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Extinction of the Human Race: Doom-Mongering or Reality?

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Abstract

The question whether or not a given consumption policy is sustainable is closely linked to the way in which nature assimilates pollution and how pollution affects life on earth. In this paper, an assimilation function is specified that is founded on the natural sciences literature. Hereby, the possibility of a break down of the life support system of the earth is explicitly taken into account. It is concluded that an optimal consumption policy need not be sustainable in a physical sense, i.e. the break down of the life support system cannot always be attributed to a market failure. In the model I consider a kind of externality that cannot be linked to any specific variable in either the utility or production function. It means that in some cases a traditional pigouvian tax alone cannot correct for the externality. This has to be done by making use of an extra tax instrument, the overendowment tax.

1 Introduction

The last two decades sustainable development has been one of the major topics in the academic literature (see e.g. Meadows et al. (1972)), Cole et al. (1973) and the World Conservation Strategy (IUCN, 1980)). Nevertheless, there are widely different opinions as to what is precisely meant by the notion of sustainable development (see Pezzey (1989)). The by now best known definition seems to be the one formulated by the Brundtland Commission¹ in their 1987 report. There, sustainable development is defined as *'development that meets the needs of the present without compromising the ability of future generations to meet their own needs.'* Common interpretations of this definition are that consumption and/or the stock of natural capital, which includes natural resources as well as environmental quality, should exhibit a non-decreasing trend over time. Nordhaus (1993) argues that sustainable development is a redundant concept: *'[...]although the concept of sustainable growth has a superficial attractiveness – akin to the adage reminding ourselves not to eat all our capital or foul our nests – it contributes little new to mainstream approaches to intertemporal choice.'* He

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¹The full name of the commission reads: The World Commission on Environment and Development.

shows that the interpretations of sustainable development given above are either inappropriate or suffer from internal inconsistencies. His basic conclusion is that within a (neoclassical) model the optimal consumption path is by definition sustainable, irrespective of the fact if consumption is increasing or decreasing over time. The validity of Nordhaus' conclusion hinges on the fact whether the neoclassical growth model describes reality in an adequate way. If so, sustainability can indeed be called a redundant concept, if not, it might have something to add to the discussion about intertemporal choice.

Now, one of the features where the neoclassical growth model is most prone to critique, is the description of nature's assimilative capacity and the consequences its depletion has. For example, in most models the possibility of a break down of the earth's life support system is not considered. In other words, within the model, the life support system of the earth will stay intact regardless of the pollution level. Not only is the optimal consumption path in a neoclassical model sustainable in the sense that it constitutes "the best" that one can do, but it might also be called Physically Sustainable (PS), since a break down of the life support system will (by definition) not occur along the optimal consumption path.

In neoclassical models the assimilation capacity is usually described by an assimilation function. The question is whether the assimilation functions that we use, describe the assimilation process in an adequate way. In this paper we will analyze what the consequences might be, when the assimilation functions currently used in the economic literature, are replaced by one that is more closer to reality. In order to be able to do this, it is necessary to take a closer look at nature's assimilation process. In section 2 we will derive an assimilation function for the acidification problem founded on the chemical, physical and biological literature on this issue. The implications of including this assimilation function in a rudimentary economic model are being discussed in section 3. More in particular, the properties of PS consumption plans as well as Non Physically Sustainable (NPS) consumption plans will be derived. Since we explicitly consider the possibility of a break down of the life support system, which can be caused by the implementation of a NPS policy, the question arises whether extinction of the human race can be optimal. Whether or not the social planner's optimum derived in section 3, can be replicated in a market economy, is the central question in section 4. Special attention is thereby given to the role that taxes and the interest rate play in this framework. Finally, section 5 contains the major conclusions of the paper and some suggestions for future research.

2 Modelling the environment

In optimal control models the assimilation of pollution is usually described by means of an assimilation function. The two types of functions most frequently used are a linear version (see e.g. Van der Ploeg and Withagen (1991)) and a concave version (see e.g. Forster (1977) and Hediger (1991)). Both functions have been used before by economists. The linear version to describe capital accumulation, the concave version to describe the growth of fish populations (see e.g. Clark (1990)). This familiarity seems to be the main reason for using these functions to model the assimilation of pollution. However, Cesar and De Zeeuw (1994) have shown that even small variations of these function may result in dramatic changes of the steady state values or, more dramatically, the absence of a steady state. An underpinning from the biological literature of this assimilation function thus seems warranted.

For the problem of acidification Aalbers (1993)² derived the following assimilation function

$$B_{t+1} = B_t - \max\{p_t - p_{cl}, 0\}, \quad t = 1, 2, \dots \quad (2.1)$$

where B_t = total buffer capacity at the start of period t ,

p_t = flow of acid at period t ,

p_{cl} = critical load level of pollution.

Although acid is assimilated in many different ways, the underlying principle is the same: each molecule of acid has to be combined with a base (=neutralization). For our purposes it suffices to distinguish two categories of acid neutralizing processes. Those that are renewable and those that are exhaustible. Equation (2.1) tells us that any amount of acid deposited up to the critical load is being neutralized by renewable processes, whereas any pollution in excess of the critical load will be neutralized by exhaustible processes.

