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**IDENTIFICATION AND ESTIMATION OF EXCHANGE RATE
MODELS WITH UNOBSERVABLE FUNDAMENTALS**

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Identification and Estimation of Exchange Rate Models with Unobservable Fundamentals

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Abstract: This paper is concerned with issues of model specification, identification, and estimation in exchange rate models with unobservable fundamentals. We show that the model estimated by Gardeazabal, Regúlez and Vázquez (*International Economic Review*, 1997) is not identified and demonstrate how to specify an identified model in-keeping with their intended approach. Estimates of the identified model are reported for five currencies over two time spans, and a restriction suggested by the asset market view of exchange rate determination is not rejected for any currency or time span. The forecasting performance of the model is also examined and is found to compare favourably with forecasts generated by a random walk with drift.

Keywords: Exchange rates; unobservable fundamentals; identification; temporal aggregation.

J.E.L. Numbers: C32, F31.

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1. INTRODUCTION

The concept of fundamentals has played an important role in economic theories of nominal exchange rate determination. Monetary models of the exchange rate, for example, explicitly define which macroeconomic variables constitute fundamentals, and furthermore, they determine the functional form which relates them to the exchange rate (which is typically log-linear). Unfortunately, the empirical performance of monetary models has not, in general, been good, despite their common adoption in theoretical work. The widely cited results of Meese and Rogoff (1983), which demonstrate that the predictive ability of monetary models with certain chosen fundamentals is inferior to a simple random walk model, largely remain valid today; for a critical survey of empirical results in this area, see Frankel and Rose (1995).

The asset market view of exchange rates, which relates the nominal exchange rate to a measure of economic fundamentals and to a term that reflects the expected rate of change in the exchange rate, has also found widespread use in theoretical work, being adopted, in particular, by the target zones literature; see, for example, Froot and Obstfeld (1991). One of the problems encountered by empirical researchers in attempting to test the asset market model, however, concerns the specification of the fundamentals process in terms of observable economic variables. The theoretical model provides little guidance as to precisely which variables to include or the appropriate functional form to use, with the result that empirical testing tends to be *ad hoc* and is likely to suffer from misspecification biases.

One noteworthy attempt at making the asset market model operational was proposed by Gardeazabal, Regúlez and Vázquez (1997) (hereafter denoted GRV).¹ Their version of the model treats the fundamentals as an unobservable, or latent, process, with the intention of retaining the principle of the asset market view while avoiding the potential misspecification biases associated with a particular choice of macroeconomic variables as fundamentals together with particular functional forms. GRV use simulation techniques as a means of accounting for the unobservable process in estimating and testing their model, and claim empirical support for this approach based on monthly data for five currencies (relative to the U.S. dollar) over the period 1974 to 1992.

Allowing the fundamentals to be unobservable is not without precedent. For example, Diebold and Nerlove (1989) find that unobservable (latent) factors are able to capture common movements in volatility across exchange rates, while King, Sentana and Wadhvani (1994) find that changes in correlations between national stock markets are driven primarily by movements in unobservable variables. The results of GRV would further seem to suggest

¹GRV refer to the asset market model as the canonical model following Krugman (1992).

that treating fundamentals as an unobservable process is a promising approach to nominal exchange rate modelling. Unfortunately, as we demonstrate in this paper, the model estimated by GRV is *not identified*, which not only casts extreme doubt upon their results, but also serves as a warning to researchers who use simulation techniques without first checking the identifiability of their model. This lack of identifiability in GRV's model raises two immediate questions. The first is whether it is possible, in the spirit of GRV, to specify an identified model of exchange rates with unobservable fundamentals, and one that can embody the principle of the asset market view. Answering in the affirmative, we then examine a second question of how well the model performs empirically, in a way that is not conjoint with either the choice or specification of the fundamentals.

The (identified) model that we specify consists of two equations. The first specifies changes in the exchange rate as a function of a drift term and a feedback term which measures the deviation of the exchange rate from the fundamentals. The second equation models the fundamentals process as a random walk with drift. The lack of identification in the GRV model essentially boils down to two features. First, they incorporated a scaling parameter between the exchange rate and the fundamentals in the feedback term. Because fundamentals are unobservable, any changes in the scale of the fundamentals are absorbed by a change in the scaling parameter – it is not possible to identify both at the same time. Effectively, we are setting the scale parameter to unity, which is consistent with the underlying asset market view. The second feature leading to underidentification in the GRV model is the presence of the feedback term in the equation governing the evolution of the fundamental. Our remedy for this is to allow the unobservable fundamentals process to evolve independently of such feedbacks, our justification being that macroeconomic fundamentals evolve more sluggishly than the exchange rate, this being a model of exchange rate determination rather than one of fundamentals determination. Both of these features, when combined, render the GRV model unidentified.

We estimate our model using two datasets on the same five currencies as in GRV, namely those for Britain, Germany, Japan, Italy, and Spain. The first dataset covers the same span as in GRV, the second covers a longer time span by extending it beyond 1992. We employ maximum likelihood estimation methods, having substituted out the unobservable fundamentals process from the model, thereby avoiding the computational costs of simulation. We find that the restriction on the model implied by the asset market model is not rejected for any currency or time span, and that the fit of the model is good. Furthermore, the dynamic forecasts produced by the model are typically close to those obtained from a random walk with drift, though they are notably inferior at longer horizons (24 months) for Italy and Spain.

The paper is organised as follows. Section 2 describes the GRV model and its relationship to the asset market approach to exchange rates, while Section 3 is concerned with issues relating to identification and estimation. Section 4 describes our identified model and contains the empirical results, and section 5 concludes the paper. Proofs of the three propositions contained in Section 3 are contained in the Appendix.

2. EXCHANGE RATES AND UNOBSERVABLE FUNDAMENTALS

A common approach to exchange rate determination has its roots in the finance literature in which an asset price can be regarded as consisting of a fundamental value plus a term that reflects expectations of future changes in the asset price. In the case of exchange rates the asset is foreign currency and the price of the asset is the exchange rate itself. The essence of this approach is captured by a linear relationship between the logarithm of the exchange rate,² $s(t)$, and a scalar measure of macroeconomic fundamentals, $f(t)$, of the form

$$(1) \quad s(t) = f(t) + \alpha \frac{E[ds(t)|\Omega(t)]}{dt},$$

where α is a scalar parameter and $\Omega(t)$ denotes the agents' information set at time t , which includes current and past values of s and f . This equation states that the exchange rate is equal to a fundamental determinant plus a speculative term that is proportional to the expected percentage change in the exchange rate. Froot and Obstfeld (1991) observe that α can be interpreted as the (negative of the) semi-elasticity of money demand with respect to the nominal interest rate, and hence there are good *a priori* grounds for expecting α to be positive, at least from the perspective of the monetary model. If $\alpha > 0$, and if speculative bubbles are excluded, the equation may be solved forwards to express the equilibrium exchange rate path as the present discounted value of expected future fundamentals,

$$(2) \quad s(t) = \frac{1}{\alpha} \int_t^\infty e^{(t-r)/\alpha} E[f(r)|\Omega(t)] dr.$$

The exchange rate models below are formulated in continuous time to reflect that trading occurs almost continuously against the fact that our data is obtained at much coarser intervals of time. Although the exchange rate dynamics evolve on a much finer timescale compared with the frequency of the observations, our methodology ensures the restrictions of the continuous time model are exactly incorporated in the distribution of the discrete time data.

