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Volume Flexibility and Capacity Investment

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Publication date:
2015

Document Version
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Wen, X., Kort, P. M., & Talman, A. J. J. (2015). *Volume Flexibility and Capacity Investment: A Real Options Approach*. (CentER Discussion Paper; Vol. 2015-022). Econometrics.

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No. 2015-022

**VOLUME FLEXIBILITY AND CAPACITY INVESTMENT:
A REAL OPTIONS APPROACH**

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2 April 2015

ISSN 0924-7815
ISSN 2213-9532

Volume flexibility and capacity investment: A real options approach*

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Abstract

This paper considers the investment decision of a firm where it has to decide about the timing and capacity. We obtain that in a fast growing market, right after investment the firm produces below capacity, where the utilization rate (the proportion of capacity that is used for production right after the investment) increases with market uncertainty for a very big market trend, and shows no monotonicity for a moderately large market trend. On the other hand we get that, for a slowly growing or shrinking market, the firm produces up to capacity right after investment. In the intermediate case, the firm produces up to capacity right after investment when uncertainty is low and below capacity when uncertainty is high, whereas the utilization rate decreases with the market uncertainty.

Keywords: Investment under Uncertainty; Monopoly; Capacity Choice; Volume Flexibility

JEL Classification: D81, D92.

1 Introduction

When entering a market, it is not only the timing (investment trigger or threshold) that is important, but also the size of the production capacity

*The authors would like to thank Verena Hagspiel for her valuable comments.

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with which the firm enters. By investing in a large capacity, the firm has high cost for investment but gains a high revenue in case of high demand. On the other hand, when being dedicated to producing at full capacity, the firm has to deal with a low market price in case of a low demand realization. We, however, allow for volume flexibility, where the firm can operate profitably at different output levels according to Sethi and Sethi (1990). This enables the firm to produce less in the latter case, and keep part of the invested capacity idle. In this way, volume production reduces the downside risk that a firm takes.

Most of the real option models focus on the optimal investment timing, taking the capacity as given (see Dixit and Pindyck (1994); Trigeorgis (1996) for an overview). In this paper, we determine not only the optimal timing but also the optimal capacity size. From the literature, we know that a monopoly firm invests later with larger capacity for higher market uncertainty (see Manne (1961); Bar-Ilan and Strange (1999)). For duopoly firms, Huisman and Kort (2015) summarize this stream of literature and propose a strategic real option model with entry deterrence/accommodation extensions.

How the flexibility in production affects the investment decision has been studied by both discrete and continuous time models, considering different kinds of flexibility. For example, in a static model, Van Mieghem and Dada (1999) look at the effect of postponement in capacity, output and price decisions to the moment that uncertainty is solved. Compared with production postponement, the price postponement makes the investment decision relatively insensitive to uncertainty. Chod and Rudi (2005) consider a firm that can use one flexible resource to produce two goods in a two-stage model. The optimal capacity of flexible resource is found to be always increasing in both demand variability and demand correlation. In a three-stage model, Anupindi and Jiang (2008) study the flexibility that production can be decided before or after the demand realization, but the capacity decisions are made *ex ante* and pricing decisions *ex post*. They find that a more volatile market results in a larger capacity investment size. By discretizing the dynamic of demand through binomial lattice, Fontes (2008) compares a fixed capacity strategy with a flexible capacity strategy and finds that an increase in flexibility leads to a higher predicted value of the project. In continuous time models, Brennan and Schwartz (1985), McDonald and Siegel (1985), and Adkins and Paxson (2012) consider the possibility to switch from operation to suspension and back to operation at a certain cost. The flexibility studied in our work, however, is that the firm can adjust production between zero and the invested capacity level at any time.

This paper is closely related to Dangl (1999) and Hagspiel et al. (2012). However, Dangl (1999) does not take into account the possibility that the

market demand is so high that the firm produces up to capacity right after the investment, while Hagspiel et al. (2012) do consider such a possibility and derive that the utilization rate decreases when the market uncertainty increases. Compared to Hagspiel et al. (2012), we adopt a slightly different demand function, which, however, leads to different results. The difference in demand function is that the market size is unbounded in the work of Hagspiel et al. (2012), while we have a bounded market size. The latter can be motivated by the presence of market maturity. Many mature markets can be seen as bounded in size since the market demand does not evolve rapidly, and sometimes fluctuates as reported about the global PC market by Statista (2015).

Our main results are the following. First, we find that under a very large market trend, right after the investment an utilization rate results that is increasing with uncertainty. This is due to the fact that when market trend is very large and uncertainty is low, the firm invests in a capacity relatively close to the maximal size of the market. Higher uncertainty makes it optimal to invest later, i.e. when demand is larger. Larger demand raises production, while at the same time capacity increases, but not so much because it was already large. Consequently, it turns out that production increases more than capacity does with the uncertainty. This leads to the at first sight counterintuitive result that the utilization rate increases with uncertainty. However, an intermediate market trend still results in an utilization rate being decreasing with uncertainty as in Hagspiel et al. (2012), and a moderately large market trend yields a non-monotonic utilization rate.

We also find that, when the market trend is large, the firm does not produce up to capacity right after the investment; when the market trend is small, the firm produces up to capacity; when the market trend is intermediate, there exists a threshold uncertainty level such that the firm produces below capacity right after the investment above this threshold and produces up to capacity below this threshold. The third contribution lies in the analysis of how market trend affects investment decision: the optimal timing is delayed for a larger trend in a less volatile environment, and accelerated in a more volatile environment. This results from the large capacity installment for small market trend under high market uncertainty. As the market grows faster, due to the bounded market size we imposed, the capacity does not increase a lot. When the market trend increases, the firm then actually prefers to invest earlier.

The rest of the paper is structured as follows. Section 2 describes the monopoly investment problem. The optimal investment decision is determined and analyzed in Section 3. A numerical analysis is provided in Section 4. Section 5 concludes.

2 Model Setup

A monopoly firm is considering to undertake an investment to enter a market, where the market price at any time $t, t \geq 0$, is given by

$$p(t) = \theta(t)(1 - \gamma q(t)),$$

with $q(t)$ being the firm's output and γ ($\gamma > 0$) a constant. Note that in this inverse demand function, the market is bounded above in such a way that $q(t) \leq 1/\gamma$.¹ Demand uncertainty is modeled by $\{\theta(t)\}$ following the geometric Brownian motion

$$d\theta(t) = \alpha\theta(t)dt + \sigma\theta(t)dW_t,$$

where $\theta(0) > 0$, α is the trend parameter, σ ($\sigma > 0$) is the volatility parameter, and dW_t is the increment of a Wiener process. The firm is risk-neutral and the discount rate r is assumed to satisfy $r > \alpha$ and $r > \sigma^2 - \alpha$. The first inequality is standard because then it is not optimal for the firm to always delay the investment (see Dixit and Pindyck (1994)). The second inequality is because the Brownian motion process $\{1/\theta(t)\}$ ² has a trend of $\sigma^2 - \alpha$, which should also be smaller than r to avoid optimally investing at infinity.

Once the investment is made, the firm becomes active and can decide on the production level, which is bounded from above by the installed capacity $K, K \geq 0$. The unit cost for acquiring capacity is δ ($\delta > 0$), and the unit cost for production is c ($c > 0$).

3 Optimal Investment Decision

This section is about the investment decision of the monopoly firm. In particular, the investment problem is solved as an optimal stopping problem in dynamic programming, and can be formalized as

$$\max_{T \geq 0, K \geq 0} E \left[\int_T^\infty \pi(\theta(t), K) \exp(-rt) dt - \delta K \exp(-rT) \mid \theta(0) = \theta \right], \quad (1)$$

which is conditional on the available information at time 0 with $\theta(0)$ set equal to θ , where T is the moment of investment and $\pi(\theta(t), K)$ is the maximum profit of the firm at time $t \geq T$ if capacity K has been invested.

¹There is no upper bound for the demand function $p(t) = \theta(t) - \gamma q(t)$ adopted in Hagspiel et al. (2012).

²It should be noted that the optimal output, corresponding profit and option value of the firm are proportional to $1/\theta$ when the firm produces below capacity right after the investment, as can be inferred from (6), (7) and (8) later on.

