergame, where the continuation payoff at history $h_k$ is computed according to strategies $(e_1^h, e_2^h)$, and for any other strategy $(\delta_1, \delta_2)$, it is true that for all $\epsilon > 0$, either $u_i(\delta_i^*, \delta_2) + \epsilon > u_i(\delta_i, \delta_2)$.

Condition (i) implies that $(e_1^h, e_2^h)$ is the unique Nash equilibrium of the second-stage game that starts at history $h_k$. Since at every possible history $h_k$ the set of feasible actions $\mathcal{A}_i^s$ consists at most of two elements, uniqueness is equivalent to say that either $e_1^h$ is a dominant strategy for borrower 1 or $e_2^h$ is a dominant strategy for borrower 2.10 The same argument applies to condition (ii): a contract is incentive compatible if $(\delta_1^*, \delta_2^*)$ is the unique Nash equilibrium in the first-stage, meaning that either $\delta_1^*$ is a dominant strategy for borrower 1 or $\delta_2^*$ is a dominant strategy for borrower 2.

**Definition.** A group lending contract $(R_{ii}^{n, s})_{i=1,2}$ is optimal if it is incentive compatible for strategies $\delta^*_1, \{e_i^h\}_{i=1}^{12}$, and it maximizes bank’s expected revenues, where the expectation is computed according to the probability distribution that is induced by the strategy profiles $\delta^*_1, \{e_i^h\}_{i=1}^{12}$. Let $\alpha(h_k; \delta)$ denote the probability induced over history $h_k$ by the strategy profile $\delta = (\delta_1, \delta_2)$, and $\rho(h_k, s_2; e)$ be the probability over states $(s_1, s_2)$ induced by the strategy profile $\delta = (e_1, e_2)$. The optimal contract solves:

$$\max_{R_{ii}^{n, s}} \sum_{h_k} \mathbb{E}(h_k; \delta) \left[ \sum_{(s_1, s_2)} \rho(h_k, s_2; e) (R_{12}^{n, s} + R_{21}^{n, s}) \right]$$

10 Notice that since repayments are continuous variables, we require that strategy $e_i^h$ is an $\epsilon$-dominant strategy. This is an innocent caveat in the definition of uniqueness, as in Winter (2004).
In this section, at first we characterize second-stage Nash-equilibria that are compatible with an \((m,m)\)-equilibrium in the first-stage. After that we solve for the optimal contract that induces a unique \((m,m)\)-equilibrium and analyze its properties. Two key features of the optimal contract will prevail: (i) joint-liability and (ii) asymmetric treatment of borrowers.

5.1. The symmetric monitoring contract \((m,m)\)

In this subsection, we present the payoffs of the second-stage contract at history \(h_2\) and analyze its properties. Two key features of the optimal contract will prevail: (i) joint-liability and (ii) asymmetric treatment of borrowers.

5.1.1. The second-stage game at history \(h_1\)

At the second-stage game that follows history \(h_1\), the set of feasible strategies for borrower \(i\) is \(\mathcal{A}_{1i}(\cdot) = \{0\}\), i.e. borrower \(i\) can only behave diligently. Therefore, \(e_i^1, e_i^2\) are \((0, 0)\) is the only feasible outcome at \(h_1\). Then, constraints \((1c)\) and \((1d)\) can be ignored for history \(h_1\). The ensuing payoffs following history \(h_1\) are

\[
p(\theta - pR_i^{HH} - (1-p)R_i^{HL}) - \psi > 0
\]

for borrower 1 and

\[
p(\theta - pR_i^{HH} - (1-p)R_i^{HL}) - \psi > 0
\]

for borrower 2.

5.1.2. The second-stage game at history \(h_2\)

Table 2 presents the payoffs of the second-stage game at history \(h_2\), where borrower 1 monitors her peer whereas borrower 2 does not. Hereafter, we use the convention that the row player is borrower 1, whereas the column player is borrower 2. At history \(h_2\), the second-stage feasible strategy space for borrower 2 is \(\mathcal{A}_{22} = \{0\}\); and therefore, \(e_2^2 = 0\). This means constraints \((1c)\) and \((1d)\) can be ignored for borrower 2 at history \(h_2\). For borrower 1 the feasible-strategy set at \(h_2\) is \(\mathcal{A}_{11} = \{0\}\) implying that constraints \((1c)\) and \((1d)\) are identical. In the following lemma we prove that \((\delta_i, \delta_j) = (m, m)\) in the first-stage requires \((e_i^1, e_i^2) = (0, 0)\) to be the unique equilibrium in the second-stage game.

**Lemma 5.1.** Any feasible contract that implements \((m,m)\) as a Nash equilibrium in the first-stage game must have \((0,0)\) as the unique Nash equilibrium at the second-stage game that follows the history \(h_2\).

**Proof.** See Appendix A. The intuition behind this result is the following: if at \(h_2\) the borrowers were choosing the same strategies as at \(h_1\), there would be no point for borrower 2 to monitor her peer in the first-stage. In order to discourage borrower 2 from shirking to monitor, at history \(h_2\) an incentive compatible contract has to encourage borrower 1 to take the private benefit action. Therefore, in order to implement peer monitoring in the first-stage game, constraint \((1c)\) must hold, which implies:

\[
p_2[\theta - pR_i^{HH} - (1-p)R_i^{HL}] - \psi + B \geq p(\theta - pR_i^{HH} - (1-p)R_i^{HL}) - \psi.
\]

This provides us with the following Lemma.

**Lemma 5.2.** When the repayments \((R_i^{h_2})_{i=1,2}\) satisfy condition \((4)\), the strategy profile \((0,0)\) is the unique Nash equilibrium of the second-stage game that follows history \(h_2\).

5.1.3. The second-stage game at history \(h_3\)

If borrower 1 does not monitor and borrower 2 monitors in the first-stage, the game will be at history \(h_3\). This history is symmetric to what we have studied at \(h_2\), which implies that any feasible contract that implements \((m,m)\) as a Nash equilibrium in the first-stage game, must implement \((0,0)\) as the unique Nash equilibrium in the second-stage game that follows history \(h_3\). Therefore, if the bank desires to implement a peer monitoring equilibrium in the first-stage, then constraint \((1c)\) must hold for borrower 2, requiring the following:

**Table 2**

<table>
<thead>
<tr>
<th>Second-stage game at history (h_2)</th>
<th>0</th>
<th>(p(\theta - pR_i^{HH} - (1-p)R_i^{HL}) - \psi &gt; p(\theta - pR_i^{HH} - (1-p)R_k^{HL}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>(p(\theta - pR_i^{HH} - (1-p)R_i^{HL}) - \psi &gt; B, p(\theta - pR_i^{HH} - (1-p)R_k^{HL}))</td>
<td></td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{4} e_i^1 \left( \sum_{(s_1, s_2) \in \mathcal{S}} p(s_1, s_2) e_i^1 [\theta_1^H - R_i^{HS} + e_i^1] \right) - \psi > 0
\]

\[
\sum_{i=1}^{4} e_i^2 \left( \sum_{(s_1, s_2) \in \mathcal{S}} p(s_1, s_2) e_i^2 [\theta_1^H - R_i^{HS} + e_i^2] \right) - \psi > 0
\]

\[
\sum_{i=1}^{4} e_i^1 \left( \sum_{(s_1, s_2) \in \mathcal{S}} p(s_1, s_2) e_i^1 [\theta_1^H - R_i^{HS} + e_i^1] \right) - \psi > 0
\]

\[
0 \leq R_i^{HS} - \theta_1^H > 0
\]
\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} \]  
\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} - B \] 
\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B \]

(5)

5.1.4. The second-stage game at history \( h_a \)

When neither of the borrowers monitor in the first-stage, the game ends up at history \( h_a \), where the feasible set of second-stage actions are \( A_{h_a}^1 = A_{h_a}^2 = \{0, B\} \). The set of actions that are feasible and the payoff matrix at \( h_a \) are presented in Table 3. Based on our parameter assumptions we first establish the following result.

**Lemma 5.3.** Any profitable contract that implements \((\delta_1, \delta_2) = (m, m)\) as a Nash equilibrium in the first-stage will induce the strategy profile \((e_{h_a}^1, e_{h_a}^2) = (B, B)\) as the unique Nash equilibrium in the second-stage game following history \( h_a \).

**Proof.** See Appendix A. Since the optimal contract induces \((e_{h_a}^1, e_{h_a}^2) = (B, B)\), as the only Nash equilibrium at history \( h_a \), we can ignore constraints (1c) and (1d) for both borrowers at history \( h_a \).

5.1.5. Implementation in the first-stage

Having derived all second-stage continuation-game equilibria, we can proceed with the first-stage implementation problem. Table 4 illustrates the expected payoff matrix of the super-game. In each entry of this payoff matrix, expected utilities are computed according to the equilibrium strategies that we derived in previous subsections.

Based on this super-game representation, if the bank implements any profitable contract that implements \((\delta_1, \delta_2) = (m, m)\) as the unique Nash Equilibrium of the first-stage, constraint (1e) requires

\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} + B = \psi \]

and furthermore, constraint (1f) implies

either

\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B = \psi \]

or

\[ p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B = \psi \]

We can finally state the problem of the bank that aims to maximize its profits, implementing as the unique equilibrium of the two stage game the one in which borrowers monitor each other in the first-stage game and act diligently in the second-stage:

\[ \max_{R^{HI}, R^{HH}, R^{HI}, R^{HH}} p[R^{HI} + R^{HI}] + (1 - p)[R^{HI} + R^{HI}] \]  
\[ s.t. \quad 0 \leq R^{HI}, R^{HI}, R^{HI}, R^{HI} \leq \theta \]

\[ p R^{HI} + (1 - p) R^{HI} \geq \theta - B \]

\[ p R^{HI} + (1 - p) R^{HI} \geq \theta - B \]

\[ R^{HI} - R^{HI} \geq \frac{\psi}{p(p - \psi)} \]

\[ R^{HI} - R^{HI} \geq \frac{\psi}{p(p - \psi)} \]

We can easily note that the limited liability constraint (1g) should bind for the repayment of a successful borrower whose peer is unsuccessful. This relaxes all constraints ((4), (5), (6b), (7a)) and raises bank’s profits. Therefore, we determine

\[ R^{HI} = R^{HI} = \theta. \]

Moreover, constraints (1g) and (7a) together imply that a solution exists if and only if (7a) is non-violated for \( R^{HI} = 0 \). This means we can ignore the constraint (1g) for \( R^{HI} \) and \( R^{HI} \), whereas (6b) and (7a) bind and determine \( R^{HI} \) and \( R^{HI} \) as

\[ R^{HI} = \theta - \frac{\psi}{p(p - \psi)} \]

\[ R^{HI} = \theta - \frac{\psi}{p(p - \psi)} \]

