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On the Contributions of John C. Harsanyi, John F. Nash and Reinhard Selten

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The Royal Swedish Academy of Sciences awarded the 1994 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel to John C. Harsanyi, John F. Nash and Reinhard Selten for “their pioneering analysis of equilibria in the theory of non-cooperative games”. The Academy justifies giving the prize in economics to three game theorists by the argument that strategic interaction characterizes many economic situations and that, therefore, game theory has proved very useful in economic analysis. Today, 50 years after the publication of Von Neumann and Morgenstern’s *Theory of Games and Economic Behavior* (1944), game theory, and, in particular, its non-cooperative branch has become a dominant tool for analyzing economic issues. In this note, I briefly review and appraise the laureates’ fundamental contributions to game theory and the economic sciences¹.

1 Equilibria in Non-Cooperative Games

The principal aspect of non-cooperative game theory is the equilibrium concept introduced by John Nash in his PhD thesis: “Non-cooperative Games” (Nash (1950b)). The thesis also introduces the fundamental distinction between cooperative and non-cooperative games. In games of the latter type, players are unable to conclude enforceable agreements outside the formal rules of the game. As Nash puts it: “The basic requirement for a non-cooperative game is that there should be no-preplay communication among the players [unless it has no bearing on the game]. Thus, by implication, there are no coalitions and no side-payments” (Nash 1950b, p. 21).

Nash’s equilibrium concept is a generalization of the minimax solution introduced in Von Neumann (1928) for the two-person zero-sum game. Nash’s main mathematical result is the proof of existence in any finite, strategic form game, of at least one Nash equilibrium. In published work (Nash (1950a, 1951)), Nash provides two alternative, elegant proofs, one based on Kakutani’s fixed point theorem, the other based directly on Brouwer’s theorem. These techniques have inspired many other existence proofs, for example, in the area of general equilibrium theory. (See Debreu (1984).)

In the section “Motivation and Interpretation” of his thesis, Nash discusses two interpretations of his equilibrium concept. In the first, “mass-action” interpretation of

¹ For more extensive descriptions see Van Damme and Weibull (1995) and Güth (1994).

equilibria “It is unnecessary to assume that the participants have full knowledge of the structure of the game, or the ability and inclination to go through any complex reasoning processes. But the participants are supposed to accumulate empirical information on the relative advantages of the various pure strategies at their disposal” (Nash 1950b, p. 21). If the frequencies with which the various strategies are played settle down, then these stable frequencies must constitute an equilibrium. As Nash notes, the interest in this interpretation derives from the fact that “There are situations in economics or international politics in which, effectively, a group of interests are involved in a non-cooperative game without being aware of it, the non-awareness helping to make the game truly non-cooperative” (Nash 1950b, p. 23).

In contrast, the second (and more familiar) “rationalistic interpretation” requires that the players are rational and know the full structure of the game. It originates from considering the question: “What would be a “rational” prediction of the behavior to be expected of rational playing the game in question?” (Nash 1950b, p. 23). A theory of rational behavior that provides a unique strategy recommendation for each player has to prescribe to play a Nash equilibrium, since otherwise it is not immune to “theory absorption”, i.e. it is self-destroying. Of course, this argument does not imply that any theory that prescribes to play a Nash equilibrium is a satisfactory one, nor that such a satisfactory theory necessarily exists. (See von Neumann and Morgenstern (1944, Sects. 4.2, 4.3 and 17.3). In particular the founding fathers insist that the solution from this *indirect argument* be independently justified by a *direct argument*.)

Since the uniqueness assumption is crucial for the rationalistic justification² of Nash’s equilibrium concept, the fact that a game frequently has multiple equilibrium points is problematic for that interpretation. In order to cope with this multiplicity, Nash introduces several auxiliary concepts (such as interchangeability, solutions and sub-solutions), however, many games do not admit solutions. Nevertheless, Nash argues that it sometimes happens that good heuristic reasons can be given for narrowing down the set of equilibria to a single solution. In particular, in an important application to the study of cooperative games, Nash (1953) suggests to discriminate among equilibria by studying their relative stabilities. Specifically, he perturbs a game and studies the limiting behavior of the associated equilibria as the perturbations vanish. Nash shows that the unique necessary limit is the outcome that he had derived earlier by means of the axiomatic method (Nash (1950c)). Nash’s suggestion to cut down on the number of equilibria in this way has been extensively followed in the literature.

