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INFORMATION SHARING GAMES

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In this paper we study information sharing situations. For every information sharing situation we construct an associated cooperative game, which we call an information sharing game. We show that the class of information sharing games coincides with the class of cooperative games with a population monotonic allocation scheme.

Keywords: Information sharing; cooperative game; PMAS.

1. Introduction

In this paper we combine two fields of research in game theory, knowledge and cooperative decision making. Knowledge is one of the areas in *non-cooperative* decision making that has received a considerable amount of attention recently. However, less attention has been paid to the relation between knowledge and *cooperative* decision making. In this paper we will associate cooperative games with situations that deal with information sharing. Our main result is that the resulting cooperative games exhaust the class of cooperative games with a population monotonic allocation scheme (cf. Sprumont (1990)).

The work of Aumann (1999a,b) has initiated a considerable amount of research dealing with knowledge and information. Here, we follow Aumann (1999a) by assuming that players do not have perfect information on the true state of the world. Additionally, we assume that players have to make decisions, where the outcome of their decisions does depend on the true state of the world. Sharing information between different players who have to make decisions and who have different information might increase joint (expected) profits. Such an information sharing situation naturally results in a cooperative game with transferable utilities. The resulting game will be called an information sharing game.

The approach taken in this paper falls under the general approach of associating cooperative games with specific (economic) situations. Such an approach

was also taken by Shapley and Shubik (1969). They studied the class of market games, i.e. games that were derived from an exchange economy. Their main result states that the class of market games coincides with the class of totally balanced games. Many similar and related results have been derived since. Kalai and Zemel (1982b) showed such a relation between flow games and non-negative totally balanced games. Subsequently, in Kalai and Zemel (1982a) they studied several generalized network problems that yield totally balanced games. Non-negative balanced games were shown to be exhausted by flow games with committee control in Curiel *et al.* (1989). In van den Nouweland *et al.* (1993) it is shown that monotonic cooperative games are spanning network games. Finally, we mention economies with land. Legut *et al.* (1994) showed that every cooperative game that results from an economy with land has a population monotonic allocation scheme. Reijnierse (1995) showed that a subclass of these games, in which initial endowments of the players are disjoint parcels of land with veto control, exhaust the class of games with a population monotonic allocation scheme. The current paper is in the same style as these papers, dealing with information sharing games and cooperative games with a population monotonic allocation scheme.

The set-up of this paper is as follows. Section 2 deals with preliminaries regarding cooperative games, population monotonic allocation schemes, and information partitions. In Sec. 3 we introduce information sharing situations and associate a cooperative game with each information sharing situation, which we call an information sharing game. The main result, which states that the class of cooperative games with a population monotonic allocation scheme coincides with the class of information sharing games, can be found in Sec. 4. We conclude in Sec. 5 with some remarks on assumptions that are made in this paper and on possible generalizations to a setting without these assumptions.

2. Preliminaries

A cooperative game with transferable utility (TU-game) is a pair (N, v) where $N = \{1, \dots, n\}$ denotes the set of players and v is a real-valued function on the family 2^N of all subsets of N with $v(\emptyset) = 0$. The function v is called the *characteristic function* of the cooperative game (N, v) .

Sprumont (1990) introduced population monotonic allocation schemes (PMAS): a vector $(y_{i,S})_{i \in S, S \subseteq N}$ is a *population monotonic allocation scheme* for the cooperative game (N, v) if it satisfies the following conditions:

- (a) $\sum_{i \in S} y_{i,S} = v(S)$ for all $S \subseteq N$.
- (b) $y_{i,S} \leq y_{i,T}$ for all $S, T \subseteq N$ with $S \subseteq T$ and all $i \in S$.

Knowledge of a player can be modeled in several ways. Aumann (1999a) presents and discusses five equivalent formalizations of the idea of knowledge:

signal functions, information functions, information partitions, knowledge operators, and knowledge ufields (universal fields). Here we will concentrate on information partitions. The set of all possible states of the world will be denoted by Ω . For notational convenience and clarity of exposition we will assume throughout this paper that $|\Omega| < \infty$.^a For example, if the roll of a dice is studied the set of states of the world is the set of possible results of rolling a dice, $\Omega = \{1, 2, 3, 4, 5, 6\}$.

