MACRO WELFARE EFFECTS OF FLEXIBLE LABOR CONTRACTS

By

José Gabo Carreño, Burak Uras

1 November 2021

ISSN 0924-7815
ISSN 2213-9532
Macro Welfare Effects of Flexible Labor Contracts*

José Gabo Carreño† Burak Uras‡

February 16, 2022

Abstract

We develop a small open economy DSGE model to study the macro welfare effects of flexible labor contracts for an economy in a currency union. The framework exhibits two sectors: a fixed sector and a flex sector. The fixed sector offers contracts that exhibit rigidities in working-hours and wages while the flex sector offers flex contracts in both dimensions. We find that the flex sector has a welfare-enhancing role in accommodating shocks, if the fixed sector’s hour adjustment exhibits a high degree of rigidity; and moreover, the presence of the flex sector reduces the desirability of rigid wages in the fixed sector. The welfare analysis also reveals an optimal flex sector size. With the baseline parameterization, a flex sector of 20% of the overall employment maximizes the macro welfare. Our results have important policy implications for a wide range of countries in European-Monetary-Union - characterized by growing and large flex sectors.

Keywords: two-sector DSGE model, wage flexibility, currency union, loss function, labor market frictions, flexible-hour labor contracts.

JEL Codes: E12, E24, E52, J41.

*Acknowledgments: For their suggestions and comments we would like thank to Ata Can Bertay, Harry Huizinga, Hugo van Buggenum and seminar participants at Tilburg University. All remaining errors are ours.

†Tilburg University, Department of Economics. Contact: j.g.carrenobustos@tilburguniversity.edu
‡Tilburg University, Department of Economics. Contact: r.b.uras@tilburguniversity.edu
1 Introduction

In developed countries, flexible labor contracts have gained increasing popularity over the last decade (Katz and Krueger, 2017, 2019; Mas and Pallais, 2017). For instance, 28% of the employees in the Netherlands were under flexible labor arrangements as of 2016.\(^1\) The surge of flexible labor contracting is likely to entail important macro-welfare implications. In particular, compared to fixed contracts, flexible labor contracts may provide firms with adjustment mechanisms - not only in hours-work but also in wage compensations. While compelling evidence shows that firms indeed rely on contractual flexibility in coping with economic shocks (Boeri et al., 2020; Borowczyk-Martins and Lalé, 2019; Burdin and Pérotin, 2019; Fontaine, 2019; Kyyrä et al., 2019; Schaefer and Singleton, 2019), a formal macro-analysis of flexible labor contracts has been absent in the literature. Our aim in this paper is to fill this important gap.

In this paper, we develop a small open economy DSGE model to understand the welfare effects of flexible contracts in an economy characterized by labor market frictions. Our paper focuses on the context of a small open economy under a currency union regime, because the role of labor market flexibility is viewed as being particularly important in the context of economies that have joined a currency union - as in those cases the exchange rate is no longer available as an adjustment mechanism (Farhi et al., 2014; Galí and Monacelli, 2016; Schmitt-Grohé and Uribe, 2016).

What makes flex contracts important for the macro welfare relates to the fact that firms may use flex workers to reduce labor costs - in both working-hours and salary payment margins - during a downturn (Adams-Prassl et al., 2020; Josten and Vlasbom, 2018). As an empirical support for this argument, in a recent paper, Grajales-Olarte et al. (2021) study the role of different types of labor contracts in explaining wage rigidities in the Netherlands. Consistent with the view that firms use flex contracts to reduce labor costs, the authors find that (1) flex-hour labor contracts exhibit substantially more wage flexibility, and (2) working-hours of the flex-hour labor contracts are more likely to reduce during a downturn compared to all other contracts.\(^2\)

\(^1\)Flex workers are defined as individuals who have either fixed-term or flexible-hour contracts. This group comprises of seven different categories: Temporary employees with prospects for permanent employment and a fixed number of hours; Temporary employees with a fixed-term contract for 1 year or more and fixed hours; Temporary employees with a fixed-term contract for less than 1 year and fixed hours; On-call and replacement employees; Workers hired through temporary employment agencies; Permanent employees with flexible hours; and Temporary employees with flexible hours.

\(^2\)In a related paper, we also show that flex workers receive a lower hourly wage, face higher worked hours volatility, and are more time unemployed than permanent workers (Bertay et al., 2022).
To incorporate these empirical facts into a theoretical framework, we develop a small open economy DSGE model with labor market rigidities and two-sectors: a fixed sector and a flex sector. The fixed sector offers imperfectly rigid contracts in working-hours and wages and the flex sector offers flex contracts in both dimensions. In order to understand the interactions between frictions (in the fixed sector) and flex contracts, we utilize Galí and Monacelli (2016)’s one-sector framework as a baseline model. The baseline analysis à la Galí and Monacelli (2016) reveals that wage flexibility in a one-sector model - exhibiting only the fixed sector - could reduce the welfare of a small open economy in a currency union characterized by staggered price and wage setting. The intuition for this result is that the effectiveness of labor cost reductions as a means to stimulate employment is much smaller in a currency union.

We then extend the baseline model into a two-sector set-up, where the fixed sector contracts offer rigid working-hours and sticky wages, while flex sector contracts always offer working-hours and wages that are set optimally in each period. Our analysis delivers important qualitative and quantitative findings. First, whenever the fixed sector’s working-hour rigidity is low, the flex sector has a limited welfare effect on the macroeconomy, as in that case the only economic contribution of the flex sector stems from offering a higher wage renegotiation frequency. Yet, there is plenty of evidence that the adjustment through working-hours may be limited in the fixed sector. Fixed long-term contracts are difficult to renegotiate and working-hours are not easy to modify. This empirical fact moves us to our next result. If the fixed sector cannot respond to shocks sufficiently by adjusting working hours, the flex sector assumes a more important welfare-enhancing role to accommodate shocks - by allowing for adjustment margins in both working-hours and wages.

While some degree of flexibility is beneficial for the performance of the macroeconomy, excessive fluctuations in working-hours and wages in the flex sector may be costly for the households - in terms of the implied consumption volatility. Confirming this intuition we also find a non-monotone relationship between the macro welfare losses and the size of the flex sector: with the baseline parameterization, a flex sector larger than 20% of the overall employment in the economy reduces welfare. Finally, as a surprising

---

3In our framework, we will assume that labor can be reallocated freely across firms in the same sector but cannot flow across sectors. This assumption is motivated by empirical studies showing that factor reallocation is more flexible within sectors rather than across sectors (Davis and Haltiwanger, 1992; Parent, 2000).
result, we show that a large flex sector size increases the desirability of wage flexibility in the fixed sector: because of a general equilibrium interaction, higher wage flexibility (lower wage rigidity) in the fixed sector helps to reduce the volatility of the working-hours in the flex sector, which is a notorious feature of flex-hour contracts (Grajales-Olarte et al., 2021). In this respect, different from the findings by Galí and Monacelli (2016), we show that even in a currency union, an increase in wage flexibility in the fixed sector could monotonically increase welfare as long as there is a large flex sector in that economy. This policy-relevant conclusion implies that we should not study the macro-welfare consequences of wage flexibility without incorporating flexible labor contracts in general equilibrium models, if such contracts are present to a large extent, as in the case of EMU economies.

The remainder of the paper is organized as follows. Section 2 reviews the related literature, whereas section 3 sets up the baseline model. Section 4 develops the two-sector models that we use to study the macro effects of flex labor contracts. Finally, section 5 concludes. The Appendix contains the derivation of the models presented in sections 3 and 4.

2 Literature Review

Our paper is related to several strands of research in economics. The first is the literature that studies the advantages and disadvantages of contractual flexibility. There are a number of papers studying the impact of contractual flexibility on wages (Cörvers et al., 2011; Drenik et al., 2020; Goldschmidt and Schmieder, 2017), wage gender gap (Chung, 2019), career opportunities (David and Houseman, 2010; Gebel, 2013), and monetary policy (Björklund et al., 2019). We add to this literature by studying the welfare implications of flexible labor contracts in an open economy DSGE setting. In a paper closely related to ours, Dolado et al. (2019) develop an equilibrium search and matching model to evaluate the pros and cons of zero-hour contracts. They analyze how the UK’s low-pay segment of the labor market would respond to a ban on zero-hour contracts. They find that zero-hour contracts are valuable for both firms and employees as firms utilize them to decrease labor costs when working hours are low. We take a different angle and explore the welfare consequences of an increase in the share of flex contracts and draw important general equilibrium conclusions related to the interaction of flex contracts with wage and working-hour rigidities in fixed contracts.
In that respect, this paper is also related to the literature on nominal wage rigidities. Since Christiano et al. (2005), the medium-scale macro-models have used the sticky wage assumption rather extensively. This is because nominal wage rigidities are more important than nominal price rigidities in explaining the dynamic effects of monetary policy shocks. The relevance of nominal wage rigidity is even more fundamental to understand the macro-dynamics of economies that have joined a currency union, because in those cases the exchange rate is no longer available as an adjustment mechanism (Farhi et al., 2014; Galí and Monacelli, 2016; Schmitt-Grohé and Uribe, 2016). Because the rise in flex contracts could change the role of nominal wage rigidities in macro-models, we add to this literature by studying the interaction between nominal wage rigidities and flex contracts.

Our work is also related to the literature that measures rigid wages using high-frequency wage data (Barattieri et al., 2014; Grajales-Olarte et al., 2021; Sigurdsson and Sigurardottir, 2016). A particularly important paper for us is Grajales-Olarte et al. (2021), in which the authors study the role of different types of labor contracts in explaining rigid wages in the Netherlands. They find that (1) flex-hour labor contracts exhibit substantially more wage volatility; and, (2) working-hours of the flex-hour labor contracts are more likely to reduce during a downturn compared to all other contracts. This evidence motivates our paper, and thus, we contribute to this literature by formalizing the macro welfare implications of flex contracts.

Our paper is also related to multisector sticky-price DSGE models (Aoki, 2001; Carvalho et al., 2021; Carvalho and Nechio, 2016; Singh and Beetsma, 2018). These papers incorporate heterogeneity in price-setting behavior in dynamic macro-models. Different from these papers, we incorporate heterogeneity in wage-setting behavior across two sectors. We do this by assuming two sectors, one with nominal wage rigidities and another one with full wage flexibility. As in Carvalho et al. (2021), we assume a segmented labor market: labor can be reallocated freely across firms in the same sector but cannot flow across sectors. This assumption is motivated by empirical studies showing that factor reallocation is relatively more flexible within sectors rather than across sectors (Davis and Haltiwanger, 1992; Parent, 2000). In this way, our labor market specification falls between the two extreme assumptions often made in the macro literature, namely, firm-specific factor markets and economy-wide factor markets.\footnote{See Carvalho and Nechio (2016) for a detailed discussion about factor specificity.} Two papers that take similar approaches as ours are Faia and Pezone (2020) and Eijffinger et al.
These papers study the relationship between wage rigidity and monetary policy in DSGE models. While they focus on different research questions, we share with them the multi-sector environment with heterogeneous wage rigidity across different sectors.

3 Baseline One-Sector Model

In this section we describe the baseline one-sector economy, à la Gáli and Monacelli (2016), which we then extend into a two-sector model in the following section.

