Tilburg University

On Incentive Compatibility and Budget Balancedness in Public Decision Making

Suijs, J.P.M.

Publication date:
1995

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Suijs, J. P. M. (1995). *On Incentive Compatibility and Budget Balancedness in Public Decision Making*. (CentER Discussion Paper; Vol. 1995-53). CentER.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

On incentive compatibility and budget balancedness in public decision making

JEROEN SUIJS^{1 2}

May 24, 1995

Abstract

Holmström (1979) showed that Groves' schemes are the unique incentive compatible transfer schemes for public decision making problems if the domain of preferences is smoothly connected. In this paper we will show that this result can be extended to public decision making problems with a connected domain of preferences and a finite number of alternatives. Moreover, we give a class of decision making problems for which a budget balanced Groves' scheme exists. Finally, we show that for this particular class a budget balanced and incentive compatible transfer scheme cannot satisfy a specific individual rights property.

¹CentER and Department of Econometrics, Tilburg University, P.O. box 90153, 5000 LE Tilburg

²The author would like to thank Peter Borm, Herbert Hamers and Eric van Damme for their helpful comments on earlier versions of this paper.

1 Introduction

It is by now well known that in the area of public decision making the individual behaviour of agents is a non negligible factor. While the public decision maker tries to optimize social welfare given the preferences revealed by the agents, the agents try to reveal their preferences in such a way that their own individual welfare is maximized.

The decision rule which determines the public decision can lead to behaviour of agents, where they reveal other preferences than their true ones, in order to maximize their individual utility. As a result, the public decision could be socially inefficient, that is, the public decision does not maximize the true total utility of the agents. To prevent this from happening, a scheme of transfer payments can be used. A transfer payments scheme assigns to each preference profile a vector of transfer payments. A transfer payments scheme is called incentive compatible if revealing the true preferences is a dominant strategy for each agent, whatever his true preferences are.

For situations with transferable utility *Groves (1973)* introduced an incentive compatible scheme of transfer payments. This transfer scheme has led to a whole class of incentive compatible transfer schemes, the so called Groves' schemes. Subsequently, *Green and Laffont (1977)* showed that Groves schemes are the unique incentive compatible transfer schemes when the domain of preferences is unrestricted. This condition of unrestricted domains was weakened by *Holmström (1979)* to smoothly connected domains. Roughly speaking, a domain of preferences is smoothly connected if the domain of preferences for each agent is connected and satisfies some differentiability conditions.

The incentive compatibility property, however, has its price. *Green and Laffont (1977)* showed that the transfer payments of a Groves' scheme need not be budget balanced when the domain of preferences is unrestricted. This means that there remains a surplus or deficit of money. As a consequence, one will lose the incentive compatibility property when trying to divide this surplus or deficit among the agents.

So an incentive compatible transfer scheme can for these cases result in a non Pareto optimal situation.

In this paper we focus on public decision making problems with a finite number of alternatives only. In Section 2 we give a formal description of the problem and recall some of the standard terminology. Moreover, we will give the exact definition of smoothly connected domains of preferences. In Section 3 we will extend Holmström's result to connected domains of preferences. We will show that Groves' schemes are the unique incentive compatible transfer schemes on connected domains of preferences, when the number of alternatives is finite. Finally, in Section 4 we will give an example to show that if the public decision problem is 'nice' enough a budget balanced Groves' scheme does exist. The example we consider is the following.

Consider a large firm with several divisions which all have to make use of the same facility - for example a repair and maintenance facility - which can only serve one division at a time. When a number of divisions ask for service of this facility, each division incurs costs for the time it is down. In this situation, the firm is the social decision maker, which wants to serve the divisions in such an order that the total cost during downtime is minimized, while each division wants to be served as soon as possible. The firm determines the order in which divisions will be served on the costs which divisions incur during downtime. These costs, however, are unknown to the firm and have to be revealed by the divisions. If we assume that the costs of the divisions are linear in time, then we can construct a budget balanced incentive compatible transfer scheme for this type of decision making problems.

2 Public decision making

In this paper we consider public decision making problems (N, A, V) , where $N = \{1, 2, \dots, n\}$ is the set of agents, A the set of alternatives, and $V = V^1 \times V^2 \times \dots \times V^n$ the domain of preferences. The preferences of agent $i \in N$ are quasi-linear preferences,

so that we can write $w^i(a, t_i) = v^i(a) + t_i$, where $v^i : A \rightarrow \mathbb{R}$ describes agent's i preferences over the alternatives and t_i equals the amount of money the agent receives. The set of all preferences v^i which agent i can reveal is denoted by V^i . Given the revealed preferences of the agents, the social decision maker selects the alternative which maximizes the total utility of the agents, i.e.

$$a^*(v) = \operatorname{argmax}_{a \in A} \sum_{i \in N} v^i(a) \quad (1)$$

for each $v \in V$.

