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STABILITY OF A DISCRETE-TIME, MACROECONOMIC DIS-
EQUILIBRIUM MODEL.

B. Kaper

STABILITY OF A DISCRETE-TIME, MACROECONOMIC DISEQUILIBRIUM MODEL.

ABSTRACT.

We investigate the stability of a macroeconomic monetary discrete-time model with a constraint on the market for bankcredit. A theorem is proved on asymptotic stability of a piecewise linear discrete-time system in $\mathbb{R}^2$ which is not overall linear.

1. INTRODUCTION.

In [3] macroeconomic monetary models have been developed with constraints on the market for bankcredit. In studying the dynamics of these disequilibrium models we met the problem of asymptotic stability of discrete-time systems which are not standard in the theory of difference equations, [5]. Similar problems arise in continuous-time disequilibrium problems (c.f. for instance [1]). Some advancements on that field of research have been made by e.g. Laroque [4], v.d. Heuvel [2]. In this paper we present a theorem on asymptotic stability of a piecewise linear discrete-time system in $\mathbb{R}^2$ which is not overall linear. This theorem is applicable to a macroeconomic monetary disequilibrium model that has been abstracted from [3]. In order to get a connection as well as possible with the disequilibrium models in Koning we will use the same type of variables.

2. A MACRO ECONOMIC DISEQUILIBRIUM MODEL.

We consider a macroeconomic monetary model with constraints on the market for bankcredit. The variables of the model represent relative deviations of their values on paths of balanced growth. If $\tilde{x}$ symbolises the actual value and $\tilde{x}_e$ the balanced-growth value of a variable of the model then the relative deviation of the actual value with respect to the balanced-growth value of the variable (x) is

$$x := \frac{\tilde{x} - \tilde{x}_e}{\tilde{x}_e}$$

In the neighborhood of the equilibrium the first difference (Δx),

$$\Delta x := x - x_{-1} \quad 1)$$

is approximately equal to the difference of the actual- and the balanced-growthrate of the variable x in the past period. This is also known as the extra growthrate of the variable in question (cf. the appendix).

If in the model all exogeneous variables are zero and there is equality between the actual and balanced-growth value the relative variables of the model will persist in zero-value at all time. If any exogeneous variable is given a non zero value relative variables of the model will leave their zero position. In order to analyse the effect of a permanent pulse on any of the exogeneous variables we will multiply these variables with the Heavisidefunction H ,

$$\begin{aligned} H(x) &= 0 & , \text{ if } x < 0 \\ &= 1 & , \text{ if } x \geq 0. \end{aligned}$$

The value of all relative variables of the model will be equal to zero if all exogeneous variables are not effective in the model.

The following quantities are involved by the model (the characters in brackets represent the exogeneous part of the variable in question):

y	national income
$c(\gamma)$	demand for consumption
$i(1)$	demand for investment goods

1) The subscript -1 indicates a retardation of one period.

r_b interest rate of bankcredit
 b_d demand for bankcredit
 $b_s(\sigma)$ supply of bankcredit (σ : discount rate)

The model is given by the following set of equilibrium- and adjustment equations:

$$(2.1) \quad c = \gamma_1 y + \gamma H$$

$$(2.2) \quad i = -\iota_1 r_{b-1} + \iota_2 (y - y_{-1}) - \iota_3 (b_{d-1} - b_{-1}) + \iota H$$

$$(2.3) \quad y = \eta_1 c + \eta_2 i$$

$$(2.4) \quad b_d = -\delta_1 r_b + \delta_2 y$$

$$(2.5) \quad b_s = \sigma_1 r_b - \sigma H$$

$$(2.6) \quad r_b = r_{b-1} + \rho_1 (b_{d-1} - b_{s-1})$$

where

$$b = \min \{b_d, b_s\}$$

The greek characters provided with a subindex are positive (adjustment) constants of the model. The model will be reduced to a set of first order difference equations in $\mathbb{R}^2$. From (2.1) - (2.3) we get an equation for y ,

