Tilburg University

Comment on: Identification in the linear errors in variables model

Bekker, P.A.

Publication date:
1984

Document Version
Publisher's PDF, also known as Version of record

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):
Bekker, P. A. (1984). *Comment on: Identification in the linear errors in variables model*. (Research Memorandum FEW). Faculteit der Economische Wetenschappen.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM
R

7626
1984
159

Bestemming

ECO

TIJDSCHRIFTENBUREAU
BIBLIOTHEEK
KATHOLIEKE
HOGESCHOOL
TILBURG

Nr.

R41

subfaculteit der econometrie

RESEARCH MEMORANDUM

* C I N O O 4 1 8 *

TILBURG UNIVERSITY
DEPARTMENT OF ECONOMICS

Postbus 90153 - 5000 LE Tilburg
Netherlands

COMMENT ON: IDENTIFICATION IN THE
LINEAR ERRORS IN VARIABLES MODEL

By Paul A. Bekker¹⁾

1) The author wishes to thank Arie Kapteyn and Tom Wansbeek for their stimulating discussions on the subject. Financial support by the Netherlands Organization for the Advancement of Pure Research (ZWO) is gratefully acknowledged.

1. Introduction

Kapteyn and Wansbeek [1] considered the following multiple linear regression model with errors in variables:

$$(1.1) \quad y_j = \xi_j' \beta + \varepsilon_j, \quad (j = 1, \dots, n)$$

$$(1.2) \quad x_j = \xi_j + v_j,$$

where ξ_j , x_j , v_j and β are k -vectors, y_j , ε_j are scalars. The ξ_j are unobservable variables: instead the x_j are observed. The measurement errors v_j are unobservable as well and it is assumed that $v_j \sim N(0, \Omega)$ and $\varepsilon_j \sim N(0, \sigma^2)$ for all j . The v_j and ε_j are mutually independent and independent of ξ_j . The ξ_j are considered as random drawings from some, as yet unspecified, multivariate distribution with zero mean.

For the case $k=1$ Reiersøl [2] has shown that normality of ξ_j is the only distributional assumption which spoils identification. For the case $k > 1$ and the components of ξ_j are mutually independent, Willassen [3] has shown that none of the components of ξ_j should be normally distributed to guarantee identifiability of β . Kapteyn and Wansbeek [1] did not assume independency of the components of ξ_j and they stated the following proposition: the parameter vector β is identified if and only if there does not exist a linear combination of ξ_j which is normally distributed. The necessity part in this proposition is incorrect, i.e. it may well be that a normally distributed linear combination of ξ_j does not spoil the identifiability of β . Here I present necessary and sufficient conditions for identification of β .

2. Statement of the Result and Proof

Proposition: Under the assumptions above, the parameter vector β is identified if and only if there does not exist a nonsingular matrix $A = (a_1, A_2)$ such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$.

Proof: We first show that nonidentifiability of β implies the existence of the matrix A . Let s be a scalar and t a k -vector. The characteristic function $\phi_{y_j, x_j}(s, t)$ of y_j and x_j is

$$(2.1) \quad \phi_{y_j, x_j}(s, t) = \exp\{-\frac{1}{2}(\sigma^2 s^2 + t' \Omega t)\} \phi_{\xi}(\beta s + t),$$

where $\phi_{\xi}(\cdot)$ is the characteristic function of ξ_j . Assuming that β is not fully identified amounts to saying that there exist parameter sets $\{\beta, \sigma^2, \Omega\}$ and $\{\beta^*, \sigma^{*2}, \Omega^*\}$, with $\beta \neq \beta^*$, generating the same distribution of y_j, x_j . Consequently $\phi_{y_j, x_j}(s, t)$ should be the same for both sets of parameters:

$$(2.2) \quad \exp\{-\frac{1}{2}(\sigma^2 s^2 + t' \Omega t)\} \phi_{\xi}(\beta s + t) = \exp\{-\frac{1}{2}(\sigma^{*2} s^2 + t' \Omega^* t)\} \phi_{\xi}^*(\beta^* s + t).$$

