The Role of Collateral in a Model of Debt Renegotiation

by Helmut Bester

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HOW DOES THE PROSPECT OF FUTURE DEBT renegotiation affect the lender's security interests at the contracting date? We study this question in a simple model of borrowing and lending with asymmetric information. A risk-neutral entrepreneur needs to raise capital for a risky investment project. The project outcome, however, cannot directly be observed by the creditors. The optimal loan arrangement is a debt contract with a bankruptcy clause that acts as a payment incentive for the entrepreneur. The institution of bankruptcy allows the creditor to take possession of some of the entrepreneur's assets in the event of default. We show that the extent of the entrepreneur's liabilities in the optimal loan contract depends upon the creditor's commitment to impose bankruptcy should default ever occur. If the creditor is precommitted not to forgive any portion of the outstanding debt, a limited liability arrangement is optimal. This means that default should entitle the creditor to liquidate only the assets remaining from the project that has been financed by the loan. In the absence of precommitment, however, such limitation of liability may no longer be optimal. Instead, debt may efficiently be secured by additional outside assets.

Although outside collateral increases the total amount of assets liquidated in the event of bankruptcy, it may lower the expected dead-weight loss associated with inefficient asset liquidation. We show that collateral requirements make it more likely that the initial debt contract is renegotiated and some part of the debt forgiven in case the entrepreneur declares himself unable to pay his debt in full. Thus, favoring

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debt renegotiation, collateral may help avoiding an inefficient change in project ownership. Indeed, a recent study by Asquish, Gertner, and Scharfstein (1992) shows that companies under financial distress frequently restructure their debt through direct negotiations. They find that that of seventy-six companies in their sample, fifty-nine restructured their bank debt in some way. Moreover, the bank’s restructuring incentives are found to be positively related to the degree of collateralization.

Renegotiation will occur when the borrower-lender relationship reaches a point where the initial contract stipulates an ex post inefficient outcome. Usually the creditor is less efficient as manager of the project’s assets than is the borrower so that bankruptcy may prove ex post inefficient. The contracting parties may achieve a Pareto-improvement by writing a new contract under which the entrepreneur maintains project ownership at a reduced debt level. The possibility of renegotiation implies that default will not always be penalized by bankruptcy and both parties to the loan realize this. Knowing that there is a chance of debt forgiveness, the borrower may falsely claim that the debt exceeds the investment's return and that he is forced to default. This motive for cheating is weakened when collateral has been posted. The higher the degree of collateralization, the more inclined is the creditor to believe that the project return actually is low when he observes default. Consequently he finds the option of taking over the project less profitable in comparison to forgiving a portion of the debt. In this way, outside collateral may reduce the expected cost of bankruptcy. Its benefit is positively related to the size of the dead-weight loss resulting from project liquidation. Especially high-risk firms will find it advantageous to offer collateral to their potential creditors.

A convincing explanation of the existence of secured debt must demonstrate that its use may provide gains that exceed its costs.¹ If collateral merely redistributed wealth between the borrower and lender in the event of default, other contractual devices that avoid costly liquidation of collateralized assets would prove advantageous. To compensate the lender for the risk of default, firms would be better off by paying interest rates that reflect their risk category instead of selling secured debt. The recent literature on credit contracts with asymmetric information shows that this argument fails if the lender knows less than the borrower about the investment’s riskiness. In credit markets with moral hazard or adverse selection outside collateral may serve as an incentive or screening device (See Besanko and Thakor 1987; Bester 1985, 1987; and Chan and Kanatas 1985). Outside collateral increases the punishment for default. If the borrower can choose among a variety of projects with different riskiness, collateral enforces the selection of less-risky projects. Similarly, as a response to adverse selection, lenders may offer a menu of contracts to sort loan loan applicants into risk categories. Entrepreneurs with low probability of default then reveal themselves by accepting collateral requirements that would be unattractive for high risks. In summary, this literature predicts a negative relation between default risk and the amount of collateral. This prediction is opposite to the

¹ Schwartz (1981) gives a critical review of explanations of the existence of secured debt.
conventional wisdom that high-risk firms have to issue security in order to attract creditors.\footnote{This view is empirically supported by Leetó and Scott's (1989) analysis of small business loans.}

To focus on the impact of renegotiation on the terms of the initial debt contract, we consider a model where all parties have ex ante symmetric information. The investment's return distribution and the entrepreneur's ability to pay his debt are not affected by the terms of the loan agreement. In contrast to the incentive or screening explanation, we find that collateral is more likely to be used for financing high-risk investments. Indeed, we conclude that renegotiation may seriously undermine the role of collateral as a screening device. The reason is that low-risk entrepreneurs can no longer distinguish themselves by posting collateral if collateralization becomes attractive also for high-risk entrepreneurs.

