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A Poisson log-bilinear regression approach to the construction of projected lifetables

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Abstract

This paper implements Wilmoth's [Computational methods for fitting and extrapolating the Lee–Carter model of mortality change, Technical report, Department of Demography, University of California, Berkeley] and Alho's [North American Actuarial Journal 4 (2000) 91] recommendation for improving the Lee–Carter approach to the forecasting of demographic components. Specifically, the original method is embedded in a Poisson regression model, which is perfectly suited for age–sex-specific mortality rates. This model is fitted for each sex to a set of age-specific Belgian death rates. A time-varying index of mortality is forecasted in an ARIMA framework. These forecasts are used to generate projected age-specific mortality rates, life expectancies and life annuities net single premiums. Finally, a Brass-type relational model is proposed to adapt the projections to the annuitants population, allowing for estimating the cost of adverse selection in the Belgian whole life annuity market.

Keywords: Life insurance; Age–sex-specific mortality; Lifetable functions; Annuities; Adverse selection

1. Introduction and motivation

1.1. Mortality on the move

Mortality improvements are viewed as a positive change for individuals and as a substantial social achievement. Nevertheless, they pose a challenge for the planning of public retirement systems as well as for the private life annuities business. More generally, all the components of social security systems, including disability and survivorship benefits, as well as medical care for the aged, are affected by mortality trends, not only old-age pensions. Similarly, other insurance products sold by private companies are influenced by improvements in longevity. A prime example is post-retirement sickness cover (in particular medical expenses cover indemnifying the insured from his retirement age on for the cost incurred in obtaining medical treatment).

During the 20th century, the human mortality globally declined. To have an idea of this evolution, Table 1 displays increases in life expectancies at birth (e_0) and at age 65 (e_{65}) calculated from Belgian period lifetables

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Table 1
Evolution of e_0 and e_{65} in Belgium

Period	e_0		e_{65}	
	Men	Women	Men	Women
1880–1890	43.29	46.51	10.67	11.60
1928–1932	56.03	59.80	11.42	12.56
1946–1949	62.03	67.26	12.32	13.87
1959–1963	67.15	73.18	12.43	14.83
1968–1972	67.78	74.20	12.59	15.28
1979–1982	70.03	76.80	12.94	16.91
1988–1990	72.42	79.12	14.02	18.30
1991–1993	72.99	79.77	14.50	18.79
1994–1996	74.06	80.75	15.21	19.58
1997–1999	74.76	81.17	15.62	19.85

(source: National Institute of Statistics, Brussels); for more details about the evolution of mortality in Belgium during 1880–1999, see Brouhns and Denuit (2001a). Notice that e_0 and e_{65} have significantly increased for both sexes, although progresses have occurred at an uneven rate.

Since 1970, the main factor driving continued gains in life expectancy in industrialized countries is a reduction of death rates among the elderly. Based on available demographic evidence, the human life span shows no sign of approaching a fixed limit imposed by biology or other factors. Rather, both the average and the maximum life span have increased steadily during the 20th century. For more details, we refer the interested reader to Wilmoth (1997) and Wilmoth et al. (2000). The complexity and historical stability of these changes suggest that the most reliable method of predicting the future is merely to extrapolate past trends, as pointed out by Wilmoth (2000).

1.2. Projected lifetables

When living benefits are concerned, the calculation of expected present values (needed in pricing and reserving) requires an appropriate mortality projection in order to avoid underestimation of future costs. This is because mortality trends at adult/old ages reveal decreasing annual death probabilities; see, e.g. Benjamin and Soliman (1993), and references therein. Mortality improvements have obvious effects on pricing and reserving for life annuities; see, e.g. Marocco and Pitacco (1998), Olivieri (2001) and Coppola et al. (2000). More generally, such trends affect any insurance cover providing some kind of "living benefits", such as long term care benefits or lifetime sickness benefits, as pointed out in Olivieri and Pitacco (1999, 2001). Olivieri and Pitacco (2000) discussed solvency requirements for life annuities.

In order to protect the company from mortality improvements, actuaries have to resort to lifetables including a forecast of the future trends of mortality (the so-called projected tables). Different approaches for building these technical bases have been developed by actuaries and demographers. Since Cramér and Wold (1935), the evolution over time of graduated mortality curves is popular for the purpose of extrapolation. One classical procedure is based on the projection of parameters (see, e.g. Benjamin and Soliman (1993)). In the authors' opinion, this approach suffers from serious drawbacks. Indeed, it first heavily relies on the appropriateness of the retained parametric models (as Makeham, for instance). Secondly, the estimated parameters are often strongly dependent so that univariate extrapolations may be misleading. Building a multivariate time series for the parameters is theoretically possible but seriously complicates the approach. The method applied in this paper avoids these problems, being simultaneously distribution-free and defining a univariate mortality index to be forecasted for generating mortality projections.

1.3. Extending the Lee–Carter approach

Lee and Carter (1992) proposed a simple model for describing the secular change in mortality as a function of a single time index. The method describes the log of a time series of age-specific death rates as the sum of an age-specific component that is independent of time and another component that is the product of a time-varying parameter reflecting the general level of mortality, and an age-specific component that represents how rapidly or slowly mortality at each age varies when the general level of mortality changes. This model is fitted to historical data. The resulting estimate of the time-varying parameter is then modeled and projected as a stochastic time series using standard Box–Jenkins methods. From this forecast of the general level of mortality, the actual age-specific rates are derived using the estimated age effects.

This paper aims to investigate possible improvements of the powerful Lee–Carter method, in the spirit of Wilmoth (1993) and Alho (2000). Specifically, we switch from a classical linear model to a generalized linear model, substituting Poisson random variation for the number of deaths for an additive error term on the logarithm of mortality rates. It is worth to mention that the Poisson distribution is well suited to mortality analyses; see, e.g. Brillinger (1986) and McDonald (1996a–c) for more details. It has been successfully applied by Renshaw and Haberman (1996) and Sithole et al. (2000) to the forecasting of mortality trends. As in the Lee–Carter method, time series are used to make long-run forecasts of age–sex-specific mortality. We believe that this improvement makes the model more intuitively acceptable. The two approaches will be compared on the basis of Belgian mortality data.

1.4. Agenda

The paper is organized as follows. Section 2 makes precise the notation and assumption used throughout this paper. It also briefly describes the data to be analyzed in the empirical part of this paper. In Section 3, we present the main features of the classical Lee–Carter methodology for projecting mortality. Section 4 is devoted to the variant of the Lee–Carter methodology studied in this paper. We carefully enhance the similarities and differences between the two approaches for readers' convenience. In Section 5, we apply both models on Belgian data. Section 6 deals with the adverse selection, particularly important in the annuities market. A Brass-type relational model is used to adapt forecasts to the annuitants' mortality (reflected in the statistics gathered by the Belgian regulatory authorities). Section 7 gives the final conclusions.

2. Notation, assumption and data

2.1. Notation

We analyze the changes in mortality as a function of both age x and time t. This "period analysis" is known to be more appropriate than a "cohort analysis"; we refer the interested reader, e.g. to Tuljapurkar and Boe (1998) for more details. Henceforth, $\mu_x(t)$ will denote the force of mortality at age x during calendar year t. We denote as D_{xt} the number of deaths recorded at age x during year t, from an exposure-to-risk E_{xt} (i.e., E_{xt} is the number of person years from which D_{xt} occurred).