**Households.** Consider a small open economy with a representative household. The representative household is made up of a continuum of members, each specialized in a differentiated occupation, indexed by $j \in [0, 1]$. The representative household seeks to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \ln(C_t) - \frac{1}{1 + \varphi} \int_0^1 N_t(j)^{1 + \varphi} dj \right) Z_t \right],$$

where $C_t$ is a consumption index, $N_t(j)$ is employment (or working-hours supplied) by the member-$j$ of the household at the nominal wage-rate $W_t(j)$. $Z_t$ is the exogenous preference shifter. Parameter $\beta \equiv \frac{1}{1 + \varphi} \in [0, 1]$ is the discount factor. The composite consumption index $C_t$ is defined as $C_t \equiv \Upsilon(C_{H,t}, C_{F,t})^{1 - \upsilon}(1 - \eta)\upsilon$, where $Y = 1 / ((1 - \upsilon)(1 - \eta))$, $C_{H,t} \equiv \left( \int_0^1 C_{H,t} \frac{\varepsilon_p - 1}{\varepsilon_p} di \right) \frac{\varepsilon_p}{\varepsilon_p - 1}$ is the domestic goods consumption, and $C_{F,t}$ is the consumption of the foreign good. In this specification, parameter $\varepsilon_p > 1$ captures the elasticity of substitution between varieties produced domestically. Parameter $\upsilon \in [0, 1]$ can be interpreted as a measure of openness. The log preference shifter, $z_t = \log Z_t$, is assumed to follow an exogenous AR(1) process:

$$z_t = \rho z_{t-1} + \varepsilon_t^2.$$  

In our analysis, we refer $z_t$ shocks as the demand shocks. The budget constraint of the representative household is given by

$$\int_0^1 P_{H,i}(i)C_{H,t}(i)di + P_{F,t}C_{F,t} + E_t \{ Q_{t,t+1}D_{t,t+1} \} \leq D_t + \int_0^1 W_t(j)N_t(j) dj - \Lambda_t,$$

for $t = 0, 1, 2, ..., $ where $P_{H,i}$ is the price of domestic variety $i$. The variable $P_{F,t}$ is the price of the imported good, expressed in domestic currency. $D_t$ is the nominal payoff in period $t$ resulting from the asset portfolio held by the end of the period.
Finally, $\Lambda_t$ denotes lump-sum taxes. The relevant stochastic discount factor for one-period nominal payoffs is $Q_t \equiv \beta(C_t/C_{t+1})(P_t/P_{t+1})$.\(^5\) Prices are set in domestic currency and the law of one price holds. This implies that the price of imported goods is given by $P_{F,t} = \mathcal{E}_t P^\ast_t$, where $\mathcal{E}_t$ is the nominal exchange rate and $P^\ast_t$ is the foreign price level - expressed in foreign currency. We assume that $P^\ast_t = 1$.

**Wage-setting.** Labor contracts are subject to nominal adjustment frictions, such that the so-called wage stickiness prevails. Specifically, in each period only a fraction $(1 - \theta_w)$ of members of the household (labor types), drawn at random get to reset their nominal wage. Equivalently, in each period the nominal wage for any worker remains unchanged with probability $\theta_w$. Parameter $\theta_w \in [0,1]$ is thus a measure of nominal wage stickiness, à la Calvo. Therefore, higher wage flexibility is associated with a lower $\theta_w$.

**Firms.** A continuum of consumption variety producers (firms), indexed by $i \in [0,1]$, operate in the home economy. A typical domestic firm produces a differentiated good using the technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where $Y_t$ is output and $N_t(i) = \left( \int_0^1 N_t(i,j) \frac{\epsilon_w}{1-\epsilon_w} dj \right)^\frac{\epsilon_w}{1-\epsilon_w}$ is a constant elasticity of substitution (CES) function of the quantities $N_t(i,j)$ of the different types of labor services, $j \in [0,1]$, employed - with labor services to be chosen optimally in each period. Parameter $\epsilon_w > 1$ denotes the elasticity of substitution between differentiated labor services supplied by the members of the representative household. $A_t$ is a stochastic technology parameter, common to all firms. Its logarithm, $a_t \equiv \log A_t$, follows an exogenous AR(1) process, $a_t = \rho a_{t-1} + \epsilon^a_t$.

**Price-setting.** In each period, a subset of firms of measure $(1 - \theta_p)$, drawn randomly get to reoptimize the price of their good, subject to a sequence of isoelastic demand schedules. The remaining fraction $\theta_p$ keep their prices from the previous period. Parameter $\theta_p \in [0,1]$ thus captures the degree of pricing rigidities, à la Calvo.

**Demand for exports.** We assume in our baseline model that households have access to a complete set of state-contingent securities, traded internationally.\(^6\) As shown by

\(^5\) $Q_t$ denotes the price of a one-period discount bond paying off one unit of domestic currency in all states at $t+1$.

\(^6\) Under a complete set of state-contingent securities, agents are allowed to smooth consumption not only across time but also across different states of nature. This implies that a complete market for state-
Galí and Monacelli (2016), this assumption implies the following relationship between domestic and world consumption:

\[ C_t = C^*_t Q_t Z_t, \]  

(5)

where \( Q_t = \frac{E_t P^*_t}{P_t} \) is the real exchange rate, \( C^*_t \) is (per capita) world consumption.\(^7\) Under the assumption that the small economy has an infinitesimal size relative to the rest of the world, \( C^*_t = Y^*_t \), where \( Y^*_t \) is the world output (in per capita terms). The demand for export of good \( i \in [0, 1] \) is assumed to be given by

\[ X_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} X_t, \]  

(6)

where \( \epsilon_p > 1 \) denotes the elasticity of substitution between varieties produced domestically and \( X_t \) is an aggregate export index. The latter is assumed to be given by

\[ X_t = \nu \left( \frac{P_{F,t}}{P_{H,t}} \right) Y^*_t \]  

(7)

= \nu S_t Y^*_t.

where \( S_t \equiv \frac{P_{F,t}}{P_{H,t}} \) denotes the terms of trade. In equilibrium, world output, \( Y^*_t \), equals world consumption, \( C^*_t \). We consider a symmetric steady state with \( S_t = 1 \) and thus \( X = \nu Y^*_t \).\(^8\)

**Monetary regime.** In order to represent the situation of a currency union in the international context, as in Galí and Monacelli (2016), we assume that the home country pegs the exchange rate indefinitely (and credibly) to the world currency. Letting \( e_t \equiv \log E_t \) denote the log nominal exchange rate, we assume without loss of generality

\[ e_t = 0, \forall t. \]  

(8)

Under this regime, that role of wage flexibility as a stability factor is going to be much less important as any change in domestic wages has a limited impact on employment. This is because the local interest rate cannot deviate from its international counterpart contingent securities at world level leads to a simple relationship linking domestic consumption with world output and the terms of trade.

\(^7\)For a full derivation, see Galí and Monacelli (2016) appendix or Chapter 8 of Galí (2015).

\(^8\)Moreover, given that in steady state \( C_F = \nu C \), and that \( C = C^* = Y^*_t \), it follows that trade is balanced at the symmetric steady state.
and thus accommodate adverse shocks in the economy. In this context, a flex sector may play a key role on stabilizing the economy - as we will delineate in the next section.

**Equilibrium.** We combine the optimality conditions for both the households and firms with the goods market clearing condition, \( Y_t(i) = C_t(i) + X_t(i) \), to solve a system of difference equations with 26 endogenous variables (lower cases denoting natural logarithms): \( y_t, y^n_t, \tilde{y}_t, c_t, c^n_t, \tilde{c}_t, n_t, n^d_t, \tilde{n}_t, a_t, z_t, i_t, w_t, \omega_t, \omega^n_t, \tilde{\omega}_t, \pi_{t, H}^p, \pi_{t, H}^w, \tilde{s}_t, s^n_t, \tilde{e}, \) and \( \tau_t \). We list the set of equations that characterize the equilibrium of the small open economy in Appendix B.a.\(^9\)

Two important equations characterizing the aggregate supply block are the domestic inflation equation

\[
\pi_{t, H}^p = \beta \mathbb{E}_t \{ \pi_{t+1, H}^p \} + \frac{\lambda_p \alpha}{1 - \alpha} \tilde{y}_t + \lambda_p \tilde{\omega}_t + \lambda_p \nu \tilde{e}_t, \quad (9)
\]

and the wage inflation equation

\[
\pi_{t}^w = \beta \mathbb{E}_t \{ \pi_{t+1}^w \} + \frac{\lambda_w \varphi}{1 - \alpha} \tilde{y} + \lambda_w \tilde{\varepsilon}_t - \lambda_w \tilde{\omega}_t. \quad (10)
\]

Both forward-looking functions are increasing in the output gap (\( \tilde{y} \)) and real wage gap (\( \tilde{\omega}_t \)). The domestic inflation equation also depends on the terms of trade gap (\( \tilde{s}_t \)), which is an important factor to accommodate shocks in the open economy setting (e.g., by the depreciation of the terms of trade). Finally, the wage inflation equation also depends on the consumption gap (\( \tilde{c}_t \)), as households care about the marginal rate of substitution between consumption and employment.

**Welfare function.** The final ingredient of the baseline characterization is the welfare function. Under the assumption of an efficient steady state, the average period utility losses of the small open economy’s representative household are given, up to a second-order approximation, by the following function

\[
L_{GM} \sim \left( \frac{1 - \nu}{2} \right) \left[ (1 + \phi) \text{var}(\tilde{n}_t) + \left( \frac{\epsilon_p}{\lambda_p (1 - \alpha)} \right) \text{var}(\pi_{t}^p) + \left( \frac{\epsilon_w}{\lambda_w} \right) \text{var}(\pi_{t}^w) \right], \quad (11)
\]

which is a linear combination of the variances of employment deviations (from the

\(^9\)See Galí (2010); Galí and Monacelli (2005, 2016) for further details.
steady state), \( \hat{n_t} \), price inflation, \( \pi^p_t \), and wage inflation, \( \pi^w_t \). The parameter \( \lambda_p \) corresponds to \( \lambda_p \equiv \frac{(1-\theta_p)(1-\beta \theta_p)}{\theta_p \theta \rho_p \rho \theta + \rho \theta \rho_p} \).

We note that the relative weight of the employment fluctuations in the loss function is increasing in \( \phi \). This is because larger values of this utility-curvature parameter amplify the effect of any given deviation from its efficient level. The weight of inflation fluctuations is increasing in the elasticity of substitutions among goods, \( \epsilon_p \), and the degree of price stickiness, \( \theta_p \) (which is inversely related to \( \lambda_p \)), since a greater stickiness amplifies the degree of price dispersion associated with any given deviation from zero inflation, and in \( \alpha \), which controls the shape of the production function. In the same way, the weight of wage inflation fluctuations is increasing in the elasticity of substitutions among different types of labor services, \( \epsilon_w \), in the degree of wage stickiness, \( \theta_w \) (which is inversely related to \( \lambda_w \)), since a greater wage stickiness amplifies the degree of wage dispersion associated with any given deviation from zero wage inflation. Finally, the term \( (1 - \nu) \) weights for the degree of openness in the economy.