Our basic model is inspired by Gale and Hellwig (1985) and Diamond (1984), who derive debt contracts as optimal arrangements under asymmetric information about project outcomes. Their analysis, however, presumes precommitment so that contracts may include ex-post inefficiencies that are common knowledge. Huberman and Kahn (1988) study debt renegotiation in a model where borrower and lender have symmetric information but return realizations are not verifiable. In this context, there is no ex-post inefficient bankruptcy and the institution of limited liability suffices to encourage the entrepreneur to pay his debt. Bergman and Callen (1991) model debt renegotiation as a bargaining game between debtholders and shareholders. The shareholders may force concessions from the creditors by threatening to run down the firm's assets. The creditors anticipation of the bargaining outcome creates an upper bound on the amount of debt in the financial structure of the firm. Hart and Moore (1989) show that debt renegotiation may involve inefficient asset liquidation despite symmetric information. In their multiperiod model this may happen because the entrepreneur cannot commit credibly to pay a certain amount of money in the future. Much of the literature on debt renegotiation deals with the case of sovereign debt (see, for example, Bulow and Rogoff 1989; Gale and Hellwig 1989; and Fernandez and Rothenthal 1988). The basic assumption of this literature is that there is no third party enforcement of contracts. This restricts the possibility of secured lending because in the event of default the creditor has at most limited access to the borrower's assets.

The remainder of this paper consists of four sections. Section 1 presents an extensive game of contract design and renegotiation. Section 2 studies the case of precommitment as a point of reference. Optimal contracts in the absence of precommitment are analyzed in section 3. Section 4 concludes.

1. THE BASIC MODEL

Consider a risk-neutral entrepreneur who is endowed with a project. The project requires some fixed initial investment $I$ and yields the random return $X$. With proba-
probability $0 < p < 1$ the project is successful and the return realization is $X_s$; if the project fails, the return is $X_f$, with $X_s > X_f > 0$.

The entrepreneur has no liquid funds to finance the investment. He raises the amount $I$ by issuing debt. As in Diamond (1984) or in Gale and Hellwig (1985), this form of finance results from the assumption that borrower and lender have asymmetric information. The entrepreneur observes the return realization at no cost. The creditor receives this information only after taking over the project. However, such a transfer of ownership is costly. The creditor’s net valuation of the project return $X$ is $\alpha X$, with $0 < \alpha < 1$. The cost $(1 - \alpha)X$ arises because the original entrepreneur has more ability to complete the project or because monitoring and liquidating the project is costly for the creditor. Also we will assume that outsiders remain uninformed about the project outcome even when the creditor becomes owner of the project. Since return realizations are not verifiable to outsiders, the borrower’s repayment obligation $R$ cannot be conditioned on the project outcome. Notice that the entrepreneur’s private information about the investment return is the only source of informational asymmetries in the model; there is no uncertainty about the riskiness of the project or the valuations of assets. Let

$$\alpha[pX_s + (1 - p)X_f] > I. \quad (1)$$

Thus the expected foreclosure value of the project exceeds the investment cost and the creditor’s expected profit from making a loan can be made positive simply by allowing him to foreclose on the project in the event of default.

While the entrepreneur has no liquid funds, he owns some amount $W$ of collateralizable wealth. This wealth cannot be used to finance investment directly, say, because it consists of illiquid assets, or it represents the entrepreneur’s future income outside the project. However, the creditor may use $W$, or any fraction thereof, as collateral $C$ for a loan. The lender’s and the borrower’s valuation of $C$ are not the same. Taking possession of and liquidating $C$ typically involves transaction costs. These costs will be represented by a factor $1 - \beta$, with $0 < \beta < 1$, so that the creditor’s net valuation of $C$ equals $\beta C$. Through collateralization the creditor can receive additional assets outside the project which otherwise would not be legally attachable. A main focus of our analysis is to investigate why it might be optimal to assign such a right to the creditor in the event of default.3