Precisely what determines the fundamental $f(t)$ is open to some debate. Simple monetary models of the exchange rate suggest variables such as income and the money stock, both

²The exchange rate is defined as the price of foreign currency in terms of a unit of the domestic currency.

in logarithmic form, as fundamentals, but the poor empirical performance of such models is indicative of misspecification in the choice of variables or functional form or both. Specifying realistic models of the exchange rate in which fundamentals can be treated as unobservable but which are still consistent with the underlying asset market approach has considerable appeal. The principal contribution in this vein is provided by Gardeazabal, Regúlez and Vázquez (1997) in which fundamentals are treated as a latent, or unobservable, process. The model proposed by GRV is the bivariate system of stochastic differential equations³

$$(3) \quad ds(t) = \mu_1 dt + \gamma_1 [f(t) - bs(t)] dt + \sigma_1 dW_1(t),$$

$$(4) \quad df(t) = \gamma_2 [f(t) - bs(t)] dt + \sigma_2 dW_2(t),$$

where μ_1 is a drift parameter, σ_1 and σ_2 are non-negative coefficients, and W_1 and W_2 are independent Wiener processes. This representation is based on considerable (though not universal) evidence that exchange rates behave like integrated processes and embodies a reduced rank assumption that implies cointegration between s and f .⁴ The general model represented by (3) and (4) is, therefore, a cointegrated vector autoregression in continuous time, which is most easily seen by writing (3) and (4) as the system

$$(5) \quad dx(t) = [\mu + Ax(t)]dt + \Sigma dW(t),$$

where $x(t) = [s(t), f(t)]'$, $W(t) = [W_1(t), W_2(t)]'$, Σ is a diagonal matrix with σ_1 and σ_2 on the diagonal, $\mu = [\mu_1, 0]'$, and A may be written in the form $A = \gamma\beta'$, where $\gamma = [\gamma_1, \gamma_2]'$ and $\beta = [-b, 1]'$. The reduced rank (cointegration) property can be seen from the form of the matrix $A = \gamma\beta'$, which is a 2×2 matrix of rank one. The elements of γ are adjustment parameters while β' is the cointegrating vector, so that $\beta'x(t) = f(t) - bs(t)$ is stationary even though $s(t)$ and $f(t)$ might be integrated processes. The non-standard feature in this system is that one of the variables, $f(t)$, is unobservable, even in discrete time.

Proposition 1 of GRV (1997) asserts that (3) and (4) are compatible with (1) provided that $\mu_1 = 0$, $b = 1$ and $\gamma_1 < 0$. This can be demonstrated by taking the conditional expectation of $ds(t)$ in (3) and solving for $s(t)$ to yield

$$s(t) = \frac{\mu_1}{\gamma_1 b} + \frac{f(t)}{b} - \frac{1}{\gamma_1 b} \frac{E[ds(t)|\Omega(t)]}{dt},$$

the coefficients of which can be compared directly to those in (1). Clearly, the first restriction, $\mu_1 = 0$, eliminates the intercept, while the restriction $b = 1$ ensures that the fundamentals

³We adopt the notation of GRV for the coefficients of the model but note that they use $x(t)$ to denote the exchange rate and $k(t)$ to denote the fundamentals.

⁴The importance of the cointegration assumption in empirical work with nominal exchange rates has recently been demonstrated by Rapach and Wohar (2002).

process remains unscaled. The restriction $\gamma_1 < 0$ then yields $\alpha > 0$. GRV's estimates of the unrestricted model in (3) and (4) for five currencies do, in fact, satisfy the requirement for $\alpha > 0$, although their estimates of μ_1 tend to be significantly different from zero and their estimates of b range from -12.73 to $+6.66$ and are significantly different from unity for four of the five currencies. Despite this, GRV claim (p.391) that "the results obtained support the rational expectations hypothesis imposed" by the model i.e. that the restrictions on (3) and (4) that yield (1) are supported by the data. Their claim has to be interpreted with caution, however, because, as we show in the next section, the model that they estimate is not identified.

3. ISSUES OF IDENTIFICATION AND ESTIMATION

The model in Section 2 is formulated in continuous time. Published observations on exchange rates are recorded at discrete intervals of time, for example daily, weekly or monthly, and so our first objective is to relate the parameters of the continuous time model to the discrete time observations. It is convenient to combine the unknown parameters of interest into the vector $\theta = [\gamma_1, \gamma_2, b, \mu_1, \sigma_1, \sigma_2]'$. The exact discrete time analogue of (5) at integer points of t is presented below because it is central to the discussion of both GRV's model as well as our own.⁵

PROPOSITION 1. *Let $x(t)$ ($t > 0$) be generated by (5), and let $x_t = x(t)$ for integer values of $t = 1, \dots, T$, where T denotes sample size. Then x_t satisfies the first-order difference equation*

$$(6) \quad \Delta x_t = \phi(\theta) + \Phi(\theta)x_{t-1} + u_t, \quad t = 1, \dots, T,$$

where $\Delta x_t = x_t - x_{t-1}$, $\Phi(\theta) = e^A - I = A\psi(1)$, $\psi(r) = (e^{r\beta'\gamma} - 1)/(\beta'\gamma)$,

$$\phi(\theta) = \left[\int_0^1 e^{rA} dr \right] \mu = \left[I + A \left(\frac{\psi(1) - 1}{\beta'\gamma} \right) \right] \mu,$$

and $u_t \sim N(0, V(\theta))$ with $E(u_t u_s') = 0$ for all $t \neq s$ and

$$V(\theta) = \int_0^1 e^{rA} \Sigma^2 e^{rA'} dr = \Sigma^2 + (\Sigma^2 A' + A \Sigma^2) \left(\frac{\psi(1) - 1}{\beta'\gamma} \right) + A \Sigma^2 A' \left(\frac{0.5\psi(2) - 2\psi(1) + 1}{(\beta'\gamma)^2} \right),$$

where $\Sigma^2 = \Sigma \Sigma'$.

The discrete time representation (6) has the form of a co-integrated VAR(1) system whose coefficient and covariance matrices embody *exact* restrictions imported from the

⁵GRV employed alternative expressions which can be found in the Appendix to their paper. We believe, however, that our expressions are more straightforward than those used by GRV, but they can be shown to be identical; details are available from the authors.

continuous time model. We stress that it is the model (with probability one) that equispaced data generated by (5) satisfy independently of the frequency with which the data are recorded.⁶ It is important to note that (6) is not obtained from (5) simply by replacing $dx(t)$ with Δx_t , $x(t)$ with x_{t-1} and $\Sigma dW(t)$ with u_t . Although ϕ and Φ are proportional to μ and A respectively, they also depend on more complicated functions of the underlying parameters, as can be seen from the form of the proportionality factors. The same is true of the covariance matrix, V , of the discrete time disturbance vector u_t , which is not simply equal to $\Sigma\Sigma'$ but also involves nonlinear functions of the elements of γ and β . Note that Φ embodies the reduced rank property of the continuous time coefficient matrix A , and hence the elements of x_t (s_t and f_t) are cointegrated.