2.2. Piecewise constant forces of mortality

In this paper, we assume that the age-specific mortality rates are constant within bands of time, but allowed to vary from one band to the next. Specifically, given any integer age x and calendar year t, it is supposed that

$$\mu_{x+\tau}(t) = \mu_x(t) \quad \text{for } 0 \leq \tau < 1. \tag{2.1}$$

This is best illustrated with the aid of a coordinate system that has calendar time as abscissa and age as coordinate. Such a representation is called a Lexis diagram after the German demographer who introduced it. Both time scales are divided into yearly bands, which partition the Lexis plane into rectangular segments. Model (2.1) assumes that the mortality rate is constant within each rectangle, but allows it to vary between rectangles.

Henceforth, we denote as $p_x(t)$ the probability that an individual aged x in year t reaches age $x+1$, as $q_x(t) = 1 - p_x(t)$ the corresponding death probability, as $e_x(t)$ the expected remaining lifetime of an individual reaching age x during year t, and as $a_x(t)$ the net single premium relating to a life annuity sold to an individual aged x in year t. Tedious but straightforward computations show that under (2.1), we have for integer age x and calendar year t:

$$p_x(t) = \exp(-\mu_x(t)) = 1 - q_x(t),$$

$$e_x(t) = \frac{1-\exp(-\mu_x(t))}{\mu_x(t)} + \sum_{k\geq 1}\left\{\prod_{j=0}^{k-1}\exp(-\mu_{x+j}(t+j))\right\}\frac{1-\exp(-\mu_{x+k}(t+k))}{\mu_{x+k}(t+k)},$$

$$a_x(t) = \sum_{k\geq 0}\left\{\prod_{j=0}^{k} p_{x+j}(t+j)\right\} v^{k+1},$$

where $v = (1+i)^{-1}$ is the discount factor corresponding to the yearly interest rate i. Throughout this paper, we have taken $i = 4\%$ for the numerical illustrations.

2.3. Data

The models presented in this paper are fitted to the matrix of Belgian death rates, 1960–1998. Data relating to the entire Belgian population have been provided by the National Institute of Statistics. In addition to these national data, we resort to market data to quantify the impact of adverse selection on the price of life annuities. These data have been supplied by the Belgian regulatory authorities (Office de Contrôle des Assurances, OCA; Controle Dienst der Verzekeringen, CDV). They are only available for a few years (1997–1999) but are of excellent quality. Indeed, OCA–CDV requires the companies to provide exposure-to-risk measured in person years, together with observed deaths in each age group. These data allow for an accurate estimation of age–sex-specific forces of mortality.

3. Lee–Carter classical methodology

3.1. Model

A powerful and elegant approach to mortality forecasts has been pioneered by Lee and Carter (1992). Those authors proposed a remarkably simple model for mortality projections, specifying a log-bilinear form for the force of mortality $\mu_x(t)$. The method is in essence a relational model

$$\ln \widehat{\mu_x}(t) = \alpha_x + \beta_x\kappa_t + \epsilon_x(t), \tag{3.1}$$

where $\widehat{\mu_x}(t)$ denotes the observed force of mortality at age x during year t, the $\epsilon_x(t)$'s are homoskedastic centered error terms and where the parameters are subject to the constraints

$$\sum_t \kappa_t = 0 \quad \text{and} \quad \sum_x \beta_x = 1 \tag{3.2}$$

ensuring model identification.

When the model (3.1) is fit by ordinary least-squares (OLS), interpretation of the parameters is quite simple:

α_x: the fitted values of α_x exactly equals the average of $\ln \widehat{\mu_x}(t)$ over time t so that $\exp \alpha_x$ is the general shape of the mortality schedule;

β_x: represents the age-specific patterns of mortality change. It indicates the sensitivity of the logarithm of the force of mortality at age x to variations in the time index κ_t. In principle, β_x could be negative at some ages x, indicating that mortality at those ages tends to rise when falling at other ages. In practice, this does not seem to happen over the long run.

κ_t: represents the time trend. The actual forces of mortality change according to an overall mortality index κ_t modulated by an age response β_x. The shape of the β_x profile tells which rates decline rapidly and which slowly over time in response of change in κ_t.

The error term $\epsilon_x(t)$, with mean 0 and variance σ_ϵ^2 reflects particular age-specific historical influence not captured in the model.

3.2. OLS estimation

The model (3.1) is fitted to a matrix of age-specific observed forces of mortality using singular value decomposition (SVD). Specifically, the $\hat{\alpha}_x$'s, $\hat{\beta}_x$'s and $\hat{\kappa}_t$'s are such that they minimize

$$\sum_{x,t} (\ln \widehat{\mu_x}(t) - \alpha_x - \beta_x \kappa_t)^2. \tag{3.3}$$

It is worth mentioning that model (3.1) is not a simple regression model, since there are no observed covariates in the right-hand side. The minimization of (3.3) consists in taking for $\widehat{\alpha_x}$ the row average of the $\ln \widehat{\mu_x}(t)$'s, and to get the $\widehat{\beta_x}$'s and $\widehat{\kappa_t}$'s from the first term of an SVD of the matrix $\ln \widehat{\mu_x}(t) - \widehat{\alpha_x}$. This yields a single time-varying index of mortality κ_t.

Before proceeding directly to modeling the parameter $\hat{\kappa}_t$ as a time series process, the $\hat{\kappa}_t$'s are adjusted (taking $\hat{\alpha}_x$ and $\hat{\beta}_x$ estimates as given) to reproduce the observed number of deaths $\sum_x D_{xt}$, i.e., the $\hat{\hat{\kappa}}_t$'s solve

$$\sum_x D_{xt} = \sum_x E_{xt} \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\hat{\kappa}}_t). \tag{3.4}$$

So, the κ_t's are reestimated so that the resulting death rates (with the previously estimated $\hat{\alpha}_x$ and $\hat{\beta}_x$), applied to the actual risk exposure, produce the total number of deaths actually observed in the data for the year t in question. There are several advantages to make this second-stage estimate of the parameters κ_t. In particular, it avoids sizable discrepancies between predicted and actual deaths (occurring because the first step is based on logarithms of death rates). Other advantages are discussed by Lee (2000).

3.3. Modeling the index of mortality

An important aspect of Lee–Carter methodology is that the time factor $\hat{\hat{\kappa}}_t$ is intrinsically viewed as a stochastic process. Box–Jenkins techniques are then used to estimate and forecast κ_t within an ARIMA times series model. These forecasts in turn yield projected age-specific mortality rates, life expectancies and annuities single premiums.

3.4. Comments

The original Lee–Carter method was used to aggregate (sexes combined) US data. Carter and Lee (1992) implemented their model for males and females separately, showing that the two series are best treated as declining independently. Wilmoth (1996) applied Lee–Carter methods to forecast Japanese mortality and also experimented

with variants of this model. Lee and Nault (1993) applied Lee–Carter methods to model Canadian mortality, Lee and Rofman (1994) fitted model (3.1) to Chilean data, and Brouhns and Denuit (2001b) did the same for Belgian statistics.