Results. We are now ready to lay out the baseline argument - re-establishing Galí and Monacelli (2016) conclusion - based on the model calibration in Galí and Monacelli (2016), which we present in Table 1. In the face of an adverse shock, a reduction in domestic wage leads to a higher demand for exports in equilibrium through the terms of trade depreciation (which is called competitiveness channel) and to a higher employment (which is called endogenous policy channel). The wage reduction is therefore an important mechanism to deal with an adverse economic shock.

As in Galí and Monacelli (2016), wage reductions affect labor demand through the endogenous (interest rate) policy channel - as long as there is an active monetary authority influencing the interest rates. Whenever wages decline, this reduces the marginal cost of firms - suppressing the price inflation. In this scenario, if the Central Bank decides to lower the monetary policy rate, it can contain deflation. If the Central Bank decreases the monetary policy rate (induced by the initial wage reduction), it would then lead to an increase in labor demand and employment. On the contrary, if the Central Bank cannot take an action, as would be the case in a currency union, the labor demand and employment would not be affected by the initial wage reduction. Therefore, in this scenario, the benefits of higher wage flexibility (i.e., output stabilization) are likely to get

---

10 See Galí (2015) for a derivation of the welfare loss function of the small open economy with sticky prices and sticky wages.
offset by the costs of wage flexibility (i.e., higher wage volatility). Consequently, this result, an important finding from Gali and Monacelli (2016) challenges the conventional view that wage flexibility is universally desirable.

We observe Gali and Monacelli (2016)’s non-monotone wage-flexibility welfare effect in Figure 1, which describes the relationship between welfare losses and the degree of wage rigidity in a currency union with demand shocks. We plot the total welfare loss, $L_{GM}$, alongside the three components of the welfare loss function, namely, the welfare losses associated with employment deviations, $(1 + \varphi)\text{var}(\hat{n}_t)$, price inflation, $(\epsilon_p \lambda_p (1 - \alpha)) \text{var}(\pi_p^t)$, and wage inflation, $(\epsilon_w \lambda_w) \text{var}(\pi_w^t)$. We weigh all components by $(1 - \nu^2)$ to sum up the total welfare losses. Therefore, total welfare losses correspond to the vertical sum of the components in the figure. We express the welfare losses as a ratio to those under the setting with a degree of wage stickiness of 0.8 - the benchmark value for the degree of wage rigidity from Gali and Monacelli (2016), at which welfare losses equal to 1.

Figure 1 shows a hump-shaped relationship between the degree of wage rigidity and welfare losses. On the one hand, starting from a value of $\theta_w$ close to unity, a reduction in that parameter (i.e., higher wage flexibility) monotonically increases welfare losses. On the other hand, if wages are sufficiently flexible to begin with (i.e., $\theta_w$ is sufficiently low), a further increase in wage flexibility always leads to a contraction in welfare losses. Thus, in the context of a currency union an increase in wage flexibility may raise or lower welfare, depending on the initial degree of wage rigidity.

4 Two-Sector Models: Fixed- and Flexible Contract Sectors

In this section we extend the baseline Gali-Monacelli set-up that we re-capped in Section 3 into two-sector economies - with alternative fixed and flexible contract arrangements and explore the macro welfare effects of flex contracts.

4.a Fixed Contracts with Flexible Hours

We first consider a two-sector economy, where each sector has available only one type of labor contract: the fixed contract sector, with Calvo-style staggered wages but flexible hours, and the flex contract sector, with full-fledged wage and hour flexibility. Since as

---

11For the sake of simplicity, we only focus on demand shocks in this paper as technology shocks generate the same pattern.
**Figure 1:** Relationship between welfare losses and wage rigidity under a currency union for demand shocks. The case of one-sector model.

**Notes:** This figure replicates Figure 4 from Gali and Monacelli (2016). The figure shows the relationship between welfare losses and wage rigidity, $\theta_w$, under a currency union regime with demand shocks. We calculate the welfare losses by using the function $L_{GM} \sim \left(1 - \frac{\nu}{2}\right) \left(1 + \phi \right) \text{var}(\hat{n}_t) + \left(\frac{\epsilon_p}{\lambda_p(1 - \alpha)} \right) \text{var}(\pi^p_t) + \left(\frac{\epsilon_w}{\lambda_w} \right) \text{var}(\pi^w_t)$, which is a linear combination of the variances of employment deviations (from the steady state), $\hat{n}_t$, price inflation, $\pi^p_t$, and wage inflation, $\pi^w_t$. We plot the total welfare loss, $L_{GM}$, together with the three components of the welfare loss function, namely, the welfare losses associated to employment deviations, $(1 + \phi \text{var}(\hat{n}_t))$, price inflation, $(\epsilon_p/\lambda_p(1 - \alpha) \text{var}(\pi^p_t))$, and wage inflation, $(\epsilon_w/\lambda_w \text{var}(\pi^w_t))$. We weigh all components by $(1 - \nu)/2$ to get the total welfare losses. Therefore, total welfare losses correspond to the vertical sum of the components in the figure. We express the welfare losses as a ratio to those under a setting with degree of wage stickiness equaling to 0.8 (Gali-Monacelli benchmark), at which level the welfare losses equal to 1.

In the baseline model working-hours are flexible in both sectors, we are considering one margin of heterogeneity among the two sectors stemming from the heterogeneity in wage flexibility. We also assume that labor can be reallocated freely across firms in the same sector but cannot flow in-between sectors.

Our research question is whether the non-monotone effects of wage-flexibility survive in an economy which is partly made of employment relationships with fully flexible wages. From a theoretical perspective, on the one hand, wage flexibility in the fixed sector may become less relevant in terms of welfare losses as the economy’s adjustment can rely on the flex sector. On the other hand, wage flexibility may become even more important through general equilibrium interaction of hours-worked in the flex sector.
with rigidity of wages in the fixed sector, inducing wage flexibility to be relatively more desirable compared to an economy without a flex sector.

**Households.** The representative consumer derives utility from a composite good, supplies different types of labor to firms in different sectors. Importantly, income is pooled within the household, which thus acts as a risk-sharing mechanism. Subject to the sequence of budget constraints, the representative household maximizes

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \ln(C_t) - \sum_{s \in S} \frac{1}{1 + \phi} N_{s,t}^{1+\phi} \right) Z_t \right],$$

(12)

where $S = \{fixed, flex\}$. $N_{s,t}$ is the number of working-hours supplied to sector $s$.\(^{12}\)

The sequence of budget constraints of the household is given by

$$\int_0^1 P_{H_i}(i)C_{H,t}(i)di + P_{F,i}C_{F,t} + E_t\{Q_{t+1}D_{t+1} - \Lambda_t\} \leq D_t + \sum_{s \in S} W_s^t N_{s,t} - \Lambda_t,$$

(13)

for $t = 0, 1, 2, ..., $ where $S = \{fixed, flex\}$.\(^{13}\)

**Intermediate goods producers (firms).** A continuum of firms, indexed by $i \in [0, 1]$, are assumed to operate in the home economy: the firms in the fixed sector are located in the sub-interval $[0, sh)$ and the flexible sector firms are located in the sub-interval $[sh, 1]$. Firms in both sectors change prices based on the Calvo price-setting. Since we are not interested in studying price-setting heterogeneity in our model,\(^{14}\) we assume that the same fraction of firms change prices in each sector, $(1 - \theta_p)$. A typical domestic intermediate-firm produces a differentiated good using the same technology as in the baseline model. The production technology for intermediate producer $i$ in sector $s$ is thus given by

$$Y_{s,i} = A_i N_{s,i}^{1-\alpha},$$

(14)

where $N_{s,i}$ is the worked hours supplied for producer $i$ in the sector $s = \{fixed, flex\}$. $A_i$ is the technology shock - common to both sectors. The intermediate good producer faces two problems. Given the demand for its output and taking the prices of the production factors as given, she minimizes the cost of production. This yields a marginal

\(^{12}\)It is important to emphasize that this model considers one single labor decision: employment (or working-hours). In this extension, we assume that households only decide on work hours.

\(^{13}\)The pooling income assumption is important here because it excludes precautionary savings due to insurance purposes.

\(^{14}\)See for instance Carvalho et al. (2021); Singh and Beetsma (2018).
cost, for the individual firm, equaling to
\[
\Psi_s^i(i) \equiv \frac{1}{(1 - \alpha) A_t N_{s,t}(i)^{-\alpha}} \times W_s^t.
\]  

(15)

This characterization implies that the marginal cost, \(\Psi_{s,t}(i)\), equals to the inverse of the marginal productivity of labor times the cost of each labor unit.

**Final good producer.** The final good is used for consumption and is produced in two steps. First, using a Dixit-Stiglitz CES aggregation technology various (heterogeneous) intermediate outputs are combined into a sectoral product. Second, through a CES aggregation technology the two sectoral aggregates are combined into the final product of the economy,
\[
Y_t = \iota Y_{fixed,t}^{1-\eta} Y_{flex,t}^\eta,
\]  

(16)

where \(\iota\) is a constant. \(Y_{fixed,t}\) and \(Y_{flex,t}\) are the sectoral aggregate outputs, and the parameter \(\eta\) captures the weight of sector \(s\) in the final product. The price of one unit of the final product is then \(P_{H,t} = (P_{H,t}^{fixed})^\eta (P_{H,t}^{flex})^{1-\eta}\). The final output producers’ problem is to minimize the cost of production, resulting in the following demands for input from the fixed sector,
\[
Y_{fixed,t} = \eta \left( \frac{P_{fixed,t}}{P_{flex,t}} \right)^{\eta-1} Y_t,
\]  

(17)

and the flex sector
\[
Y_{flex,t} = (1 - \eta) \left( \frac{P_{fixed,t}}{P_{flex,t}} \right)^{\eta} Y_t.
\]  

(18)

Combining the previous two equations, we obtain the following relationship:
\[
\frac{Y_{fixed,t}}{Y_{flex,t}} = \frac{\eta}{1 - \eta} \left( \frac{P_{fixed,t}}{P_{flex,t}} \right)^{-1}.
\]  

(19)

The aggregate sector-level outputs are characterized as:
\[
Y_{fixed,t} = \left( \frac{1}{sH} \right)^{\eta} \int_0^{sH} Y(i) \left( \frac{\epsilon_{fixed,t}}{Y_{fixed,t}} \right)^{\eta-1} di
\]  

\(\epsilon_t = \).  