Nash argues that game theory’s cooperative (axiomatic) and non-cooperative approaches are complementary, that each helps to justify and clarify the other. If a solution can be obtained from a convincing set of axioms, this indicates that the solution might be appropriate in a wider variety of situations than those formally captured by a specific non-cooperative model. On the other hand, by following the so-called *Nash program*, i.e. by reformulating cooperative games as non-cooperative ones and by solving for the Nash equilibria of the latter, an abstract discussion about the rea-

² In the remainder, attention will be confined to this interpretation. Recently, the mass-action interpretation has regained attention. For a partial review of recent developments that build on it, the reader is referred to Weibull (1994).

sonableness of certain outcomes or principles can be replaced by a more mundane discussion about the appropriateness of the rules of the game.

While the Nash program has the attractive feature of enabling theory unification, in practice it encounters severe difficulties because of the multiplicity problem. Consequently, major breakthroughs (such as Rubinstein (1982)) were obtained only after substantial advances had been made in addressing this multiplicity issue so as to enable analysis of more complex models. It is in this regard that the work of John Harsanyi and Reinhard Selten was of fundamental importance.

2 Dynamic Games and Equilibrium Refinements

Since each extensive form game can be reduced to a game in strategic form, Nash's theorem implies that any finite extensive form game has at least one equilibrium. Selten (1965), however, notes that some equilibria are "unreasonable" since they rely on "incredible" threats. These equilibria are stable only if players are able to commit themselves to their strategies, but such commitments are impossible in a non-cooperative game. Selten (1965) argues that behavior in a subgame should constitute a Nash equilibrium in this subgame, since otherwise at least one player would have an incentive to deviate once the subgame is reached. Equilibria with this property are called *subgame perfect equilibria*. They exist and can be found by backwards induction (Kuhn (1953)). Selten's criterion of subgame perfection has been widely applied in the economics literature and it has yielded considerable insights. For example, one may refer to bargaining theory (Binmore et al. (1992)) and to the various multistage models of imperfect competition in the literature on industrial organization (Tirole (1988)).

A conceptual difficulty associated with the backwards induction procedure is that it assumes that players will behave rationally (i.e. act in accordance with the theory) even after players have deviated from that theory. Indeed, this difficulty was already noted by Von Neumann and Morgenstern (*op cit.*, Sect. 15.8). At the same time, the subgame perfectness concept is not restrictive enough to eliminate all "unreasonable" equilibria. Selten (1975) resolves both difficulties in one below by viewing perfect rationality as a limiting case of incomplete rationality, by introducing the possibility that players may make mistakes when choosing their actions. He writes: "There cannot be any mistakes if the players are absolutely rational. Nevertheless, a satisfactory interpretation of equilibrium points in extensive games seems to require that the possibility of mistakes is not completely excluded. This can be achieved by a point of view which looks at complete rationality as a limiting case of incomplete rationality." (Selten 1975, p. 35). Selten defines a (*trembling hand*) *perfect equilibrium* as an equilibrium that can be rationalized by *some* form of slight mistakes, hence, s is perfect if there exists a sequence of perturbed games of which an associated sequence of equilibria converges to s . The perfect equilibria constitute a nonempty subset of the subgame perfect equilibria.

A slight, but influential modification of perfect equilibrium, the concept of *sequential equilibrium*, was introduced by Kreps and Wilson (1982). This concept makes explicit the beliefs that are generated by the trembles. Since the language using beliefs is more congenial to economists than that using trembles, and since sequential equilibria are somewhat easier to work with than perfect equilibria, the sequential equilibrium concept has been used more frequently in applications. In fact, it has been used very frequently, in particular, in connection with information revelation through time and reputation building. (See Wilson (1985).)

Note that, while Nash insisted on stability with respect to *all* perturbations (in pay-offs), Selten is satisfied with stability with respect to *some* perturbations (in strategies). In general, there does not exist an equilibrium that is stable with respect to all strategy trembles, however, Kohlberg and Mertens (1986) have suggested to focus attention on minimal sets of equilibria that are stable in this respect. The Kohlberg and Mertens' stability concept has found many applications and has led to significant spin-offs, for example, in connection with (applications of) signalling games. (See Kreps and Sobel (1994)). At the same time, Harsanyi and Selten (1988, Postscript) argue that, if one insists on such strong stability, one has to give up some fundamentally important principles of game theoretic rationality (such as uniqueness, or subgame consistency) and they feel that this is a too high price to pay. At present, the debate about whether (and which) strong stability requirements do indeed capture necessary requirements of rational behavior is still going on. (See Van Damme (1994) for further details.)