If f_t was observed in addition to s_t , then the estimation of θ could proceed straightforwardly by maximising the likelihood function corresponding to (6). The difficulty arises because f_t is unobservable, and a number of estimation strategies are available. GRV, for example, employed a simulated method of moments estimator, the moments being those of the observed exchange rate series. Alternatively, the Kalman filter could be used to construct the likelihood function, thereby avoiding the computational burdens associated with simulation. Another likelihood-based approach constructs the likelihood function based on the observed exchange rate series once the unobservable variable has been substituted out. Such an approach was advocated by Bergstrom and Chambers (1990) and is adopted here, based on the following key result. It is convenient to let ϕ_j , Φ_{jk} and V_{jk} denote the j -th element of $\phi(\theta)$, the (j, k) -th element of $\Phi(\theta)$, and the (j, k) -th element of $V(\theta)$, respectively.

PROPOSITION 2. *Let $s(t)$ and $f(t)$ be generated by (3) and (4), as represented in (5), and let $s_t = s(t)$ be observed at $t = 1, \dots, T$. Then s_t satisfies the ARMA(2,1) process*

$$(7) \quad s_1 = \alpha_{10}(\theta) + \alpha_{11}(\theta)s_0 + \alpha_{12}(\theta)f_0 + \eta_1,$$

$$(8) \quad s_2 = \alpha_{20}(\theta) + \alpha_{21}(\theta)s_1 + \alpha_{22}(\theta)s_0 + \alpha_{23}(\theta)f_0 + \eta_2,$$

$$(9) \quad s_t = \alpha_0(\theta) + \alpha_1(\theta)s_{t-1} + \alpha_2(\theta)s_{t-2} + \eta_t, \quad t = 3, \dots, T,$$

where (suppressing the dependence on θ for convenience)

$$\begin{aligned} \alpha_{10} &= \phi_1, & \alpha_{11} &= \Phi_{11}, & \alpha_{12} &= \Phi_{12}, \\ \alpha_{20} &= \phi_1 + \Phi_{12}\phi_2, & \alpha_{21} &= \Phi_{11}, & \alpha_{22} &= \Phi_{12}\Phi_{21}, & \alpha_{23} &= \Phi_{12}\Phi_{22}, \\ \alpha_0 &= \phi_1(1 - \Phi_{22}) + \Phi_{12}\phi_2, & \alpha_1 &= \Phi_{11} + \Phi_{22}, & \alpha_2 &= \Phi_{12}\Phi_{21} - \Phi_{11}\Phi_{22}. \end{aligned}$$

⁶Estimates obtained from models naively specified in discrete time, like most exchange rate models that have appeared to date, are subject in principle to being distorted by temporal aggregation bias. For an empirical example where temporal aggregation bias is found to be important, see McCrorie and Chambers (2003).

Furthermore, η_t ($t = 1, \dots, T$) is a zero mean Gaussian MA(1) process with variances and autocovariances given by

$$\begin{aligned} E(\eta_1^2) &\equiv \omega_{11}(\theta) = V_{11}, \quad E(\eta_1\eta_2) \equiv \omega_{12}(\theta) = \Phi_{12}V_{12}, \quad E(\eta_2^2) \equiv \omega_{22}(\theta) = V_{11} + \Phi_{12}^2V_{22}, \\ E(\eta_t^2) &\equiv \omega_0(\theta) = (1 + \Phi_{22}^2)V_{11} + \Phi_{12}^2V_{22} - 2\Phi_{12}\Phi_{22}V_{12}, \\ E(\eta_t\eta_{t-1}) &\equiv \omega_1(\theta) = \Phi_{12}V_{12} - \Phi_{22}V_{11}, \quad t = 3, \dots, T. \end{aligned}$$

The coefficients of the ARMA representation in Proposition 2, including the autocovariances, are nonlinear functions of the parameters of the underlying continuous time model (5) owing to the process of temporal aggregation and our substituting out the unobservable fundamentals process. Note, too, that the representations for s_1 and s_2 both depend on the initial values $s_0 = s(0)$ and $f_0 = f(0)$. The former of these, s_0 , is observable while the second, f_0 , could, in principle, be estimated (but not consistently). Here, instead, we set $f_0 = s_0$ in accordance with the equilibrium condition in (1).

The likelihood function can be derived straightforwardly from the ARMA representation for s_t in Proposition 2. Let $\eta(\theta) = (\eta_1(\theta), \dots, \eta_T(\theta))'$ denote the $T \times 1$ disturbance vector with typical element $\eta_t(\theta) = s_t - \alpha_0(\theta) - \alpha_1(\theta)s_{t-1} - \alpha_2(\theta)s_{t-2}$ ($t = 3, \dots, T$), let $\Omega(\theta)$ denote its associated $T \times T$ covariance matrix, and let $s = (s_0, s_1, \dots, s_T)'$ denote the $(T+1) \times 1$ vector of observations. Note that $\Omega(\theta)$ is a Toeplitz matrix because $\eta_t(\theta)$ is an MA(1) process, its typical diagonal element being $\omega_0(\theta)$ with $\omega_1(\theta)$ in adjacent positions. Then the log-likelihood function is given by

$$(10) \quad \ln L(s, \theta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \ln \det[\Omega(\theta)] - \frac{1}{2} \eta(\theta)' \Omega(\theta)^{-1} \eta(\theta).$$

This representation of the likelihood function also aids the investigation of the identification of the parameter vector θ , which is assumed to belong to a parameter space Θ . Following Rothenberg (1971), we shall say that two parameter points θ_1 and θ_2 are *observationally equivalent* if $L(s, \theta_1) = L(s, \theta_2)$ for all $s \in \mathcal{R}^T$. A parameter point $\theta_0 \in \Theta$ is then said to be *identifiable* if there is no other $\theta \in \Theta$ which is observationally equivalent.

PROPOSITION 3. *Let $\theta^* = [\gamma_1^*, \gamma_2, b^*, \mu_1, \sigma_1, \sigma_2^*]'$, where $\gamma_1^* = \gamma_1/\lambda$, $b^* = \lambda b$ and $\sigma_2^* = \lambda \sigma_2$ for some constant λ . Then $\ln L(s, \theta) = \ln L(s, \theta^*)$ for all s and λ , and hence θ is not identifiable.*

