HETEROGENEITY IN WAGE SETTING BEHAVIOR IN A NEW-KEYNESIAN MODEL

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Heterogeneity in Wage Setting Behavior in a New-Keynesian Model∗

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Abstract

In this paper we estimate a New-Keynesian DSGE model with heterogeneity in price and wage setting behavior. In a recent study, Coibion and Gorodnichenko (2011) develop a DSGE model, in which firms follow four different types of price setting schemes: sticky prices, sticky information, rule of thumb, or flexible prices. We enrich Coibion and Gorodnichenko (2011) framework by incorporating heterogeneity in nominal wage setting behavior among households. We solve this DSGE model and estimate it using Bayesian techniques for the United States economy for the period of 1955-2014. Our results confirm the previous findings in the literature regarding the importance of nominal rigidity in wages to better match the macroeconomic data. More importantly, we identify qualitative as well as quantitative business cycle features allowed by the heterogeneity in wage rigidity, such as the persistence in price and the wage inflation, which a standard New Keynesian model with only Calvo-type wage rigidity fails to achieve. We also show that modelling wage rigidity heterogeneity - as oppose to standard-Calvo-wages - amplifies the macroeconomic output fluctuations resulting from a technology shock whereas it mitigates the output fluctuations following a monetary tightening.

JEL Classification: C11, E24, E31, E32, E52.

Keywords: Heterogeneity; Price, Wage and Information Stickiness; Bayesian Estimation.

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1 Introduction

It is well documented that DSGE models require nominal rigidities (in addition to real frictions) to fit the data better and to replicate the dynamics observed in stylized facts. In particular, in order to be able to generate the observed persistence in output and inflation - a common result in theory-free VAR models, nominal rigidities are necessary. Nominal rigidities—which differentiate NK models from Real Business Cycle models—are usually included through an exogenous price setting behavior of firms and wage setting behavior of households. For instance, the widely applied nominal rigidity scheme proposed by Calvo (1983) assumes that in every period only a fraction of randomly selected agents are allowed to set their prices or wages. This approach, despite its convenience, is unappealing for two reasons. First, Calvo-type nominal rigidity leaves the mechanism unexplained by which a subset of firms are allowed to re-optimize their prices (or the wages for the case of households), while others are unable to do so. Second, the underlying assumption of the Calvo scheme, where agents that re-optimize their prices or wages, do in fact re-optimize at the same time is at odds with the behavior of firms as well as workers observed in microdata.

Alternative approaches have been proposed in the DSGE literature to address the above mentioned two limitations of the Calvo scheme. These alternatives include, for example, the use of state-dependent pricing and modeling of the nominal frictions using information rigidities instead of price rigidities; and, the utilization of more general specifications, which maintain the Calvo approach but incorporate some refinements and extensions to increase modelling flexibility. To this end, for instance, Coibion and Gorodnichenko (2011) suggest that modeling an economy by assuming a uniform price setting rule across firms has non-trivial consequences. In particular, consequences related to the ability of the model to fit the data; and, more importantly, for the determination of optimal monetary policy. Our paper is related to this latter strand of literature and its contribution is twofold. First, we complement the studies dealing with the general equilibrium implications of heterogeneity in nominal rigidities. This is achieved by allowing heterogeneity in prices akin to Coibion and Gorodnichenko (2011) and extending their specification simultaneously to the wage setting heterogeneity among households. Specifically, we develop a New Keynesian DSGE model where firms and households are divided in four types and use their monopolistic power to set prices and wages according to four different rules. As in Coibion and Gorodnichenko (2011), the rules that we consider are as follows: Sticky prices a la Calvo, sticky information, full flexibility, and indexation. We use Bayesian techniques to estimate our model for the United States economy for the period of 1955-2014. We also estimate alternative model specifications, where nominal-rigidity heterogeneity is considered only partially for prices as well as wages. The second contribution of our work is related to the use of the estimated models to investigate the macroeconomic impact of

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1Woodford (2003).
2There is a profusion of literature on DSGE models using the Calvo scheme for prices and wages, with Smets and Wouters (2007) being one of the most influential.
3The same dynamic for price inflation could be obtained using a model with stronger microeconomics linkages given by quadratic cost of price adjustment, as in Rotemberg (1982).
the heterogeneity in prices and wages over the business cycle — as in e.g. Aoki (2001), Carvalho (2006) and Stahlschmidt (2007).

Our key estimation results can be summarized as follows: We find that 90% of firms follow a Calvo-type price stickiness or informational stickiness when setting their prices - a quantitative result matching the findings of Coibion and Gorodnichenko (2011). On household side we find that only a total of 58% of the workers set their wages via Calvo and information stickiness rules. We find that nearly 1/5 of the households follow a flexible-wage rule, where they re-optimize their wages in every period.

We conduct a business cycle analysis utilizing our estimated model. A key result in this regard is observed against models which only consider flexible wages. The models, which don't incorporate any wage rigidity, generate implausible fluctuations of macroeconomic variables, such as for wage inflation. The impulse-responses reveal that allowing heterogeneity in wage rigidity amplifies the macroeconomic output fluctuations resulting from a technology shock whereas it mitigates the output fluctuations following a monetary tightening. We also identify interesting qualitative business cycle dynamics generated by the heterogeneity in wage rigidity, such as price and wage inflation persistence, which standard models with only Calvo-type wage rigidity fail to achieve. Finally, we also find that the model with heterogeneity in wage and price setting fits the data the best compared to models where wages are flexible and prices are set according to heterogeneous price-setting rules as in Coibion and Gorodnichenko (2011).

A key - highly policy relevant - conclusion from our analysis is that wage flexibility is decisive for both the effectiveness and costs of monetary policy. Moreover, the heterogeneity in wage staggeredness proves to be an important channel through which nominal rigidities could determine the level of monetary policy effectiveness. In this respect, our paper is in line with the mantra of the ECB about the necessity of reforms across Europe to increase labor market flexibility. Building upon this policy debate, there is an increasing attention to understand the relevance of more wage flexibility for the macroeconomy. Two important papers in this literature are Gali (2013) and Gali and Monacelli (2014). While Gali (2013) shows that in the context of a closed economy model, macroeconomic consequences of wage rigidities depend on the type of the monetary policy rule, Gali and Monacelli (2014) study the gains from wage flexibility for the case of an open economy and show that an increase in wage flexibility could reduce welfare - especially in economies under an exchange rate peg. We contribute to this policy debate by emphasizing that it is not only the level of aggregate labor market flexibility but also its sectoral composition what matters for the macroeconomic policy making.

The remainder of this paper is divided into five sections. Section 2 provides a summary of empirical findings related to the presence of heterogeneity in price and wage setting behavior. Section 3 summarizes some finding about the relevance of wage rigidity for business cycles. In Section 4, the model specification, its log-linear form, and equilibrium conditions are sketched. Sections 5 and 6 discuss the estimation methodology and present the results, respectively, while in Section 7, concluding remarks and suggestions for further research are presented.
2 Evidence on heterogeneity in price and wage setting

An extensive empirical literature shows that the price setting behavior is not homogeneous across firms and shows great variability. This variability depends, among other things, on factors like the firm size, the presence of economies of scope or the existence of explicit or implicit contracts which impede even minor adjustments in prices. Similarly, firms can be reluctant to change prices guided by the notion that consumers could wrongly relate a reduction in price with a reduction in the quality of the products, deterring in this way downward correction in prices. In addition, competition stances or failures in coordination among firms could make a firm not willing to change the price of its products for fear that competitors will not do the same. Finally, different pricing behavior could be a consequence of the traditionally studied implication of high costs (menu costs) for changing prices, which are different among firms (Hoeberichts and Stokman, 2006).

One study that is particularly relevant for this paper is Coibion and Gorodnichenko (2011), where the authors solve and estimate a traditional NK DSGE model, in which wages are assumed to be flexible but heterogeneity in price setting behavior among firms is allowed. In particular, Coibion and Gorodnichenko (2011) consider that the economy is composed of four sectors whose only difference is the constraint (or lack of it) faced when setting prices. The four sectors considered are firms that use traditional sticky prices a la Calvo, firms that face sticky information, those that adjust their prices every period according to the last period’s inflation, and firms that have flexible prices. Coibion and Gorodnichenko (2011) find that the presence of this kind of heterogeneity in prices improves the data fit performance of the model (casting doubts on specifications that only include one kind of rigidity or none at all). In addition, the authors find that the non-consideration of the heterogeneity can hinder the task of the monetary authority and complicate the achievement of its stabilization objectives.

In contrast with a large body of literature dealing with heterogeneity in rigidity of prices among firms, there are relatively fewer papers that empirically investigate the presence of heterogeneity in wage setting behavior. The scarcity of research on this topic was due to a great extent to the lack of appropriate data. Recently, high frequency wage data started to become increasingly available - leading a new strand of research to gain momentum. For example, in the case of France, using survey data for the period of 1998-2005, Bihan, Montornès and Heckel (2012) find evidence of heterogeneity in the frequency of wage adjustments across sectors, occupations, and firm size. Their findings indicate that wages are more rigid in the service sector, especially for managers, and in firms with 20-49 employees. The authors also show a seasonal pattern in wage changes, which are more likely to occur in the first quarter for total wages and in the third quarter for wages closer to the minimum wage. In addition to the usual downward rigidity, the paper also finds a non-flat hazard function for wages having a peak at the fourth quarter.

In a similar vein, using monthly administrative data from Luxembourg for the period of 2001-2006, Lünnemann and Wintr (2009) find a high degree of heterogeneity in wage flexibility for the nearly 11,000 firms considered in the sample. The authors show that the heterogeneity can be explained by firm size, whether the firm is private or public, and by industry. On top of that, the authors also document a clear seasonal pattern for changes in wages - clustered in January and October - with wage changes being more evident for civil servants and white-collar workers.

Similar results are found by Sigurdsson and Sigurdardottir (2012), who conduct an empirical analysis using administrative monthly data for Iceland between 1998 and 2010. The authors find evidence of heterogeneity in wage flexibility across industries and occupations and strong support for this heterogeneity based on firm size. With respect to industries, their findings reveal that wages are more flexible in transport and trade - and relatively more rigid in financial services. Regarding occupations, the analysis points out to lower rigidity in sales and support positions and higher rigidity for the managerial level. With regard to firm size, bigger firms tend to change their wages more frequently than smaller firms. Additionally, the authors also show evidence of a seasonal pattern in the changes of wages, with nominal wages tending to increase in January and June.

Complementing these findings, Druant et al. (2012) find evidence of heterogeneity in wage rigidity across sectors based on survey information from the Wage Dynamic Network initiative for 17 European countries. In particular, the authors illustrate that wages change more frequently in construction and least frequently in trade and market services. In the same line, by means of an econometric exercise, the authors conclude that wages are more flexible in larger firms and where the share of white-collar workers is small. Besides, their findings also suggest that competitive pressures within the sector are not relevant for wage flexibility, and that the share of white-collar workers is positively associated with wage stickiness. With respect to the seasonal pattern of wage changes, the paper documents a clear cluster of firms that change their wages principally in January, followed by a smaller cluster changing their prices in July. Druant et al. (2012) also shows that the sectors that more clearly show a seasonal pattern in their wage adjustments are market and financial services, while construction is the sector with the least evident pattern. Finally, the authors uncover a strong synchronization between price and wage changes.

Research regarding heterogeneity in wages for the United States has been scarce. Nonetheless, Barattieri, Basu and Gottschalk (2014) documented heterogeneity in the flexibility of wage setting for the U.S. between 1996 and 2004. Using data for four-month periods (infra-annual frequency) based on the results of the Survey of Income and Program Participation (SIPP), the authors show that wages are stickier in the manufacturing sector and more flexible in agriculture, mining, and services. Similarly, wages tend to be less flexible for workers in managerial occupations compared with workers in direct production. As in other studies, the authors uncover empirical evidence of a seasonal pattern in the frequency of wage adjustments - that the frequency of wage changes is slightly higher in the second half of the year, and additionally,
that the hazard function of a nominal wage is not constant, having a peak at 12 months. This behavior of the hazard function contradicts the Calvo framework.

The above mentioned evidence on heterogeneity in wages is complemented by studies that include this heterogeneity in macroeconomic models. Particularly with respect to the seasonal pattern, Olivei and Tenreyro (2007) use a VAR model and find that in quarters where wages are more flexible (3rd and 4th), the responses of GDP and GDP deflator after a monetary shock are weaker than in quarters where wages are less flexible. The authors extend the model used in Christiano, Eichenbaum and Evans (2005) to take into account the seasonal heterogeneity in wages by assuming that households set their wages according to a Calvo framework. In contrast to traditional specifications, Olivei and Tenreyro (2007) assume that the probability of resetting wages is different in each quarter (this probability is calibrated according to the empirical data for the United States). The authors show that this extended DSGE model generates impulse responses that quantitatively match those found in the actual data. In contrast to Olivei and Tenreyro (2007), where the seasonal heterogeneity is addressed, Dixon and Bihan (2010) investigate the effects of heterogeneity in the rigidity of wages across industries. The authors implement a variation of the model used by Smets and Wouters (2003), allowing for more flexibility in both price and wage setting. Using data from France, the authors calibrate two independent variations for nominal rigidities: a so-called Generalized Taylor Economy (GTE), where firms have wage spells of different durations, and a Generalized Calvo Economy (GCE), where firms have different probabilities of resetting their wage (price). As the authors highlight, these two mechanisms "allow the distribution of durations implied by the pricing model to be exactly the same as the distribution found in the actual micro-data." As a benchmark, Dixon and Bihan (2010) consider the traditional Calvo specification and set the probability of resetting prices and wages equal to the average probability for different firms. The paper finds that after the inclusion of these general specifications, the model can replicate the persistence of inflation and output observed in data. The authors also show that the GTE specification is able to generate a hump-shaped response of inflation and output that is not present in the standard Calvo framework - but widely observed in Business Cycle data.

3 Wage Rigidity and Business Cycles

There is a non-exhaustive list of studies arguing for the importance of staggered wage adjustments - alongside nominal price rigidities - in explaining business cycle data. For instance, Christiano, Eichenbaum and Evans (1999) and Christiano, Eichenbaum and Evans (2005) document that real wages are acyclical - or at best slightly procyclical, which implies that any New-Keynesian model with nominal price frictions is consistent with business cycle real wage data if nominal wage rigidities are also incorporated (Figure 1). Similarly, Christiano, Eichenbaum and Evans (1999) criticize sticky-price flexible-wage models because they imply an implausibly sharp real-wage decline after a negative monetary shock. Furthermore, Christiano,
Eichenbaum and Evans (2005), Atlig et al. (2002) and Smets and Wouters (2007) find out that models that incorporate both wage and price stickiness do a better job fitting the impulse responses - observed in the data - following a monetary shock.

In terms of the persistence, Andersen (1998) and Huang and Liu (1999) have argued that the presence of sticky wages generates more persistent real effects after a monetary perturbation compared to models of sticky prices. Finally, the model with both wage and price stickiness can capture richer wage dynamics than it can be achieved in a model with flexible-wages. For instance, in Figure 2 we depict the response of real wages to a monetary shock. In Figure 2 we can see that the size of the real-wage rise associated for a given size of effect on output declines as the degree of wage stickiness increases.

These arguments rationalize the incorporation of nominal wage frictions in a New-Keynesian DSGE framework alongside incomplete nominal price adjustments. In this paper, our aim is to model and estimate business cycle implications of the heterogeneity in simultaneous nominal adjustment frictions in wages and prices.

4 The model

Let us consider the following New-Keynesian Economy. There are three types of agents: households, firms, and a monetary authority. Firms and households interact in a framework of monopolistic competition in labor provision and in production. There is heterogeneity in the way firms set their prices and the households set their wages. Specifically, four different rules for setting prices and wages are considered among firms as well as among households. This contrasts with traditional specifications used, e.g. in Smets and Wouters (2007), where staggered price and wage setting behavior - a la Calvo (1983) - is homogeneous across all economic agents.