As long as the level of pollution stays below the critical load, no harmful effects arise. It is only at pollution levels above the critical load that acidification affects the ecosystem. To list a few: plants and trees will have difficulties in taking up nutrients and aluminium is being released from the soil affecting groundwater quality and poisoning the vegetation. But although the depletion of the buffer is irreversible, the damage arising from this depletion is not irreversible. As long as the buffer has not been depleted below a certain level (in the present model this level is zero), a return to the initial situation remains possible. But whenever the buffer is depleted too far (in our model: $B_t < 0$) the damage becomes irreversible: the ecosystem will stop functioning after a certain period of time. Hence, a sufficient condition for the ecosystem to survive is that the buffer never gets fully depleted ($\forall_t : B_t \geq 0$). This requires that in the stationary state $\max\{p_t - p_{cl}, 0\} = 0$. That is, a necessary condition to keep an ecosystem intact is that the excess pollution must eventually be reduced to zero.

Formally, a feasible path is called Physically Sustainable (PS) if $\forall_t : B_t \geq 0$, and Non Physically Sustainable (NPS) if $\exists_t$ s.t. $B_t < 0$. Hence, on a feasible path the ecosystem will stay intact, whereas on a NPS path this will not be the case.

3 The problem of the social planner

In this section the basic problem of the paper, whether extinction is, or is not optimal, will be dealt with from the point of view of a benevolent dictator. He will take full account of any externality present in the economy, meaning that if extinction is optimal, this cannot be attributed to any market failure. The basic economic model including the above assimilation function is presented and solved in section 3.1. Section 3.2 contains some conclusions.

3.1 A simple model of the economy with environment

Before presenting the model we will take a look at the channels in which pollution, in our case acidification, can effect the economy. Basically, there are three ways. The first channel

²This paper is available from the author on request.

is the amenity effect. Since acidification will deteriorate the forests and consumers care about them, pollution will have a negative effect on utility (see e.g. Vousden (1973), D'Arge and Kogiku (1973), Forster (1977) and Luptacik and Schubert (1982)). The second channel is the productivity effect, i.e. pollution effects production negatively. (see e.g. Kamien and Schwartz (1982), Cesar (1993)). The third channel is that an excess of pollution may put serious strain or even destroy the earth's life support system. A consequence of this might be that production and consumption are no longer possible, since the earth can no longer provide mankind with the essential prerequisites for life itself.

Of course the third channel can only become operative if both the first and the second channel fail to prevent the decline of the life support system. With respect to the productivity effect serious doubts arise. The only sector in the economy that is seriously affected by acidification is forestry. A sector which in most countries is of none or little importance. Whether the amenity effect can prevent a decline of the life support system is doubtful. Arguments in favour and against this proposition are both conceivable. Here, we will assume that the amenity effect is absent³.

Another important feature of pollution is that there exists a considerable delay before the effects of pollution manifest themselves. This has (almost?) been completely ignored in the economic literature. Yet, in reality the delay is often not only substantial⁴, but as we will see in section 3.2, it can change the features of the optimal consumption path quite dramatically. Here, we will assume that exceeding the buffer capacity during period $\mathcal{T} + 1$, will lead to an inevitable decline of the life support system at the end of period $\mathcal{T} + \tau$. So, whenever the consumer chooses to exceed the buffer capacity, he implicitly chooses a finite life span. Time $\mathcal{T} + \tau$ might be called the "extinction time of mankind"⁵. Note that extinction is not unavoidable, since the consumer can choose a consumption plan such that the buffer is never exceeded. Moreover, we will assume that the agent is unconcerned with the fact whether his life span is finite or infinite, i.e. the only value that the agent attaches to his life is the utility he derives from consumption while he lives. In section 3.2 we will however come back to the implications of this assumption. We now turn to the model.

The model used here, is that of a simple one good economy extensively used in the literature (see e.g. Blanchard and Fisher (1989)). A (representative) consumer is assumed to derive utility from consumption only⁶. At the beginning of each period he makes a decision how much to consume (and save) from that period's supply of the composite good. This composite good is assumed to be a perfect substitute with regard to consumption and production. The

³Notice that the results of the analysis will not change if we were to allow for an amenity effect.

⁴Not only does it take decades before the stock of pollutants will have been stabilized, but it will also take decades before the destructive effect of the pollution has taken its toll on the ecosystem. Examples of the first type of delay are the depletion of the ozone layer for which this delay is estimated at 50 years or more (see Van der Woerd and Slaper (1991)) and the greenhouse gas problem for which it is estimated at 30 years (see De Vries et al. (1991)).

⁵An alternative, but less dramatic way to interpret this assumption is that consumers value the decline of the ecosystem as if they would become extinct. In that case the preference structure is lexicographic: consumption only attributes something to welfare if the environment is above some threshold.

⁶The results will not change if the amenity value is included in the utility function. This follows from the fact that marginal utility of nature does not go to infinity, if the buffer approaches zero. For if the buffer would satisfy the Inada conditions ($\lim_{B \rightarrow 0} u_B(c, B) = \infty$), this would have to entail that the ecosystem stops functioning when the buffer is fully depleted. However, if the buffer has been fully depleted the ecosystem will still function, be it somewhat less attractive and productive than before.

consumer maximize welfare not only with respect to consumption, but also with respect to the buffer depletion time, $\mathcal{T}$. There is no depreciation of the capital stock and production takes place according to a production technology, $f(\cdot)$. Furthermore, it is assumed that pollution is generated by consumption only⁷. We abstract from both technological progress in the direction of clean technologies as well as abatement. This means that the value of α , the rate at which consumption generates pollution, must be the same in all periods and that pollution cannot be cleaned up after it has been emitted: the only way to pollute less, is to consume less. Finally, the way in which nature assimilates pollution is described by equation (2.1). This gives us the following model

$$V(c, \mathcal{T}) \equiv \max_{c_t, \mathcal{T}} \sum_{t=1}^{\mathcal{T}} \theta^{t-1} u(c_t) + \theta^{\mathcal{T}} S(k_{\mathcal{T}+1}, \tau), \quad (3.1)$$

