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Demand Management in multi-stage distribution chain

A.G. de Kok ¹ and F.B.S.L.P. Janssen ²

Abstract

In this paper we discuss demand management problems in a multi-stage distribution chain. We focus on distribution chains where demand processes have high variability due to a few large customer orders. We give a possible explanation, and suggest two simple procedures that help to smooth demand. It is shown that these procedures yield stock reductions of 40%-50% in practical situations. The quantitative results are based on the analysis of the underlying model related to the two procedures proposed, called large order overflow, applicable if the supplying organization executes a multi-stage distribution chain, and delivery splitting, applicable to any situation.

1. Introduction.

Since Forrester's Industrial Dynamics (1961) a lot has been published on the control of multi-stage logistics chains. The problems signalled by Forrester with respect to amplification of demand fluctuations upstream in the logistic chain have been understood widely and Material Requirements Planning (MRP), see, for example, Orlicky (1975), and Distribution Requirements Planning (DRP), for example, Martin (1990), systems are used throughout industry to eliminate this amplification as much as possible. Yet a closer look reveals that these systems typically operate within industrial and retail organizations, but seldom, if ever, across different organizations in the logistics chain. Although it is claimed by various authors, such as Martin (1990), that the tight coupling of MRP systems of end product manufacturers with DRP systems of component manufacturers should solve or at least alleviate these problems, it is still rare that such an approach is implemented.

This paper focuses on the management of the supply chain across the organization of a supplier of fast moving consumer goods and the organizations of its customers, that is, power

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retailers, wholesalers and retailers. The study presented in the paper is motivated by one of the authors' 10-year experience at a European electronics manufacturer supplying the global market. The case material presented originates from an internal consulting project in the Consumer Electronics Division of this company. During these years a number of projects were started aimed at implementing integrated supply chain management in Europe. The major steps taken were the establishment of a European sales and marketing organization with tight control on the local sales organization, and the implementation of European Distribution Centers when appropriate, for example, for audio products manufactured in the Far-East.

One of the major problems concerning operational control of the supply chain was the so-called 'big order' issue. The company was regularly confronted with unexpected big orders for particular products. At the same time, investigations at their customers showed that their demands were relatively stable. Apparently the stable consumer demand was changed into erratic customer demand at the interface of the supplier and its customers. Based on the empirical research it was found that the major reason for the erratic behavior of the customer demand was due to large lot sizes caused by both the supplier's sales men and the customers' buyers: the sales men needed the large lot sizes of particular products in order to achieve their monthly sales budgets, while the buyers waited for discounts associated with these large lot sizes in order to maximize their sales turnover with the given purchasing budget.

In this paper we address this problem mainly from the supplier's point of view. The supplier's supply chain consists of a factory, a Regional Distribution Center (RDC), and local sales organizations with local stock. We model this supply chain as a two-echelon divergent system. To take into account the customers perspective, we make economic trade offs. The main idea is to smooth demand at the local sales organizations by offering different customer service conditions for large orders and small orders. This may be at the expense of the customer. Yet we claim that the savings made by the supplier are of such a magnitude that this provides funding for discounts to customers to stimulate the acceptance of a differentiated customer service policy. It is our belief that differentiated customer service policy, based on the capabilities of the supplier and the customers requirements, substantially reduces the

total supply chain costs. This belief is based on a number of recent master's thesis projects at various industrial companies in the Netherlands, and the customer service strategy of the Consumer Electronics company defined for the European market based on their successful customer service strategy in the USA, which yielded substantial reduction in supply chain costs, while increasing market share.

A lot of research has been devoted to supply chain management in the last five years, Lee and Billington (1992, 1993). Their research is based on a methodology similar to our research: a combination of empirical research and the applications of quantitative models. Another related paper by Vastag et al. (1994) gives a general overview of the costs involved in the management of supply chains. The literature on quantitative modelling of supply chains is huge. For an excellent overview of the research in this area until 1992 we refer to Graves et al. (1993). It should be noted that the research reviewed in Graves et al. (1993) is focused on minimization of total costs, consisting of holding, ordering, and penalty costs. Other research focused on cost minimization subject to service level constraints. Papers based on the latter approach are Lagodimos (1992), Lagodimos and Anderson (1989), De Kok (1990), and De Kok and Verrijdt (1995). In this paper we also focus on cost minimization subject to service level constraints. A subject related to our work is risk pooling as described in Eppen and Schrage (1981), and Jönsson and Silver (1987). Risk pooling arises in our situation when we reroute large customer orders to the RDC to stabilize demand at the stock points of the sales organizations. The economic theory on discount policies, as discussed in Silver and Peterson (1985) and Tersine (1994), is not the subject of research in this paper. Merely we would like to draw attention to the impact of discount policies on demand variability and to give an estimate of the cost reduction caused by employing a strategy aimed at stabilizing customer demand. This cost reduction can be used to give discounts to customers that operate according to the service strategy of the supplier.