Let $\ell = \beta^* s + t$, then $\phi_{\xi}(\beta s + t) = \phi_{\xi}((\beta - \beta^*)s + \ell) = \phi_{\xi'(\beta - \beta^*), \xi}(s, \ell)$. Thus

(2.2) carries over into

$$(2.3) \quad \phi_{\xi'(\beta - \beta^*), \xi}(s, \ell) = \exp\{-\frac{1}{2}[s^2(\sigma^{*2} - \sigma^2) + (\ell - \beta^* s)'(\Omega^* - \Omega)(\ell - \beta^* s)]\} \phi_{\xi}^*(\ell).$$

The characteristic function corresponding to the marginal distribution of $\xi_j'(\beta - \beta^*)$ is found by setting $\ell = 0$

$$(2.4) \quad \phi_{\xi'(\beta-\beta^*), \xi}(s) = \exp\left\{-\frac{1}{2}s^2(\sigma^{*2} - \sigma^2 + \beta^{*'}(\Omega^* - \Omega)\beta^*)\right\},$$

which is the characteristic function of a normally distributed variable.

In addition to this result, which was obtained by Kapteyn and Wansbeek [1], it will now be shown that nonidentifiability of β also implies the existence of a matrix A_2 such that (a_1, A_2) is nonsingular and $\xi_j' a_1$ is distributed independently of $\xi_j' A_2$. The characteristic function corresponding to the marginal distributions of ξ_j is found by setting $s=0$ in (2.3):

$$(2.5) \quad \phi_{\xi}(\ell) = \exp\left\{-\frac{1}{2}\ell'(\Omega^* - \Omega)\ell\right\} \phi_{\xi}^*(\ell).$$

Thus, we may rewrite (2.3) as

$$(2.6) \quad \phi_{\xi'(\beta-\beta^*), \xi}(s, \ell) = \phi_{\xi'(\beta-\beta^*), \xi}(s) \phi_{\xi}(\ell) \exp\{s\beta^{*'}(\Omega^* - \Omega)\ell\}.$$

Let B be a $(k \times (k-1))$ -matrix of full column rank such that $\beta^{*'}(\Omega^* - \Omega)B = 0$. Equality (2.6) holds for all possible values of s and ℓ . In particular (2.6) holds if we let ℓ vary such that $\ell = Bm$, where m is a $(k-1)$ -vector:

$$(2.7) \quad \phi_{\xi'(\beta-\beta^*), \xi}(s, Bm) = \phi_{\xi'(\beta-\beta^*), \xi}(s) \phi_{\xi}(Bm),$$

or equivalently,

$$(2.8) \quad \phi_{\xi'(\beta-\beta^*), B}(s, m) = \phi_{\xi'(\beta-\beta^*), \xi}(s) \phi_{\xi, B}(m).$$

Thus nonidentifiability of β implies the existence of a matrix $(\beta-\beta^*, B)$ such that $\xi_j'(\beta-\beta^*)$ is distributed normally and independently of $\xi_j' B$. If $\text{rank}(\beta-\beta^*, B) = k$ then a matrix A is given by $(\beta-\beta^*, B)$. In the trivial case where $\text{Rank}(\beta-\beta^*, B) = k-1$, the variable $\xi_j'(\beta-\beta^*)$ is distributed independently of itself and must therefore be equal to zero identically (which is also considered as a normal distribution). In that case any nonsingular matrix A whose first column equals $\beta-\beta^*$ will do.

To prove the necessity part of the Proposition we assume that there exists a nonsingular matrix $A = (a_1, A_2)$ such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$. If we substitute $t = A\ell = a_1 \ell_1 + A_2 \ell_2$ and $\beta = A\tilde{\beta} = a_1 \tilde{\beta}_1 + A_2 \tilde{\beta}_2$ (ℓ_1 and $\tilde{\beta}_1$ are scalars, ℓ_2 and $\tilde{\beta}_2$ are $(k-1)$ -vectors) in (2.1), then the characteristic function of y_j, x_j takes the following form:

$$(2.9) \quad \phi_{y_j, x_j}(s, A\ell) = \exp\left[-\frac{1}{2}(\sigma^2 s^2 + \ell' A' \Omega A \ell)\right] \phi_{\xi}(A(\tilde{\beta} s + \ell)).$$