It should be noted that the Lee–Carter method does not attempt to incorporate assumptions about advances in medical science or specific environmental changes; no information other than previous history is taken into account. This means that this approach is unable to forecast sudden improvements in mortality due to the discovery of new medical treatments or revolutionary cures including antibiotics. Similarly, future deteriorations caused by epidemics, the apparition of new diseases or the aggravation of pollution cannot enter the model. The actuary has to keep this in mind when he sets his reinsurance program.

The Lee–Carter methodology is a mere extrapolation of past trends. All purely extrapolative forecasts assume that the future will be in some sense like the past. Some authors (see, e.g. Gutterman and Vanderhoof (2000)) severely criticized this approach because it seems to ignore underlying mechanisms. As pointed out by Wilmoth (2000), such a critique is valid only in so far as such mechanisms are understood with sufficient precision to offer a legitimate alternative method of prediction. The understanding of the complex interactions of social and biological factors that determine mortality levels being still imprecise, the extrapolative approach to prediction is particularly compelling in the case of human mortality.

4. Poisson modeling for the number of deaths and Lee–Carter methodology

4.1. Model

According to Alho (2000), the model described in equation (3.1) is not well suited to the situation of interest. As already mentioned, the main drawback of the OLS estimation via SVD is that the errors are assumed to be homoskedastic. This is related to the fact that for inference we are actually assuming that the errors are normally distributed, which is quite unrealistic. The logarithm of the observed force of mortality is much more variable at older ages than at younger ages because of the much smaller absolute number of deaths at older ages.

Because the number of deaths is a counting random variable, according to Brillinger (1986), the Poisson assumption appears to be plausible. In order to circumvent the problems associated with the OLS method, we now consider that

$$D_{xt} \sim \text{Poisson}(E_{xt}\mu_x(t)) \quad \text{with } \mu_x(t) = \exp(\alpha_x + \beta_x\kappa_t), \tag{4.1}$$

where the parameters are still subjected to the constraints (3.2). The force of mortality is thus assumed to have the log-bilinear form $\ln\ \mu_x(t) = \alpha_x + \beta_x\kappa_t$. The meaning of the α_x, β_x, and κ_t parameters is essentially the same as in the classical Lee–Carter model.

4.2. Maximum likelihood estimation

Instead of resorting to SVD for estimating α_x, β_x and κ_t, we now determine these parameters by maximizing the log-likelihood based on model (4.1), which is given by

$$L(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\kappa}) = \sum_{x,t}\{D_{xt}(\alpha_x + \beta_x\kappa_t) - E_{xt}\exp(\alpha_x + \beta_x\kappa_t)\} + \text{constant}.$$

Because of the presence of the bilinear term $\beta_x\kappa_t$, it is not possible to estimate the proposed model with commercial statistical packages that implement Poisson regression. However, the LEM program (Vermunt, 1997a,b) can be used for this purpose. In Appendix A, we give the quite simple LEM input files that we used for our analyses.

The algorithm implemented in LEM to solve the likelihood equations is a uni-dimensional or elementary Newton method. Goodman (1979) was the first who proposed this iterative method for estimating log-linear models with bilinear terms. In iteration step $\nu+1$, a single set of parameters is updated fixing the other parameters at their current

estimates using the following updating scheme

$$\hat{\theta}^{(\nu+1)} = \hat{\theta}^{(\nu)} - \frac{\partial L^{(\nu)}/\partial\theta}{\partial^2 L^{(\nu)}/\partial\theta^2},$$

where $L^{(\nu)} = L^{(\nu)}(\hat{\theta}^{(\nu)})$.

In our application, there are three sets of parameters, i.e., the α_x, the β_x, and the κ_t terms. The updating scheme is as follows, starting with $\hat{\alpha}_x^{(0)} = 0$, $\hat{\beta}_x^{(0)} = 1$, and $\hat{\kappa}_t^{(0)} = 0$ (random values can also be used)

$$\hat{\alpha}_x^{(\nu+1)} = \hat{\alpha}_x^{(\nu)} - \frac{\sum_t (D_{xt} - \hat{D}_{xt}^{(\nu)})}{-\sum_t \hat{D}_{xt}^{(\nu)}}, \qquad \hat{\beta}_x^{(\nu+1)} = \hat{\beta}_x^{(\nu)}, \qquad \hat{\kappa}_t^{(\nu+1)} = \hat{\kappa}_t^{(\nu)},$$

$$\hat{\kappa}_t^{(\nu+2)} = \hat{\kappa}_t^{(\nu+1)} - \frac{\sum_x (D_{xt} - \hat{D}_{xt}^{(\nu+1)})\hat{\beta}_x^{(\nu+1)}}{-\sum_x \hat{D}_{xt}^{(\nu)}(\hat{\beta}_x^{(\nu+1)})^2}, \qquad \hat{\alpha}_x^{(\nu+2)} = \hat{\alpha}_x^{(\nu+1)}, \qquad \hat{\beta}_x^{(\nu+2)} = \hat{\beta}_x^{(\nu+1)},$$

$$\hat{\beta}_x^{(\nu+3)} = \hat{\beta}_x^{(\nu+2)} - \frac{\sum_t (D_{xt} - \hat{D}_{xt}^{(\nu+2)})\hat{\kappa}_t^{(\nu+2)}}{-\sum_t \hat{D}_{xt}^{(\nu+2)}(\hat{\kappa}_t^{(\nu+2)})^2}, \qquad \hat{\alpha}_x^{(\nu+3)} = \hat{\alpha}_x^{(\nu+2)}, \qquad \hat{\kappa}_t^{(\nu+3)} = \hat{\kappa}_t^{(\nu+2)},$$

where $\hat{D}_{xt}^{(\nu)} = E_{xt}\exp(\hat{\alpha}_x^{(\nu)} + \hat{\beta}_x^{(\nu)}\hat{\kappa}_t^{(\nu)})$, or the estimated number of deaths after iteration step ν. The criterion used to stop the procedure is a very small increase of the log-likelihood function (the default value of LEM is 10^{-6}, but it can be recommmended to set the criterion a little bit sharper, so to 10^{-10}).

After updating the κ_t parameters, we have to impose a location constraint. LEM uses the centering constraint $\sum_t \hat{\kappa}_t = 0$, which is the same constraint as in the Lee–Carter parameterization. This constraint is specified with a design matrix, namely the *spe*() statement in the code given in Appendix A. After updating the β_x parameters, a scaling constraint has to be imposed. The scaling constraint used by LEM is $\hat{\beta}_1 = 1$, which is different from the Lee–Carter parameterization. In order to obtain the Lee–Carter parameterization in which $\sum_x \hat{\beta}_x = 1$, one has to divide the LEM estimates for β_x by $\sum_x \hat{\beta}_x$ and multiply the LEM estimates for κ_t by the same number.

Another option to take the constraint $\sum_t \kappa_t = 0$ into account consists in computing the updates for the κ_t's without constraints and centering the updates before really updating the κ_t's. This simple method only works because we are dealing with an identification constraint (not a model restriction).

Contrarily to the classical Lee–Carter approach (where SVD is applied to transformed mortality rates), the error applies directly on the number of deaths in the Poisson regression approach. There is thus no need of a second-stage estimation like (3.4).

4.3. Modeling the index of mortality

We do not modify the time series part of the Lee–Carter methodology. Estimates of α_x and β_x are used with forecasted κ_t to generate other lifetable functions.