An information partition is a partition of Ω . A player with some information partition cannot distinguish between the states of the world that belong to the same partition element. For example if a player will be told that the result of rolling a dice is odd or even, his information partition is $\{\{1, 3, 5\}, \{2, 4, 6\}\}$. So, he cannot distinguish between states 1, 3 and 5, nor can he distinguish between states 2, 4 and 6. Denote the set of all partitions of Ω by Π^Ω . For all information partitions $I \in \Pi^\Omega$ and all $\omega \in \Omega$ we denote the element of I containing ω by $P(\omega, I)$. For every two information partitions, say I and I^* , we call I a *refinement* of I^* if every element of I is a subset of an element of I^* . If I is a refinement of I^* then we call I^* a *coarsening* of I .

Consider two players, say 1 and 2, with information partitions $I_1, I_2 \in \Pi^\Omega$, respectively. Then, if ω^* is the true state of the world, player 1 knows that the true state of the world is an element of $P(\omega^*, I_1)$, whereas player 2 knows that the true state of the world is an element of $P(\omega^*, I_2)$. So, if they share their information they know that the true state of the world belongs to $P(\omega^*, I_1) \cap P(\omega^*, I_2)$.

The joint information partition of these two players, if they share their information, is the coarsest partition of Ω that is a refinement of both I_1 and I_2 :

$$I_1 \vee I_2 = \{P_1 \cap P_2 \mid P_1 \in I_1, P_2 \in I_2, P_1 \cap P_2 \neq \emptyset\}.$$

The partition $I_1 \vee I_2$ is called the *join* of I_1 and I_2 . For a set of players S with information partitions $(I_i)_{i \in S}$ the join of their information partitions is defined similarly:

$$\bigvee_{i \in S} I_i = \{\bigcap_{i \in S} P_i \mid P_i \in I_i \text{ for all } i \in S, \bigcap_{i \in S} P_i \neq \emptyset\}. \quad (2.1)$$

We assume that every player has some prior probability measure on the set of possible states of the world. The prior of player i is denoted by π_i . Note that $\pi_i = \pi$ for all $i \in N$ corresponds to a situation with a common prior. Furthermore, we will assume that every player thinks that every state occurs with positive probability. So, $\pi_i(\omega) > 0$ for all $\omega \in \Omega$ and all $i \in N$. The probability that player i assigns to an event $A \subseteq \Omega$ is denoted by $P^{\pi_i}(A) = \sum_{\omega \in A} \pi_i(\omega)$. Of course, $P^{\pi_i}(\Omega) = 1$. Once player i knows that the true state of the world is an element of $P_i \in I_i$ he can deduce the posterior probabilities from prior probabilities by Bayesian updating:

$$P^{\pi_i}(\omega | P_i) = \frac{P^{\pi_i}(\{\omega\} \cap P_i)}{P^{\pi_i}(P_i)} = \begin{cases} \frac{P^{\pi_i}(\{\omega\})}{P^{\pi_i}(P_i)} & \text{if } \omega \in P_i; \\ 0 & \text{otherwise.} \end{cases}$$

^aSee Sec. 5 for some remarks on assumptions made in this paper.

3. The Model

In this section we will introduce information sharing situations, which describe economic agents who have to make decisions based on the information they have. The rewards the players receive depend on their own choices and on the true state. We start with an example that illustrates some notions we will encounter in this section and which will be used throughout the remainder of this work. Subsequently, we will formally describe information sharing situations. Finally, we will construct a cooperative game for each information sharing situation.

Example 3.1. Consider three retailers, all of them selling (a variant of) product X . All three retailers buy the product from the same supplier and have to place their orders well in advance of the selling period. Retailer 2 has a specific brandname for the product. Therefore, he pays some marketing expenses for a campaign that will start before orders are placed and, consequently, he expects to be able to demand a higher price. All three retailers have already determined their selling price. Product X is complementary to products Y and Z . There is uncertainty whether products Y and Z will be launched before or after the coming selling period. An early launch of these products would boost total sales of product X in this period. Retailers 1 and 2 are both part of larger companies. Other divisions of these two companies are preparing the launch of product Y and product Z , respectively. Retailer 3 has a solid knowledge about preferences of consumers over different retailers and, therefore, after seeing the campaign of retailer 2, he can very well predict the division of total sales of product X over the different retailers, specifically whether the brand name of retailer 2 in combination with a higher selling price will result in more or less than average sales. It seems obvious that all retailers are interested in maximizing their expected profits. Furthermore, it seems obvious that sharing information about the date of launch of products Y and Z and consumer preferences will result in better orders of the retailers and, hence, in more total profit. $\diamond$