Let θ^* be the value of the Brownian motion where the firm is indifferent between continuation and stopping, and let the corresponding acquired capacity be K^* . For $0 < \theta(0) < \theta^*$, the firm is in the continuation region and waits with investing until $\theta(t)$ reaches θ^* ; for $\theta(0) > \theta^*$, the firm invests immediately. Denote by $V(\theta, K)$ the value of the firm given that the level of the Brownian motion is θ and capacity K has been installed. Then $V(\theta, K)$ can be determined by the Bellman equation

$$rV = \pi + \frac{1}{dt}E[dV], \quad (2)$$

and Ito's Lemma

$$dV = \frac{\partial V}{\partial \theta} \alpha \theta dt + \frac{\partial^2 V}{\partial \theta^2} \frac{1}{2} \sigma^2 \theta^2 dt + \frac{\partial V}{\partial \theta} \sigma \theta dW. \quad (3)$$

Substitution of (3) into (2) results in the following differential equation:

$$\frac{1}{2} \sigma^2 \theta^2 \frac{\partial^2 V(\theta, K)}{\partial \theta^2} + \alpha \theta \frac{\partial V(\theta, K)}{\partial \theta} - rV(\theta, K) + \pi(\theta, K) = 0. \quad (4)$$

We first determine the firm's optimal instantaneous profit $\pi(\theta, K)$. Once the firm becomes active in the market with installed capacity $K \geq 0$, it chooses at level θ of an output to maximize the profit flow, i.e.

$$\pi(\theta, K) = \max_{0 \leq q \leq K} (p - c)q = \max_{0 \leq q \leq K} [\theta(1 - \gamma q) - c]q. \quad (5)$$

A straightforward analysis leads to the next proposition.

Proposition 1. *For invested capacity $K \geq 0$, and level $\theta > 0$, the optimal monopoly production output is*

$$q(\theta, K) = \begin{cases} 0 & 0 < \theta < c, \\ \frac{\theta - c}{2\gamma\theta} & \theta \geq c \text{ and } K > \frac{\theta - c}{2\gamma\theta}, \\ K & \theta \geq c \text{ and } 0 \leq K \leq \frac{\theta - c}{2\gamma\theta}. \end{cases} \quad (6)$$

The corresponding profit is

$$\pi(\theta, K) = \begin{cases} 0 & 0 < \theta < c, \\ \frac{(\theta - c)^2}{4\gamma\theta} & \theta \geq c \text{ and } K > \frac{\theta - c}{2\gamma\theta}, \\ [\theta(1 - \gamma K) - c]K & \theta \geq c \text{ and } 0 \leq K \leq \frac{\theta - c}{2\gamma\theta}. \end{cases} \quad (7)$$

Figure 1: Comparison of investment capacity K and optimal production outputs $q(\theta, K)$. In Region 1, $q(\theta, K) = 0$; in Region 2, $q(\theta, K) = (\theta - c)/(2\gamma\theta)$; and in Region 3, $q(\theta, K) = K$.

The comparison between production output and investment capacity is illustrated in Figure 1, where the line $\theta = c$ and the curve $(\theta - c)/(2\gamma\theta)$ divide the (θ, K) -space into three regions. In Region 1, where $0 < \theta < c$, there is no production. Region 2 is to the right of $\theta = c$ and above the curve $(\theta - c)/(2\gamma\theta)$. It is the region where the optimal output level is lower than the invested capacity. Region 3 is below the curve $(\theta - c)/(2\gamma\theta)$, where the production is constrained by the capacity and the firm produces an output level being equal to the installed capacity.

Substitution of (7) into (4), and employing value matching and smooth pasting give the value of the project:

$$V(\theta, K) = \begin{cases} L(K) \theta^{\beta_1} & 0 < \theta < c, \\ M_1(K) \theta^{\beta_1} + M_2 \theta^{\beta_2} & \\ + \frac{1}{4\gamma} \left[\frac{\theta}{r-\alpha} - \frac{2c}{r} + \frac{c^2}{(r+\alpha-\sigma^2)\theta} \right] & \theta \geq c \text{ and } K > \frac{\theta-c}{2\gamma\theta}, \\ N(K) \theta^{\beta_2} + \frac{(1-\gamma K)K}{r-\alpha} \theta - \frac{cK}{r} & \theta \geq c \text{ and } 0 \leq K \leq \frac{\theta-c}{2\gamma\theta}, \end{cases} \quad (8)$$

in which

$$L(K) = \frac{c^{1-\beta_1} [1 - (1 - 2\gamma K)^{1+\beta_1}]}{4\gamma(\beta_1 - \beta_2)} F(\beta_2), \quad (9)$$

$$M_1(K) = -\frac{c^{1-\beta_1}(1-2\gamma K)^{1+\beta_1}}{4\gamma(\beta_1-\beta_2)}F(\beta_2), \quad (10)$$

$$M_2 = \frac{c^{1-\beta_2}}{4\gamma(\beta_1-\beta_2)}F(\beta_1), \quad (11)$$

$$N(K) = \frac{c^{1-\beta_2}[1-(1-2\gamma K)^{1+\beta_2}]}{4\gamma(\beta_1-\beta_2)}F(\beta_1), \quad (12)$$

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (13)$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (14)$$

with

$$F(\beta) = \frac{2\beta}{r} - \frac{\beta-1}{r-\alpha} - \frac{\beta+1}{r+\alpha-\sigma^2}. \quad (15)$$

The derivation of $L(K)$, $M_1(K)$, M_2 and $N(K)$ is given in A.1. We find the optimal investment decision in two steps. First, for any given level of the Brownian motion θ , the optimal value of K is found by maximizing $V(\theta, K) - \delta K$. Second, the optimal investment threshold and capacity level are derived. The two steps are summarized in the following proposition, where $\bar{\sigma} > 0$ is such that

$$\bar{\sigma}^2 = \frac{-2(\Lambda - \alpha^2)(2r - \alpha) - 4\sqrt{r\Lambda(\Lambda - \alpha^2)(r - \alpha)}}{\Lambda - (2r - \alpha)^2}, \quad (16)$$

with $\Lambda = \left(\frac{2\delta r(r-\alpha)-\alpha c}{c}\right)^2$. The proof can be found in A.2.

Proposition 2. *There are two possibilities regarding the firm's investment decision:*

- Suppose $\alpha > \delta r^2/(c + \delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma > \bar{\sigma}$. Then the firm does not produce up to capacity right after the investment. For any $\theta \geq c$, the optimal value of K that maximizes $V(\theta, K) - \delta K$ is

$$K(\theta) = \frac{1}{2\gamma} \left[1 - \frac{c}{\theta} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right], \quad (17)$$

and the optimal investment threshold θ^* is implicitly determined by

$$\frac{\beta_1 - \beta_2}{\beta_1} M_2 \theta^{\beta_2} + \frac{1}{4\gamma} \left[\frac{\theta(\beta_1 - 1)}{\beta_1(r - \alpha)} - \frac{2c}{r} + \frac{c^2(\beta_1 + 1)}{\beta_1(r + \alpha - \sigma^2)\theta} \right] - \delta K(\theta) = 0. \quad (18)$$

If $\theta(0) < \theta^*$, the optimal capacity is $K^* = K(\theta^*)$. If $\theta(0) \geq \theta^*$, the firm invests in capacity $K^* = K(\theta(0))$ immediately at $t = 0$.

- Suppose $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma \leq \bar{\sigma}$. Then the firm produces up to capacity right after the investment. For any $\theta \geq c$, the optimal value of K that maximizes $V(\theta, K) - \delta K$, $K(\theta)$, satisfies

$$\frac{(\beta_2 + 1) F(\beta_1) (1 - 2\gamma K)^{\beta_2}}{2(\beta_1 - \beta_2) c^{\beta_2 - 1}} \theta^{\beta_2} + \frac{1 - 2\gamma K}{r - \alpha} \theta - \frac{c}{r} - \delta = 0, \quad (19)$$

and the optimal investment threshold θ^* is implicitly determined by

$$\frac{\beta_1 - \beta_2}{\beta_1} N(K) \theta^{\beta_2} + \frac{\beta_1 - 1}{\beta_1} \frac{(1 - \gamma K) K \theta}{r - \alpha} - \frac{cK}{r} - \delta K = 0, \quad (20)$$

with $K = K(\theta)$. If $\theta(0) < \theta^*$, the firm invests in capacity $K^* = K(\theta^*)$. If $\theta(0) \geq \theta^*$, the firm invests in capacity $K^* = K(\theta(0))$ immediately at $t = 0$.