To prevent agents from revealing any other preferences than the true ones, we introduce a transfer payment scheme $t : V \rightarrow \mathbb{R}^n$, which assigns to each preference profile $v \in V$ a vector of transfer payments. The interpretation is that agent i has to pay the amount $t_i(v)$ if $t_i(v) < 0$ and he receives the amount $t_i(v)$ if $t_i(v) > 0$. A transfer payments scheme is called incentive compatible if revealing the true preferences is a dominant strategy for each agent, whatever his true preferences are. Formally this means that for all $i \in N$ and all $v \in V$ and any true preference $\nu \in V$ we have

$$\nu^i(a^*(\nu^i, v^{-i})) + t_i(\nu^i, v^{-i}) \geq \nu^i(a^*(v^i, v^{-i})) + t_i(v^i, v^{-i}), \quad (2)$$

where $v^{-i} = (v^1, \dots, v^{i-1}, v^{i+1}, \dots, v^n)$.

Examples of incentive compatible transfer schemes are Groves' schemes. A transfer scheme t is called a Groves' scheme if for each $i \in N$ there exists a function $h_i : V^{-i} \rightarrow \mathbb{R}$ such that for each $v \in V$ it holds that

$$t_i(v) = \sum_{j \in N \setminus \{i\}} v^j(a^*(v)) + h_i(v^{-i}). \quad (3)$$

A transfer scheme t is called budget balanced if for all $v \in V$ it holds that $\sum_{i \in N} t_i(v) = 0$. So, for a budget balanced Groves' scheme, all the money needed to ensure agents revealing their true preferences is paid by the agents themselves. Finally, we give the formal definition of a smoothly connected domain of preferences, as used in *Holmström (1979)*. A domain V of preferences is called smoothly connected

if for any $v^i, \hat{v}^i \in V^i$ and any $v^{-i} \in V^{-i}$ there exists a collection $\{v^i(a; y_i)\}_{y_i \in [0,1]} \subset V^i$ such that for all $a \in A$

- (i) $v^i(a; 0) = v^i(a)$, $v^i(a; 1) = \hat{v}^i(a)$,
- (ii) $\partial v^i(a; y_i)/\partial y_i$ exists for all $y_i \in [0, 1]$, where for the endpoints the existence of one-sided derivatives is sufficient,
- (iii) there exists a $K \in (0, \infty)$ such that for all $y_i \in [0, 1]$ and all $\hat{a}$ with $\hat{a} = a^*(v^i(a; y_i), v^{-i})$ for some $y_i \in [0, 1]$ it holds that

$$\left| \frac{\partial v^i(\hat{a}, y_i)}{\partial y_i} \right| \leq K.$$

Holmström (1979) showed that Groves' schemes are the unique incentive compatible transfer schemes when the public decision making problem has a smoothly connected domain of preferences. Note that this result holds for both a finite and an infinite number of alternatives.

3 Groves' schemes on connected domains

In this section we only consider public decision making problems with a finite number of alternatives. We will show that the Groves' schemes are the unique incentive compatible transfer schemes when the domain of preferences V is connected.

Since A is finite, the preferences $v^i \in \mathbb{R}^A$ of agent i can be represented by $|A|$ -dimensional vectors. Consequently, the domain of preferences V^i is a subset of the $|A|$ -dimensional Euclidean space. Then, the domain of preferences V is called connected if for all $i \in N$ the set V^i is a connected subset of the $|A|$ -dimensional Euclidean space. Note that a smoothly connected domain of preferences is connected.

Next, let for each $i \in N$, $v^{-i} \in V^{-i}$ and each $a \in A$ the set $V^i(a, v^{-i}) \subseteq V^i$ contain the preferences v^i of agent i for which the decision rule a^* selects alternative a , i.e.

$$V^i(a, v^{-i}) = \{v^i \in V^i \mid a = a^*(v^i, v^{-i})\}.$$

Further, define for each $i \in N$, $v^{-i} \in V^{-i}$ and each $a, \hat{a} \in A$ the distance between $V^i(a, v^{-i})$ and $V^i(\hat{a}, v^{-i})$ as

$$d(V^i(a, v^{-i}), V^i(\hat{a}, v^{-i})) = \inf \{ \max_{\tilde{a} \in A} |v^i(\tilde{a}) - \hat{v}^i(\tilde{a})| \mid v^i \in V^i(a, v^{-i}), \hat{v}^i \in V^i(\hat{a}, v^{-i}) \}.$$

Before we turn to the main result of this section we prove the following two lemmas.

Lemma 3.1 If t is an incentive compatible transfer scheme then for each $i \in N$ there exists a function $h_i : A \times V^{-i} \rightarrow \mathbf{R}$ such that

$$t_i(v) = \sum_{j \in N \setminus \{i\}} v^j(a^*(v)) + h_i(a^*(v), v^{-i}) \quad (4)$$

for all $v \in V$.