$$(2.7) \quad y = \alpha [-\eta_2 \iota_1 r_{b-1} - \eta_2 \iota_2 y_{-1} - \eta_2 \iota_3 (b_{d-1} - b_{-1}) + (\eta_1 \gamma + \eta_2 \iota) H]$$

where

$$\alpha := (1 - \eta_1 \gamma_1 - \eta_2 \iota_2)^{-1}$$

and

$$b_{d-1} - b_{-1} = \min \{0, (\delta_1 + \sigma_1) r_{b-1} - \delta_2 y_{-1} - \sigma H\}.$$

From (2.4) - (2.6) we derive an equation for r_b ,

$$(2.8) \quad r_b = r_{b-1} + \rho_1 (-\delta_1 r_{b-1} + \delta_2 y_{-1} - \sigma_1 r_{b-1} + \sigma H).$$

Let us introduce a new set of variables,

$$x_1(n) := y_{-1}$$

$$x_2(n) := r_{b-1}$$

We get a first-order system in $\mathbb{R}^2$:

$$(2.9) \quad \begin{cases} x_1(n+1) = -\alpha\eta_2 l_2 x_1(n) - \alpha\eta_2 l_1 x_2(n) - \alpha\eta_2 l_3 \min\{0, \\ -\delta_2 x_1(n) + (\delta_1 + \sigma_1)x_2(n) - \sigma H(n)\} + \alpha(\eta_1 \gamma + \eta_2 l)H(n) \\ x_2(n+1) = \rho_1 \delta_2 x_1(n) + (1 - \delta_1 \rho_1 - \sigma_1 \rho_1)x_2(n) + \rho_1 \sigma H(n). \end{cases}$$

Assuming that all variables were equal to zero up to the zeroth period we get at period 1

$$\begin{cases} x_1(1) = -\alpha\eta_2 l_3 \min\{0, -\sigma\} \\ x_2(1) = \rho_1 \sigma \end{cases}$$

This will be taken as the initial value of the first order system (2.9). The Heaviside function then might be replaced by the value 1. System (2.9) consists of two linear subsystems whereas the system itself is ofcourse not overall linear:

Subsystem 1, if $-\delta_2 x_1(n) + (\delta_1 + \sigma_1)x_2(n) - \sigma < 0$

$$\underline{x}(n+1) = A_1 \underline{x}(n) + \underline{b}_1$$

where

$$A_1 = \begin{bmatrix} -\alpha\eta_2 l_2 + \alpha\eta_2 l_3 \delta_2 & -\alpha\eta_2 l_1 - \alpha\eta_2 l_3 (\delta_1 + \sigma_1) \\ \rho_1 \delta_2 & 1 - (\delta_1 + \sigma_1) \rho_1 \end{bmatrix}$$

$$\underline{b}_1 = [\alpha\eta_2 l_3 \sigma + \alpha(\eta_1 \gamma + \eta_2 l), \rho_1 \sigma]^T$$

and

$$\underline{x}(n) := [x_1(n), x_2(n)]^T;$$

Subsystem 2, if $-\delta_2 x_1(n) + (\delta_1 + \sigma_1)x_2(n) - \sigma \geq 0$

$$\underline{x}(n+1) = A_2 \underline{x}(n) + \underline{b}_2$$

where

$$A_2 = \begin{bmatrix} -\alpha\eta_2 l_2 & -\alpha\eta_2 l_1 \\ \rho_1 \delta_2 & 1 - (\delta_1 + \sigma_1) \rho_1 \end{bmatrix}$$

and

$$\underline{b}_2 = [\alpha(\eta_1\gamma + \eta_2\iota), \rho_1\sigma].$$

The equilibrium position $\underline{x}$ of the overall system (2.9) is just equal to the common equilibrium solution of both subsystems provided it exists (c.f. the appendix),