The proof of Proposition 3 is based on a demonstration that the autoregressive parameters and autocovariances of the ARMA(2,1) representation of s_t are identical when based on θ and θ^* and, hence, the values of the likelihood function are identical. Such a phenomenon

is not confined to the continuous time model analysed here, but applies more generally to models formulated directly in discrete time. Suppose, for example, that the observed exchange rate, s_t , and the unobservable fundamental, f_t , satisfy the VAR(1) system

$$(11) \quad s_t = a_{11}s_{t-1} + a_{12}f_{t-1} + \epsilon_{1t},$$

$$(12) \quad f_t = a_{21}s_{t-1} + a_{22}f_{t-1} + \epsilon_{2t},$$

where, for $i = 1, 2$, ϵ_{it} is an IID($0, \sigma_i^2$) random disturbance and $E(\epsilon_{1t}\epsilon_{2t}) = \sigma_{12}$. If both s_t and f_t were observed, then the parameters of (11) and (12) could be estimated consistently by ordinary least squares. When f_t is unobserved the essence of the identification problem is, perhaps, most easily demonstrated by solving out the unobservable variable f_{t-1} in (11) using (12). This yields an ARMA(2,1) representation for s_t of the form⁷

$$(13) \quad s_t = \delta_1 s_{t-1} + \delta_2 s_{t-2} + v_t,$$

where $\delta_1 = a_{11} + a_{22}$, $\delta_2 = a_{12}a_{21} - a_{11}a_{22}$, and $v_t = \epsilon_{1t} - a_{22}\epsilon_{1,t-1} + a_{12}\epsilon_{2,t-1}$ is an MA(1) disturbance with variance $c_0 = (1 - a_{22}^2)\sigma_1^2 + a_{12}^2\sigma_2^2 - 2a_{12}a_{22}\sigma_{12}$ and autocovariance $c_1 = -a_{22}\sigma_1^2 + a_{12}\sigma_{12}$. From a sample of observations on s_t it is possible to estimate the four unknowns δ_1 , δ_2 , c_0 and c_1 . However, these four quantities are functions of the *seven* underlying parameters of interest, these being the a_{ij} ($i, j = 1, 2$), σ_1^2 , σ_2^2 , and σ_{12} . Without further *a priori* identifying information or assumptions, the seven unknown parameters of interest can not be determined from observations on s_t alone. Although we have highlighted the problem of identification using a simple VAR(1) with an unobservable variable, the problem of underidentification obviously persists more generally in higher-order discrete time VARs.

The non-identifiability of the parameters of the model estimated by GRV therefore challenges their claims concerning the empirical validity of the model. It also asks the question of whether we can provide an identified exchange rate model with unobservable fundamentals that has economic meaning, in the spirit of GRV. In the next section, we do so by way of a restriction implied by the asset market view of exchange rates and obtain empirical results using exchange rate data for five countries covering two spans of data.

4. AN IDENTIFIED MODEL AND ITS EMPIRICAL PERFORMANCE

An implication of Proposition 3 is that an infinite number of fundamentals processes are consistent with the GRV model in (3) and (4). Put another way, there is not a unique

⁷Note that this is a special case of Proposition 2 but where the coefficients are not restricted by temporal aggregation considerations.

unobservable fundamentals process associated with an observed exchange rate series in the GRV model. This can be seen by multiplying (4) by the arbitrary constant λ and defining $f^*(t) = \lambda f(t)$, $b^* = \lambda b$ and $\sigma_2^* = \lambda \sigma_2$, yielding $df^*(t) = \gamma_2[f^*(t) - b^*s(t)]dt + \sigma_2^*dW_2(t)$. This equation is empirically indistinguishable from (4), while defining $\gamma_1^* = \gamma_1/\lambda$ means that (3) can be written in terms of $f^*(t)$ as $ds(t) = \mu_1dt + \gamma_1^*[f^*(t) - b^*s(t)]dt + \sigma_1dW_1(t)$. This is the underlying reason why $L(s, \theta) = L(s, \theta^*)$, and so we now focus on the issue of whether, and how, it might be possible to formulate a model with unobservable fundamentals, in the spirit of the GRV model, that is identified.

Part of the problem with the GRV model is that, in equilibrium, the unobservable process $f(t)$ is related to $s(t)$ by the unknown parameter b . Writing this relationship as $s(t) = f(t)/b$ it is clear that $s(t) = f^*(t)/b^*$ also satisfies the requirement. Motivated by (1) it seems reasonable to impose the condition that $f(t) = s(t)$ in equilibrium. It is then natural to base our specification of the model on the triangular representation of cointegrated systems developed by Phillips (1991a) and employed in a continuous time setting in Phillips (1991b). This also has the advantage that it automatically brings us into Phillips's optimal inference framework which facilitates hypothesis testing using standard (e.g. chi-square) distributions. The model is defined by the two stochastic differential equations

$$(14) \quad ds(t) = \mu_1dt + \gamma_1[f(t) - s(t)]dt + \sigma_1dW_1(t),$$

$$(15) \quad df(t) = \mu_2dt + \sigma_2dW_2(t),$$

where the variables are as defined previously. In this model, fundamentals evolve as a random walk with drift μ_2dt in continuous time, as depicted in (15), while the exchange rate responds to the disequilibrium $f(t) - s(t)$ in addition to containing its own drift term μ_1dt . The model achieves identification by reducing the number of unknown parameters by one, to five, and by requiring $f(t) = s(t)$ in equilibrium. Put another way, it is not possible to replace $f(t)$ by $f^*(t)$ and obtain an observationally equivalent model, because the form of the disequilibrium term in (14) precludes such a possibility. Note that the specification (14) remains consistent with the asset market relationship in (1), because

$$s(t) = \frac{\mu_1}{\gamma_1} + f(t) - \frac{1}{\gamma_1} \frac{E[ds(t)|\Omega(t)]}{dt},$$

yielding (1) if $\mu_1 = 0$. This restriction can be easily tested empirically. Note, too, that the coefficient α in (1) is equal to $-\gamma_1^{-1}$ above, and hence a positive value for α requires γ_1 being negative. However, this latter condition means that the system comprising (14) and (15) is unstable. To see this, write the system as in (5), and note that the matrix A is still given by $A = \gamma\beta'$ but where now $\gamma = (\gamma_1, 0)'$ and $\beta = (-1, 1)'$. The system is stable if the eigenvalues of A have negative real parts. Here, the eigenvalues are equal to zero and $-\gamma_1$,

the zero eigenvalue reflecting the unit root in the system. If $\gamma_1 < 0$ the system is clearly unstable and inference based on estimates of the system would be unreliable.

Our empirical results are based upon estimating and testing the identified system comprising (14) and (15) using two data sets. The first corresponds to the data set used by GRV and covers the period January 1974 through September 1992 (225 month-end observations) for five currencies, namely those for Britain, Germany, Japan, Italy, and Spain, all measured relative to the U.S. Dollar. The second data set represents an extension of the first, with the currencies for Britain and Japan taken through to December 2000 (324 observations), while those for Germany, Italy and Spain are extended through to December 1998 (300 observations).⁸

Table 1 presents ordinary least squares estimates of the simple random walk with drift model for the logarithms of the exchange rates, given by $\Delta s_t = \delta + \epsilon_t$, where δ denotes the drift parameter and ϵ_t is assumed to be a serially uncorrelated innovation process. The estimated drift parameters are simply the average percentage change in the exchange rates over the relative sample, and indicate that the currencies of Britain, Germany and Japan depreciated relative to the dollar in both sample periods, while the currencies of Italy and Spain appreciated relative to the dollar. The Table also reports Lagrange Multiplier (LM) statistics⁹ for testing for serial correlation and ARCH effects in the disturbances. For both sample periods and for all five currencies there is no evidence of serial correlation or ARCH effects with the exception of the extended sample period for Germany in which ARCH effects seem to be highly prevalent. With this one exception, the logarithm of exchange rates appears to be well described as a simple random walk with drift. This could help to explain to a large extent the findings of Meese and Rogoff (1983) and others that a simple random walk model dominates fundamentals-based (monetary) models in terms of forecasting performance, despite the apparently good in-sample fit of such models. We shall return to this issue later.