Suppose the creditor breaks even only if the loan contract specifies a repayment obligation $R$ that exceeds $X_f$. Since the entrepreneur has private information about the project outcome, he must have some contractual incentives to pay $R$ when the return is $X_s$. This incentive can be created by giving the creditor the right to seize some of the debtor’s assets in the event of default. Some models of the credit market assume that the borrower repays his loan only if the value of the collateral exceeds his debt (see, for example, Barro 1976 and Benjamin 1978). Under this arrangement

3. In principle, a contract could transfer some of the entrepreneur’s illiquid assets independently of whether he fulfills his repayment obligation or not. It is easy to see, however, that such an arrangement would be suboptimal if $\beta < 1$. 
the creditor is allowed to liquidate the collateral \( C \) but not the project assets. In the event of project failure, the entrepreneur pays \( X_f \) and loses \( C \leq W \). As long as the incentive restriction \( R \leq C + X_f \) is satisfied, the successful entrepreneur is better off by paying \( R \) than by defaulting. Of course, this solution works only if \( W \) is sufficiently large. We are interested in the case where collateralization is insufficient to provide appropriate payment incentives. The creditor’s right to foreclose on the project assets becomes then essential to induce the borrower to pay his debt. This will create a role for renegotiation because liquidating the project is ex post inefficient. Accordingly, we will assume

\[ I > W + X_f. \]  

Thus the creditor cannot recover the amount \( I \) in case of project failure even when he takes over all of the entrepreneur’s assets. Therefore, the debt contract must specify a repayment obligation \( R \geq I > W + X_f \). Collateralization cannot be used as a payment incentive as the successful entrepreneur would rather pay \( X_f \) and give up his wealth \( W \) than pay \( R \).

In summary, a debt contract \( \Gamma = (R, C) \) obliges the entrepreneur to pay the amount \( R \); failure to fulfill this obligation entitles the creditor to take over the project and the collateral \( C \leq W \). The contract \( \Gamma \) employs the threat of bankruptcy to induce the successful entrepreneur to pay \( R \) even though the creditor is unable to observe the project return. This threat, however, may commit the parties to an inefficient outcome. In the case of project failure it implies the dead-weight cost \((1 - \alpha)X_f + (1 - \beta)C\). Therefore the creditor may wish to renegotiate the original contract \( \Gamma \) and to forgive some part of the debt after the entrepreneur announces project failure. If actually the project has failed, he would maximize his payoff by making the take-it-or-leave-it offer \( \Delta = (X_f, C) \) which reduces the firm’s debt to \( X_f \) and makes the creditor owner of the collateralized assets \( C \). Accepting this proposal leaves the entrepreneur no worse off because the original contract allows the creditor to take possession of the project and the collateral \( C \). While contract renegotiation of this kind may avoid an inefficient allocation of project ownership, it has a negative impact on the successful entrepreneur’s incentives to pay his debt. If he pretends project failure and the creditor concedes to renegotiate, he gains \( R - X_f - C \).

To analyze borrower and lender behavior in such a situation we will adopt the following stylized game of contract design and renegotiation. Potential creditors compete by offering contracts of the form \( \Gamma = (R, C) \). For completeness, each creditor also has the option of not making an offer, which ensures him a profit of zero. If the firm finds none of the contracts acceptable, the game ends and all parties get zero payoff. Otherwise the entrepreneur undertakes the investment by accepting the offer of one of the creditors. All other would-be creditors receive a payoff of zero. As a result of competition, the equilibrium contract maximizes the entrepreneur’s expected payoff subject to the condition that the creditor earns zero expected profits. Of course, these payoffs depend on the borrower’s and lender’s behavior after the contract has been signed. Figure 1 describes their moves in the remaining game,
where we allow the players to adopt mixed strategies. The significance and interpretation of such strategies will be discussed in combination with the equilibrium in section 3.

- In stage one of the borrower-lender relationship $X_s$ is realized with probability $p$ and $X_f$ is realized with probability $1 - p$. This is observed by the entrepreneur while the creditor remains uninformed.

- In stage two only the successful entrepreneur can pay $R$ as $X_f < R < X_s$. Thus after observing $X_f$, the entrepreneur is forced to default. In the event of success he has two choices: He can make his debt payment or he can claim project failure and default. He chooses a possibly mixed strategy so that he defaults with probability $0 < d < 1$ and pays $R$ with probability $1 - d$. In the event of repayment the game ends with payoffs $X_s - R$ and $R - I$ for the entrepreneur and the creditor, respectively.