Following Cobion and Gorodnichenko (2011) we assume that firms are divided into four types. Firms following a standard Calvo (1983) scheme, which set their prices by taking into account the likelihood of not being able to re-optimize in the following period. Firms that follow a scheme of sticky information a la Mankiw and Reis (2002) and therefore set their prices in every period but do it using an outdated information set. Firms with flexible prices, which set prices optimally in every period; and finally, firms following a rule-of-thumb which set prices mimicking the aggregate price inflation of the previous period.
We deviate from Coibion et al. (2011) by assuming that the above mentioned four types of rules are also present in the case of wage setting among a distribution of heterogeneous household/workers who offer differentiated labor services to firms. This specification, therefore, makes it possible to encompass the heterogeneity in price and wage setting behavior simultaneously - as observed in the empirical data, which we discussed in the previous section.

The key features of our model are presented below. The full derivation can be found in the Appendix.\(^6\)

### 4.1 Firms

There is a continuum of firms indexed by \(i \in [0,1]\), producing differentiated products using the following decreasing returns to scale production function,

\[
Y_t(i) = A_t N_t(i)^{1-\alpha}.
\]

Firms share the same total productivity, which in log terms follows the AR(1) process \(a_t = \rho a_t - 1 + \epsilon_a^t\). In this setup \((1-\alpha)\) represents the elasticity of production with respect to labor while

\[
N_t(i) = \left( \int_0^1 N_t(i,j) \frac{\epsilon_w^{-1}}{\epsilon_w} d\epsilon_w \right)^{\frac{1}{\epsilon_w}}
\]

corresponds to an index of labor input, with \(N_t(i,j)\) being the demand of labor type \(j\) by firm \(i\) and \(\epsilon_w\) measures the elasticity of substitution between labor types. The labor types, which will be defined below, belong to households differing in their wage setting rules - with sticky wages, sticky information in wages, flexible wages and full indexed wages. It should be noted that, in this model, each firm uses all types of labor. Standard optimization of firms yields the demand for a particular type of labor as

\[
N_t(i,j) = N_t(i) \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w}
\]

where \(W_t(j)\) is the wage paid to that labor type while the aggregate wage rate is defined as

\[
W_t = \left( \int_0^1 W_t(j)^{1-\epsilon_w} d\epsilon_w \right)^{-\frac{1}{1-\epsilon_w}}.
\]

Firms use their monopolistic power to set product prices. In order to model the heterogeneity in the price setting behavior, we assume that there are four different "sectors" in the economy

\(^6\)The solution and estimation of the model was done using Dynare (Adjemian, Bastani and Juillard, 2011). The code is available upon request.
and that each sector sets the price of its product following a different rule. In particular, two sectors face rigidities: a fraction \( s_{sp} \) of firms follows a Calvo pricing mechanism (sticky prices), while a fraction \( s_{sp} \) follows a sticky information approach. Additionally, a fraction \( s_{sp} \) of firms sets prices in a flexible way, while a share \( s_{sp} \) of firms adjusts prices following a rule-of-thumb approach. Within the latter group, firms set prices according to the price inflation observed in the previous period.

4.1.1 Sticky prices

In the sector with sticky prices, firms follow a Calvo pricing mechanism. This means that in each period the probability of changing prices is \( (1 - \theta_{cp}) \) where \( \theta_{cp} \) is the probability of being unable to do so. The objective of these firms is to maximize the present value of their profits (1), with respect to the optimal price \( P_t^* \),

\[
\sum_{k=0}^{\infty} \theta^k_{cp} E_t \left[ Q_{t+k} \left( P_t^* Y(i)_{t+k|t} - \Psi(i)_{t+k} \{ Y(i)_{t+k|t} \} \right) \right],
\]

where \( Q_{t+k} \) is the stochastic discount factor (see the household problem below) and \( \Psi(i)_{t+k} \) is the total nominal cost. Firms optimize subject to the demand for their product, which is given by

\[
Y(i)_{t+k|t} = C_{t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p},
\]

where \( \epsilon_p \) measures the elasticity of substitution between products. The solution to this problem yields the following optimal pricing rule

\[
P_t^* = \frac{\epsilon_p}{\epsilon_p - 1} \sum_{k=0}^{\infty} \beta^k \theta_{cp}^k E_t \left[ C_{t+k}^{1-\sigma} \left( P_t^* \right)^{1-\sigma} Y(i)_{t+k|t} \right],
\]

where \( \Psi_{t+k|t} \) is the nominal marginal cost. After log-linearization, the following equation is obtained (hereafter, small caps represent the log of the variables and variables without time index represent steady state values):  

\[
p_t^* = \mu_p + (1 - \beta \theta_{cp}) \sum_{k=0}^{\infty} \beta^k \theta_{cp}^k E_t \left[ mc(i)_{t+k|t} + p_{t+k} \right],
\]

where \( mc(i)_{t+k|t} \) is the log of the real marginal cost for firm \( i \), and \( \mu_p = \frac{1}{\epsilon_p - 1} \). In Equation (2) the optimal price depends on the real marginal cost of a particular firm, which is different from the aggregate real marginal cost given that the production function exhibits decreasing return to scale. It is possible to re-write (2) as a function of the average real marginal cost of the whole economy; therefore, yielding
\[ p^*_t = (1 - \beta \theta_{cp}) \sum_{k=0}^{\infty} p^k \theta_{cp} E_t [\Theta \tilde{m}c_{k+1} + p_{k+1}] \]  

with \( \tilde{m}c_{k+1} = mc_{t+k} - mc \) and \( \Theta = \frac{a-1}{a - ac_p - 1} \). The aggregate price in this sector is then expressed by

\[ P_i = \left( \theta_{cp} P_{t-1}(i)^{1-c_p} + (1 - \theta_{cp}) (p^*_{t})^{1-c_p} \right)^{\frac{1}{1-c_p}}, \]  

where it is clear that firms that do not optimize in a given period leave their prices unchanged. Equation (4) can be used to calculate the log-linear version of price inflation in this sector as

\[ \pi^{p,3p}_t = (1 - \theta_{cp}) \{ p^*_t - p_{t-1} \}. \]

Then, using the optimal price found in (3), we get

\[ \pi^{p,3p}_t = \beta E_t [\pi_{t+1}] + \frac{(\theta_{cp} - 1)}{\theta_{cp}} \Theta (\beta \theta_{cp} - 1) \tilde{m}c_t, \]

or in terms of the output gap \( \tilde{y}_t = y_t - y^n_t \) and the real wage gap \( \tilde{\omega}_t = \omega_t - \omega^n_t \):

\[ \pi^{p,3p}_t = \beta E_t [\pi_{t+1}] \frac{(\theta_{cp} - 1)(\beta \theta_{cp} - 1) \tilde{\omega}_t}{\theta_{cp}(1 - \alpha + \alpha c_p)} + \frac{(\theta_{cp} - 1)(\beta \theta_{cp} - 1) \tilde{\pi}_{t}}{\theta_{cp}(1 - \alpha + \alpha c_p)}. \]  

Equation (5) corresponds to the standard New Keynesian Philips curve.

### 4.1.2 Sticky information

Following Mankiw and Reis (2002), in a given period only a fraction \((1 - \theta_{ip})\) of the firms in the \textit{sticky information} sector update their information, while the remaining \(\theta_{ip}\) fraction does not, so the new information takes time to reach all firms. In this scenario, the aggregate price for the sector is

\[ p_t = \sum_{j=0}^{\infty} (1 - \theta_{ip}) \theta_{ip}^{j} p_{t-j, t}, \]  

where \( p_{t-j, t} \) is the price set in period \( t \) using the outdated information from \( t - j \). Thus, \( \theta_{ip} \) is a measure of the degree of information stickiness in the sectors, i.e. if \( \theta_{ip} = 0 \) no firms use old information.
Denoting \( \{P_{t,t+k}\}_{k=0}^{\infty} \) as the future prices for a firm, which revises its price plan in period \( t \), the problem of this firm is to choose a price plan that maximizes

\[
\sum_{k=0}^{\infty} \theta_i^k E_t \{Q_{t,t+k} \{P_{t,t+k} Y(i)_{t+k} - \Psi(i)_{k+1}(Y(i)_{t+k})\}\}
\]

subject to the demand for its product,

\[ Y(i)_{t+k} = C_{t+k} \left( \frac{P_{t,t+k}}{P_{t+k}} \right)^{\epsilon_p}. \]

The solution to this problem is of the form

\[ P_{t,t+k} = \frac{\epsilon_p}{\epsilon_p - 1} E_t [MC(i)_{t+k} + P_{t+k}], \]

for \( k = 0, 1, 2, \ldots \). After log-linerization yielding

\[ p_{t,t+k} = E_t \left[ p_{t+k} - \frac{(a - 1) \hat{mc}_{t+k}}{\alpha \epsilon_p - \alpha + 1} \right], \]

which, together with the aggregate price for this sector (Equation (6)) provides us with the inflation rate in the sector with information stickiness:

\[
\pi_{p,si}^t = \frac{1 - \theta_{ip}}{\theta_{ip}} \sum_{j=1}^{\infty} \theta_{ij}^t E_{t-j} \left[ \frac{a \Delta \bar{y}_t - (a - 1) \Delta \bar{\omega}_t}{a(\epsilon_p - 1) + 1} + \pi_i^p \right] + \frac{1 - \theta_{ip}}{\theta_{ip}} \left( \frac{a \bar{y}_t - (a - 1) \bar{\omega}_t}{a(\epsilon_p - 1) + 1} \right). \quad (7)
\]

### 4.1.3 Flexible price sector

In the flexible price sector, all firms optimize prices in every period. In this case, each firm has a one-period objective function and the optimal price in log-terms is given by

\[ p_t^* = mc(i)_t - m_c + p_t, \]

while the sectoral inflation is

\[
\pi_t^{p,fl} = p_t^* - p_{t-1}.
\]

After some algebra and by using

\[ mc(i)_t = mc_t + \frac{\alpha \epsilon_p (p_t^* - p_{t-1})}{\alpha - 1} \]

and

\[ \hat{mc}_t = mc_t - m_c = \frac{a \bar{y}_t}{\alpha - 1} - \bar{\omega}_t, \]

we derive:

\[
\pi_t^{p,fl} = \pi_t^p - \frac{(a - 1) \bar{\omega}_t}{\alpha \epsilon - \alpha + 1} + \frac{a \bar{y}_t}{\alpha \epsilon - \alpha + 1}. \quad (8)
\]
4.1.4 Rule-of-thumb firms

Firms in the rule-of-thumb sector set their prices according to the prevailing price inflation rate in the previous period. Therefore, the inflation rate in this sector is simply given by:

\[ \pi_t^{p,rot} = \pi_{t-1}^p. \]

4.1.5 Aggregate price inflation

Aggregate inflation is the weighted average of the inflation prevailing in each sector:

\[ \pi_t^p = s_{sp}^p \pi_t^{p,sp} + s_{si}^p \pi_t^{p,si} + s_{fl}^p \pi_t^{p,fl} + s_{rot}^p \pi_t^{p,rot}. \] (9)

It is worth to mention that in our framework sectoral inflation rates are dependent on the aggregate price inflation.\(^7\)

4.2 Households

There is a continuum of household/workers indexed by \( j \in [0,1] \) who maximize their utility given by

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t(j), N_t(j)) \right]. \]

The instantaneous utility of each household is represented by

\[ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}. \] (10)

where \( \sigma \) and \( \phi \) denote the inverse elasticity of intertemporal substitution and the Frisch elasticity of labor supply, respectively. Households value a consumption index given by

\[ C_t(j) = \left( \int_0^1 C_t(i, j) \pi_t^{p,sp} \pi_t^{p,si} \pi_t^{p,fl} \pi_t^{p,rot} \right) d(i). \]

\(^7\)In order to guarantee that the estimated values for the share of each sector are between zero and one and add to 1, in the estimation step \( s_{sp}^p, s_{si}^p \) and \( s_{fl}^p \) were considered. Therefore \( s_{sp}^p = \hat{s}_{sp}^p, s_{si}^p = s_{si}^p (1 - \hat{s}_{sp}^p), s_{fl}^p = \hat{s}_{fl}^p \left( \hat{s}_{sp}^{p+1} - 1 \right) \left( \hat{s}_{si}^{p+1} - 1 \right) \) and \( s_{rot}^p = \left( 1 - \hat{s}_{sp}^p \right) \left( \hat{s}_{si}^{p+1} - 1 \right) \left( \hat{s}_{fl}^{p+1} - 1 \right). \) In spite of this, the reported values presented below corresponds to the shares in (9).
The consumption decision is a standard two-stage problem where in the first stage each household’s demand for a variety \( j \) is given by

\[
C_t(i, j) = C_t(j) \left( \frac{P_t(i)}{P_t(j)} \right)^{-\epsilon_p} \text{ with } P_t = \int_0^1 P_t(i)^{1-\epsilon_p} \ di \right)^{-\frac{1}{\epsilon_p}}.
\]

In the second stage the household maximizes its utility function with respect to the consumption index and labor subject to the budget constraint

\[
P_t C_t(j) + Q_t B_t \leq B_{t-1} + W_t N_t + T_t,
\]

where \( B_{t-1} \) is the number of bonds purchased in the previous period, \( Q_t = \frac{1}{1+i_t} \) is the price of the bond and \( T_t \) is a lump-sum transfer representing taxes and dividends.

The solution to the second stage problem yields the standard first order conditions, which in log terms are expressed as

\[
w_t - p_t = \sigma c_t + \phi n_t, \quad (11)
\]

\[
c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho), \quad (12)
\]

where \( i_t = -\ln Q_t \) and \( \rho = -\ln \beta \).

Given that households have monopolistic power over their differentiated labor they can set their wage rate. By analogy to the case of the price setting behavior of firms, we assume heterogeneity in wage setting behavior as well where different fractions of households set their wages following different types of rules. Specifically, a fraction \( s_w^{\text{cw}} \) of all households follows a Calvo mechanism (sticky wages) while another fraction \( s_w^{\text{si}} \) follows a sticky information approach. Additionally, a fraction \( s_w^{\text{fl}} \) is allowed to set flexible wages while the remaining \( s_w^{\text{rot}} \) adjust wages according the rate of wage inflation observed in the previous period. We label this latter group as rule-of-thumb households.