Given the repayment terms of the group loan contract we can express the expected utility of borrowers 1 and 2 net of monitoring costs, and the expected profit of the bank as follows

| Table 3 |
| --- | --- |
| Second-stage game at history \( h_a \). |

<table>
<thead>
<tr>
<th>m</th>
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<tbody>
<tr>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} - B )</td>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B )</td>
</tr>
<tr>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B )</td>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B )</td>
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<table>
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<tr>
<th>Table 4</th>
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<tr>
<td>Super-game.</td>
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<tr>
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<tr>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} - \psi )</td>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + \psi )</td>
</tr>
<tr>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B - \psi )</td>
<td>( p_\theta \theta - p R^{HH} - (1 - p) R^{HH} - (1 - p) R^{HI} + B + \psi )</td>
</tr>
</tbody>
</table>
Proposition 5.4. A group loan contract with peer monitoring exists if and only if

\[
\psi \leq \frac{2 P B}{\theta p (p - p_b)} \cdot \frac{2 p_p (\theta - 1) (p - p_b)}{p^2 + \theta p_b}.
\]

(14)

The optimal contract exhibits joint-liability of borrowers, i.e. \( R^{III}_{left} > R^{III}_{right} \) and \( R^{III}_2 > R^{III}_1 \). Moreover, although borrowers are ex-ante symmetric in all characteristics, repayments are ranked as \( R^{III}_{left} < R^{III}_2 < R^{III}_1 = R^{III}_{right} \). This implies that from her participation at the group loan contract borrower 1 enjoys a higher net expected utility compared to her peer, i.e. \( EV(m,m) > EV(m,m) \). Proposition 5.4 contains two important results. First, many studies in the literature on group lending with social enforcement assume that the effort cost of peer monitoring should be low enough in order to motivate efficient group lending mechanisms. The threshold that we characterized for \( \psi \) in Proposition 5.4 provides the highest value that this key parameter can take for the monitoring equilibrium with asymmetric contractual terms to be implementable and profitable from the perspective of a microfinance institution.

Second, and more importantly, the distinctive feature of the optimal group loan contract that we characterized is the endogenous emergence of the asymmetric treatment of borrowers, i.e. the (relatively) higher expected utility that the borrower 1 enjoys when compared to her peer (as long as \( p > p_b > 0 \) and \( \psi > 0 \)).

5.2. Other contracts: \((b,n)\) and \((m,n)\) implementability

There can be other contract options for the bank. One possibility is not to rely on peer monitoring at all. In the next proposition we show that given the parameter restrictions of the model (listed at assumptions A1 and A3), this is never profitable.

Lemma 5.5. The implementation of the no-monitoring \((\delta_1, \delta_2) = (a, n)\) equilibrium in the first-stage results in negative profits for the bank.

Proof. See Appendix B.

Another possibility for the bank is to rely on peer monitoring only partially. This happens when only one of the borrowers monitor her peer, yielding an \((m,n)\) (or \((n,m)\)) equilibrium to prevail in the first-stage game. When the cost of monitoring is sufficiently small, the MFI prefers that both borrowers monitor each other. This result is formally stated in the next proposition.

Lemma 5.6. If the peer monitoring cost of a borrower \((\psi)\) satisfies\(^{11}\)

\[
\psi \leq \min \left\{ \frac{p_b B}{p_b + p}, \frac{p - p_b}{p} \right\},
\]

then the MFI prefers the optimal contract that implements the symmetric \((\delta_1, \delta_2) = (m, m)\) equilibrium over a contract that implements an asymmetric equilibrium \((\delta_1, \delta_2) = (m, n)\) (or \((n,m)\)).

Proof. See Appendix B.

These two results allow us to conclude with the following Proposition.

Proposition 5.7. When conditions at Lemmas 5.5 and 5.6 are met, then the group lending contract that we characterized in Section 5.1 with asymmetric terms of repayment is the unique optimal contract.

6. Discussion

For the remainder of our analysis we assume that the cost of peer monitoring is small enough to satisfy the conditions that we derived at Proposition 5.4 and Lemma 5.6 - yielding the optimal contract to induce the unique \((m,m)\)-equilibrium in the first-stage with asymmetric terms of repayment. In this section we discuss two key features of the optimal contract - the joint-liability and the group leadership - and also delineate on the implementability of a lending scheme with asymmetric terms of repayment as an outcome of a fair contract.

6.1. Optimality of joint-liability with a group leader

We start with the joint-liability feature. Joint-liability is a key property of many group lending mechanisms studied in the literature. In our framework, removing the joint-liability scheme from the optimal contract implies that borrowers will have no incentives to monitor each other: if \( R^{III}_{left} = R^{III}_{right} \) and \( R^{III}_1 = R^{III}_2 \), the incentive compatibility constraints associated with spending peer monitoring effort in the first-stage ((6b) and (7a)) are violated. In this case, the no-monitoring equilibrium will become the unique equilibrium of the dynamic game - under which case expected bank profits are always negative - and group lending breaks down.

The novel property proposed by our optimal contract design is the asymmetric treatment of borrowers. We interpret this feature as the endogenous emergence of a group leader concept. Intuitively, the optimal contract that we solved sets one agent (borrower 1) to do the right thing under any condition, which formally implies for this particular borrower to monitor her peer regardless of her beliefs. This is achieved by providing the leader borrower with extra information rents. Differently, the second agent (borrower 2) is rewarded just enough to monitor only if she believes that her peer will do the same.

In order to understand the importance of the asymmetric contract-

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\(^{11}\) This condition may or may not overlap with the condition characterized at (14).
ing, we investigate the following. If the MFI were to remove the uniqueness requirement (1f) from the optimal contracting problem, agents would be treated symmetrically (with $R^{III}_{1} = R^{III}_{2}$ and $R^{III}_{1} = R^{III}_{2}$). In this case bank’s maximization problem would be the same as in (1a)–(7a), except for one difference: constraint (7a) would be replaced with

$$R^{III}_{1} - R^{III}_{2} \geq \frac{\theta p - \eta}{p(p - \eta)}.$$  \hspace{1cm} (16)

In this case an optimal contract would exist if $\theta \leq B$, $R^{III}_{1} = R^{III}_{2} \geq 0$, and the expected bank profits are non-negative. We argue that the latter will not be always guaranteed under a symmetric treatment contract. In order to observe this, we first derive the terms of the symmetric treatment contract:

$$R^{III}_{1} = R^{III}_{2} = \theta,$$

$$R^{III}_{1} = R^{III}_{2} = \theta - \frac{\psi}{p(p - \eta)}.$$  \hspace{1cm} (17)

The super-game matrix for this contract specification is presented in Table 5. We can see that the no-monitoring action for borrower 1 is a best response to no-monitoring of borrower 2 if

$$p\eta \left[ \theta - \eta \theta + \frac{p_{n} - \psi}{p (p - \eta)} - (1 - \eta) \theta \right] + B \geq p\eta \left[ \theta - \eta \theta + \frac{\psi}{p (p - \eta)} - (1 - \eta) \theta \right] + B - \psi.$$  \hspace{1cm} (18)

This condition holds (strictly) because $(p - \eta)^{2} > 0$.

Because the borrowers’ objectives are symmetric to each other, no-monitoring for borrower 2 is a best response to no-monitoring of borrower 1, and (no monitor, no monitor) turns out to be a Nash equilibrium of the super-game presented in Table 5. Therefore, the super-game in Table 5 exhibits two Nash equilibria: a good one in which both borrowers monitor each other and behave diligently, and a bad one in which no borrower spends resources into monitoring her peer, both borrowers shirk, and the bank incurs negative expected profits. We would like to note that there is no mixed-strategy Nash-equilibrium in the first-stage of this symmetric treatment game. We formally prove this result in the Online-Appendix. The intuition can also be understood from Table 5. Consider borrower 1, i.e. the row player. On the one hand, if she expects borrower 2 to play $m$, she is indifferent between $m$ and $n$. On the other hand, if she expects borrower 2 to play $n$, she strictly prefers $n$. Then, if she expects borrower 1 to randomize between $m$ and $n$, she will always play $n$ with probability 1. But then there can not be any mixed strategy equilibrium. One can see that such a result is related to $(m,m)$ being the only equilibrium that survives the equilibrium refinement that we extensively discuss below.

A natural question to ask is the following: “Which equilibrium does the bank expect to emerge?” If the bank were to expect borrowers to coordinate on the good equilibrium, then multiplicity might not be a concern. However, if the bank expects the bad equilibrium to emerge then multiplicity is clearly an issue. We address this question by refining the equilibria in Table 5 using the notion of “Strong Nash Equilibrium”.

**Definition.** $(\delta^{*}_{1}, \delta^{*}_{2}) \in [m,n]^{2}$ is a strong Nash Equilibrium of the super-game in Table 5 if and only if it is a Nash Equilibrium, and for $\delta_{1} \neq \delta^{*}_{1}$ and $\delta_{2} \neq \delta^{*}_{2}$, either $U_{1}(\delta_{1}, \delta^{*}_{2}) \geq U_{1}(\delta^{*}_{1}, \delta^{*}_{2})$, or $U_{2}(\delta^{*}_{1}, \delta_{2}) \geq U_{2}(\delta^{*}_{1}, \delta^{*}_{2}).$ The notion of Strong Nash Equilibrium means that the two borrowers cannot jointly deviate in a way that benefits both. However, by symmetry in our environment a strategy profile is a Strong Nash equilibrium if a joint deviation would make both borrowers worse off. Based on this notion we prove the following important result.

Table 5

<table>
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<tr>
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<tbody>
<tr>
<td>$p\eta \left[ \theta - \eta \theta + \frac{p_{n} - \psi}{p (p - \eta)} - (1 - \eta) \theta \right] + B$</td>
<td>$p\eta \left[ \theta - \eta \theta + \frac{\psi}{p (p - \eta)} - (1 - \eta) \theta \right] + B - \psi$</td>
</tr>
</tbody>
</table>

**Proposition 6.1.** $(m,m)$ is the Strong Nash Equilibrium of the game presented in Table 5 induced by the group contract which has no group leader.

**Proof.** See Appendix C.

This result shows that multiplicity may be a severe problem for the MFI, since the bad equilibrium with no peer monitoring survives the refinements above, while the good equilibrium does not: the no-monitoring equilibrium in Table 5 is the preferred equilibrium by the two borrowers. Thus, treating borrowers symmetrically could lead to unprofitable lending and eventually to the break-down of group lending. Asymmetric treatment of borrowers through terms of repayment, which is the result of the empowerment of one of the borrowers with strong monitoring incentives, rules out such an undesirable equilibrium multiplicity situation. Therefore, we suggest asymmetric group loan contracting as a vital tool to enhance stable microfinance lending.