An *information sharing situation* is a tuple $(\Omega, N, (\pi_i)_{i \in N}, (I_i)_{i \in N}, (A_i)_{i \in N}, (r_i)_{i \in N})$. Here, Ω is a (finite) set describing all possible states, $N = \{1, \dots, n\}$ is the set of players, and, for any $i \in N$, π_i is the probability distribution over Ω describing the prior of player i with a positive probability for all states of the world. For a player $i \in N$, I_i describes a partition of Ω . Two states are in the same partition element in I_i if player i cannot distinguish between these states. The set A_i contains all actions of player $i \in N$. We will assume that A_i is finite for all $i \in N$. Finally, r_i is a function that assigns to every pair $(\omega, a) \in \Omega \times A_i$ a reward $r_i(\omega, a) \in \mathbb{R}$.

Example 3.1 (continued) We will define an information sharing situation describing this example. The set of states in the example describe whether or not product Y is launched early, whether or not product Z is launched early, and whether or not the brand name of retailer 2 is appealing to the target group of customers. Each state can be denoted by a binary code of length 3, where the first coordinate

denotes whether product Y will be launched early (1) or not (0). Similarly, the second coordinate denotes whether product Z will be launched before the selling season (1) or not (0). The third coordinate denotes whether the brand name of retailer 2 will be successful (1) or not (0). So, $\Omega = \{0, 1\}^3$ and, for example, 001 refers to the situation in which products Y and Z are not launched early and the brand name of retailer 2 attracts additional customers. The three players are contained in player set $N = \{1, 2, 3\}$. Beforehand, every player believes that all eight possibilities have the same probability, i.e. $\pi_i(\omega) = \frac{1}{8}$ for all $i \in N$ and $\omega \in \Omega$. For notational convenience we will denote the two elements of the information partition of player 1 by $\overline{\mathcal{Y}}$ and $\underline{\mathcal{Y}}$, where $\overline{\mathcal{Y}}$ contains all states in which Y is launched early and $\underline{\mathcal{Y}}$ all states in which Y is not launched early. Similarly, we introduce $\overline{\mathcal{Z}}$ and $\underline{\mathcal{Z}}$ for product Z . Finally, we denote $\overline{\mathcal{B}}$ and $\underline{\mathcal{B}}$ for the sets of states in which the brand name of retailer 2 is successful or not, respectively. Now, the information partitions of the players are easily described,

$$I_1 = \{\overline{\mathcal{Y}}, \underline{\mathcal{Y}}\};$$

$$I_2 = \{\overline{\mathcal{Z}}, \underline{\mathcal{Z}}\};$$

$$I_3 = \{\overline{\mathcal{B}}, \underline{\mathcal{B}}\}.$$

Retailers 1 and 2 will get to know the information that is available within the company and retailer 3 will learn the consumer preferences. Before we can describe the possible actions and the reward functions we present some additional information. If none of products Y and Z is launched before the selling season total demand for product X will equal 600. If product Y is introduced before the selling season total demand will increase by 400, whereas an early introduction of Z will increase total demand by 600. If the marketing campaign of retailer 2, which will cost 50, is successful then retailers 1, 2 and 3 will have market shares, $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$, respectively. Otherwise, these shares will be $\frac{3}{8}$, $\frac{1}{4}$ and $\frac{3}{8}$, respectively. Retailers buy the product for a price of 3, retailers 1 and 3 sell it for 5, while retailer 2 sells it for 7.

Combining the information above, we see that sales of retailer 1 can *a priori* be equal to $\frac{1}{4}$ or $\frac{3}{8}$ times the possible total selling quantities of 600, 1000, 1200 and 1600. Hence, they can be equal to $150(= 600 \cdot \frac{1}{4})$, $225(= 600 \cdot \frac{3}{8})$, 250, 300, 375, 400, 450 and 600. We remark that the set of order sizes that maximize the expected profit of this retailer will always contain at least one of those selling quantities. Hence, we define