Besides presenting the optimal investment threshold and capacity level, Proposition 2 also shows how the market affects the flexible firm's production decision right after the investment. This is illustrated in Figure 2. If the market is growing fast ($\alpha > \delta r^2/(c + \delta r)$), right after the investment the firm chooses an output below the installed capacity. The initially unused capacity can be employed later to meet an increased market demand. However, if the market is growing very slowly or even shrinking ($\alpha \leq r - c/\delta$), the firm produces at full capacity right after investment.³ If the market trend is at an intermediate level ($r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$), the market uncertainty plays a decisive role in the decision of whether to produce up to full capacity right after the investment. In a more volatile environment ($\sigma > \bar{\sigma}$), producing below capacity makes the extra capacity an "option" to implement when the price level is higher. For a more certain environment ($\sigma \leq \bar{\sigma}$), such an extra capacity is not needed, and the firm produces up to full capacity right after the investment.

³Note that we allow for $r < c/\delta$.

Figure 2: Illustration of market trend α and uncertainty σ affecting the firm to produce below capacity (Region 2), or up to capacity (Region 3), right after the investment.

4 Numerical Analysis

This section focuses on the influence of the market trend and uncertainty on the investment decision and the utilization rate right after the investment. The utilization rate is equal to the ratio q^*/K^* with $q^* = q(\theta^*, K^*)$. It has been shown that for the unbounded demand function $p(t) = \theta(t) - \gamma q(t)$, at given level of the market trend, the utilization rate decreases significantly with market uncertainty (see Hagspiel et al (2012)). However, this section shows that for our model, if the market trend is large enough, the utilization rate increases with market uncertainty.

4.1 Market trend

We first look at how market trend affects the optimal investment timing and capacity when the firm produces below capacity right after the investment. As shown in Figure 3 we have that, when the market uncertainty is low, both the optimal investment time and the investment capacity increase with market trend. This is because when deciding how much to invest in a less volatile environment, the firm considers the market increase after the investment and installs a large capacity in case of a high market demand, which makes it reasonable to invest later. However, when the market uncertainty

is high, Figure 3a shows that the firm invests slightly earlier for a larger market trend. The reason is that in a highly volatile environment, the firm still invests in a larger capacity for a larger market trend. But since the capacity level is already at a high level when the market grows slowly (Figure 3b), with a larger market trend the capacity does not increase a lot, and the resulting effect on investment timing is low.⁴ This makes that in a higher uncertain environment, the firm prefers to invest earlier when the market trend goes up, because the firm is more eager to invest in such a market.

The influence of the market trend α on the utilization rate q^*/K^* is shown in Figure 4. Regardless of the uncertainty level, the utilization rate decreases with α . This is because when deciding on capacity, the future market is considered. This implies that for a larger α , a larger capacity will be installed. At the same time, only the current market is important when deciding about the production amount. The current market size is small compared to the future market size when α is large. This makes that the production level is low compared to capacity, hence a low utilization rate results. Moreover, the utilization rate decreases less fast with α for larger σ . The intuition is that the rate of increase in the installed capacity is lower than that in production output for larger σ , since, as before, the optimal capacity is already close to its upper bound $1/(2\gamma)$ when σ is large.

4.2 Market uncertainty

When the market trend α is small, the utilization rate equals to 1 and is unaffected by the market uncertainty. When α is at an intermediate level, Figure 5 shows that the utilization rate is 1 for small market uncertainty σ , as is also illustrated in Figure 2, and decreases as market uncertainty σ increases. When α is very large, the utilization rate increases with σ , and when α is moderately large, the utilization rate can both increase and decrease with σ . The intuition behind this is as follows.

If α is at intermediate level, i.e. $r - c/\delta < \alpha \leq \delta r/(c + \delta r)$. Then the firm invests later in a larger capacity when σ goes up. This is shown in Figure 6. Given the other parameter values (r , γ , c and δ) in Figure 6, the firm produces below capacity for $\alpha = 0.02, \sigma > 0.2866$ and $\alpha = 0.03, \sigma > 0.1473$; and up to capacity for $\alpha = 0.02, \sigma \leq 0.2866$ and $\alpha = 0.03, \sigma \leq 0.1473$, right after the investment. The firm invests in a larger capacity in a more volatile environment, because more future uncertainty makes excess capacity desirable to match upward demand shocks. Since the required capacity at the moment of investment is larger, so is the investment cost, the firm wants

⁴Note that with this demand function, the optimal capacity size is always below $1/(2\gamma)$.

(a) Investment timing

(b) Investment capacity

Figure 3: Illustration of investment timing and capacity as function of market trend α under different uncertainty levels σ when producing below capacity right after the investment. Parameter values are $r = 0.1$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 4: Illustration of utilization rate as function of market trend α under different uncertainty levels σ when producing below capacity right after the investment. Parameter values are $r = 0.1$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 5: Illustration of utilization rate as function of the market uncertainty level σ under different market trends α . Parameter values are $r = 0.1$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

the output price to be higher when it invests, implying that the optimal threshold increases also. The delayed timing suggests that the output level is also increasing. But for an intermediate α , this happens in a more gradual way. Thus, the utilization rate decreases with uncertainty when the firm produces below capacity right after the investment. The utilization rate being decreasing with uncertainty for an intermediate α , $\alpha = 0.02$ for example, is consistent with the findings in Hagspiel et al. (2012).

If α is large, i.e. $\alpha > \delta r^2 / (c + \delta r)$, there are two possibilities. When α is very large, take $\alpha = 0.06$ for example, in Figure 6b, the optimal capacity is already at a high level for small σ . Then the capacity upper bound of $1/(2\gamma)$ is relatively close, so the capacity increases slowly with σ . However, the optimal investment timing is delayed a lot compared with the optimal capacity. This implies the output right after the investment increases quite a lot. Thus, for a very large α , the utilization rate increases with uncertainty. That the utilization rate can be increasing with uncertainty does not show up in the work of Hagspiel et al. (2012), because of different demand functions. The market is not bounded in their work, whereas in this paper, we have that a positive market price requires the quantity to be always below $1/\gamma$. When α is moderately large, for example, $\alpha = 0.035$ and 0.04 in Figure 5, the utilization rate does not change monotonically with uncertainty. In fact, the opposite effects for intermediate α and very large α above occur here, and this explains the non-monotonicity.

5 Conclusion

This paper analyzes the investment decisions of a monopoly firm with access to volume flexibility in a dynamic uncertain environment. In such an environment, not only the uncertainty, but also the market trend has significant qualitative effects on the timing, the investment capacity size, and the decision whether to produce up to capacity right after the investment. We show that a large (small) market trend corresponds to producing below (up to) capacity right after the investment. An intermediate market trend and an uncertainty level above (below) a certain threshold yields an output level below (up to) capacity right after the investment. The utilization rate is increasing with market uncertainty when the trend is very large, shows no monotonicity when the trend is moderately large, and decreases with uncertainty when the trend is intermediate. Moreover, we find that capacity increases and the utilization rate decreases with the market trend. However, the investment timing is delayed in a more certain market, but accelerated in a more volatile market.

(a) Investment timing

(b) Investment capacity

Figure 6: Illustration of investment timing and capacity as functions of uncertainty level σ under different market trends α . Parameter values are $r = 0.1$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

A limitation of the model is that the firm can only invest once. If the firm can undertake several investments during its life time, then the decision to produce up to/below capacity after investment is probably going to be affected by the frequency and moments of investments, which could be an interesting topic for future research. Another interesting topic is to introduce competition by studying a duopoly framework. Then Huisman and Kort (2015), where firms are obliged to produce up to capacity, is extended by allowing the firms to produce below capacity. The implication is that the firm can no longer commit to a high production level, which leads to a significant change in the resulting strategic interactions.