5. An application to Belgian population mortality statistics

5.1. Model selection

Table 2 reports the value of the likelihood-ratio statistic (L^2) for various models we estimated using the Belgian population mortality statistics. It is obtained by comparing the current model with the saturated model, i.e.:

$$L^2 = 2\sum_x \sum_t D_{xt} \ln\left(\frac{D_{xt}}{\widehat{D_{xt}}}\right),$$

Table 2
Testing results for the estimated Poisson models

Model	Women		Men	
	L^2	ΔL^2	L^2	ΔL^2
α_x	82 313		44 674	
$\alpha_x + \kappa_t$	13 150	0.840	11 133	0.751
$\alpha_x + \beta \cdot t$	15 712	0.809	15 856	0.645
$\alpha_x + \beta_x \kappa_t$	7 083	0.914	6 395	0.857
$\alpha_x + \beta_x \cdot t$	10 430	0.873	12 677	0.716

where $\widehat{D_{xt}}$ is the estimated number of deaths in the model concerned. It is a real badness-of-fit measure: the smaller is L^2 the better is the model. Model 4 is the Lee–Carter model. In order to get an impression on its performance in describing the time trend in the age-specific death rates, we also estimated four more restricted models. These models assume time-constant age-specific rates (1), age-independent trend (2), age-independent linear trend (3), and age-dependent linear trend (5). The fit measures show that our bilinear model outperforms these more parsimonious specifications. More precisely, both the assumption of an age-independent and a linear time trend is too restrictive.

The ΔL^2 measures denote the proportional reduction of L^2 compared to the model with time-constant mortality rates (model 1). It indicates which proportion of the observed change in rates over time can be explained by a model with a time trend. As can be seen, the Lee–Carter model reduces the L^2 with 91.4% among females and 85.7% among males. These proportions can be increased by including additional terms to the model such as, for example, a second bilinear term. If we extend the model with a second bilinear term, we obtain ΔL^2 values of 96.3 and 93.6% for females and males, respectively. The inclusion of a second bilinear term moderately improves the fit but seriously complicates the analysis (because two dependent time indices have now to be extrapolated in the future). Therefore, we confine our study to the single bilinear term model.

5.2. Parameter estimates

For the sake of comparison, we give on all figures both the results obtained with the classical Lee–Carter methodology (dashed lines) and the ones obtained with the Poisson modeling described in Section 4 (solid lines).

Fig. 5.1 plots the estimated α_x, β_x and κ_t (for the female population). This clearly illustrates the fact that similar trends are observed even if the way to calculate α_x, β_x and κ_t are different. Appendices B, C and D contain the detailed numerical values.

5.3. Forecasting

Box–Jenkins methodology (identification–estimation–diagnosis) is used to generate the appropriate ARIMA time series model for the male and female mortality indexes. The estimated models (ARIMA(0,1,1)) are

$$\kappa_t - \kappa_{t-1} = C_{\mathrm{m}} + \varepsilon_t + \theta_{\mathrm{m}} \varepsilon_{t-1} \tag{5.1}$$

for males and

$$\kappa_t - \kappa_{t-1} = C_{\mathrm{f}} + \varepsilon_t + \theta_{\mathrm{f}} \varepsilon_{t-1} \tag{5.2}$$

for females. The constant terms (C_{m} and C_{f}) indicate the average annual change of κ_t, and it is this change that drives the forecasts of the long-run change in mortality. The ε_t is the independent disturbance (random error). The resulting values for the parameters of the models are given in Table 3, both for the κ_t's obtained via the classical Lee–Carter method and for the Poisson case.

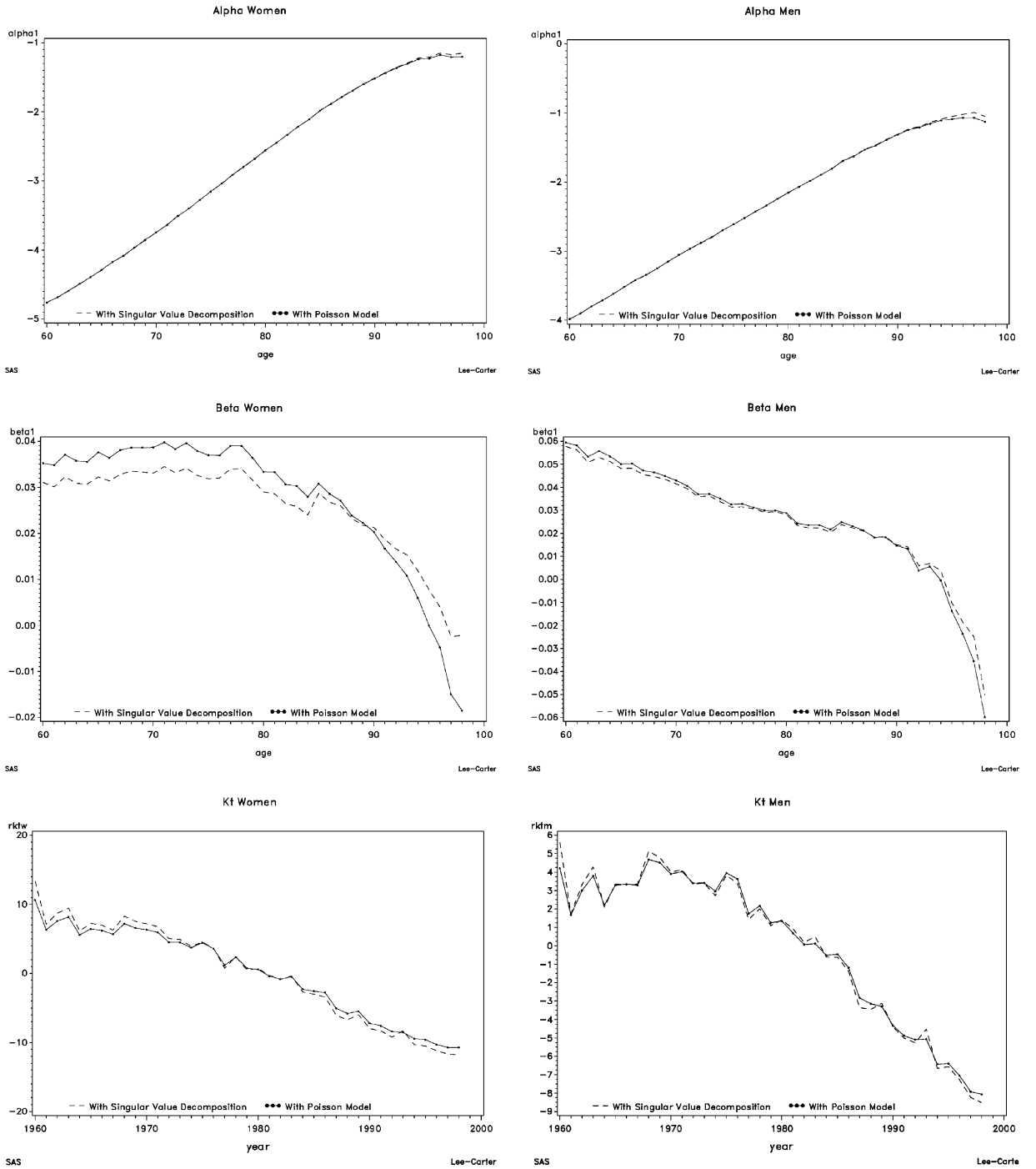

Fig. 5.1. Estimations of α_x, β_x and κ_t.