(20)
and
\[ Y_{\text{fixed},t} = \left( \frac{1}{1 - \text{sh}} \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \int_{\text{sh}}^{1} Y(i)_{\text{fixed},t}^\frac{\epsilon_p - 1}{\epsilon_p} dt, \]  

(21)

where \( Y(i)_{s,t} \) is the variety output by firm \( i \) in sector \( s = \{ \text{fixed, flex} \} \). The parameter \( \epsilon_p \geq 0 \) is the elasticity of substitution within a sector (i.e., in-between pairs of intermediate goods within a sector). The demand for each intermediate product is
\[ Y_{\text{fixed},t}(i) = \frac{1}{\text{sh}} \left( \frac{P_{\text{fixed},t}(i)}{P_{\text{fixed},t}} \right)^{\epsilon_p} Y_{\text{fixed},t}, \]  

(22)

and
\[ Y_{\text{flex},t}(i) = \frac{1}{1 - \text{sh}} \left( \frac{P_{\text{flex},t}(i)}{P_{\text{flex},t}} \right)^{-\epsilon_p} Y_{\text{flex},t}, \]  

(23)

where \( Y_{\text{fixed},t}(i) \) and \( Y_{\text{flex},t}(i) \) are the demand schedules faced by intermediate good producers. We close this part by defining the unit price charged by producer \( i \) in sector \( s \), and the ideal price index in sector \( s \):
\[ p_{\text{fixed},t}^\text{H} = \left[ \frac{1}{\text{sh}} \int_0^{\text{sh}} P_{\text{fixed},t}(i)^{1-\epsilon_p} di \right]^\frac{1}{1-\epsilon_p}. \]  

(24)

and
\[ p_{\text{flex},t}^\text{H} = \left[ \frac{1}{1 - \text{sh}} \int_{\text{sh}}^{1} P_{\text{flex},t}(i)^{1-\epsilon_p} di \right]^\frac{1}{1-\epsilon_p}. \]  

(25)

**Price setting behaviour.** The price setting is going to be similar to the one in the baseline model except for the definition of the real wage. We first illustrate the pricing behaviour in the fixed sector. When choosing the price \( p_{\text{fixed},t}^\text{H} \), each firm seeks to maximize its value, subject to the sequence of demand constraints. The resulting optimality condition around the zero-inflation steady state yields
\[ p_{\text{fixed},t}^\text{H} = (1 - \beta \theta_p) \sum_{k=0}^{\infty} (\beta \theta_p)^k E_t \{ \psi_{t+k}^\text{fixed} \}, \]  

(26)

where \( p_{\text{fixed},t}^\text{H} \) is the log of the price newly set by firms adjusting prices in period \( t \), \( \log(p_{\text{fixed},t}^\text{H}) \); \( \psi_{t+k}^\text{fixed} \) is the log of the nominal marginal cost, \( \log(\psi_{t+k}^\text{fixed}) \).\footnote{Although the price decision of the fixed sector is disconnected to the price decision of the flex sector at nominal level, we will see later that firms in both sectors take in consideration the general price of the home economy (weighted average of the price-decisions of both sectors) to decide the prices at sector level.} Firms in the fixed sector adjust their prices as a function of the desired markup over a weighted av-
verage of current and future nominal marginal costs in the fixed sector. By calculating
the marginal cost of firms, we can express the pricing equation
\[
\bar{p}_{H,t}^{\text{fixed}} = (1 - \beta \theta_p) \sum_{k=0}^{\infty} (\beta \theta_p)^k E_t \{ p_{H,t+k}^{\text{fixed}} - \Theta \mu_t^{\text{fixed},p} \},
\]
(27)

where \( \mu_t^{\text{fixed},p} \equiv \mu_t^{\text{fixed},p} - \mu_t^{\text{fixed},p} \) is the deviation between the average and desired
markups, with \( \Theta \equiv \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon} \in (0, 1] \) and
\[
\mu_t^{\text{fixed},p} \equiv p_{H,t}^{\text{fixed}} - \Psi_t^{\text{fixed}}.
\]
(28)

Since we want to derive the real marginal cost, we need to find an expression relating
\( p_{H,t} \) and \( p_{H,t}^{\text{fixed}} \). Combining the relative price index, \( \Gamma_t = p_{H,t}^{\text{fixed}} - p_{H,t}^{\text{flex}} \), together with
the aggregate price index, we obtain
\[
p_{H,t}^{\text{fixed},p} = p_{H,t} + (1 - \eta) \Gamma_t.
\]
(29)

This equation makes visible that price-decisions in both sectors are connected through
the aggregate price index \( p_{H,t} \). Combining equations (28) and (29), we can write
\[
\mu_t^{\text{fixed},p} \equiv p_{H,t} - \Psi_t^{\text{fixed}} + (1 - \eta) \Gamma_t.
\]
(30)

The additive term in the real marginal cost of the fixed sector, \( (1 - \eta) \Gamma_t \), is the only dif-
fERENCE compared to what we obtained in the baseline model. Finally, we characterize
the inflation equation for the fixed sector as:
\[
\pi_{H,t}^{\text{fixed},p} = \beta E_t \{ \pi_{H,t+1}^{\text{fixed},p} \} + \lambda_p \frac{\alpha}{1 - \alpha} \tilde{y}_{t}^{\text{fixed}} + \lambda_p \tilde{\alpha}_{t}^{\text{fixed}} + \lambda_p v \tilde{s}_{t} - (1 - \eta) \tilde{\Gamma}_t.
\]
(31)

This price inflation equation states that the inflation in the fixed sector is increasing in
inflation expectations, output gap in the fixed sector, real wage gap in the fixed sector,
terms of trade gap weighted by the level of openness of the economy, and in the price
deviations across sectors. For the interpretation of the results, it is crucial to under-
stand that the term \( (1 - \eta) \Gamma_t \) means that firms in the fixed sector would like to keep
their prices in line with other firms’ prices - including those in the flex sector. This
connection in-between the two sector is the cross-sectoral strategic complementarity
(Carvalho et al., 2021).

Next we consider the pricing behaviour in the flex sector. While the derivation of the
price-inflation equation follows the same logic as the one of the fixed sector, we notice that the real wage in the flex sector is set optimally, as \( w^\text{flex}_t - p_t = c_t + \varphi n^\text{flex}_t \). Therefore, there is no real-wage-gap term in the price-inflation equation in the flex sector. We provide the derivation of the price-inflation in Appendix A.a, and refer here to the price inflation for the flex sector, which is characterized as

\[
\pi^\text{flex,p}_{H,t} = \beta \mathbb{E}_t \left\{ \pi^\text{flex,p}_{H,t+1} \right\} + \left( \frac{\lambda_p (1 + \varphi)}{1 - \alpha} \right) g^\text{flex}_t - 2 \lambda_p \bar{s}_t. \tag{32}
\]

**Wage setting.** Wages in the flex sector are always set optimally, whereas as in the baseline model, the wage inflation equation in the fixed sector is given by

\[
\pi^\text{fixed,\bar{w}}_t = \beta \mathbb{E}_t \left\{ \pi^\text{fixed,\bar{w}}_{t+1} \right\} + \frac{\lambda_\bar{w} \varphi}{1 - \alpha} g^\text{fixed} + \lambda_\bar{w} \bar{c}_t - \lambda_\bar{w} \bar{\Omega}^\text{fixed}_t, \tag{33}
\]

where \( \lambda_\bar{w} \equiv \frac{(1 - \theta_\bar{w})(1 - \beta \theta_w)}{\theta_\bar{w}(1 + \epsilon_\bar{w} \varphi)} \). Importantly, this wage inflation equation depends on the level of wage flexibility, \( \lambda_\bar{w} \), and the output gap, which is connected to the flex sector thorough general equilibrium interactions, as we delineate when discussing the results.

**Welfare function.** The welfare loss function for the two-sector model, which we derive in Appendix A.b, is expressed as a linear combination of the variance of the employment gap, price inflation, and wage inflation:

\[
\hat{L} \sim \left( \frac{1 - \varphi}{2} \right) \left[ (1 + \varphi) \text{var}(\bar{n}_t) + \left( \frac{\epsilon_p}{\lambda_p (1 - \alpha)} \right) \text{var}(\pi^p_t) + \eta \left( \frac{\epsilon_\bar{w}}{\lambda_\bar{w}} \right) \text{var}(\pi^\text{fixed,\bar{w}}_t) \right]. \tag{34}
\]

We have now a welfare loss function that depends on employment and price inflation volatility that is determined in both sectors and on the wage inflation, which is only determined in the fixed sector and thus, weighted by \( \eta \). The welfare loss function, \( \hat{L} \), encompasses several other welfare loss specifications.\(^{16}\) If we assume a one-sector economy with only the fixed sector and sticky wages, i.e., \( \eta = 1 \) and \( \theta_w < 1 \), the expression for the welfare losses collapses to the one in the baseline one-sector model, i.e., Galí and Monacelli (2016). If we assume a one-sector economy with only a fixed sector and wage flexibility, i.e., \( \eta = 1 \) and \( \theta_w = 1 \), the expression for the welfare losses becomes the same as in Galí and Monacelli (2005). Finally, if we assume a one-sector model with a fixed sector in a closed-economy with \( \theta^\text{fixed}_p \neq \theta^\text{flex}_p \), then the expression for the welfare

\(^{16}\)It is important to keep in mind two key features. First, the welfare loss coming from the fixed sector is weighted by \( \eta \), the share of the fixed sector. Second, price inflation, \( \pi^p_t \), is a function of the terms of trade, \( s_t \), and price inflation in the home economy, \( \pi^p_{H,t} \), which is \( \pi^p_{H,t} \equiv \eta \pi^\text{fixed,\bar{w}}_{H,t} + (1 - \eta) \pi^\text{flex,p}_{H,t} \).
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>ϕ</td>
<td>Curvature of labor disutility</td>
<td>2.2</td>
</tr>
<tr>
<td>α</td>
<td>Index of decreasing return to labor</td>
<td>0.26</td>
</tr>
<tr>
<td>εₚ</td>
<td>Elasticity of substitution (goods)</td>
<td>3.8</td>
</tr>
<tr>
<td>εₚₜ</td>
<td>Elasticity of substitution (labor)</td>
<td>3.8</td>
</tr>
<tr>
<td>θₚ</td>
<td>Calvo index of price rigidities</td>
<td>0.8</td>
</tr>
<tr>
<td>θₚₜ</td>
<td>Calvo index of wage rigidities</td>
<td>0.8</td>
</tr>
<tr>
<td>v</td>
<td>Openness</td>
<td>0.3</td>
</tr>
<tr>
<td>ρₐ</td>
<td>Autocorrelation technology shocks</td>
<td>0.9</td>
</tr>
<tr>
<td>ρₑ</td>
<td>Autocorrelation demand shocks</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Notes: The table reports the calibration of the baseline one-sector model and two-sector models based on Gali and Monacelli (2016) (Table 1, pp. 3839).

losses becomes equivalent to that of Singh and Beetsma (2018).

Results. We calibrate the two-sector model as in the baseline one-sector model. Table 1 reports the calibrated values and Appendix B.b lists the set of conditions characterizing the equilibrium of the extended model. Figure 2 repeats the exercise we presented in Figure 1 for the two-sector model but considering a range of values for the size of the flex sector, 1 − η. Some intuitive results are easily observable. First, we replicate the Gali and Monacelli (2016) effect when the flex sector size is small enough, namely, the hump-shaped relationship between the degree of wage rigidities and welfare losses. Second, we observe that the hump-shaped relationship holds for almost any size of the flex-sector, except when the flex sector represents 100% of the economy.

This brings us naturally to the question of why the flex-sector does not break the relationship between the degree of wage stickiness and welfare losses. This is because the only difference between the two-sector model and the baseline model is the measure of workers resetting nominal wages in a given period of time. While in the baseline one-sector model this measure equals to 1 − θₚₜ, the measure of workers resetting wages in the two-sector model is 1 − ηθₚₜ ≥ 1 − θₚₜ for {η, θₚₜ} ∈ [0, 1]. This is why we do not observe too much of a change in the predictions of the model when we compare the current specification of the two-sector model against the baseline specification. As a

17 The total number of workers resetting nominal wages is η(1 − θₚₜ) + (1 − η) = 1 − ηθₚₜ.
Figure 2: Relationship between welfare losses, wage rigidity, and the share of the flex sector under a currency union for demand shocks. The case of two-sector model with flexible hours in both sectors.