$$A_1 = \{150, 225, 250, 300, 375, 400, 450, 600\}.$$

Similarly, we define

$$A_2 = \{150, 250, 300, 400, 500, 600, 800\};$$

$$A_3 = \{150, 225, 250, 300, 375, 400, 450, 600\}.$$

Note that retailer 2 sells 300 with probability $\frac{1}{4}$, whereas all other selling quantities occur with *a priori* probability $\frac{1}{8}$. Let $s_i(\omega)$ denote the demand for product X at retailer $i \in N$ if $\omega \in \Omega$ is the true state of the world. Then the reward functions of the players are given by

$$r_1(\omega, a) = -3a + 5 \min\{a, s_1(\omega)\};$$

$$r_2(\omega, a) = -3a + 7 \min\{a, s_2(\omega)\} - 50;$$

$$r_3(\omega, a) = -3a + 5 \min\{a, s_3(\omega)\}.$$

◇

A player will get to know only the element of his information partition containing the true state. This implies that a player will get to know the true state only if this state is the single element of an information partition element. Together with the prior probability distribution over the set of states the player can determine the posterior probability distribution over the elements of the partition element, which is a conditional distribution. Assuming risk-neutrality of the player, this player will choose the strategy that maximizes his expected payoff. *A priori*, a player can already make up his mind what his choice will be, for any possible element of his information set.

The *a priori* strategy of player i can formally be described by a function x_i from the set of states Ω into the set of strategies A_i . Since a player cannot distinguish between states in the same partition element, this function has to be constant on every element of his information partition. If I denotes such an information partition (possibly $I = I_i$) then this function has to be constant on every $P \in I$. The set of all such functions for player i will be denoted by $X_i(A_i, I)$. Since Ω is a finite set it follows that I contains a finite number of partition elements. Together with the finiteness of A_i it follows that the set $X_i(A_i, I)$ is finite. The (*a priori*) value of player i with information partition I , notation $v^I(\{i\})$, is defined as his maximum expected payoff, i.e.,

$$v^I(\{i\}) = \max_{x_i \in X_i(A_i, I)} E_{\pi_i}[r_i(\cdot, x_i(\cdot))], \quad (3.1)$$

where E_{π_i} represents the expected value with respect to the prior distribution π_i . This maximum is well defined by the finiteness of $X_i(A_i, I)$. Initially, with information partition I_i of player i , the maximum expected payoff of this player is $v^{I_i}(\{i\})$.

Example 3.1 (continued) Player 1 (retailer 1) knows whether product Y will be launched early or not. Given the reward function of player 1, an optimal action of player 1 is to order 375 if Y is launched early and 225 otherwise. Then the expected

payoff for player 1 is

$$\begin{aligned}
 v^{I_1}(\{1\}) &= \frac{1}{8}((-675 + 5 \min\{225, 150\}) + (-675 + 5 \min\{225, 225\}) \\
 &\quad + (-675 + 5 \min\{225, 300\}) + (-675 + 5 \min\{225, 450\}) \\
 &\quad + (-1125 + 5 \min\{375, 250\}) + (-1125 + 5 \min\{375, 375\}) \\
 &\quad + (-1125 + 5 \min\{375, 400\}) + (-1125 + 5 \min\{375, 600\})) \\
 &= 475.
 \end{aligned}$$

◇

For any information sharing situation we can determine the value of a player. However, players might have an incentive to cooperate with each other by sharing information. Sharing information can result in higher joint profits for the players. We will construct a cooperative game with each information sharing situation, representing the idea that cooperation between the players consists of sharing information. Sharing information between the players results in a refined information partition for the players that share the information. If players i and j share their information, both end up with information partition $I_i \vee I_j$. With this information player i can expect value $v^{I_i \vee I_j}(\{i\})$. So, together players i and j can expect

$$v^{I_i \vee I_j}(\{i\}) + v^{I_i \vee I_j}(\{j\}).$$

Obviously, this idea can be extended to every set of players. This results in a cooperative game which we call the *information sharing game* (N, v^κ) associated with information situation κ , where

$$v^\kappa(S) = \sum_{i \in S} v^{\vee_{j \in S} I_j}(\{i\}) \quad \text{for all } S \subseteq N.$$