Table 3
Estimation of the parameters of the models (5.1) and (5.2)

	Men		Women	
	$\hat{C}_m$	$\hat{\theta}_m$	$\hat{C}_f$	$\hat{\theta}_f$
SVD	−0.34988	−0.39603	−0.63191	−0.46299
Poisson	−0.31324	−0.27881	−0.54574	−0.48978

The sex-specific estimated values of κ_t are shown with their 95% interval forecasts in Fig. 5.2. Appendix D gives the complete results in tabular form. The fitted ARIMA(0,1,1) model generates mortality forecasts by first forecasting κ_t. The reconstituted sex-specific forces of mortality are then used to generate sex-specific life expectancies and life annuities. Most of the variance over time at any given age is explained by the parameter κ_t. Proportions of the variance accounted for by the model (ratio of the variance of differences between the actual and fitted rates to the variance for the actual rates) over the years 1960–1998 at different ages are given in Table 4. We see that for Belgian data, the proportion of the total temporal variance in mortality rates accounted by both models ((3.1) and (4.1)) through ages 60–98 is in most of the cases above the 90%. The Poisson model performs better at the highest ages (over 90). An overall measure of goodness-of-fit proposed by Lee and Carter (1992) is obtained by summing all the

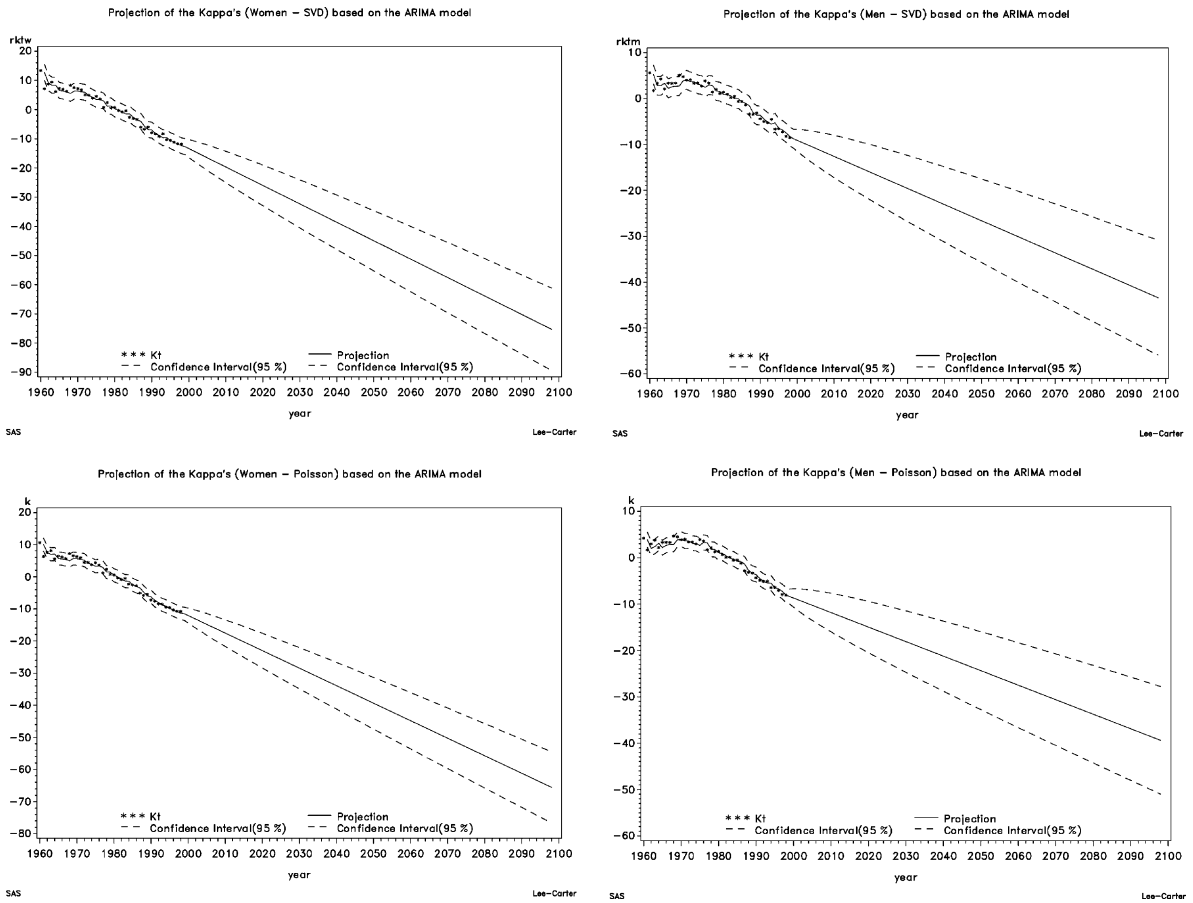

Fig. 5.2. Estimated values of κ_t for the different cases.

Table 4
Proportions of the variance accounted for by the model

Age	SVD		Poisson	
	Men	Women	Men	Women
60	0.9613	0.9483	0.9662	0.9562
61	0.9778	0.9670	0.9798	0.9689
62	0.9767	0.9784	0.9770	0.9809
63	0.9806	0.9821	0.9801	0.9820
64	0.9828	0.9804	0.9845	0.9803
65	0.9864	0.9784	0.9873	0.9787
66	0.9817	0.9837	0.9836	0.9827
67	0.9817	0.9832	0.9848	0.9846
68	0.9850	0.9921	0.9873	0.9941
69	0.9855	0.9922	0.9893	0.9942
70	0.9779	0.9823	0.9818	0.9857
71	0.9818	0.9884	0.9849	0.9904
72	0.9714	0.9907	0.9738	0.9925
73	0.9792	0.9920	0.9805	0.9939
74	0.9772	0.9884	0.9759	0.9893
75	0.9851	0.9922	0.9843	0.9931
76	0.9895	0.9914	0.9896	0.9935
77	0.9806	0.9835	0.9799	0.9867
78	0.9806	0.9819	0.9780	0.9837
79	0.9874	0.9905	0.9858	0.9931
80	0.9835	0.9934	0.9814	0.9950
81	0.9922	0.9888	0.9905	0.9910
82	0.9879	0.9913	0.9842	0.9922
83	0.9729	0.9895	0.9689	0.9915
84	0.9712	0.9914	0.9662	0.9926
85	0.9842	0.9883	0.9827	0.9915
86	0.9808	0.9878	0.9796	0.9906
87	0.9828	0.9893	0.9826	0.9921
88	0.9733	0.9832	0.9741	0.9856
89	0.9547	0.9843	0.9565	0.9875
90	0.9504	0.9810	0.9528	0.9864
91	0.9677	0.9823	0.9708	0.9876
92	0.9316	0.9704	0.9408	0.9781
93	0.8965	0.9359	0.9129	0.9466
94	0.8538	0.9300	0.8818	0.9464
95	0.8866	0.9166	0.9090	0.9384
96	0.8714	0.8396	0.9027	0.8760
97	0.8966	0.7506	0.9158	0.8038
98	0.7369	0.8200	0.8228	0.8500
Overall	0.8859	0.8951	0.9079	0.9138

unexplained age group variances and taking their ratio to the sum of total variances over ages. These results can be found in the last line of Table 5. The Poisson model accounts for slightly more variability than its SVD counterpart.