We have argued that the problem of erratic demand is caused by ordering policies, which can be considered to be non-rational from an inventory management point of view. Yet, these policies may be quite rational from the perspective of short term cost minimization or

other incentives. Because we advocate a long-term perspective, we follow the line of thought advocated in the Just In Time literature (e.g. Hall (1983)), where all waste should be avoided. Apparently such non-rational policies increase the amount of stock held in the supply chains, which is counterproductive from a long-term perspective.

The paper is organized as follows. In section 2 we present a two-echelon model of the supply chain under consideration. Furthermore, we introduce the demand process, the lead time structure, and the cost structure on which the trade offs are made. In section 3 we exploit the two-echelon supply chain structure to divert large orders from local stockpoints to the RDC. We show the effect on overall supply chain costs, subject to a service level constraint. In section 4 we concentrate on a particular local stockpoint only. We discuss the effect of so-called delivery splitting (the delivery of large orders in consecutive smaller lots to the customer) on the costs of the sales organization. In section 5 we summarize our conclusions in relation to the case and discuss further research.

2. Model description

To illustrate the effects of large order overflow and delivery splitting we consider a two-echelon distribution chain. The supply chain consists of a factory supplying a regional distribution center (RDC), possibly near the factory, and N local stockpoints (LSP), wherefrom customers demand is satisfied (see Figure 1). Typically, the RDC holds seasonal stocks and replenishes the local stockpoints.

In this paper we do not take into account seasonal demand processes, and therefore we ignore the seasonal stock.

The lead time from factory to RDC, denoted by L_F , represents the sum of planning lead time, production and distribution lead time between a (Far-East) factory and a (European) RDC. The transportation lead times between the RDC and the LSP's are assumed to be identical, and are denoted by L_R . We assume that all stockpoints have compound Poisson demand processes. More specifically, the customers arrive according to a Poisson process

Figure 1: Supply chain with RDC and local stockpoints

with rate λ_i ($i = 1, \dots, N$). Let D denote the demand of an arbitrary customer. We assume the demands per customer are independent and identically distributed non-negative random variables with distribution function $F_D(\cdot)$. The mean and standard deviation of the demand size D is denoted by $\mathbb{E}D$ and $\sigma(D)$ respectively. Furthermore, we denote the coefficient of variation of D by c_D , that is, $c_D = \frac{\sigma(D)}{\mathbb{E}(D)}$. The coefficient of variation is a measure for the variability of demand. Often it is assumed that if $c_D < 1$, then demand is stable and if $c_D > 1$, then demand is erratic. Note that the Poisson process assumption has been shown to hold in most practical situations with regard to customer arrival streams (for example. Tijms (1994)).

The replenishments at the RDC as well as the LSP's are controlled by (s, Q) policies, that is, when the inventory position (physical stock minus backlog plus on order) drops below the reorder point s we order a multiple of Q , such that the inventory position after ordering is between s and $s+Q$. As a service performance measure the P_2 service measure (often denoted as the fill rate) is used: the long-run fraction of demand delivered directly from shelf, see for example Silver and Peterson (1985), Tijms and Groenevelt(1984). Demands which can not be delivered directly from shelf are backordered. The aim is to minimize the costs incurred in the supply chain subject to a P_2 service level constraint.

The purpose of this paper is to analyze the cost differences of the proposed demand management policies. Therefore, we need specifications for the costs that are dependent of these policies, which are: the costs of carrying items in inventory, the ordering or setup costs, and the transportation costs. We assume that the carrying costs are proportional to the average inventory level in the distribution chain, where the costs of having one item stocked for one time unit is identical along the supply chain, and is denoted by h (\$ / time unit). Assuming constant carrying cost along the supply chain is based on the fact that once a product has been completed the material value remains constant while further distributing through the supply chain. The ordering costs are fixed per replenishment (that is independent of the size of the replenishment), and are denoted by A_R at the RDC and A_L at the LSP's. We assume that transportation costs are independent of the transportation size, and are denoted by T_R at the RDC and T_L at the LSP's. This is typically the case for transportation of fast moving goods, where mostly Full (mixed) Truck Loads (FTL) are guaranteed.

3. Rerouting large orders to upstream stockpoints; large order overflow.

In general, a stockpoint in a multi-echelon distribution chain satisfies all customers that arrive at that particular stockpoint, where customers are defined as the external customers as well as replenishment orders of downstream stockpoints in the distribution chain. However, in case large order overflow is applied, customers with large demand are not satisfied by the stockpoint at which they arrive, but by an upstream stockpoint. Thus for each stockpoint a maximal customer order quantity Q_c and an alternative source τ is defined such that customers with demand larger than Q_c are satisfied by source τ . Note that the case in which large order overflow is not allowed can be identified by the situation where Q_c is equal to ∞ . However, re-routing orders to another source implies in most cases increasing lead times of customers with large demand. On the other hand, the number of internal replenishments decreases. Of course it may not be easy to persuade customers to accept this new regime. It

may be needed to give a discount for the customers' willingness to collaborate. How much discount can be given depends on the cost savings realized. An alternative approach to neutralize the increasing lead times is faster transportation. This also results in a trade off between the costs of fast transportation and the costs savings due to large order overflow, that is savings of holding costs.