- In stage three, upon default the creditor either imposes bankruptcy or offers the new contract $\Delta = (X_f, C)$. Again we allow for random strategies and $0 < b < 1$ denotes the probability of bankruptcy. In the case of bankruptcy the creditor takes over the project so that his payoff is either $\alpha X_s + \beta C - I$ or $\alpha X_f + \beta C - I$, depending upon the entrepreneur's type. The entrepreneur's payoff equals $-C$. By contract renegotiation the creditor ensures himself a payoff of $X_f + \beta C - I$; the entrepreneur's payoff from $\Delta$ depends upon his type and is either $X_s - X_f - C$ or $-C$.

We assume that bankruptcy entitles the lender to liquidate the entire project. Therefore, by imposing bankruptcy on a lying entrepreneur the lender may get a payoff that exceeds $R - I$. It is easy to see that this assumption is consistent with an optimal loan arrangement. If the creditor could appropriate only some part of the project, then cheating would become more attractive for the successful entrepreneur and expected bankruptcy costs would be increased. Our stylized model gives the creditor two options in the event of default: He can either exercise his right to foreclose on the debtor's assets or forgive a portion of the debt. Of course, the renego-
tiation procedure could be modeled by a more complicated bargaining game with additional stages. For instance, one might allow the creditor to delay the bankruptcy decision in stage three to give the entrepreneur the chance to pay his debt in stage four. If he fails to do so, then in stage five the creditor faces the same decision problem as in stage three before. The outcome of this extended game will be identical to our three-stage version. In general, we are confident that our results continue to hold in a number of variations of the basic theme.

An interesting point is that even the renegotiated contract $\Delta$ involves costly liquidation of the entrepreneur's outside assets as long as the original contract $\Gamma$ entails collateral requirements. The reason is that after project failure the debtor has no liquid funds in excess of $X_f$ to compensate the creditor for a reduction in $C$. A Pareto-improving move that avoids the cost $(1 - \beta)C$ is not feasible. The debtor's liquidity constraint may thus result in an inefficient liquidation of assets. This phenomenon appears to be a typical characteristic of debt renegotiation and has been observed in a different context by Aghion and Bolton (1992) and Hart and Moore (1989).

2. OPTIMAL CONTRACTS WITHOUT RENEGOTIATION

First, we want to take a look at the contracting problem in the absence of debt renegotiation. We thus study the subgame-perfect equilibrium of the game described in the foregoing section under the exogenous restriction $\beta = 1$. This serves to illustrate the relation between renegotiation and collateralization. It should not suggest that the creditors would prefer to commit themselves not to renegotiate if they had the means for such a commitment. The question of whether ex ante commitment of this kind is actually desirable will be addressed in section 3.

Note that our description of debt contracts precludes the use of random devices. The creditor's right in the event of default is deterministic; he cannot impose bankruptcy with some contractually specified probability. As noted by Townsend (1979) and Mookherjee and Png (1989), stochastic auditing may be preferable in situations with costly monitoring of income realizations so that the assumption of deterministic contracts may be restrictive. Loan contracts specifying a random allocation of ownership rights, however, are hardly observed in reality. As a theoretical justification we assume that random devices are not verifiable so that stochastic outcomes are not contractible. It is important to bear in mind that as a result of this assumption the initial contract is incomplete.

When the creditor always uses his right to foreclose on the debtor's assets in the event of default, the successful entrepreneur is better off by paying his debt as long as $R < X_f$. Commitment not to forgive any part of the debt constitutes a strong enforcement mechanism that induces the entrepreneur to reveal his type truthfully. As in Gale and Hellwig (1985), the threat of bankruptcy serves to satisfy incentive-compatibility conditions that make sure that the entrepreneur tells the truth for each return realization. Enforcing truth-telling behavior, however, has its cost. With
probability \(1 - p\), project ownership does not rest with the entrepreneur. The following result deals with the optimality of collateralization in this situation.

**PROPOSITION 1:** Assume that creditors are committed not to forgive any debt so that \(b = 1\). Then in equilibrium a loan contract \(\Gamma^*\) is signed which satisfies \(C^* = 0\).