Notes: The figure shows the relationship between welfare losses, wage rigidity, $\theta_w$, and the share of the flex sector, $1 - \eta$, under a currency union for demand shocks. We calculate the welfare losses by using the function
\[
\hat{L} \sim \left( \frac{1 - \eta}{\theta_w} \right) \left[ (1 + \varphi)\text{var}(\hat{n}_t) + \left( \frac{\varpi}{\lambda_{w1-\eta}} \right) \text{var}(\pi^p_t) + \eta \left( \frac{\varpi}{\lambda_{w1-\eta}} \right) \text{var}(\pi^{\text{fixed},w}_t) \right],
\]
which is a linear combination of the variances of worked hours deviations (from the steady state), $\hat{n}_t$, price inflation, $\pi^p_t$, and wage inflation, $\pi^{\text{fixed},w}_t$. We plot the total welfare losses, $\hat{L}$. We also express the welfare losses as a ratio to those under a setting with degree of wage stickiness equal to 0.8.

result, we can consider the two-sector model with flexible hours in the fixed sector as a one-sector model but with higher wage renegotiation frequency, $1 - \eta \theta_w$, compared to the baseline.

Nonetheless, the fundamental reason for why the flex sector has a limited effect on the economy is because firms in the fixed sector adjust working-hours as they please. Yet, there is plenty of evidence that the adjustment through working-hours may be limited in the fixed-sector. Fixed long-term contracts are difficult to renegotiate and working-hours are not easy to modify. For instance, in a recent paper, Borowczyk-Martins and Lalé (2019) find that firms adjust working-hours through lumpy transitions of workers between full-time and part-time work. Therefore, it is empirically not plausible to assume that the working-hours adjustment is equal to each other in fixed- and flex sectors. For instance, Grajales-Olarte et al. (2021) study the role of different types of labor contracts in explaining rigid wages in the Netherlands. They find that (1) flexible-hour
labor contracts exhibit substantially more wage volatility; and (2) working-hours of the flex-hour labor contracts are more likely to reduce during a downturn compared to all other contracts.

Therefore, in the next section we further extend the two-sector model by assuming that the working-hours are fixed in the fixed sector while they vary in the flex sector. Based on this further extension we develop important insights regarding the macro welfare consequences of wage flexibility and flex contracts.

4.b Fixed Contracts with Fixed Hours

Let us now assume that working-hours are fixed in the fixed sector and cannot respond to shocks while they remain flexible in the flex sector as we covered in the previous section. Appendix B.c lists the set of conditions characterizing the equilibrium of this two-sector model alternative. Figure 3 depicts the results, and delivers three main findings. First, Figure 3 shows that in the case of a negligibly small flex sector size (1%) and fixed-hours in the fixed sector, we take Galí and Monacelli (2016)’s argument to an extreme. Since working-hours in the fixed sector are not responsive to wage reductions, we observe that higher wage flexibility increases welfare losses rapidly: while going from full wage rigidity to a mediocre level wage flexibility raises welfare losses from 0.7 to 1.2 in the baseline one-sector model (by 71%, see Figure 1), we observe that, for the same level of a change in wage rigidity in the current two-sector model, the welfare losses go from 0.1 to 1.8 (by 1600%).

As a key and novel finding we note that the flex sector makes wage flexibility in the fixed sector more important for the macro welfare - reducing the desirability of the wage rigidity in the fixed sector. As we raise the relative size of the flex sector, $1 - \eta$, the impact of wage flexibility on welfare changes from a hump-shaped pattern into a monotone relationship - altering the conclusion by Galí and Monacelli (2016). While wage flexibility in the fixed sector cannot affect working-hours in the fixed sector, it has a general equilibrium impact on hours-worked of the flex sector. This mechanism prevails in the following way: Higher wage flexibility in the fixed-sector lowers the marginal cost of firms in the fixed sector during a downturn and reduces prices of those firms in the event of an adverse shock. This channel in turn positively influences the working-hours in the flex sector. We can see this by combining the output relationship between two sectors

$$n_t^{\text{flex}} = \frac{1}{1 - \alpha} (y_t^{\text{flex}} - y_t^{\text{fixed}}) \quad (35)$$
**Figure 3:** Relationship between welfare losses and wage rigidity under a currency union for demand shocks. The case of two-sector model with fixed hours in the fixed sector.

**Notes:** The figure shows the relationship between welfare losses, wage rigidity, $\theta_w$, and the share of the flex sector, $1-\eta$, under a currency union for demand shocks. We assume that the employment is fixed in the fixed sector. We calculate the welfare losses by using the function $\hat{L} \sim \left(\frac{1+\phi}{2}\right)\left(\frac{1}{1-\alpha}\right)^{\alpha}\left(\frac{1}{\lambda}\right)^{\beta}\hat{n}_t\left[\var(\pi_{p}^{t}) + \eta\left(\frac{\lambda}{\var(\pi_{f}^{fixed, w})}\right)\right]$ which is a linear combination of the variances of worked hours deviations (from the steady state), $\hat{n}_t$, price inflation, $\pi_{p}^{t}$, and wage inflation, $\pi_{f}^{fixed, w}$. We plot the total welfare losses, $\hat{L}$. We also express the welfare losses as a ratio to those under a setting with degree of wage stickiness equal to 0.8.

with the price relation between sectors to obtain

$$n_{t}^{flex} = \frac{1}{1-\alpha}\left(p_{H,t}^{flex} - p_{H,t}^{fixed}\right),$$

where $n_{t}^{flex}$ corresponds to the log of working-hours in the flex sector. Therefore, the two-sector model - with fixed hours in the fixed sector - creates a general equilibrium connection between the nominal rigidities in the fixed sector and output of the flex sector. This connection induces wage flexibility to be welfare reducing through the effect on output (i.e., worked hours in the flex sector), because wage flexibility in the fixed sector helps to stabilize the output in the flex sector, thus the aggregate output. We can explore this relationship further by studying the welfare losses exclusively associated with working-hour volatility and its relationship with wage rigidity in the fixed sector. We calculate the share of the welfare losses associated exclusively with working-hours
volatility as

\[ \hat{L}_{\hat{n}_t} = \frac{(1 + \varphi)\text{var}(\hat{n}_t)}{(1 + \varphi)\text{var}(\hat{n}_t) + \left(\frac{\epsilon_p}{\lambda_p(1 - \alpha)}\right)\text{var}(\pi^p_t) + \eta\left(\frac{\epsilon_w}{\lambda_w}\right)\text{var}(\pi^{\text{fixed},w})}. \] (37)

Figure 4 depicts \( \hat{L}_{\hat{n}_t} / \hat{L} \) for different degrees of wage stickiness. We observe that even when the size of the fixed sector is small (i.e., 90% flex sector), higher wage flexibility in the fixed sector rapidly reduces welfare losses. To compensate for wage stickiness in the fixed sector, the flex sector uses working-hours to cope with shocks. For that reason, higher wage flexibility reduces working-hours volatility in the flex sector. This high volatility of working-hours has a strong empirical support. As noted earlier, flex-hour contracts are relatively more volatile in both wages and working-hours, as documented by Grajales-Olarte et al. (2021).

All in all, these results are in line with flex-hour contracts accommodating aggregate shocks. Additionally, we obtain the important novel result that wage flexibility in the fixed sector has a more beneficial role in an economy with a large flexible sector size. Therefore, different from the findings by Galí and Monacelli (2016), we show that even in a currency union small open economy, wage flexibility may reduce welfare losses as long as a large flex sector is present in that economy.

4.c Fixed Hours in Both Fixed and Flex Contracts

We continue our analysis by studying whether the welfare contribution of the flex sector results from the working-hour flexibility or from wage flexibility. In order to address this question, we consider a final extension of the two-sector model by assuming that the working-hours are fixed in both sectors. This specification gives us a two-sector model with fixed working-hours overall but with wage rigidities in the fixed sector and full wage flexibility in the flex sector. Appendix B.d provides the set of conditions characterizing the equilibrium of this extension.

In this particular case working-hours is not responsive to wage reductions in both sectors and thus, we should expect the non-monotone effect of high wage flexibility on aggregate welfare losses to prevail. We confirm this quantitative effect in Figure 5, which depicts the relationship between welfare losses and wage rigidities for different values of the flex sector, \( 1 - \eta \). We observe a strong hump-shaped relationship between wage rigidities and welfare losses, reaching back to Galí and Monacelli (2016)
Figure 4: Relationship between welfare losses associated to the employment gap and wage rigidity under a currency union for demand shocks. The case of two-sector model with fixed hours in the fixed sector.

Notes: The figure shows the relationship between welfare losses associated to the employment gap, wage rigidity, \( \theta_w \), and the share of the flex sector, \( 1 - \eta \), under a currency union for demand shocks. We assume that the employment is fixed in the fixed sector. We calculate the welfare losses by using the function
\[
\hat{L} \sim \left( \frac{1 - \eta}{2} \right) \left[ (1 + \varphi) \text{var}(\hat{n}_t) + \left( \frac{\epsilon_p}{\pi_t (1 - \eta)} \right) \text{var}(\pi^p_t) + \eta \left( \frac{\epsilon_w}{\bar{n}_w} \right) \text{var}(\pi^w_{\text{fixed}}) \right]
\]
which is a linear combination of the variances of worked hours deviations (from the steady state), \( \hat{n}_t \), price inflation, \( \pi^p_t \), and wage inflation, \( \pi^w_{\text{fixed}} \). We plot the total welfare losses, \( \hat{L} \). We also express the welfare losses as a ratio to those under a setting with degree of wage stickiness equal to 0.8. We then calculate the ratio \( \hat{L}_n / \hat{L} \), where
\[
\hat{L}_n \sim \left( \frac{1 - \eta}{2} \right) \left[ (1 + \varphi) \text{var}(\hat{n}_t) \right],
\]
for different values of \( 1 - \eta \).

Conclusion. This result confirms that the “gains from higher wage flexibility in the fixed sector” induced by a large flex sector, results from the ability of firms adjusting their working-hours in the flex sector and not due the ability to adjust wages in that sector.

4.d Optimal Size of the Flex Sector

We conclude our analysis by studying the optimal size of the flex sector in the version of the two-sector model with fixed working-hours in the fixed sector. Figure 6 (a) shows the relationship between welfare losses (and their components) and the share of the flex sector in the economy. We assume the baseline value for wage rigidity of \( \theta_w = 0.8 \) and obtain a non-monotone relationship between welfare losses and the relative size of the flex sector. On the one hand, starting from a one-sector model with only the fixed sector, an increase in the size of the flex sector decreases welfare losses. On the other
Figure 5: Relationship between welfare losses and wage rigidity under a currency union for demand shocks. Two-sectors model with worked hours fixed in both sectors.