The characteristic function $\phi_{\xi}(A(\tilde{\beta} s + \ell))$ can be rewritten as follows:

$$(2.10) \quad \begin{aligned} \phi_{\xi}(A(\tilde{\beta} s + \ell)) &= \phi_{\xi, A}(\tilde{\beta} s + \ell) = \phi_{\xi, a_1}(\tilde{\beta}_1 s + \ell_1) \phi_{\xi, A_2}(\tilde{\beta}_2 s + \ell_2) \\ &= \exp\left[-\frac{1}{2}(\tilde{\beta}_1 s + \ell_1)^2 \text{Var}(\xi' a_1)\right] \phi_{\xi, A_2}(\tilde{\beta}_2 s + \ell_2). \end{aligned}$$

Using (2.10), (2.9) carries over into

$$(2.11) \quad \phi_{y_j, x_j}(s, A\ell) = \exp\left[-\frac{1}{2}(s, \ell') C (s, \ell')'\right] \phi_{\xi, A_2}(\tilde{\beta}_2 s + \ell_2),$$

where

$$(2.12) \quad C \equiv \begin{bmatrix} \sigma^2 & 0 \\ 0 & A' \Omega A \end{bmatrix} + \text{Var}(\xi' a_1) \begin{bmatrix} \tilde{\beta}_1^2 & e_1' \tilde{\beta}_1 \\ e_1 \tilde{\beta}_1 & e_1 e_1' \end{bmatrix}.$$

The $\frac{1}{2}k(k+1) + 2$ nonzero elements of C are functions of $\frac{1}{2}k(k+1) + 3$ parameters in σ^2 , Ω , $\tilde{\beta}_1$ and $\text{Var}(\xi' a_1)$, whereas the function $\phi_{\xi, A_2}(\tilde{\beta}_2 s + \ell_2)$ is not affected by these parameters. Clearly, different parameter values generate the same distribution of y_j, x_j . The existence of a nonsingular transformation such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$ thus implies nonidentifiability of β . Q.E.D.

3. Discussion

Compared to Kapteyn and Wansbeek's proposition, the sufficiency part of the proposition proved here is stronger. Nonidentifiability does not only imply the existence of a normally distributed linear combination $\xi'_{j1}a_1$, but also the existence of A_2 such that $\xi'_{j1}a_1$ and $\xi'_{j2}A_2$ are mutually independent. Consequently, the necessity part of their proposition must be wrong, because they fail to invoke the existence of a matrix A_2 such that $\xi'_{j1}a_1$ and $\xi'_{j2}A_2$ are mutually independent.

As an example, consider the model with two regressors ξ_{j1} and ξ_{j2} , the first of which is normally distributed, $\xi_{j1} \sim N(0, \sigma^2)$, and the second is a function of the first $\xi_{j2} = \xi_{j1}^2 - \sigma^2$. According to Kapteyn and Wansbeek this model would not be identified since ξ_{j1} is normally distributed. However, this point of view would be too pessimistic. Clearly there is no nonsingular transformation (a_1, a_2) such that $(\xi_{j1}, \xi_{j2})a_1$ is distributed independently of $(\xi_{j1}, \xi_{j2})a_2$ and so the model is identified.

References

- [1] Kapteyn, A. and T.J. Wansbeek: "Identification in the Linear Errors in Variables Model", *Econometrica* 51 (1983), 1847-49.
- [2] Reiersøl, O.: "Identifiability of a Linear Relation Between Variables Which are Subject to Error", *Econometrica* 18 (1950), 375-389.
- [3] Willassen, Y.: "Extension of Some Results by Reiersøl to Multivariate Models", *Scandinavian Journal of Statistics* 6 (1979), 89-91.