We interpret the empowerment with strong monitoring incentives as the assignment of one borrower as a group leader. The rationale for our interpretation is as follows. Group leadership is a practice commonly applied by many microfinance institutions around the globe. The function of group leaders varies across microfinance institutions. However, the most basic role of a group leader is maintaining discipline in a group lending environment and making sure that repayments are maximized. Repayment probabilities are maximized in our framework when one of the borrowers in the group is set to do the right thing (monitoring) under any circumstance (including off-the-equilibrium scenarios), which reinforces also her peer’s equilibrium monitoring. Therefore, we think of a borrower with such strong monitoring incentives as a leader. The important feature of our optimal contract design is that the vitality of a leader borrower is apparent even when borrowers are homogenous in all ex-ante characteristics.

The final question that we would like to address in this section is the following: “Why would the bank not impose the analog of the constraint (7a) for borrower 2 (and make monitoring a dominant strategy for borrower 2) - as doing this clearly gets rid off the equilibrium multiplicity property, too?” The answer is that imposing this additional constraint would set the repayments of two borrowers equal such that $R^{III}_{1} = R^{III}_{2} \equiv R^{I}$ and $R^{III}_{1} = R^{III}_{2} \equiv R^{I}$ with:

$$R^{I} = \theta - \frac{\psi}{p\eta (p - \eta)},$$  \hspace{1cm} (17)

$$R^{I} = \theta.$$  \hspace{1cm} (18)

Comparing Eqs. (17) and (18) against (8)–(10) shows that when monitoring is a dominant strategy for both borrowers, the expected aggregate repayment that the bank would obtain from the group is lower compared to the case where monitoring is a dominant strategy for only one borrower. The intuition is that incentivizing monitoring for an additional borrower increases the amount of rents that need to be redistributed to the borrowers - without any further benefits to the end of the bank. Therefore, repayment heterogeneity and involvement of a leader-borrower turns out to be an outcome of an optimal group lending contract.

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[12] For a detailed discussion of Strong Nash Equilibria see Bernheim et al. (1987).
6.2. Implementing group leadership with fair contracts

The group loan contract that we derived in Section 5.1.5 produces the outcome that borrower 1’s expected payoff is higher than that of borrower 2, although the two borrowers exert the same monitoring effort and undertake the same project choice in equilibrium. The asymmetric contracts with an appointment of the leader can be implemented in a way that ex-ante expected borrower payoffs are equalized. Specifically, the MFI can introduce a lottery (i.e. a randomization device) to specify which person will serve as the group leader after the group loan contract is signed.

Formally, let us consider a lottery that returns \( \lambda = \lambda_1 \) and \( \lambda = \lambda_2 \) with equal probabilities. A group loan contract with randomization is a collection of repayment functions \( \{R_{i\lambda}^{ss}\}_{i=1,2} \) that are conditioned on the outcome of the lottery. Fig. 2 presents the modified timing which incorporates the lottery that takes place after the group loan contract is signed. In Appendix C we prove the following result.

**Proposition 6.2.** Let \( R_{ij}^{ss} \) be given by (8)–(10), and let \( (\delta, \epsilon_0) \) be the strategies induced by such a contract according to (1c)–(1f).

Define

\[
\pi_{i\lambda} = \begin{cases} 
R_{i\lambda}^{ss} & \text{if } \lambda = \lambda_1 \\
R_{i\lambda}^{ss} & \text{if } \lambda = \lambda_2 
\end{cases}
\]

Then the following is true:

(i) \( R_{i\lambda}^{ss} \) induces \( \delta_i = \delta_i \) and \( \epsilon_0^{i\lambda} = \epsilon_0^{i\lambda} \) for all \( \lambda \).

(ii) \( \pi_{i\lambda} \) solves the modified contracting problem shown in Fig. 2.

(iii) \( \sum\frac{1}{2}\{p(1-p)R_{2}\} - (1-p)\sum\frac{1}{2}\{p(1-p)R_{2}\} - \psi \}

\[
= \sum\frac{1}{2}\{p(1-p)R_{2}\} - (1-p)\sum\frac{1}{2}\{p(1-p)R_{2}\} - \psi 
\]

**Proof.** See Online Appendix. Proposition 6.2 shows that the MFI can implement a fair contract by entitling the leadership of the group by the outcome of a lottery whose realization is unknown at the time the contract is signed. Borrowers receive a lottery over two group-loan contracts, where the repayment terms of each group-loan contract are the same as in Section 5.1.5. However, the person entitled to the low repayment (the group leader) is determined by the realization of the lottery. If \( \lambda = \lambda_1 \), the contract \( \{R_{i\lambda}^{ss}\}_{i=1,2} \) is implemented and borrower 1 is entitled to the low repayment. If \( \lambda = \lambda_2 \) the contract \( \{R_{i\lambda}^{ss}\}_{i=1,2} \) is implemented and borrower 2 is entitled to the low repayment. When the lottery is fair, meaning that \( \text{Prob}(\lambda = \lambda_1) = \text{Prob}(\lambda = \lambda_2) = \frac{1}{2} \), the theoretical analysis remains as in Section 5.1.5 except that the ex-ante borrower payoffs get equalized to each other.

7. Alternative specifications

In this section we relax two key model assumptions. Specifically, in Section 7.1 we relax the assumption that the monitoring decision is a binary choice, whereas in Section 7.2 we relax the assumption of a monopolistic Microfinance Institution. We characterize equilibrium outcomes in these alternative environments and show that the equilibrium multiplicity property of our dynamic game survives these modifications.

7.1. Continuous monitoring effort

Different from our benchmark model, suppose that borrowers can monitor their peer with an intensity of \( a_i \in [0, 1] \). Borrower 1’s monitoring intensity determines the private benefit that borrower –i enjoys from taking action-B. Specifically, when borrower 1 monitors with intensity \( a_i \), borrower –i’s private benefit from action-B becomes \((1 - a_i)\cdot B\). In this alternative environment, a strategy for borrower i is a pair \((a_i, \epsilon_0)\) where \( a_i : \{\Omega\} \rightarrow [0, 1] \) is the monitoring intensity, and \( \epsilon_0 : [0, 1] \rightarrow (0, B) \) is a function that maps histories \((a_i, \epsilon_0)\) into borrower i’s decision over investment projects, namely the action-0 and the action-B.

We assume that the bank wants to induce both borrowers to undertake the action-0 in the second-stage, ignoring the issue of multiplicity. In Appendix D we solve for the optimal contract that induces the profit maximizing monitoring efforts \((a_i^*, a_0^*)\) so that borrowers will choose \( \epsilon(a_0^*, a_0^*) = 0 \). Then, we show that such a contract cannot rule out a second Nash equilibrium, where borrowers don’t monitor each other and then undertake the action-B, which allows us to state the following proposition.13

**Proposition 7.1.** When monitoring effort is a continuous choice variable and the bank ignores the multiplicity problem, the game induced by the optimal contract exhibits two equilibria. In the first equilibrium borrowers choose \((a_1, a_2)\), where \( a_1^* = a_2^* = \frac{p_{BB}}{p_{BB} + p_{FB}} > 0 \), and then take the action-0; in the second equilibrium borrowers choose \((a_1, a_2) = (0, 0)\), and then both borrowers take the action-B. Furthermore, \((a_1, a_2) = (0, 0)\) followed by \((\epsilon_0^*, \epsilon_0^*) = (B, B)\) is the Nash equilibrium that maximizes both borrowers’ payoffs.

**Proof.** See Appendix D.

When monitoring is a continuous variable, the main features of the model with a binary monitoring choice remain. Specifically, the game that is induced by the revenue maximizing contract has two equilibria: an equilibrium where borrowers monitor their peer, and another equilibrium where borrowers don’t monitor and do not behave diligently. Furthermore, the equilibrium where borrowers do not monitor each other is a stronger equilibrium (preferred by the borrowers), as it yields a higher level of utility to both borrowers. Therefore, as in the case with a binary monitoring choice, the contract needs to adjust to break down the equilibrium multiplicity property of the lending environment.

The model that we present in this section is closely related to that of Ghatak and Guinnane (1999), where as in their framework we model an ex-ante moral hazard environment and allow for a continuous action set for the peer monitoring activity. We provide a full-fledged characterization of equilibrium outcomes based on this framework and point out an equilibrium multiplicity problem resulting from the symmetric group loan contract and propose the need for symmetry-breaking.
7.2. Perfect competition in the MFI Sector

In this section we relax the assumption of a monopolist MFI and assume that there is free entry in the MFI loan market. As a result of perfect competition, optimal contracts maximize the (equally) weighted payoff of group members, subject to a participation constraint for the MFI. Using the same notation as in Section 4, a strategy for borrower \( i \) consists of a monitoring decision \( \delta_i \in [m, n] \) and a project choice \( e_h^i \in A_h^i \), where the latter is defined for all possible histories \( h_k \) (\( k = 1, 2, 3, 4 \)). Then, we say that a contract satisfies the MFI participation constraint for the strategy profile \( (\delta_i, e_h^i)_{1\leq i \leq n} \) if
\[
\begin{align*}
[p(e_h^{i_1, j_1}(\theta_p))p(e_h^{i_2, j_2}|e_h^{i_1, j_1})R_{1_H}^H + & (1 - p(e_h^{i_2, j_2}))(R_{2_H}^H - 1) \\
+ & [p(e_h^{i_3, j_3}(\theta_p))p(e_h^{i_4, j_4}|e_h^{i_3, j_3})R_{2_H}^H + (1 - p(e_h^{i_4, j_4}))(R_{2_H}^H - 1)] \geq 0.
\end{align*}
\]

where the MFI’s opportunity cost of lending is normalized to 1, i.e. the loan’s size. In general, there could be different strategy profiles that satisfy the MFI participation constraint (19). In the next lemma we introduce sufficient conditions which guarantee that the participation constraint (19) can be satisfied only if both borrowers monitor their peer and then take the action-0.

**Lemma 7.2.** If \( \frac{p}{\theta} - B > 2(\theta - 1) \), and \( \theta - 1 < 1 - pB \), the MFI participation constraint (19) can be satisfied only by contracts that are feasible and incentive compatible for the strategy profiles \( \delta_1 = \delta_2 = m \) and \( e_h^1 = e_h^2 = 0 \).

**Proof.** See Appendix D.

The two conditions in Lemma 7.2 are analogous to the conditions in assumptions A1 and A3, with the difference that the conditions in Lemma 7.2 are defined for group-loans, whereas A1 and A3 were defined for individual loans. Specifically, if \( \frac{p}{\theta} - B > 2(\theta - 1) \), then it is too costly for the MFI to provide an unmonitored borrower with enough information rents to take the action-0. If \( \theta - 1 < 1 - pB \), then it is too costly for the MFI to let an unmonitored borrower undertake the action-B.