Notes: The figure shows the relationship between welfare losses, wage rigidity, \( \theta_w \), and the share of the flex sector under a currency union for demand shocks. We calculate the welfare losses by using the function
\[
L' \sim \left( \frac{1-\nu}{\nu} \right) \left[ \frac{\epsilon_p}{\lambda_p(1-\alpha)} \right] \text{var}(\pi^p_t) + \eta \left( \frac{\epsilon_w}{\lambda_w} \right) \text{var}(\pi^\text{fixed}_w) \]
which is a linear combination of the variances of price inflation, \( \pi^p_t \), and wage inflation, \( \pi^\text{fixed}_w \). We plot the total welfare losses, \( L' \). We also express the welfare losses as a ratio to those under a setting with degree of wage stickiness equal to 0.8.

hand, if the flex sector is large enough to start with (around 20%), a further increase in the share of flex sector increases welfare losses. This is because large fluctuations in consumption (i.e., working hours gap), associated with a higher flexibility, are costly for households. We confirm this result by investigating the components of the welfare losses and observing that while wage inflation losses decrease with a larger flex sector size, the losses associated with the volatility of working-hours and price inflation grow exponentially with the size of the flex sector. This is a natural implication of the larger volatility implied by the larger flex sector size.

Next we show that the optimal size of the flex sector depends on the wage rigidity level in the fixed sector. As shown in the previous subsections, there is a close relationship between the welfare effects of fixed sector’s wage flexibility and the size of the flex sector. Figure 6 (b) depicts the relationship of welfare losses with the flex sector size for three different values of wage rigidity: (i) Low wage-rigidity case (or high wage
**Figure 6:** Relationship between welfare losses and the size of the flex sector. Two-sectors model with employment fixed in the fixed sector.

(a) Optimal flex sector with $\theta_w = 0.8$

(b) Optimal flex sector for different $\theta_w$

**Notes:** The figure shows the relationship between welfare losses and the share of flex sector in the economy under a currency union for demand shocks. Figure (a) shows the relationship between welfare losses (and their components) and the share of flex sector for a wage rigidity equal to $\theta_w = 0.8$. Figure (b) shows the relationship between welfare losses and the share of flex sector for different values of $\theta_w$. We calculate the welfare losses by using the function $\hat{L} \sim \left( \frac{1}{2} \right) \left( 1 + \nu \right) \text{var}(\hat{n}_t) + \left( \frac{\epsilon_p}{\lambda_p (1 - \alpha)} \right) \text{var}(\pi_p^f) + \eta \left( \frac{\epsilon_w}{\lambda_w} \right) \text{var}(\pi_{\text{fixed},w}^f)$ which is a linear combination of the variances of worked hours deviations (from the steady state), $\hat{n}_t$, price inflation, $\pi_p^f$, and wage inflation, $\pi_{\text{fixed},w}^f$. For all figures, we express the welfare losses as a ratio to those under a setting with share of the flex sector equal to 0.2. Therefore, the welfare losses is equal to one when the share of the flex sector is 0.2.

Flexibility case), $\theta_t = 0.05$; (ii) Baseline case, $\theta_t = 0.8$; and (iii) High wage-rigidity case, $\theta_t = 0.95$. This analysis reveals that for the extreme levels of rigid wages in the fixed sector, increasing the flexible sector size does not raise macro welfare. For the case of highly flexible wages in the fixed sector ($\theta_t = 0.05$), the fixed sector can accommodate adverse shocks by itself without the need of a sector with large volatility of working-hours in the flex sector. For the case of highly rigid wages in the fixed sector ($\theta_t = 0.95$), the induced consumption volatility for flex sector workers turns out to be so high, such that due to this consumption volatility the presence of a flex sector is not desirable. For interim values of wage rigidities there is an optimal size of the flex sector.

## 5 Conclusion

This paper has analyzed the macro-welfare gains from flex labor contracts for a small open economy in a currency union. The rise of flex contracts, especially that of flex-hour labor arrangements, among EMU countries has been a prevalent feature of these
economies over the last decade. Motivated with this stylized fact, we generalize the currency union framework by Galí and Monacelli (2016), who show that in a currency union the welfare implications of wage flexibility could be non-monotone.

We incorporate into Galí and Monacelli (2016)’s model two sectors with differing degrees of flexibility of contractual arrangements. The highlighted result from our analysis shows that an increase in wage flexibility would monotonically increase welfare as long as there is a large flex sector in the economy - altering the conclusion by Galí and Monacelli (2016). This is because higher wage flexibility in the fixed sector helps to reduce the high-volatility of the working-hours in the flex sector. However, we also find that a large flex sector size may not be desirable. Flex contracts are costly for households because these contracts imply a large volatility of income and thus consumption. This policy relevant result implies that a high degree of contractual flexibility could be desirable for many countries in the context of EMU provided that the number of workers under flex arrangements does not exceed a threshold. Under our baseline parameterization, a flex sector larger than 20% reduces welfare.
6 Bibliography


APPENDIX

A Solving the model

A.a Price setting behavior

Fixed sector: The average nominal marginal cost is

\[
\Psi_t = \frac{W_{t\text{fixed}}(1 + \tau_t)}{(1 - \alpha)A_t(N_{t\text{fixed}})^{\alpha}},
\] (A38)

and the average price markup is

\[
\mu_{t\text{fixed}} = p_{H,t} - \psi_{t\text{fixed}} + (1 - \eta)\Gamma_t
\]

with \( \psi_t = \log \Psi_t \).

Therefore, the average price markup for the fixed sector is given by

\[
\mu_{t\text{fixed},p} = p_{H,t} - \psi_{t\text{fixed}} - w_{t\text{fixed}} - \log(1 - \alpha) + a_t - \alpha n_{t\text{fixed}} - \alpha - \alpha \tilde{\omega}_{t\text{flex}} - \tilde{\psi}_{t\text{flex}} - \tilde{\psi}_{s}\tilde{t} + (1 - \eta)\tilde{\Gamma}_t.
\]

(A40)

where \( \omega_{t\text{fixed}} = w_{t\text{fixed}} - p_t \). Hence,

\[
\mu_{t\text{fixed},p} - \mu_{t\text{fixed}} = -\frac{\alpha}{1 - \alpha} \tilde{y}_{t\text{flex}} - \tilde{\omega}_{t\text{flex}} - \tilde{\psi}_{t\text{flex}} - \tilde{\psi}_{s}\tilde{t} + (1 - \eta)\tilde{\Gamma}_t.
\]

(A41)

Therefore, we have that

\[
\pi_{t\text{fixed},H,t} = \beta \mathbb{E}_t \{ \pi_{t\text{fixed},H,t+1} \} - \lambda_p (\mu_{t\text{fixed},p} - \mu_{t\text{fixed}})
\]

where \( \lambda_p = \frac{(1 - \theta_p)(1 - \beta_p)}{\theta_p(1 - \alpha + \alpha \gamma_p)} \).

Flex sector: The only particular difference with the derivation of the marginal cost in the flex sector is that we can use

\[
w_{t\text{flex}} - p_t = c_t + q n_{t\text{flex}}^f,
\]

(A42)
which introduces the consumption variable in the equation. We use then the following relationships

\[ y_t = (1 - v)c_t + v(2 - v)\zeta s_t + vy_t^* \tag{A43} \]

and

\[ c_t = y_t^* + (1 - v)s_t + z_t, \tag{A44} \]

where we can get a relationship between \( c_t \) and \( y_t \). Introducing this relationship into the real wage in the flex sector and using \( n_{flex} = \frac{y_{flex} - a_t}{1 - \alpha} \) we get

\[ w_{t}^{flex} - p_t = y_t + s_t v(1 - v - \zeta(2 - v)) + z_t + \frac{\varphi y_{flex} - a_t}{1 - \alpha}. \tag{A45} \]

We need again an expression for \( y_t \). Note that the demand for input from the flex sector is

\[ Y_{flex} = (1 - \eta)\Gamma_t^\eta Y_t, \tag{A46} \]

and the log form is (ignoring the constant)

\[ y_{flex} = \eta \Gamma_t + y_t, \tag{A47} \]

and thus

\[ y_t = y_{flex} - \eta \Gamma_t. \tag{A48} \]

The real wage in the flex sector is then

\[ w_{t}^{flex} - p_t = y_{flex} - \eta \Gamma_t + s_t \vartheta + vz_t + \frac{\varphi y_{flex} - a_t}{1 - \alpha} - \frac{\varphi a_t}{1 - \alpha}, \tag{A49} \]

where \( \vartheta = (1 - v - \zeta(2 - v)) \). Next, we derive the marginal cost in the flex sector. The average nominal marginal cost is

\[ \Psi_t = \frac{W_{t}^{flex}(1 + \tau_t)}{(1 - \alpha)A_t(N_{t}^{flex})^{-\alpha}}, \tag{A50} \]

and the average price markup is \( \mu_{t}^{flex,p} = p_{H,t} - \psi_{t}^{flex} - \eta \Gamma_t \). Therefore, the average price markup for the flex sector is given by

\[ \mu_{t}^{flex,p} = p_{H,t} - \tau_t - w_{t}^{flex} - \log(1 - \alpha) + a_t - an_{t}^{flex} - \eta \Gamma_t \]

\[ = -(w_{t}^{flex} - p_t) - vs_t - \tau_t - \log(1 - \alpha) + a_t - an_{t}^{flex} - \eta \Gamma_t \]

\[ = \frac{1 + \varphi y_{t}^{flex}}{1 - \alpha} - s_t \vartheta(\vartheta - 1) - vz_t + \frac{1 + \varphi a_t}{1 - \alpha} - \tau_t - \log(1 - \alpha). \tag{A51} \]
Hence,
\[
\mu_{fixed,t}^p - \mu_{fixed,t}^p = -\left(\frac{1 + \varphi}{1 - \alpha}\right)\tilde{y}_t^{flex} - \bar{v}_t(\bar{v} - 1).
\] (A52)

Therefore, we have that
\[
\pi_{flex,t}^{H,t} = \beta E_t\{\pi_{flex,t+1}^{H,t}\} - \lambda_p(\mu_{flex,t}^{p} - \mu_{flex,t}^{p})
\]
\[
= \beta E_t\{\pi_{flex,t+1}^{H,t}\} - \lambda_p\left(\frac{1 + \varphi}{1 - \alpha}\right)\tilde{y}_t^{flex} + \bar{v}_t(\bar{v} - 1)
\] (A53)

The natural level of a variable, namely, its equilibrium value under flexible prices and wages is going to be identical between sectors, which converts the two-sectors model into a one-sector model and thus, identical to the one derived by Galí and Monacelli (25).

