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INEFFICIENCY OF CREDIBLE STRATEGIES IN OLIGOPOLISTIC
RESOURCE MARKETS WITH UNCERTAINTY

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ABSTRACT

The implications of different information patterns for firms in oligopolistic resource markets are considered. The traditional open-loop Nash equilibrium with static information sets is one of many possible Nash equilibria and is not suitable for stochastic environments. When shocks to resource growth are serially uncorrelated, there are no gains from conditioning the harvest on past stock levels and the feedback or credible Nash equilibrium is the appropriate Nash equilibrium concept. This credible equilibrium assumes that firms have knowledge of current stocks of reserves, which typically leads to more rapid extraction of the resource and possibly extinction. Since the open-loop Nash equilibrium is efficient when demand is iso-elastic and extraction costs are zero, it is clear that an increase in information can be detrimental to firms in the industry.

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Appendix

1. Introduction

When a given number of firms deplete an exhaustible resource with zero extraction costs and iso-elastic demand, it has been argued that the oligopoly and cartel outcomes are efficient and that firms deplete according to Hotelling's rule (Khalatbari, 1977; Dasgupta and Heal, 1979, Section 14.5). This implies that dynamic oligopolies and cartels cannot be distinguished from perfect competition and that firms act as if there are well-defined private property rights. These results are somewhat counter-intuitive and cannot explain the phenomena of 'wild-catting'. One reason for excessive extraction rates in oligopolistic resource markets may be that firms are worried that, if they announce to extract efficiently, one of their rivals with access to current stock levels will have an incentive to deplete more rapidly. The fact that firms have access to current and past stocks of untapped reserves can lead to excessive extraction rates. Such inefficiencies are a consequence of credible extraction strategies and illustrate that an increase in information can be detrimental to firms in the industry. They are not considered in most of the existing literature on market imperfections in resource markets (e.g., Salant, 1976; Khalatbari, 1977; Dasgupta and Heal, 1979; Bolle, 1980), because there firms are implicitly assumed to condition their harvests on the initial stock of reserves only. This is a dubious assumption in stochastic environments, but is due to the game-theoretic structure, also unrealistic in deterministic environments. It turns out that stochastics are essential for obtaining a unique symmetric Nash equilibrium solution, so that this paper can be viewed as a game-theoretic extension of Pindyck (1984). Eswaran and Lewis (1984) also point to the importance of credible extraction strategies, but they use a deterministic discrete-time model with no renewal (exhaustible resources), slow seepage, iso-elastic demand and zero extraction costs. Levhari and Mirman (1982) discuss a dynamic

programming solution for a renewable resource market, but restrict their analysis to a deterministic discrete-time (two-period) framework with zero extraction costs. The main goals of this paper are to analyze a stochastic model of renewable resource markets, which gives rise to a unique symmetric Nash equilibrium solution, and to compare open-loop and sub-game perfect Nash equilibrium solutions. A similar stochastic analysis can be found in Clemhout and Wan (1985b), but they focus on multi-resource models and provide explicit solutions to different examples.

The objective of this paper is to consider the implications of dynamic information patterns for oligopolistic resource markets with restricted entry and exit and, more specifically, to compare open-loop and feedback or credible oligopolies with efficient outcomes. Section 2 sets up a continuous-time problem of oligopolistic resource depletion and allows for renewable, as well as exhaustible, resources and extraction costs that decline with the stock of untapped reserves. Section 3 compares the oligopoly with static information patterns with the efficient and cartel outcomes and shows that oligopolies with more dynamic information patterns have an infinite number of Nash equilibria. Section 4 argues that, when there are serially uncorrelated shocks to the natural replenishment rate, the sub-game perfect Nash or credible equilibrium is not dominated by any of the other Nash equilibria. The resulting stochastic credible arbitrage equation is compared with the open-loop arbitrage equation and with the efficient and cartel outcomes. Section 4 also shows that, when there are isolated markets, demand is iso-elastic and extraction costs are zero, the sub-game perfect Nash equilibrium solution is inefficient. Section 5 considers an example that permits an explicit analytical solution and Section 6 considers policies to correct for the stock externalities. Section 7 concludes the paper with some observations on the problem of time-inconsistency.