For the rest of this section assume that the conditions in Lemma 7.2 hold. Then, the only feasible contracts are the ones that induce both borrowers to monitor their peer and we can build our analysis on most of the results from Section 5. First, the results in Lemma 5.1 apply and hence optimal contracts should satisfy Eqs. (4) and (5). Second, Lemma 5.3 applies and (B,B) is the equilibrium strategy profile at history \( h_k \). Incentive compatibility over monitoring decisions requires that Eqs. (6a) and (6b) hold. Equilibrium uniqueness of the first-stage monitoring decisions requires that either (7a) or (7b) hold. Therefore, a perfectly competitive MFI will offer a group-loan contract that solves the following problem:

\[
\begin{align*}
(P^C) & \max_{R_{1_H}^H, R_{2_H}^H} \left\{ \frac{1}{2}[p\theta - pR_{1_H}^H - (1 - p)R_{2_H}^H] - \psi \right\} \\
& + \left( \frac{1}{2}[p\theta - pR_{2_H}^H - (1 - p)R_{2_H}^H] - \psi \right) \tag{20}
\end{align*}
\]  

s. t. \( [pR_{1_H}^H + (1 - p)R_{2_H}^H] - 1 + [pR_{2_H}^H + (1 - p)R_{2_H}^H] - 1] \geq 0 \)  
(21)

\( pR_{1_H}^H + (1 - p)R_{2_H}^H \geq \theta - \frac{B}{p - p_B} \)  
(22)

\( pR_{2_H}^H + (1 - p)R_{2_H}^H \geq \theta - \frac{B}{p - p_B} \)  
(23)

\( R_{1_H}^H - R_{2_H}^H \geq \frac{\psi}{p_B(p - p_B)} \)  
(24)

\( R_{2_H}^H - R_{2_H}^H \geq \frac{\psi}{p_B(p - p_B)} \)  
(25)

either \( R_{1_H}^H - R_{2_H}^H \geq \frac{\psi}{p_B(p - p_B)} \) or \( R_{2_H}^H - R_{2_H}^H \geq \frac{\psi}{p_B(p - p_B)} \) (26)

0 \leq R_{1_H}^H, R_{2_H}^H; \quad R_{1_H}^H, R_{2_H}^H \leq \theta \)  
(27)

In problem \((P^C)\) we refer to constraint (26) as either \( \psi \) or \( \frac{\psi}{p_B(p - p_B)} \) is non-negative. This is without loss of generality, since in Section 6.2 we discussed how to introduce contracts with asymmetric repayment terms using fair contracts. By (26) we can ignore constraint (24).

We note that the linearity of preferences (20) and of the participation constraint (21) suggest that the problem may have many different solutions. In the rest of the analysis we ask when a solution to problem \((P^C)\) exists and more importantly if there exists at least a contract with symmetric repayments across borrowers that solves such a problem. If such a contract does not exist, it means that optimal contracts have endogenously asymmetric repayment terms, as in the case of a monopolistic MFI.

**Proposition 7.3.** Suppose that all conditions in Eq. (14) of Proposition 5.4 hold. Then, there exists an optimal contract solving \((P^C)\). Further, assume that \( \frac{p}{\theta} > \theta - 1 \); then

i. if \( \psi \leq \frac{\psi}{p_B(p - p_B)} \), there exists a symmetric contract that solves problem \((P^C)\);  
ii. if \( \psi > \frac{\psi}{p_B(p - p_B)} \), a solution to problem \((P^C)\) requires asymmetric repayment terms, satisfying \( R_{1_H}^H < R_{2_H}^H \), and \( pR_{1_H}^H + (1 - p)R_{2_H}^H < pR_{2_H}^H + (1 - p)R_{2_H}^H \).

**Proof.** See Appendix D.

**Proposition 7.3.** Proposition 7.3 proves that an optimal contract exists if \( \psi \) is not too large, where the upper bound on \( \psi \) for existence of an optimal contract is defined in Eq. (14). This is not surprising: existence of an optimal contract hinges on the same conditions both with perfect competition and with a monopolist MFI.

However there is an important difference with Section 5: perfect competition makes the delegation of monitoring activity simpler. In Section 5, asymmetric contracts were optimal for two reasons: first, they guaranteed uniqueness of equilibrium strategies in the peer-monitoring game; second, they minimized the information rents that the MFI had to pay to the borrowers. With perfect competition, the second argument ceases to be relevant; the MFI maximizes borrowers’ expected payoff and is not concerned with minimizing information rents. This is the reason why there always exists a symmetric contract that solves problem \((P^C)\) when the cost of monitoring is small. In these environments the MFI can offer the same contract to both borrowers, where such a contract guarantees that monitoring is a dominant strategy for both borrowers. Differently from Section 5, there is no additional cost for the MFI associated with the off-equilibrium incentive compatibility constraint (26).

However, when \( \psi \) is neither too small nor too large, no symmetric contract can guarantee uniqueness of first-stage monitoring strategies. In such environments optimal contracts require asymmetric repayment terms: borrower 1, i.e. the designated leader, is entitled to lower expected repayments. This is similar to the case of a monopolist MFI, even though the result relies on a different mechanism. To gain intuition for this result, notice that when \( \psi > \frac{\psi}{p_B(p - p_B)} \) there is no symmetric contract that guarantees incentive compatibility (26) when the MFI’s participation constraint (21) is satisfied. The difference between \( R_{1_H}^H - R_{2_H}^H \) that guarantees incentive compatibility (26) is incompatible with the MFI participation constraint (21) within a symmetric contract. Then, if \( R_{1_H}^H \) and \( R_{2_H}^H \) need to satisfy constraint (26), i.e. monitoring needs to be a dominant strategy for borrower 1, the repayment terms \( R_{1_H}^H \) and \( R_{2_H}^H \) have to adjust to satisfy the MFI participation constraint. Importantly, \( R_{1_H}^H \) and \( R_{2_H}^H \) cannot satisfy the
incentive compatibility constraint (26), but they can still satisfy the incentive compatibility constraint (25). Then, as in the case of a monopolist MFI, optimal contracts induce monitoring as borrower 1’s dominant strategy, whereas monitoring is borrower 2’s best response only when she expects borrower 1 to monitor as well.

8. Conclusion

Group lending is an extensively used lending practice in environments with weak institutions and lack of collateral. Group-leadership is commonly applied by microfinance institutions as part of joint-liability agreements. For instance, Gramin Vikash Bank, a leading microfinance institution in India, states on its web-site the following: “The (group) leader fosters a sense of unity, oversees and maintains discipline, shares information and facilitates repayments. For the bank, she is the focal point for group activities.” The existing theoretical literature has focused on joint-liability lending contracts without dealing explicitly with the specific role of the group leader in fostering the performance of group lending mechanisms. In this paper we fill this gap by introducing the concept of endogenously asymmetric group loan contracts.

We characterize the optimal group loan contract in an environment with moral hazard and costly peer monitoring. The optimal contract has two inseparable properties. It exhibits a joint liability scheme, and it requires the figure of a group leader. Notably, both properties are necessary for the efficient implementation of group lending contracts. The desirability of a group leader survives even within a group of borrowers with homogeneous characteristics. Indeed, the introduction of a group-leader is the efficient endogenous mechanism that rules out the possibility of coordination failures in the implementation of the efficient contract. We obtain this result within the most simple setup that we could consider, but then provide extensions to show that the multiplicity of equilibria is not the result of a special-case framework. In our benchmark model we assume that the bank implements actions in dominant strategies and we assume that there is no uncertainty in fundamentals. To this end, in future research, it would be interesting to generalize the coordination game. A possibility would be to model a global game environment, where borrowers receive noisy signals on the enforcement choice of other borrowers. We conjecture that the noise structure will select the equilibrium of play, and the role of a group leader will depend on how noisy the signal is. It would be also interesting to generalize our model to the case of an arbitrary N-number of borrowers as in Ahlin (2015): Given such a specification, would there be just one group-leader? And how would the optimal incentive structure look like? Would there be a hierarchy among the borrowers? These are all interesting extensions that go beyond the purpose of the current paper and we leave them to future research.

Appendix A. Optimal group loan contract with unique (m,m)-equilibrium

Proof of Lemma 5.1. Suppose by contradiction that there is a contract that implements \((\delta_1, \delta_2) = (m, m)\) as a Nash equilibrium in the first-stage game and at history \(h_2\) (which is off the equilibrium) such a contract implements actions \((e_1^{h_2}, e_2^{h_2}) = (0, 0)\). Then, borrower-2’s payoff at history \(h_2\) would be

\[
p(\theta - pR_2^{III} - (1 - p)R_2^{LH}).
\]

But then constraint (1e) is violated: the strategy \((\delta_1, \delta_2) = (m, m)\) is a Nash equilibrium in the first-stage game if borrower 2 monitors her peer when she expects borrower 1 to monitor her as well. Using the payoffs at histories \(h_1\) and \(h_2\) this requires for constraint (1e)

\[
p(\theta - pR_2^{III} - (1 - p)R_2^{LH} - \psi) \geq p(\theta - pR_2^{III} - (1 - p)R_2^{LH}),
\]

which is never possible for any \(\psi > 0\).

Proof of Lemma 5.3. Consider a contract that implements \((\delta_1, \delta_2) = (m, m)\) as a Nash equilibrium in the first-stage.

(i) First we show that \((e_1^{h_2}, e_2^{h_2}) = (B, B)\) is a Nash equilibrium at history \(h_2\) for any contract that induces \((\delta_1, \delta_2) = (m, m)\). This means for any borrower, action-B is a best response to her peer’s choice of action-B. Consider a contract that induces \((\delta_1, \delta_2) = (m, m)\), then we know from Lemma 5.1 that \((e_1^{h_2}, e_2^{h_2}) = (B, 0)\) is the Nash equilibrium at history \(h_2\). Then, it is easy to show that constraint (1e) yields \(R_1^{III} > R_1^{III}\). Moreover, constraint (4) holds. Putting the two together we get:

\[
p_\theta(\theta - pR_2^{III} - (1 - p)R_2^{III}) + B \geq (\theta - pR_2^{III} - (1 - p)R_2^{III}) \geq (\theta - pR_2^{III} - (1 - p)R_2^{III}),
\]

which proves that action-B is a best response given peer’s action-B.