PROOF: Let t be an incentive compatible transfer scheme. We have to prove that $t_i(v) - \sum_{j \in N \setminus \{i\}} v^j(a^*(v))$ depends only on $a^*(v)$ and v^{-i} , for all $v \in V$ and all $i \in N$. Now suppose the lemma is not true. Then there exist $i \in N$, $v^{-i} \in V^{-i}$ and $v^i, \tilde{v}^i \in V^i$ such that $a^*(v^i, v^{-i}) = a^*(\tilde{v}^i, v^{-i})$ and $t_i(\tilde{v}^i, v^{-i}) < t_i(v^i, v^{-i})$. This implies

$$\tilde{v}_i(a^*(\tilde{v}^i, v^{-i})) + t_i(\tilde{v}^i, v^{-i}) < \tilde{v}_i(a^*(v^i, v^{-i})) + t_i(v^i, v^{-i}),$$

which contradicts the incentive compatibility of t when $\tilde{v}_i$ represents the true preferences of agent i . $\square$

Note that if h_i in (4) is also independent of $a^*(v)$ for all $v \in V$ and all $i \in N$, then t is a Groves' scheme.

Lemma 3.2 Let t be an incentive compatible transfer scheme. Take $i \in N$, $v^{-i} \in V^{-i}$ and $a, \hat{a} \in A$ such that $V^i(a, v^{-i}) \neq \emptyset$ and $V^i(\hat{a}, v^{-i}) \neq \emptyset$. Let h_i satisfy expression (4). Then the following statement holds.

$$\text{If } d(V^i(a, v^{-i}), V^i(\hat{a}, v^{-i})) = 0, \text{ then } h_i(a, v^{-i}) = h_i(\hat{a}, v^{-i}).$$

PROOF: By Lemma 3.1, incentive compatibility of t implies

$$v^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) + h_i(a, v^{-i}) \geq v^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a}) + h_i(\hat{a}, v^{-i})$$

when v^i represents the true preferences of agent i , and

$$\hat{v}^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a}) + h_i(\hat{a}, v^{-i}) \geq \hat{v}^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) + h_i(a, v^{-i})$$

when $\hat{v}^i$ represents the true preferences of agent i . Rearranging terms yields

$$v^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) - (v^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a})) \geq h_i(\hat{a}, v^{-i}) - h_i(a, v^{-i}) \quad (5)$$

and

$$\hat{v}^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) - (\hat{v}^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a})) \leq h_i(\hat{a}, v^{-i}) - h_i(a, v^{-i}). \quad (6)$$

Since $d(V^i(a, v^{-i}), V^i(\hat{a}, v^{-i})) = 0$ we can find for each $\varepsilon > 0$ preferences $v_\varepsilon^i \in V_i(a, v^{-i})$ and $\hat{v}_\varepsilon^i \in V^i(\hat{a}, v^{-i})$ such that $|v_\varepsilon^i(a) - \hat{v}_\varepsilon^i(a)| < \varepsilon$ and $|v_\varepsilon^i(\hat{a}) - \hat{v}_\varepsilon^i(\hat{a})| < \varepsilon$. From $a^*(v_\varepsilon^i, v^{-i}) = a$ we can derive that $\sum_{j \in N \setminus \{i\}} v^j(a) \geq v_\varepsilon^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a}) - v_\varepsilon^i(a)$. Hence,

$$\begin{aligned} \hat{v}_\varepsilon^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) - (\hat{v}_\varepsilon^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a})) &\geq \hat{v}_\varepsilon^i(a) - v_\varepsilon^i(a) - \hat{v}_\varepsilon^i(\hat{a}) + v_\varepsilon^i(\hat{a}) \\ &\geq -2\varepsilon. \end{aligned} \quad (7)$$

Similarly, $a^*(\hat{v}_\varepsilon^i, v^{-i}) = \hat{a}$ implies $\sum_{j \in N \setminus \{i\}} v^j(\hat{a}) \geq \hat{v}_\varepsilon^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) - \hat{v}_\varepsilon^i(\hat{a})$.

Hence,

$$\begin{aligned} v_\varepsilon^i(a) + \sum_{j \in N \setminus \{i\}} v^j(a) - (v_\varepsilon^i(\hat{a}) + \sum_{j \in N \setminus \{i\}} v^j(\hat{a})) &\leq v_\varepsilon^i(a) - \hat{v}_\varepsilon^i(a) + \hat{v}_\varepsilon^i(\hat{a}) - v_\varepsilon^i(\hat{a}) \\ &\leq 2\varepsilon. \end{aligned} \quad (8)$$

Combining the inequalities (7) and (8) with (5) and (6) we find

$$-2\varepsilon \leq h_i(\hat{a}, v^{-i}) - h_i(a, v^{-i}) \leq 2\varepsilon$$

for each $\varepsilon > 0$. This proves the result. $\square$

With these two lemmas we can prove the following theorem.

Theorem 3.3 Let (N, A, V) be a public decision making problem. If A is finite and the domain V of preferences is connected, then Groves' schemes are the unique incentive compatible transfer schemes.