In our numerical experiment we consider 10 local stockpoints (LSP). The lead time from factory to RDC, L_F , equals 40 days and the lead times between the RDC and the LSP's, L_R , are equal to 3 days. For sake of simplicity we assume that all stockpoints have an identical demand process, with arrival rate 1 customer each day, an expected demand ($\mathbb{E}D$) equal to 100, and the coefficient of variation c_D is equal to $\sqrt{3}$. The value for the coefficient of variation is based on an extensive analysis of about 10.000 consumer electronics products. We varied the P_2 service measure between 0.90, 0.95 and 0.99. Furthermore, we assumed that the reorder quantity Q is 10 and 5 days of demand at the RDC and the LSP's, respectively.

Given this information about the distribution chain, we determine the control policies that yield a prespecified P_2 service measure for both the RDC and the LSP's, see Appendix 1. We used a heuristic method to find the control variables. The heuristic is analogously to the approach followed by van der Heijden (1992), who analyzed divergent logistic networks with local (R, S) inventory control. The two key elements in his approach are the decomposition of the multi-echelon distribution chain into single echelon inventory models, and the calculation of the replenishment order delay due to stockouts at upstream stockpoints. Discrete event simulation is used to compute the following performance measures, which will be used to evaluate the possible benefit of large order overflow:

- X := the average cumulative stocks at the LSP's and the RDC;
- β := the actual service level averaged over the LSP's and RDC;
- N := the transportation frequency between the RDC and LSP's plus the number of diverted customers per time unit.

Note that the service at the RDC is also incorporated in β , because diverted customers will

be served by the RDC. In the simulation experiment we sampled 20 runs of 50.000 time units (corresponding to 500.000 customers each run). For each of the performance measures, defined as above, the average value and the corresponding 95% confidence interval over the 20 runs are tabulated in Appendix 2. Moreover we used a mixture of Erlang distributions to sample the demand size (see Tijms(1994) pp. 358).

In Figure 2 we show the average cumulative stock needed to guarantee a target customer service level of 90%, 95% and 99% for various values of Q_c . Figure 3 illustrates the relative stock savings $((X(\infty) - X(Q_c))/X(\infty) \times 100\%)$.

Figure 2: Average cumulative stock needed with large order overflow.

Figure 3: Relative stock reduction with large order overflow.

The average cumulative stock needed, in the distribution chain which is not controlled by large order overflow, to guarantee a prespecified P_2 service level of 90%, 95% and 99% is about 16, 21 and 30 days demand respectively (horizontal lines in figure 2). We notice that by diverting large orders to the RDC if demands are larger than 3 times the average customer order size the cumulative stock needed to guarantee a service level of 90%, 95% and 99% equals 9, 11 and 16 days of demand respectively. By taking Q_c equal to the average customer order size we apparently discriminate between small and large orders leading to

relative cumulative stock savings of about 50%.

The analysis reveals another important point. Without large order overflow we need very high stocks to maintain a 99% service level, as is usually proposed as a standard in the Operations Management literature. In practice we find such high stocks unacceptable from an economic point of view. Thus one accepts a lower service level or one applies procedures such as large order overflow or delivery splitting, which is discussed in the next section. Such procedures (and others) are applied on an ad hoc basis, especially in situations where stock-outs are likely to occur in the near future. However, this usually comes as a surprise to the customer. We advocate the routine use of such procedures, where customers know the conditions, and target stock levels are set taking into account the benefits of the differentiated procedures. Notice, that the stock needed in case $\beta = 0.99$ and $Q_c = 200$ equals the stock needed when $\beta = 0.90$ and $Q_c = \infty$, i.e. in case customers with demands larger than 2 times the average demand are diverted to the RDC, a service of 99% can be realized with stock required to guarantee a service of 90% in case large order overflow is not applied.

It is clear that large order overflow decreases the average stock on hand enormously. Moreover, the number of internal deliveries due to replenishment orders of downstream stockpoints decrease. In fact the total amount shipped between the RDC and LSP's decreases with an amount which equals the total amount shipped from the RDC directly to the customers. Notice that the number of shipments to customers remain the same. It is clear that for rigorous treatment of the model we need to incorporate the safety stock at the customers. As motivated before we assume that the consequences for the customers are neutralized by discounts for customers accepting this new regime of large order overflow.