$$\underline{x} = (I - A_i)^{-1} \underline{b}, \quad i = 1 \text{ or } 2.$$

Finally we apply a linear transformation to system (2.9),

$$\underline{y}(n) = \underline{x}(n) - \underline{x},$$

which transforms (2.9) into a homogeneous first-order system in $\mathbb{R}^2$,

$$(2.10) \quad \begin{cases} y_1(n+1) = -\alpha\eta_2\iota_2 y_1(n) - \alpha\eta_2\iota_1 y_2(n) - \min \{0, -\delta_2 y_1(n) + (\delta_1 + \sigma_1) y_2(n)\} \\ y_2(n+1) = \rho_1 \delta_2 y_1(n) + (1 - \delta_1 \rho_1 - \sigma_1 \rho_1) y_2(n) \end{cases}$$

or equivalently into two linear subsystems,
if

$$-\delta_2 y_1(n) + (\delta_1 + \sigma_1) y_2(n) < 0$$

$$\underline{y}(n+1) = A_1 \underline{y}(n),$$

else

$$\underline{y}(n+1) = A_2 \underline{y}(n).$$

The minimum function in (2.10) implies continuity of the right hand side of (2.10). In the next section it will be shown that the $\underline{0}$ solution of such a homogeneous piecewise linear system in $\mathbb{R}^2$ will be asymptotically stable if both subsystems are asymptotically stable.

CONCLUSION: The equilibrium of (2.10) is asymptotically stable if both subsystems are asymptotically stable, i.e. if

$$|\operatorname{tr} A_i| - 1 < \det A_i < 1, \quad i \in \{1, 2\}$$

Or equivalently, the equilibrium position of the model (2.1) - (2.6) is asymptotically stable if

$$(2.11) \quad \begin{aligned} & | -\alpha\eta_2\iota_2 + \alpha\eta_2\iota_3\delta_2 + 1 - (\delta_1 + \sigma_1)\rho_1 | - 1 < (-\alpha\eta_2\iota_2 + \alpha\eta_2\iota_3\delta_2)(1 - [\delta_1 + \sigma_1]\rho_1) \\ & + (\alpha\eta_2\iota_1 + \alpha\eta_2\iota_3[\delta_1 + \sigma_1])\rho_1\delta_2 < 1 \end{aligned}$$

and

$$(2.12) \quad | -\alpha\eta_2\iota_2 + 1 - (\delta_1 + \sigma_1)\rho_1 | - 1 < (-\alpha\eta_2\iota_2)(1 - [\delta_1 + \sigma_1]\rho_1) + \alpha\eta_2\iota_1\rho_1\delta_2 < 1$$

NUMERICAL EXAMPLE: If we take the following set of coefficients in the model (2.1) - (2.6) then the conditions (2.11) and (2.12) are satisfied and the model is asymptotically stable:

	γ	ι	η	δ	σ	ρ
exog.	0	0	-	-	0.6	-
1	0.75	1	0.8	1	1	0.2
2	-	0.5	0.2	1	-	-
3	-	1	-	-	-	-

$$(\alpha = (0.3)^{-1}).$$

3. PIECEWISE-LINEAR DISCRETE DYNAMIC SYSTEMS IN $\mathbb{R}^2$.

We consider the following autonomous piecewise-linear difference equation in $\mathbb{R}^2$

$$(3.1) \quad \underline{x}_{n+1} = A \underline{x}_n$$

where

$$A := A_i \text{ if } \underline{x} \in C_i, \quad i \in I_n := \{1, 2, \dots, n\},$$

C_i be closed cones in $\mathbb{R}^2$ with vertices in the origin, with disjoint interiors, $\bigcup_{i=1}^n C_i = \mathbb{R}^2$; Let the numbering of the cones around the origin be anticlockwise

$$C_i \cap C_{i+1} := \{\lambda \underline{q}_{i+1} \mid \|\underline{q}_{i+1}\| = 1, \lambda \geq 0\}, \quad i \in I_n, \quad n+1 := 1$$

We will prove asymptotic stability of the zero solution of system (3.1) if $\underline{0}$ is an asymptotically stable solution of each of the linear subsystems

$$\underline{x}_{n+1} = A_i \underline{x}_n$$

and the function in the right hand side of (3.1) is continuous.