A Identification of $L(K)$, $M_1(K)$, M_2 , and $N(K)$ and Proof of Proposition 2

A.1 Identification of $L(K)$, $M_1(K)$, M_2 , and $N(K)$

Given the value function in different regions and according to the value matching and smooth pasting conditions at $\theta_1 = c$ and $\theta_2 = \frac{c}{1-2\gamma K}$, it holds that

$$\begin{aligned} L(K) \theta_1^{\beta_1} = & M_1(K) \theta_1^{\beta_1} + M_2 \theta_1^{\beta_2} \\ & + \frac{\theta_1}{4\gamma(r-\alpha)} - \frac{2c}{4\gamma r} + \frac{c^2}{4\gamma(r+\alpha-\sigma^2)\theta_1}, \end{aligned} \quad (21)$$

$$\begin{aligned} L(K) \beta_1 \theta_1^{\beta_1-1} = & \beta_1 M_1(K) \theta_1^{\beta_1-1} + \beta_2 M_2 \theta_1^{\beta_2-1} \\ & + \frac{1}{4\gamma(r-\alpha)} - \frac{c^2}{4\gamma(r+\alpha-\sigma^2)\theta_1^2}, \end{aligned} \quad (22)$$

$$\begin{aligned} M_1(K) \theta_2^{\beta_1} + M_2 \theta_2^{\beta_2} + & \frac{\theta_2}{4\gamma(r-\alpha)} - \frac{2c}{4\gamma r} + \frac{c^2}{4\gamma(r+\alpha-\sigma^2)\theta_2} \\ = & N(K) \theta_2^{\beta_2} + \frac{(1-\gamma K)K}{r-\alpha} \theta_2 - \frac{cK}{r}, \end{aligned} \quad (23)$$

$$\begin{aligned} \beta_1 M_1(K) \theta_2^{\beta_1-1} + \beta_2 M_2 \theta_2^{\beta_2-1} + & \frac{1}{4\gamma(r-\alpha)} - \frac{c^2}{4\gamma(r+\alpha-\sigma^2)\theta_2^2} \\ = & \beta_2 N(K) \theta_2^{\beta_2-1} + \frac{(1-\gamma K)K}{r-\alpha}. \end{aligned} \quad (24)$$

Take

$$\begin{aligned} F(\beta) &= \frac{2\beta}{r} - \frac{\beta-1}{r-\alpha} - \frac{\beta+1}{r+\alpha-\sigma^2} \\ &= \frac{\beta(2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) + r(2\alpha - \sigma^2)}{r(r-\alpha)(r+\alpha-\sigma^2)}. \end{aligned}$$

From (21) and (22), we get

$$\begin{aligned} M_2 &= \frac{\theta_1^{-\beta_2}}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2c\beta_1}{r} - \frac{\theta_1(\beta_1 - 1)}{r - \alpha} - \frac{c^2(\beta_1 + 1)}{(r + \alpha - \sigma^2)\theta_1} \right] \\ &= \frac{c^{1-\beta_2}}{4\gamma(\beta_1 - \beta_2)} \left(\frac{2\beta_1}{r} - \frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1 + 1}{r + \alpha - \sigma^2} \right) \\ &= \frac{c^{1-\beta_2}}{4\gamma(\beta_1 - \beta_2)} F(\beta_1). \end{aligned} \tag{25}$$

$M_1(K)$ can be derived from (23) and (24) as

$$\begin{aligned} M_1(K) &= \frac{\theta_2^{-\beta_1}}{\beta_1 - \beta_2} \left\{ \frac{1}{4\gamma} \left[\frac{\theta_2(\beta_2 - 1)}{r - \alpha} + \frac{c^2(\beta_2 + 1)}{(r + \alpha - \sigma^2)\theta_2} - \frac{2c\beta_2}{r} \right] \right. \\ &\quad \left. + \frac{cK\beta_2}{r} + \frac{(1 - \beta_2)(1 - \gamma K)K\theta_2}{r - \alpha} \right\} \\ &= \frac{\theta_2^{-\beta_1}}{\beta_1 - \beta_2} \left[\frac{\theta_2(\beta_2 - 1)(1 - 2\gamma K)^2}{4\gamma(r - \alpha)} + \frac{c^2(\beta_2 + 1)}{4\gamma\theta_2(r + \alpha - \sigma^2)} \right. \\ &\quad \left. - \frac{2c\beta_2(1 - 2\gamma K)}{4\gamma r} \right] \\ &= -\frac{\theta_2^{-\beta_1}c(1 - 2\gamma K)}{4\gamma(\beta_1 - \beta_2)} \left(\frac{2\beta_2}{r} - \frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2 + 1}{r + \alpha - \sigma^2} \right) \\ &= -\frac{c^{1-\beta_1}(1 - 2\gamma K)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} F(\beta_2). \end{aligned} \tag{26}$$

We get $L(K)$ from (21) as

$$\begin{aligned} L(K) &= M_1(K) + M_2\theta_1^{\beta_2-\beta_1} + \frac{\theta_1^{-\beta_1}}{4\gamma} \left[\frac{\theta_1}{r - \alpha} - \frac{2c}{r} + \frac{c^2}{(r + \alpha - \sigma^2)\theta_1} \right] \\ &= M_1(K) + \frac{\theta_1^{-\beta_1}}{4\gamma} \left[\frac{\theta_1}{r - \alpha} - \frac{2c}{r} + \frac{c^2}{(r + \alpha - \sigma^2)\theta_1} \right] \\ &\quad + \frac{\theta_1^{-\beta_1}}{4\gamma} \left[\frac{2c\beta_1}{r(\beta_1 - \beta_2)} - \frac{\theta_1(\beta_1 - 1)}{(r - \alpha)(\beta_1 - \beta_2)} - \frac{c^2(\beta_1 + 1)}{(r + \alpha - \sigma^2)\theta_1(\beta_1 - \beta_2)} \right] \end{aligned}$$

$$\begin{aligned}
&= M_1(K) + \frac{\theta_1^{-\beta_1}}{4\gamma} \left[\frac{\theta_1}{r-\alpha} \frac{1-\beta_2}{\beta_1-\beta_2} + \frac{2c}{r} \frac{\beta_2}{\beta_1-\beta_2} - \frac{c^2}{(r+\alpha-\sigma^2)\theta_1} \frac{\beta_2+1}{\beta_1-\beta_2} \right] \\
&= M_1(K) + \frac{c^{1-\beta_1}}{4\gamma(\beta_1-\beta_2)} \left(\frac{2\beta_2}{r} - \frac{\beta_2-1}{r-\alpha} - \frac{\beta_2+1}{r+\alpha-\sigma^2} \right) \\
&= \frac{c^{1-\beta_1}[1-(1-2\gamma K)^{1+\beta_1}]}{4\gamma(\beta_1-\beta_2)} F(\beta_2).
\end{aligned}$$

From (23), we get $N(K)$ as

$$\begin{aligned}
N(K) &= M_1(K) \theta_2^{\beta_1-\beta_2} + M_2 + \theta_2^{-\beta_2} \left[\frac{\theta_2}{4\gamma(r-\alpha)} - \frac{2c}{4\gamma r} + \frac{c^2}{4\gamma(r+\alpha-\sigma^2)\theta_2} \right. \\
&\quad \left. + \frac{cK}{r} - \frac{(1-\gamma K)K}{r-\alpha} \right] \\
&= M_2 + \frac{\theta_2^{-\beta_2}}{\beta_1-\beta_2} \left\{ \frac{1}{4\gamma} \left[\frac{\theta_2(\beta_2-1)}{r-\alpha} + \frac{c^2(\beta_2+1)}{(r+\alpha-\sigma^2)\theta_2} - \frac{2c\beta_2}{r} \right] + \frac{cK\beta_2}{r} \right. \\
&\quad \left. + \frac{(1-\beta_2)(1-\gamma K)K\theta_2}{r-\alpha} \right\} + \theta_2^{-\beta_2} \left\{ \frac{1}{4\gamma} \left[\frac{\theta_2}{r-\alpha} - \frac{2c}{r} + \frac{c^2}{(r+\alpha-\sigma^2)\theta_2} \right] \right. \\
&\quad \left. + \frac{cK}{r} - \frac{(1-\gamma K)K\theta_2}{r-\alpha} \right\} \\
&= M_2 + \frac{\theta_2^{-\beta_2}}{\beta_1-\beta_2} \left\{ \frac{1}{4\gamma} \left[\frac{\theta_2}{r-\alpha} (\beta_1-1) - \frac{2c\beta_1}{r} + \frac{c^2}{(r+\alpha-\sigma^2)\theta_2} (1+\beta_1) \right] \right. \\
&\quad \left. + \frac{c\beta_1 K}{r} + \frac{(1-\gamma K)K\theta_2}{r-\alpha} (1-\beta_1) \right\} \\
&= M_2 - \frac{\theta_2^{1-\beta_1}(1-2\gamma K)^2}{4\gamma(\beta_1-\beta_2)} \left(\frac{2\beta_1}{r} - \frac{\beta_1-1}{r-\alpha} - \frac{\beta_1+1}{r+\alpha-\sigma^2} \right) \\
&= \frac{c^{1-\beta_2}[1-(1-2\gamma K)^{1+\beta_2}]}{4\gamma(\beta_1-\beta_2)} F(\beta_1). \tag{27}
\end{aligned}$$

A.2 Proof of Proposition 2

The proof of Proposition 2 consists of two parts. The first part derives the optimal investment timing and investment capacity for producing below and producing up to capacity right after the investment. The second part derives conditions when the firm will produce up to or below capacity right after the investment.