5.4. *Forecasting e_{65} and a_{65}*

Lee and Carter (1992, Appendix B) reported that for life expectancy forecasts, it is reasonable to restrict attention to the errors in forecasting the mortality index and to ignore those in fitting the mortality matrix, even for short run forecasts. Therefore, we have based the CI on the variability relating to the mortality index. The resulting life

Table 5
Life expectancies at the age of 65, in function of the year this age is reached

Year	Men	CI	Women	CI
SVD				
1999	16.01	[15.03, 16.96]	21.21	[19.89, 22.09]
2000	16.09	[15.06, 17.08]	21.33	[19.98, 22.24]
2001	16.17	[15.10, 17.21]	21.46	[20.07, 22.39]
2002	16.25	[15.14, 17.33]	21.59	[20.16, 22.54]
2003	16.33	[15.18, 17.44]	21.72	[20.25, 22.68]
2004	16.41	[15.22, 17.55]	21.84	[20.34, 22.82]
2005	16.49	[15.26, 17.67]	21.97	[20.44, 22.96]
Poisson				
1999	15.91	[15.02, 16.78]	21.03	[19.99, 21.82]
2000	15.99	[15.04, 16.90]	21.15	[20.08, 21.96]
2001	16.06	[15.07, 17.02]	21.26	[20.17, 22.10]
2002	16.14	[15.11, 17.13]	21.38	[20.26, 22.24]
2003	16.21	[15.14, 17.24]	21.49	[20.35, 22.37]
2004	16.28	[15.18, 17.35]	21.60	[20.44, 22.50]
2005	16.36	[15.22, 17.46]	21.72	[20.53, 22.63]

Table 6
Life annuities at the age of 65, in function of the year this age is reached

Year	Men	CI	Women	CI
SVD				
1999	10.68	[10.17, 11.17]	13.18	[12.64, 13.62]
2000	10.72	[10.19, 11.24]	13.24	[12.68, 13.69]
2001	10.77	[10.21, 11.31]	13.30	[12.72, 13.76]
2002	10.81	[10.23, 11.37]	13.36	[12.77, 13.83]
2003	10.86	[10.25, 11.43]	13.41	[12.81, 13.90]
2004	10.90	[10.27, 11.49]	13.47	[12.85, 13.96]
2005	10.94	[10.30, 11.55]	13.53	[12.90, 14.02]
Poisson				
1999	10.63	[10.17, 11.08]	13.15	[12.71, 13.53]
2000	10.68	[10.18, 11.15]	13.21	[12.75, 13.60]
2001	10.72	[10.20, 11.21]	13.26	[12.79, 13.67]
2002	10.76	[10.22, 11.27]	13.31	[12.83, 13.73]
2003	10.80	[10.24, 11.33]	13.37	[12.87, 13.79]
2004	10.84	[10.25, 11.39]	13.42	[12.92, 13.85]
2005	10.88	[10.28, 11.45]	13.47	[12.96, 13.91]

expectancies at the age of 65 are given in Table 5, while the resulting life annuities can be found in Table 6. Both methods are also compared here. It is interesting to note that the Poisson approach gives lower forecasts compared to the classical Lee–Carter model.

6. Measuring the impact of adverse selection

6.1. Log-linear approach

A peculiarity of the mortality projection problem, which seems not to have been examined so far in the literature, and which is crucial for actuaries, is the adverse selection characterizing life annuities markets. We resort here to a

Brass-type relational model to quantify the impact of this phenomenon on annuity premiums. The idea is to build a function $f(\mu_x)$ and to relate the mortality in a population under study (the annuitants, in our case) to that in a reference population whose mortality rates are μ_x^{ref} (the whole Belgian population, in our case), so that

$$f(\mu_x) = \vartheta_1 + \vartheta_2 f(\mu_x^{\text{ref}}).$$

Examples of the function $f(\cdot)$ include logarithm and logit. Note that we have dropped the reference to the calendar time t since the paucity of market data often forces the actuary to concentrate on a particular period of time.

Usually, the actuary has some mortality statistics about annuitants at his disposal, either market statistics, or data from some insurance portfolio. Annuitants mortality data are often much more scarce than official statistics. It is therefore hopeless to reproduce an approach in the spirit of the Lee–Carter method.

In Brouhns and Denuit (2001c), a clear linear relationship between forces of mortality relating to the entire Belgian population (μ_x^{NIS}) and the annuitants reflected in the statistics gathered by the regulatory authorities (μ_x^{RA}) has been detected for the period 1997–1999 (explaining more than 96% of the variation, for both males and females). Specifically, the model

$$\ln \mu_x^{\text{RA}} = \vartheta_1 + \vartheta_2 \ln \mu_x^{\text{NIS}} \tag{6.1}$$

has been estimated on the basis of the 1997–1999 period lifetable (the last available from the Belgian National Institute of Statistics). The results are displayed in Table 7.

Henceforth, let us denote as $\mu_x^{\text{NIS}}(t)$ (resp. $\mu_x^{\text{RA}}(t)$) the mortality force of the Belgian population at age x during year t, as reflected in the NIS data (resp. of the Belgian annuitants at age x during year t, as reflected in the data collected by the Belgian regulatory authorities). Assuming that the relation (6.1) relating national and annuitants forces of mortality remains valid over time, we get

$$\mu_x^{\text{RA}}(t) = \exp(\vartheta_1)\{\mu_x^{\text{NIS}}(t)\}^{\vartheta_2},$$

where we insert the estimates of Table 7.

6.2. *Poisson modelling*

We can also address this problem with a Brass-type relational model, embedded in a Poisson regression framework. We assume here that

$$D_x^{\text{RA}} \sim \text{Poisson}(E_x \mu_x^{\text{RA}}) \quad \text{with } \mu_x^{\text{RA}} = \exp\{\varrho_1 + \varrho_2 \ln \mu_x^{\text{NIS}}\},$$

where the μ_x^{NIS}'s are treated as known constants. The model has been fitted to the data relating to the period 1997–1999. The parameter estimation via the SAS procedure GENMOD gives the results summarized in Table 8. This is comparable to Table 7, except that we have gained in precision: the confidence intervals are in this case smaller than with the linear approach.