(ii) Next, we show that there is no other strategy profile \((e_1^{h_2}, e_2^{h_2})\) which can be a Nash-equilibrium at history \(h_4\). From (A-1), the action-B is the unique best response to action-B. Then the only other possible Nash equilibrium is \((e_1^{h_2}, e_2^{h_2}) = (0, 0)\). The action-0 is in borrower 1’s best response when she expects borrower 2 to choose the action-0, which implies

\[
pR_1^{III} + (1 - p)R_1^{III} \leq \theta - \frac{B}{p - p_\theta}.
\]

Similarly, the action-0 must be in borrower 2’s best response when she expects borrower 1 to play the action-0:

\[
pR_2^{III} + (1 - p)R_2^{III} \leq \theta - \frac{B}{p - p_\theta}.
\]

Because of Assumption A3, the MFI’s profits are negative.

\[14\] http://www.bgvb.co.in/JoinLIabilityGroup.aspx.
\[ \{ p[R_{1}^{HIH} + (1 - p)R_{1}^{HL}] - 1 \} + \{ p[R_{2}^{HIH} + (1 - p)R_{2}^{HL}] - 1 \} \leq \left\{ p[\theta - \frac{B}{p - p_{h}}] - 1 \right\} + \left\{ p[\theta - \frac{B}{p - p_{h}}] - 1 \right\} < 0. \]

**Appendix B. Other contracts**

**Proof of Lemma 5.5.** [The symmetric \((n, n)\) equilibrium implementation] When neither of the borrowers monitor the peer, the first-stage game ends up at history \(h_{4}\). Consider a contract that implements \((e_{1}^{b_{1}}, e_{2}^{b_{1}}) = (0, 0)\) as a Nash equilibrium at \(h_{4}\). Then constraint \((1c)\) implies that action-0 must be in borrower 1’s best response when she expects borrower 2 to choose 0:

\[ p[\theta - pR_{1}^{HIH} - (1 - p)R_{1}^{HL}] \geq p_{h}[\theta - pR_{1}^{HIH} - (1 - p)R_{1}^{HL}] + B. \]

Similarly, because action-0 must be in borrower 2’s best response when she expects borrower 1 to choose action-0 we also have

\[ p[\theta - pR_{2}^{HIH} - (1 - p)R_{2}^{HL}] \geq p_{h}[\theta - pR_{2}^{HIH} - (1 - p)R_{2}^{HL}] + B. \]

Re-writing the two constraints we get

\[ pR_{1}^{HIH} + (1 - p)R_{1}^{HL} \leq \theta - \frac{B}{p - p_{h}}, \quad pR_{2}^{HIH} + (1 - p)R_{2}^{HL} \leq \theta - \frac{B}{p - p_{h}}. \]

As a result, the expected profits of the bank are expressed as

\[ EV_{b} = \{ p[pR_{1}^{HIH} + (1 - p)R_{1}^{HL}] - 1 \} + \{ p[pR_{2}^{HIH} + (1 - p)R_{2}^{HL}] - 1 \} \leq \left\{ p[\theta - \frac{B}{p - p_{h}}] - 1 \right\} + \left\{ p[\theta - \frac{B}{p - p_{h}}] - 1 \right\} < 0, \]

which by Assumption 3 implies negative profits.

Next we investigate the implementability of the \((e_{1}^{b_{1}}, e_{2}^{b_{1}}) = (0, 0)\) at \(h_{4}\). Since this is the mirror case to the \((B, 0)\) equilibrium, we study the consequences of \((0, B)\) equilibrium only. At such an equilibrium, constraint \((1c)\) requires

\[ p_{h}R_{1}^{HIH} + (1 - p_{h})R_{1}^{HL} \leq \theta - \frac{B}{p - p_{h}}. \]

Furthermore, the limited liability constraint for borrower 2 must hold. As a result bank profits can be expressed as

\[ EV_{b} = \{ p[p_{h}R_{1}^{HIH} + (1 - p_{h})R_{1}^{HL}] - 1 \} + \{ p[p_{h}R_{2}^{HIH} + (1 - p_{h})R_{2}^{HL}] - 1 \} \leq \left\{ p[\theta - \frac{B}{p - p_{h}}] - 1 \right\} + \left[ p_{h}\theta - 1 \right] < 0, \]

where the first term on the RHS is negative by Assumption A3 and the second term is negative by Assumption A1.

Finally by Assumption A1 it trivially follows that \((e_{1}^{b_{1}}, e_{2}^{b_{1}}) = (B, B)\) also yields negative profits for the bank. \(\Box\)

**Proof of Lemma 5.6.** [The asymmetric \((m, n)\) equilibrium implementation] In order to characterize the profitability of an equilibrium which exhibits asymmetric monitoring behavior of borrowers for MFI, without loss of generality we pick the borrower 1 to be the monitoring peer in the group loan contract. This means we investigate the implementability and profitability of the \((\delta_{1}, \delta_{2}) = (m, n)\) equilibrium in the first-stage game. There are two possible equilibria to take into account: one associated with the strategy profile \((e_{1}^{b_{1}}, e_{2}^{b_{1}}) = (0, 0)\) and the second associated with the strategy profile \((e_{1}^{b_{2}}, e_{2}^{b_{2}}) = (B, 0)\).

1. Consider first any contract that induces \((e_{1}^{b_{1}}, e_{2}^{b_{2}}) = (0, 0)\). Constraint \((1c)\) implies

\[ p[\theta - pR_{1}^{HIH} - (1 - p)R_{1}^{HL}] \geq p_{h}[\theta - pR_{1}^{HIH} - (1 - p)R_{1}^{HL}] + B, \]

which simplifies to

\[ pR_{1}^{HIH} + (1 - p)R_{1}^{HL} \leq \theta - \frac{B}{p - p_{h}}. \] \((A-2)\)

As a result because of limited liability and the condition \((15)\) on \(\psi\) in Lemma 5.6, expected profits of the bank in this case are less than the expected profits induced by the optimal contract under symmetric \((m, m)\) equilibrium, given in Eq. \((13)\):

\[ EV_{b} \leq \left\{ p[\theta - 1 - \frac{pB}{p - p_{h}}] \right\} + \left( p[\theta - 1] - \frac{pB}{p - p_{h}} \right) \leq \left\{ p[\theta - 1 - \frac{pB}{p - p_{h}}] \right\} + \left( p[\theta - 1] - \frac{pB}{p - p_{h}} \right) \]

2. Next consider a contract that induces \((e_{1}^{b_{1}}, e_{2}^{b_{2}}) = (B, 0)\). Constraint \((1c)\) implies

\[ pR_{1}^{HIH} + (1 - p)R_{1}^{HL} \geq \theta - \frac{B}{p - p_{h}}. \] \((A-3)\)

The incentive compatibility constraint \((1c)\) can be defined for four possible off-the-equilibrium strategies at history \(h_{4}\): \((e_{1}^{b_{1}}, e_{2}^{b_{1}}) = (B, B), (e_{1}^{b_{1}}, e_{2}^{b_{2}}) = (0, B), (e_{1}^{b_{2}}, e_{2}^{b_{1}}) = (B, 0)\) and \((e_{1}^{b_{2}}, e_{2}^{b_{2}}) = (0, 0)\).

(a) First, if the contract implements \((e_{1}^{b_{1}}, e_{2}^{b_{2}}) = (B, B)\), then constraint \((1c)\) requires:
(A-4)

\[ p_R^{RH} + (1 - p_B)R_{HI} \geq \theta - \frac{B}{p - p_B} \]

(A-5)

\[ p_R^{RH} + (1 - p_B)R_{HI} \geq \theta - \frac{B}{p - p_B} \]

whereas (1e) implies:

\[ R_{HI}^{HL} - R_{HI}^{HH} \geq \frac{\psi}{p_B(p - p_B)} \]

(A-6)

We can note that (A.3)–(A.5) are not relevant for the maximization problem. As a result, the constraint (1g) binds for \( R_{HI}^{HL} \) and (A-6) should bind as well. Hence:

\[ R_{HI}^{HL} = \theta, R_{HI}^{HH} = \theta - \frac{\psi}{p_B(p - p_B)} \]

Moreover, it is straightforward to show that by choosing

\[ R_{HI}^{HH} = R_{HI}^{HL} = \theta, \]

constraint (1e) is satisfied for borrower 2. For this contract to exist we need to check whether (A.3)–(A.5) are satisfied. Easily, (A-5) holds and (A-3) implies (A-4). Then, such a contract exists if (A-3) holds. Because of the condition (15) on \( \psi \) in Lemma 5.6, expected profits of the bank in this case are less than the expected profits induced by the optimal contract under symmetric \((m, m)\) equilibrium, given in Eq. (13):

\[
EV_{\text{bank}} = \left\{ p_B \theta - p \frac{\psi}{p - p_B} - 1 \right\} + \left\{ p \theta - 1 - p \frac{\psi}{p - p_B} \right\} + \left\{ p \theta - 1 - p \frac{\psi}{p - p_B} \right\}.
\]

(b) Second, any contract that would implement \((e_1^{b}, e_2^{b}) = (0, B)\) is dominated by the optimal contract that induces \((e_1^{b}, e_2^{b}) = (B, B)\). The reason is that the relevant constraints for the previous contract were (A.3) and (A.6). When the contract would implement off-the-equilibrium strategies \((e_1^{b}, e_2^{b}) = (0, B)\) the constraint

\[ p_R^{RH} + (1 - p_B)R_{HI} \leq \theta - \frac{B}{p - p_B} \]

has to be satisfied and the constraint (1e) imposes a tighter restriction than (A-6).

(c) Finally, any contract that would implement \((e_1^{b}, e_2^{b}) = (B, 0)\) or \((e_1^{b}, e_2^{b}) = (0, 0)\) violates (1e). The former is straightforward as \((e_1^{b}, e_2^{b}) = (e_1^{b}, e_2^{b})\). In the latter case, at \( h_1 \) the constraint (1e) implies that the constraint (A-3) needs to hold with equality. Replacing (A-3) with equality into (1e) we obtain

\[
\frac{p}{p - p_B} B - B \geq \frac{p}{p - p_B} B,
\]

which is a contradiction.

Appendix C. Discussion of the optimal group loan contract with asymmetric terms of repayment

Proof of Proposition 6.1. We already know \((n, n)\) is a Nash Equilibrium. We only have to check that\n
\[ U_i(n, n) \geq U_i(m, m) \]

holds for \( i \in \{1, 2\} \). Using the payoffs in Table 5 we obtain that the following condition should hold for borrower 1 and for borrower 2

\[ pB \geq p_B \psi. \]

(A-8)

As we have shown at Proposition 5.4 the condition (A-8) is satisfied whenever a contract exists.