PROOF: We have to prove that an incentive compatible transfer scheme must be a Groves' scheme. Let t be an incentive compatible transfer scheme and let h_i , $i \in N$, be as in (4). It is sufficient to show that $h_i(a, v^{-i})$ is independent of a , for each $i \in N$ and each $v^{-i} \in V^{-i}$. Therefore, take $i \in N$ and $v^{-i} \in V^{-i}$. Now, let k be the number of alternatives for which $V^i(a, v^{-i}) \neq \emptyset$. If $k = 1$, we have that $h_i(a, v^{-i})$ is independent of a . If $k \geq 2$, the finiteness of A and the connectedness of V^i imply that there exist alternatives $a_1, a_2 \in A$ such that $d(V^i(a_1, v^{-i}), V^i(a_2, v^{-i})) = 0$. Lemma 3.2 then implies $h_i(a_1, v^{-i}) = h_i(a_2, v^{-i})$. If $V^i(a_1, v^{-i}) \cup V^i(a_2, v^{-i}) = V^i$ the proof is finished. If not, by the finiteness of A and the connectedness of V^i there exist an alternative $a_3 \in A$ such that $d(V^i(a_3, v^{-i}), V^i(a_j, v^{-i})) = 0$ for some $j \in \{1, 2\}$. Then, by Lemma 3.2, we have $h_i(a_3, v^{-i}) = h_i(a_j, v^{-i})$ for $j = 1, 2$. Again, if $\cup_{j \in \{1, 2, 3\}} V^i(a_j, v^{-i}) = V^i$ the proof is finished. Repeat this argument m times until $\cup_{j \in \{1, 2, \dots, m\}} V^i(a_j, v^{-i}) = V^i$ and the proof is finished. Note that such an m exists by the finiteness of A . $\square$

Since smoothly connected domains of preferences are connected Theorem 3.3 extends the result of Holmström for public decision making problems with a finite number of alternatives.

As already mentioned in the introduction, the incentive compatibility property has its price. In many cases the transfer payments of an incentive compatible transfer scheme are not budget balanced. This means that there is a surplus or deficit of money. Unfortunately, in many cases this amount of money cannot be redistributed among the agents without losing the incentive compatibility property. In the next section, however, we will show that if the decision making problem is 'nice' enough, a transfer scheme exists which is both incentive compatible and budget balanced. Moreover, we will show that an impossibility occurs when transfer schemes also have to satisfy a specific individual rights property.

4 A budget balanced Groves' scheme

Consider a large firm with several divisions which all have to make use of the same facility - for example a repair and maintenance facility - which can only serve one division at the time. When divisions ask for service of the repair and maintenance facility, each division incurs cost for the time it is waiting for service.

This situation gives rise to a decision making problem, where the divisions are the 'individuals' and the firm is the social decision maker, determining in which order the repair and maintenance facility should serve the several divisions of the firm. The firm's interest is minimizing the total costs of downtime of the divisions, while the divisions individually want to minimize their own cost, that is, they want to be served as soon as possible. These costs, however, are as opposed to the service times not common knowledge but have to be revealed by the divisions.

The scheduling problem mentioned above can be described with a public decision making problem (N, Π, V) , where $N = \{1, 2, \dots, n\}$ is the set of divisions of the firm, $\Pi = \{\sigma : N \rightarrow \{1, 2, \dots, n\} \mid \forall_{i,j \in N} : \sigma(i) = \sigma(j) \Leftrightarrow i = j\}$ the set of all possible serving orders for the divisions, and $V^i \subset \mathbb{R}^\Pi$ the domain of preferences of division i . Note that $\sigma(i) = j$ means that division i is in position j of the serving order σ . We

assume that the costs of each division are linear in time. Then V^i can be written as

$$V^i = \{v^i \in \mathbb{R}^\Pi \mid \exists_{\alpha_i > 0} \forall_{\sigma \in \Pi} : v^i(\sigma) = - \sum_{j: \sigma(j) < \sigma(i)} p_j \alpha_i\},$$

where p_j denotes the service time of division j , α_i denotes division's i costs per time unit, and $v^i(\sigma)$ represents the valuation of the serving order σ for division i . Since the firm wants to minimize the total costs of downtime, while in public decision making the decision maker wants to maximize the total valuation of the alternatives, we have that $v_i(\sigma)$ equals minus the waiting costs at order σ . Note that preferences $v^i \in V^i$ are determined by specifying the costs per time unit α_i . Therefore, we write the preferences of division i as $v_{\alpha_i}^i$ instead of v^i , and a preference profile as $v_\alpha = (v_{\alpha_i}^i)_{i \in N} \in V$ instead of $v = (v^i)_{i \in N} \in V$.