A.2.1 Derivation of optimal investment timing and capacity

First, for any $\theta > 0$ find the optimal value of the investment capacity, $K(\theta)$, that maximizes the option value minus the cost of investment $V(\theta, K) - \delta K$. Then the optimal investment timing θ^* is derived by using this optimal value. For $\theta < \theta^*$, assume the value of the investment option in the continuation region is $A\theta^{\beta_1}$. According to value matching and smooth pasting conditions at θ^* , we have

$$\begin{cases} A\theta^{*\beta_1} &= V(\theta^*, K(\theta^*)) - \delta K(\theta^*), \\ \beta_1 A\theta^{*\beta_1-1} &= \frac{d}{d\theta} [V(\theta^*, K(\theta^*)) - \delta K(\theta^*)], \end{cases}$$

so θ^* is a solution of the equation

$$V(\theta, K(\theta)) - \delta K(\theta) = \frac{\theta}{\beta_1} \frac{d}{d\theta} [V(\theta, K(\theta)) - \delta K(\theta)] = \frac{\theta}{\beta_1} \frac{\partial V(\theta, K)}{\partial \theta}, \quad (28)$$

because

$$\frac{\partial V(\theta, K(\theta)) - \delta K(\theta)}{\partial K} \frac{dK(\theta)}{d\theta} = 0.$$

- If the firm does not produce right after the investment, then $K(\theta)$ should maximize $V(\theta, K) - \delta K$, which is

$$\frac{c^{1-\beta_1} \theta^{\beta_1} [1 - (1 - 2\gamma K)^{1+\beta_1}]}{4\gamma(\beta_1 - \beta_2)} F(\beta_2) - \delta K.$$

The first order condition implies

$$\frac{(1 + \beta_1) F(\beta_2) c^{1-\beta_1} (1 - 2\gamma K(\theta))^{\beta_1}}{2(\beta_1 - \beta_2)} \theta^{\beta_1} - \delta = 0.$$

Then

$$K(\theta) = \frac{1}{2\gamma} \left[1 - \frac{c}{\theta} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1) F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right]. \quad (29)$$

The second order partial derivative with respect to K is negative,⁵ so there is a global maximum for $V(\theta, K) - \delta K$ when the firm does not produce right after the investment.

Determine the optimal investment timing θ^* according to (28), then θ^* is the solution for the following equation,

$$\frac{c^{1-\beta_1} \theta^{\beta_1} [1 - (1 - 2\gamma K(\theta))^{\beta_1}]}{4\gamma(\beta_1 - \beta_2)} F(\beta_2) - \delta K(\theta)$$

⁵From B.1, $\beta_1 > 1$, $\beta_2 < 0$, $F(\beta_1) < 0$, and $F(\beta_2) > 0$.

$$= \frac{c^{1-\beta_1} \theta^{\beta_1} [1 - (1 - 2\gamma K(\theta))^{1+\beta_1}]}{4\gamma(\beta_1 - \beta_2)} F(\beta_2),$$

which is equivalent to $K(\theta^*) = 0$, contradicting to the assumption that firm invests but does not produce. So the firm does not invest if there is no production right after the investment.

- If the firm produces below capacity right after the investment, then the option value of the project is

$$V(\theta, K) = M_1(K) \theta^{\beta_1} + M_2 \theta^{\beta_2} + \frac{\theta}{4\gamma(r - \alpha)} - \frac{2c}{4\gamma r} + \frac{c^2}{4\gamma(r + \alpha - \sigma^2) \theta}$$

with $M_1(K)$, M_2 as in (26) and (25). Letting the first order partial derivative of $V(\theta, K) - \delta K$ with respect to K equal 0 at $K(\theta)$ gives $K(\theta)$, the same as (29). Because the second order partial derivative of $V(\theta, K) - \delta K$ with respect to K is negative, there is a global maximum at $K(\theta)$.

Next, we determine the optimal investment timing θ^* . If (28) has admissible solutions, then we get

$$\begin{aligned} & M_1(K(\theta^*)) \theta^{*\beta_1} + M_2 \theta^{*\beta_2} + \frac{\theta^*}{4\gamma(r - \alpha)} - \frac{2c}{4\gamma r} \\ & + \frac{c^2}{4\gamma(r + \alpha - \sigma^2) \theta^*} - \delta K(\theta^*) \\ & = \frac{\theta^*}{\beta_1} \left[\frac{\partial V(\theta^*, K(\theta^*))}{\partial \theta} + \frac{\partial V(\theta^*, K(\theta^*))}{\partial K} \frac{dK(\theta^*)}{d\theta} \right] \\ & = \frac{\theta^*}{\beta_1} \left[\beta_1 M_1(K(\theta^*)) \theta^{*\beta_1-1} + \beta_2 M_2 \theta^{*\beta_2-1} + \frac{1}{4\gamma(r - \alpha)} - \frac{c^2}{4\gamma(r + \alpha - \sigma^2) \theta^{*2}} \right] \\ & = M_1(K(\theta^*)) \theta^{*\beta_1} + \frac{\beta_2}{\beta_1} M_2 \theta^{*\beta_2} + \frac{\theta^*}{4\gamma\beta_1(r - \alpha)} - \frac{c^2}{4\gamma\beta_1(r + \alpha - \sigma^2) \theta^*}. \end{aligned}$$

So θ^* should satisfy the implicit expression

$$\begin{aligned} & \frac{\beta_1 - \beta_2}{\beta_1} M_2 \theta^{*\beta_2} - \delta K(\theta^*) \\ & + \frac{1}{4\gamma} \left[\frac{\beta_1 - 1}{\beta_1} \frac{\theta^*}{r - \alpha} - \frac{2c}{r} + \frac{\beta_1 + 1}{\beta_1} \frac{c^2}{(r + \alpha - \sigma^2) \theta^*} \right] = 0. \end{aligned}$$

In case the derived $K(\theta^*)$ is such that $K(\theta^*) \leq \frac{\theta^* - c}{2\gamma\theta^*}$, i.e. the capacity is not bigger than the optimal output, then it contradicts to that the firm produces below capacity right after the investment. Thus, the firm would not invest for this case.

- If the firm produces up to capacity right after the investment, then the value of the project is

$$V(\theta, K) = N(K) \theta^{\beta_2} + \frac{(1 - \gamma K) K}{r - \alpha} \theta - \frac{cK}{r},$$

where $N(K)$ is as in (27). The first order condition of $V(\theta, K) - \delta K$ with respect to K implies that the optimal value for capacity, $K(\theta)$, should implicitly satisfy

$$\frac{(\beta_2 + 1) F(\beta_1) (1 - 2\gamma K(\theta))^{\beta_2}}{2(\beta_1 - \beta_2) c^{\beta_2 - 1}} \theta^{\beta_2} + \frac{1 - 2\gamma K(\theta)}{r - \alpha} \theta - \frac{c}{r} - \delta = 0. \quad (30)$$

In order to check the second order partial derivative of $V(\theta, K) - \delta K$ with respect to K , we let

$$\begin{aligned} \mathcal{F}(\theta, K) &= \frac{dN(K)}{dK} \theta^{\beta_2} + \frac{1 - 2\gamma K}{r - \alpha} \theta - \frac{c}{r} - \delta \\ &= \frac{F(\beta_1) c^{1 - \beta_2} (\beta_2 + 1)}{2(\beta_1 - \beta_2)} (1 - 2\gamma K)^{\beta_2} \theta^{\beta_2} + \frac{1 - 2\gamma K}{r - \alpha} \theta - \frac{c}{r} - \delta, \end{aligned}$$

and from Appendix B,

$$\frac{\partial \mathcal{F}(\theta, K)}{\partial K} < 0.$$

So the second order partial derivative of $V(\theta, K) - \delta K$ with respect to K is negative, implying if equation (30) has an admissible solution, there is a maximum $V(\theta, K(\theta)) - \delta K(\theta)$. If (30) does not have any admissible solution, then $V(\theta, K) - \delta K$ is increasing or decreasing with K , and the firm would not invest for this case. We can rule out the increasing case, because it implies more capacity is better. Particularly, capacity that is bigger than $(\theta - c)/(2\gamma\theta)$ is better. This suggests the firm should invest for the case of producing below capacity right after the investment. For the decreasing case, it implies that the optimal investment capacity is 0, we can also rule out the decreasing case.