$$\text{s.t. } k_{t+1} = k_t + f(k_t) - c_t, \quad t = 1, 2, \dots, \mathcal{T}, \quad (3.2)$$

$$B_{t+1} = B_t - \max\{p_t - p_{cl}, 0\}, \quad t = 1, 2, \dots, \mathcal{T}, \quad (3.3)$$

$$p_t = \alpha c_t, \quad t = 1, 2, \dots, \mathcal{T}, \quad (3.4)$$

$$k_{\mathcal{T}+1} \text{ free, } B_{\mathcal{T}+1} \geq 0, \quad k_1, B_1 \text{ given,} \quad (3.5)$$

where the scrap-value $S(k_{\mathcal{T}+1}, \tau)$ is defined as follows

$$S(k_{\mathcal{T}+1}, \tau) = \begin{cases} \max_{c_t} \sum_{t=\mathcal{T}+1}^{\mathcal{T}+\tau} \theta^{t-\mathcal{T}-1} u(c_t) & \text{if } \mathcal{T} < \infty, \\ \text{s.t. } k_{t+1} = f(k_t) + k_t - c_t, \\ k_{\mathcal{T}+\tau+1}, B_{\mathcal{T}+\tau+1} \text{ free, } B_{\mathcal{T}+2} < 0, k_{\mathcal{T}+1} \text{ given} & \\ 0 & \text{if } \mathcal{T} = \infty. \end{cases}$$

Here c_t and k_t denote respectively consumption and capital formation in period t , k_1 and B_1 are the initial values of the state variables, u is a concave utility function which satisfies the Inada conditions, θ is the discount factor ($0 < \theta < 1$) and $S(\cdot, \cdot)$ is a scrap value function, that gives the utility the agent can obtain after the buffer capacity has been exceeded. For finite $\mathcal{T}$ the consumer can consume and produce for an extra τ periods. For infinite $\mathcal{T}$ the scrap value is zero. Unfortunately, the model above cannot be solved directly by making an appeal to the maximum principle. The reason is that the function $S(k_{\mathcal{T}+1}, \tau)$ has a discontinuity point at $\mathcal{T} = \infty$. However, the model can be solved if it is split up into a Non Physically Sustainable (NPS) problem with a finite time horizon and a Physically Sustainable (PS) problem with an infinite time horizon⁸. Usually, in the literature the term 'sustainability' is used to describe consumption paths that are PS and additionally satisfy certain distributional criteria, e.g. $\forall_t : c_t \geq c_{t-1}$ (see e.g. Pezzey (1989)). However, in this paper we will not be concerned with the distributional aspects of sustainability. A sustainable solution is simply a solution that is PS.

⁷Pollution could also be modelled originating from production. Note that in an one good economy, whatever is produced, must also be consumed. The only difference is that the pollution will occur at a different point in time. In reality, pollution is of course caused by both consumption and production.

⁸From now on the superscript NPS (PS) is used to indicate that a variable belongs to the NPS (PS) case. But in cases where it is clear what is meant the superscript will be omitted.

A priori it is possible to distinguish two solutions in the above model. One in which the steady state level of consumption, c_∞ , lies below the critical load level of consumption, $\frac{p_{cl}}{\alpha}$, and one in which it lies above this level. In the first case the economy is not productive enough in order to destroy the life support system. Effectively, the environment poses no restrictions on the economy. Hence we will ignore this case further on.

3.1.1 Non Physically Sustainable policies

Given that the agent chooses a NPS consumption plan, we get the following Hamiltonian^{9 10}

$$H = u(c_t) + \lambda_{t+1}(f(k_t) + k_t - c_t) + \mu_{t+1}(B_t + p_{cl} - \alpha c_t). \quad (3.6)$$

According to the maximum principle an optimal solution must satisfy the following criteria

$$u'(c_t) - \lambda_{t+1} - \alpha\mu_{t+1} = 0, \quad t = 1, 2, \dots, T^*, \quad (3.7)$$

$$\mu_t = \mu_{t+1}\theta, \quad \lambda_t = \lambda_{t+1}(1 + f'(k_t))\theta, \quad t = 1, 2, \dots, T^*, \quad (3.8)$$

$$\mu_{T^*+1} \geq 0 \quad (= 0, \text{ if } B_{T^*+1} > 0), \quad B_{T^*+2} < 0, \quad \lambda_{T^*+1} = S_k(k_{T^*+1}, \tau), \quad (3.9)$$

$$T^* = \arg \max_{\mathcal{T}} \left\{ \sum_{t=1}^{\mathcal{T}} \theta^{t-1} u(c_t^{NPS}(\mathcal{T})) + \theta^{\mathcal{T}} S(k_{\mathcal{T}+1}^{NPS}, \tau) \right\}. \quad (3.10)$$

(3.7) says that on the margin the utility of consumption at time t should be equal to the benefits of delaying consumption for one period. These benefits not only consist of increased consumption possibilities with respect to the expansion of the capital stock, but also with respect to the expansion (= non-depletion) of the buffer. (3.8) displays the usual arbitrage conditions. Notice that since the environment is not productive, its shadow price increases at an exponential rate, θ . The transversality condition for the capital stock (3.9) says that the shadow price of capital at the time the buffer is crossed, should be equal to the marginal product of the scrap value (= future utility) with respect to capital.

The optimal buffer depletion time, T^* , is given by condition (3.10). Furthermore, at the begin of period $T^* + 1$ the buffer can not be overdepleted yet, i.e. $B_{T^*+1} \geq 0$, whereas it must be overdepleted at the start of the subsequent period, i.e. $B_{T^*+2} < 0$. For simplicity we will assume that none of the environmental constraints are binding at the optimum. The solution to the NPS problem is an internal solution, i.e. $\forall_t : \mu_t = 0$.