### 4.2.1 Sticky wages

Following a Calvo scheme, in every period only a fraction \( (1 - \theta_{cw}) \) of the households in the sticky wage sector can change their wages while the remaining \( \theta_{cw} \) fraction keeps the same wage as in the previous period. That means the household maximizes

\[
E_t \left[ \sum_{k=0}^{\infty} \beta^k \theta_{cw} U(C_{t+k|t}, N_{t+k|t}) \right],
\]

with respect to \( W^*_t \) subject to the constraints given by the demand for labor and the budget constraint,

\[
N(j)_{t+k|t} = \left( \frac{W^*_t}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}
\]
The solution to this problem is

\[ W_t^* = \frac{\epsilon_w}{\epsilon_w - 1} \sum_{k=0}^{\infty} E_t \left[ u_c \beta^k N_{t+k} \theta_{cw}^{k+1} \frac{MRS_{t+k}}{p_{t+k}} \right] - \sum_{k=0}^{\infty} E_t \left[ u_c N_{t+k} \theta_{cw}^{k+1} W_{t+k}^* \right], \]

where \( MRS_t = \frac{\mu_{ct}}{u_c} \) is the marginal rate of substitution. In log-terms, the equation for the optimal wage in the sticky-wage sector then becomes

\[ w_t^* = \left( 1 - \beta \theta_{cw} \right) \left( w_t - \frac{\bar{\mu}_{ct}}{\phi \epsilon_w + 1} \right) + \beta \theta_{cw} E_t w_{t+1}^*, \]

with \( \bar{\mu}_{ct} = \mu_{ct} - \mu_w \) and \( \mu_{ct} = (w_t - p_t) - mrs_t \). We can express the wage inflation for this sector as in the following:

\[
\pi_t^{w,si} = \beta E_t \left[ \pi_{t+1}^w \right] - \frac{(\theta_{cw} - 1) \bar{\omega}_t}{\theta_{cw}} \left( \frac{\beta \theta_{cw} - 1}{(\phi \epsilon_w + 1)} \right) + \frac{(\theta_{cw} - 1) \bar{\varphi}_t}{\theta_{cw}} \left( \frac{(\alpha - 1) \sigma - \phi}{(\alpha - 1) \phi \epsilon_w + 1} \right).
\]

**4.2.2 Sticky information in wages**

With sticky information in wages, the household chooses the following wage path

\[ w_{t+k} = E_t \left[ w_{t+k} - \frac{\bar{\mu}_{ct+k}}{\phi \epsilon_w + 1} \right], \]

yielding an aggregate wage rate for the sector as

\[ w_t = \sum_{j=0}^{\infty} (1 - \theta_{iw}) \theta_{iw}^j w_{i,t+j}, \]

which allows us to calculate the wage inflation for the sticky information sector as

\[
\pi_t^{w,si} = \frac{1 - \theta_{iw}}{\theta_{iw}} \sum_{j=1}^{\infty} E_{t-j} \theta_{iw}^j \left[ \pi_t^w + \frac{\Delta \bar{y}_t ((\alpha - 1) \sigma - \phi) - (\alpha - 1) \Delta \bar{\omega}_t}{(\alpha - 1) \phi \epsilon_w + 1} \right] + \frac{(1 - \theta_{iw}) \bar{\varphi}_t ((\alpha - 1) \sigma - \phi) - (\alpha - 1) \bar{\omega}_t}{(\alpha - 1) \phi \epsilon_w + 1}.
\]

(13)
According to Equation (13), wage inflation for households with informational frictions is a weighted sum of expectations of the current wage inflation for up to $j$ past periods, changes in the real wage gap, and changes in the output gap. In order to make this specification implementable, we set $j = 8$.\footnote{The value chosen for $j$ was determined for a technical reason related to the increasingly computational burden associated with higher values for $j$ in the estimation process. As noted by Verona and Wolters (2013), in papers dealing with sticky information, the truncation point ranges from 3 to 252 periods.}

### 4.2.3 Flexible wages sector

In order to calculate the optimal wage in the flexible wage sector, the easiest procedure is to solve an objective similar to the one with sticky wages but by acknowledging that the objective is now a one-period program. The solution to this program yields the following wage inflation for the flexible-wages sector:

$$\pi^w_{t}^{fl} = \pi^w_t + \frac{\bar{y}_t (a\sigma - a - \phi)}{(a - 1) [\phi e^w + 1]} - \frac{\bar{\omega}_t}{\phi e^w + 1}. \tag{14}$$

### 4.2.4 Rule-of-thumb wages

In the rule-of-thumb wages sector, households set their wages according to the wage inflation rate observed in the previous period. Therefore, the wage inflation in this sector is simply given by

$$\pi^w_{t}^{rot} = \pi^w_{t-1}.$$

### 4.2.5 Aggregate wage inflation

Analogously to the price case, the aggregate wage inflation is computed as the weighted average of the sectoral wage inflations.\footnote{The remarks in footnote 7 also apply in this case.}

$$\pi^w_t = s^w_{sp} \pi^w_{t}^{sp} + s^w_{si} \pi^w_{t}^{si} + s^w_{fl} \pi^w_{t}^{fl} + s^w_{rot} \pi^w_{t}^{rot}.$$
\[ i_t = \lambda i_{t-1} + (1-\lambda) \left( \rho + \phi_p \pi_r^P + \phi_u \pi_u^P + \phi_y \psi_t \right) + v_t, \]  

where the disturbance term is an AR(1) process with \( v_t = \rho_s v_{t-1} + \epsilon_t^y \). The parameter \( \lambda \) captures the degree of interest rate smoothing.

4.4 Equilibrium conditions

The derivation of the IS curve completes the model. This relation is derived, in output gap terms, using the Euler equation of the household (12) together with the market clearance condition \( y_t = c_t \). The IS equation then gets expressed as

\[ \tilde{y}_t = -\frac{1}{\sigma} \left( i_t - E_t \left[ \pi_{t+1}^P \right] - r_t^p \right) + E_t \left[ \tilde{y}_{t+1} \right], \]  

where the output gap - \( \tilde{y}_t = y_t - y_t^n \) - is measured with respect to the natural output and the production level prevailing in the absence of nominal rigidities is given by \( y_t^n = \psi_{ya} n_t + \theta_y \psi_t \) with \( \psi_{ya} = \frac{1+\phi}{\sigma(1-a)+\phi+a} \) and \( \theta_y = -\frac{(1-a)\mu_s-\ln(1-a)}{(1-a)(\alpha+\phi+a)} \).

Additionally, the natural interest rate is defined as \( r_t^n = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1} \), while the natural wage follows \( w_t^n = \psi_{wa} a_t + \theta_w \psi_t \) where \( \psi_{wa} = \frac{1-\alpha\psi_{wa}}{1-a} \) and \( \theta_w = -\frac{(\mu_s-\ln(1-a))(\alpha+\phi)}{\sigma(1-a)+\phi+a} \). Finally, wage and price inflations are linked according to \( w_t = w_{t-1} + \pi_{t-1}^w - \pi_t^P \).

4.5 Summary of the equations

The complete model consists of 18 endogenous variables (\( \tilde{y}_t, y_t^n, y_t, r_t^n, n_t, \pi_t^P, \pi_t^u, w_t, w_t^n, i_t, \pi_t^{p,sp}, \pi_t^{p,si}, \pi_t^{p,fi}, \pi_t^{p,rot}, \pi_t^{w,sp}, \pi_t^{w,si}, \pi_t^{w,fi}, \pi_t^{w,rot} \)), 26 parameters (\( \sigma, \alpha, \rho, \phi_p, \phi_u, \psi_t, \psi_{ya}, \psi_{wa}, \beta, \theta_{cp}, \theta_{ip}, \epsilon_p, \theta_{cw}, \theta_{iw}, \epsilon_w, \phi, \lambda, s_{sp}, s_{si}, s_{fi}, s_{rot}, s_{si}, s_{fi}, \mu_s \)) and two exogenous process (\( a_t \) and \( v_t \)). The equations of the model are re-listed in the following for ease of reference:

\[
\begin{align*}
\tilde{y}_t &= \frac{-1}{\sigma} \left( i_t - E_t \left[ \pi_{t+1}^P \right] - r_t^p \right) + E_t \left[ \tilde{y}_{t+1} \right], \\
y_t &= y_t^n - \phi_{ya} a_t + \theta_y^n \\
y_t^n &= \psi_{ya} a_t + \theta_y^n \\
y_t &= a_t + (1-\alpha) n_t \\
r_t^n &= \rho + \sigma \psi_{ya} E_t \Delta a_{t+1} \\
\pi_t^P &= \pi_{t,sp}^{p,sp} + \pi_{t,si}^{p,si} + \pi_{t,fi}^{p,fi} + \pi_{t,rot}^{p,rot}
\end{align*}
\]
\[
\pi_t^w = s_{wp}a_t + s_{w,sw} + s_{w,si} + s_{w,fl} + s_{w,rot}^{w}\rho_t
\]

\[
u_t = w_{t-1} + \pi_t^w - \pi_t^p
\]

\[
\psi_{w} = \psi_{w,a} + \theta_{w}
\]

\[
i_t = \lambda(t) + (1 - \lambda)\left(\rho + \phi_{p} n_t + \phi_{w} \pi_t^w + \phi_{r} y_t\right) + \nu_t
\]

\[
a_t = \rho_{a} a_{t-1} + \epsilon_t^{a}
\]

\[
v_t = \rho_{v} v_{t-1} + \epsilon_t^{v}
\]

\[
\pi_{t,s}^{p,s} = \beta E_{t}[\pi_{t+1}^{p,s}] - \frac{\theta_{cp}^{-1} (a - 1) (\beta \theta_{cp} - 1) \omega_t}{\theta_{cp}^{-1} (1 - a + \alpha c_{p})} + \left(\theta_{cp}^{-1} (1 - a + \alpha c_{p}) \right) + \frac{1 - \theta_{ip} (a y_t - (a - 1) \omega_t)}{\theta_{ip} (a (c_p - 1) + 1)}
\]

\[
\pi_{t,s}^{p,si} = \pi_{t,s}^{p} - (a - 1) \omega_t + \frac{a y_t}{a - a + 1}
\]

\[
\pi_{t,rot}^{p,rot} = \pi_{t,rot}^{p}
\]

\[
\pi_{t,w,sw} = \beta E_{t}[\pi_{t+1}^{w,sw}] - \frac{\theta_{cw} (a - 1) \omega_t (\beta \theta_{cw} - 1)}{\theta_{cw} (1 - a + \alpha c_{p})} + \left(\theta_{cw}^{-1} \phi_{e_w}^{-1} (a - 1) \Delta \omega_t + 1\right) + \frac{1 - \theta_{ip} (a y_t - (a - 1) \omega_t)}{\theta_{ip} (a (c_p - 1) + 1)}
\]

\[
\pi_{t,w,si} = \pi_{t,w} + \frac{\Delta \omega_t (a - 1) \sigma - \phi) - (a - 1) \Delta \omega_t}{(a - 1) \phi_{e_w}^{-1} (a - 1) \Delta \omega_t}
\]

\[
\pi_{t,w,fl} = \pi_{t,w} + \frac{\Delta \omega_t (a - 1) \sigma - \phi) - (a - 1) \Delta \omega_t}{(a - 1) \phi_{e_w}^{-1} (a - 1) \Delta \omega_t}
\]

\[
\pi_{t,w,rot} = \pi_{t,w}^{w,rot} - \pi_{t-1}^w
\]

## 5 Empirical Analysis

In order to investigate the empirical implications of the heterogeneity in wage and price setting behavior we estimate the model using Bayesian techniques. We utilize the Bayesian estimation methodology because of the well-known advantages of this full information procedure over other estimation methods. On the one hand, Bayesian techniques allow to perform the estimation using pre-sample information stemming from data or past research. The pre-sample information, which is included in the estimation procedure as priors for the distribution of the parameters, is not easy to accommodate in classical estimation methodologies like Maximum Likelihood or Generalized Method of Moments. The pre-sample information is highly valuable because it helps to improve the identification of the parameters (Dejong, Ingram and Whiteman, 2000). On the other hand, Bayesian methodology, being a full information procedure, appears more appealing than procedures based on limited information such as the Simulated Method of Moments (McFadden, 1989) or a calibration exercise. The advantage over the former is that because Simulated Method of Moments is based only on a limited number of moments, and the disadvantage of calibration is that it lacks a formal statistical founda-
We estimate the benchmark model with heterogeneity in prices and wages (hereafter the HpHw model) based on the standard priors used in the literature (Table 1). Specifically, mean, standard error, and the distribution for the share of capital in production ($\alpha$), the parameters governing the reaction of the monetary policy with respect to the output gap ($\phi_y$), to price inflation ($\phi_p$) and the degree of interest rate smoothing ($\lambda$) were set according to Smets and Wouters (2007). The standard error and the distribution for the inverse of the elasticity of intertemporal substitution ($\sigma$) are also based on Smets and Wouters (2007). The prior mean for this parameter, however, was set according to Galí (2008) who assumes log-utility for consumer’s preferences. Regarding the standard deviation of technology and policy shocks as well as the standard deviation of the measurement errors in the output gap and in wage inflation we take the priors from Rabanal and Rubio-Ramirez (2001). The parameters measuring the degree of price and wage stickiness a la Calvo ($\theta_{cp}$ and $\theta_{cw}$) follow a Beta distribution with a mean that incorporates the finding in the literature that wages tend to be less flexible than prices. By symmetry, the priors for the parameters ruling information rigidity in prices and wages ($\theta_{ip}$ and $\theta_{iw}$) were set alike the Calvo case. Similarly, the autoregressive parameters for the evolution of monetary and technology processes ($\rho_a$ and $\rho_v$) have the same first moment as in Smets and Wouters (2007) but with a smaller standard deviation for the case of the monetary process. The latter we set close to the value used in Galí (2008). Finally, the population share of each sector - with respect to price and wage setting behavior - is assumed to be equal to 0.25 with standard deviations of 0.15. We do not estimate some of the parameters because they present problems for the identification of our model. To this end, we set the Frisic elasticity of the labor supply ($\phi$) to 1 as in Coibion and Gorodnichenko (2011) and the discount factor to 0.99 as in Rabanal and Rubio-Ramirez (2001). Finally, the elasticity of substitution between goods and labor is set according to Smets and Wouters (2007).

Table 1 about here.

Following the tradition in the literature, we allow for measurement error for output gap and wage inflation. We have a total number of 4 shocks in the model. Hence, when estimating we use four observed variables: output gap, price inflation, wage inflation, and nominal interest rate. Our sample covers 1955:1 to 2014:1 in quarterly periodicity for the United States. With the exception of the interest rate, we transform all variables using the natural logarithm and detrend them using a one-sided Hodrick-Prescott filter (Stock and Watson, 1999). All data observations are from the Federal Reserve Bank of St. Louis database.

In our Bayesian estimation we use one chain - composed of 250,000 draws, 50,000 of which are discarded during the burning period. To check the stability of the chain, Geweke (1999) procedure was implemented. As it can be seen in the Appendix A.2, all parameters exhibited

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10 A brief description of Bayesian Methods and their application in DSGE models can be found in Fernández-Villaverde (2010) and An and Schorfheide (2007). A textbook approach is available in Canova (2007) and DeJong and Dave (2011).

11 We set a smaller prior standard deviation due to implausible estimated results for this parameter using a looser prior.
substantial convergence. The acceptance rate is 25.39%, which is sufficiently close to what it is considered optimal (around one forth, Koop (2003)). The (theoretical) identification of all structural components are checked using the local identification analysis described by Iskrev (2010).

6 Results

Table 2 presents the estimation results for the posterior distributions of the benchmark model with heterogeneity in price and wage setting (HpHw). In general, our estimates are in line with what have been found by other studies. For most of the parameters the standard deviation of the prior distribution is greater than the one estimated in the posterior distribution. This indicates that the data is very informative for the estimation of model parameters.

Regarding the estimated values, the model exhibits a value for the elasticity of intertemporal substitution of 0.21, which is similar to the value found by Rabanal and Rubio-Ramirez (2001) and in between what was found by Andrés, López-salido and Nelson (2005) and Rabanal and Rubio-Ramirez (2005). The share of capital in production ($\alpha$) closely matches the values previously estimated in the literature for the United States. Although we used different priors when estimating the degrees of price and wage stickiness a la Calvo, the estimated values end up being very similar to each other. The average duration of contracts with Calvo is 4 and 3.6 quarters for wages and prices, respectively. These values are comparable to what it is usually found in the literature, e.g. within the boundaries estimated by Christiano, Eichenbaum and Evans (2005). These average durations are also similar to the findings in Smets and Wouters (2007) for wages and the findings in Rabanal and Rubio-Ramirez (2001) for prices. With respect to information rigidity, the estimated values show that this is less pronounced compared to the traditional rigidity a la Calvo. In particular, wage and price contracts with informational frictions have average durations of 3.6 and 2.5 quarters, respectively. With regard to the parameters capturing the persistence of the technology and monetary disturbance processes, the estimated values are in line with the estimates argued by others in the literature. In particular, the technology process shows very high persistence while the persistence of the monetary process is much more moderate, where both estimates are highly consistent with the findings of Rabanal and Rubio-Ramirez (2005) and Gali (2008).

The results indicate that the majority of firms and households follow the Calvo rule. This supports the findings from several papers, e.g. Klenow and Kryvtsov (2008) and Carlsson and Skans (2012), which show that the Calvo specification in prices fits the data best. Both in terms

12In comparison, Coibion and Gorodnichenko (2011) found and average duration for price contracts with informational frictions ranging from 4 to 20 quarters.
of prices and wages, the share of agents who use the sticky information is relatively small -
where the fraction is smaller for price setters than wage setters. It is interesting to note that
compared to prices (and reinforcing what was found with respect to the stickiness of wages
and prices), wages appear to be more flexible given that one fifth of the households set their
wages using the flexible wage setting rule. Finally, the share of the households with indexed
wages is greater than the corresponding fraction for indexed price setters.

In order to investigate the macroeconomic implications of the heterogeneity in price and wage
setting behavior, we estimate five alternative models in addition to the benchmark Heterogeneous-
Prices-Heterogenous-Wages (hereafter HpHw) model. Specifically, we estimate two models of
flexible wage setting - one where heterogeneity in price setting (a model labeled as HpFw) is
assumed and another one with only Calvo type price setting (labeled as CpFw). Additionally,
we estimate two other models where wages are set a la Calvo. In one case of this second group,
all price setters follow a Calvo scheme as well (labeled as CpCw) and in another case we allow
for heterogeneity in the price setting behavior (labeled as HpCw). Finally, we also estimate a
model where prices are flexible while there is heterogeneity in the wage setting behavior (la-
beled as FpHw). As Table 3 illustrates, the restricted specifications can be derived from the
general model by fixing some corresponding parameters.