In order to make the proper trade offs we now introduce the Differential Total Relevant Costs (DTRC) which is defined as the difference between the total relevant costs in the situations with or without large order overflow. In the DTRC we need to incorporate the transportation cost between the RDC and LSP's and between the RDC and external customers, the holding costs and the possible extra ordering costs. Hence, the situation is considered that the customers which are diverted have extra ordering costs $A_R - A_L$ and extra

transportation costs T_R . Then recall the definition of X and N which are obviously functions of Q_c . Then $DTRC(Q_c) = h(X(\infty) - X(Q_c)) + A(N(\infty) - N(Q_c))$, where h denotes the stock keeping costs per time unit and $A := A_R - A_L + T_R$.

The profitability then depends on the transportation-holding cost ratio TH, where $TH = \frac{A}{h}$. It is clear that for large TH-ratios large order overflow is not profitable, whereas for small TH-ratios the opposite holds. In figure 4 we illustrate the DTRC for $\beta = 0.99$ where we varied the TH-ratio. The indifference curves give good insight in profitability of large order overflow.

Figure 4: TRC as function of the TH-ratior

For example, in case $Q_c = 300$, $A = 500$ the holding costs decrease by 1186 (\$/day), where the transportation costs increase by 570, hence the overall profit is \$ 615 each day. Moreover, the curves can be used as a graphical aid in determining the optimal Q_c .

4. Delivery splitting.

In this section we analyze the effect of delivery splitting on average physical stock, customer service level, and delivery frequencies. We restrict ourself to the analysis of a single local stockpoint (LSP). The large order overflow procedure aims at reducing variability of demand to be satisfied from local stockpoints. Indeed, supply chain stocks are dramatically reduced by this procedure. Yet, the customer orders are all shipped in one lot. This is likely to cause high stocks at the customers. Therefore we propose the following procedure which

may induce a beneficial result for both supplier and customers. Determine a maximum shipment lot Q_s and an intershipment time T for consecutive lots of the same customer order. Then large demands will not be delivered in one single batch, even in case the inventory level is sufficiently large. The customer receives only a limited quantity, Q_s , at a time. If the demand size is larger than Q_s , starting at the demand epoch, an amount of Q_s is delivered in a number of shipments which are T time units apart. Consequently all quantities are equal to Q_s except possibly the last. For example, when a customer arrives at time t , with demand of size D , results in the following delivery scheme for that customer:

$$\begin{cases} \text{deliver } Q_s & \text{on epoch } t + j T & (j = 0, \dots, n-1) \\ \text{deliver } D - nQ_s & \text{on epoch } t + n T \end{cases} \quad (1)$$

where $n := \max\{m \in \mathbb{N} | mQ_s \leq D\}$.

In practice, situations occur in which delivery splitting is not feasible, for example, when large orders are required immediately. However, in the situations where quantity discounts generate the large customer demand sizes, delivery splitting is especially suitable. Note, that the case in which delivery splitting is not allowed we set Q_s equal to ∞ . The focus of delivery splitting is to reduce the variance in the demand process and to achieve stock reductions at the customers as a by-product, for the reduction of the inventory at the customers see Chiang and Chiang (1996).

Closely related to delivery splitting is the concept of order splitting (see, for example, Sculli and Wu (1981) and Lau and Lau (1994)). Order splitting, however, primarily aims at reducing of lead time uncertainties by splitting the replenishment orders over more than one supplier, and is therefore a lead time management strategy. Hong and Hayya (1992) give a chronological summary of the literature about order splitting. The papers about order splitting stress the trade off between the increase in delivery costs and the decrease in holding costs, which is also an important issue when delivery splitting is applied.

To quantify the effects of delivery splitting we use simulation. We specified the distribution function of the demand size $F_D(\cdot)$ as a mixture of Erlang distributions, as in the case of large order overflow. The simulation experiments are composed of 25 runs of 100.000 time units to guarantee a 95% confidence interval of maximal 1% of the actual service level. We consider

240 cases as follows. The average customer demand size ($\bar{D}$) is fixed at 50, the coefficient of variation of the customer demand size (c_D) varies as 1, 2 and 4 to emphasize the high variability in demand. The average number of customer orders per day (λ) is equal to 1. The lead time of replenishment orders from the RDC to the LSP (L) varies between 10 and 20 days. The replenishment order quantity is equal to 1000 for all cases. The target customer service level is varied between 0.90 and 0.99. The intershipment time T is varied between 0.3, 0.5, 1 and 1.5 times the lead time. The maximal lot size of a shipment (Q_s) is varied between 0.5, 1, 2 and 4 times the average customer demand (see Appendix 3 for detailed specification). We used a heuristic method to find the control variables. The key elements in this heuristic are expressions for the arrival intensity of the splitted deliveries, and the first two moments of the sizes of such splitted deliveries. For a more detailed description of the heuristic see Janssen et al. (1995)). Discrete event simulation is used to compute the following performance measures:

- X := the average physical stock at the LSP;
- β := the actual service level at the LSP;
- N := the delivery frequency to the customer per unit of time.