Appendix D. Alternative specifications

Proof of Proposition 7.1. Let \( p(e_i) \in [p, p_B] \) be the probability of success of borrower \( i \)'s project when she undertakes action \( e_i \). We say that \((e_1^a(a_1, a_2), e_2^a(a_1, a_3))\) is a Nash equilibrium of the second-stage game that starts at history \((a_1, a_2)\) if

\[
e_1^a(a_1, a_2) \in \arg\max_{e_2^a(\cdot)} p(e_1^a(a_1, a_2))R_{HI}^{e_1^a e_2^a} + (1 - p(e_1^a(a_1, a_2)))R_{HI}^{e_1^a e_2^a} + (1 - a_1^a)Be_1^a(a_1, a_3) - \psi a_1^a.
\]

(A-9)

Similarly \((a_1^a, a_2^a)\) is a Nash equilibrium of the first-stage super-game if

\[
a_1^a \in \arg\max_{e_2^a(\cdot)} p(e_2^a(a, a_2^a)) \cdot \theta - p(e_2^a(a, a_2^a))R_{HI}^{e_1^a e_2^a} + (1 - p(e_2^a(a, a_2^a)))R_{HI}^{e_1^a e_2^a} + (1 - a_2^a)Be_2^a(a, a_3^a) - \psi a_2^a.
\]

(A-10)

where we used the notation that \((a_1, a_2^a) = (a, a_2)\) if \( i = 1 \) and \((a_1^a, a_2) = (a_1^a, a)\) if \( i = 2 \). In the remainder of the analysis, we restrict our attention to contracts that exhibit joint-liability, meaning \( R_{HI}^{HL} \geq R_{HI}^{HH} \) and \( R_{HI}^{HH} \geq R_{HI}^{HL} \). Later we will show that this assumption is indeed satisfied.

We begin by characterizing the second-stage equilibria. Let \( e_i^a(c_i); (a_1, a_3) \) be borrower \( i \)'s best response at history \((a_1, a_3)\) when borrower \(-i\) is choosing \( e_{-i} \) and define the following thresholds:
where \( \psi_i \) is the degree of monitoring intensity of borrower \( i \) that keeps the borrower \(-i\) indifferent between choosing the bad project (with private benefits worth of \((1 - \psi_i)B\)) and the good project - conditional on borrower \( i \) choosing the good project, whereas \( \pi_i \) is the degree of monitoring intensity of borrower \( i \) that keeps the borrower \(-i\) indifferent between choosing the bad project (with private benefits worth of \((1 - \pi_i)B\)) and the good project - conditional on borrower \( i \) choosing the bad project. In the next lemma we show that borrower \( i \)'s best response depends on her peer's monitoring intensity.

**Lemma A4.1.** If the contract exhibits joint-liability, then borrower \( i \)'s best response \( e_i^*(e_{-i}; (a_i, a_{-i})) \) at history \((a_i, a_{-i})\) is

\[
e_i^*(e_{-i}; (a_i, a_{-i})) = \begin{cases} 0 & \text{if } a_{-i} > \pi_{-i} \\ 0 & \text{if } a_{-i} \in [a_{-i}, \pi_{-i}] \text{ and } e_{-i} = 0 \\ B & \text{if } a_{-i} \in [a_{-i}, \pi_{-i}] \text{ and } e_{-i} = B \\ B & \text{if } a_{-i} < a_{-i} \end{cases}
\]

**Proof.** Let \((a_i, a_{-i})\) be a generic history at time 2. From (A-9) it follows that

\[
0 \in e_i^*(0; (a_i, a_{-i})) \text{ iff } a_{-i} \geq \psi_i, 0 \in e_i^*(B; (a_i, a_{-i})) \text{ iff } a_{-i} \geq \pi_i, B \in e_i^*(0; (a_i, a_{-i})) \text{ iff } a_{-i} \leq \psi_i \text{ or } B \in e_i^*(B; (a_i, a_{-i})) \text{ iff } a_{-i} \leq \pi_i
\]

where the joint-liability implies that \( \pi_i \geq \psi_i \). Then the conclusion easily follows.

According to **Lemma A4.1**, borrower 1 has a dominant strategy if she is monitored intensely, i.e. \( a_2 \geq \pi_2 \), or if she is monitored inadequately, i.e. \( a_2 < \psi_2 \). Similarly, borrower 2 has a dominant strategy if either \( a_1 > \pi_1 \) or \( a_1 < \psi_1 \). As a result, second-stage games have unique Nash equilibria, except for the games that start at histories \((a_1, a_2)\) for \( a_1 \in [\psi_1, \pi_1] \) and \( a_2 \in [\psi_2, \pi_2] \). The continuation games that follow histories \((a_1, a_2)\) and \((a_2, a_1)\) have two Nash equilibria: \((0, 0)\) and \((B, B)\). In the remainder of the analysis we assume that in such second-stage games borrowers coordinate on the Strong Nash Equilibrium (that maximizes the sum of their payoffs) to be consistent with the rest of the paper. In the next lemma we show that this is the \((0, 0)\)-equilibrium.

**Lemma A4.2.** If the contract exhibits joint-liability and \( a_1 \in [\psi_1, \pi_1] \) and \( a_2 \in [\psi_2, \pi_2] \), the \((0, 0)\) - equilibrium maximizes the sum of borrowers' payoffs at the second-stage game which starts at \((a_1, a_2)\).

**Proof.** Since the problem is symmetric across borrowers, it is enough to show that borrower 1 prefers the \((0, 0)\) equilibrium over the \((B, B)\) equilibrium when \( \pi_2 \geq a_2 \geq \psi_2 \). For all \( a_1 \geq \psi_1 \), we have

\[
p_\theta[\theta - p_\theta R_{1i}^{Hj} - (1 - p_\theta)R_{1i}^{Lj}] + B(1 - a_2) \leq p_\theta[\theta - p_\theta R_{1i}^{Hj} - (1 - p_\theta)R_{1i}^{Lj}] + B(1 - a_2) \leq p_\theta[\theta - p_\theta R_{1i}^{Hj} - (1 - p)R_{1i}^{Lj}] + B(1 - a_2)
\]

where the first inequality comes from \( a_2 \geq \psi_2 \), the second one comes from joint-liability and the (last) equality comes from the fact that at any \((a_1, a_2)\) borrower 1 is indifferent between taking the action-B and the action-0 which concludes the proof.

Using the results from **Lemmas A4.1 and A4.2** we can partition the space of histories \((a_1, a_2)\) according to the prevailing second-stage Nash equilibria - as we illustrate in Fig. 3. If the MFI desires to induce both borrowers to undertake the action-0 in the second-stage, from Fig. 3 we see that the contract has to implement the monitoring efforts \( a_1^\ast = \psi_1 \) and \( a_2^\ast = \psi_2 \). In the second-stage both borrowers will undertake the action-0 only if \( a_1^\ast \geq \psi_1 \) and \( a_2^\ast \geq \psi_2 \). Moreover, all histories \((a_1, a_2)\) that yield the same equilibrium strategies in the second-stage game. Then condition (A-10) implies that borrowers will never choose \( a_1 > \psi_1 \), since any effort in excess of \( \psi_1 \) would not produce any benefit in return to the additional disutility \( \psi(a_1 - \psi_1) \) incurred. Replacing \( a_1^\ast = \psi_1 \) in expression (A-11), we can rewrite the problem of the profit-maximizing MFI:

\[
\max_{(a_1^\ast, a_2^\ast)\in [\psi_1, \pi_1] \times [\psi_2, \pi_2]} p[R_1^{Hj} + (1 - p)R_1^{Lj}] + p[R_2^{Hj} + (1 - p)R_2^{Lj}]
\]

s. t. \[
pR_1^{Hj} + (1 - p)R_1^{Lj} = \theta - \frac{(1 - a_1^\ast)B}{p - p_\theta}
\]

\[
pR_2^{Hj} + (1 - p)R_2^{Lj} = \theta - \frac{(1 - a_2^\ast)B}{p - p_\theta}
\]
\[
p[\theta - pR_{1}^{HH} - (1 - p)R_{1}^{HL}] - \psi a_{1}^{*} \geq p_{y}[\theta - p_{y}R_{1}^{HH} - (1 - p_{y})R_{1}^{HL}] + (1 - a_{2}^{*})B \tag{A-16}
\]
\[
p[\theta - pR_{2}^{HH} - (1 - p)R_{2}^{HL}] - \psi a_{2}^{*} \geq p_{y}[\theta - p_{y}R_{2}^{HH} - (1 - p_{y})R_{2}^{HL}] + (1 - a_{1}^{*})B \tag{A-17}
\]
\[
0 \leq R_{i}^{HL} \leq \theta a_{i}^{*}, \quad a_{i}^{*} \in [0, 1] \tag{A-18}
\]

In this maximization problem, constraints (A.14) and (A.15) guarantee that \( a_{i}^{*} = \theta a_{i} \), so that both borrowers will undertake the action-0 in the second-stage game. Both expressions come directly from the condition (A-11). Constraints (A.16) and (A.17) guarantee that the monitoring intensity (\( a_{1}^{*}, a_{2}^{*} \)) is a Nash equilibrium of the first-stage game. Both constraints come from the condition (A-10) when we replace the second-stage equilibrium strategies. The right-hand-side of both constraints (A.16) and (A.17) are each borrower’s most profitable unilateral deviation payoff. It is easy to observe that borrower i’s most profitable deviation is to choose \( a_{i}^{*} = 0 \), where in the second-stage game following such a deviation both borrowers undertake the action-B.

We solve constraints (A.14) and (A.15) for \( R_{1}^{HH} \) and \( R_{2}^{HH} \) and then replace those values in the objective function and in the incentive compatibility constraints (A.16) and (A.17) in order to derive the result summarized in the following Lemma.

**Lemma A4.3.** The optimal contract satisfies \( R_{1}^{HL} = \min \left\{ 0, \theta - \frac{(1 - a_{1}^{*})B}{\frac{p - R_{1}^{HL}}{p}} \right\} \) and \( R_{2}^{HL} = \max \left\{ 0, \theta - \frac{(1 - a_{2}^{*})B}{\frac{p - R_{2}^{HL}}{p}} \right\} \). Similarly, \( R_{1}^{IH} = \min \left\{ 0, \theta - \frac{(1 - a_{1}^{*})B}{\frac{p - R_{1}^{IH}}{p}} \right\} \) and \( R_{2}^{IH} = \max \left\{ 0, \theta - \frac{(1 - a_{2}^{*})B}{\frac{p - R_{2}^{IH}}{p}} \right\} \).