Table 4 presents the parameter estimates for all six models. In general, and despite the differ-
ences in specifications, the estimated parameter values are comparable across models. To give
examples for the parameters that are robust to different specifications are the degree of infor-
mation stickiness for prices and the parameter measuring the reactions of the monetary policy
to changes in the output gap. The stability of the estimates for the elasticity of intertemporal
substitution across models with rigidity in prices is also noteworthy. The average duration for
price contracts a la Calvo ranges from 2 to 3.6 quarters while for price information rigidities
the interval goes from 2.3 to 2.6 quarters. In this respect, we find that overall, Calvo price stick-
iness is higher in models that incorporate an heterogeneity in prices, and the estimate of this
kind of price stickiness becomes higher when the heterogeneity is extended to wages. Average
Calvo wage contract duration, in turn, gets estimated values between 2.6 and 6.1 quarters
and, as in the case of price duration, it becomes higher once we allow for heterogeneity in the
way wages are set. It is also worth noting that even though the priors in the three models that
take into account Calvo stickiness in prices and wages (HpHw, CpCw and HpCw) come along
with a greater stickiness in wages than in prices, the estimated posteriors for models CpCw
and HpCw show the opposite result. This is in contrast with findings in the literature but is in
line with the degree of stickiness found, for example, by Rabanal and Rubio-Ramirez (2001),
In (Table 4) we can also observe that there is a large variance in the parameter estimates measuring the reaction of the monetary policy rule with respect to the changes in price inflation. To this end, it is clear that in models where wages are assumed to be flexible, the reaction to price inflation is double the reaction observed in models where some kind of rigidity is considered in wages.

One of the main advantages of the models considering heterogeneity in the setting of prices and wages is that they offer the possibility to calculate the shares of each sector—shares that are clearly not directly observable in the data. In this respect, it is not easy to do an extensive comparison due to the scarce availability of papers considering heterogeneity; however, it is interesting to compare the estimated results with those of Coibion and Gorodnichenko (2011). Coibion and Gorodnichenko (2011) considered a model with heterogeneity in price setting only with flexible wages, which is pretty close to our model estimate labeled as HpFw. In their paper, Coibion and Gorodnichenko (2011) estimate the population fractions of 62%, 21%, 8%, and 9% for sticky prices, sticky information, flexible, and rule-of-thumb prices, respectively. These estimated shares contrast with the almost equal weights found in the HpFw model (third column of Table 4). This results, at least for the case of informational frictions, can be explained in part given that, compared to Coibion and Gorodnichenko (2011), in HpFw model the higher estimated share for this sector is compensated by a lower degree of information rigidity. Unfortunately, the specific cause of this difference is not easy to isolate given that, despite the similarities, the two models have important differences, not only in their specifications but also in the estimation methodology and the sample data used. However, it is interesting to note that once heterogeneity in wages is considered (HpHw model), the estimated shares become quite comparable. Specifically, we find in our HpHw model estimation that the majority of the firms follow a Calvo price setting behavior while the share of the firms that face some kind of stickiness in prices (sticky prices + sticky information) sum up to a solid 92%, where both results are in line with the findings of Coibion and Gorodnichenko (2011). The estimated population shares in two papers are also similar for the case of flexible prices (5% in our study vs. 8% in Coibion and Gorodnichenko (2011)). Another result to highlight is the drastic effect of the inclusion of rigidity on the estimated shares in models with heterogeneity in prices. Specifically, comparing model HpFw with models HpHw and HpCw (a model with flexible wages versus two models with some kind of rigidity in wages) we can observe that the inclusion of stickiness in wages, irrespective of the presence of heterogeneity in this stickiness, substantially increases the estimated share of firms that follow a sticky price rule. The fact that the data favors a higher share of sticky price firms once sticky wages are included suggests that these two types of nominal rigidities reinforce each other when matching the macroeconomic data, and also confirms the findings in previous empirical studies such as Druant, Fabiani and Kezdi (2009). In addition, the strong contrast among the estimated shares of firms and households in model HpHw suggest that the traditional approach of modeling prices and wages symmetrically is not appropriate.

Finally, a basic model comparison confirms the usual result in the literature that models which consider only flexible prices or wages are dominated by specifications where those are as-
sumed to be rigid (last row of Table 4). In the same line, the comparison among models shows that the posterior model probability favors the model which incorporates the simultaneous heterogeneity in price and wage setting behavior (Table 5).

6.1 Implications for monetary and technology shocks

Figures 3-6 illustrate the responses of the four key model variables with respect to a positive (tightening) monetary shock using the estimated monetary policy reaction function. For the output gap variable, the six models can be separated into two broad groups based on the incorporation of nominal wage rigidities. This is somewhat an expected result given that the estimated parameters across models within each group (nominal wage rigidity group vs. flexible wages group) are very similar, in particular the parameters measuring the population fractions of sectors with differing price setting behavior.

Figure 5 illustrates the implausibly strong reaction of the wage inflation in models with flexible wages which is, as we delineated before, at odds with macro data. This strong reaction is followed by an implausibly large reaction into the opposite direction starting the second period. This effect is more persistent in the models where no heterogeneity in prices is considered because there is not a sector with flexible prices that can be adjusted. The muted behavior of wage inflation in models with sticky wages reduces the implication of the monetary shock for real wages. In this way it helps to counteract (through the marginal cost) the effect of such shocks on inflation resulting from the decline in the output gap. As a result, models with rigid wages exhibit more subtle reactions of price inflation (Figure 4). The lower the reaction of the price inflation, the higher and more persistent response of the nominal interest rate (Figure 6). This property brings as its consequence sharper contractions in output gap following monetary shocks (Figure 3).

As a key contribution of our analysis, we find that the incorporation of heterogeneity in wage setting behavior yields impulse-responses that are different than what one obtains using a
more standard model where Calvo-type nominal wage setting is uniform across households. Particularly, given pricing behavior heterogeneity (Hp) following a monetary contraction; the output gap contracts less and recovers more quickly in the model with heterogeneous wage setters compared to the model with uniform Calvo-type wage setting. With respect to the nominal price changes; when heterogeneity in wage setting is allowed, the pricing inflation exhibits some level of persistence following a monetary contraction, whereas no such qualitative price inflation persistence is observed in any of the other alternative models (with our without price heterogeneity rigidity). Inflation persistence is an important characteristic of the business cycle data. Similarly, following a monetary contraction the model with wage setting heterogeneity generates wage inflation persistence, too. As a matter of fact the model produces initially declining wage inflation and then a follow-up stagnation in wage inflation and then rising wage inflation. None of the alternative models are capable of generating this qualitative cyclical property either. With respect to the nominal interest rate, the behavior of the set-up with heterogeneous wage rigidity shows qualitative similarities with the rest of the sticky-wages class of models. To this end, there is a striking quantitative difference though: The model with heterogeneous wage setting produces a nominal interest rate, which reacts to the monetary contraction significantly less compared to the models with standard Calvo-type nominal wage setting. Finally, none of the estimated models are able to reproduce the hump shape in the response of output gap. This anomaly can be explained by taking into account that the models in the current paper are very stylized and do not take into account other rigidities, particularly consumption habits. Having said that, we would like to note that among all the models in hand, the one with heterogeneous wage and price setting behavior is the one, which gets closest to producing a hump-shaped impulse response function for the case of the output gap.

The responses of the key macro variables following a technology shock are presented in Figures 7 to 10. As in the case of a monetary shock, the response to a technology shock are differentiated according to the presence of rigidity of nominal wages. It is worth noting that the responses for all variables are more persistent (compared to a monetary shock) given that for all models the estimated value for the persistence of the technology shock ($\rho_a$) is almost double compared to the persistence of a monetary perturbation ($\rho_\nu$). Qualitatively, the wage inflation persistence property of the heterogeneous wage setting model is observable following a technology shock as well. Price inflation behavior is comparable across the class models, which incorporate nominal wage rigidities. Quantitatively, we obtain that the output gap and the nominal interest rate in the model with heterogeneous wage setting reacts more compared to the model with standard Calvo-type nominal wage setting.

Our results indicate that the incorporation of heterogeneity in wage rigidity - in addition to the heterogeneity in price setting - enriches the business cycle implications of a standard New-Keynesian model qualitative as well as quantitatively.
Figure 7 illustrates a strong decline in the output gap for the three models incorporating nominal rigidities in wages and a comparatively more pronounced decline once heterogeneity in wages is considered (HpHzw). This reduction is explained by a strong rise in natural output, that is proportionally stronger in the model with heterogeneity in wage setting behavior. This is not a direct result of heterogeneity though, but rather a consequence of the estimated parameter values resulting from the model specification, particularly the lower estimated value of the inverse elasticity of substitution. Unsurprisingly, output gap is more persistent in models where wages present some degree of rigidity. With respect to the price inflation (Figure 8), we can observe that all of the model specifications can capture the fall in price inflation following a technology shock. The response of price inflation (to a technology shock) closely correlates with the response of the output gap, where the response of price inflation to a technology shock is stronger whenever there is a large decline in output gap (except for the model with flexible prices). Figure 9 exhibits a larger variance in the response of wage inflation - with respect to a technology shock - in those models with flexible wages, which is a consequence of the pronounced positive response observed in the first periods - followed by even a more strong decline wages in between periods three to seven. The dynamics of the nominal interest rate (Figure 10) is in line with the response of the output gap and the inflation in wages and prices. In particular, we can show that the reaction rule of the monetary policy in models with only flexible prices implies a strong reduction of the interest rate two periods following the technology shock in order to compensate for the large and more persistence reductions observed in the wage inflation.

6.2 Conclusions for monetary policy and wage flexibility

Our analysis builds upon the estimated DSGE model of Coibion and Gorodnichenko (2011) with strategic interaction among heterogeneous price setting by incorporating not only nominal wage rigidities but also heterogeneity in the wage setting behavior. That makes our model more complete and more relevant for analyzing the interaction between price and wage rigidities. Some authors (e.g. Christoffel and Linzert (2010)) separate the production process in two steps with only wage rigidities in the intermediate sector and price rigidities in the final goods sector and thus not allowing for an interaction between price and wage rigidities in the model - unlike our set-up.

In our model there is a real interaction between price and wage rigidities - reinforcing the dynamics generated by each type of friction. More price rigidities lead to more wage rigidities and vice versa. Companies have less need to adjust their price setting if wages and their
marginal costs change less frequently. Our Impulse Response Functions (IRF) in Figures 3-6 show large differences between flexible and rigid wages as already concluded in the previous section, while the difference between Calvo wages and Heterogeneity in staggered wage adjustment (respectively Cw and Hw) seems to be less obvious than the difference with Flexible wages (Fw). This is from an intuitive point of view quite understandable.

On the basis of the IRF for the output gap after a monetary shock in Figure 3 and the IRF for price inflation after a monetary shock in Figure 4 we may conclude that given price rigidities it matters a lot for monetary policy whether there are wage rigidities or not. If wages are flexible, monetary policy is much more effective than if wages are sticky. In case of wage flexibility the costs of monetary policy in terms of the output gap are also smaller that with wage rigidities. We also show that the effectiveness of monetary policy is higher with the standard-Calvo compared to the Heterogeneous wage setting model. Therefore, it turns out that heterogeneous wage setting behavior is decisive for both the effectiveness and costs of monetary policy. This supports the known mantra of the ECB President in his Introductory Statement after a regular meeting of the Governing Council on monetary policy decisions about the necessary reforms of the labor market, i.e. more labor market flexibility. Two important papers recently studied the importance of wage flexibility for the conduct of monetary policy and macroeconomic performance: Gali (2013) studies a closed economy New Keynesian model and shows that macroeconomic consequences of wage flexibility depends on the type of the monetary policy rule choice of the Central Bank. For the context of an open economy, Galí and Monacelli (2014) illustrate that an increase in wage flexibility could reduce macroeconomic welfare in economies under an exchange rate peg. We contribute to this policy debate by emphasizing that it is not only the level of aggregate labor market flexibility but also the distribution of wage setting behavior across the economy what matters for the macroeconomic policy making.

7 Conclusions

Nominal rigidities in prices and wages have proven to be two useful mechanisms that allow New Keynesian DSGE models to replicate several stylized facts observed in the data. For instance, the presence of nominal rigidities increases the persistence of some key variables after a shock and besides it allows for non-neutrality of monetary policy in the short run. However, the way this friction is usually incorporated in the DSGE models—staggering price and wage setting following Calvo (1983)—is somehow arbitrary, restrictive, and in contradiction to empirical findings regarding the timing of price and wage adjustments across economic sectors and occupations.

In order to bypass these shortcomings, several alternatives have been considered. These include models where the rigidity in prices and wages is not time dependent as in Calvo (1983) or Taylor (1999), but it is state dependent as in Caplin and Leahy (1991) or setups where nominal rigidities are the consequence of informational frictions (Mankiw and Reis, 2002). In addition,
some authors have preferred a specification where the Calvo scheme is preserved but at the same time extended to include greater heterogeneity in price and wage setting by allowing, for example, for sectoral differences in firms and types of households.

This paper can be framed within this latter strand of research. Therefore, an otherwise standard new Keynesian DSGE model is considered, but it is assumed that four types of firms (sectors) and four types of households set their prices and wages according to four different rules. These rules are: sticky prices, sticky information, full flexibility, and indexation to previous price or wage inflation. The linearized model was estimated using Bayesian techniques for the United States for the period of 1955-2014, and was then compared to more restricted specifications. Subsequently, the models were utilized to investigate the business cycle implications of wage-and-price setting heterogeneity.

Our estimation results indicate that the majority of firms and households face rigidities in prices and wages either in a time-dependent manner or as a consequence of informational frictions. At the same time, the share of households with flexibility in wages is five times greater than the corresponding share of firms with flexible prices. The estimated model is capable of producing impulse-responses that are in line with the dynamics observed in actual data. Furthermore, a key impulse-response result is observed against models which only consider flexible wages. The models, which don’t incorporate any wage rigidity, generate implausible fluctuations of macroeconomic variables, such as in wage inflation. The impulse-responses also reveal that allowing heterogeneity in wage rigidity amplify the macroeconomic fluctuations resulting from a technology shock whereas they mitigate the macroeconomic fluctuations resulting from a monetary tightening. We also identify interesting qualitative business cycle dynamics generated by the heterogeneity in wage rigidity, such as price and wage inflation persistence, which standard models with only Calvo-type wage rigidity fail to achieve. Moreover, our results suggests that the standard approach of modeling prices and wages symmetrically is a very strong assumption whose effects are not trivial. Finally, we find that the model with heterogeneity in wage and price setting fits the data the best compared to the models where wages are allowed to be flexible and prices are set according to heterogeneous price-setting rules as in Coibion and Gorodnichenko (2011).