Two versions of the system comprising (14) and (15) are estimated for each currency and for each data set. The first version imposes no restrictions except those that arise from the process of temporal aggregation, and will be referred to as the *unrestricted model* in what follows. The second imposes the restriction $\mu_1 = 0$ and will be referred to as the *restricted model*. Maximum likelihood estimates are obtained by maximising the log-likelihood function (10) with respect to the unknown parameters μ_1 , γ_1 , σ_1 , μ_2 and σ_2 . The exchange rate observation for January 1974 was taken to be s_0 and it was assumed that

⁸The German, Italian and Spanish currencies were linked to the euro with effect from January 1, 1999.

⁹The F versions of the LM statistics are reported rather than the chi-square versions following the recommendation of Kiviet (1986). Note, too, that the conventional goodness-of-fit statistics, R^2 , are zero for each of these regressions, and hence are not reported.

$f_0 = s_0$ (corresponding to the equilibrium condition); hence the effective sample size (T) is 224 in the GRV sample and either 323 or 299 in the extended sample.

The estimates of the model obtained using the GRV data set appear in Table 2 while those obtained with the extended data are reported in Table 3. In addition to the estimated parameter values, the Tables also report two goodness-of-fit measures, namely the usual R^2 statistic computed with respect to s_t , and an alternative measure, denoted R_D^2 , which is computed with respect to Δs_t . Defining $RSS = \sum_{t=1}^T \hat{\eta}_t^2$ to be the sum of squared residuals, then

$$R^2 = 1 - \frac{RSS}{\sum_{t=1}^T (s_t - \bar{s})^2} \quad \text{and} \quad R_D^2 = 1 - \frac{RSS}{\sum_{t=1}^T (\Delta s_t - \bar{\delta})^2},$$

where $\bar{s} = T^{-1} \sum_{t=1}^T s_t$ and $\bar{\delta} = T^{-1} \sum_{t=1}^T \Delta s_t$. The statistic R_D^2 was suggested by Harvey (1989, p. 268) because it enables the fit of the estimated model to be related directly to that of a random walk with drift, which Table 1 suggests is a good representation of the data. For $0 < R_D^2 \leq 1$ the model is providing a better fit than the random walk with drift; for $R_D^2 = 0$ the fit is the same; while for $R_D^2 < 0$ the fit of the model is worse than the random walk with drift. Note that in this last case R_D^2 is not just confined to a unit interval and can, in principle, take on any negative value. Tables 2 and 3 also report a portmanteau-type statistic for assessing model adequacy based on the residuals $\hat{\eta}_t$. The test was proposed by Bergstrom (1990, ch. 7) as a test of dynamic specification for continuous time models, and is based on a transformation of $\hat{\eta}_t$ that, under the null hypothesis that the model is correctly specified, yields a vector of random variables having zero mean and an identity covariance matrix. Taking $\hat{P}$ to be the lower triangular Cholesky factorisation of the estimated $T \times T$ covariance matrix $\hat{\Omega}$ of $\hat{\eta}$ such that $\hat{P}\hat{P}' = \hat{\Omega}$, the statistic $S(p)$ is based on the elements $\hat{\epsilon}_t$ of the random vector $\hat{\epsilon} = \hat{P}^{-1}\hat{\eta}$ as follows:

$$S(p) = \frac{1}{(T-p)} \sum_{j=1}^p \left(\sum_{t=p+1}^T \hat{\epsilon}_t \hat{\epsilon}_{t-j} \right)^2.$$

Asymptotically, as p and $T-p$ both tend to infinity, $S(p)$ has a chi-square distribution with p degrees of freedom. The statistic is effectively testing for the absence of serial correlation of up to order p in the vector of residuals, and is reported for a value of p equal to 24.¹⁰ The test is actually quite stringent because the matrix Ω used to construct $\hat{\epsilon}$ denotes the theoretical covariance matrix of η assuming the model to be correctly specified. Departures from the assumed model will cause the actual covariance properties of $\hat{\eta}$ to diverge from Ω and hence $\hat{\epsilon}$ will not be standardised in the correct way, thereby leading to a significant value of the test statistic. The standardised residuals could, of course, be computed using the sample

¹⁰We also calculated the statistic for other values of p but the qualitative results remain the same.

covariance matrix of $\hat{\eta}$, but this results in a less stringent test of the model.¹¹ Note also that, if the model is correctly specified, the residuals $\hat{\eta}_t$ will be MA(1), and implementing a test for the absence of serial correlation in $\hat{\eta}_t$ makes no sense in this context. Hence the test is for the absence of serial correlation in the normalised residuals $\hat{\epsilon}_t$, which should be serially uncorrelated if the model provides an adequate description of serial correlation in the data.

Turning to the results themselves, Tables 2 and 3 indicate that the model provides an adequate description of the serial correlation properties of the data in both samples, with the possible exception of the Japanese Yen, where the $S(24)$ statistics are significant at the 5% level. Although the model fit is high for all currencies, as measured by the R^2 statistics, in all cases the fit is inferior to that of a simple random walk with drift, as witnessed by the R_D^2 statistics being negative, although R_D^2 is only marginally negative for Britain under the restricted model in the extended sample. Imposition of the restriction $\mu_1 = 0$ also seems to have little impact on the estimates of the other parameters, and the resulting fall in the log-likelihood is small in all cases, as we would expect if the restrictions were true. A formal test of the restriction can be carried out using the likelihood ratio test based on the maximised likelihood values in Tables 2 and 3. The resulting statistics, denoted LR , along with their marginal probability values, are reported in Table 4. In no case is the null hypothesis ($\mu_1 = 0$) rejected at the 5% significance level, although it is a close call for Germany in the GRV sample. Table 4 also reports the Schwarz Criteria for the restricted and unrestricted models, denoted SC^R and SC^U respectively. In all cases the Schwarz Criterion is minimised for the restricted model, a further indication that the restriction is valid.

One of the preoccupations in the exchange rate literature since the publication of Meese and Rogoff (1983) has been trying to explain why the forecasting performance of fundamentals-based (monetary) models is so poor relative to simple random walk models. Table 1 suggests a possible explanation, which is simply that the logarithm of exchange rates is very well approximated by a random walk with drift. Another potential explanation is that the specification of the fundamentals process used in applied work has been incorrect, although the poor forecasting performance of the models is perhaps hard to justify in view of the good fit of the models in-sample. In allowing the fundamentals to be a latent or unobservable process we are agnostic about the choice and true functional form of the fundamentals yet at the same time we allow, in principle, for complicated, even non-linear, relationships between macroeconomic variables and the fundamentals, subject to (15) being satisfied. This approach also has the potential for variables not usually considered in monetary models to be a component of the fundamentals term as well. A further, and rather

¹¹The commonly used multivariate portmanteau statistic of Hosking (1980) is based on the sample covariances but is not considered here for the reasons just outlined in the text.

important, test of the model with unobservable fundamentals is therefore to assess its ability to forecast exchange rates beyond the sample period.