If (28) has admissible solutions, then the optimal investment threshold θ^* is the solution of the following equation,

$$\begin{aligned} &N(K(\theta)) \theta^{\beta_2} + \frac{(1 - \gamma K(\theta)) K(\theta)}{r - \alpha} \theta - \frac{cK(\theta)}{r} - \delta K(\theta) \\ &= \frac{\theta}{\beta_1} \left[\beta_2 N(K(\theta)) \theta^{\beta_2 - 1} + \frac{(1 - \gamma K(\theta)) K(\theta)}{r - \alpha} \right]. \end{aligned}$$

Rearranging terms gives that θ^* implicitly satisfies

$$\begin{aligned} & \frac{\beta_1 - \beta_2}{\beta_1} N(K(\theta^*)) \theta^{*\beta_2} + \frac{\beta_1 - 1}{\beta_1} \frac{(1 - \gamma K(\theta^*)) K(\theta^*) \theta^*}{r - \alpha} \\ & - \frac{cK(\theta^*)}{r} - \delta K(\theta^*) = 0. \end{aligned}$$

If this equation does not give any admissible solution or gives a solution that is smaller than c , or $K(\theta^*) > (\theta^* - c)/(2\gamma\theta^*)$, then the firm would not invest in this case.

A.2.2 Derivation of conditions for producing up to or below capacity right after the investment

If the firm produces below capacity right after the investment (Region 2), then for $\theta \geq c$, it holds that

$$K(\theta) = \frac{1}{2\gamma} - \frac{c}{2\gamma\theta} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} > \frac{\theta - c}{2\gamma\theta},$$

which is equivalent to

$$\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} < 1.$$

For any $\theta \geq c$, right after the investment, the firm either produces below capacity or up to capacity. So the firm produces up to capacity right after the investment if

$$\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \geq 1.$$

At the boundary of the two regions, the equality holds. For Region 2, we get the optimal value for investment capacity at the boundary is

$$K(\theta) = \frac{\theta - c}{2\gamma\theta}.$$

The optimal value of investment capacity at the boundary for Region 3 is the solution to (30) when

$$2\delta(\beta_1 - \beta_2) = c(1 + \beta_1)F(\beta_2). \quad (31)$$

Then (30) can be written as

$$0 = \frac{c(\beta_2 + 1)F(\beta_1)}{2(\beta_1 - \beta_2)} \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta$$

$$\begin{aligned}
&= \left[\frac{c(\beta_2 + 1)F(\beta_1)}{2(\beta_1 - \beta_2)} - \delta \right] \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \delta \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} \\
&\quad + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \frac{c(\beta_2 + 1)F(\beta_1) - 2(\beta_1 - \beta_2)\delta(\theta - 2\theta\gamma K)^{\beta_2}}{2(\beta_1 - \beta_2)c^{\beta_2}} \\
&\quad + \delta \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \frac{c(\beta_2 + 1)F(\beta_1) - c(\beta_1 + 1)F(\beta_2)(\theta - 2\theta\gamma K)^{\beta_2}}{2(\beta_1 - \beta_2)c^{\beta_2}} \\
&\quad + \delta \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \frac{(\beta_1 - \beta_2)(2\alpha\sigma^2 - 2\alpha^2 - 2\alpha r)}{2(\beta_1 - \beta_2)} \frac{c}{r(r - \alpha)(r + \alpha - \sigma^2)} \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} \\
&\quad + \delta \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \frac{-\alpha c}{r(r - \alpha)} \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \delta \frac{(\theta - 2\theta\gamma K)^{\beta_2}}{c^{\beta_2}} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \left(\frac{c}{r} - \frac{c}{r - \alpha} + \delta \right) \left(\frac{\theta - 2\theta\gamma K}{c} \right)^{\beta_2} + \frac{\theta - 2\theta\gamma K}{r - \alpha} - \frac{c}{r} - \delta \\
&= \left(\frac{c}{r} + \delta \right) \left[\left(\frac{\theta - 2\theta\gamma K}{c} \right)^{\beta_2} - 1 \right] + \frac{\theta - 2\theta\gamma K}{r - \alpha} \left[1 - \left(\frac{\theta - 2\theta\gamma K}{c} \right)^{\beta_2 - 1} \right].
\end{aligned}$$

$K(\theta) = \frac{\theta - c}{2\gamma\theta}$ is a solution for this equation, implying there is smooth transfer from Region 2 and Region 3. However, there is no overlap at the boundary because the optimal capacity is equal to production at the boundary, implying it is Region 3 defined.

Next, we determine $\bar{\sigma}$ such that equation (31) holds as illustrated in Figure 2. Substituting β_1, β_2 and $F(\beta_2)$ from (13)–(15) into (31) gives

$$c(2r\bar{\sigma}^2 + 2\alpha^2 - \alpha\bar{\sigma}^2) = [2\delta r(r - \alpha) - \alpha c] \sqrt{(\bar{\sigma}^2 - 2\alpha)^2 + 8r\bar{\sigma}^2}. \quad (32)$$

Two cases are considered:

- (a) $2\delta r(r - \alpha) \leq \alpha c$ (or $\alpha \geq \frac{2\delta r^2}{c + 2\delta r}$). According to (32), for all $\sigma > 0$ such that $r + \alpha > \sigma^2$, then

$$[2\delta r(r - \alpha) - \alpha c] \sqrt{(\bar{\sigma}^2 - 2\alpha)^2 + 8r\bar{\sigma}^2} \leq 0 < c(2r\bar{\sigma}^2 + 2\alpha^2 - \alpha\bar{\sigma}^2),$$

which implies

$$2\delta(\beta_1 - \beta_2) < c(1 + \beta_1)F(\beta_2),$$

and it is Region 2 defined.

(b) $2\delta r(r - \alpha) > \alpha c$ (or $\alpha < \frac{2\delta r^2}{c+2\delta r}$). Then (32) becomes

$$\left((\bar{\sigma}^2 - 2\alpha)^2 + 8r\bar{\sigma}^2\right)(2\delta r(r - \alpha) - \alpha c)^2 = c^2(2r\bar{\sigma}^2 + 2\alpha^2 - \alpha\bar{\sigma}^2)^2,$$

which can also be written as

$$(\Lambda - (2r - \alpha)^2)\bar{\sigma}^4 + 4(\Lambda - \alpha^2)(2r - \alpha)\bar{\sigma}^2 + 4\Lambda\alpha^2 - 4\alpha^4 = 0, \quad (33)$$

with

$$\Lambda = \left(\frac{2\delta r(r - \alpha) - \alpha c}{c}\right)^2.$$

The discriminant for (33) is

$$\Delta = 64\Lambda r(\Lambda - \alpha^2)(r - \alpha),$$

and the possible solutions for $\bar{\sigma} > 0$ are supposed to satisfy either of the following

$$\begin{aligned} \bar{\sigma}_1^2 &= \frac{-2(\Lambda - \alpha^2)(2r - \alpha) - 4\sqrt{r\Lambda(\Lambda - \alpha^2)(r - \alpha)}}{\Lambda - (2r - \alpha)^2}, \\ \bar{\sigma}_2^2 &= \frac{-2(\Lambda - \alpha^2)(2r - \alpha) + 4\sqrt{r\Lambda(\Lambda - \alpha^2)(r - \alpha)}}{\Lambda - (2r - \alpha)^2}. \end{aligned}$$

Then we have the following subcases.

- If $0 < \Lambda < \alpha^2$, which is $\alpha > \frac{r^2\delta}{c+r\delta}$, then $\Delta < 0$ and (33) has no solution for $\bar{\sigma}^2$. Then for all $\sigma > 0$ with $r + \alpha > \sigma^2$,

$$(\Lambda - (2r - \alpha)^2)\sigma^4 + 4(\Lambda - \alpha^2)(2r - \alpha)\sigma^2 + 4\Lambda\alpha^2 - 4\alpha^4 < 0,$$

which implies

$$2\delta(\beta_1 - \beta_2) < c(1 + \beta_1)F(\beta_2).$$

So, it is Region 2 defined.