Figure 5: Stock reductions obtained by delivery splitting.

In figure 5 we show the relative stock reductions obtained by delivery splitting, expressed in the percentage of the stock on hand that is needed when no delivery splitting is applied. To be more precise the figure represents the quantity $(X(\infty) - X(Q_s))/X(\infty) \times 100\%$. We conclude that for c_D relatively small ($c_D = 1$) delivery splitting does not reduce the average stock on hand much. However, for c_D large we notice the enormous stock reductions that can be obtained by delivery splitting.

On the other hand the delivery frequency increases, and thereby possibly the transportation costs. Hence, to make a trade off between applying delivery splitting or not we need to include the transportation costs, as was the case for order splitting. Because the number of customer demands remains the same we need not include the ordering costs at the LSP. We again consider the Differential Total Relevant Costs (DTRC) which is now defined as the difference between the total relevant costs in the situations using delivery splitting or not. Then $DTRC(Q_s) = h(X(\infty) - X(Q_s)) + A(N(\infty) - N(Q_s))$, where $A := T_R$. The profitability of delivery splitting depends on the ratio A/h . Figure 6 illustrates DTRC (\$/day) where h is fixed at 0.2\$/unit/day and A is varied as 15, 30, ..., 150.

Figure 6: Differential total relative costs (\$/day).

If the ratio A/h is high, delivery splitting is not profitable at all. On the other hand when the ratio A/h is small, delivery splitting is indeed profitable. To investigate the profitability

of delivery splitting for practical situations and to evaluate a number of scenarios we need a fast method to calculate $DTRC$. We refer to Janssen et al.(1995) for such a method.

So far we considered a replenishment policy which is only based on the inventory position, that is, physical inventory level plus stock on order minus backorders. However, in case delivery splitting is applied, explicit knowledge about the occurrences of future deliveries of the splitted orders is available. This knowledge could be used to improve the performance of the inventory system. To compare the effectiveness of using information about future deliveries we analyzed the same 240 cases as above. Again we used a heuristic method (Janssen et al.(1995)) to find the control parameters, whereas discrete event simulation is used to compute the performance measures (see Appendix 4). In figure 7 we present the relative stock reductions obtained by delivery splitting in case we use information about future deliveries over the stock reductions obtained by delivery splitting without using information about future deliveries. To be more precise the figure represents the quantity $(X(Q_s) - X_i(Q_s))/(X(\infty) - X(Q_s)) \times 100\%$ where X_i denotes the average stock on hand level with delivery splitting using information about future deliveries explicitly.

Figure 7: Additional stock reductions obtained by using information about future deliveries

It is clear that the additional stock reductions are dependent of Q_s . Actually, we conjecture

that there exists a Q_s for which the additional stock reductions are maximal. The additional stock reduction increase for small Q_s because of the increasing amount known to be delivered during the lead time (the number of deliveries within the lead time remains the same but the quantity per delivery increases). Thus the information about future deliveries is used more effectively. On the other hand, for large Q_s the total amount known to be delivered during the lead time decreases, because the number of future deliveries decreases. In deciding whether to implement delivery splitting with or without using the information about future deliveries a trade off has to be made between the additional stock on hand savings and the extra cost due to a more complex replenishment strategy.

5. Conclusions.

In this paper we discussed the problem of demand management in a multi-stage distribution chain. We argued the organizational causes for high demand variations in intermediate stages, in spite of stable end-customer demand. This effect is another example of Forrester's findings (1961), yet it differs from the usual interpretation of Forrester's conclusions, which are more related to batch sizing and information delays (see Silver and Peterson (1985)). We discussed ways of resolving the problems caused by these high variations; our discussion was based on the insight that both supplier and customer benefit from stability of the demand process at intermediate stages by stock reduction. We introduced and analyzed two simple procedures that could be applied in the order processing systems. We emphasize that the resulting procedures are customer-oriented; hence the procedures chosen are product-customer dependent. Our analysis showed the large impact of the two procedures from the point of view of the supplying stage.

We consider the two proposed procedures as powerful tools for management to decrease variabilities in demand and therefore decrease safety stocks. In that sense we deliberately used the notion of demand management, which is pro-active, as opposed to inventory management. Further research is required with respect to the implementation of the procedures and the validation of our stock savings predictions. Yet these predictions are in accordance with the

savings reported by companies that implemented DRP-systems with Available-To-Promise capabilities. This constitutes another subject of further research, that is, the relation of the demand management procedures defined in this paper and the ATP-capability.