Example 3.1 (continued) Denote the information sharing situation that was described before by κ . We already showed that $v^\kappa(\{1\}) = 475$. Now, consider coalition $\{1, 3\}$. After players 1 and 3 have shared their information, player 1 knows not only whether product Y is launched early or not, but also whether the brand name of retailer 2 will be successful or not. Depending on the info they receive, they will make a decision. For example, if product Y is launched early and the brand name of retailer 2 is not successful then player 1 will order 375 and player 3 will order 375 as well (note that their payoff functions are the same and, consequently, if they have the same information then they will have the same optimal choices). So, with probability $\frac{1}{4}$ they will each obtain an expected profit of 750. Combining this with a similar analysis for each of the other elements in their joint information partition we find that the *ex ante* profit of each of the players equals 500. Hence, $v^\kappa(\{1, 3\}) = 1000$. Similar calculations show that the information sharing

game associated with κ is described by

$$v^\kappa(T) = \begin{cases} 0 & \text{if } T = \emptyset; \\ 475 & \text{if } T = \{1\}; \\ 1137,5 & \text{if } T = \{2\}; \\ 468,75 & \text{if } T = \{3\}; \\ 1737,5 & \text{if } T = \{1, 2\}; \\ 1000 & \text{if } T = \{1, 3\}; \\ 1937,5 & \text{if } T = \{2, 3\}; \\ 2975 & \text{if } T = N. \end{cases} \quad (3.2)$$

◇

4. Population Monotonic Allocation Schemes

In this section we show that the class of information sharing games coincides with the class of cooperative games that have a population monotonic allocation scheme. Firstly, consider the following example.

Example 4.1. Consider Example 3.1. It can be checked that $(v^{\vee_{j \in S} I_j}(\{i\}))_{i \in S, S \subseteq N}$ is a population monotonic allocation scheme for the game (N, v^κ) . This population monotonic allocation scheme is represented in Table 1.

Table 1. A population monotonic allocation scheme for the game (N, v^κ) in Example 4.1.

	1	2	3
$\{1\}$	475	*	*
$\{2\}$	*	1137,5	*
$\{3\}$	*	*	468,75
$\{1,2\}$	550	1187,5	*
$\{1,3\}$	500	*	500
$\{2,3\}$	*	1375	562,5
N	687,5	1600	687,5

According to this population monotonic allocation scheme player 2, for example, receives 1375 if coalition $\{2, 3\}$ would happen to result. ◇

Example 4.1 describes a direct way to determine a population monotonic allocation scheme in an information sharing game, using the structure of the underlying information sharing situation. The following theorem shows that this holds in general.

Theorem 4.1. *Let $\kappa = (\Omega, N, (\pi_i)_{i \in N}, (I_i)_{i \in N}, (A_i)_{i \in N}, (r_i)_{i \in N})$ be an information sharing situation and (N, v^κ) the associated information sharing game.*

Then $(v^{\vee_{j \in S} I_j}(\{i\}))_{i \in S, S \subseteq N}$ is a population monotonic allocation scheme for the game (N, v^κ) .

Proof. We will check that $(v^{\vee_{j \in S} I_j}(\{i\}))_{i \in S, S \subseteq N}$ satisfies the conditions in the definition of population monotonic allocation schemes.

- (a) Let $S \subseteq N$. Then $\sum_{i \in S} v^{\vee_{j \in S} I_j}(\{i\}) = v^\kappa(S)$ by definition of $v^\kappa(S)$.
- (b) Let $S, T \subseteq N$ with $S \subseteq T$. Then $\vee_{j \in S} I_j$ is coarser than $\vee_{j \in T} I_j$, i.e. every element of $\vee_{j \in S} I_j$ is partitioned in one or more elements of $\vee_{j \in T} I_j$, since $\vee_{j \in T} I_j = (\vee_{j \in S} I_j) \vee (\vee_{j \in T \setminus S} I_j)$. This implies that every function that is constant on every element of $\vee_{j \in S} I_j$ is also constant on every element of $\vee_{j \in T} I_j$:

$$X_i(A_i, \vee_{j \in S} I_j) \subseteq X_i(A_i, \vee_{j \in T} I_j) \quad \text{for all } i \in S.$$

Hence, for all $i \in S$ it holds that

$$\begin{aligned} v^{\vee_{j \in S} I_j}(\{i\}) &= \max_{x_i \in X_i(A_i, \vee_{j \in S} I_j)} E_{\pi_i}[r_i(\cdot, x_i(\cdot))] \\ &\leq \max_{x_i \in X_i(A_i, \vee_{j \in T} I_j)} E_{\pi_i}[r_i(\cdot, x_i(\cdot))] \\ &= v^{\vee_{j \in T} I_j}(\{i\}). \end{aligned}$$

This completes the proof. □

Theorem 4.1 implies that every information sharing game has a PMAS. In the following theorem we will deal with the reverse question, i.e. is every game with a PMAS an information sharing game? We show that this question can be answered in the affirmative.