We address this issue by computing forecasts from the restricted model, spanning horizons up to 24 months, beginning with the models estimated up to September 1992, the last month of the GRV sample (these estimates are contained in Table 2). The model is then re-estimated for each currency, using one extra month of data, and forecasts produced up to 24 months ahead. This is repeated a total of 48 times in addition to the initial estimates, so that there are 49 forecasts produced for each horizon. Table 5 reports the root mean square errors (RMSEs) of these forecasts at horizons 1, 3, 6, 12 and 24, as well as the ratios of these RMSEs to those obtained from forecasts based on the random walk with drift, labelled RMSE(RWD). Note that these RMSEs are estimates of the k -period ahead forecast RMSEs, as opposed to the more commonly reported RMSEs that are averages across *all* horizons up to horizon k .

Table 5 reveals that, for most horizons, the forecasts from the model are only slightly inferior to those from the random walk with drift, in many cases the RMSE ratios being only slightly greater than unity. In this respect the model is doing reasonably well, particularly when compared to the implied ratios of the RMSEs of forecasts from monetary models with respect to those from a random walk that are reported in Meese and Rogoff (1983). But, even so, the model is unable convincingly and consistently to outperform the random walk forecasts. The forecast performance is particularly poor relative to the random walk at horizon 24 for Italy and Spain, the RMSE ratios rising to 1.1435 and 1.2680 respectively.

5. CONCLUSIONS

The relationship between the exchange rate and fundamentals has been of interest for many years and, as yet, no clear consensus has emerged. Much of the debate has centred around which variables actually constitute fundamentals even though, from the point of view of econometric testing, any form we take for the fundamentals is likely to be misspecified. A promising line of research, as exemplified by GRV, is to allow the fundamentals to enter the model in a latent or unobservable manner, but we have demonstrated that the model they proposed is not identifiable. Accordingly, we have proposed a model of exchange rates with unobservable fundamentals whose parameters are identified, and in the spirit intended by GRV. We have estimated the model for five currencies against the dollar using both their original data set and an updated (extended) data set, and for all currencies in both cases have found that a restriction that makes the model compatible with the asset market view of exchange rate determination is not rejected. We also found that the fit of our model and its forecasting performance is broadly in line with, though slightly inferior to, a random walk

with drift. Nonetheless, our results compare favourably with extant results from exchange rate models based on explicit, pre-specified fundamentals. Our finding with respect to our restricted model – that the data corroborates the asset market view of exchange rates – is based on a hypothesis test that is not conjoint with the choice or specification of the fundamentals, and the estimates we obtained, derived from the exact discrete analogue of the exchange rate model, are not subject to temporal aggregation bias. These two properties taken together are not, as far as we are aware, shared by any other econometric analysis of exchange rate models other than GRV's.

Advocates of monetary models might argue that they know the variables that constitute fundamentals and so there is little or no need to allow them to be unobservable. While we feel a direct comparison with such models is beyond the scope of this paper, we would argue these models have been widely criticised over the years precisely because they do not provide an adequate description of exchange rates; see Flood and Rose (1995). The restricted and unrestricted models estimated in this paper do seem to provide an adequate description of the data within-sample and are only narrowly inferior to a random walk with drift post-sample. Our results also corroborate the asset market view of exchange rates in what is a genuine test of it.

APPENDIX

PROOF OF PROPOSITION 1. Integrating (5) from 0 to t yields

$$(16) \quad x(t) - x(0) = \int_0^t [\mu + Ax(r)] dr + \int_0^t \Sigma dW(r).$$

For any given $x(0)$, (16) has a solution, unique (with probability one) in the class of mean square continuous processes, given by

$$(17) \quad x(t) = e^{tA}x(0) + \int_0^t e^{(t-r)A}\mu dr + \int_0^t e^{(t-r)A}\Sigma dW(r);$$

see Bergstrom (1984). From (17) we obtain

$$(18) \quad \begin{aligned} x(t) &= e^A \left[e^{(t-1)A}x(0) + \int_0^{t-1} e^{(t-1-r)A}\mu dr + \int_0^{t-1} e^{(t-1-r)A}\Sigma dW(r) \right] \\ &\quad + \int_{t-1}^t e^{(t-r)A}\mu dr + \int_{t-1}^t e^{(t-r)A}\Sigma dW(r) \\ &= e^A x(t-1) + \phi + u_t, \end{aligned}$$

which yields (6) upon subtracting $x(t-1)$ from both sides. Noting that $u_t = \int_0^1 e^{rA}\Sigma dW(r)$ yields the first expression for V . To derive the simplified expressions for Φ , ϕ and V , note

first that, because $A = \gamma\beta'$,

$$\begin{aligned}
e^{rA} &= I + \sum_{j=1}^{\infty} \frac{(r\gamma\beta')^j}{j!} \\
&= I + \sum_{j=1}^{\infty} \frac{r^j}{j!} (\beta'\gamma)^{j-1} \gamma\beta' \quad (\text{because } (\gamma\beta')^j = (\beta'\gamma)^{j-1} \gamma\beta' \text{ for } j \geq 1) \\
(19) \quad &= I + \sum_{j=1}^{\infty} \frac{(r\beta'\gamma)^j}{j!} \frac{\gamma\beta'}{\beta'\gamma} = I + \frac{(e^{r\beta'\gamma} - 1)}{\beta'\gamma} \gamma\beta' = I + A\psi(r),
\end{aligned}$$

and hence $\Phi = e^A - I = A\psi(1)$ as required. Next, consider

$$(20) \quad \int_0^1 e^{rA} dr = \int_0^1 [I + A\psi(r)] dr = I + A \left(\frac{\psi(1) - 1}{\beta'\gamma} \right),$$

which yields the expression for ϕ . Finally, the expression for V follows by noting that

$$e^{rA} \Sigma^2 e^{rA'} = [I + A\psi(r)] \Sigma^2 [I + A'\psi(r)] = \Sigma^2 + (\Sigma^2 A' + A \Sigma^2) \psi(r) + A \Sigma^2 A' \psi(r)^2,$$

and that $\int_0^1 \psi(r)^2 dr = [0.5\psi(2) - 2\psi(1) + 1]/(\beta'\gamma)^2$. □

PROOF OF PROPOSITION 2. From the discrete time representation in (6) we obtain

$$(21) \quad s_t = \phi_1 + \Phi_{11}s_{t-1} + \Phi_{12}f_{t-1} + u_{1t},$$

$$(22) \quad f_t = \phi_2 + \Phi_{21}s_{t-1} + \Phi_{22}f_{t-1} + u_{2t},$$

where $u_t = (u_{1t}, u_{2t})'$. Lagging (21) by 1 period and solving for f_{t-2} yields

$$(23) \quad f_{t-2} = \frac{1}{\Phi_{12}}(s_{t-1} - \phi_1 - \Phi_{11}s_{t-2} - u_{1,t-1}),$$

while lagging (22) by 1 period and using (23) to substitute for f_{t-2} yields

$$f_{t-1} = \phi_2 + \Phi_{21}s_{t-2} + \frac{\Phi_{22}}{\Phi_{12}}(s_{t-1} - \phi_1 - \Phi_{11}s_{t-2} - u_{1,t-1}) + u_{2,t-1},$$

which can be rearranged to give

$$\begin{aligned}
(24) \quad \Phi_{12}f_{t-1} &= (\phi_2\Phi_{12} - \phi_1\Phi_{22}) + \Phi_{22}s_{t-1} + (\Phi_{12}\Phi_{21} - \Phi_{11}\Phi_{22})s_{t-2} \\
&\quad + \Phi_{12}u_{2,t-1} - \Phi_{22}u_{1,t-1}.
\end{aligned}$$