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**Appendix 1 : Control parameters for the 2-echelon
inventory model with large order overflow.**

Q_{max} β		0.90		0.95		0.99	
		s	Q	s	Q	s	Q
50	RDC	41593	10000	43438	10000	46893	10000
	LDC	75	57	93	57	135	57
100	RDC	41635	10000	43480	10000	46935	10000
	LDC	136	107	168	107	243	107
200	RDC	41720	10000	43565	10000	47099	10000
	LDC	234	175	290	175	425	175
300	RDC	41808	10000	43693	10000	47306	10000
	LDC	336	238	418	238	613	238
400	RDC	41937	10000	43862	10000	47554	10000
	LDC	436	296	543	296	797	296
500	RDC	42026	10000	44031	10000	47803	10000
	LDC	526	345	653	345	961	345
600	RDC	42154	10000	44159	10000	48011	10000
	LDC	603	384	750	384	1102	384
700	RDC	42241	10000	44286	10000	48178	10000
	LDC	668	415	830	415	1221	415
800	RDC	42297	10000	44371	10000	48303	10000
	LDC	722	438	897	438	1319	438
900	RDC	42371	10000	44455	10000	48427	10000
	LDC	765	455	952	455	1398	455
1000	RDC	42375	10000	44498	10000	48470	10000
	LDC	801	468	995	468	1463	468
∞	RDC	42462	10000	44586	10000	48637	10000
	LDC	892	500	1125	500	1674	500

**Appendix 2 : Performance measures for the 2-echelon
inventory model with large order overflow.**

Q_{max}	β	0.90	0.95	0.99
50	X	7108 [6399 , 7817]	9489 [9408 , 9570]	13295 [13225 , 13365]
	N	5.87 [5.87 , 5.88]	5.87 [5.87 , 5.88]	5.87 [5.87 , 5.88]
	β	0.9034 [0.9030 ,0.9039]	0.9557 [0.9554 ,0.9559]	0.9941 [0.9940 ,0.9942]
100	X	7934 [7474 , 8393]	10204 [10149 , 10258]	13954 [13303 , 14606]
	N	4.43 [4.43 , 4.44]	4.43 [4.43 , 4.44]	4.44 [4.43 , 4.44]
	β	0.9039 [0.9034 ,0.9044]	0.9552 [0.9549 ,0.9555]	0.9935 [0.9934 ,0.9936]
200	X	9103 [9018 , 9188]	11053 [10398 , 11708]	15848 [15184 , 16513]
	N	3.47 [3.47 , 3.48]	3.47 [3.47 , 3.47]	3.47 [3.47 , 3.48]
	β	0.9037 [0.9032 ,0.9043]	0.9544 [0.9541 ,0.9547]	0.9934 [0.9933 ,0.9935]
300	X	10152 [10075 , 10228]	12705 [12644 , 12766]	18276 [18180 , 18372]
	N	2.94 [2.94 , 2.94]	2.94 [2.93 , 2.94]	2.94 [2.94 , 2.94]
	β	0.9046 [0.9041 ,0.9051]	0.9545 [0.9542 ,0.9548]	0.9932 [0.9931 ,0.9933]
400	X	11207 [11113 , 11302]	14053 [13983 , 14123]	20248 [20184 , 20311]
	N	2.58 [2.58 , 2.58]	2.58 [2.58 , 2.58]	2.58 [2.58 , 2.59]
	β	0.9061 [0.9056 ,0.9066]	0.9552 [0.9549 ,0.9555]	0.9932 [0.9931 ,0.9933]
500	X	12081 [12004 , 12157]	14923 [14262 , 15585]	22117 [21987 , 22247]
	N	2.34 [2.34 , 2.34]	2.34 [2.34 , 2.34]	2.34 [2.34 , 2.35]
	β	0.9076 [0.9071 ,0.9081]	0.9559 [0.9557 ,0.9561]	0.9933 [0.9932 ,0.9934]
600	X	12652 [11995 , 13309]	16342 [16236 , 16447]	23355 [22695 , 24015]
	N	2.18 [2.18 , 2.18]	2.18 [2.18 , 2.18]	2.18 [2.18 , 2.18]
	β	0.9087 [0.9082 ,0.9092]	0.9562 [0.9559 ,0.9564]	0.9933 [0.9932 ,0.9934]
700	X	13291 [12630 , 13952]	16757 [16055 , 17459]	24602 [23941 , 25262]
	N	2.06 [2.06 , 2.07]	2.06 [2.06 , 2.07]	2.07 [2.06 , 2.07]
	β	0.9093 [0.9089 ,0.9097]	0.9523 [0.9445 ,0.9601]	0.9931 [0.9931 ,0.9932]
800	X	14241 [14147 , 14335]	18021 [17960 , 18082]	26012 [25923 , 26100]
	N	1.99 [1.98 , 1.99]	1.99 [1.98 , 1.99]	1.99 [1.99 , 1.99]
	β	0.9100 [0.9095 ,0.9105]	0.9564 [0.9560 ,0.9567]	0.9930 [0.9929 ,0.9931]
900	X	14395 [13735 , 15054]	18567 [18445 , 18689]	26943 [26855 , 27031]
	N	1.93 [1.93 , 1.93]	1.93 [1.93 , 1.93]	1.93 [1.93 , 1.93]
	β	0.9098 [0.9094 ,0.9103]	0.9563 [0.9560 ,0.9567]	0.9928 [0.9927 ,0.9929]
1000	X	15032 [14942 , 15123]	18662 [17996 , 19328]	27664 [27564 , 27764]
	N	1.89 [1.89 , 1.89]	1.89 [1.89 , 1.89]	1.89 [1.89 , 1.89]
	β	0.9101 [0.9096 ,0.9107]	0.9559 [0.9556 ,0.9562]	0.9926 [0.9925 ,0.9927]
∞	X	16278 [16177 , 16378]	20676 [20555 , 20797]	30135 [30008 , 30262]
	N	1.80 [1.80 , 1.80]	1.80 [1.80 , 1.80]	1.80 [1.80 , 1.80]
	β	0.9048 [0.9044 ,0.9052]	0.9517 [0.9514 ,0.9520]	0.9900 [0.9898 ,0.9901]