Theorem 4.2. *Let (N, v) be a cooperative game with a population monotonic allocation scheme. Then there exists an information sharing situation κ such that $v^\kappa = v$.*

Proof. Let $(y_{i,S})_{i \in S, S \subseteq N}$ be a population monotonic allocation scheme for (N, v) . Define the information sharing situation $\kappa = (\Omega, N, (\pi_i)_{i \in N}, (I_i)_{i \in N}, (A_i)_{i \in N}, (r_i)_{i \in N})$ in the following way.

- 1) The set Ω is the collection of all maps $\omega : 2^N \rightarrow 2^N$ that satisfy $\omega(S) \subseteq S$ for every $S \in 2^N$. A state of the world ω can be seen as a mechanism that assigns to every coalition a selected subgroup.
- 2) For all players i the prior probability distribution π_i is defined by $\pi_i(\omega) = 1/|\Omega|$ for every $\omega \in \Omega$. So π_i assigns equal probabilities to all states in Ω and $(\pi_i)_{i \in N}$ constitute a common prior.
- 3) In order to define the information partition I_i of player $i \in N$ we introduce the equivalence relation $\sim_i$ on Ω : $\omega \sim_i \omega'$ if for all $S \subseteq N$ with $i \in S$ we have $i \in \omega(S)$ iff $i \in \omega'(S)$. The information partition I_i consists of all equivalence

classes of $\sim_i$. The information partition I_i can be interpreted as follows: player i only knows for every coalition to which he belongs whether he is part of the selected subgroup or not.

- 4) The action set A_i of player $i \in N$ is defined by $A_i = \{S \subseteq N \mid i \in S\} \times \{0, 1\}$. An action $a = (a_1, a_2)$ of player i consists of a choice of a coalition a_1 to which he belongs together with a guess a_2 whether the size of the selected subgroup $\omega(a_1)$ is even ($a_2 = 0$) or odd ($a_2 = 1$).
- 5) The reward function r_i of player $i \in N$ is defined by

$$r_i(\omega, a) = \begin{cases} y_{i,a_1} & \text{if } |\omega(a_1)| \bmod 2 = a_2; \\ 2y_{i,\{i\}} - y_{i,a_1} & \text{otherwise} \end{cases}$$

for every $\omega \in \Omega$ and $a \in A_i$. So, player i receives y_{i,a_1} if he guesses correctly whether the size of the selected subgroup of the coalition which he chose (a_1) is even or odd. Otherwise, he receives the smaller amount $2y_{i,\{i\}} - y_{i,a_1}$.

In order to show that $v^\kappa = v$ it suffices to prove for every $S \subseteq N$, $i \in S$ that $v^{\vee_{j \in S} I_j}(\{i\}) = y_{i,S}$. Since then we have, for every $S \subseteq N$,

$$v^\kappa(S) = \sum_{i \in S} v^{\vee_{j \in S} I_j}(\{i\}) = \sum_{i \in S} y_{i,S} = v(S).$$

So, let $S \subseteq N$, $i \in S$. Let $P \in \vee_{j \in S} I_j$. Note that, for every $T \subseteq S$, the selected subgroup does not depend upon $\omega \in P$, i.e. for every $\omega, \omega' \in P$ we have $\omega(T) = \omega'(T)$. We will show that the maximal conditional expected reward for player i , conditional upon P , equals $y_{i,S}$. Thereby we distinguish three cases.

First, consider the action $a^* = (a_1^*, a_2^*) \in A_i$ with $a_1^* = S$ and $a_2^* = |\omega(S)| \bmod 2$ for some $\omega \in P$. Note that a_2^* is well-defined since $\omega(S) = \omega'(S)$ for every $\omega, \omega' \in P$. Clearly, $r_i(\omega, a^*) = y_{i,S}$ for every $\omega \in P$, so the conditional expected reward for player i , if he chooses action a^* , equals $y_{i,S}$.