The incorporation of heterogeneity in prices and wages in an otherwise standard DSGE model allows for compelling qualitative as well as quantitative results that deserve further attention. One natural path to follow is related to the implementation of the specification proposed in this paper, but in a medium-scale DSGE model in which other nominal, real and financial frictions are considered. Taking into account these additional frictions would help to isolate the implications of heterogeneity in prices and wages and additionally facilitate the performance of data fitting across models. Another more substantial extension would be to make the heterogeneity in the price and wage setting behavior endogenous by, for example, linking it to financial constraints faced by firms and households. These extensions we leave to future research.
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A Appendix

A.1 Complete derivation of the model

A.1.1 Households

Households decide the amount of consumption, labor and, giving that they have monopolistic power over the labor, their wage. With respect to consumption and labor, households maximize

$$\max_{C_t(j),N_t(j)} E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t(j), N_t(j)) \right]$$  \hspace{1cm} (17)

subject to their budget constraint (18) and the consumption index (19):

$$\int_0^1 P_t(i) C_t(i, j) di + Q_t B_t \leq B_{t-1} + W_t N_t(j) + T_t$$ \hspace{1cm} (18)

$$C_t(j) = \left( \int_0^1 C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}}.$$ \hspace{1cm} (19)

This is a standard two stage problem where in the first stage households choose how to allocate their consumption among the different goods. Therefore, their problem is this stage is

$$\min_{C_t(i, j)} \int_0^1 P_t(i) C_t(i, j) di \text{ s.t. } C_t(j) = \left( \int_0^1 C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}},$$

which can be solve using the Lagrangian

$$\mathcal{L} = \int_0^1 P_t(i) C_t(i, j) di + \lambda_t \left[ C_t(j) - \left( \int_0^1 C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}} \right],$$

and calculating the F.O.C.s:

$$\frac{\partial \mathcal{L}}{\partial C_t(i, j)} = P_t(i) - \lambda_t \left( \int_0^1 C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}} - \frac{\epsilon_{p-1}}{\epsilon_p} C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} - 0$$
\[ P_t(i) - \lambda_t \left( \int_0^1 C_t(i, j) \frac{\epsilon_{p-1}}{\epsilon_p} \, di \right)^{\frac{1}{\epsilon_p}} C_t(i, j)^{-\frac{1}{\epsilon_p}} = 0 \]

\[ P_t(i) - \lambda_t C_t(j)^{\frac{1}{\epsilon_p}} C_t(i, j)^{-\frac{1}{\epsilon_p}} = 0 \]

\[ C_t(i, j) = \left( \frac{P_t(i)}{\lambda_t} \right)^{-\epsilon_p} C_t(j). \quad (20) \]

Replacing (20) in (19),

\[ C_t(j) = \left( \int_0^1 C_t(i, j)^{\frac{\epsilon_{p-1}}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}} = \left( \int_0^1 \left( \left( \frac{P_t(i)}{\lambda_t} \right)^{-\epsilon_p} C_t(j) \right)^{\frac{\epsilon_{p-1}}{\epsilon_p}} \, di \right) \]

\[ = C_t(j) \left( \frac{1}{\lambda_t} \right)^{-\epsilon_p} \left( \int_0^1 P_t(i)^{1-\epsilon_p} \, di \right)^{\frac{\epsilon_p}{\epsilon_{p-1}}} \]

\[ \lambda_t = \left( \int_0^1 P_t(i)^{1-\epsilon_p} \, di \right)^{-\epsilon_p} \equiv P_t, \quad (21) \]

which is the price index and can be used in (20) to get the demand of each good as

\[ C_t(i, j) = C_t(j) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p}. \quad (22) \]

Finally, using (21) and (22) in \( \int_0^1 P_t(i)C_t(i, j) \, di \):

\[ \int_0^1 P_t(i)C_t(i, j) \, di = \int_0^1 P_t(i)C_t(j) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p} \, di \]

\[ = C_t(j)P_t^{\epsilon_p} \int_0^1 P_t(i)^{1-\epsilon_p} \, di \]

\[ = C_t(j)P_t^{\epsilon_p} P_t^{1-\epsilon_p} \]

\[ = C_t(j)P_t, \quad (23) \]

makes it possible to eliminate the integral in the budget constraint used in (18) and in Section 4.2.

The second stage of the household's problem is to maximize (17) subject to (18) (after use (23)). The Lagrangian in this case is:

\[ \mathcal{L} = E_0 \sum_{t=0}^{\infty} \left[ \beta^t U(C_t(j), N_t(j)) - \lambda_t \left( C_t(j)P_t + Q_tB_t - B_{t-1} - W_t N_t(j) - T_t \right) \right], \]
with the following F.O.C.s

\[
\frac{\partial \mathcal{L}}{\partial C_t(j)} = \beta_t U_{C_t} - P_t \lambda_t = 0, \quad (24)
\]

\[
\frac{\partial \mathcal{L}}{\partial N_t(j)} = \beta_t U_{N_t} + \lambda_t W_t = 0, \quad (25)
\]

\[
\frac{\partial \mathcal{L}}{\partial B_t} = E_t \left[ \lambda_{t+1} \right] - Q_t \lambda_t = 0. \quad (26)
\]

From (26)

\[
\frac{1}{1 + i_t} \equiv Q_t = \frac{E_t \left[ \lambda_{t+1} \right]}{\lambda_t}. \quad (27)
\]

From (24) \( U_{C_t} = \frac{P_t \lambda_t}{\beta_t} \) and \( U_{C_{t+1}} = \frac{E_t \left[ P_{t+1} \lambda_{t+1} \right]}{\beta_{t+1}} \) so

\[
\frac{\beta_t P_t \lambda_t}{E_t \left[ P_{t+1} \lambda_{t+1} \right]} = \frac{U_{C_t}}{E_t \left[ U_{C_{t+1}} \right]}, \quad \frac{\beta_t P_t}{E_t \left[ P_{t+1} Q_t \right]} = \frac{U_{C_t}}{E_t \left[ U_{C_{t+1}} \right]}, \quad \frac{\beta_t \left(1 + i_t \right) P_t}{E_t \left[ P_{t+1} \right]} = \frac{U_{C_t}}{E_t \left[ U_{C_{t+1}} \right]}, \quad \frac{\beta_t \left(1 + i_t \right) P_t E_t \left[ U_{C_{t+1}} \right]}{P_{t+1} U_{C_t}} = 1. \quad (27)
\]

Equating (25) and (24)

\[
W_t P_t = \frac{U_{N_t}}{U_{C_t}}. \quad (28)
\]

Using the assumed utility function giving by (10), (27) and (28) become (hereafter, to simplify notation, the expectation operator is omitted when there is no ambiguity)

\[
\frac{\beta_t \left(1 + i_t \right) P_t C_t^\varphi C_{t+1}^{-\varphi}}{P_{t+1}} = 1 \quad (29)
\]

\[
\frac{W_t}{P_t} = C_t^\varphi N_t^\phi. \quad (30)
\]

The two previous equation are then log-linearized around the steady state. This procedure, which is repeatedly used in this Appendix, is sketched here for illustrative purposes. As mentioned, lower caps represent natural logarithm and variables without time index stand for
steady state values. Following the steps presented by Uhlig (1995), in particular the fact that a variable \( X_t = X e^{\hat{x}_t} \) where \( \hat{x}_t = x_t - x \) and \( e^{\hat{x}_t} \approx 1 + \hat{x}_t \), Equation (29) can be written then as

\[
\frac{\beta e^{p_t - \hat{p}_{t+1} - \hat{Q}_t} \left( C e^{\hat{c}_t} \right) \left( C e^{\hat{c}_{t+1}} \right)^{-\sigma}}{Q} = 1
\]

\[
\frac{\beta e^{\hat{x}_t - \hat{x}_{t+1} + \hat{p}_t - \hat{p}_{t+1} - \hat{Q}_t}}{Q} = 1
\]

\[
\frac{\beta \left( \sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{p}_t - \hat{p}_{t+1} - \hat{Q}_t + 1 \right)}{Q} = 1.
\]  

(31)

In steady state (29) becomes \( Q = \beta \) which can be used in (31) to get

\[
\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{p}_t - \hat{p}_{t+1} - \hat{Q}_t + 1 = 1
\]

\[
\sigma (c_t - c) - \sigma (c_{t+1} - c) + p_t - p_{t+1} - \ln(Q_t) + \ln(Q) + 1 = 1
\]

\[
c_t = \frac{\sigma c_{t+1} - p_t + p_{t+1} + \ln(Q_t) - \ln(Q)}{\sigma}.
\]

Using \( \ln(Q) = \ln(\beta) \), \( \ln(Q_t) = -i_t \) and \( \ln(\beta) = -\rho \) the last equation can be written as

\[
c_t = \frac{\sigma c_{t+1} - i_t - p_t + p_{t+1} + \rho}{\sigma}
\]

\[
c_t = E_t \left[ c_{t+1} \right] - \frac{1}{\sigma} \left( i_t - E_t \left[ \pi^p_{t+1} \right] - \rho \right),
\]

(32)

which is Equation (12) in the paper. A similar procedure could be used to log-linearized Equation (30) and get (11).

Households can use their market power to set their wage according to the following rules:

**Sticky wages a la Calvo**  Households maximize, choosing their wage \( (W^*_t) \), the expected present value of the utilities generated over the period when they cannot change their wage

\[
E_t \left[ \sum_{k=0}^{\infty} \beta^k \theta_k U(C_{t+k|t}, N(j)_{t+k|t}) \right],
\]

(33)

subject to the demand for labor (34) and their budget constraints:
The first order condition for this problem is given by (again, the expectation operator is omitted when there is no ambiguity)

\[
\frac{\sum_{k=0}^{\infty} \beta^k \theta_u^w U_t \left( N_{k+1} W_t^{\epsilon_u} W_{k+1}^{\epsilon_w} - \epsilon_u N_{k+1} W_t^{\epsilon_u} W_{k+1}^{\epsilon_w} \right)}{P_{k+1}} - U_W \epsilon_N W_{k+1} W_t^{\epsilon_u-1} W_{k+1}^{\epsilon_w} = 0
\]

Equation (36) is binding giving the concavity of the utility function and therefore can be solve for \( C_{t+k|t} \). This, together with (34), can be inserted in (33) to obtain an unconstrained problem

\[
\max_{W_t} \left[ \sum_{k=0}^{\infty} \beta^k \theta_u^w U_t \left( D_{k+1|t} + T_{k+1|t} + W_t^{\epsilon_u} N_{k+1|t} \right) - D_{k+1|t+1} Q_{k+1|t+1} \right] - U_W \epsilon_N W_{k+1} W_t^{\epsilon_u-1} W_{k+1}^{\epsilon_w}.
\]

The first order condition for this problem is given by (again, the expectation operator is omitted when there is no ambiguity)

\[
W_t^* \sum_{k=0}^{\infty} \frac{\left( U_t \beta^k N_{k+1} W_{k+1}^{\epsilon_u} \right) - U_W \epsilon_N W_{k+1} W_t^{\epsilon_u-1} W_{k+1}^{\epsilon_w}}{P_{k+1}} = \epsilon_u \epsilon_w \sum_{k=0}^{\infty} \frac{W_t^{\epsilon_u-1} W_{k+1}^{\epsilon_w}}{P_{k+1}} - U_W \epsilon_N W_{k+1} W_t^{\epsilon_u-1} W_{k+1}^{\epsilon_w}.
\]

Defining \( MRS_{t+k|t} = -\frac{U_W}{\epsilon_u} \), the previous equation can be written as

\[
W_t^* = \frac{\epsilon_u}{\epsilon_u - 1} \frac{\sum_{k=0}^{\infty} U_t \beta^k N_{k+1} W_{k+1}^{\epsilon_u} MRS_{t+k|t} W_{k+1}^{\epsilon_w}}{\sum_{k=0}^{\infty} U_t \beta^k N_{k+1} W_{k+1}^{\epsilon_u} W_{k+1}^{\epsilon_w}}.
\]
which is the optimal wage for households with sticky wages and it is the equation that should be log-linearized. Noting that with flexible wages \( \theta_w = 0 \), the solution to the previous problem is instead \( W^*_t = -\frac{U_N P}{\epsilon_w c_{w-1}} \) which can be written as \( W^*_t = \frac{P MRS_{t1}}{\epsilon_w - 1} \), therefore, in steady state:

\[
\frac{W}{P} = \frac{\epsilon_w MRS}{\epsilon_w - 1},
\]

(38)
equation that is useful for the log-linearization step. Log linearization of (37) yields

\[
\sum_{k=0}^{\infty} \frac{\beta^k \theta_w^k W^{c_{w+1}} U_c \left( \bar{\epsilon}_k + \epsilon \bar{w}_k + \bar{u}_t^* + 1 \right)}{p} = \sum_{k=0}^{\infty} \frac{U_c MRS \epsilon_w \theta_w^k W^{c_w} \left( \bar{mrs}_{k+t|t} + \bar{\epsilon}_k + \epsilon \bar{w}_k + 1 \right)}{\epsilon_w - 1},
\]

(39)
using (38) in (39)

\[
\sum_{k=0}^{\infty} N \beta^k \theta_w^k W^{c_{w+1}} U_c \left( \bar{\epsilon}_k + \epsilon \bar{w}_k + \bar{u}_t^* + 1 \right) = \sum_{k=0}^{\infty} N U_c \frac{\epsilon_w \theta_w^k W^{c_w} \left( \bar{mrs}_{k+t|t} + \bar{\epsilon}_k + \epsilon \bar{w}_k + 1 \right)}{\epsilon_w - 1},
\]

which can be solved for \( w_t^* \) (taking into account that, for example, \( \bar{p}_{k+t} = p_{k+t} - p \)) to get

\[
w_t^* = \left( 1 - \beta \theta_w \right) \sum_{k=0}^{\infty} \beta^k \theta_w^k (mrs_{k+t|t} + \bar{p}_{k+t}) - mrs + (w - p);
\]
defining \( \mu_w = (w - p) - mrs = \ln \left( \frac{\epsilon}{\epsilon_w - 1} \right) \), the last equation becomes

\[
w_t^* = \mu_w + \left( 1 - \beta \theta_w \right) \sum_{k=0}^{\infty} \beta^k \theta_w^k (mrs_{k+t|t} + \bar{p}_{k+t}).
\]

(40)
Using (10), it is easy to calculate \( MRS_{k+t|t} = C_{k+t|t}^\phi N_{k+t|t}^\phi \) and therefore \( mrs_{k+t|t} = \phi n_{k+t|t} + \sigma \epsilon_{k+t} \). Additionally, from (34),

\[
n_{k+t|t} = n_{k+t} - \epsilon_w \left( w_t^* - w_{k+t} \right).
\]

(41)
Defining the economy’s average marginal rate of substitution as \( mrs_{k+t} = \phi n_{k+t} + \sigma c_{k+t} \) it is possible to calculate

\[
\begin{align*}
    mrs_{k+t+1} - mrs_{k+t} &= \phi n_{k+t+1} - (\sigma c_{k+t} + \phi n_{k+t}) + \sigma c_{k+t} \\
    mrs_{k+t} &= \phi (n_{k+t} - n_{k+t}) + mrs_{k+t}.
\end{align*}
\] (42)

Using (41) in (42):

\[
\begin{align*}
    mrs_{k+t} &= mrs_{k+t} - \phi \epsilon_w (w_t^* - w_{k+t}).
\end{align*}
\] (43)

Replacing (43) in (40) and solving for \( w_t^* \)

\[
\begin{align*}
    w_t^* &= \frac{(1 - \beta \theta_w) \sum_{k=0}^{\infty} \beta^k \theta_w^k (mrs_{k+t} + p_{k+t} + \phi \epsilon_w w_{k+t} + \mu_w)}{\phi \epsilon_w + 1},
\end{align*}
\] (44)

as before, defining \( \mu_{w,k+t} = (w_{k+t} - p_{k+t}) - mrs_{k+t} \) or equivalently \( p_{k+t} = mrs_{k+t} - \mu_{w,k+t} + w_{k+t} \) can be replaced in (44) to get the optimal wage for households with sticky wages as

\[
\begin{align*}
    w_t^* &= (1 - \beta \theta_w) \left( w_t - \frac{\hat{\mu}_{w,t}}{\phi \epsilon_w + 1} \right) + \beta \theta_w \epsilon_t w_{t+1}^*.
\end{align*}
\] (45)

The aggregate wage in this sector is given by

\[
\begin{align*}
    W_t = \left( \theta_w w_{t-1}^{1-\epsilon_w} + (1 - \theta_w) W_{t-1}^{1-\epsilon_w} \right)^{1/\epsilon_w},
\end{align*}
\]

which in log terms becomes

\[
\begin{align*}
    w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*.
\end{align*}
\] (46)

and can be used to calculate the inflation in this sector

\[
\begin{align*}
    \pi_t^{w,sw} = (\theta_w - 1) (w_{t-1} - w_t^*).
\end{align*}
\] (47)

Finally, using (45), (46) and (47) it is possible to calculate the inflation in this sector as:

\[
\begin{align*}
    \pi_t^{w,sw} &= \beta E_t \pi_{t+1}^{w,sw} - \frac{\hat{\mu}_{w,t} (\theta_w - 1) (\beta \theta_w - 1)}{\theta_w (\phi \epsilon_w + 1)}.
\end{align*}
\] (48)

From the production function in log form \( (y_t = a_t + (1 - a)n_t) \) it is possible to solve for labor as
\[ n_t = \frac{y_t - a_t}{1 - \alpha}, \quad (49) \]

it was already established that

\[ mrs_t = \sigma c_t + \phi n_t, \quad (50) \]

and

\[ \mu_{t,w} = (w_t - p_t) - mrs_t. \quad (51) \]