**PROOF:** As a result of competition, \(\Gamma^*\) maximizes the entrepreneur's expected payoff subject to the lenders' break-even constraint. Define \(R^*\) by

\[
pR^* + (1 - p)\alpha X_f = l.
\]

Then (1) implies \(R^* < X_s\), so that the successful entrepreneur with contract \(\Gamma^* = (R^*, 0)\) optimally chooses \(d^* = 0\) in stage two. Consequently the lender's expected payoff from proposing \(\Gamma^*\) is zero and the entrepreneur's expected payoff is

\[
p(X_s - R^*) = pX_s + (1 - p)\alpha X_f - l > 0.
\]

Now consider any other contract \(\Gamma\) which gives positive expected payoffs to the firm. Then \(R < X_s\) again implies \(d = 0\) so that the lender breaks even if

\[
pR + (1 - p)(\alpha X_f + \beta C) = l.
\]

Given (5), the entrepreneur's profit from \(\Gamma\) equals

\[
p(X_s - R) - (1 - p)C = pX_s + (1 - p)(\alpha X_f - (1 - \beta)C) - l < p(X_s - R^*).
\]

This proves that any contract \(\Gamma\) with \(C > 0\) is suboptimal and that in equilibrium the project is financed by \(\Gamma^*\). Q.E.D.

Collateral cannot improve efficiency if bankruptcy occurs solely as a result of project failure. In this case it only increases the dead-weight cost of the change in firm ownership. For collateral to become effective, it must have an impact upon the equilibrium probability of bankruptcy. As we shall see, this may happen when debt renegotiation is possible.

### 3. Renegotiation and the Optimality of Collateral

In the absence of precommitment the appropriate solution concept for the contracting game is the perfect Bayesian equilibrium. This basically means that each agent's behavior has to be optimal given the other party's behavior in every stage of the game. Moreover, in the final game stage the creditor's beliefs about the actual project outcome have to be consistent with updating of prior probabilities according to Bayes' rule. When default occurs, the creditor remains uninformed about the project outcome. From Bayes' rule the posterior probability that \(X_s\) has been realized is
\( \pi(d) = pd((pd + 1 - p)) \),

because the successful entrepreneur defaults with probability \( d \). The probability \( d \) is determined endogenously by optimizing behavior on the part of the successful entrepreneur. In equilibrium the creditor forms rational expectations so that after observing default he concludes that the project return is \( X \), with probability \( \pi(d) \) and \( X_r \) with probability \( 1 - \pi(d) \).

As a first step toward investigating the features of equilibrium contracts we consider the subgame following the realization of \( \tilde{X} \). Suppose a loan has been made. In addition, let \( X_f + W < R < X_s \). The motivation for the first inequality is that a contract with \( R \leq X_f + W \) would not allow the lender to break even because of (2). Clearly the precommitment solution studied in the foregoing section is inconsistent with sequential rationality. As the entrepreneur reacts to \( \tilde{d} = 1 \) by setting \( d = 0 \), Bayesian updating requires the creditor to conclude that the project has failed when he observes default. Given this information, however, imposing bankruptcy is suboptimal because the payoff from renegotiating \( \Gamma \) is higher by the amount \((1 - \alpha)X_r \).

The equilibrium concept precludes the use of incredible threats to enforce repayment. The equilibrium then prescribes the parties to adopt random strategies in stages two and three. Indeed, we already have seen that \( b = 1 \) can no longer be part of an equilibrium path. The following argument reveals that \( b = 0 \) cannot represent equilibrium behavior either: Expecting that the creditor always concedes to \( \Delta \) in the final stage, the successful entrepreneur would optimally default as \( X_s - R < X_s - X_f - C \) and so \( d = 1 \). But given the posterior probability \( \pi(d) = \pi(1) = p \), the creditor prefers liquidation of the project to the reduced debt payment \( X_f \) because \( \alpha[pX_s + (1 - p)X_f] > X_f \) by (1) and (2). This means the creditor optimally chooses \( b = 1 \), which contradicts the entrepreneur’s expectation that \( b = 0 \). This leaves \( 0 < b < 1 \) as the remaining candidate for equilibrium. Accordingly, the creditor must be indifferent about imposing bankruptcy or proposing \( \Delta \). This is the case if

\[
\alpha[\pi(d)X_s + (1 - \pi(d))X_f] = X_f. \tag{8}
\]

In equilibrium the borrower forms rational expectations about the lender’s behavior. Therefore, the successful entrepreneur’s expected payoff from defaulting is \((1 - b)(X_s - X_f) - C \). He loses the collateral \( C \) but with probability \((1 - b) \) he maintains ownership of the firm by paying the reduced debt \( X_f \). It follows from (8) that \( 0 < d < 1 \) and so also the borrower randomizes after observing \( \tilde{X} = X_s \). For him to be indifferent between default and repayment, it must be the case that

\[
X_s - R = (1 - b)(X_s - X_f) - C. \tag{9}
\]