**Wage setting.** The baseline wage inflation equation is
\[
\pi_{fixed}^{w,t} = \beta E_t\{\pi_{fixed}^{w,t+1}\} + \lambda_w(\mu_{fixed}^{w,t} - \mu_{fixed}^{w,t}),
\] (A54)

where \(\pi_{fixed}^{w,t} \equiv w_{t}^{fixed} - w_{t-1}^{fixed}\) is wage inflation, \(\mu_{fixed}^{w,t} = w_t - p_t - c_t - \varphi n_t^{fixed}\), denotes the log average wage markup, and \(\lambda_w \equiv \frac{(1-\varphi)\beta}{\varphi(1+cw\varphi)}\). We can write the average wage markup as
\[
\mu_{fixed}^{w,t} \equiv \omega_{fixed,t} - (c_t + \varphi n_t^{fixed}).
\] (A55)

Thus, in deviations from the natural equilibrium, it becomes
\[
\mu_{fixed}^{w,t} - \mu_{fixed}^{w,t} = \tilde{\omega}_{t}^{fixed} - \tilde{c}_t - \frac{\varphi}{1 - \alpha}\tilde{y}_t^{fixed},
\] (A56)

where \(\tilde{\omega}_{t}^{fixed} \equiv \omega_{t}^{fixed} - \omega_{n,t}^{fixed}\), \(\tilde{y}_{t}^{fixed} \equiv y_{t}^{fixed} - y_{n,t}^{fixed}\), and \(\tilde{c}_t \equiv c_t - c_{n,t}\). We can thus rewrite the wage inflation equation as
\[
\pi_{fixed}^{w,t} = \beta E_t\{\pi_{fixed}^{w,t+1}\} + \frac{\lambda_w\varphi}{1 - \alpha}\tilde{y}_t^{fixed} + \lambda_w\tilde{c}_t - \lambda_w\tilde{\omega}_t^{fixed}.
\] (A57)

### A.b Derivation of the Utility-Based Loss Function

We linearize around a steady state - with flexible prices and wages - in which shocks are absent. We set subsidies to eliminate the monopolistic competition distortions, and thus, the gross markups in the two sector are equal to one. The expression for the period
$t$ utility is

$$U(C_t, N_t) = \ln(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi}. \quad (A58)$$

Taking a second-order approximation of period $t$ utility, evaluating it at steady state, and then neglecting the third - and higher order terms, we get

$$U_t - \bar{U} = \hat{c}_t - (\hat{h}_t + \frac{1+\varphi}{2}\hat{h}_t^2) + \mathcal{O}(||z^3||), \quad (A59)$$

where $\bar{U}$ is the utility evaluated at the steady state; $\mathcal{O}(||z^3||)$ corresponds to the third - and higher order terms. The objective is to write the expression in terms of the output gap and both price and wage inflation. We start with the term $\hat{c}_t$. Since $\hat{y}_t = \hat{c}_t$ and $y^*_t = c^*_t$, we have that

$$\hat{c}_t = \hat{y}_t + y^*_t. \quad (A60)$$

We now focus on the second term of equation (A59). The labor-market clearing condition is

$$N_t = N_{fixed,t} + N_{flex,t}, \quad (A61)$$

and we have that

$$N_{fixed,t} = \left(\frac{Y_{fixed,t}}{A_t}\right)^{\frac{1}{\gamma}} D_{fixed,t}^p D_{fixed,t}^w, \quad (A62)$$

and

$$N_{flex,t} = \left(\frac{Y_{flex,t}}{A_t}\right)^{\frac{1}{\gamma}} D_{flex,t}^p, \quad (A63)$$

where $D_{fixed,t}^p$, $D_{fixed,t}^w$, and $D_{flex,t}^p$ are the measures of price and wage dispersions in each sector.\(^18\) We take logarithms on both sides of the fixed sector expression for employment to then subtract the steady state counterpart and obtain

$$(1 - \alpha)\hat{h}_{fixed,t} = \hat{y}_{fixed,t} + d_{fixed,t}^w + d_{fixed,t}^p. \quad (A64)$$

There is no dispersion in steady state, so we do not need the hat for the dispersion terms. From Galí (23), we have that

$$d_{fixed,t}^p \simeq \frac{\epsilon_p}{2\Theta} \text{Var}\{p_{fixed,t}(i)\}, \quad (A65)$$

\(^18\) These terms come from the aggregation of employment.
where \( \Theta = \frac{1-a}{1-\alpha + \alpha \epsilon_p} \). We also have that
\[
d_{\text{fixed},t}^w \simeq \frac{(1-a)\epsilon_w}{2} \text{Var}\{w_{\text{fixed},t}(j)\}.
\] (A66)

These two expressions allow us to write equation (A64) as follows
\[
(1-a)\hat{n}_{\text{fixed},t} = \hat{y}_{\text{fixed},t} + \frac{(1-a)\epsilon_w}{2} \text{Var}\{w_{\text{fixed},t}(j)\} + \frac{\epsilon_p}{2\Theta} \text{Var}\{p_{\text{fixed},t}(i)\}.
\] (A67)

Analogous for the flex sector, we get the following expression.
\[
(1-a)\hat{n}_{\text{flex},t} = \hat{y}_{\text{flex},t} + \frac{\epsilon_p}{2\Theta} \text{Var}\{p_{\text{flex},t}(i)\}.
\] (A68)

We combine equations (A67) and (A68) by using
\[
\hat{n}_t = \eta\hat{n}_{\text{fixed},t} + (1-\eta)\hat{n}_{\text{flex},t},
\] (A69)

and thus, we get
\[
(1-a)\hat{n}_t = \eta \hat{y}_{\text{fixed},t} + (1-\eta)\hat{y}_{\text{flex},t} + \frac{\eta(1-a)\epsilon_w}{2} \text{Var}\{w_{\text{fixed},t}(j)\} + \frac{\epsilon_p}{2\Theta} \left[ \eta \text{Var}\{p_{\text{fixed},t}(i)\} + (1-\eta)\text{Var}\{p_{\text{flex},t}(i)\} \right].
\] (A70)

If we use that \( \hat{y}_t = \eta \hat{y}_{\text{fixed},t} + (1-\eta)\hat{y}_{\text{flex},t} \) and \( \hat{y}_t = \tilde{y}_t + y^*_t \), we then have that
\[
(1-a)\hat{n}_t = \tilde{y}_t + y^*_t + \frac{\eta(1-a)\epsilon_w}{2} \text{Var}\{w_{\text{fixed},t}(j)\} + \frac{\epsilon_p}{2\Theta} \left[ \eta \text{Var}\{p_{\text{fixed},t}(i)\} + (1-\eta)\text{Var}\{p_{\text{flex},t}(i)\} \right].
\] (A71)

We are ready to calculate the expression \( \left( \hat{n}_t + \frac{1+\varphi}{2} \hat{n}^2_t \right) \). This expression becomes
\[
\hat{n}_t + \frac{1+\varphi}{2} \hat{n}^2_t = \frac{\tilde{y}_t}{1-a} + \frac{y^*_t}{1-a} + \frac{(1+\varphi)\tilde{y}^2_t}{2(1-a)^2} + \frac{(1+\varphi)y^*_ty^*_t}{2(1-a)^2} + \frac{\eta(1-a)\epsilon_w}{2} \text{Var}\{w_{\text{fixed},t}(j)\} + \frac{\epsilon_p}{2\Theta} \left[ \eta \text{Var}\{p_{\text{fixed},t}(i)\} + (1-\eta)\text{Var}\{p_{\text{flex},t}(i)\} \right] + t.i.p + O(||z^3||)
\] (A72)

Replacing this expression and the expression for consumption into equation (A59), we
$$U_t - \bar{U} = -\left[\frac{(1 + \varphi)\tilde{y}_t^2}{2(1 - \alpha)^2} + \frac{\eta c_w}{2} \text{Var}\{w_{fixed,t}(j)\} + \frac{\epsilon_p}{2(1 - \alpha)} \theta \left[\eta \text{Var}\{p_{fixed,t}(i)\} + (1 - \eta) \text{Var}\{p_{flex,t}(i)\}\right] + t.i.p + O(||z^3||).$$

(A73)

Considering that we are studying an open economy setting, and using that $\tilde{n}_t^2 = \frac{\tilde{q}_t^2}{(1 - \alpha)^2}$, we can write the loss function as

$$U_t - \bar{U} = -\left[\frac{(1 - v)}{2} \left[(1 + \varphi)\tilde{n}_t^2 + \eta c_w \text{Var}\{w_{fixed,t}(j)\} + \frac{\epsilon_p}{1 - \alpha} \theta \left[\eta \text{Var}\{p_{fixed,t}(i)\} + (1 - \eta) \text{Var}\{p_{flex,t}(i)\}\right] + t.i.p + O(||z^3||).$$

(A74)

We only need to express the dispersions $\text{Var}\{p_{fixed,t}(i)\}$, $\text{Var}\{p_{fixed,t}(i)\}$, and $\text{Var}\{p_{flex,t}(i)\}$ in terms of inflation rates, which is a rather standard approach (see Gali (23)). Thus, we get

$$U_t - \bar{U} = -\left[\frac{(1 - v)}{2} \left[(1 + \varphi)\tilde{n}_t + \eta c_w \text{Var}\{\pi_{fixed,t}^w\} + \frac{\epsilon_p}{\lambda_w(1 - \alpha)} \theta \left[\eta \text{Var}\{\pi_{fixed,t}^w\} + (1 - \eta) \text{Var}\{\pi_{flex,t}^p\}\right] + t.i.p + O(||z^3||).$$

(A75)

where $\lambda_w \equiv \frac{(1 - \theta_w)(1 - \beta_{\pi w})}{\delta_{\pi w}(1 + c_w \varphi)}$ and $\lambda_p \equiv \frac{(1 - \theta_p)(1 - \beta_{\pi p})}{\delta_{\pi p}} \Theta$. 

36
B Set of equations corresponding to the models developed in the paper

B.a Set of equations corresponding to GM2016.

\[ y_t = (1 - v)c_t + v(2 - v)s_t \] (A76)
\[ c_t = (1 - v)s_t + z_t \] (A77)
\[ c_t = \mathbb{E}_t\{c_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\}) + (1 - \rho_z)z_t \] (A78)
\[ s_t \equiv e_t - p_{H,t} \] (A79)
\[ n_t = \frac{1}{1 - \alpha}(y_t - a_t) \] (A80)
\[ \pi_{H,t}^p = \beta \mathbb{E}_t\{\pi_{H,t+1}^p\} + \frac{\lambda_p\alpha}{1 - \alpha} \tilde{y}_t + \lambda_p\tilde{\omega}_t + \lambda_puv_t \] (A81)
\[ \pi_{H,t}^w \equiv p_{H,t} - p_{H,t-1} \] (A82)
\[ \pi_{t}^p \equiv p_t - p_{t-1} \] (A83)
\[ p_t = p_{H,t} + vs_t \] (A84)
\[ \pi_{t}^w = \beta \mathbb{E}_t\{\pi_{t+1}^w\} + \frac{\lambda_w\phi}{1 - \alpha} \tilde{y}_t + \lambda_w\tilde{\epsilon}_t - \lambda_w\tilde{\omega}_t \] (A85)
\[ \pi_t^a \equiv w_t - w_{t-1} \] (A86)
\[ \omega_t \equiv w_t - p_t \] (A87)
\[ \tilde{c}_t = c_t - c_t^n \] (A88)
\[ \tilde{y}_t = y_t - y_t^n \] (A89)
\[ \tilde{\omega}_t = \omega_t - \omega_t^n \] (A90)
\[ \tilde{s}_t = s_t - s_t^n \] (A91)
\[ \tilde{n}_t = n_t - n_t^n \] (A92)
\[ n_t^n = \frac{v}{1 + \phi}(-z_t) \] (A93)
\[ y_t^n = a_t + (1 - \alpha)n_t^n \] (A94)
\[ s_t^n = a_t - z_t - (\alpha + \phi)n_t^n \] (A95)
\[ c_t^n = z_t + (1 - v)s_t^n \] (A96)
\[ \omega_i^n = a_t - \alpha n_i^n - v s_i^n \]  
(A97)

\[ r_t = i - \pi_{H,t+1}^p \]  
(A98)

\[ z_t = \rho_z z_{t-1} + \epsilon^z_t \]  
(A99)

\[ a_t = \rho_a a_{t-1} + \epsilon^a_t \]  
(A100)

\[ \epsilon_t = 0 \]  
(A101)
B.b Extending GM2016 I

We incorporate a flex sector with flex working-hours and full wage flexibility. We also assume that working-hours are flexible in the fixed sector but with wage rigidities a la Calvo (like in GM2016).