By definition, this public decision making problem has a finite number of alternatives and a (smoothly) connected domain of preferences. Hence, the only incentive compatible transfer schemes for this type of problems are Groves' schemes. Moreover, *Smith (1956)* showed that the total costs of downtime are minimal for the firm if the divisions are served in decreasing order with respect to α_i/p_i . Since the firm wants to minimize the total costs of downtime, the optimal serving order is determined by the revealed costs α_i . So, by revealing higher costs α_i division i will move forward in the serving order.

In the next theorem we will show that for this type of public decision making problems a budget balanced and incentive compatible transfer scheme does not exist if the number of divisions equals two.

Theorem 4.1 Let (N, Π, V) be a public decision making problem that arises from a scheduling problem. If $|N| = 2$ then there exists no transfer scheme which satisfies budget balancedness and incentive compatibility.

PROOF: Suppose t is incentive compatible and budget balanced. Take $v_{\alpha_1}^1, v_{\hat{\alpha}_1}^1 \in V^1$ and $v_{\alpha_2}^2, v_{\hat{\alpha}_2}^2 \in V^2$ such that $\alpha_1/p_1 > \alpha_2/p_2 > \hat{\alpha}_2/p_2 > \hat{\alpha}_1/p_1$. Then the optimal or-

ders $\sigma^*(v_{\alpha_1}^1, v_{\alpha_2}^2)$, $\sigma^*(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2)$, $\sigma^*(v_{\hat{\alpha}_1}^1, v_{\hat{\alpha}_2}^2)$ and $\sigma^*(v_{\hat{\alpha}_1}^1, v_{\hat{\alpha}_2}^2)$ are $(1, 2)$, $(1, 2)$, $(2, 1)$ and $(2, 1)$, respectively. By Theorem 3.3 we know that t is a Groves' scheme. Expression (3) implies for $i = 1$ and $v^{-i} = v_{\alpha_2}^2$ that

$$t_1(v_{\alpha_1}^1, v_{\alpha_2}^2) - v_{\alpha_2}^2(\sigma^*(v_{\alpha_1}^1, v_{\alpha_2}^2)) = t_1(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2) - v_{\alpha_2}^2(\sigma^*(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2)).$$

Since $\sigma^*(v_{\alpha_1}^1, v_{\alpha_2}^2) = (1, 2)$ and $\sigma^*(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2) = (2, 1)$ we have

$$t_1(v_{\alpha_1}^1, v_{\alpha_2}^2) + p_1 \alpha_2 = t_1(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2).$$

Budget balancedness implies $t_1(v_{\alpha_1}^1, v_{\alpha_2}^2) = -t_2(v_{\alpha_1}^1, v_{\alpha_2}^2)$ and $t_1(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2) = -t_2(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2)$.

Hence,

$$t_2(v_{\alpha_1}^1, v_{\alpha_2}^2) - p_1 \alpha_2 = t_2(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2).$$

Since t is a Groves' scheme, we can find a function h_2 (cf. (3)) such that $t_2(v_{\alpha_1}^1, v_{\alpha_2}^2) = 0 + h_2(v_{\alpha_1}^1)$ and $t_2(v_{\hat{\alpha}_1}^1, v_{\alpha_2}^2) = -p_2 \hat{\alpha}_1 + h_2(v_{\hat{\alpha}_1}^1)$. Substituting these expressions yields

$$-p_1 \alpha_2 = h_2(v_{\alpha_1}^1) - p_2 \hat{\alpha}_1 - h_2(v_{\hat{\alpha}_1}^1).$$

Similarly, one derives for $i = 1$ and $v^{-i} = v_{\hat{\alpha}_2}^2$

$$-p_1 \hat{\alpha}_2 = h_2(v_{\hat{\alpha}_1}^1) - p_2 \hat{\alpha}_1 - h_2(v_{\alpha_1}^1).$$

Combining the two equalities above yields $p_1 \alpha_2 = p_1 \hat{\alpha}_2$. Obviously, this is a contradiction. $\square$

If, however, the number of divisions exceeds two, then there does exist a transfer scheme which satisfies both budget balancedness and incentive compatibility. Such a transfer scheme is the following. Define for each $i \in N$ and each $v_\alpha \in V$

$$\begin{aligned} \tau_i(v_\alpha) = & \sum_{j: \sigma_\alpha(j) < \sigma_\alpha(i)} \frac{1}{2} p_i \alpha_j - \sum_{j: \sigma_\alpha(j) > \sigma_\alpha(i)} \frac{1}{2} p_i \alpha_j \\ & - \frac{1}{2(n-2)} \sum_{j \in N \setminus \{i\}} \sum_{k \in N \setminus \{i\}: \sigma_\alpha(k) > \sigma_\alpha(j)} p_k \alpha_j - p_j \alpha_k, \end{aligned} \tag{9}$$

where σ_α denotes the optimal order for preference profile v_α . Note that the final term in (9) is independent of the preference $v_{\alpha_i}^i$ of agent i . This follows from the fact that division k is served before division j when $\alpha_k/p_k > \alpha_j/p_j$ (cf. *Smith (1956)*). Hence, the set $\{j, k \in N \setminus \{i\} \mid \sigma_\alpha(k) > \sigma_\alpha(j)\}$ is independent of $v_{\alpha_i}^i$. Consequently, incentive compatibility should be guaranteed by the first two terms. The third term is used to make the transfer scheme budget balanced. Let us try to interpret expression (9).