THEOREM.

Consider system (3.1). If

$$(3.2) \quad |\operatorname{tr} A_i| - 1 < \det A_i < 1, \quad \forall i \in I_n,$$

and

$$(3.3) \quad A_i \underline{q}_i = A_{i-1} \underline{q}_i$$

then system (3.1) is asymptotically stable.

PROOF. We will make use of the concept of a Liapunov function (c.f. [5]).

The form of the Liapunov function is based on the one constructed by Laroque for piecewise linear differential systems (c.f. [4]).

A refinement of Laroque's function has been introduced by van den Heuvel in his thesis [2]. Let $V : \mathbb{R}^2 \rightarrow \mathbb{R}$, defined by

$$V(\underline{x}) := \det^2[\underline{x}, A\underline{x}] + \epsilon \|\underline{x}\|^4$$

where ϵ is defined by (3.5). Because of (3.3) V is a continuous function that is positive definite on $\mathbb{R}^2$. Relative to the system (3.1) define

$$\dot{V}(\underline{x}) = V(A\underline{x}) - V(\underline{x}).$$

Then

$$\dot{V}(\underline{x}) = \det^2[A\underline{x}, A^2\underline{x}] + \varepsilon \|A\underline{x}\|^4 - \det^2[\underline{x}, A\underline{x}] - \varepsilon \|\underline{x}\|^4$$

$$\stackrel{1)}{=} \{\det^2(A_j) - 1\} \det^2[\underline{x}, A\underline{x}] + \varepsilon \{\|A\underline{x}\|^4 - \|\underline{x}\|^4\}$$

The expression $\det^2[\underline{x}, A\underline{x}]$ equals zero if $\underline{x}$ is a real eigenvector of A . Consider the case that for some index $i \in I_n$ A_i has an eigenvalue with a real eigenvector $\underline{y}$ that belongs to $\text{Int}(C_i)$. By virtue of (3.2) we have $|\lambda| < 1$.

Choose γ such that $0 < \gamma < 1 - \lambda^4$. Then the set K_y defined by

$$K_y := \{\underline{x} \in \mathbb{R}^2 \mid \|A\underline{x}\|^4 - \|\underline{x}\|^4 < -\gamma \|\underline{x}\|^4\} \cap \text{Int}(C_i)$$

contains y . K_y is an open cone. If the eigenvector of A_i just equals $\underline{q}_i$ then by the continuity property of $A \underline{q}_i$ is also an eigenvector of A_{i-1} :

$$\lambda \underline{q}_i = A_i \underline{q}_i = A_{i-1} \underline{q}_i.$$

In that case in the definition of $K_{\underline{q}_i}$ the intersection should be taken with

$$\text{Int}(C_{i-1} \cup C_i).$$

Let V be the set of all eigenvectors of A satisfying the above conditions,

$$V := \{\underline{y} \mid \exists i \in I_n : A_i \underline{y} = \lambda \underline{y}, \lambda \in \mathbb{R}, \lambda \neq 0 \wedge \underline{y} \in C_i\}.$$

To each $\underline{y} \in V$ an open cone K_y can be assigned. Define

$$K := \bigcup_{\underline{y} \in V} K_y.$$

Then by assumption (3.2) and the definition of K we have for each $\underline{x} \in K$

$$(3.4) \quad \dot{V}(\underline{x}) < -\varepsilon \gamma \|\underline{x}\|^4.$$

Let us determine next the scalars α , β , and ε

$$\alpha := \min \{\det^2[\underline{x}, A\underline{x}] \mid \|\underline{x}\| = 1, \underline{x} \notin K\}$$

$$\beta := \max \{\|A\underline{x}\| \mid \|\underline{x}\| = 1\}$$

1) Cayley-Hamilton theorem: $A^2 = \text{tr}(A) A - \det(A) I$.