\[ y_t = (1 - v)c_t + v(2 - v)s_t \]  
(A102)

\[ c_t = (1 - v)s_t + z_t \]  
(A103)

\[ c_t = \mathbb{E}_t \{c_{t+1}\} - (\bar{v} - \mathbb{E}_t \{\pi_{t+1}\}) + (1 - \rho_z)z_t \]  
(A104)

\[ s_t \equiv e_t - p_{H,t} \]  
(A105)

\[ n_t^{\text{flex}} = \frac{1}{1 - \alpha} (y_t^{\text{flex}} - a_t) \]  
(A106)

\[ n_t^{\text{fixed}} = \frac{1}{1 - \alpha} (y_t^{\text{fixed}} - a_t) \]  
(A107)

\[ y_t = \eta y_t^{\text{fixed}} + (1 - \eta)y_t^{\text{flex}} \]  
(A108)

\[ y_t^{\text{fixed}} - y_t^{\text{flex}} = -rp \]  
(A109)

\[ \pi_{H,t}^{\text{fixed},p} = \beta \mathbb{E}_t \{\pi_{H,t+1}^{\text{fixed},p}\} + \frac{\lambda_p}{1 - \alpha} y_t^{\text{fixed}} + \lambda_p \bar{w}_t^{\text{fixed}} + \lambda_p v_s t - \lambda_p (1 - \eta)rp \]  
(A110)

\[ \pi_{H,t}^{\text{fixed},p} \equiv p_{H,t}^{\text{fixed}} - p_{H,t-1}^{\text{fixed}} \]  
(A111)

\[ \pi_t^p \equiv p_t - p_{t-1} \]  
(A112)

\[ p_t = p_{H,t} + v s_t \]  
(A113)

\[ rp_t \equiv p_{H,t}^{\text{fixed}} - p_{H,t}^{\text{flex}} \]  
(A114)

\[ p_{H,t} = \eta p_{H,t}^{\text{fixed}} + (1 - \eta)p_{H,t}^{\text{flex}} \]  
(A115)

\[ \pi_{H,t}^{\text{flex},p} = \beta \mathbb{E}_t \{\pi_{H,t+1}^{\text{flex},p}\} + \frac{\lambda_p}{1 - \alpha} y_t^{\text{flex}} + \lambda_p v(\bar{v} - 1)s_t \]  
(A116)

\[ \pi_{H,t}^{\text{flex},p} \equiv p_{H,t}^{\text{flex}} - p_{H,t-1}^{\text{flex}} \]  
(A117)

\[ \pi_t^{\text{flex},w} = \beta \mathbb{E}_t \{\pi_{t+1}^{\text{flex},w}\} + \frac{\lambda_w}{1 - \alpha} y_t^{\text{flex}} + \lambda_w \bar{c}_t - \lambda_w \bar{c}_t^{\text{fixed}} \]  
(A118)

\[ \pi_t^{\text{flex},w} \equiv w_{t}^{\text{fixed}} - w_{t-1}^{\text{fixed}} \]  
(A119)

\[ \bar{c}_t = c_t - c_t^n \]  
(A120)
\[ y^\text{fixed}_t = y^\text{fixed}_t - y^n_t \]  
\[ y^\text{flex}_t = y^\text{flex}_t - y^n_t \]  
\[ \omega^\text{fixed}_t = \omega^\text{fixed}_t - \omega^n_t \]  
\[ s_t = s_t - s^n_t \]  
\[ n_t = \eta n^\text{fixed}_t + (1 - \eta)n^\text{flex}_t \]  
\[ \bar{n}_t = n_t - n^n_t \]  
\[ n^n_t = \frac{v}{1 + \varphi}(-z_t) \]  
\[ y^n_t = a_t + (1 - \alpha)n^n_t \]  
\[ s^n_t = a_t - z_t - (\alpha + \varphi)n^n_t \]  
\[ e^n_t = z_t + (1 - v)s^n_t \]  
\[ \omega^n_t = a_t - \alpha n^n_t - vs^n_t \]  
\[ r_t = i - (\eta \pi^\text{fixed,}H_t+1 + (1 - \eta)\pi^\text{flex,}H_t+1) \]  
\[ z_t = \rho z_{t-1} + e^z_t \]  
\[ a_t = \rho a_{t-1} + e^a_t \]  
\[ e_t = 0 \]
B.c Extending GM2016 II

We incorporate a flex sector with flex working-hours and full wage flexibility. Yet, we assume that working-hours are fixed in the fixed sector - unlike the specification in GM2016. We also assume wage rigidities a la Calvo.

\[ y_t = (1 - v)c_t + v(2 - v)s_t \]  
\[ c_t = (1 - v)s_t + z_t \]  
\[ c_t = \mathbb{E}_t\{c_{t+1}\} - (i_t - \mathbb{E}_t\{r_{t+1}\}) + (1 - \rho)z_t \]  
\[ s_t \equiv e_t - p_{H,t} \]  
\[ n_t^{\text{flex}} = \frac{1}{1 - \alpha}(y_t^{\text{flex}} - y_t^{\text{fixed}}) \]  
\[ y_t = \eta y_{t}^{\text{fixed}} + (1 - \eta)y_t^{\text{flex}} \]  
\[ y_t^{\text{flex}} - y_t^{\text{fixed}} = -rp \]  
\[ \pi_{t}^{\text{fixed},p} = \beta \mathbb{E}_t\{\pi_{t+1}^{\text{fixed},p}\} + \lambda_p \omega_t^{\text{fixed}} + \lambda_p v \bar{s}_t - \lambda_p (1 - \eta)rp \]  
\[ \pi_{t}^{\text{fixed},p} \equiv p_t^{\text{fixed}} - p_{t-1}^{\text{fixed}} \]  
\[ p_t = p_{H,t} + v s_t \]  
\[ rp_t \equiv p_t^{\text{fixed}} - p_{H,t}^{\text{flex}} \]  
\[ p_{H,t} = \eta p_{H,t}^{\text{fixed}} + (1 - \eta)p_t^{\text{flex}} \]  
\[ \pi_{t}^{\text{flex},p} = \beta \mathbb{E}_t\{\pi_{t+1}^{\text{flex},p}\} + \frac{\lambda_p (1 + \varphi)}{1 - \alpha}y_t^{\text{flex}} + \lambda_p v(\bar{s} - 1)\bar{s}_t \]  
\[ \pi_{t}^{\text{flex},p} \equiv p_t^{\text{flex}} - p_{t-1}^{\text{flex}} \]  
\[ \pi_{t}^{\text{fixed},w} = \beta \mathbb{E}_t\{\pi_{t+1}^{\text{fixed},w}\} + \lambda_w \bar{c}_t - \lambda_w \bar{c}_t^{\text{fixed}} \]  
\[ \pi_{t}^{\text{fixed},w} \equiv w_t^{\text{fixed}} - w_{t-1}^{\text{fixed}} \]  
\[ \omega_t^{\text{fixed}} \equiv w_t^{\text{fixed}} - p_t \]  
\[ \bar{c}_t = c_t - c_t^n \]  
\[ \bar{y}_t^{\text{flex}} = y_t^{\text{flex}} - y_t^n \]
\[ \omega_t^{\text{fixed}} = \omega_t^{\text{fixed}} - \omega_t^n \]  
(A157)

\[ \tilde{s}_t = s_t - s_t^n \]  
(A158)

\[ n_t = (1 - \eta)n_t^{\text{flex}} \]  
(A159)

\[ \tilde{n}_t = n_t - n_t^n \]  
(A160)

\[ n_t^n = \frac{v}{1 + \varphi}(-z_t) \]  
(A161)

\[ y_t^n = a_t + (1 - \alpha)n_t^n \]  
(A162)

\[ s_t^n = a_t - z_t - (\alpha + \varphi)n_t^n \]  
(A163)

\[ c_t^n = z_t + (1 - v)s_t^n \]  
(A164)

\[ \omega_t^n = a_t - \alpha n_t^n - vs_t^n \]  
(A165)

\[ r_t = i - (\eta \pi_{t,\phi+1}^{\text{fixed},p} + (1 - \eta)\pi_{t,\phi+1}^{\text{flex},p}) \]  
(A166)

\[ z_t = \rho_z z_{t-1} + e_t^z \]  
(A167)

\[ a_t = \rho_a a_{t-1} + e_t^a \]  
(A168)

\[ e_t = 0 \]  
(A169)
B.d Extending GM2016 III

We incorporate a flex sector with fixed working-hours and full wage flexibility. We also assume that the working-hours are fixed in the fixed sector and wage rigidities a la Calvo. The only difference in-between the two sectors is the flexibility of wages.

\[ y_t = (1 - v)c_t + v(2 - v)s_t \]  \hspace{1cm} (A170)
\[ c_t = (1 - v)s_t + z_t \]  \hspace{1cm} (A171)
\[ c_t = \mathbb{E}_t\{c_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\}) + (1 - \rho_z)z_t \]  \hspace{1cm} (A172)
\[ s_t \equiv e_t - p_{H,t} \]  \hspace{1cm} (A173)
\[ \pi^p_{H,t} = \beta \mathbb{E}_t\{\pi^p_{H,t+1}\} + \lambda p \tilde{c}^n_t + \lambda p \nu s_t \]  \hspace{1cm} (A174)
\[ \pi^p_{H,t} \equiv p_{H,t} - p_{H,t-1} \]  \hspace{1cm} (A175)
\[ \pi^n_t \equiv p_t - p_{t-1} \]  \hspace{1cm} (A176)
\[ p_t = p_{H,t} + \nu s_t \]  \hspace{1cm} (A177)
\[ \pi^n_t = \beta \mathbb{E}_t\{\pi^n_{t+1}\} + \lambda w \tilde{c}^n_t - \lambda w \tilde{p}^n_t \]  \hspace{1cm} (A178)
\[ \pi^n_t \equiv w_{t}^{fixed} - w_{t-1}^{fixed} \]  \hspace{1cm} (A179)
\[ \omega^n_t \equiv w_t^{fixed} - p_t \]  \hspace{1cm} (A180)
\[ \tilde{c}_t = c_t - c^n_t \]  \hspace{1cm} (A181)
\[ \tilde{\omega}_t^{fixed} = \omega_t^{fixed} - \omega^n_t \]  \hspace{1cm} (A182)
\[ s_t = s_t - s^n_t \]  \hspace{1cm} (A183)
\[ s^n_t = a_t - z_t - (\alpha + \varphi)n^n_t \]  \hspace{1cm} (A184)
\[ c^n_t = z_t + (1 - v)s^n_t \]  \hspace{1cm} (A185)
\[ \omega^n_t = a_t - \alpha n^n_t - \nu s^n_t \]  \hspace{1cm} (A186)
\[ r_t = i - \pi^p_{H,t+1} \]  \hspace{1cm} (A187)
\[ z_t = \rho_z z_{t-1} + e^z_t \]  \hspace{1cm} (A188)
\[ a_t = \rho_a a_{t-1} + e^a_t \]  \hspace{1cm} (A189)
\[ e_t = 0 \]  \hspace{1cm} (A190)