Appendix 3 : Performance measures for delivery splitting without using information about future deliveries

T	Q_s	L=10						L=20					
		$\beta=0.90$			$\beta=0.99$			$\beta=0.90$			$\beta=0.99$		
		$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$
0.3 L	∞	533	769	1652	930	1469	3442	1115	1404	2381	1619	2299	4534
		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		543	783	1672	932	1469	3433	631	928	1926	1118	1801	4039
	25	259	142	67	496	343	226	601	370	221	897	620	419
		2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6100
		453	437	424	683	631	578	485	455	431	771	698	621
	50	403	279	132	719	582	364	873	618	339	1269	994	625
		1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.6900
		496	492	453	804	788	678	554	540	476	937	906	754
	100	492	472	285	862	923	680	1041	944	583	1506	1497	1058
		1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.2800
		526	598	552	887	1038	941	603	688	614	1054	1228	1079
	200	523	644	613	918	1242	1326	1102	1226	1051	1599	1952	1878
		1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		538	701	805	923	1287	1508	624	832	931	1105	1541	1743
0.5 L	25	528	723	1090	928	1402	2234	1112	1365	1680	1614	2196	3000
		1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0200
		541	749	1199	930	1414	2331	629	906	1393	1115	1716	2694
	50	186	98	49	379	261	189	458	282	186	699	484	357
		2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6100
		437	427	420	623	584	555	454	436	424	687	631	589
	100	342	205	99	616	444	283	756	481	275	1101	778	502
		1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.6900
		476	461	439	743	694	617	519	489	451	853	779	671
	200	468	377	199	818	742	488	995	780	439	1435	1231	791
		1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.2800
		516	543	497	857	900	779	586	609	531	1013	1048	875
L	25	520	578	437	912	1107	952	1095	1118	791	1587	1765	1405
		1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		536	658	669	918	1176	1177	621	771	751	1097	1405	1354
	50	528	706	867	928	1366	1792	1112	1334	1377	1614	2136	2441
		1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0200
		541	736	1012	930	1383	1928	629	886	1164	1115	1669	2215
	100	115	60	32	267	194	157	317	207	152	505	370	301
		2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6100
		424	420	418	571	550	538	432	424	420	612	582	564
	200	261	143	75	485	330	229	601	365	230	882	597	419
		1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.7500	1.5800	1.6900	1.6900
		452	440	430	670	620	580	479	454	437	750	679	621
1.5 L	25	420	276	139	735	556	352	907	605	337	1302	953	598
		1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.3500	1.1500	1.2800	1.2800
		497	490	462	804	764	670	555	531	480	938	870	734
	50	511	476	286	894	910	642	1079	949	567	1559	1481	992
		1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		532	594	557	904	1019	908	614	679	607	1078	1200	1023
	100	528	665	609	928	1272	1254	1112	1266	1027	1614	2006	1785
		1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0200
		541	707	799	929	1301	1438	629	842	906	1115	1565	1654
	200	109	56	31	260	191	155	311	203	150	498	366	299
		2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6500	2.5400	2.6100	2.6100
		419	417	417	564	546	536	426	421	419	605	577	561

In each cel the top, the middle and the bottom elements denote the reorder point calculated by the method described in Janssen et al. (1995), the associated delivery frequency and the average stock position, respectively.