3 Incomplete Information

The rationalistic interpretation of Nash equilibrium is based on assumption of complete information, that each player knows the full structure of the game. Obviously, this idealization is only rarely met in actual situations. Incomplete information (about the strategies, knowledge, or payoffs of the others) is the rule rather than the exception. This observation led Luce and Raiffa (1957, p. 55) to remark that, rather than the rationality assumption "the knowledge assumption appears to be the real source of unreality in the model". Unfortunately, until the mid 1960s no general methods were available to analyze games with incomplete information³. Such games seemed untractable since they seemed to give rise to an infinite regress in reciprocal expectations.

Harsanyi (1967–1968) cuts this Gordian knot by introducing the concept of "player-type". He postulates that each player may be of several possible *types* where a type specifies all the information that the player has about the game. In particular,

³ However, the important paper Vickrey (1961) should be noted. Among others, Vickrey analyses first (and second) price auctions for the case where bidders' values are independently distributed. He models the situation as a game and computes its Nash equilibria. He discusses "revenue equivalence" results and notes that a player whose value is drawn from a continuous distribution does not need to randomize in equilibrium.

a player's type specifies a probability distribution over the types of the other players as well as over the exogenous uncertainty associated with the game. Intuitively, a player's type may be thought of as the player's state of mind and a vector of types (one for each player) as a state of the world. Harsanyi introduces the term *Bayesian game* for the formulation of the incomplete information game by means of player types. Under a consistency requirement, viz. that players' types can be derived from a common prior distribution on the set states of the world, this Bayesian game is equivalent to an ordinary game with complete, but imperfect information: First nature distributes information across the players (i.e. informs each player about his type), next players choose actions. Harsanyi argues that it should always be possible to trace differences in players' beliefs to differences in their information (the so-called *Harsanyi doctrine*), hence, that it should be possible to construct a consistent model. (See Mertens and Zamir (1985) for elaboration and for the formal demonstration that Harsanyi's construction is completely general, i.e. that well-defined type spaces can always be constructed.)

Although incomplete information games can be handled by standard methods, their analysis reveals that they have substantially different properties. For example, in a complete information environment one would expect unrestricted bargaining to produce efficient outcomes. When there is incomplete information, ex post inefficiencies are unavoidable (Myerson and Satterthwaite (1983)) and Harsanyi's techniques allow us to quantify the transaction costs associated with different trading mechanisms.

Harsanyi's conceptual breakthrough provided the theoretical foundation that enabled information economics – the study of strategic interaction in economies in which information is asymmetrically distributed, to take off and since then many important insights have been obtained in this area. As a practical example, one may refer to the recent auction of radio spectrum organized by the FCC (Federal Communications Commission) in the US. The auction form that was used was proposed by the leading auction theorists Paul Milgrom, Robert Wilson and Preston McAfee, authors who rely essentially on Harsanyi's model in their theoretical work. The auction was considered to be a big success by both taxpayers and firms. As the *Wall Street Journal* noted on August 1, 1994. "By combining cutting-edge economic theory and auction procedures, the FCC wrung top prices out of bidders." (See McMillan (1994) for further details.)

Harsanyi's work also had important spin-offs for pure game theory. Among others, it led to a reinterpretation of mixed strategies that eliminated the intuitive difficulties and seeming instabilities associated with them. In a perturbed game that takes explicitly into account the fact that each player always has a slight amount of private information concerning his own payoffs, all equilibria are essentially pure. Furthermore, all regular equilibria of the original game can be approximated by the beliefs generated by such pure equilibria of slightly perturbed games (Harsanyi (1973)). The point of view that a mixed strategy of player i can be interpreted as the beliefs (conjectures) of the opponents concerning i 's behavior has been further developed by Robert Aumann. It has led to such important concepts as correlated equilibria (Aumann (1974)) and common knowledge (Aumann (1976)), and it has yielded a better understanding of the epistemic foundations of the Nash equilibrium concept (Aumann and Brandenburger (1991)).

4 Other Contributions

In the above, attention was confined to the Laureates' main contributions to the theory of non-cooperative games that were mentioned in the official press announcement of the prize by the Royal Swedish Academy of Sciences. Harsanyi, Nash and Selten have made several additional outstanding contributions to game theory, mathematics and the social sciences, of which some will be briefly considered now. The reader should consult the laureates' lists of publications in the January 1995 issue of the *Scandinavian Journal of Economics* for further details.