and

$$(3.5) \quad \varepsilon := \frac{\alpha}{2\beta^4} [1 - \max \{\det^2(A_i) \mid i \in I_n\}].$$

Note that the scalars α and β exist by the continuity of the respective functions on compact sets. For each $\underline{x} \notin K$ we have

$$\dot{V}(\underline{x}) \leq \{\det^2(A_j) - 1\}\alpha\|\underline{x}\|^4 + \frac{\alpha}{2\beta^4}(1 - \max \{\det^2(A_i) \mid i \in I_n\}) \cdot \beta^4\|\underline{x}\|^4.$$

and hence

$$(3.6) \quad \dot{V}(\underline{x}) \leq \frac{1}{2}\alpha\{\det^2(A_j) - 1\}\|\underline{x}\|^4,$$

From the definition of V and the inequalities (3.4) and (3.6) we may conclude that V is a Liapunov function in the classical case: $V(\underline{x})$ and $-\dot{V}(\underline{x})$ are positive definite. Then $\underline{0}$ is (globally) asymptotically stable. $\square$

Appendices.

1. Let us denote the actual and balanced growth rate in the past period by g respectively g_e . Then

$$\tilde{x} = (1+g)\tilde{x}_{-1} \quad \text{and} \quad \tilde{x}_e = (1+g_e)\tilde{x}_{e-1}.$$

The first difference of the relative variable x is

$$\Delta x = \frac{\tilde{x} - \tilde{x}_e}{\tilde{x}_e} - \frac{\tilde{x}_{-1} - \tilde{x}_{e-1}}{\tilde{x}_{e-1}} = \frac{\tilde{x}_{-1}}{\tilde{x}_{e-1}} \cdot \frac{1}{1+g_e} \cdot (g - g_e) \approx g - g_e.$$

2. The equilibrium positions of the linear subsystems are equivalent if there holds the relation

$$(a.1) \quad (I - A_{e1})^{-1} \underline{b}_1 = (I - A_2)^{-1} \underline{b}_2$$

Let us define $\tilde{A}$, $\tilde{b}$, where $A_1 = A_2 + \tilde{A}$ and $\underline{b}_1 = \underline{b}_2 + \tilde{b}$,

$$\tilde{A} = \begin{bmatrix} \alpha \eta_{213} \delta_2 & -\alpha \eta_{213} (\delta_1 + \sigma_1) \\ 0 & 0 \end{bmatrix}$$

and

$$\tilde{b} = [\alpha \eta_{213} \sigma, 0]^T.$$

Relation (a.1) will successively be reduced into the following relations:

$$\underline{b}_2 + \tilde{b} = (I - A_2 - \tilde{A})(I - A_2)^{-1} \underline{b}_2$$

or eventually

$$\tilde{b} = -\tilde{A}(I - A_2)^{-1} \underline{b}_2.$$

The last relation can be checked by straight forward substitution.

3. The equilibrium position is given by $\underline{x} = [x_1, x_2]^T$,

$$x_1 = \beta^{-1} \{ (\delta_1 + \sigma_1) \rho_1 \alpha (\eta_1 \gamma + \eta_2 \iota) - \alpha \eta_{21} \rho_1 \sigma \},$$

$$x_2 = \beta^{-1} \{ \rho_1 \delta_2 \alpha (\eta_1 \gamma + \eta_2 \iota) + \rho_1 \sigma (1 + \alpha \eta_{21} \iota) \},$$

where

$$\beta = (1 + \alpha \eta_{21}^1) (\delta_1 + \sigma_1) \rho_1 + \alpha \eta_{21}^1 \rho_1 \delta_2.$$

In the numerical example the equilibrium is given by $\underline{x} = (-0.12, 0.24)$. The equilibrium quantities of the model are $c = -0.09$, $i = 0.24$, $y = -0.12$, $b = b_d = b_s = -0.36$, $r_b = 0.24$.

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