Appendix 4 : Performance measures for delivery splitting using information about future deliveries

T	Q_s	L=10						L=20					
		$\beta=0.90$			$\beta=0.99$			$\beta=0.90$			$\beta=0.99$		
		$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$	$c_D = 1$	$c_D = 2$	$c_D = 4$
0.3 L	∞	533	769	1652	930	1469	3442	1115	1404	2381	1619	2299	4534
		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		543	783	1672	932	1469	3433	631	928	1926	1118	1801	4039
	25	477	500	513	766	812	835	1025	1056	1072	1389	1448	1480
		2.5400	2.6100	2.6400	2.5400	2.6100	2.6400	2.5400	2.6100	2.6500	2.5400	2.6100	2.6100
		486	508	522	766	813	836	537	569	582	889	950	980
	50	504	571	621	846	981	1067	1069	1156	1218	1498	1670	1780
		1.5800	1.7000	1.7400	1.5800	1.7000	1.7500	1.5800	1.7000	1.7500	1.5800	1.6900	1.6900
		514	580	631	846	980	1065	583	668	728	998	1171	1284
	100	519	653	806	897	1180	1447	1096	1268	1450	1570	1916	2250
		1.1600	1.2900	1.3500	1.1600	1.2800	1.3500	1.1600	1.2800	1.3500	1.1600	1.2800	1.2800
		531	663	813	897	1180	1448	611	785	966	1071	1417	1753
	200	527	712	1075	922	1340	2012	1109	1349	1766	1606	2117	2905
		1.0200	1.1000	1.1600	1.0200	1.0900	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		538	726	1088	923	1341	2013	625	868	1286	1107	1616	2408
0.5 L	25	528	737	1360	928	1423	2691	1112	1390	2091	1614	2231	3634
		1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0700	1.0000	1.0200	1.0200
		541	752	1376	929	1426	2691	630	914	1612	1115	1734	3135
	50	449	454	456	697	709	714	984	991	994	1297	1314	1322
		2.5400	2.6100	2.6400	2.5400	2.6100	2.6400	2.5400	2.6100	2.6400	2.5400	2.6100	2.6100
		457	462	466	697	709	716	496	502	508	798	815	823
	100	481	502	514	788	836	861	1035	1065	1082	1422	1488	1521
		1.5800	1.7000	1.7500	1.5800	1.6900	1.7400	1.5800	1.7000	1.7500	1.5800	1.6900	1.6900
		491	512	524	788	836	865	547	578	592	923	989	1026
	200	510	576	622	871	1012	1103	1081	1171	1232	1535	1721	1840
		1.1600	1.2900	1.3500	1.1600	1.2900	1.3500	1.1600	1.2800	1.3400	1.1600	1.2900	1.2900
		521	586	633	872	1011	1102	597	687	752	1036	1219	1339
L	25	525	662	804	919	1220	1490	1106	1288	1466	1599	1980	2328
		1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		538	676	818	919	1221	1489	622	805	985	1100	1482	1837
	50	528	724	1072	928	1391	2054	1112	1373	1785	1614	2183	2991
		1.0000	1.0200	1.0600	1.0000	1.0200	1.0700	1.0000	1.0200	1.0700	1.0000	1.0200	1.0200
		541	739	1087	929	1392	2052	629	892	1312	1115	1687	2491
	100	422	422	422	623	623	623	942	942	942	1195	1196	1196
		2.5400	2.6100	2.6300	2.5400	2.6100	2.6400	2.5400	2.6100	2.6400	2.5400	2.6100	2.6100
		430	429	433	623	624	623	452	452	453	695	698	698
	200	446	447	448	704	708	710	985	987	987	1312	1316	1319
		1.5800	1.6900	1.7500	1.5800	1.7000	1.7500	1.5800	1.6900	1.7500	1.5800	1.7000	1.7000
		456	457	459	705	708	710	497	500	502	812	814	824
1.5 L	25	483	494	499	810	839	851	1044	1061	1068	1457	1498	1514
		1.1600	1.2800	1.3400	1.1600	1.2800	1.3500	1.1600	1.2800	1.3500	1.1600	1.2800	1.2800
		494	505	512	810	840	851	558	574	583	958	1000	1014
	50	517	572	596	899	1033	1091	1096	1175	1211	1577	1756	1834
		1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1600	1.0200	1.1000	1.1000
		529	585	608	899	1034	1092	612	692	728	1078	1256	1331
	100	528	670	768	928	1266	1487	1112	1309	1441	1613	2055	2340
		1.0000	1.0200	1.0700	1.0000	1.0200	1.0600	1.0000	1.0200	1.0600	1.0000	1.0200	1.0200
		541	686	784	928	1266	1487	628	828	964	1114	1556	1841
	200	422	422	422	623	623	623	942	942	942	1195	1196	1196
		2.5400	2.6100	2.6400	2.5400	2.6100	2.6500	2.5400	2.6100	2.6400	2.5400	2.6000	2.6000
		430	429	431	623	624	622	452	453	455	696	699	699

In each cell the top, the middle and the bottom elements denote the reorder point calculated by the method described in Janssen et al. (1995), the associated delivery frequency and the average stock position, respectively.