3.1.2 Physically Sustainable policies

The objective function for the PS problem is split up into two parts. During the first time period, $t \in \{1, 2, \dots, S\}$, consumption lies above its critical load level, while during the second part, $t \in \{S + 1, S + 2, \dots\}$, consumption is equal to its critical load level. It then reads

⁹Notice that the agent will always pollute at least up to the critical load level. This follows from non-satiation. Hence it is possible to drop the max expression in 3.3.

¹⁰Here λ_t and μ_t denote respectively the shadow price of capital and the environment.

$$\max_{c_t, S} \sum_{t=1}^S \theta^{t-1} u(c_t) + \frac{\theta^S}{1-\theta} u\left(\frac{p_{cl}}{\alpha}\right). \quad (3.11)$$

That is, next to the optimal consumption path, the optimal depletion time, S , of the buffer has to be determined. Furthermore let $\bar{k}$ denote the minimum capital stock that suffices to produce the critical load level of consumption, i.e. $f(\bar{k}) = \frac{p_{cl}}{\alpha}$. The Hamiltonian for this problem is the same as the one of the NPS problem¹¹. According to the maximum principle an optimal solution must satisfy the following conditions

$$u'(c_t) - \lambda_{t+1} - \alpha\mu_{t+1} = 0, \quad t = 1, 2, \dots, T, \quad (3.12)$$

$$\mu_t = \mu_{t+1}\theta, \quad \lambda_t = \lambda_{t+1}(1 + f'(k_t))\theta, \quad t = 1, 2, \dots, T, \quad (3.13)$$

$$\mu_{T+1} \text{ free}, \quad \lambda_{T+1} \geq 0, \quad (= 0, \text{ if } k_{T+1} > \bar{k}), \quad (3.14)$$

$$T = \arg \max_S \left\{ \sum_{t=1}^S \theta^{t-1} u(c_t^{PS}(S)) + \frac{\theta^S}{1-\theta} u\left(\frac{p_{cl}}{\alpha}\right) \right\}. \quad (3.15)$$

Two cases can be distinguished. Either the environment is the limiting production factor ($\lambda_{T+1} = 0$) or both the environment as well as the production capacity are limiting production factors ($\lambda_{T+1} > 0$)¹². In the first case consumption is a decreasing function over time as long as the buffer has not been fully depleted. This can be seen from (3.12) - (3.14), which gives $\theta u'(c_t) = u'(c_{t-1})$, i.e. the marginal rate of substitution equals one. Since nature's assimilative capacity cannot be enlarged by the agent, he has no other option as to equalize the marginal rate of substitution to the marginal rate of transformation of the environment, which is one. A few remarks are in order. First, the economy behaves, as if it were an exchange economy. Capital formation becomes a tool for shifting consumption between periods instead of a tool for expanding consumption possibilities, i.e. there exists an overendowment. Second, the marginal rate of capital in such an economy must be equal to zero. For if an extra amount, however small, is invested, there will be an increase in both consumption and pollution. Since nature's assimilative capacity has already been fully depleted, this will mean a break down of the life support system, which renders the extra investment worthless.

In the second case the equation of motion of consumption that can be derived from (3.12)-(3.13) equals the one of the NPS case. But the actual path is different since the transversality conditions of the two cases are not the same. The equation of motion is

$$\theta u'(c_t)(1 + f'(k_t)) = u'(c_{t-1}) + \alpha\mu_t f'(k_t), \quad (3.16)$$

which says that along the optimal path consumption in period t and $t-1$ should be such that the discounted marginal utility of consumption in period t times the rate of transformation equals the marginal utility of consumption in period $t-1$ *plus* the marginal burden that substituting consumption between period t and $t-1$ gives. This burden arises, because on the one hand

¹¹But remember that the transversality conditions of the NPS problem are different.

¹²Remember that the case in which the productive capacity is the only limiting production factor has been excluded from the analysis.

substituting consumption enlarges the consumption possibilities, while on the other hand they are limited by nature's exhaustible assimilative capacity.

The following propositions shed some light on the characteristics of the optimal path and the optimal buffer depletion time

Proposition 1 *As long as the buffer has not been fully depleted ($B_t > 0$), it holds that once consumption starts decreasing over time, it will stay decreasing over time, i.e. if $c_t < c_{t-1}$ then $\forall_{s>t} : c_{s+1} < c_s$.*

This means that when $\lambda_{T+1} = 0$ consumption will be constantly decreasing, whereas when $\lambda_{T+1} > 0$ the trajectory will be single peaked. In the first case the proof is straightforward and will be omitted here. In the second case, suppose that $c_t < c_{t-1}$ and $c_{t+1} > c_t$. From (3.16) evaluated at period t and $t + 1$ and (3.13), it is possible to derive that

$$\theta u'(c_t)(f'(k_t) - f'(k_{t+1})) < \alpha \mu_{t+1}(f'(k_t) - f'(k_{t+1})). \quad (3.17)$$

First assume that $f'(k_t) - f'(k_{t+1}) > 0$, i.e. $\theta u'(c_t) < \alpha \mu_{t+1}$. This gives

$$\theta u'(c_{t+1})(1 + f'(k_{t+1})) = u'(c_t) + \alpha \mu_{t+1} f'(k_{t+1}) > \theta u'(c_{t+1})(1 + f'(k_{t+1})), \quad (3.18)$$

which is a contradiction. Second, assume that $f'(k_t) - f'(k_{t+1}) < 0$, i.e. $\theta u'(c_t) > \alpha \mu_{t+1}$. Then

$$\begin{aligned} u'(c_{t+1}) - u'(c_t) &= (u'(c_t) - u'(c_{t+1}))f'(k_{t+1}) + \theta^{-1}(u'(c_t) - u'(c_{t-1})) + \\ &\quad u'(c_t)(f'(k_t) - f'(k_{t+1})) + \alpha \theta^{-1}(\mu_{t+1}f'(k_{t+1}) - \mu_t f'(k_t)) > \\ &\quad (u'(c_t) - u'(c_{t+1}))f'(k_{t+1}) + \theta^{-1}(u'(c_t) - u'(c_{t-1})) + \\ &\quad \alpha(1 - \theta)\theta^{-1}\mu_{t+1}f'(k_t) > 0 \end{aligned}$$

again leading to a contradiction.