Substituting (24) into (21) yields (9) with $\eta_t = u_{1t} - \Phi_{22}u_{1,t-1} + \Phi_{12}u_{2,t-1}$, from which the variance and autocovariance expressions for η_t follow. The equation relating s_1 to s_0 and f_0 is obtained directly by setting $t = 1$ in (6), which results in $\eta_1 = u_{11}$. The equation for s_2 is derived similarly by setting $t = 2$ in (6) and substituting the expression for f_1 derived in

the previous step. The resulting disturbance term is $\eta_2 = u_{12} + \Phi_{12}u_{21}$ which gives rise to the stated variance and autocovariance terms. $\square$

PROOF OF PROPOSITION 3. We shall first establish the effect of the arbitrary parameter λ on $\phi(\theta^*)$, $\Phi(\theta^*)$ and $V(\theta^*)$. It is convenient to use a superscript $*$ to denote a quantity based on θ^* . First note that $(\beta^*)'\gamma^* = (-\lambda b, 1)(\gamma_1/\lambda, \gamma_2)' = \gamma_2 - \gamma_1 b = \beta'\gamma$ and hence $\psi^*(r) = \psi(r)$ for all θ^* . Next, observe that

$$A^* = \gamma^*(\beta^*)' = \begin{bmatrix} -\gamma_1 b & \gamma_1/\lambda \\ -\lambda\gamma_2 b & \gamma_2 \end{bmatrix}.$$

It then follows from the above and the fact that $\Phi = A\psi(1)$ that

$$\Phi_{11}^* = \Phi_{11}, \quad \Phi_{12}^* = \Phi_{12}/\lambda, \quad \Phi_{21}^* = \lambda\Phi_{21}, \quad \text{and} \quad \Phi_{22}^* = \Phi_{22}.$$

Now, because $\mu = [\mu_1, 0]'$ and $\phi = [I + A(\psi(1) - 1)/(\beta'\gamma)]\mu$, it follows immediately that

$$\phi_1^* = \phi_1 \quad \text{and} \quad \phi_2^* = \lambda\phi_2.$$

Furthermore, from the definition of the covariance matrix V , and the above results, it can be established that $V_{11}^* = V_{11}$, $V_{12}^* = \lambda V_{12}$, $V_{21}^* = \lambda V_{21}$, and $V_{22}^* = \lambda^2 V_{22}$. Applying these results to the coefficients of the ARMA(2,1) representation in Proposition 2, as well as to the variances and autocovariances, establishes that all are equal when based on θ and θ^* , with the exception of $\alpha_{12}^* = \alpha_{12}/\lambda$ and $\alpha_{23}^* = \alpha_{23}/\lambda$. However, both of these coefficients multiply the initial value of the unobserved fundamentals process, f_0 , which is implicitly also scaled by λ , so that $f_0^* = \lambda f_0$. To see this, note that the differential equation for $f(t)$, (4), when based on θ^* , requires multiplying through by λ , resulting in $df^*(t) = \gamma_2[f^*(t) - b^*s(t)]dt + \sigma_2^*dW_2(t)$. Given that the discrete time ARMA(2,1) process for s_t remains unchanged when evaluated at θ^* as compared with θ , this implies that $L(s, \theta) = L(s, \theta^*)$ for all s and λ , thereby establishing the claim in Proposition 3. $\square$

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TABLE 1
ESTIMATES OF THE RANDOM WALK WITH DRIFT MODEL
FOR THE LOGARITHM OF EXCHANGE RATES[†]

	Britain	Germany	Japan	Italy	Spain
<i>January 1974–September 1992</i>					
Drift	−0.0011 (0.0027)	−0.0030 (0.0023)	−0.0041 (0.0022)	0.0028 (0.0021)	0.0023 (0.0021)
Serial(1)	0.5252 [0.4694]	0.0028 [0.9578]	1.1814 [0.2782]	0.3431 [0.5586]	0.2195 [0.6399]
Serial(12)	0.8827 [0.5654]	1.4916 [0.1290]	1.2056 [0.2807]	1.1989 [0.2854]	0.8459 [0.6031]
ARCH(12)	0.6390 [0.8074]	0.5933 [0.8464]	1.5360 [0.1131]	0.8422 [0.6069]	0.3045 [0.9881]
<i>Extended Samples</i>					
Drift	−0.0013 (0.0017)	−0.0017 (0.0021)	−0.0030 (0.0019)	0.0031 (0.0018)	0.0029 (0.0019)
Serial(1)	2.1231 [0.1461]	0.6219 [0.4310]	2.2506 [0.1345]	2.7089 [0.1008]	0.9614 [0.3276]
Serial(12)	0.8686 [0.5795]	0.5424 [0.8860]	1.3788 [0.1746]	1.3764 [0.1765]	0.9262 [0.5210]
ARCH(12)	1.4674 [0.1351]	3.5083 [0.0001]	1.2209 [0.2673]	1.3018 [0.2166]	0.1559 [0.9995]

[†] Extended samples: January 1974–December 2000 (Britain, Japan), January 1974–December 1998 (Germany, Italy, Spain). Serial(p) denotes the Lagrange Multiplier statistic for testing for serial correlation up to order p in the disturbances, distributed as $F(p, T - p - 1)$ under the null of no serial correlation; ARCH(q) denotes the Lagrange Multiplier statistic for testing for ARCH effects of up to order q , distributed as $F(q, T - q - 1)$ under the null of no ARCH effects; and figures in square brackets denote marginal probability values.

TABLE 2
ESTIMATES OF UNRESTRICTED AND RESTRICTED
MODELS, JANUARY 1974–SEPTEMBER 1992[†]