Table 7
Results for the linear regression model

	$\widehat{\vartheta_1}$	$\sigma(\widehat{\vartheta_1})$	95% CI	$\widehat{\vartheta_2}$	$\sigma(\widehat{\vartheta_2})$	95% CI
Women	−0.9512	0.1014	[−1.1499, −0.7525]	0.9453	0.0300	[0.8865, 1.0041]
Men	−1.2928	0.0733	[−1.4365, −1.1491]	0.8322	0.0261	[0.7810, 0.8842]

Table 8
Parameter estimation for the Poisson model

	$\widehat{\varrho_1}$	$\sigma(\widehat{\varrho_1})$	95% CI	$\widehat{\varrho_2}$	$\sigma(\widehat{\varrho_2})$	95% CI
Women	−0.8706	0.0588	[−0.9859, −0.7553]	0.9522	0.0170	[0.9190, 0.9855]
Men	−1.2401	0.0593	[−1.3564, −1.1238]	0.8422	0.0167	[0.8094, 0.8750]

Table 9
Comparison life expectancies and life anuities calculated with and without adjusting for antiselection

Year 65 is reached	e_{65} Poisson	e_{65}^{RA} SVD	e_{65}^{RA} Poisson	Deviation (%)	a_{65} Poisson	a_{65}^{RA} SVD	a_{65}^{RA} Poisson	Deviation (%)
Women								
1999	21.03	26.90	25.44	21.0	13.15	15.44	15.07	14.6
2000	21.15	26.99	25.52	20.7	13.21	15.48	15.11	14.4
2001	21.26	27.07	25.61	20.4	13.26	15.52	15.15	14.2
2002	21.38	27.15	25.69	20.2	13.31	15.55	15.18	14.0
2003	21.49	27.24	25.77	19.9	13.37	15.59	15.22	13.9
2004	21.60	27.32	25.85	19.7	13.42	15.62	15.25	13.7
2005	21.72	27.39	25.93	19.4	13.47	15.65	15.29	13.5
2006	21.83	27.47	26.01	19.1	13.53	15.69	15.32	13.3
2007	21.94	27.55	26.09	18.9	13.58	15.72	15.36	13.1
2008	22.05	27.63	26.16	18.7	13.63	15.75	15.39	12.9
2009	22.16	27.70	26.24	18.4	13.68	15.79	15.42	12.8
2010	22.27	27.77	26.31	18.2	13.73	15.82	15.46	12.6
Men								
1999	15.91	22.84	21.94	37.9	10.63	13.66	13.43	26.3
2000	15.99	22.89	22.00	37.6	10.67	13.69	13.46	26.1
2001	16.06	22.95	22.05	37.3	10.72	13.71	13.49	25.9
2002	16.14	23.00	22.11	37.0	10.76	13.74	13.52	25.7
2003	16.21	23.05	22.16	36.7	10.80	13.77	13.54	25.4
2004	16.28	23.10	22.22	36.4	10.84	13.79	13.57	25.2
2005	16.36	23.16	22.27	36.2	10.88	13.82	13.60	25.0
2006	16.43	23.21	22.33	35.9	10.92	13.84	13.62	24.8
2007	16.50	23.26	22.38	35.6	10.96	13.87	13.65	24.6
2008	16.58	23.31	22.43	35.3	11.00	13.89	13.67	24.4
2009	16.65	23.36	22.49	35.1	11.03	13.92	13.70	24.2
2010	16.72	23.41	22.54	34.8	11.07	13.94	13.73	23.9

6.3. Incorporating adverse selection in the price list

In order to be aware of the consequences of adverse selection on pure premiums, we have computed the difference between life expectancies and life annuities calculated either on the whole Belgian population, either on the insured population. The results are summarized in Table 9. Relative deviations were calculated as follows: $(e_{65}^{RA} - e_{65}^{NIS})/e_{65}^{NIS}$ (with e_{65}^{RA} calculated in the Poisson framework) and similarly $(a_{65}^{RA} - a_{65}^{NIS})/a_{65}^{NIS}$. We see that the impact of adverse selection on pure premiums relating to life annuities may be as large as 15% for women and 26% for men. As above, the adaptation based on the Poisson model gives lower premiums than the linear regression approach.

7. Conclusion

This paper proposes a new method for building projected lifetables. We substituted a log-bilinear Poisson regression model for SVD in the Lee–Carter approach, implementing a suggestion of Alho (2000). The results are

in accordance with those produced by SVD, but the Poisson approach allows for many other applications in life insurance, in particular the projection of future cash flows.

In this paper, we have used the likelihood ratio statistic for model selection (i.e., to decide on retaining or deleting some effects in the model). Another descriptive measure that could be used is the absolute difference between observed and estimated cell counts (called the dissimilarity index DI in LEM). For example, for males model 1 in Table 2 has a value of 0.0637 and model 4 a value of 0.0180. A model with a constant rate of mortality across time and age has a value of 0.2961. For model 4, this means that the discrepancy between the observed deaths table and its estimated counterpart is only 1.8%.

Recently, Renshaw and Haberman (2002) investigated the feasibility of constructing mortality forecasts on the basis of the first two sets of SVD vectors, rather than just on the first set of such vectors, as in the Lee–Carter approach. These authors also considered generalized linear and bilinear models with Poisson error structures. We refer the readers to this excellent paper for more details.

Mortality trends may differ from the forecasted trend. This originates the longevity risk. The longevity risk is thus attributable to systematic deviations of the mortality from the projected mortality assumed in the calculation basis (used in pricing and reserving). In a companion paper Brouhns et al. (2002), we show how the model proposed in this paper may be used to deal with the longevity risk.

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Appendix A. LEM input files

This is the LEM input file that estimates the Poisson version of the Lee–Carter model:

```
man 2
dim 40 39
lab X T
mod{wei(XT), X, spe(T,1a,X,b)}
dat deaths.dat
sta wei(XT) exposures.dat
```

The command “`man`” indicated the number of (manifest) variables, in this case 2. With “`dim`”, one specifies the number of levels of the variables. For females, we had 40 age groups and 39 time points. The command “`lab`” is used to specify variable labels. The “`mod`” statement is used to specify the three relevant model terms: the exposures [`wei(XT)`], the age effect [`X`], and the bilinear term [`spe(T,1a,X,b)`]. It is assumed that the files “`deaths.dat`” and “`exposures.dat`” contain the tables with observed counts and exposure times. The commands “`dat`” and “`sta`” are used to specify these data files.

Appendix B. Values of α

Age	SVD		Poisson	
	Men	Women	Men	Women
60	−3.98	−4.76	−3.99	−4.76
61	−3.90	−4.69	−3.90	−4.68
62	−3.80	−4.59	−3.80	−4.59
63	−3.72	−4.49	−3.71	−4.49
64	−3.62	−4.39	−3.62	−4.39
65	−3.52	−4.29	−3.52	−4.29
66	−3.42	−4.18	−3.42	−4.18
67	−3.34	−4.09	−3.34	−4.09
68	−3.25	−3.97	−3.25	−3.97
69	−3.15	−3.86	−3.15	−3.86
70	−3.06	−3.75	−3.05	−3.75
71	−2.97	−3.64	−2.97	−3.64
72	−2.88	−3.51	−2.88	−3.51
73	−2.80	−3.40	−2.80	−3.40
74	−2.70	−3.28	−2.70	−3.28
75	−2.61	−3.15	−2.61	−3.15
76	−2.52	−3.04	−2.52	−3.04
77	−2.43	−2.91	−2.43	−2.91
78	−2.34	−2.80	−2.34	−2.80
79	−2.25	−2.68	−2.24	−2.68
80	−2.16	−2.56	−2.15	−2.56
81	−2.07	−2.45	−2.07	−2.45
82	−1.98	−2.34	−1.98	−2.33
83	−1.89	−2.21	−1.89	−2.21
84	−1.81	−2.11	−1.81	−2.11
85	−1.69	−1.98	−1.70	−1.98
86	−1.62	−1.88	−1.63	−1.89
87	−1.53	−1.78	−1.53	−1.79
88	−1.47	−1.69	−1.47	−1.69
89	−1.38	−1.59	−1.39	−1.60
90	−1.31	−1.51	−1.32	−1.52
91	−1.23	−1.43	−1.25	−1.44
92	−1.20	−1.36	−1.21	−1.37
93	−1.14	−1.29	−1.15	−1.30
94	−1.09	−1.22	−1.11	−1.24
95	−1.05	−1.21	−1.09	−1.23
96	−1.02	−1.15	−1.07	−1.18
97	−0.99	−1.17	−1.07	−1.21
98	−1.05	−1.15	−1.13	−1.20