Proposition 2 *The optimal buffer depletion time, T , will be finite.*

In both cases the proof is an indirect one. First, consider the case where $\lambda_{T+1} > 0$. Suppose that T were infinity. Now, $c_t \rightarrow \frac{p_{cl}}{\alpha}$ and $k_t \rightarrow f^{-1}(\frac{p_{cl}}{\alpha})$ as $t \rightarrow \infty$. However, this leads to a contradiction, since $\mu_t \rightarrow \infty$ as $t \rightarrow \infty$. Thus (3.16) cannot hold for all t , meaning that T must be finite. Second, consider $\lambda_{T+1} = 0$. Again suppose that T equals infinity. Rewriting the first order conditions we get

$$c_{t+1} = (u')^{-1}(\theta^{-t} u'(c_1)), \quad (3.19)$$

which means that along the optimal path consumption will go to zero as t goes to infinity. But this is clearly not optimal since the consumer could have chosen to consume a higher level of consumption (still below the critical load level, $\frac{p_{cl}}{\alpha}$), since capital is present in abundance. This contradicts the fact that the policy given by (3.19) is optimal. Hence T must be finite.

3.2 Comparing the candidate optimal solutions

In figure 1 possible trajectories of the optimal PS consumption path and the optimal NPS consumption path have been drawn. Because of the discontinuity at $\mathcal{T} = \infty$ of the scrap value function, it has not been possible to solve (3.1) - (3.5) integrally. The following proposition does, however, shed some light on the overall optimum consumption plan

Proposition 3 *The optimal consumption path is*

- (i) PS, if $S(k_{T^*+1}, \tau) < \frac{1}{1-\theta} u(\frac{p_{cl}}{\alpha})$,
- (ii) NPS, if $S(k_{T+1}, \tau) > \frac{1}{1-\theta} u(\frac{p_{cl}}{\alpha})$,

where T^* is the optimal NPS buffer depletion time and T is the optimal PS depletion time. In case $T \neq T^*$ proposition 3 gives only a partial solution to problem (3.1) - (3.5).

The proof of part (i) goes as follows. Suppose the optimal path were NPS. Then $V(c^{NPS}, T^*) > V(c^{PS}, \infty)$. Consider the following consumption path: $\bar{c}_t = c_t^{NPS}$, $t = 1, 2, \dots, T^*$ and $\bar{c}_t = \frac{p_{cl}}{\alpha}$, $t = T^* + 1, T^* + 2, \dots$. Then by (i), $V(\bar{c}, \infty) > V(c^{NPS}, T^*)$. But $\bar{c}_t$ is a PS consumption path. Hence $V(c^{PS}, \infty) > V(\bar{c}, \infty) > V(c^{NPS}, T^*)$. Contradiction. The proof of part (ii) is based on a similar argument. Suppose the optimal path were PS. Then $V(c^{PS}, \infty) > V(c^{NPS}, T^*)$. Construct the following path: $\tilde{c}_t = c_t^{PS}$, $t = 1, 2, \dots, T$ and $\tilde{c}_t$, $t = T + 1, T + 2, \dots, T + \tau$, where the latter is the optimal consumption path of the $S(k_{T+1}, \tau)$ problem. Then by (ii), $V(\tilde{c}, T) > V(c^{PS}, \infty)$. But $\tilde{c}$ is a NPS path, hence $V(c^{NPS}, T^*) > V(\tilde{c}, T) > V(c^{PS}, \infty)$. Contradiction.

The intuition is clear. If at time T^* at the optimal NPS path, the remaining discounted utility from the NPS policy is less than the remaining discounted utility of the PS policy, the NPS policy cannot be optimal. On the other hand, if at time T at the optimal PS path, the remaining discounted utility of the NPS option is larger than the remaining discounted utility of the PS path, the PS path cannot be optimal.

So preferences (and also the production possibility set) do play an important role in the sense that they can have a decisive impact on the shape (PS or NPS) of the economy's optimal consumption path. More specifically, in cases where the NPS consumption path is optimal, the destruction of the life support system cannot be contributed to a market failure! This characteristic is not to be found in the literature, where the PS consumption path is always optimal (see e.g. Nordhaus (1993)). Coming to this point the attentive reader might say: "Surely extinction can be optimal here, since there is no allowance of either an amenity value in the utility function or caring about life itself¹³." However, including such variables in the utility function does not necessarily imply that extinction can be excluded as an option, *unless* the agent attaches an infinite amount of welfare to his own life. But why is it then that extinction can be optimal in present model and is never optimal in the models we encounter in the literature? The reason is that at least one of the following assumptions is made

- $\tau = 0$. Pollution leads instantaneously to an increase in the stock of pollution, which in turn instantaneously affects social welfare.

¹³This could be modeled by including the agent's life span as a variable in the utility function, i.e. $u(c_t, \mathcal{T})$.

Figure 1: Examples of NPS and PS consumption (pollution) paths.