Defining the real wage as \( \omega_t = w_t - p_t \), noting that in this model \( y_t = c_t \) and using (49), (50) and (51):

\[
\hat{\mu}_{w,t} = \mu_{t,w} - \mu_w = (w_t - p_t) - mrs_t - (w - p) - mrs
\]

\[
= \omega_t - mrs_t - (\omega^n - mrs)
\]

\[
= (c\sigma + n\phi) - (\sigma c_t + \phi n_t) + \omega_t - \omega^n
\]

\[
= (y^n\sigma + n\phi) - (\sigma c_t + \phi n_t) + \omega_t - \omega^n
\]

\[
= \frac{\phi (y^n - a_t)}{1 - \alpha} - \frac{\phi (y_t - a_t)}{1 - \alpha} - \sigma y_t + \omega_t + \sigma y^n - \omega^n
\]

\[
= \frac{(\alpha \sigma - \sigma - \phi) (y^n - y_t)}{\alpha - 1} + \omega_t - \omega^n
\]

\[ \hat{\mu}_{w,t} = \frac{\tilde{\omega}_t - (\alpha \sigma - \sigma - \phi) (y^n - y_t)}{\alpha - 1} \quad (52) \]

where the superscript \( n \) denotes natural levels which will be defined below, and variables with tilde stand for gaps, e.g. \( \tilde{y}_t = y_t - y^n \). Finally, replacing (52) in (48) the wage inflation for sticky wages is obtained:

\[
\pi^{w,sw}_t = \beta E_t [n_{t+1}^{w}] - \frac{(\theta_{w} - 1)}{\theta_w} \left( \tilde{\omega}_t (\beta \theta_w - 1) \right) \frac{(\phi c_w + 1)}{\theta_w} \left( \tilde{y}_t (\alpha \sigma - \sigma - \phi) (\beta \theta_w - 1) \right) \frac{(\alpha - 1)}{\theta_w} \left( \phi c_w + 1 \right).
\]

**Sticky information in wages** For this type of household, a fraction \( 1 - \theta_w \) adjusts its wage while \( \theta_w \) does not do it, therefore in every period the wage for each household is given by

\[
W(j)_{t} = (1 - \theta_w) W(j)_{t,t} + (1 - \theta_w) \theta_w W(j)_{t-1,t} + (1 - \theta_w) \theta_w^2 W(j)_{t-2,t} - \cdots = \sum_{j=0}^{\infty} (1 - \theta_w) \theta_w^j W(j)_{t-j,t}.
\]

(53)
where $W_{t-j,t}$ represents the wage chosen for period $t$ in period $t-j$. As will be shown below the aggregate wage in this sector is given by $W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w}dj\right)^{1/\epsilon_w}$, which combined with (53) and after log-linearization yields:

$$w_t = \sum_{j=0}^{\infty} (1-\theta_w)\theta_w^{j} w_{t-j,t},$$

(54)

Households in this sector seek to maximize, choosing $W_{t,k+t}^*$ for $k = 0, 1, 2, \ldots$

$$E_t \left[ \sum_{k=0}^{\infty} \theta_w^{k} U(C_{t+k}, N(j)_{t+k}) \right],$$

subject to

$$N(j)_{k+t} = N_{k+t} \left( \frac{W_{t,k+t}^*}{W_{k+t}} \right)^{-\epsilon_w}$$

(55)

$$C_{k+t}P_{k+t} + D_{k+t+1}Q_{k+t,k+t+1} = N_{k+t}W_{t,k+t}^* + D_{k+t} + T_{k+t}.$$ 

Inserting the two constraints in the objective function the problem becomes an unrestricted one given by

$$E_t \sum_{k=0}^{\infty} \theta_w^{k} U \left( \frac{D_{k+t} + T_{k+t} - D_{k+t+1}Q_{k+t,k+t+1} + N_{k+t}W_{t,k+t}^* \left( \frac{W_{t,k+t}^*}{W_{k+t}} \right)^{-\epsilon_w}}{P_{k+t}}, N_{k+t} \left( \frac{W_{t,k+t}^*}{W_{k+t}} \right)^{-\epsilon_w} \right).$$

The F.O.C. with respect to $W_{t,k+t}^*$ is

$$U_C \left( N_{k+t}W_{t,k+t}^{\epsilon_w} W_{t,k+t}^{\epsilon_w-\epsilon_w} - \epsilon_w N_{k+t}W_{t,k+t}^{\epsilon_w} W_{t,k+t}^{\epsilon_w-\epsilon_w} \right) = -U_N \epsilon_w N_{k+t}W_{t,k+t}^{\epsilon_w} W_{t,k+t}^{\epsilon_w-\epsilon_w-1} = 0,$$

or after solving for $W_{t,k+t}^*$

$$W_{t,k+t}^* = -\frac{U_N \epsilon_w P_{k+t}}{U_C (\epsilon_w - 1)},$$

(56)

Defining $MRS(j)_{k+t} = -\frac{U_N}{U_C} \epsilon_w$, (56) can be written as

39
\[
W_{t,k+t}^* = \frac{e_w MRS(j)_{k+t} P_{k+t}}{\epsilon_w - 1},
\]

with steady state given by \(W^* = \frac{e_w MRS P}{\epsilon_w - 1}\). Log-linearization of (57) yields

\[
w_{t,k+t}^* = mrs(j)_{k+t} + p_{k+t} + \mu_w,
\]

where as before \(\mu_w = (w - p) - mrs\).

Similarly to the case of sticky wages, the marginal rate of substitution of household \(j\) is \(mrs(j)_{k+t} = \phi n(j)_{k+t} + \sigma c_{k+t}\) while the aggregate marginal rate of substitution of the economy is given by \(mrs_{k+t} = \phi n_{k+t} + \sigma c_{k+t}\). The using the log-linear version of (55), that is,

\[
n(j)_{k+t} = n_{k+t} - \epsilon_w \left( w_{t,k+t}^* - w_{k+t} \right),
\]

it is possible to calculate

\[
mrs(j)_{k+t} - mrs_{k+t} = \phi n(j)_{k+t} - \left( \sigma c_{k+t} + \phi n_{k+t} \right) + \sigma c_{k+t} \]

\[
mrs(j)_{k+t} = \phi \epsilon_w \left( w_{k+t} - w_{t,k+t}^* \right) + mrs_{k+t}.
\]

Using (59) in (58) and solving for \(w_{t,k+t}^*\) gives

\[
w_{t,k+t}^* = \frac{mrs_{k+t} + p_{k+t} + \phi \epsilon_w w_{k+t} + \mu_w}{\phi \epsilon_w + 1}.
\]

From \(\mu_{w,k+t} = (w_{k+t} - p_{k+t}) - mrs_{k+t}\), \(p_{k+t}\) can be solved and replaced in (60) to get

\[
w_{t,k+t}^* = \frac{w_{k+t} \left( \phi \epsilon_w + 1 \right) + \mu_w - \mu_{w,k+t}}{\phi \epsilon_w + 1},
\]

or defining \(\bar{\mu}_{w,k+t} = \mu_{w,k+t} - \mu_w\),

\[
w_{t,k+t}^* = E_t \left[ w_{k+t} - \frac{\bar{\mu}_{w,k+t}}{\phi \epsilon_w + 1} \right].
\]

Equation (61) can be used in (54) to calculate wage inflation for households with sticky information. In this case
\[ w_t = \sum_{j=0}^{\infty} (1 - \theta_w) \theta_w^j E_{t-j} \left( w_t - \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \right), \quad (62) \]

and

\[ w_{t-1} = \sum_{j=0}^{\infty} (1 - \theta_w) \theta_w^j E_{t-1-j} \left( w_{t-1} - \frac{\tilde{\mu}_{w,t-1}}{\phi \epsilon_w + 1} \right). \quad (63) \]

It is convenient to write Equation (62) as

\[ w_t = \left( w_t - \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \right) (1 - \theta_w) + \sum_{j=0}^{\infty} (1 - \theta_w) \theta_w^j E_{t-j} \left( w_t - \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \right), \quad (64) \]

and multiply both sides of (63) by \( \theta_w \) and then calculate \( w_{t-1} - \theta_w w_t \). After some algebra the following equation in obtained

\[ \theta_w \pi_{w,si} = \sum_{j=0}^{\infty} \sum_{j=1}^{\infty} (1 - \theta_w) \theta_w^j E_{t-j} \left( \pi_{w,t} + \Delta \tilde{y}_{t-1} \sigma - \sigma - \phi \right) \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \]

or after rearrangement,

\[ \pi_{w,si} = \sum_{j=1}^{\infty} \sum_{j=1}^{\infty} \theta_w^j E_{t-j} \left( \pi_{w,t} + \Delta \tilde{y}_{t-1} \sigma - \sigma - \phi \right) \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \]

Using (52) twice it is possible to calculate

\[ \Delta \tilde{y}_{t} = \Delta \tilde{y}_{t-1} (\alpha \sigma - \sigma - \phi) \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \]

Finally, replacing (52) and (66) in (65) the wage inflation for households with sticky information in terms of the output gap and the real wage gap is:

\[ \pi_{w,si} = \frac{1 - \theta_w}{\theta_w} \sum_{j=1}^{\infty} E_{t-j} \theta_w^j \left[ \pi_{w,t} + \Delta \tilde{y}_{t} (\alpha - 1) \sigma - \sigma - \phi \right] \frac{\tilde{\mu}_{w,t}}{\phi \epsilon_w + 1} \]

which is Equation (13) in the paper.
**Flexible wages** For this type of household the aggregate wage is defined, again, by
\[ W_t = \left( \int_0^1 W_t(j)^{1-\epsilon_w} d j \right)^{1/1-\epsilon_w}. \]
Letting \( W_t^* \) represent the optimal wage set by this household, the wage inflation with flexible wages is given, after log-linearization by
\[ \pi^{w, fl}_t = W_t^* - W_{t-1}. \tag{67} \]

The problem of households with flexible wage is similar to the problem in the sticky wages but with a new framework a la Calvo framework but taking into account that in this case it becomes a one period problem. Therefore, after replacing the constraints in the objective function, the problem to solve is

\[
\max_{W_t^*} \left( D_{t+1} + T_t + N_t W_t^* \left( \frac{W_t^*}{W_t} \right)^{-\epsilon_w} - D_{t+1} Q_{t+1} - N_t \left( \frac{W_t^*}{W_t} \right)^{-\epsilon_w} \right),
\]
whose first order condition is
\[
U_C \left( \frac{N_t W_t^* \left( \frac{W_t^*}{W_t} \right)^{-\epsilon_w}}{P_t} - \frac{N_t W_t^* \epsilon_w \left( \frac{W_t^*}{W_t} \right)^{-\epsilon_w-1}}{W_t} \right) = U_N N_t \epsilon_w \left( \frac{W_t^*}{W_t} \right)^{-\epsilon_w-1} = 0.
\]
Solving the previous equation for \( W_t^* \) the optimal wage is found as
\[ W_t^* = -\frac{U_N P_t \epsilon_w}{U_C (\epsilon_w - 1)}. \tag{68} \]
Defining \( MRS_{t|t} = -\frac{U_N}{U_C} \), (68) can be written as
\[ W_t^* = \frac{MRS_{t|t} P_t \epsilon_w}{\epsilon_w - 1}, \]
which, in log-linear terms becomes
\[ w_t^* = \mu_w + mrs_{t|t} + p_t, \tag{69} \]
where as defined previously,
\[ \mu_w = (w - p) - mr s. \tag{70} \]
Using \( mrs_{t|t} = mrs_t + \phi c_w \left( w_t - w^*_t \right) \), which allows to write the marginal rate of substitution of this particular household as a function of the average rate in the economy, solving \( p_t \) from (70) and replacing in (69) the following equation for the optimal wage in log-terms is obtained:

\[
\begin{align*}
\hat{m}_{w,t} &= \mu_{w,t} - \mu_w, \\
\end{align*}
\]

with \( \hat{m}_{w,t} = \mu_{w,t} - \mu_w \). In order to get Equation (14) two additional step are necessary. First, \( \hat{m}_{w,t} \) can be replaced using (52) to get the optimal wage as a function of the output gap and real wage gap,

\[
\begin{align*}
\hat{w}^*_t &= \frac{\bar{y}_t (\alpha \sigma - \sigma - \phi)}{(\alpha - 1) (\phi c_w + 1)} - \frac{\bar{\omega}_t}{\phi c_w + 1} + w_t, \\
\end{align*}
\]

and second, (71) can be replaced in (67) to finally get

\[
\begin{align*}
\hat{\pi}^{w,fl}_t = \hat{\pi}^w_t + \frac{\bar{y}_t (\alpha \sigma - \sigma - \phi)}{(\alpha - 1) (\phi c_w + 1)} - \frac{\bar{\omega}_t}{\phi c_w + 1}.
\end{align*}
\]

A.1.2 Firms

In the specification presented in this papers firms share the same production function given by

\[
\begin{align*}
Y_t(i) = A_t N_t(i)^{1-a}, \\
\end{align*}
\]

with total labor given in turn by

\[
\begin{align*}
N_t(i) = \left( \int_0^1 N_t(i, j)^{\frac{\sigma - 1}{\sigma}} \, dj \right)^{\frac{\sigma w}{\sigma w - 1}},
\end{align*}
\]

where \( N_t(i, j) \) is the quantity of labor of type \( j \) employed. It can be seen that the problem of firms is similar to the one faced by households and therefore can be solved in two stages. In the first stage firms, giving the labor cost, find the demand for each type of labor. The solution to this problem can be find in an analogously way an in the case of consumers and yields the demand for labor as

\[
\begin{align*}
N_t(i, j) = N_t(i) \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w},
\end{align*}
\]

43
where $W_t = \left( \int_0^1 W_j(j)^{1-\epsilon_w} \, dj \right)^{1-\epsilon_w}$ represents the aggregate wage. In addition, and as in the case of consumption expenditures, the following result can be derived,

$$
\int_0^1 W(j) \, N(i, j) \, dj = W_t \, N(i_t).
$$ (73)

In the second step, firms choose the price, following the four different rules, in order to maximize their profits.