John Nash also made seminal contributions to cooperative game theory. In his (1950c) paper on bargaining he departed radically from the orthodox economic theory that considered the bargaining problem to be indeterminate. In contrast, Nash assumes that bargaining between rational players leads to a unique outcome and he determines this outcome by imposing that it satisfies several desirable properties. Nash's formulation of the bargaining problem and his bargaining solution constitute the foundation of modern bargaining theory. Much less known are Nash's contributions to the early literature in experimental economics. (See Roth (1993).) Among others, Nash makes important comments concerning the observed cooperation in the finitely repeated prisoners' dilemma. In addition, the pioneering work of Nash, together with Kalisch, Milnor and Nering, on characteristic function game experiments (Kalisch et al. (1954)) was an important stimulus for Reinhard Selten to start doing experiments in Germany (Selten (1993)). This work led to the flourishing German school in experimental economics, which contributed substantially to establishing experimental economics as a separate field. Last, but not least, John Nash made important contributions to pure mathematics.

Besides his contributions to non-cooperative game theory, John Harsanyi also made outstanding contributions to cooperative game theory. In particular, one may mention his contributions to bargaining (such as linking Nash's solution to that of Zeuthen), his construction of a value for NTU-games and his reformulation of the Von Neumann-Morgenstern stable set concept (the latter being inspired by his attempt to provide a non-cooperative foundation for the concept). In addition, Harsanyi made prominent contributions to the foundations of welfare economics and moral decision making (by providing, among others, a justification of utilitarianism) and to the philosophy of science. Harsanyi's work displays a remarkable conceptual consistency. The principles of Bayesian rationality serve as the starting point of his analysis of various diverse problems.

In contrast, Selten's *oeuvre* is characterized by a methodological dualism. Selten advocates a sharp distinction between normative, rationalistic game theory and positive, descriptive game theory. While Selten considers the rationalistic theory to be of great philosophical significance, he is doubtful of the descriptive relevance of a theory that is based on strong rationality assumptions. He is convinced that a new kind of descriptive, behavioral theory, that is firmly based on empirical observations, will have to be developed for economic applications. A great part of Selten's work is devoted to the construction of building blocks for such a theory of boundedly rational decision making. Laboratory experimentation plays an important role in this

research, in relation to which Selten devised the strategy method to gain more insight into the structure of human decision making. Significant insights that were obtained thus far are that equal division is not only a fairness norm but also a natural point of departure for boundedly rational behavior under time constraints, and that there are not only cognitive but also motivational bounds to human rationality (Selten (1978)). Selten also contributed significantly to evolutionary game theory by showing how the Maynard Smith and Price (1973) concept of evolutionary stable strategies (which had originally been defined only for 2-person symmetric games) could be extended to general 2-person extensive form games, thereby considerably enlarging the scope of evolutionary theory. Finally, let me mention Selten's pioneering work on multistage game models in the theory of industrial organization.

In joint work, that builds on ideas of Nash, Harsanyi and Selten (1988) have developed the first general, coherent theory of equilibrium selection. The theory singles out a unique equilibrium for each non-cooperative game and, hence, enables a more successful application of the Nash program. While it is clear that the Harsanyi and Selten (1988) theory is not the unique possible theory of equilibrium selection (indeed both Harsanyi (1995) and Selten (1995) describe alternatives), the original theory has already seen many applications (see Selten (1993) for a partial overview). In addition, its seminal building blocks, such as the tracing procedure and the notion of risk dominance, also figure prominently in other contexts.

5 Conclusion

The final paragraph of the press release announcing the 1994 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel reads "Through their contributions to equilibrium analysis in non-cooperative game theory, the three laureates constitute a natural combination: Nash provided the foundations for the analysis, while Selten developed it with respect to dynamics and Harsanyi with respect to incomplete information". I hope to have shown this natural unity above.

The conceptual and technical breakthroughs established by Harsanyi, Nash and Selten have enabled game theory to fulfill the high expectations that were expressed for the theory immediately after the publication of Von Neumann and Morgenstern's *Theory of Games and Economic Behavior*. Game theoretic reasoning is by now established as the foundation of major parts of economics and it is rapidly intruding related disciplines such as accounting, finance, political science and sociology. The contributions of Harsanyi, Nash and Selten constitute important milestones in the development of game theory and in establishing a common methodology for analyzing strategic interaction within all the social sciences.

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