- There are no thresholds in the model: every level of emissions is sustainable, that is there exists no level of emissions at which the life support system of the earth can break down.

By making the first assumption, a choice for extinction will never be optimal. From proposition 3 and $S(k_{T^*+1}, 0) = 0$ it follows immediately that the PS consumption plan is optimal. The second assumption excludes extinction by definition. So, either of these two assumptions will make the agent *a priori* prefer the optimal PS consumption plan over any NPS consumption plan.

Finally, the propositions below elucidate the role of the delay, τ , and the rate of time preference, θ , play in the question whether the optimal policy is PS or NPS.

Proposition 4 *Consider a society with given tastes and production opportunities. Then there exists delays, $\underline{\tau}$ and $\bar{\tau}$ ($0 < \underline{\tau} \leq \bar{\tau}$) such that*

- (i) *for $\tau < \underline{\tau}$ the optimal policy is PS,*
- (ii) *for $\tau > \bar{\tau}$ the optimal policy is NPS.*

This follows from $S(k_{T^*+1}, 0) < \frac{1}{1-\theta}u(\frac{p_{cl}}{\alpha})$, $S(k_{T+1}, \infty) > \frac{1}{1-\theta}u(\frac{p_{cl}}{\alpha})$, and $\frac{\partial S(\cdot, \tau)}{\partial \tau} > 0$.

Proposition 5 *If a society cares "sufficiently" for the future (in the sense that its rate of time preference, θ , is large enough), its optimal consumption plan will always be PS.*

This follows from the fact that $\lim_{\theta \rightarrow 1} V(c^{NPS}, T^*) < \infty$ and $\lim_{\theta \rightarrow 1} V(c^{PS}, \infty) = \infty$ ¹⁴. Table 1 contains some simulation results. Total discounted utility of both NPS and PS plans

¹⁴This is so, because by definition of a NPS plan T^* is finite, and $\lim_{t \rightarrow \infty} u(c_t^{PS}) > 0$

θ	$V(c, T)$			
	NPS			PS
	$\tau = 25$	$\tau = 50$	$\tau = 100$	
0.91	104.1	105.7	105.9	102.1
0.92	115.7	118.3	118.7	113.4
0.93	130.3	134.5	135.3	127.9
0.94	148.5	155.9	157.9	146.9
0.95	172.7	185.2	190.3	173.0
0.96	205.2	227.9	240.3	211.5
0.97	249.8	293.0	325.3	274.0
0.98	315.8	400.1	488.4	395.3
0.99	429.7	595.7	859.2	743.2

Table 1: Simulation results

were calculated for a number of parameter sets¹⁵. A few things are worth remarking. First, for a sufficiently large delay, τ , the optimal consumption plan is NPS. Second, for the rate of time preference, θ , sufficiently large, the optimal consumption plan will be PS¹⁶. In case $\tau = 100$ this entails that $\theta > 0.99$. Notice that these results are in line with propositions 4 and 5.

4 The market economy and environmental constraints

In the previous section we have described the command optimum of the economy given by (3.1) - (3.5). In this section attention is focused on the question how these optimal solutions (PS or NPS) can be replicated in a market economy. More specifically, the origins of the environmental externality will be explored, whereafter the role that taxation can play in correcting for these externalities is discussed.

4.1 The decentralized version

In the economic literature it has been common to associate the concept of externality with one or more real variables in either the utility or production function of an individual. According to Baumol and Oates (1988, p. 17) an externality is said to be present, "whenever some individual's (say A's) utility or production function include real (that is, nonmonetary) variables, whose values are chosen by others (persons, corporations, governments) without giving particular attention to the effects on A's welfare". In our model agents do not care for the environment,

¹⁵The following functional forms and parameters were used: $f(k) = 0.11k$, $u(c) = \ln c$, $\alpha = 0.001$, $B_1 = 250$, $p_{cl} = 1$. In the NPS case it was assumed that none of the environmental constraints were binding. Hence, the reported discounted utility may underestimate the 'true' discounted utility. Furthermore the initial capital stock was chosen so large that $\lambda_{T+1} = 0$.

¹⁶Strictly speaking, this can not be deduced from table 1. Since the initial capital stock for which $\lambda_{T+1} = 0$ varies, as θ varies, it is not correct to compare the NPS figures for different values of θ . However, the figures presented here are in line with the true figures.

neither is there any effect of pollution on production. The externality can thus not be related to any specific variable in the utility or production function. Nevertheless an externality is present, for if individual A would like to choose the PS consumption plan, whereas all the others are choosing the NPS consumption plan, his welfare is certainly being effected. The point here is that whenever the others are choosing the NPS consumption plan, it is impossible for individual A to choose an infinite lifetime, since his own influence on the total amount of pollution is negligible. The actions of all other individuals are thus affecting the welfare of individual A, since it will be impossible for him to consume after the breakdown of the earth's life support system.

The economy under consideration is made up out of N identical consumers and producers, where N is large. The individual consumer realizes that his own influence on the total amount of pollution is negligible and consequently, he does not take account of the environment. Consumers maximize life-time utility according to (3.1) subject to the budget constraint, $c_t + s_t = (1 + r_t)s_{t-1} + w_t$, where they take the interest rate, r_t , previous period savings, s_{t-1} and the wage rate, w_t , as given¹⁷. Labour is supplied inelastically. At the beginning of each period it is decided how much to save and to consume. At the optimum consumers equalize the marginal rate of substitution to the level of the discount factor, i.e. $\theta \frac{u'(c_t)}{u'(c_{t-1})} = \frac{1}{1+r_t}$.