For the moment consider a situation with only two divisions j and k . Suppose that the real costs of divisions j and k are such that $\alpha_j/p_j > \alpha_k/p_k$. Then the firm would like to serve division j before division k . If the firm wants the divisions to reveal their true preferences, she could implement the transfer scheme in (9) with the third term omitted. Then the costs of the firm would equal $\frac{1}{2}p_k\alpha_j - \frac{1}{2}p_j\alpha_k$, since division j pays an amount $\frac{1}{2}p_j\alpha_k$ and division k receives an amount $\frac{1}{2}p_k\alpha_j$. Obviously, this transfer scheme is not budget balanced. But it can be made budget balanced if there are more than two divisions. Since these costs are independent of the preferences $v_{\alpha_i}^i$, revealed by the other divisions i , the costs $\frac{1}{2}p_k\alpha_j - \frac{1}{2}p_j\alpha_k$ can be allocated to the remaining $n - 2$ divisions, without disturbing the incentive compatibility property. So, the transfer payment scheme τ allocates the costs $\frac{1}{2}p_k\alpha_j - \frac{1}{2}p_j\alpha_k$, for serving divisions j and k in the right order, to the remaining $n - 2$ divisions, which explains the third term in (9). Finally, note that by the incentive compatibility of τ and Theorem 3.3 that τ is Groves' scheme.

Theorem 4.2 Let (N, Π, V) be a public decision making problem arising from a scheduling situation. If $|N| \geq 3$, then the transfer scheme τ defined in (9) satisfies budget balancedness and incentive compatibility.

PROOF: For the proof of incentive compatibility, take $i \in N$, $v_{\alpha_i}^i, v_{\alpha_i}^i \in V^i$ and $v_{\alpha_i}^{-i} \in V^{-i}$. Next, denote with σ the optimal order for preferences $(v_{\alpha_i}^i, v_{\alpha_i}^{-i})$ and with ρ the optimal order for preferences $(v_{\alpha_i}^i, v_{\alpha_i}^{-i})$. We have to show that

$$v_{\alpha_i}^i(\sigma) + \tau_i(v_{\alpha_i}^i, v_{\alpha_{-i}}^{-i}) \geq v_{\alpha_i}^i(\rho) + \tau_i(v_{\alpha_i}^i, v_{\alpha_{-i}}^{-i}).$$

Using (9) and the definition of $v_{\alpha_i}^i$, this is equivalent with showing that

$$\begin{aligned} & - \sum_{j: \sigma(j) < \sigma(i)} p_j \alpha_i + \sum_{j: \sigma(j) < \sigma(i)} \frac{1}{2} p_i \alpha_j - \sum_{j: \sigma(j) > \sigma(i)} \frac{1}{2} p_i \alpha_j \geq \\ & - \sum_{j: \rho(j) < \rho(i)} p_j \alpha_i + \sum_{j: \rho(j) < \rho(i)} \frac{1}{2} p_i \alpha_j - \sum_{j: \rho(j) > \rho(i)} \frac{1}{2} p_i \alpha_j. \end{aligned}$$

Rearranging terms yields

$$\begin{aligned} & - \sum_{j: \substack{\sigma(j) < \sigma(i) \\ \rho(j) > \rho(i)}} p_j \alpha_i + \sum_{j: \substack{\sigma(j) > \sigma(i) \\ \rho(j) < \rho(i)}} p_j \alpha_i + \sum_{j: \substack{\sigma(j) < \sigma(i) \\ \rho(j) > \rho(i)}} \frac{1}{2} p_i \alpha_j - \sum_{j: \substack{\sigma(j) > \sigma(i) \\ \rho(j) < \rho(i)}} \frac{1}{2} p_i \alpha_j \\ & + \sum_{j: \substack{\sigma(j) < \sigma(i) \\ \rho(j) > \rho(i)}} \frac{1}{2} p_i \alpha_j - \sum_{j: \substack{\sigma(j) > \sigma(i) \\ \rho(j) < \rho(i)}} \frac{1}{2} p_i \alpha_j \geq 0. \end{aligned}$$

Rewriting then gives

$$\sum_{j: \substack{\sigma(j) < \sigma(i) \\ \rho(j) > \rho(i)}} (p_i \alpha_j - p_j \alpha_i) + \sum_{j: \substack{\sigma(j) > \sigma(i) \\ \rho(j) < \rho(i)}} (p_j \alpha_i - p_i \alpha_j) \geq 0.$$

This inequality holds, since $\sigma(j) < \sigma(i)$ implies $\alpha_j/p_j \geq \alpha_i/p_i$ and $\sigma(j) > \sigma(i)$ implies $\alpha_j/p_j \leq \alpha_i/p_i$.