Secondly, consider action $a = (a_1, a_2)$ with $a_1 \subsetneq S$. Keep in mind that $\omega(a_1) = \omega'(a_1)$ for every $\omega, \omega' \in P$. If $a_2 = |\omega(a_1)| \bmod 2$ for every $\omega \in P$ then $r_i(\omega, a) = y_{i,a_1}$ for every $\omega \in P$ and hence the conditional expected reward for player i , if he chooses action a , equals $y_{i,a_1} \leq y_{i,S}$. If $a_2 \neq |\omega(a_1)| \bmod 2$ for every $\omega \in P$ then $r_i(\omega, a) = 2y_{i,\{i\}} - y_{i,a_1}$ for every $\omega \in P$ and hence the conditional expected reward for player i , if he chooses action a , equals $2y_{i,\{i\}} - y_{i,a_1} \leq y_{i,a_1} \leq y_{i,S}$.

Thirdly, consider an action $a = (a_1, a_2) \in A_i$ with $a_1 \not\subseteq S$. Let $j \in a_1 \setminus S$. With every $\omega \in \Omega$ we can associate $\omega' \in \Omega$, $\omega \neq \omega'$ in the following way: $\omega'(T) = \omega(T)$ for every $T \neq a_1$, $\omega'(a_1) = \omega(a_1) \setminus \{j\}$ if $j \in \omega(a_1)$ and $\omega'(a_1) = \omega(a_1) \cup \{j\}$ if $j \notin \omega(a_1)$. Note that $\omega' \sim_i \omega$ for every $i \in N \setminus \{j\}$. In particular $\omega' \sim_i \omega$ for every $i \in S$. Therefore, we have $\omega \in P$ iff $\omega' \in P$. Moreover, $|\omega(a_1)|$ is even iff $|\omega'(a_1)|$ is odd. Since $(\omega')' = \omega$ for every $\omega \in \Omega$ it follows that for half of the states $\omega \in P$ we get $r_i(\omega, a) = y_{i,a_1}$ and for the other half of the states $\omega \in P$ we get $r_i(\omega, a) = 2y_{i,\{i\}} - y_{i,a_1}$. So, the conditional expected reward for player i , if he chooses action a , equals $\frac{1}{2}y_{i,a_1} + \frac{1}{2}(2y_{i,\{i\}} - y_{i,a_1}) = y_{i,\{i\}} \leq y_{i,S}$. We conclude that

the maximal conditional expected reward for player i equals $y_{i,S}$. Since this is true for every $P \in \bigvee_{j \in S} I_j$ we get

$$v^{\bigvee_{j \in S} I_j}(\{i\}) = y_{i,S}.$$

This finishes the proof. $\square$

In the proof above we see that the players in information sharing situation κ have a common prior. Hence, we have the following corollary.

Corollary 4.1. *Let (N, v) be a cooperative game with a population monotonic allocation scheme. Then there exists an information sharing situation κ with a common prior such that $v^\kappa = v$.*

5. Remarks

We conclude in this section with some remarks on assumptions made in the paper. Firstly, we concentrate on finiteness-assumptions and, secondly, on the *ex ante* perspective.

For several reasons we have assumed finiteness of several sets in our model. Firstly, because of notational convenience and clarity of exposition. Extending to a setting with Ω possibly being infinite demands for the use of a σ -algebra of subsets of Ω , a probability measure on this σ -algebra, and a reward function that is bounded and measurable with respect to this σ -algebra. In our opinion this will distract attention from the main ideas in this paper, though extending our model in this direction is very well possible.

Secondly, non-finiteness of the number of actions of a player would demand for a slight change in the model, by taking the supremum rather than the maximum in (3.1) or by making some additional assumptions on A_i and r_i , e.g. compactness of A_i and continuity of r_i . Again we note that such a generalization is possible.

We remark that for details on extensions as described above, the interested reader can turn to Brânzei *et al.* (2001), who introduce a model that is similar to our model, but with one decision maker only. Brânzei *et al.* (2001) do not make any finiteness-assumptions.

In the *ex ante* perspective in the paper, cooperation (information sharing) between the players is analyzed independent of the information each player will actually receive. Alternatively, we could have looked at an *in-mediis* stage, where the cooperation is analyzed after each player has received his share of information. This implies that each player has learned that the true state of the world belongs to some specific element of his information partition. Though conceptually very different this *in-mediis* stage can be described as an information sharing situation, as introduced in this paper, with adjusted individual prior probabilities. The adjustments are Bayesian updates of the original prior probabilities of all players to account for the information these players have received. Though many states of the world may now have zero probabilities according to several players, the result that

the information sharing situation results in an information sharing game that has a population monotonic allocation scheme still holds.

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