**Sticky prices a la Calvo** In this sector firms set their price in period $t$ taking into account that they have a probability $\theta^k$ of having the same price $k$ periods in the future. Therefore these firms seek to maximize, choosing $P^*_t$, the current value of their profits generated while the price remains effective, subject to the demand from their product which was derived in the household problem in A.1.1. The problem is therefore

$$
\max_{P^*_t} \sum_{k=0}^\infty \theta^k E_t \left[ Q_{t,k+1} \left( P^*_t \, y(i)_{k+l_t} - \psi(i)_{k+l_t} \left( Y(i)_{k+l_t} \right) \right) \right]
$$

subject to $Y(i)_{k+l_t} = C_{k+t} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p}$,

where $Q_{t,k+1}$ is the stochastic discount factor (it can be derived from (27)) and $\psi(i)_{k+l_t} \left( Y(i)_{k+l_t} \right)$ is the total nominal cost. After replacing this problem becomes an unrestricted one given by

$$
\max_{P^*_t} \sum_{k=0}^\infty \theta^k \left( \frac{C_{k+t}}{C_t} \right)^{-\sigma} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p} \psi(i)_{k+l_t} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p} - \sum_{k=0}^\infty \epsilon_p \beta^k \theta^k \psi(i)_{k+l_t} \left( \frac{C_{k+t}}{C_t} \right)^{-\sigma} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p} - \sum_{k=0}^\infty \epsilon_p \beta^k \theta^k \psi(i)_{k+l_t} \left( \frac{C_{k+t}}{C_t} \right)^{-\sigma} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p} + \sum_{k=0}^\infty \beta^k \theta^k \psi(i)_{k+l_t} \left( \frac{C_{k+t}}{C_t} \right)^{-\sigma} \left( \frac{P^*_t}{P_{k+t}} \right)^{-\epsilon_p} = 0,
$$

where $\psi(i)_{k+l_t}$ is the nominal marginal cost. Solving $P^*_t$ from the previous equation the nominal optimal price is found as

44
\[ P_t^* = \frac{\epsilon_p}{\epsilon_p - 1} E_t \left[ \sum_{k=0}^{\infty} \beta^k \theta^k C_{k+t}^{1-\sigma} p_{k+t}^{\sigma-1} \psi(i_{k+t}) \right], \]

or, in order to obtain the that equation in terms of the real marginal cost \( MC(i_{k+t}) \):

\[
\frac{P_t^*}{P_{t-1}} = \frac{\epsilon_p}{\epsilon_p - 1} E_t \left[ \sum_{k=0}^{\infty} \beta^k \theta^k C_{k+t}^{1-\sigma} p_{k+t}^{\sigma-1} \psi(i_{k+t}) \frac{MC(i_{k+t})}{P_{t-1}} \right],
\]

\[
\frac{P_t^*}{P_{t-1}} = \frac{\epsilon_p}{\epsilon_p - 1} E_t \left[ \sum_{k=0}^{\infty} \beta^k \theta^k C_{k+t}^{1-\sigma} p_{k+t}^{\sigma-1} \psi(i_{k+t}) \right]. \tag{74}
\]

From Equation (74) it is found that in steady state, the real marginal cost is given by

\[ MC_p = \frac{\epsilon_p - 1}{\epsilon_p} \to -mc = -\ln \left( \frac{\epsilon_p - 1}{\epsilon_p} \right) \equiv \mu_p. \tag{75} \]

Log-linearization of (74) and making use of the definition of \( \mu_p \) given in (75) yields:

\[ p_t^* = \mu_p + (1 - \beta \theta) \sum_{k=0}^{\infty} \beta^k \theta^k \left( MC(i_{k+t}) + p_{k+t} \right). \tag{76} \]

In order to find and expression for the real marginal cost it should be taken into consideration that from (72) \( N_t(i) = \frac{Y_t(i)}{A_t} \frac{1}{\tau^\alpha} \) and therefore total cost is simply \( \Psi(i_t) = W_t \left( \frac{Y_t(i)}{A_t} \right)^{\tau^\alpha} \) while real marginal cost is given by

\[ MC(i_t) = \frac{W_t A_t^{-\tau^\alpha} Y_t(i)^{-\tau^\alpha}}{(1 - \alpha) P_t}, \]

which after log-linearization becomes

\[ mc(i_t) = w_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha) - p_t, \tag{77} \]

while, by the same token, the average real marginal cost in the economy is

\[ mc = w_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha) - p_t. \tag{78} \]
The real marginal cost of a particular firm as a function of the average real marginal cost can be calculated using (77), (78) and the log-linear form of the output of the firm (Equation (22)) which is

\[ y(i)_t = y_t - \epsilon_p (p^*_t - p_t), \]

therefore

\begin{align*}
mc(i)_{k+t|t} - mc_{k+t} &= \frac{ae_p (p^*_t - p_{k+t})}{a - 1},
mc(i)_{k+t|t} &= mc_{k+t} + \frac{ae_p (p^*_t - p_{k+t})}{a - 1},
\end{align*}

(79)

Before continuing with the derivation of the optimal price it is necessary to calculate the price inflation in this sector. The aggregate price is composed by the price of firms that optimize their price and by the price of firms that let their price unchanged with respect to the price in the previous period, that is,

\[ P_t = (\theta_p p_{t-1}^* + (1 - \theta_p) p_{t-1}^*) \frac{1}{\epsilon_t}. \]

This equation can be used to calculate log-linear form of the price inflation for this sector as:

\[ \pi_p^{sp} = (1 - \theta_p) (p_t^* - p_{t-1}). \]

(80)

At this point, it is possible to use (76), (79) and (80) to obtain the new Keynesian Phillips curve:

\[ \pi_p^{sp} = \frac{(\theta_p - 1) \Theta (\beta \theta_p - 1) \tilde{mc}_t}{\theta_p} + \beta E_t [\pi_t^{p}], \]

(81)

where \( \tilde{mc}_t = mc_t - mc \) and \( \Theta = \frac{a - 1}{a - \epsilon + a - 1} \).

From (78) \( mc_t = w_t - \frac{ay_t}{\alpha - 1} - \frac{a}{1 - \alpha} - \ln(1 - \alpha) - p_t \) and by analogy \( mc = w_{n}^n - \frac{ay_{n}}{\alpha - 1} - \frac{a}{1 - \alpha} - \ln(1 - \alpha) - p_{n}^n \), where as before, the superscript denotes natural values, that is, the values each variable would take in absence of rigidities in prices and wages. Therefore, the difference of the real marginal cost from its steady state as a function of both output and real wage gap is given by

\[ \tilde{mc}_t = \frac{ay_t}{\alpha - 1} - \bar{\omega}_t. \]

(82)

Finally, replacing (82) in (81), Equation (5), the new Keynesian Phillips curve, is obtained.
Sticky information in prices  In exactly the same way as in the case of sticky information in wages shown in A.1.1, the log-linearized aggregate price for the sector with sticky information in prices is given by

\[ p_t = \sum_{j=0}^{\infty} (1 - \theta_w)\theta^j w_t p_{t-j}. \]  
\[ \text{(83)} \]

The problem for the firm is

\[ \max_{P_{t+k}, k} \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[ Q_{t,k} \left( P_{t,k} Y(i)_{k+t} - \Phi(i)_{k+t} (Y(i)_{k+t}) \right) \right], \]

or, after replacing the demand for its production and the stochastic discount factor:

\[ \max_{P_{t+k}, k} \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[ P_{t,k} \left( C_{k+1} - \sigma \left( \epsilon P_{t,k} - \epsilon P_{k+1} \right) \right) / P_{k+1} \right] \]

The F.O.C.s for \( k = 0, 1, 2, \ldots \) are given by

\[ \beta^k \theta^k \left( \frac{C_{k+1}}{C_t} \right)^{-\sigma} \right] \left[ \frac{\epsilon C_{k+1} \left( P_{t,k}^{t+k} / P_{k+1}^{t+k} \right)^{-\epsilon P_{t,k}^{t+k} / P_{k+1}^{t+k}}} {P_{k+1}^{t+k}} \right] = 0, \]

which, after simplifying and dividing by \( P_{t+k} \) to obtain the real marginal cost \( MC(i)_{k+t} \), becomes

\[ \frac{P_{t,k+t}}{P_{k+t}} \frac{\epsilon P_{t,k+t}}{\epsilon - 1} = 0. \]  
\[ \text{(84)} \]

Log-linearization of (84) gives:

\[ p_{t,k+t} = \mathbb{E}_t \left[ \frac{\epsilon P_{t,k+t} + p_{k+t}}{\epsilon - 1} \right] + \mu_p, \]  
\[ \text{(85)} \]

where, as before, \( \mu_p = -mc \) was used. Using (79) appropriately, that is,

\[ mc(i)_{k+t} = mc_{k+t} + \frac{a \epsilon (p_{t,k+t} - p_{k+t})}{a - 1}, \]
in (85) the following equation is obtained: $\hat{m} \tilde{c}_{k+t}$

$$p_{t,k+t} = E_t \left[ \frac{(1-\alpha)\hat{m} \tilde{c}_{k+t}}{\alpha e - \alpha + 1} + p_{k+t} \right].$$

(86)

Equation (86) can be replaced in (83) getting

$$p_t = \sum_{j=0}^{\infty} (1-\theta_p) \theta_p^j E_{t-j} \left[ \frac{(1-\alpha)\hat{m} \tilde{c}_t}{\alpha e - \alpha + 1} + p_t \right],$$

(87)

which is equivalent to

$$p_t = (1-\theta_p) \left( \frac{(1-\alpha)\hat{m} \tilde{c}_t}{\alpha e - \alpha + 1} + p_t \right) + \sum_{j=0}^{\infty} (1-\theta_p) \theta_p^{j+1} E_{t-1-j} \left[ \frac{(1-\alpha)\hat{m} \tilde{c}_{t-1}}{\alpha e - \alpha + 1} + p_{t-1} \right].$$

Multiplying one lag of (87) by $\theta_p$,

$$\theta_p p_t = \theta_p \sum_{j=0}^{\infty} (1-\theta_p) \theta_p^j E_{t-1-j} \left[ \frac{(1-\alpha)\hat{m} \tilde{c}_{t-1}}{\alpha e - \alpha + 1} + p_{t-1} \right],$$

it is possible to calculate $p_t - \theta_p p_t$, yielding

$$\pi_{t}^{p.si} = \left( \frac{1-\theta_p}{\theta_p} \right) \left( \frac{(1-\alpha)\hat{m} \tilde{c}_t}{\alpha e - \alpha + 1} + p_t \right) + \sum_{j=1}^{\infty} \left( \frac{1-\theta_p}{\theta_p} \right) \theta_p^{j-1} E_{t-j} \left[ \frac{(1-\alpha)\hat{m} \tilde{c}_{t-j}}{\alpha e - \alpha + 1} + \pi_{t-j}^{p} \right].$$

Finally, using (82) twice the price inflation for the sector with sticky information in then

$$\pi_{t}^{p.si} = \left( \frac{1-\theta_p}{\theta_p} \right) \sum_{j=1}^{\infty} \theta_p^j E_{t-j} \left[ \frac{\alpha \Delta \tilde{y}_t - (\alpha - 1) \Delta \tilde{\omega}_t}{\alpha (e_p - 1) + 1} + \pi_{t-j}^{p} \right] + \frac{1-\theta_p}{\theta_p} \frac{(\alpha \tilde{y}_t - (\alpha - 1) \tilde{\omega}_t)}{(\alpha (e_p - 1) + 1)},$$

which is Equation (7) in the paper.

**Flexible prices**  Analogously to the case of flexible wages presented in A.1.1, price inflation is given by

$$\pi_t^{p.fl} = p_t^* - p_{t-1},$$

(88)

where $p_t^*$ is the optimal price.

The problem for the firm is
\[
\max_{P_t} C_t P_t^* \left( \frac{P^*_t}{P_t} \right)^{-\epsilon_p} - \Psi(i)_t \left( C_t \left( \frac{P^*_t}{P_t} \right)^{-\epsilon_p} \right),
\]

whose F.O.C. is

\[
\frac{\epsilon_p C_t \left( \frac{P^*_t}{P_t} \right)^{-\epsilon_p-1}}{P_t} \psi(i)_t - \frac{\epsilon_p C_t P_t^* \left( \frac{P^*_t}{P_t} \right)^{-\epsilon_p-1}}{P_t} + C_t \left( \frac{P^*_t}{P_t} \right)^{-\epsilon_p} = 0,
\]

which, solving for \( P_t^* \) and dividing by \( P_t \), to obtain the real marginal cost, becomes

\[
\frac{P_t^*}{P_t} = \frac{\epsilon_p MC(i)_t}{\epsilon_p - 1}.
\]

Log-linearization of (89) yields

\[
p_t^* = mc(i)_t - mc + p_t,
\]

which, after replacing (79) can be written as

\[
p_t^* = p_t - \frac{(\alpha - 1)\tilde{mc}_t}{\alpha - \alpha + 1}.
\]

Finally, replacing (82) in (90) and subsequently (90) in (88) yields Equation (8) in the paper.

### A.1.3 Other relationships

**Natural Output** The natural output, that is, the level of production obtained in absence of rigidities in prices and wages can be constructed using the equations representing the marginal cost (78), the optimal labor decision (11) and labor from the production function (49). In this way

\[
mc_t = w_t - p_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha),
\]

\[
= \sigma c_t + \phi n_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha),
\]

\[
= \sigma c_t + \phi \frac{y_t - a_t}{1 - \alpha} - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha),
\]

\[
mc_t = \frac{(\phi + 1)a_t}{\alpha - 1} - \ln(1 - \alpha) + \frac{y_t(\alpha \sigma - a - \sigma - \phi)}{\alpha - 1}.
\]
It was shown previous that for the case of flexible prices \( mc = -\mu_p \), and then replacing this in Equation (91):

\[
-\mu_p = \frac{(\phi + 1) a_t}{\alpha - 1} - \ln(1 - \alpha) + \frac{y_t^n(a\sigma - \alpha - \sigma - \phi)}{a - 1},
\]

which can be solved for the natural output as

\[
y_t^n = \frac{\alpha \ln(1 - \alpha) - \alpha \mu_p - \ln(1 - \alpha) + \mu_p}{a\sigma - \alpha - \sigma - \phi} - \frac{(\phi + 1) a_t}{a\sigma - \alpha - \sigma - \phi}.
\]

**Natural real wage** The natural real wage can be derived in a similar way. Using (78)

\[
mc_t = w_t - p_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha),
\]

\[
mc_t = \omega_t - \frac{\alpha y_t}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha).
\]

As shown previously, in absence of rigidities

\[
-\mu_p = \omega_t^n - \frac{\alpha y_t^n}{\alpha - 1} - \frac{a_t}{1 - \alpha} - \ln(1 - \alpha),
\]

which, after solving for the natural real wage and using (92), becomes

\[
\omega_t^n = \frac{(\mu_p - \ln(1 - \alpha))(\alpha - 1)}{a\sigma - \alpha - \sigma - \phi} - a_t(\sigma + \phi).
\]

**IS equation** Using the market clearing condition \( y_t = c_t \), the Euler equation (32) can be written as

\[
y_t = E_t [y_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}^P] - \rho).
\]

Defining the real interest rate by

\[
r_t = i_t - E_t [\pi_{t+1}^P],
\]

and replacing in (93) yields
\[ y_t = E_t [y_{t+1}] - \frac{r_t - \rho}{\sigma}, \] (95)

which, by analogy, can be used written in term of natural values as

\[ y_t^n = E_t [y_{t+1}^n] - \frac{r_t^n - \rho}{\sigma}. \]

In this way it is possible to write

\[
\begin{align*}
\tilde{y}_t &= y_t - y_t^n, \\
\tilde{y}_t &= \frac{r_t^n - \rho}{\sigma} - \frac{r_t - \rho}{\sigma} + E_t [y_{t+1} - y_{t+1}^n].
\end{align*}
\] (96)

Finally using (94) in (96), the IS equation in terms of the output gap and the real interest rate is defined as

\[
\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \frac{1}{\sigma} \left( i_t - E_t [\pi_{t+1}^p] - r_t^n \right),
\]

which is Equation (16) in the paper.

**Real interest rate** From (95) \( \Delta y_{t+1} = \frac{r_t - \rho}{\sigma} \) and therefore \( r_t = \rho + \sigma \Delta y_{t+1}, \) or equivalently

\[ r_t^n = \rho + \sigma \Delta y_{t+1}^n. \] (97)

Using (92) it is possible to calculate

\[ \Delta y_{t+1}^n = \frac{(\phi + 1) \Delta a_{t+1}}{a(-\sigma) + a + \sigma + \phi}, \]

which can be replaced in (97) to get

\[ r_t^n = \rho + \frac{\sigma(\phi + 1) \Delta a_{t+1}}{a(-\sigma) + a + \sigma + \phi}. \]
A.2 Convergence test for the simulated chain

In order to evaluate if the samples of the chain are truly representatives of the underlying stationary distribution of the Markov Chain, the procedure suggested by Geweke (1999) was implemented. This method compares the mean of two non overlapped portions of the chain, in particular, it compares draws between 5000 and 9000 to draws between 150000 and 250000. The results of a test for equality of means is presented in Table 6. It is important to note the differences in the p-value with different tapering values, this indicates the presence of autocorrelation in the draw, and therefore, as a more reliable p-value is this case is the one with 15% taper.

[Table 6 about here.]
List of Figures

1  Response of real wage and output to decrease in the nominal interest rate . . . 54
2  Response of real wage with alternative degrees of stickiness to a reduction of the nominal interest rate ........................................... 54
3  IRF for output gap after a monetary shock ........................................... 55
4  IRF for price inflation after a monetary shock .................................... 55
5  IRF for wage inflation after a monetary shock ...................................... 56
6  IRF for nominal interest rate after a monetary shock ............................. 56
7  IRF for output gap after a technology shock ....................................... 57
8  IRF for price inflation after a technology shock .................................... 57
9  IRF for wage inflation after a technology shock .................................... 58
10 IRF for nominal interest rate after a technology shock .......................... 58
Figure 1: Response of real wage and output to decrease in the nominal interest rate


Figure 2: Response of real wage with alternative degrees of stickiness to a reduction of the nominal interest rate

Source: Woodford (2003). The solid line represents the response of real GDP. In this figure $\zeta_w$ measures the degree of wage flexibility.
Figure 3: IRF for output gap after a monetary shock

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.

Figure 4: IRF for price inflation after a monetary shock

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.
Figure 5: IRF for wage inflation after a monetary shock

Figure 6: IRF for nominal interest rate after a monetary shock

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.
Figure 7: IRF for output gap after a technology shock

![Graph showing IRF for output gap after a technology shock.](image)

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.