IN 1983 REEDS VERSCHENEN

- 126 H.H. Tigelaar
Identification of noisy linear systems with multiple arma inputs.
- 127 J.P.C. Kleijnen
Statistical Analysis of Steady-State Simulations: Survey of Recent Progress.
- 128 A.J. de Zeeuw
Two notes on Nash and Information.
- 129 H.L. Theuns en A.M.L. Passier-Grootjans
Toeristische ontwikkeling - voorwaarden en systematiek; een selectief literatuuroverzicht.
- 130 J. Plasmans en V. Somers
A Maximum Likelihood Estimation Method of a Three Market Disequilibrium Model.
- 131 R. van Montfort, R. Schippers, R. Heuts
Johnson S_U -transformations for parameter estimation in arma-models when data are non-gaussian.
- 132 J. Glombowski en M. Krüger
On the Rôle of Distribution in Different Theories of Cyclical Growth.
- 133 J.W.A. Vingerhoets en H.J.A. Coppens
Internationale Grondstoffenovereenkomsten.
Effecten, kosten en oligopolisten.
- 134 W.J. Oomens
The economic interpretation of the advertising effect of Lydia Pinkham.
- 135 J.P.C. Kleijnen
Regression analysis: assumptions, alternatives, applications.
- 136 J.P.C. Kleijnen
On the interpretation of variables.
- 137 G. van der Laan en A.J.J. Talman
Simplicial approximation of solutions to the nonlinear complementarity problem with lower and upper bounds.

IN 1984 REEDS VERSCHENEN

- 138 G.J. Cuypers, J.P.C. Kleijnen en J.W.M. van Rooyen
Testing the Mean of an Asymetric Population:
Four Procedures Evaluated
- 139 T. Wansbeek en A. Kapteyn
Estimation in a linear model with serially correlated errors when
observations are missing
- 140 A. Kapteyn, S. van de Geer, H. van de Stadt, T. Wansbeek
Interdependent preferences: an econometric analysis
- 141 W.J.H. van Groenendaal
Discrete and continuous univariate modelling
- 142 J.P.C. Kleijnen, P. Cremers, F. van Belle
The power of weighted and ordinary least squares with estimated
unequal variances in experimental design
- 143 J.P.C. Kleijnen
Superefficient estimation of power functions in simulation
experiments
- 144 P.A. Bekker, D.S.G. Pollock
Identification of linear stochastic models with covariance
restrictions.
- 145 Max D. Merbis, Aart J. de Zeeuw
From structural form to state-space form
- 146 T.M. Doup and A.J.J. Talman
A new variable dimension simplicial algorithm to find equilibria on
the product space of unit simplices.
- 147 G. van der Laan, A.J.J. Talman and L. Van der Heyden
Variable dimension algorithms for unproper labellings.
- 148 G.J.C.Th. van Schijndel
Dynamic firm behaviour and financial leverage clienteles
- 149 M. Plattel, J. Peil
The ethico-political and theoretical reconstruction of contemporary
economic doctrines
- 150 F.J.A.M. Hoes, C.W. Vroom
Japanese Business Policy: The Cash Flow Triangle
an exercise in sociological demystification
- 151 T.M. Doup, G. van der Laan and A.J.J. Talman
The $(2^{n+1}-2)$ -ray algorithm: a new simplicial algorithm to compute
economic equilibria

IN 1984 REEDS VERSCHENEN (vervolg)

- 152 A.L. Hempenius, P.G.H. Mulder
Total Mortality Analysis of the Rotterdam Sample of the Kaunas-Rotterdam Intervention Study (KRIS)
- 153 A. Kapteyn, P. Kooreman
A disaggregated analysis of the allocation of time within the household.
- 154 T. Wansbeek, A. Kapteyn
Statistically and Computationally Efficient Estimation of the Gravity Model.
- 155 P.F.P.M. Nederstigt
Over de kosten per ziekenhuisopname en levensduurmodellen
- 156 B.R. Meijboom
An input-output like corporate model including multiple technologies and make-or-buy decisions
- 157 P. Kooreman, A. Kapteyn
Estimation of Rationed and Unrationed Household Labor Supply Functions Using Flexible Functional Forms
- 158 R. Heuts, J. van Lieshout
An implementation of an inventory model with stochastic lead time

Bibliotheek K. U. Brabant

17 000 01059784 8