To check budget balancedness, take $v_\alpha \in V$. Then

$$\begin{aligned} 2 \sum_{i \in N} \tau_i(\alpha) &= \sum_{i \in N} \sum_{j: \sigma(j) < \sigma(i)} p_i \alpha_j - \sum_{i \in N} \sum_{j: \sigma(j) > \sigma(i)} p_i \alpha_j \\ &\quad - \frac{1}{n-2} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \sum_{k \in N \setminus \{i\}: \sigma(k) > \sigma(j)} (p_k \alpha_j - p_j \alpha_k) \\ &= \sum_{j \in N} \sum_{i: \sigma(i) > \sigma(j)} p_i \alpha_j - \sum_{i \in N} \sum_{j: \sigma(j) > \sigma(i)} p_i \alpha_j - \sum_{j \in N} \sum_{k: \sigma(k) > \sigma(j)} (p_k \alpha_j - p_j \alpha_k) \\ &= \sum_{j \in N} \sum_{i: \sigma(i) > \sigma(j)} p_i \alpha_j - \sum_{j \in N} \sum_{i: \sigma(i) > \sigma(j)} p_j \alpha_i - \sum_{j \in N} \sum_{i: \sigma(i) > \sigma(j)} (p_i \alpha_j - p_j \alpha_i) \\ &= 0, \end{aligned}$$

where the second equality follows from the fact that each term $p_k \alpha_j - p_j \alpha_k$ turns up exactly $n - 2$ times in the summation over the set $\{j, k \in N \setminus \{i\} \mid \sigma_\alpha(k) > \sigma_\alpha(j)\}$. $\square$

Although the transfer scheme τ satisfies incentive compatibility and budget balancedness, there is also a negative aspect. Suppose that there is already an initial serving order σ_0 of the divisions, for example, the order based on the first come first served principle. Further, we assume that each division has the right to be served in the time interval, during which this division is served in the initial order σ_0 . So, each division can claim this right whenever the serving order determined by the firm would yield a higher cost. Then a transfer payments scheme t satisfies the individual rights property if it guarantees that the total costs of each division do not exceed the costs of this same division in the initial order σ_0 . Formally, this means that for each $i \in N$, $v_\alpha \in V$ it holds that

$$v_{\alpha_i}^i(\sigma^*(v_\alpha)) + t_i(v_\alpha) \geq v_{\alpha_i}^i(\sigma_0).$$

In the next example we will show that the transfer payments scheme τ does not satisfy the individual rights property.

Example 4.3 Consider the following problem with three divisions, initial order σ_0 equal to $(1, 2, 3)$ and service times $p_1 = p_2 = p_3 = 1$. Let the revealed and true costs per time unit of division 1, 2 and 3 be $\alpha_1 = 3, \alpha_2 = 1$ and $\alpha_3 = 2$, respectively. Then the optimal processing order $\hat{\sigma}$ is $(1, 3, 2)$ and the transfer payments equal

$$\begin{aligned} \tau_1(v_\alpha) &= -\frac{1}{2}p_1\alpha_3 - \frac{1}{2}p_1\alpha_2 - \frac{1}{2}(p_2\alpha_3 - p_3\alpha_2) = -2 \\ \tau_2(v_\alpha) &= \frac{1}{2}p_2\alpha_1 + \frac{1}{2}p_2\alpha_3 - \frac{1}{2}(p_3\alpha_1 - p_1\alpha_3) = 2 \\ \tau_3(v_\alpha) &= \frac{1}{2}p_3\alpha_1 - \frac{1}{2}p_3\alpha_2 - \frac{1}{2}(p_2\alpha_1 - p_1\alpha_2) = 0. \end{aligned}$$

The total costs of the divisions 1, 2 and 3 then equal -2 , 0 and -3 , respectively. Since the utility of division 1 equals 0 in the initial situation, division 1 is worse off in this new situation.

Theorem 4.4 For a decision making problem (N, Π, V) arising from a scheduling problem, there exists no transfer scheme which satisfies incentive compatibility, budget balancedness and the individual rights property.

PROOF: Let t be a transfer scheme that satisfies incentive compatibility, budget balancedness and the individual rights property. Note that t must be a Groves' scheme. Let $\sigma_0 = (1, 2, \dots, n)$ denote the initial serving order. Take $v_{\alpha_1}^1, v_{\hat{\alpha}_1}^1 \in V^1$, $v_{\alpha_2}^2, v_{\hat{\alpha}_2}^2 \in V^2$ and $v_{\alpha_j}^j \in V^j, j = 3, 4, \dots, n$ such that

$$\alpha_1/p_1 > \alpha_2/p_2 > \hat{\alpha}_1/p_1 > \tilde{\alpha}_2/p_2 > \alpha_3/p_3 > \dots > \alpha_n/p_n.$$