4. Clearly this is no longer the case when the borrower pays the lower proceeds, \( X_f \), as soon as he declares failure. If the game is modified in this way, one has to introduce some fixed cost \( K \) to ensure that project liquidation is costly. By imposing bankruptcy the creditor then gets \( \alpha(X_s - X_f) - K \) if \( X = X_s \) and \(-K \) if \( X = X_f \). Equation (8) then becomes \( \alpha \pi(d)(X_s - X_f) = K \) and the equilibrium analysis follows the same arguments.
Solving equations (8) and (9) for \( d \) and \( b \), we obtain the following result:

**Proposition 2:** Assume that the project has been financed by a loan \( R \) with \( X_f + W < R < X_s \). Then the equilibrium in the following subgame is unique and is given by

\[
d^* = \frac{(1 - p)(1 - \alpha)X_f}{p(\alpha X_s - X_f)} \quad \text{and} \quad b^* = \frac{R - X_f - C}{X_s - X_f}
\]

The mixed strategies described in Proposition 2 may be viewed as the beliefs of the two players concerning their opponents behavior (see Aumann 1987). The creditor believes that the successful entrepreneur repudiates with probability \( d^* \) and the entrepreneur expects that bankruptcy will be imposed with probability \( b^* \). In equilibrium all decisions to which a strictly positive probability is assigned are optimal, given the beliefs. An alternative interpretation is due to Harsanyi (1973), who demonstrated that mixed strategy equilibria may be viewed as the limit of pure strategy equilibria of a related "disturbed" game as the disturbances vanish. In the disturbed game each party's payoff is subject to a small random disturbance, the value of which is known only to him. Due to these exogenous random shocks, the individual's behavior appears to be random even though it is actually deterministic.

An interesting feature of the equilibrium is that, in contrast with the case of precommitment, the firm's repayment behavior no longer reveals its private information about the project outcome. As \( 0 < d^* < 1 \), there is partial pooling so that the creditor is not precisely informed about the true return realization when he observes default. This is similar to observations by Dewatripont (1989), Hart and Tirole (1988), and Laffont and Tirole (1988), who conclude that the possibility of renegotiation favors the use of mixed strategies and reduces the degree of information revelation.

Proposition 2 indicates why in the absence of precommitment it may be desirable to include collateral requirements in the loan contract. Increasing \( C \) has a dual impact on the project's overall profitability. On one hand it creates an additional deadweight loss because the entrepreneur's valuation of \( C \) exceeds the lender's valuation. On the other hand, \( b^* \) and thereby the probability of an inefficient change in project ownership is lowered. Which of these effects dominates the other depends upon the relative costs expressed by the factors \( \alpha \) and \( \beta \). Define

\[
\beta = \frac{\alpha p(X_s - X_f) - (1 - \alpha)X_f}{\alpha p(X_s - X_f)}.
\]

Note that \( 0 < \beta < 1 \). Moreover, \( \beta \) and \( \alpha \) are positively related and \( \beta \) tends to unity when \( \alpha \) approaches one.

**Proposition 3:** If \( \beta > \beta \) then in equilibrium a debt contract \( R \) is signed such that \( C^* = W \). Otherwise it is optimal to set \( C^* = 0 \).
PROOF: By (1) and (2), no creditor will offer a contract with \( R \leq X_f + W \). When a contract \( \Gamma \) with \( R > X_f + W \) is signed, Proposition 2 applies and so the creditor receives the payment \( R \) with probability \( p(1 - d^*) \). When default occurs he is indifferent between bankruptcy and renegotiation. Therefore, with probability \( pd^* + 1 - p \) the creditor receives the payoff \( X_f + \beta C - l \). Accordingly, for \( \Gamma \) to be individually rational for the creditor, it has to be the case that

\[
p(1 - d^*)R + (pd^* + 1 - p)(X_f + \beta C) \geq l.
\]

(11)

The entrepreneur’s payoff is \(-C\) when the project fails; otherwise he is indifferent between defaulting and paying \( R \). Therefore, his expected payoff from signing \( \Gamma \) is given as \( p(X_s - R) - (1 - p)C \). As a result of creditor competition, the constraint (11) must be binding in equilibrium so that substituting \( R \) from (11) yields

\[
p(X_s - R) - (1 - p)C = pX_s + \frac{pd^* + 1 - p}{1 - d^*} X_f - \frac{(pd^* + 1 - p)(1 - \beta) - d^*}{1 - d^*} C - \frac{l}{1 - d^*}.
\]

(12)

Thus maximizing the entrepreneur’s payoff with respect to \( C \) subject to \( 0 \leq C \leq W \) implies \( C^* = W \) if \( (pd^* + 1 - p)(1 - \beta) < d^* \). Using the value of \( d^* \) from Proposition 2, this condition is easily seen to be equivalent to \( \beta > \hat{\beta} \). Of course, \( C^* = 0 \) solves the maximization problem if \( \beta \leq \hat{\beta} \). Q.E.D.