On the production side producers rent capital, k_t , and one unit of labour against their respective market prices. Profits per unit of labour are maximized, i.e. $\max_{k_t} \pi_t = f(k_t) - w_t - r_t k_t$. At the optimum the marginal rate of capital is equal to the interest rate, i.e. $r_t = f'(k_t)$. Since perfect competition entails zero profits, the equilibrium wages are given by $w_t = f(k_t) - r_t k_t$. Finally, in equilibrium $s_t = k_{t+1}$.

4.2 The role of taxes

Since consumers do not take the environment into account when maximizing life-time utility, they will equalize the marginal rate of substitution to the discount factor. Thus there will be a moment in time when nature's assimilative capacity is fully depleted, leading to a breakdown of the life-support system τ periods later; the market, if left on its own, will select the NPS outcome. From the previous section it is clear that it is impossible to say which of the two candidate optima, the NPS or the PS, gives the highest welfare. Therefore, we will consider each possibility in turn.

4.2.1 The NPS consumption plan is optimal

Suppose for the moment that the overall optimal consumption plan is NPS. Since we assumed that we are dealing with an internal solution (see section 3.1.1), the social planner's first order conditions are identical with the first order conditions of the market economy. This means that there is no room for a government to interfere by imposing some kind of tax scheme in order to make agents choose a PS consumption plan: agents prefer to have a finite life span and the concomitant consumption. Extinction is optimal.

¹⁷Notice that initial savings including the return on this savings plus wage income per consumer is equal to initial endowment in the social planner's economy, i.e. $(1 + r_1)s_0 + w_1 = e_1$. Here s_0 denotes initial savings, which are given.

4.2.2 The PS consumption plan is optimal

On the other hand if the overall optimal consumption plan is PS, there is room for a government to interfere. For the market will select the NPS outcome, something which is clearly not optimal. The usual way in which externalities are internalized in a market economy is by means of levying a pigouvian tax on the externality generating good (see e.g. Baumol and Oates ch. 4 (1988)), where the level of the pigouvian tax is equal to the shadow price of the environment. The proceeds of this pigouvian tax are then returned to the consumer by means of a lump-sum subsidy. Again we have to consider two cases. At first we will consider the case where the environment is the only limiting production factor, that is, there exists an overendowment ($\lambda_{T+1} = 0$). After that we will turn to the case where both the environment and capital are limiting production factors ($\lambda_{T+1} > 0$). Consider the following proposition

Proposition 6 *Let $\lambda_{T+1} = 0$. In order to replicate the command optimum in the market economy, a government can levy the following taxes (subsidies):*

(i) *A non-unique pigouvian tax on consumption, ϕ_t , having the following properties*

ϕ_1 free, ($\phi_1 \neq -1$), $\phi_t = \beta_t(1 + \phi_{t-1})(1 + r_t) - 1$, $t = 2, 3, \dots$, where

$$\beta_t = \begin{cases} 1 & , t = 2, 3, \dots, T \\ \theta \frac{u'(p_{cl}/\alpha)}{\alpha \mu_{t+1}} & , t = T + 1 \\ \theta & , t = T + 2, T + 3, \dots \end{cases}$$

(ii) *A lump-sum subsidy, $\Phi_t = \phi_t c_t^{PS}$, $t = 2, 3, \dots$*

(iii) *An overendowment tax, $\Phi^{over} = f(k_1) + k_1 - c_1^{PS} - k_2^{PS} > 0$, which is to be levied in the first period only and is not returned to the consumer.*

The most prominent difference as compared to the literature is the presence of the overendowment tax. Another remarkable aspect is that the pigouvian tax is not uniquely defined and can even be negative!

The proposition can be derived as follows. Equate the first order condition of the command optimum to that of the market economy. The consumer's budget constraint when the government levies the taxes mentioned above looks like $(1 + \phi_t)c_t + s_t = (1 + r_t)s_{t-1} + w_t + \Phi_t - \Phi^{over}\mathcal{I}(t = 1)$ ¹⁸. The consumer's first order condition then becomes

$$\theta \frac{u'(c_t)}{u'(c_{t-1})} = \frac{1 + \phi_t}{(1 + \phi_{t-1})(1 + r_t)} = \beta_t, \quad t = 2, 3, \dots$$

which gives the first part of the proposition.

When $\lambda_{T+1} = 0$, capital is present in abundance in the PS economy, i.e. $\forall_t : c_t^{PS} + k_{t+1}^{PS} < f(k_t^{PS}) + k_t^{PS}$. However, this cannot be the case in the market economy, even if the government were levying a pigouvian tax. For the market economy will use all available capital, meaning that the resulting outcome will be NPS. Only in the case the government taxes the so-called

¹⁸Here $\mathcal{I}(\cdot)$ is an indicator function. It is equal to one when $t = 1$ and equal to zero when $t \neq 1$.

overendowment away (and does not return the proceeds to the consumer), can the command optimum be replicated in the market economy. Therefore, we set $\Phi^{over} = f(k_1) + k_1 - c_1^{PS} - k_2^{PS}$, that is, the overendowment tax is equal to that part of initial wealth which is superfluous. From the consumer's budget constraint, labour and capital market equilibrium it then follows that $\Phi_t = \phi_t c_t^{PS}$. To summarize, if capital is present in abundance, the government has to tax the overendowment away in the first period and, additionally, it has to levy a set of pigouvian taxes, which are returned in a lump-sum way to the consumer¹⁹.