Further, let $v_\alpha = (v_{\alpha_1}^1, v_{\alpha_2}^2, \dots, v_{\alpha_n}^n)$, $v_{\hat{\alpha}} = (v_{\hat{\alpha}_1}^1, v_{\hat{\alpha}_2}^2, \dots, v_{\hat{\alpha}_n}^n)$ and $v_{\tilde{\alpha}} = (v_{\tilde{\alpha}_1}^1, v_{\tilde{\alpha}_2}^2, \dots, v_{\tilde{\alpha}_n}^n)$. Denote with $\sigma, \hat{\sigma}$ and $\tilde{\sigma}$ the optimal processing orders for $v_\alpha, v_{\hat{\alpha}}$ and $v_{\tilde{\alpha}}$, respectively. Note that processing order $\hat{\sigma}$ equals $(2, 1, 3, 4, \dots, n)$ and that processing orders σ and $\tilde{\sigma}$ equal $(1, 2, 3, 4, \dots, n)$. First, consider the preferences v_α . Since σ and σ_0 are the same serving orders, the individual rights property implies $t_i(v_\alpha) \geq 0$ for all $i \in N$. Budget balancedness then gives $t_i(v_\alpha) = 0$ for all $i \in N$. The same result holds for preferences $v_{\hat{\alpha}}$. Next, consider preferences v_α and $v_{\hat{\alpha}}$. Since t is a Groves' scheme, it follows from (3) that

$$t_1(v_\alpha) - \sum_{j \in N \setminus \{1\}} v_{\alpha_j}^j(\sigma) = t_1(v_{\hat{\alpha}}) - \sum_{j \in N \setminus \{1\}} v_{\hat{\alpha}_j}^j(\hat{\sigma}),$$

or, equivalently,

$$t_1(v_\alpha) - v_{\alpha_2}^2(\sigma) = t_1(v_{\hat{\alpha}}) - v_{\hat{\alpha}_2}^2(\hat{\sigma})$$

Substituting $t_1(v_\alpha) = 0$, $v_{\alpha_2}^2(\sigma) = -p_1\alpha_2$ and $v_{\hat{\alpha}_2}^2(\hat{\sigma}) = 0$, gives $0 = t_1(v_{\hat{\alpha}}) - p_1\alpha_2$. We can follow a similar argument for division 2 if we consider the preferences $v_{\hat{\alpha}}$ and $v_{\tilde{\alpha}}$. This gives $0 = t_2(v_{\hat{\alpha}}) + p_2\hat{\alpha}_1$. Finally, the individual rights property implies $t_i(v_{\hat{\alpha}}) \geq 0$ for $j = 3, 4, \dots, n$. Then we have the following contradiction

$$0 = \sum_{i \in N} t_i(v_{\hat{\alpha}}) \geq t_1(v_{\hat{\alpha}}) + t_2(v_{\hat{\alpha}}) = p_1\alpha_2 - p_2\hat{\alpha}_1 > 0. \quad \square$$

5 Concluding remarks

In this paper we have shown that for public decision making problems with a finite number of alternatives and a connected domain of preferences, the only transfer schemes which satisfy incentive compatibility are Groves' schemes. We also showed that for a particular case a Groves scheme exists which is budget balanced. However, such a transfer scheme can never satisfy the individual rights property, as was stated in the last theorem.

As far as the individual rights property is concerned, this property is only relevant when there is an initial situation with which the agents can compare the new situation. For the scheduling situation considered in the Section 4, this was initially not the case. For, in our first description of the model we did not consider any initial serving order. So the incentive compatible and budget balanced transfer scheme τ , introduced in the previous section, is very well suited for this problem. However, the suitability of τ becomes less from the moment we assume the presence of such an initial serving order.

A last remark concerns other classes of problems for which incentive compatible and budget balanced transfer schemes may exist. An interesting question is what conditions the domain of preferences must satisfy for the existence of a budget balanced and incentive compatible transfer scheme. For example, in the scheduling problem considered in Section 4 the linearity of the costs is a crucial assumption. Since linearity implies that the order of two divisions j and k is independent of the preferences revealed by the other divisions. This property can be lost when revelation of monotonically increasing cost functions is allowed.

References

- CURIEL I., PEDERZOLI G. and TIJS S. (1989), Sequencing games, *European Journal of Operations Research*, **40**, 344-351
- GREEN J. and LAFFONT J. (1977), Characterization of satisfactory mechanisms for the revelation of preferences for public goods, *Econometrica*, **45**, 427-438
- GREEN J. and LAFFONT J. (1979), Incentives in public decision making, North Holland, Amsterdam
- GROVES T. (1973), Incentives in teams, *Econometrica*, **41**, 617-633
- HOLMSTRÖM B. (1979), Groves' scheme on restricted domains, *Econometrica*, **47**, 1137-1144
- MOULIN H. (1988), Axioms of cooperative decision making, *Econometric Society monographs*, **15**, 196-221
- SMITH W. (1956), Various optimizers for single stage production, *Naval Research Logistics Quarterly*, **3**, 59-66