**Proof.** If we solve the first two constraint for \( R_{1}^{HH} \) and \( R_{2}^{HH} \) we obtain:

\[
R_{1}^{HH} = \theta - \frac{(1 - a_{1}^{*})B}{p - p_{y}} \quad \text{and} \quad R_{2}^{HH} = \theta - \frac{(1 - a_{2}^{*})B}{p - p_{y}}.
\]

Replacing such values into the objective function and constraints (A.16)–(A.18) we get:

\[
\begin{align*}
\max_{\left\{ a_{1}^{*}, a_{2}^{*} \right\}} & \left\{ \theta - \frac{(1 - a_{1}^{*})B}{p - p_{y}} + \theta - \frac{(1 - a_{2}^{*})B}{p - p_{y}} \right\} \tag{A-19} \\
\text{s.t.} & \quad p_{y}p(\theta - p_{y}) \geq \psi a_{1}^{*} - \frac{p_{y}}{p}(1 - a_{2}^{*})B \\
& \quad R_{1}^{HH} - \theta \geq \psi a_{2}^{*} - \frac{p_{y}}{p}(1 - a_{1}^{*})B \tag{A-20} \\
R_{1}^{HL} & \leq \min \left\{ \theta, \frac{(1 - a_{1}^{*})B}{\frac{p - R_{1}^{HL}}{p}} \right\} \quad \text{and} \quad R_{2}^{HL} \leq \min \left\{ \theta, \frac{(1 - a_{2}^{*})B}{\frac{p - R_{2}^{HL}}{p}} \right\} \tag{A-21}
\end{align*}
\]

Without loss of generality we assume that \( R_{1}^{HL} = \min \left\{ \theta, \frac{(1 - a_{1}^{*})B}{\frac{p - R_{1}^{HL}}{p}} \right\} \) and \( R_{2}^{HL} = \min \left\{ \theta, \frac{(1 - a_{2}^{*})B}{\frac{p - R_{2}^{HL}}{p}} \right\} \) because doing this relaxes constraints (A.19) and (A.20) and the objective function increases in \( a_{1}^{*} \) and \( a_{2}^{*} \). Replacing these expressions in \( R_{1}^{HH} \) and \( R_{2}^{HH} \) we obtain the results of the lemma.

Next we determine the monitoring intensities \( a_{1}^{*} \) and \( a_{2}^{*} \) that the optimal contract induces in the first-stage. For the rest of the analysis we consider only the case with \( R_{1}^{HL} = R_{2}^{HL} = \theta \), and therefore \( R_{1}^{IH} = R_{2}^{IH} > 0 \) holds. It is relatively straightforward to show that \( a_{1}^{*} \) and \( a_{2}^{*} \) are pinned down by the binding incentive compatibility constraints (A.16) and (A.17), once we replace \( R_{1}^{HL} = R_{2}^{HL} = \theta \) and \( R_{1}^{IH}, R_{2}^{IH} \) from Lemma A4.3. This is proved in the next lemma.

**Lemma A4.4.** The constraints (A.16) and (A.17) hold with equality, and then borrowers’ monitoring intensities are identical and they are expressed as:

\[
a_{1}^{*} = a_{2}^{*} = \frac{p_{y}B}{p_{y}B + p_{y}}. \tag{A-21}
\]

**Proof.** Rewrite the maximization problem with \( R_{1}^{HL} = R_{2}^{HL} = \theta \):

\[
\max_{\left\{ a_{1}^{*}, a_{2}^{*} \right\} \in [0,1]^2} \left\{ \theta - \frac{(1 - a_{1}^{*})B}{p - p_{y}} + \theta - \frac{(1 - a_{2}^{*})B}{p - p_{y}} \right\} \tag{A-22} \\
\text{s.t.} & \quad \frac{p_{y}}{p}(1 - a_{1}^{*})B - \psi a_{1}^{*} \geq 0 \quad \frac{p_{y}}{p}(1 - a_{2}^{*})B - \psi a_{2}^{*} \geq 0
\]

We can observe that both constraints cannot be slack at the same time: if so, we could increase both \( a_{1}^{*} \) and \( a_{2}^{*} \), without violating any constraint, and increasing the expected revenues of the MFI. Therefore there are three possible solutions: either (i) both constraints bind, or (ii) the first one binds and the second one is slack, or (iii) the first one is slack and the second one binds. Solutions (ii) and (iii) are symmetric. Then without loss of generality all we need to characterize are solutions (i) and (ii). Suppose that we are in the scenario of solution (ii) and the first constraint binds, whereas the second constraint is slack:

\[
\frac{p_{y}}{p}(1 - a_{1}^{*})B - \psi a_{1}^{*} = 0, \tag{A-22}
\]

\[
\frac{p_{y}}{p}(1 - a_{2}^{*})B - \psi a_{2}^{*} > 0. \tag{A-23}
\]

Solving for the binding constraint we obtain
Replacing $a_1^*$ in the objective function, $a_1^*$ should solve:

$$\max_{a_1^*} p \left[ \theta - \frac{p \psi a_1^*}{p - p_B a_1^*} \right] + \left[ \theta - \frac{(1 - a_1^*) B}{p - p_B} \right], \text{ s.t. } 0 \leq a_1^* \leq \min \left\{ 1, 1 - \frac{p \psi a_1^*}{p_B} \right\}.$$ 

The first derivative with respect to $a_1^*$ is:

$$\frac{\partial EV_{\text{m}1}}{\partial a_1^*} = p \frac{p_B B - p \psi a_1^*}{(p - p_B)^2} \leq 0 \text{ if } a_1^* \geq 0,$$

$$\frac{\partial EV_{\text{m}1}}{\partial a_1^*} \geq 0 \text{ if } a_1^* \leq \min \left\{ 1, 1 - \frac{p \psi a_1^*}{p_B} \right\}.$$ 

If $p_B B - p \psi > 0$, the solution is $a_1^* = 1$ and $a_1^* = 1 - \frac{p \psi a_1^*}{p_B} > 0$. However, constraint (A-23) is violated, which is a contradiction. If $p_B B - p \psi < 0$, the solution is $a_1^* = 0$ and $a_1^* = 1$. However, (A-23) is again violated. If $p_B B = p \psi$, then any solution for $a_1^*$ is good, and w.l.o.g we can pick the solution that makes the second constraint bind as well. Then we conclude that both constraints should bind.

Replacing $a_1^*$ and $a_2^*$ in $R_i^{HH}$ and $R_i^{HL}$ from Lemma A4.3 we can prove that the optimal contract will result in a second equilibrium where borrowers choose the monitoring intensity $a_1 = a_2 = 0$ and then both take the action-B. The results obtained at Lemmas A4.1, A4.2, A4.3, and A.21 allows us to provide the following proof for Proposition 7.1.

**Proof of Proposition 7.1.** Suppose that borrower 1 expects borrower 2 to monitor with intensity $a_2 = 0$. We want to show that $a_1 = 0$ is borrower 1’s best response. Looking at Fig. 3, we see that we only have to consider the alternative monitoring choice $a_1^* = \pi_i$. The reason is that all games that would follow the choice $a_1^* < \pi_i$ result in the same second-stage game as choosing $a_1 = 0$. By definition, $a_1 = 0$ is borrower 1’s best response if

$$p_B [\theta - p_B R_i^{HH} - (1 - p_B) R_i^{HL}] + B \geq p_B [p_B R_i^{HH} + (1 - p_B) R_i^{HL}] - \psi \pi_i + B.$$ 

Replacing the values of $R_i^{HH} = \theta - \frac{(1 - a_1^*) B}{p(p - p_B)}$ and $R_i^{HL} = \theta$ the previous condition reduces to:

$$p_B^2 (1 - a_1^*) B \left( \frac{1}{p(p - p_B)} - \psi \pi_i \right) \geq p_B^2 \frac{(1 - a_1^*) B}{p(p - p_B)} - \psi \pi_i.$$

(A-24)

Since the incentive compatibility constraint (A-16) binds, we know that $p_B (1 - a_1^*) B = p \psi a_1^*$. Using this we can rewrite (A-24) as

$$\psi \pi_i - \psi \pi_i^* \geq 0,$$

which is always satisfied as $a_1^* = a_1 < \pi_i$. Therefore action-0 is the best response of borrower 1 when she expects borrower 2 to choose 0.

Finally, to see that (0, 0) is the equilibrium that maximizes the payoff of both borrowers we can observe that

$$EV(a_1^*, a_2^*) = p [\theta - p_B R_i^{HH} - (1 - p_B) R_i^{HL}] - \psi \pi_i + p_B^2 (1 - a_1^*) B \left( \frac{1}{p(p - p_B)} - \psi \pi_i \right) = p_B^2 \frac{(1 - a_1^*) B}{p(p - p_B)} + B(1 - a_1^*),$$

whereas

$$EV(0, 0) = p_B [\theta - p_B R_i^{HH} - (1 - p_B) R_i^{HL}] + B = p_B^2 \frac{(1 - a_1^*) B}{p(p - p_B)} + B(1 - a_1^*) = EV(a_1^*, a_2^*).$$

**Proofs for the perfect competition extension**

**Proof of Lemma 7.2.** Consider a contract that is incentive compatible for the strategy $(\delta_i, \delta_j, (e_i^{(\delta_i, \delta_j)}), e_j^{(\delta_i, \delta_j)})$ where $e_i^{(\delta_i, \delta_j)} = B$. Because of the limited liability, we would have

$$\{p_B [p e_i^{(\delta_i, \delta_j)}] R_i^{HH} + (1 - p e_i^{(\delta_i, \delta_j)}) R_i^{HL} - 1]\} + \{p e_j^{(\delta_i, \delta_j)} [p_B R_j^{HH} + (1 - p_B) R_j^{HL} - 1]\} \leq \{p \theta - 1\} + \{p \theta - 1\} < 0$$

and (19) would be violated, as we assumed $p \theta - 1 < 1 - p \theta$. Similarly, (19) would be violated if $e_j^{(\delta_i, \delta_j)} = B$. Therefore, if $(\delta_i, \delta_j)$ are the first-stage monitoring strategies, the second-stage project choices must be incentive compatible for $e_i^{(\delta_i, \delta_j)} = 0$.