The non-uniqueness of the pigouvian tax can be explained by taking a look at the effects a pigouvian tax has in a "traditional" model. There, it causes two effects. First, it will change the relative price between consumption and pollution within each period. As a reaction to this, the consumer will change the composition of the consumption bundle in every period (intratemporal). Second, if the current value pigouvian tax changes over time, this will result in substitution of consumption and pollution over time (intertemporal). Since the consumer derives positive utility from consumption and negative utility from pollution, there exists a unique pigouvian tax, that equates the marginal utility of consumption to the (negative of the) marginal disutility of pollution. However, in the model under consideration in this paper, pollution does not cause effects within the periods. It only imposes some restrictions on consumption over the whole planning horizon. Consequently, the pigouvian tax is not uniquely defined.

A nice parallel can be drawn to the case where an ordinary exhaustible resource, e.g. oil, is being extracted. In that case the Hotelling rule tells us that the optimal price of this resource should be exponentially increasing over time (see e.g. Pearce and Turner (1990)). Since in the present model nature is essentially modelled as an exhaustible resource, it need not surprise us that the tax on consumption shows this same property. However, in the case of an ordinary exhaustible resource the price path is uniquely determined. This is due to the fact that either there exists a substitute for the resource (the backstop technology) whose price is known, or the fact that the demand curve has the property that demand falls to zero for a finite price. Both requirements are however not met for the present model. One, there does not exist a substitute for the life support system of the earth. Two, demand does not fall to zero whatever the price of the consumption good may be. The latter effect occurs, since the revenues of the pigouvian tax are fully returned to the consumer, which means that the demand curve will not be affected by the pigouvian tax.

Finally, we turn to the case where both the environment as well as the capital stock are the limiting production factors ($\lambda_{T+1} > 0$). We have the following proposition

Proposition 7 *Let $\lambda_{T+1} > 0$. In order to replicate the command optimum in the market economy, a government can levy:*

(i) *A non unique pigouvian tax on consumption having following properties*

$$\phi_1 \text{ free, } (\phi_1 \neq -1), \phi_t = \gamma_t(1 + \phi_{t-1})(1 + r_t) - 1, \quad t = 2, 3, \dots, \text{ where}$$

¹⁹The not uniquely determined pigouvian taxes in proposition 6 can be replaced by a uniquely determined tax on capital income. The consumer's budget constraint in this case is $c_t + s_t = (1 + (1 - \phi_{r_t})r_t)s_{t-1} + w_t + \Phi_t - \Phi^{over}\mathcal{I}(t = 1)$. Here Φ^{over} is defined as in proposition 6 and $\Phi_t = \phi_t r_t k_t^{PS}$.

$$\gamma_t = \begin{cases} \frac{\alpha\mu_t/\lambda_t(1+r_t)+1}{\alpha\mu_t/\lambda_t(1+r_t)+1+r_t} & , t = 2, 3, \dots, T \\ \theta \frac{u'(p_{cl}/\alpha)}{\lambda_{t+1}+\alpha\mu_{t+1}} & , t = T+1 \\ \theta & , t = T+2, T+3, \dots \end{cases}$$

(ii) *An uniquely determined capital income tax defined as follows*

$$\phi_{r_t} = \frac{\gamma_t(1+r_t)-1}{\gamma_t r_t}.$$

The proceeds are again returned to the consumer by means of a lump-sum subsidy, Φ_t .

This can be derived in a way analog to proposition 6. The remarks concerning that proposition apply here also.

5 Conclusion and suggestions for further research

Whether or not sustainable development is an inherent goal of society or not, has been the central question of this paper. Within neoclassical theory this question is closely linked to the exact specification of the assimilation function. On the basis of the natural sciences literature, an assimilation function for the acidification problem was derived. It appeared that two important features of nature's assimilation process have been overlooked by the specification of the assimilation functions so far. First, the amount of pollution that nature can assimilate in a finite time period is finite, and second, environmental effects occur with a delay of at least several decades, or perhaps even one or two centuries.

These features of the assimilation function lead to the conclusion that sustainability does not have to be an inherent goal of society: only when society cares enough for the future, e.g. has a rate of time preference that is high enough, is sustainability an inherent goal. Preferences thus (among other things) determine the features (PS or NPS) of the economy's consumption path, meaning that extinction can be an optimal policy, i.e. it cannot be attributed to any market failure. Another factor, that influences the decision whether or not sustainability is preferred by society, is the length of the delay that exists between exceeding the buffer and the break down of the life support system. The larger this delay, and the larger the forgone consumption possibilities are when choosing the PS path, the sooner a society will choose a NPS policy. Finally, we saw that whenever the PS consumption path is optimal and there exists an overendowment, the economy will have the characteristics of an exchange economy. Production is no longer primarily a tool for expanding consumption possibilities, but mainly a tool to shift consumption over time.

If society prefers the NPS option, the government has no reason to interfere in the economy. However, if society prefers the PS option, the government has a reason to interfere, since the market will always select the NPS option. The government can do this by levying the following taxes: a pigouvian tax on consumption (or equivalently a capital income tax), a lump sum subsidy in size equal to the proceeds of the pigouvian (or capital income) tax and, if necessary, an overendowment tax. The first two taxes are known from the literature on environmental economics, but the last one is not. Yet in case there exists an overendowment,

it is an essential element of the government's tax policy: it makes sure that the consumer does not use the superfluous wealth, he owns, for consumption purposes.

A point open to debate is how sensitive the conclusions are for some of the assumptions that have been made in this paper. What would happen if the consumer could spend part of his income on abatement? Does the government in that case still need to levy an overendowment tax? Another important question that remains to be answered is, how to deal with uncertainty. A large part of the features of nature's assimilative capacity are not known with certainty or are not known at all. How is sustainable development defined in an uncertain world? And in what way could uncertainty influence consumer decisions? These and other questions need to be answered, before any policy conclusions can be drawn.

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