- If $\alpha^2 \leq \Lambda < (2r - \alpha)^2$, which is equivalent to $r - \frac{c}{\delta} < \alpha \leq \frac{\delta r^2}{c+\delta r}$, then $\Delta \geq 0$, and it holds that $\bar{\sigma}_1^2 \leq 0$ and $\bar{\sigma}_2^2 \geq 0$. So there is one solution for $\bar{\sigma} > 0$ and $\bar{\sigma} = \bar{\sigma}_2$. For any $\sigma > 0$ with $\sigma^2 < r + \alpha$, Region 3 is defined when $0 < \sigma \leq \bar{\sigma}$ and Region 2 is defined when $\sigma \geq \bar{\sigma}$.

- If $\Lambda > (2r - \alpha)^2$, which is $\alpha < r - \frac{c}{\delta}$, then $\bar{\sigma}_1^2 < 0$ and $\bar{\sigma}_2^2 < 0$. So there is no solution for $\bar{\sigma} > 0$, and for all $\sigma > 0$ with $\sigma^2 < r + \alpha$, we have

$$(\Lambda - (2r - \alpha)^2) \sigma^4 + 4(\Lambda - \alpha^2)(2r - \alpha) \sigma^2 + 4\Lambda\alpha^2 - 4\alpha^4 > 0,$$

which implies

$$2\delta(\beta_1 - \beta_2) > c(1 + \beta_1)F(\beta_2).$$

It is only Region 3 that is defined.

- If $\Lambda = (2r - \alpha)^2$, then $\delta(r - \alpha) = c$ and

$$\begin{aligned} & [2\delta r(r - \alpha) - \alpha c]^2 \left[(\sigma^2 - 2\alpha)^2 + 8r\sigma^2 \right] - c^2 (2r\sigma^2 + 2\alpha^2 - \alpha\sigma^2)^2 \\ &= c^2 (2r - \alpha)^2 \left[(\sigma^2 - 2\alpha)^2 + 8r\sigma^2 \right] - c^2 [(2r - \alpha)\sigma^2 + 2\alpha^2]^2 \\ &= c^2 (2r - \alpha)^2 (\sigma^2 - 2\alpha)^2 + 8rc^2\sigma^2(2r - \alpha)^2 - c^2\sigma^4(2r - \alpha)^2 \\ &\quad - 4\alpha^2\sigma^2c^2(2r - \alpha) - 4c^2\alpha^4 \\ &= 4c^2(2r - \alpha)^2(\alpha^2 - \alpha\sigma^2 + 2r\sigma^2) - 4c^2\alpha^2(\alpha^2 - \alpha\sigma^2 + 2r\sigma^2) \\ &= 16rc^2(r - \alpha)(\alpha^2 - \alpha\sigma^2 + 2r\sigma^2) > 0. \end{aligned}$$

It implies that

$$2\delta(\beta_1 - \beta_2) > c(1 + \beta_1)F(\beta_2),$$

and Region 3 is defined.

Summarizing the above cases, it can be concluded that when $\alpha > \delta r^2/(c + \delta r)$, it is always Region 2 that is defined. When $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$, there exists $\bar{\sigma} > 0$ such that if $\bar{\sigma}^2 < r + \alpha$, then it is Region 3 for $\sigma \leq \bar{\sigma}$ and Region 2 for $\sigma > \bar{\sigma}$; if $\bar{\sigma}^2 \geq r + \alpha$, then it is always Region 3. When $\alpha \leq r - c/\delta$, then it is Region 3 that is defined.

B Additional Proof for Proposition 2

B.1 Proof of $\partial\mathcal{F}(\theta, K)/\partial K < 0$

Before we check the sign of $\partial\mathcal{F}(\theta, K)/\partial K$, we first look at the signs for β_1, β_2 and $F(\beta_1)$. $r > 0$ and the assumption $r > \alpha$ imply $(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2r}{\sigma^2} > (\frac{1}{2} + \frac{\alpha}{\sigma^2})^2$, thus $\beta_1 > 1$ and $\beta_2 < -1$ if $\frac{1}{2} + \frac{\alpha}{\sigma^2} > 0$. If $\frac{1}{2} + \frac{\alpha}{\sigma^2} \leq 0$, then $\frac{\alpha}{\sigma^2} \leq -\frac{1}{2}$. The assumption $r + \alpha - \sigma^2 > 0$ implies $\frac{2r}{\sigma^2} > 2 - \frac{2\alpha}{\sigma^2}$. Then

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$$

$$\begin{aligned}
&< \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + 2 - \frac{2\alpha}{\sigma^2}} \\
&= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\frac{9}{4} - \frac{3\alpha}{\sigma^2} + \left(\frac{\alpha}{\sigma^2}\right)^2} \\
&= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \frac{\alpha}{\sigma^2} - \frac{3}{2} \\
&= -1.
\end{aligned}$$

Thus, it can be concluded that $\beta_2 < -1$.

Recall that

$$F(\beta) = \frac{\beta(2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) + r(2\alpha - \sigma^2)}{r(r - \alpha)(r + \alpha - \sigma^2)},$$

with $r(r - \alpha)(r + \alpha - \sigma^2) > 0$ since $r > \alpha$ and $r + \alpha > \sigma^2$.

If $2\alpha < \sigma^2$, then

$$\begin{aligned}
r(r - \alpha)(r + \alpha - \sigma^2)F'(\beta) &= 2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2 \\
&< 2\alpha\sigma^2 + \alpha\sigma^2 - \sigma^4 - 2\alpha^2 \\
&= -\sigma^4 + 3\alpha\sigma^2 - 2\alpha^2 \\
&= -(\sigma^2 - \alpha)(\sigma^2 - 2\alpha) \\
&< 0;
\end{aligned}$$

and if $\sigma^2 < 2\alpha$, then

$$\begin{aligned}
r(r - \alpha)(r + \alpha - \sigma^2)F'(\beta) &= 2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2 \\
&= \alpha\sigma^2 - r\sigma^2 + \alpha\sigma^2 - 2\alpha^2 \\
&= \sigma^2(\alpha - r) + \alpha(\sigma^2 - 2\alpha) \\
&< 0.
\end{aligned}$$

So $F'(\beta) < 0$. Define β_0 such that $F(\beta_0) = 0$, then

$$\beta_0 = \frac{r(2\alpha - \sigma^2)}{r\sigma^2 + 2\alpha^2 - 2\alpha\sigma^2}.$$

Because $F(\beta)$ decreases with β , if we can compare the values for β_0 , β_1 and β_2 , then it would be easy to get the signs for $F(\beta_1)$ and $F(\beta_2)$. Let

$$G(\beta) = \frac{\sigma^2}{2}\beta^2 + \left(\alpha - \frac{\sigma^2}{2}\right)\beta - r.$$

β_1 and β_2 are the intersection points of $G(\beta)$ and the β -axis. If we can show that $G(\beta_0) < 0$, then $\beta_1 > \beta_0 > \beta_2$, $F(\beta_1) < 0$ and $F(\beta_2) > 0$.

$$\begin{aligned}
G(\beta_0) &= \frac{\sigma^2}{2}\beta_0^2 + \left(\alpha - \frac{\sigma^2}{2}\right)\beta_0 - r \\
&= \frac{1}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} \left\{ \frac{r^2\sigma^2(2\alpha - \sigma^2)^2}{2} - r[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2 \right. \\
&\quad \left. + \frac{r}{2}(2\alpha - \sigma^2)^2[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2] \right\} \\
&= \frac{r}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} \left\{ (2\alpha - \sigma^2)^2(r\sigma^2 - \alpha\sigma^2 + \alpha^2) \right. \\
&\quad \left. - (r\sigma^2 - \alpha\sigma^2 + \alpha^2 + \alpha^2 - \alpha\sigma^2)^2 \right\} \\
&= \frac{r}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} \left\{ (2\alpha - \sigma^2)^2(r\sigma^2 - \alpha\sigma^2 + \alpha^2) \right. \\
&\quad \left. - (r\sigma^2 - \alpha\sigma^2 + \alpha^2)^2 - 2(\alpha^2 - \alpha\sigma^2)(r\sigma^2 - \alpha\sigma^2 + \alpha^2) - (\alpha^2 - \alpha\sigma^2)^2 \right\} \\
&= \frac{r[(r\sigma^2 - \alpha\sigma^2 + \alpha^2)(-r\sigma^2 - \alpha\sigma^2 + \alpha^2 + \sigma^4) - (\alpha^2 - \alpha\sigma^2)^2]}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} \\
&= \frac{r[(\alpha^2 - \alpha\sigma^2)^2 - r^2\sigma^4 + \sigma^4(r\sigma^2 - \alpha\sigma^2 + \alpha^2) - (\alpha^2 - \alpha\sigma^2)^2]}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} \\
&= \frac{r\sigma^4(r-\alpha)}{[(r-\alpha)\sigma^2 + 2\alpha^2 - \alpha\sigma^2]^2} [\sigma^2 - (r+\alpha)].
\end{aligned}$$