Figure 8: IRF for price inflation after a technology shock

![Graph showing IRF for price inflation after a technology shock.](image)

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.
Figure 9: IRF for wage inflation after a technology shock

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.

Figure 10: IRF for nominal interest rate after a technology shock

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.
# List of Tables

1. Priors ................................................................. 60
2. Estimated values for the HpHw model ............................. 61
3. Specification for the six models to estimate ....................... 62
4. Estimated values for the six models ............................... 63
5. Model comparison .................................................. 64
6. Geweke convergence test ........................................... 64
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard error</th>
</tr>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Fixed</td>
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<td>–</td>
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<tr>
<td>$\sigma$</td>
<td>Gamma</td>
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<td>0.375</td>
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<td>$\epsilon_p$</td>
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<td>10</td>
<td>–</td>
</tr>
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<td>$\theta_{cw}$</td>
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<td>0.1</td>
</tr>
<tr>
<td>$\theta_{cp}$</td>
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<td>0.6</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta_{iw}$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
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<td>$\theta_{ip}$</td>
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<td>0.05</td>
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</tr>
<tr>
<td>$s^{sp'}$</td>
<td>Beta</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>$s^{sp''}$</td>
<td>Beta</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>$s^{sw}$</td>
<td>Beta</td>
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<tr>
<td>$s^{sw'}$</td>
<td>Beta</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>$s^{sw''}$</td>
<td>Beta</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>$\lambda$</td>
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<td>$\sigma_a$</td>
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<td>$\infty$</td>
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<td>Inverse Gamma</td>
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</tr>
<tr>
<td>$\sigma^{me}_y$</td>
<td>Inverse Gamma</td>
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<td>$\infty$</td>
</tr>
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</table>

Note: The parameters are: Discount factor ($\beta$), Frisch elasticity ($\phi$), inverse of the elasticity of intertemporal substitution ($\sigma$), share of capital in production ($\alpha$), elasticity of substitution for products ($\epsilon_p$), elasticity of substitution for labor types ($\epsilon_w$), rigidity in wages a la Calvo ($\theta_{cw}$), rigidity in prices a la Calvo ($\theta_{cp}$), information rigidity in wages ($\theta_{iw}$), information rigidity in prices ($\theta_{ip}$), reaction to price inflation ($\phi_p$), reaction to wage inflation ($\phi_w$), reaction to output gap ($\phi_y$), persistence for technology process ($\rho_a$), persistence for monetary process ($\rho_y$), nominal interest rate smoothing ($\lambda$), share of firms with sticky prices ($s^{sp}$), share of firms with sticky information in prices ($s^{sp'}$), share of firms with flexible prices ($s^{sp''}$), share of firms with rule-of-thumb prices ($s^{sp'}_{rot}$), share of households with sticky wages ($s^{sw}$), share of households with sticky information in wages ($s^{sw'}$), share of households with flexible wages ($s^{sw''}$), share of households with rule-of-thumb wages ($s^{sw'}_{rot}$), standard deviation for technology process ($\sigma_a$), standard deviation for monetary process ($\sigma_y$), standard deviation for measurement error in output gap ($\sigma^{me}_y$) and standard deviation for measurement error in wage inflation ($\sigma^{me}_a$)
Table 2: Estimated values for the HpHw model

<table>
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<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>LB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
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<td>0.6473</td>
<td>4.0820</td>
<td>5.4464</td>
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<tr>
<td>$\alpha$</td>
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<td>0.0509</td>
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<td>0.4502</td>
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<td>$\theta_{cw}$</td>
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<td>0.1026</td>
<td>0.5945</td>
<td>0.9082</td>
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<tr>
<td>$\theta_{cp}$</td>
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<td>0.0507</td>
<td>0.6363</td>
<td>0.8039</td>
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<tr>
<td>$\theta_{iw}$</td>
<td>0.7217</td>
<td>0.0986</td>
<td>0.5775</td>
<td>0.8781</td>
</tr>
<tr>
<td>$\theta_{ip}$</td>
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<td>0.4467</td>
<td>0.7551</td>
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<td>–</td>
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<tr>
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<td>$s_{wp}^{rot}$</td>
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<td>–</td>
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Note: The parameters are: inverse of the elasticity of intertemporal substitution ($\sigma$), share of capital in production ($\alpha$), rigidity in wages a la Calvo ($\theta_{cw}$), rigidity in prices a la Calvo ($\theta_{cp}$), information rigidity in wages ($\theta_{iw}$), information rigidity in prices ($\theta_{ip}$), reaction to price inflation ($\phi_p$), reaction to wage inflation ($\phi_w$), reaction to output gap ($\phi_y$), persistence for technology process ($\rho_a$), persistence for monetary process ($\rho_y$), nominal interest rate smoothing ($\lambda$), share of firms with sticky prices ($s_{sp}^p$), share of firms with sticky information in prices ($s_{sp}$), share of firms with flexible prices ($s_{sp}^{rot}$), share of firms with rule-of-thumb prices ($s_{sp}^{rtt}$), share of households with sticky wages ($s_{wp}^w$), share of households with sticky information in wages ($s_{wp}$), share of households with flexible wages ($s_{wp}^{f_l}$), share of households with rule-of-thumb wages ($s_{wp}^{rot}$), standard deviation for technology process ($\sigma_a$), standard deviation for monetary process ($\sigma_v$), standard deviation for measurement error in output gap ($\sigma_{me}^w$) and standard deviation for measurement error in wage inflation ($\sigma_{me}^w$).
Table 3: Specification for the six models to estimate

<table>
<thead>
<tr>
<th>Model</th>
<th>Setting behavior</th>
<th>Wages</th>
<th>Firms</th>
<th>Household</th>
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<tbody>
<tr>
<td>HpHw</td>
<td>Heterogeneity</td>
<td>Heterogeneity</td>
<td>All estimated</td>
<td>All estimated</td>
</tr>
<tr>
<td>HpFw</td>
<td>Heterogeneity</td>
<td>Flexible</td>
<td>All estimated</td>
<td>$s_{fl}^w = 1$</td>
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<tr>
<td>CpCw</td>
<td>A la Calvo</td>
<td>A la Calvo</td>
<td>$s_{sp}^p = 1$</td>
<td>$s_{sp}^w = 1$</td>
</tr>
<tr>
<td>CpFw</td>
<td>A la Calvo</td>
<td>Flexible</td>
<td>$s_{sp}^p = 1$</td>
<td>$s_{fl}^w = 1$</td>
</tr>
<tr>
<td>HpCw</td>
<td>Heterogeneity</td>
<td>A la Calvo</td>
<td>All estimated</td>
<td>$s_{sp}^w = 1$</td>
</tr>
<tr>
<td>FpHw</td>
<td>Flexible</td>
<td>Heterogeneity</td>
<td>$s_{fl}^1 = 1$</td>
<td>All estimated</td>
</tr>
</tbody>
</table>

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages. The relevant parameters for this Table are: share of firms with sticky prices ($s_{sp}^p$), share of firms with sticky information in prices ($s_{sp}^i$), share of firms with flexible prices ($s_{fl}^p$), share of firms with rule-of-thumb prices ($s_{rot}^p$), share of households with sticky wages ($s_{sw}^w$), share of households with sticky information in wages ($s_{sw}^i$), share of households with flexible wages ($s_{fl}^w$) and share of households with rule-of-thumb wages ($s_{rot}^w$).
Table 4: Estimated values for the six models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HpHw</th>
<th>HpFw</th>
<th>CpCw</th>
<th>CpFw</th>
<th>HpCw</th>
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<tr>
<td>σ</td>
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<td>θiw</td>
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<td>0.9724</td>
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<td>φp</td>
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<tr>
<td>ρa</td>
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<td>0.7975</td>
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</tr>
<tr>
<td>ρν</td>
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<td>0.3619</td>
<td>0.5062</td>
<td>0.3224</td>
<td>0.5073</td>
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</tr>
<tr>
<td>σa</td>
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<td>0.2689</td>
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<td>1*</td>
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</tr>
<tr>
<td>σν</td>
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<td>0.2614</td>
<td>–</td>
<td>–</td>
<td>0.0479</td>
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</tr>
<tr>
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<td>0.3001</td>
<td>–</td>
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<td>1*</td>
</tr>
<tr>
<td>λ</td>
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<td>0.1695</td>
<td>–</td>
<td>–</td>
<td>0.0313</td>
<td>–</td>
</tr>
<tr>
<td>σsp</td>
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<td>–</td>
<td>1*</td>
<td>–</td>
<td>0.1777</td>
<td>–</td>
</tr>
<tr>
<td>σspf</td>
<td>0.2079</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<tr>
<td>σspw</td>
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<td>–</td>
<td>1*</td>
<td>0.0327</td>
<td>–</td>
</tr>
<tr>
<td>sprot</td>
<td>0.2203</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.1297</td>
<td>–</td>
</tr>
<tr>
<td>λ</td>
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<td>0.8052</td>
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<td>σa</td>
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<td>0.0164</td>
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<td>0.0023</td>
</tr>
<tr>
<td>σν</td>
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<td>0.0071</td>
<td>0.0024</td>
<td>0.0069</td>
<td>0.0024</td>
<td>0.0024</td>
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<tr>
<td>σym</td>
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<td>0.0158</td>
<td>0.0174</td>
<td>0.0158</td>
<td>0.0174</td>
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<tr>
<td>σym</td>
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<td>0.0095</td>
<td>0.006</td>
<td>0.01</td>
<td>0.006</td>
<td>0.0059</td>
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<tr>
<td>Log</td>
<td>3661.94</td>
<td>3513.40</td>
<td>3641.89</td>
<td>3476.02</td>
<td>3642.37</td>
<td>3593.54</td>
</tr>
</tbody>
</table>

Note: * this parameters were not estimated but fixed. Column two of this Table replicates column two of Table 2. In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages. The parameters are: inverse of the elasticity of intertemporal substitution ($σ$), share of capital in production ($α$), rigidity in wages a la Calvo ($θcw$), rigidity in prices a la Calvo ($θcp$), information rigidity in wages ($θiw$), information rigidity in prices ($θip$), reaction to price inflation ($φp$), reaction to wage inflation ($φw$), reaction to output gap ($φy$), persistence for technology process ($ρa$), persistence for monetary process ($ρν$), nominal interest rate smoothing ($λ$), share of firms with sticky prices ($sp^P$), share of firms with sticky information in prices ($sp^I$), share of firms with flexible prices ($sp^F$), share of firms with rule-of-thumb prices ($sp^T$), share of households with sticky wages ($sw^P$), share of households with sticky information in wages ($sw^I$), share of households with flexible wages ($sw^F$), share of households with rule-of-thumb wages ($sw^T$), standard deviation for technology process ($σa$), standard deviation for monetary process ($σν$), standard deviation for measurement error in output gap ($σym$) and standard deviation for measurement error in wage inflation ($σym$).
### Table 5: Model comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>HpHw</th>
<th>HpFw</th>
<th>Cpfw</th>
<th>CpFw</th>
<th>HpCw</th>
<th>FpHw</th>
</tr>
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<tbody>
<tr>
<td>Priors</td>
<td>0.166</td>
<td>0.166</td>
<td>0.166</td>
<td>0.166</td>
<td>0.166</td>
<td>0.166</td>
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<tr>
<td>Log Marginal Density</td>
<td>3661.94</td>
<td>3513.40</td>
<td>3641.89</td>
<td>3476.02</td>
<td>3642.37</td>
<td>3593.54</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: In the acronyms of the models: H-Heterogeneity, C-Calvo, F-Flexible, p-Prices and w-Wages. Then, for example, HpFw corresponds to a model with Heterogeneity in Prices and Flexible Wages.

### Table 6: Geweke convergence test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>p-value</th>
<th>p-value 4% taper</th>
<th>p-value 8% taper</th>
<th>p-value 15% taper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.997</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.000</td>
<td>0.730</td>
<td>0.724</td>
<td>0.703</td>
</tr>
<tr>
<td>$\theta_{cw}$</td>
<td>0.940</td>
<td>0.993</td>
<td>0.993</td>
<td>0.993</td>
</tr>
<tr>
<td>$\theta_{cp}$</td>
<td>0.000</td>
<td>0.464</td>
<td>0.459</td>
<td>0.411</td>
</tr>
<tr>
<td>$\theta_{iw}$</td>
<td>0.000</td>
<td>0.421</td>
<td>0.380</td>
<td>0.288</td>
</tr>
<tr>
<td>$\theta_{ip}$</td>
<td>0.000</td>
<td>0.404</td>
<td>0.382</td>
<td>0.372</td>
</tr>
<tr>
<td>$\phi_{p}$</td>
<td>0.000</td>
<td>0.639</td>
<td>0.616</td>
<td>0.631</td>
</tr>
<tr>
<td>$\phi_{w}$</td>
<td>0.134</td>
<td>0.856</td>
<td>0.855</td>
<td>0.862</td>
</tr>
<tr>
<td>$\phi_{y}$</td>
<td>0.000</td>
<td>0.078</td>
<td>0.066</td>
<td>0.061</td>
</tr>
<tr>
<td>$\rho_{a}$</td>
<td>0.000</td>
<td>0.517</td>
<td>0.488</td>
<td>0.435</td>
</tr>
<tr>
<td>$\rho_{v}$</td>
<td>0.000</td>
<td>0.099</td>
<td>0.102</td>
<td>0.081</td>
</tr>
<tr>
<td>$\hat{s}_{ip}$</td>
<td>0.000</td>
<td>0.408</td>
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<td>$\hat{s}_{fl}$</td>
<td>0.002</td>
<td>0.725</td>
<td>0.727</td>
<td>0.705</td>
</tr>
<tr>
<td>$\hat{s}_{wp}$</td>
<td>0.000</td>
<td>0.575</td>
<td>0.549</td>
<td>0.503</td>
</tr>
<tr>
<td>$\hat{s}_{w}$</td>
<td>0.000</td>
<td>0.165</td>
<td>0.130</td>
<td>0.078</td>
</tr>
<tr>
<td>$\hat{s}_{sp}$</td>
<td>0.000</td>
<td>0.316</td>
<td>0.374</td>
<td>0.392</td>
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<tr>
<td>$\lambda$</td>
<td>0.000</td>
<td>0.158</td>
<td>0.120</td>
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<tr>
<td>$\sigma_{a}$</td>
<td>0.000</td>
<td>0.343</td>
<td>0.307</td>
<td>0.257</td>
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<tr>
<td>$\sigma_{v}$</td>
<td>0.165</td>
<td>0.884</td>
<td>0.868</td>
<td>0.851</td>
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<tr>
<td>$\sigma_{me}^{y}$</td>
<td>0.718</td>
<td>0.971</td>
<td>0.971</td>
<td>0.970</td>
</tr>
<tr>
<td>$\sigma_{me}^{\mu}$</td>
<td>0.000</td>
<td>0.141</td>
<td>0.132</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Note: The parameters are: inverse of the elasticity of intertemporal substitution ($\sigma$), share of capital in production ($\alpha$), rigidity in wages a la Calvo ($\theta_{cw}$), rigidity in prices a la Calvo ($\theta_{cp}$), information rigidity in wages ($\theta_{iw}$), information rigidity in prices ($\theta_{ip}$), reaction to price inflation ($\phi_{p}$), reaction to wage inflation ($\phi_{w}$), reaction to output gap ($\phi_{y}$), persistence for technology process ($\rho_{a}$), persistence for monetary process ($\rho_{v}$), nominal interest rate smoothing ($\lambda$), share of firms with sticky prices ($\hat{s}_{ip}$), share of firms with sticky information in prices ($\hat{s}_{sp}$), share of firms with flexible prices ($\hat{s}_{fl}$), share of firms with rule-of-thumb prices ($\hat{s}_{rot}$), share of households with sticky wages ($\hat{s}_{wsp}$), share of households with sticky information in wages ($\hat{s}_{wsp}$), share of households with flexible wages ($\hat{s}_{wfl}$), share of households with rule-of-thumb wages ($\hat{s}_{wrot}$), standard deviation for technology process ($\sigma_{a}$), standard deviation for monetary process ($\sigma_{v}$), standard deviation for measurement error in output gap ($\sigma_{me}^{y}$) and standard deviation for measurement error in wage inflation ($\sigma_{me}^{\mu}$).