Whether posting collateral is optimal depends upon the size of the entrepreneur’s comparative advantage to own and manage the firm. For a given value of the parameter \( \beta \), the gains from collateralization are higher the lower the value of \( \alpha \). This means collateral becomes useful when the costs of liquidating the firm are sufficiently high. When project ownership is irrelevant as \( \alpha \) goes to one, collateral requirements turn out to be suboptimal.

The relation between project risk and the equilibrium contract provides another interesting insight. To investigate this relationship, we define the parameter

\[
\hat{\beta} = \frac{(1 - \alpha)X_f}{[(1 - \beta)\alpha(X_s - X_f)]}.
\]

(13)

Inspection of (10) and (13) shows that \( \beta > \hat{\beta} \) if and only if \( p < \hat{p} \). This leads to a simple Corollary of Proposition 3:

**Proposition 4:** If \( p < \hat{p} \), then in equilibrium a debt contract \( \Gamma^* \) is signed such that \( C^* = W \). Otherwise it is optimal to set \( C^* = 0 \).

The result has the following intuition. The prospect of debt renegotiation no longer induces truth-telling behavior on the part of the entrepreneur. In this situation the intention of collateral agreements is not to punish for project failure but to make

5. Note that \( p < \hat{p} \) is consistent with assumption (1) unless \( (1 - \beta)l > (1 - \beta\alpha)X_f \). If \( \beta > 1 \), Proposition 4 simply says that setting \( C^* = W \) is always optimal.
default less attractive in the event of success. As Proposition 2 shows, the equilibrium likelihood of dishonesty $d^*$ is inversely related to the project’s success probability $p$. Therefore a higher success rate makes it more likely that the entrepreneur will lose his outside assets because of project failure rather than because of the attempt to cheat. As a consequence, collateral is more effective with a high risk of project failure.

Interestingly, the conclusion of Proposition 4 is in direct contrast with the signaling theory of collateral, as developed in Besanko and Thakor (1987), Bester (1985, 1987), and Chan and Kanatas (1985). These models predict a positive relation between the investment’s success probability and the degree of collateralization. The underlying assumption is that the creditors are less informed about project risks than the entrepreneur. Different contracts are then used to sort loan applicants into risk classes. Entrepreneurs who are more likely to succeed are inclined to post a higher amount of collateral because they are less likely to lose it in the event of project failure. In equilibrium low-risk entrepreneurs choose debt contracts with low repayment obligations and high collateral requirements whereas high-risk entrepreneurs sign contracts with high repayment obligations and low collateral requirements.

Finally we turn to the question of whether precommitment not to renegotiate the original contract $\Gamma^*$ increases social welfare. One way of preventing debt renegotiation is to employ the aid of third parties, as suggested by Schelling (1960): The creditor signs a contract with an outsider agreeing to pay a large sum of money should he ever forgive any portion of the debt. Of course, for such a scheme to work, the outsider must be incorruptible because otherwise he could be bribed into permitting renegotiation if the debt contract prescribes an ex post inefficient liquidation of assets. Alternatively, precommitment may be enforced by reputation considerations. The concern for long-run reputation effects may induce the creditor not to forgive the debt if this is optimal ex ante, even though it may be suboptimal ex post. In what follows, we do not want to investigate the feasibility but rather the desirability of precommitment. In other words, we compare the dead-weight loss associated with bankruptcy in the two categories of equilibrium analyzed in the foregoing and the present section, respectively.

The possibility of renegotiation affects the expected cost of bankruptcy in two ways. First, default is less frequently followed by project liquidation as $b^* < 1$. This positive effect is even enlarged when setting $C^* = W$ is optimal. Second, default occurs more often because the entrepreneur may seek to cheat. Indeed, in the equilibrium described by Proposition 2 the probability of default is $pd^* + 1 - p$ compared with $1 - p$ if $b = 1$. This effect is especially harmful because in some cases the successful project is liquidated. Note that competition reduces the creditor’s expected profits to zero in any equilibrium. Therefore, the entrepreneur’s expected payoff is critical for evaluating the welfare implications of precommitment.