	Britain	Germany	Japan	Italy	Spain
<i>Unrestricted Model</i>					
μ_1	0.0290 (0.0320)	−0.0629 (0.0403)	−0.0495 (0.0441)	−0.0088 (0.0178)	−0.0046 (0.0126)
γ_1	0.4563 (0.6182)	0.5310 (0.4482)	0.6142 (0.6026)	0.1217 (0.1329)	0.0707 (0.0671)
σ_1	0.0326 (0.0021)	0.0330 (0.0020)	0.0309 (0.0021)	0.0304 (0.0016)	0.0309 (0.0016)
μ_2	−0.0014 (0.0026)	−0.0025 (0.0025)	−0.0038 (0.0026)	0.0033 (0.0033)	0.0026 (0.0040)
σ_2	0.0382 (0.0063)	0.0377 (0.0057)	0.0381 (0.0060)	0.0478 (0.0163)	0.0575 (0.0238)
R^2	0.9490	0.9586	0.9850	0.9821	0.9829
R_D^2	−0.3225	−0.3508	−0.2146	−0.6626	−0.7887
$\ln L$	441.8574	439.3972	448.4601	457.7958	454.5721
$S(24)$	18.4300 [0.7817]	26.0521 [0.3505]	37.6743 [0.0374]	12.0425 [0.9794]	16.5902 [0.8656]
<i>Restricted Model</i>					
μ_1	0.0000	0.0000	0.0000	0.0000	0.0000
γ_1	0.1099 (0.1283)	0.0760 (0.0820)	0.1732 (0.1624)	0.0891 (0.0852)	0.0624 (0.0589)
σ_1	0.0333 (0.0017)	0.0338 (0.0017)	0.0319 (0.0018)	0.0305 (0.0016)	0.0310 (0.0016)
μ_2	−0.0012 (0.0029)	−0.0034 (0.0033)	−0.0043 (0.0029)	0.0030 (0.0035)	0.0023 (0.0041)
σ_2	0.0422 (0.0123)	0.0487 (0.0178)	0.0436 (0.0105)	0.0512 (0.0184)	0.0597 (0.0256)
R^2	0.9349	0.9422	0.9803	0.9816	0.9828
R_D^2	−0.6872	−0.8894	−0.6007	−0.7132	−0.8022
$\ln L$	441.3564	437.5943	447.5956	457.6410	454.4995
$S(24)$	18.6534 [0.7702]	25.6458 [0.3714]	37.5423 [0.0386]	12.0065 [0.9798]	16.2716 [0.8780]

[†] Figures in parentheses denote asymptotic standard errors; $\ln L$ denotes the maximised log-likelihood; R^2 and R_D^2 denote, respectively, the goodness of fit statistics with respect to the level and the difference of the exchange rate; $S(p)$ denotes the multivariate portmanteau statistic (defined in the text) for testing for serial correlation of up to order p in the vector of residuals; and figures in square brackets denote marginal probability values.

TABLE 3
ESTIMATES OF UNRESTRICTED AND RESTRICTED
MODELS, EXTENDED SAMPLES[†]

	Britain	Germany	Japan	Italy	Spain
<i>Unrestricted Model</i>					
μ_1	0.0352 (0.0311)	-0.0794 (0.0481)	-0.0555 (0.0387)	-0.0280 (0.0314)	-0.0069 (0.0133)
γ_1	0.7959 (0.9183)	0.7984 (0.6865)	0.7027 (0.4864)	0.4095 (0.4359)	0.1043 (0.0849)
σ_1	0.0294 (0.0024)	0.0366 (0.0023)	0.0312 (0.0019)	0.0284 (0.0018)	0.0308 (0.0014)
μ_2	-0.0014 (0.0020)	-0.0014 (0.0020)	-0.0027 (0.0022)	0.0033 (0.0023)	0.0032 (0.0029)
σ_2	0.0349 (0.0036)	0.0341 (0.0035)	0.0392 (0.0045)	0.0390 (0.0071)	0.0490 (0.0143)
R^2	0.9546	0.9639	0.9905	0.9871	0.9828
R_D^2	-0.1276	-0.2431	-0.1623	-0.3145	-0.7089
$\ln L$	662.8804	571.1441	639.3954	620.9567	608.2370
$S(24)$	28.3089 [0.2472]	19.7372 [0.7116]	38.2152 [0.0330]	18.3027 [0.7881]	23.2970 [0.5023]
<i>Restricted Model</i>					
μ_1	0.0000	0.0000	0.0000	0.0000	0.0000
γ_1	1.9229 (2.4115)	0.0239 (0.0135)	0.2663 (0.4020)	0.1705 (0.1992)	0.0956 (0.0827)
σ_1	0.0272 (0.0055)	0.0363 (0.0015)	0.0324 (0.0020)	0.0292 (0.0015)	0.0308 (0.0014)
μ_2	-0.0013 (0.0019)	-0.0024 (0.0005)	-0.0029 (0.0024)	0.0031 (0.0026)	0.0029 (0.0029)
σ_2	0.0341 (0.0028)	0.00004 (0.0424)	0.0423 (0.0076)	0.0443 (0.0116)	0.0502 (0.0153)
R^2	0.9597	0.9403	0.9880	0.9847	0.9827
R_D^2	-0.0018	-1.0587	-0.4694	-0.5575	-0.7220
$\ln L$	662.4845	570.6474	637.9450	620.3720	608.0950
$S(24)$	28.3142 [0.2470]	21.7067 [0.5968]	38.7668 [0.0289]	18.6859 [0.7685]	23.2604 [0.5045]

[†] Figures in parentheses denote asymptotic standard errors; $\ln L$ denotes the maximised log-likelihood; R^2 and R_D^2 denote, respectively, the goodness of fit statistics with respect to the level and the difference of the exchange rate; $S(p)$ denotes the multivariate portmanteau statistic (defined in the text) for testing for serial correlation of up to order p in the vector of residuals; and figures in square brackets denote marginal probability values.

TABLE 4
STATISTICAL COMPARISON OF UNRESTRICTED AND RESTRICTED MODELS[†]

	Britain	Germany	Japan	Italy	Spain
<i>January 1974–September 1992</i>					
<i>LR</i>	1.0020 [0.3168]	3.6058 [0.0576]	1.7290 [0.1885]	0.3096 [0.5779]	0.1452 [0.7032]
<i>SC^U</i>	−856.6343	−851.7139	−869.8397	−888.5110	−882.0638
<i>SC^R</i>	−861.0483	−853.5242	−873.5268	−893.6176	−887.3346
<i>Extended Samples</i>					
<i>LR</i>	0.7918 [0.3736]	0.9934 [0.3189]	2.9008 [0.0885]	1.1694 [0.2793]	0.2840 [0.5941]
<i>SC^U</i>	−1296.8570	−1113.7693	−1249.8871	−1213.3945	−1187.9552
<i>SC^R</i>	−1301.8461	−1118.4797	−1252.7671	−1217.9289	−1193.3748

[†] *LR* denotes the likelihood ratio test statistic of the null hypothesis that $\mu_1 = 0$; *SC^U* and *SC^R* denote, respectively, the Schwarz Criterion for the unrestricted and restricted models; and figures in square brackets denote marginal probability values.

TABLE 5
FORECAST SUMMARY STATISTICS[†]

Horizon	Britain	Germany	Japan	Italy	Spain
<i>RMSE</i>					
1	0.0279	0.0291	0.0325	0.0287	0.0345
3	0.0405	0.0481	0.0699	0.0519	0.0546
6	0.0481	0.0697	0.1053	0.0689	0.0746
12	0.0637	0.1164	0.1421	0.0879	0.1189
24	0.0883	0.1839	0.2601	0.1043	0.1697
<i>Ratio of RMSE to RMSE(RWD)</i>					
1	0.9990	1.0110	1.0017	0.9957	1.0109
3	1.0021	1.0207	1.0142	1.0064	1.0169
6	1.0113	1.0194	1.0225	1.0345	0.9993
12	1.0014	1.0307	1.0081	1.0326	1.0643
24	1.0104	1.0515	1.0054	1.1435	1.2680

[†] Forecasts are derived from a sequence of 49 estimations of each model in which the sample period is extended in turn by one period covering January 1974–September 1992 through to January 1974–September 1996. RMSE denotes the root mean square error of the forecasts from the restricted continuous time model; RMSE(RWD) denotes the RMSE from a random walk with drift.