Because $r > \alpha$ and $\sigma^2 < r + \alpha$, we get $G(\beta_0) < 0$. Thus, $F(\beta_1) < 0$ and $F(\beta_2) > 0$. Next, we check the sign for $\partial\mathcal{F}(\theta, K)/\partial K$.

$$\frac{\partial\mathcal{F}(\theta, K)}{\partial K} = -\frac{\beta_2\gamma F(\beta_1)\theta^{\beta_2}(\beta_2+1)}{(\beta_1-\beta_2)c^{\beta_2-1}}(1-2\gamma K)^{\beta_2-1} - \frac{2\gamma\theta}{r-\alpha},$$

where

$$-\frac{\beta_2\gamma F(\beta_1)(\beta_2+1)}{(\beta_1-\beta_2)c^{\beta_2-1}} > 0.$$

Because Region 3 is defined such that $\theta \geq \frac{c}{1-2\gamma K}$, we have

$$\begin{aligned}
\frac{\partial\mathcal{F}(\theta, K)}{\partial K} &\leq \gamma\theta \left[-\frac{\beta_2 F(\beta_1)(\beta_2+1)}{\beta_1-\beta_2} - \frac{2}{r-\alpha} \right] \\
&= \frac{\gamma\theta}{r-\alpha} \left[-\frac{\beta_2(\beta_2+1)\beta_1(2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) + r(2\alpha - \sigma^2)}{(\beta_1-\beta_2)r(r+\alpha-\sigma^2)} - 2 \right] \\
&= \frac{\gamma\theta}{r-\alpha} \left[\frac{\beta_2+1}{\beta_1-\beta_2} \frac{\frac{2r}{\sigma^2}(2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) - r\beta_2(2\alpha - \sigma^2)}{r(r+\alpha-\sigma^2)} - 2 \right]
\end{aligned}$$

$$= \frac{\gamma\theta}{(\beta_1 - \beta_2)r(r - \alpha)(r + \alpha - \sigma^2)} \left[(\beta_2 + 1) \frac{2}{\sigma^2} (2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) - \beta_2(\beta_2 + 1)(2\alpha - \sigma^2) - 2(\beta_1 - \beta_2)(r + \alpha - \sigma^2) \right]$$

and

$$\begin{aligned} & (\beta_2 + 1) \frac{2}{\sigma^2} (2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) - \beta_2(\beta_2 + 1)(2\alpha - \sigma^2) - 2(\beta_1 - \beta_2)(r + \alpha - \sigma^2) \\ &= (2\beta_2 + 2) \left(2\alpha - r - \frac{2\alpha^2}{\sigma^2} \right) - \beta_2^2(2\alpha - \sigma^2) - \beta_2(2\alpha - \sigma^2) \\ & \quad - 2\beta_1(r + \alpha - \sigma^2) + 2\beta_2(r + \alpha - \sigma^2) \\ &= 2\beta_2 \left(2\alpha - \frac{2\alpha^2}{\sigma^2} - \frac{\sigma^2}{2} \right) - \beta_2^2(2\alpha - \sigma^2) + 2 \left(2\alpha - r - \frac{2\alpha^2}{\sigma^2} \right) - 2\beta_1(r + \alpha - \sigma^2) \\ &= 2\beta_2 \left[\frac{\alpha}{\sigma^2} (-2\alpha + \sigma^2) + \frac{1}{2} (2\alpha - \sigma^2) \right] - \beta_2^2(2\alpha - \sigma^2) + 2(\alpha - r) + \frac{2\alpha}{\sigma^2} (\sigma^2 - 2\alpha) \\ & \quad - 2\beta_1(r + \alpha - \sigma^2) \\ &= 2\beta_2(2\alpha - \sigma^2) \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right) - \beta_2^2(2\alpha - \sigma^2) + 2(\alpha - r) + \frac{2\alpha}{\sigma^2} (\sigma^2 - 2\alpha) \\ & \quad - 2\beta_1(r + \alpha - \sigma^2) \\ &= -\frac{\beta_2}{\sigma^2} (2\alpha - \sigma^2)^2 - \beta_2^2(2\alpha - \sigma^2) + 2(\alpha - r) - \frac{2\alpha}{\sigma^2} (2\alpha - \sigma^2) - 2\beta_1(r + \alpha - \sigma^2) \\ &= \frac{(2\alpha - \sigma^2)^2}{\sigma^2} \left(\frac{2\alpha - \sigma^2}{2\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \right) + 2(\alpha - r) - \frac{2\alpha}{\sigma^2} (2\alpha - \sigma^2) \\ & \quad - (2\alpha - \sigma^2) \left[2 \left(\frac{\sigma^2 - 2\alpha}{2\sigma^2} \right)^2 + \frac{2r}{\sigma^2} + \frac{2\alpha - \sigma^2}{\sigma^2} \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \right] \\ & \quad - 2\beta_1(r + \alpha - \sigma^2) \\ &= \frac{(2\alpha - \sigma^2)^3}{2\sigma^4} + \frac{(2\alpha - \sigma^2)^2}{\sigma^2} \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} - \frac{2\alpha}{\sigma^2} (2\alpha - \sigma^2) \\ & \quad - \frac{(2\alpha - \sigma^2)^3}{2\sigma^4} - \frac{2r(2\alpha - \sigma^2)}{\sigma^2} - \frac{(2\alpha - \sigma^2)^2}{\sigma^2} \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \\ & \quad - 2\beta_1(r + \alpha - \sigma^2) + 2(\alpha - r) \\ &= 2(\alpha - r) - \frac{2(2\alpha - \sigma^2)}{\sigma^2} (r + \alpha) - 2\beta_1(r + \alpha - \sigma^2) \end{aligned}$$

$$\begin{aligned}
&= 2(\alpha - r) - \frac{2(2\alpha - \sigma^2)}{\sigma^2} (r + \alpha) - 2(r + \alpha - \sigma^2) \left(\frac{\sigma^2 - 2\alpha}{2\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \right) \\
&= 2(\alpha - r) - \frac{2\alpha - \sigma^2}{\sigma^2} (r + \alpha + \sigma^2) - 2(r + \alpha - \sigma^2) \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.
\end{aligned}$$

Since $\beta_1 > 0$, $\beta_2 < 0$, $r - \alpha > 0$ and $r + \alpha - \sigma^2 > 0$, we conclude that if $2\alpha - \sigma^2 \geq 0$, then $\partial \mathcal{F}(\theta, K)/\partial K < 0$. However, if $\sigma^2 - 2\alpha > 0$, then we continue with the expression above and get

$$\begin{aligned}
&2(\alpha - r) - \frac{2\alpha - \sigma^2}{\sigma^2} (r + \alpha + \sigma^2) - 2(r + \alpha - \sigma^2) \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \\
&< 2(\alpha - r) - \frac{2\alpha - \sigma^2}{\sigma^2} (r + \alpha + \sigma^2) - 2(r + \alpha - \sigma^2) \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2} \\
&= 2(\alpha - r) - \frac{2\alpha - \sigma^2}{\sigma^2} (r + \alpha + \sigma^2) - 2(r + \alpha - \sigma^2) \frac{\sigma^2 - 2\alpha}{2\sigma^2} \\
&= 2(\alpha - r) + \frac{\sigma^2 - 2\alpha}{\sigma^2} (r + \alpha + \sigma^2) - \frac{\sigma^2 - 2\alpha}{\sigma^2} (r + \alpha - \sigma^2) \\
&= 2(\alpha - r) + 2(\sigma^2 - 2\alpha) = -2(r + \alpha - \sigma^2) < 0.
\end{aligned}$$

Thus, we conclude that

$$\frac{\partial \mathcal{F}(\theta, K)}{\partial K} < 0.$$

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