**Proposition 5:** The entrepreneur’s expected payoff is higher in the equilibrium where renegotiation is possible than in the equilibrium with precommitment not to renegotiate.
PROOF: The entrepreneur's equilibrium payoff in the two categories of equilibrium is given by (4) and (12), respectively. Suppose, contrary to the Proposition, that the expression in (12) does not exceed the expression in (4). Because $C$ is chosen to maximize (12), this implies

$$\alpha p^* X + (1 - p) X_I - I \leq (1 - d^*)(1 - p) \alpha X_I - I.$$  \hspace{1em} (14)

Using (7) and (8), it follows that (14) is equivalent to

$$\alpha (p d^* X_I + (1 - p) X_J) \leq (1 - d^*)(1 - p) \alpha X_I - d^* I.$$  \hspace{1em} (15)

But (15) implies $\alpha (p X_I + (1 - p) X_J) \leq I$, a contradiction to assumption (1). This proves that the Proposition must hold. Q.E.D.

It is important for this result that the initial contract is incomplete in that it does not allow for randomization. If stochastic debt forgiveness were contractible, the "renegotiation-proofness" principle would apply that implies that the absence of commitment lowers welfare. Proposition 5 is an example demonstrating that this principle may fail to hold when contracts are incomplete. As a result, we may expect to observe debt renegotiation in practice even when the creditors have the means to commit themselves not to forgive any debt. Competition among lenders does not favor eliminating the prospect of renegotiation. Yet, one should regard this implication of our model with caution. In particular the assumption that the contracting parties have symmetric information about the project's return characteristics seems important. When the entrepreneur knows more about the project's ex ante profitability than the creditor, renegotiation may in fact be harmful. Adverse selection may occur when the creditor cannot commit to liquidating inefficient firms in the future. Dewatripont and Maskin (1989) discuss this aspect in a model where the creditors would like to commit ex ante against refinancing in order to deter entrepreneurs from starting bad projects.

Our discussion of the conflict between Proposition 4 and the signaling motive for collateral indicates another reason why renegotiation may lead to adverse selection. In the absence of renegotiation entrepreneurs with good projects can distinguish themselves from those with bad projects by posting more collateral. But, Proposition 4 shows that also the high-risk entrepreneurs will find it advantageous to offer collateral when there is a chance of renegotiation. This means that renegotiation may preclude a separating equilibrium where collateral serves as a screening device. Good and bad projects will then be pooled and, as shown by De Meza and Webb (1987), the equilibrium will have a tendency toward a higher level of aggregate investment than is socially optimal.

4. CONCLUSION

This paper investigates how the prospect of future debt renegotiation affects the lender's security interests at the contracting date. The terms of the initial debt con-
tract play a strategic role in the development of the borrower-lender relationship; indirectly they determine the likelihood of renegotiation and the terms of the renegotiated contract. Renegotiation occurs because the absence of precommitment precludes incredible bankruptcy threats. As a result, there is a chance that the creditor responds to default by forgiving some part of the debt rather than by imposing bankruptcy. This in turn influences the borrower's default decision. In our model the creditor cannot distinguish whether the borrower defaults voluntarily or whether he is actually unable to meet his payment obligations. The chance of debt forgiveness may induce the borrower to falsely report that the investment's return is too low to pay the full amount of debt.

We show that these circumstances favor the issuance of debt that is secured by outside assets. The event of default entitles the creditor to liquidate the borrower's collateralized wealth in addition to the assets remaining from the investment project. Although outside collateral increases the total amount of assets liquidated in the case of bankruptcy, perhaps surprisingly it may actually lower the expected dead-weight loss associated with asset liquidation. The reason is that collateralization reduces the debtor's motives for voluntary default so that bankruptcy is less likely to occur. We show that this effect is especially relevant for high-risk investment projects. Therefore, such firms are more likely to be financed through loans that include collateral requirements than low-risk firms.

While debt renegotiation may simply be interpreted as resulting from the creditor's inability to precommit himself, we find that renegotiation may in fact increase welfare. This provides an efficiency explanation of why debt renegotiation is frequently observed in practice. We are careful, however, to point out that our assumption of ex ante symmetric information is essential for this result.

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