Suppose now the contract is incentive compatible for $(\delta_i, \delta_j)$, where $\delta_i \neq \delta_i$, and $e_i^{(\delta_i, \delta_i)} = e_j^{(\delta_i, \delta_i)} = 0$. By symmetry let us consider the incentive compatibility for borrower 1 only. Incentive compatibility for $e_i^{(\delta_i, \delta_i)} = 0$ requires

$$p B_i^{HH} + (1 - p) R_i^{HL} \leq \theta - \frac{B}{p - p_B},$$

which together with the limited liability constraint (27) for $R_i^{HH}$ and $R_i^{HL}$ implies that the MFI expected profits would be

$$\{p [p B_i^{HH} + (1 - p) R_i^{HL} - 1]\} + \{p [p B_i^{HH} + (1 - p) R_i^{HL} - 1]\} \leq \{p \theta - 1\} + \{p \theta - 1\} < 0,$$

where the last inequality comes from the assumption that $\frac{p}{p - p_B} B > 2(p \theta - 1)$. This violates the zero-profit condition (19), which concludes the proof.
Proof of Proposition 7.3. First, we prove existence of an optimal contract. Notice that choosing \( R_1^{HL} = R_2^{HL} = \theta \) relaxes (27)–(29), (31) and (32). Replacing these values for \( R_1^{HL} \) and \( R_2^{HL} \), constraint (21) requires
\[
|p[pR_1^{HL} + (1 - p)\theta] - 1| + |p[pR_2^{HL} + (1 - p)\theta] - 1| \geq 0.
\] (A-25)

Constraint (31) and (32) require
\[
0 \leq R_2^{HL} \leq \theta - \frac{\psi}{p(p - p_H)},
\] (A-26)

Constraint (32) and (33) require
\[
0 \leq R_1^{HL} \leq \theta - \frac{\psi}{p_H(p - p_H)},
\] (A-27)

Constraint (22) requires
\[
pR_1^{HL} + (1 - p)\theta \geq \theta - \frac{B}{p - p_H},
\] (A-28)

and finally constraint (23) requires
\[
pR_2^{HL} + (1 - p)\theta \geq \theta - \frac{B}{p - p_H},
\] (A-29)

Since \( p_H < p \), from condition (14), we have
\[
\theta - \frac{\psi}{p(p - p_H)} > \theta - \frac{\psi}{p_H(p - p_H)} \geq 0,
\]
so in (A-26) and (A-27) we can ignore the 0 on both left-hand sides.

Replacing in (A-25), (A-28) and (A-29) the conditions for \( R_1^{HL} \) from (A-27) and \( R_2^{HL} \) from (A-26) we obtain that the set of feasible contracts is non-empty if
\[
\left\{ \left[ p\theta - \frac{p\psi}{p_H(p - p_H)} - 1 \right] + \left( p\theta - \frac{p\psi}{p(p - p_H)} - 1 \right) \geq 0 \right\} 
\geq 0 \iff \psi \leq \frac{2p_H(p - p_H)(\theta - 1)}{p(p + p_H)}, \quad \frac{p\psi}{p_H(p - p_H)} \leq \frac{B}{p - p_H} < \psi \leq \frac{p_H B}{p}.
\]

and
\[
\frac{p\psi}{p(p - p_H)} \leq \frac{B}{p - p_H} < \psi \leq B.
\]

These three conditions are satisfied by condition (14). Therefore, the space of feasible contracts is non-empty, closed, and bounded, and an optimal contract solving \( (P^C) \) exists when (14) is satisfied.

Next, assume that \( \frac{\theta - 1}{p} > p\theta - 1 \).

i. Let \( (R_1^{HL}, R_1^{HL}, R_2^{HL}, R_2^{HL}) \) be a solution to problem \( (P^C) \) and suppose that \( \psi \leq \frac{2p_H(p - p_H)(\theta - 1)}{p(p + p_H)} \). Then, combining (26) and (27) we have
\[
\frac{1}{2} |p[\theta - pR_1^{HL} - (1 - p)\theta]_1| - \psi| + \frac{1}{2} |p[\theta - pR_2^{HL} - (1 - p)\theta]_2| - \psi| \leq p\theta - \psi - 1.
\] (A-30)

Consider now the symmetric contract \( (\tilde{R}_1^{HL}, \tilde{R}_1^{HL}, \tilde{R}_2^{HL}, \tilde{R}_2^{HL}) \) defined as
\[
\tilde{R}_1^{HL} = \tilde{R}_2^{HL} = \frac{p}{p_H} - \frac{1 - p}{p} \theta \tilde{R}_1^{HL} = \tilde{R}_2^{HL} = \theta.
\]

Notice that constraint (21) holds with equality:
\[
p[p\tilde{R}_1^{HL} + (1 - p)\tilde{R}_1^{HL}] = p[p\tilde{R}_2^{HL} + (1 - p)\tilde{R}_2^{HL}] = 1.
\]

Moreover constraints (28) and (29) are satisfied as long as \( p\theta - \frac{p\psi}{p_H} < 1 \). Constraints (30)–(32) are satisfied since
\[
\tilde{R}_1^{HL} = \tilde{R}_2^{HL} = \frac{p}{p_H} - \frac{1 - p}{p} \theta \tilde{R}_1^{HL} = \tilde{R}_2^{HL} = \frac{p\theta - 1}{p} - \frac{\psi}{p_H(p - p_H)}.
\]

Finally it is easy to check that constraint (27) is also satisfied. Therefore, the symmetric contract \( (\tilde{R}_1^{HL}, \tilde{R}_1^{HL}, \tilde{R}_2^{HL}, \tilde{R}_2^{HL}) \) is feasible in problem \( (P^C) \).

Moreover, as (21) holds with equality
\[
\frac{1}{2} |p[\theta - pR_1^{HL} - (1 - p)\theta]_1| - \psi| + \frac{1}{2} |p[\theta - pR_2^{HL} - (1 - p)\theta]_2| - \psi| = p\theta - \psi - 1,
\]
that combined with (A-30) implies that \( (\tilde{R}_1^{HL}, \tilde{R}_1^{HL}, \tilde{R}_2^{HL}, \tilde{R}_2^{HL}) \) must solve as well problem \( (P^C) \), concluding the proof.

ii. Suppose that \( \psi > \frac{2p_H(p - p_H)(\theta - 1)}{p(p + p_H)} \).

First, we show that we can ignore constraint (23). Using (21), (26), (27), and the assumption \( p[\theta - \frac{\psi}{p_H(p - p_H)}] < 1 \), we obtain
\[
pR_2^{HL} + (1 - p)R_2^{HL} \geq \frac{1}{p} + \frac{1}{p} - |pR_1^{HL} + (1 - p)R_1^{HL}| \geq \frac{1}{p} + \frac{1}{p} - \frac{p\psi}{p_H(p - p_H)} \geq \frac{1}{p} + \frac{1}{p} - \theta - \frac{p\psi}{p_H(p - p_H)} > \frac{1}{p} - \frac{B}{p - p_H}.
\]
Next, we show that \( pR^{HH}_1 + (1 - p)R^{HH}_2 > pR^{HH}_1 + (1 - p)R^{HH}_3 \). Suppose by contradiction that \( pR^{HH}_1 + (1 - p)R^{HH}_1 \geq pR^{HH}_2 + (1 - p)R^{HH}_2 \). Then, using constraint (26), the limited liability constraint (27), and the assumption that \( \psi > \frac{pB}{p(p - p_B)} \), we note that
\[
pR^{HH}_1 + (1 - p)R^{HH}_2 \leq p \left[ R^{HH}_1 - \frac{\psi}{p_B(p - p_B)} \right] + (1 - p)R^{HH}_2 = R^{HH}_1 - \frac{\psi}{p_B(p - p_B)} - \frac{p\psi}{p_B(p - p_B)} \leq \theta - \frac{p\psi}{p_B(p - p_B)} < \frac{1}{p}.
\]
Then, it is easy to see that if \( pR^{HH}_1 + (1 - p)R^{HH}_1 \geq pR^{HH}_2 + (1 - p)R^{HH}_2 \), then MFI participation constraint (21) is violated.

Next, we show that \( R^{HH}_1 < R^{HH}_2 \). Suppose by contradiction that \( R^{HH}_1 \geq R^{HH}_2 \). Then, using the participation constraint (21), the limited liability constraint (27) and the assumption that \( \psi > \frac{pB}{p(p - p_B)} \frac{\psi}{p} + \frac{1}{p} \), we obtain
\[
\theta \geq R^{HH}_2 \geq \frac{2}{p(1 - p)} \left[ \frac{p}{1 - p} R^{HH}_1 + \frac{p}{1 - p} R^{HH}_1 + R^{HH}_1 \right] \geq \frac{2}{p(1 - p)} \left[ \frac{2p}{1 - p} R^{HH}_1 + R^{HH}_1 \right] \geq \frac{2}{p(1 - p)} \left[ \frac{2p}{1 - p} R^{HH}_1 - \frac{\psi}{p_B(p - p_B)} + R^{HH}_1 \right]
\]
\[
\geq \frac{2}{p(1 - p)} \left[ \frac{2p}{1 - p} \theta - \frac{\psi}{p_B(p - p_B)} + \theta \right] = 2 \left\{ \frac{1}{p(1 - p)} - \frac{1}{p - p_B} \right\} \theta - \frac{2}{p(1 - p)} \theta = \theta,
\]
where the inequality in the last line comes from the assumption that \( \psi > \frac{pB}{p(p - p_B)} \frac{\psi}{p} + \frac{1}{p} \).

Next, we show that constraint (21) binds. Suppose by contradiction that (21) is slack. Then it should be that \( R^{HH}_2 = 0 \) and \( R^{HH}_2 = \frac{\psi}{pB(p - p_B)} \). Using the participation constraint (21) and the limited liability constraint (27) we obtain
\[
\{\rho(R^{HH}_1 + (1 - p)R^{HH}_2) - 1\} + \{\rho(R^{HH}_2 + (1 - p)R^{HH}_1) - 1\} \leq \{\rho(R^{HH}_1 + (1 - p)R^{HH}_2) - 1\} = (1 - p) \left[ \frac{\psi}{pB(p - p_B)} - \frac{1}{p(1 - p)} \right],
\]
which violates (21).

Finally, we show that asymmetric contracts are optimal for a non-empty interval on \( \psi \):
\[
\frac{pB}{p(p - p_B)} \frac{\theta - 1}{p} < \psi \leq \min \left\{ \frac{2p(p - p_B)}{p(p + p_B)}, \frac{pB}{pB(p - p_B)} \right\}.
\]
Notice that since \( p > p_B \), we have
\[
\frac{pB}{p(p - p_B)} \frac{\theta - 1}{p} < \frac{2p(p - p_B)}{p(p + p_B)} \frac{\theta - 1}{p} < \frac{pB}{pB(p - p_B)} \frac{\theta - 1}{p},
\]
and since \( \theta - 1 < \frac{pB}{pB} \frac{\theta - 1}{p} \), we have
\[
\frac{pB}{p(p - p_B)} \frac{\theta - 1}{p} < \frac{pB}{pB(p - p_B)} \frac{\theta - 1}{p},
\]
and finally since \( \frac{1}{1 - p} > \frac{pB}{pB} \frac{\theta - 1}{p} \), we have
\[
\frac{pB}{p(p - p_B)} \frac{\theta - 1}{p} < \frac{pB(p - p_B)}{pB},
\]
Therefore the lower bound on \( \psi \) is always smaller than the upper bound on \( \psi \), which concludes the proof.

Appendix E. Supplementary data

Supplementary data associated with this article can be found in the online version at [http://dx.doi.org/10.1016/j.jdeveco.2017.02.003](http://dx.doi.org/10.1016/j.jdeveco.2017.02.003).

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