Appendix C. Values of β

Age	SVD		Poisson	
	Men	Women	Men	Women
60	0.0576	0.0310	0.0594	0.0352
61	0.0563	0.0301	0.0582	0.0348
62	0.0508	0.0323	0.0533	0.0371
63	0.0529	0.0309	0.0557	0.0357
64	0.0512	0.0307	0.0534	0.0355
65	0.0481	0.0322	0.0501	0.0376
66	0.0482	0.0314	0.0503	0.0364
67	0.0455	0.0328	0.0473	0.0381
68	0.0447	0.0335	0.0464	0.0386
69	0.0434	0.0333	0.0448	0.0386
70	0.0415	0.0331	0.0430	0.0386
71	0.0394	0.0345	0.0407	0.0398
72	0.0358	0.0332	0.0369	0.0383
73	0.0362	0.0341	0.0371	0.0396
74	0.0337	0.0326	0.0351	0.0379
75	0.0314	0.0318	0.0325	0.0370
76	0.0314	0.0320	0.0328	0.0369
77	0.0307	0.0340	0.0313	0.0390
78	0.0292	0.0340	0.0300	0.0390
79	0.0294	0.0315	0.0299	0.0364
80	0.0281	0.0290	0.0288	0.0333
81	0.0235	0.0286	0.0243	0.0333
82	0.0222	0.0264	0.0236	0.0307
83	0.0224	0.0258	0.0236	0.0303
84	0.0204	0.0240	0.0217	0.0279
85	0.0238	0.0287	0.0248	0.0308
86	0.0224	0.0267	0.0232	0.0285
87	0.0210	0.0260	0.0213	0.0271
88	0.0183	0.0233	0.0182	0.0239
89	0.0185	0.0217	0.0183	0.0223
90	0.0150	0.0213	0.0146	0.0203
91	0.0143	0.0187	0.0132	0.0167
92	0.0059	0.0166	0.0038	0.0138
93	0.0069	0.0153	0.0056	0.0108
94	0.0038	0.0118	−0.0004	0.0059
95	−0.0102	0.0078	−0.0137	0.0000
96	−0.0186	0.0040	−0.0236	−0.0048
97	−0.0249	−0.0025	−0.0355	−0.0150
98	−0.0503	−0.0021	−0.0598	−0.0185

Appendix D. Values of κ

Estimated κ_t (1960–1998) and forecasted κ_t (1999–2040)

Year	SVD		Poisson	
	Men	Women	Men	Women
1960	5.61	13.36	4.20	10.62
1961	1.74	7.10	1.68	6.32
1962	3.34	8.75	3.01	7.59
1963	4.29	9.43	3.79	8.15
1964	2.10	6.15	2.21	5.58
1965	3.35	7.25	3.29	6.43
1966	3.32	6.99	3.34	6.21
1967	3.36	6.28	3.29	5.69
1968	5.12	8.28	4.68	7.19
1969	4.79	7.52	4.52	6.58
1970	4.02	7.16	3.90	6.32
1971	4.13	6.79	4.03	5.96
1972	3.34	5.02	3.40	4.51
1973	3.40	4.90	3.41	4.50
1974	2.72	3.92	2.97	3.74
1975	3.84	4.53	3.95	4.42
1976	3.38	3.62	3.63	3.60
1977	1.42	0.76	1.74	1.19
1978	2.01	2.42	2.17	2.36
1979	1.10	0.51	1.25	0.75
1980	1.41	0.71	1.35	0.58
1981	0.92	−0.28	0.69	−0.40
1982	0.20	−0.86	0.07	−0.83
1983	0.48	−0.37	0.12	−0.49
1984	−0.62	−2.67	−0.52	−2.29
1985	−0.63	−3.00	−0.46	−2.57
1986	−1.37	−3.41	−1.19	−2.77
1987	−3.36	−6.04	−2.83	−5.05
1988	−3.45	−6.75	−3.15	−5.82
1989	−3.13	−5.98	−3.29	−5.49
1990	−4.40	−7.97	−4.33	−7.22
1991	−5.01	−8.33	−4.88	−7.59
1992	−5.27	−9.23	−5.09	−8.41
1993	−4.52	−8.31	−5.07	−8.54
1994	−6.65	−10.31	−6.42	−9.46
1995	−6.58	−10.53	−6.39	−9.61
1996	−7.28	−11.18	−7.05	−10.31
1997	−8.24	−11.68	−7.92	−10.72
1998	−8.52	−11.82	−8.07	−10.72
1999	−8.77	−12.72	−8.38	−11.55
2000	−9.12	−13.36	−8.69	−12.09

Appendix D. (*Continued*)

Year	SVD		Poisson	
	Men	Women	Men	Women
2001	−9.47	−13.99	−9.01	−12.64
2002	−9.82	−14.62	−9.32	−13.18
2003	−10.17	−15.25	−9.63	−13.73
2004	−10.52	−15.88	−9.95	−14.27
2005	−10.87	−16.52	−10.26	−14.82
2006	−11.22	−17.15	−10.57	−15.37
2007	−11.57	−17.78	−10.89	−15.91
2008	−11.92	−18.41	−11.20	−16.46
2009	−12.27	−19.04	−11.51	−17.00
2010	−12.62	−19.67	−11.83	−17.55
2011	−12.97	−20.31	−12.14	−18.09
2012	−13.32	−20.94	−12.45	−18.64
2013	−13.67	−21.57	−12.76	−19.19
2014	−14.02	−22.20	−13.08	−19.73
2015	−14.37	−22.83	−13.39	−20.28
2016	−14.72	−23.47	−13.70	−20.82
2017	−15.07	−24.10	−14.02	−21.37
2018	−15.42	−24.73	−14.33	−21.91
2019	−15.77	−25.36	−14.64	−22.46
2020	−16.12	−25.99	−14.96	−23.01
2021	−16.47	−26.63	−15.27	−23.55
2022	−16.82	−27.26	−15.58	−24.10
2023	−17.17	−27.89	−15.90	−24.64
2024	−17.52	−28.52	−16.21	−25.19
2025	−17.87	−29.15	−16.52	−25.73
2026	−18.22	−29.79	−16.84	−26.28
2027	−18.57	−30.42	−17.15	−26.83
2028	−18.92	−31.05	−17.46	−27.37
2029	−19.27	−31.68	−17.78	−27.92
2030	−19.62	−32.31	−18.09	−28.46
2031	−19.97	−32.94	−18.40	−29.01
2032	−20.32	−33.58	−18.72	−29.56
2033	−20.67	−34.21	−19.03	−30.10
2034	−21.02	−34.84	−19.34	−30.65
2035	−21.37	−35.47	−19.66	−31.19
2036	−21.72	−36.10	−19.97	−31.74
2037	−22.07	−36.74	−20.28	−32.28
2038	−22.42	−37.37	−20.60	−32.83
2039	−22.77	−38.00	−20.91	−33.38
2040	−23.12	−38.63	−21.22	−33.92

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