2. A common resource problem with restricted entry and exit

Let x , u_i , N and Y denote the untapped stock of common resource, the harvesting rate of firm i , the total number of firms in the industry and the industry harvest ($Y = \sum_{i=1}^N u_i$), respectively. The dynamics of resource depletion follow a stochastic pattern (e.g., Pindyck, 1984) and are given by

$$dx = \{H(x) - Y\}dt + \sigma(x)dw, \quad (2.1)$$

where $dw = \varepsilon(t)\sqrt{dt}$, $\varepsilon(t) \sim IN(0, 1)$ and $w(t)$ is Brownian motion.

Assumption 1: $H(x)$ is continuously differentiable on R_+ ; $H(0) = 0$;

$$\exists x_c > 0 \text{ s.t. } H(x_c) = 0 \text{ and } H'(x_c) < 0; \quad H''(x) > 0,$$

$$\forall x \geq 0; \quad \sigma(0) = 0; \quad \sigma'(x) \geq 0, \quad \forall x \geq 0.$$

$H(x)$ is concave and describes the renewal rate of the resource in the absence of depletion and shocks. The logistic and Gompertz growth curves (e.g., Dasgupta and Heal, 1979, Chapter 5) are particularly relevant for *renewable* resources (such as animal, bird or fish populations), since then the saturation effects might arise from a fixed food supply. The special case $H(x) = 0$ corresponds to *exhaustible* resources (such as fossil fuels or minerals). The case $H''(x) = 0$ corresponds to exponential growth or decline of the resource and may be relevant for small populations with insignificant saturation effects. A *stationary* catch policy is defined by $H(x) = Y$. In the absence of depletion or shocks, the stock level x_c is the natural carrying capacity of the common and is a stable equilibrium. The stock level $x_b = 0$ is called the biotic potential, which in the absence of depletion is unstable and therefore there is no danger of extinction in the absence of depletion. The pool as modelled is truly common to all firms, so that there are no explicit private property rights.¹

¹ Alternatively, there are private property rights and seepage occurs at an infinite speed. The more general case of private pools with sluggish seepage will not be discussed here (Sinn, 1984; Eswaran and Lewis, 1984).

We assume that all firms harvesting the common face identical costs. The effort (e.g., vessels, fish nets or man-power), c_i , required to achieve a catch u_i when the common stock is x diminishes when untapped reserves are large or the required harvest is low. This is captured by the cost function $c_i = c(x)u_i$, which displays constant marginal cost. The inverse of the industry demand function is given by $P = p(Y)$, where P is the price of the harvest.

Assumption 2: (a) $p(Y)$ and $c(x)$ are continuously differentiable on R_+ ; $p'(Y) < 0$, $\forall Y \geq 0$; $c'(x) < 0$, $c''(x) \leq 0$, $\forall x \geq 0$;
(b) $\lim_{Y \rightarrow 0} p(Y) = \infty$.

If the common resource can be freely traded in the industry, the instantaneous profit function of each firm may be written as

$$\pi_i(x, u_1, \dots, u_N) = d(Y)u_i - c(x)u_i. \quad (2.2)$$

Observe that the profit function allows for both stock and flow externalities.¹

Each firm maximizes the expected value of its discounted stream of profits,

$$\max_{u^i} J^i(u^1, \dots, u^N) \equiv E_\varepsilon \Pi_i(u^1, \dots, u^N, \varepsilon), \quad \Pi_i = \int_0^T \pi_i(x, u_1, \dots, u_N) e^{-rt} dt, \quad (2.3)$$

subject to (2.1)-(2.2), its conjectures regarding the actions of rival firms,

$u_i \geq 0$ and the initial condition $x(0) = x_0$, where $u^i = \{u_i(t), 0 \leq t \leq T\}$

and $\varepsilon = \{\varepsilon(t), 0 \leq t \leq T\}$. When the optimal actions of the firms do not lead to extinction, the planning horizon is infinite ($T \rightarrow \infty$). Otherwise,

$x(T) = 0$ gives an additional boundary condition where T denotes the date of

¹ We assume that each firm has sufficient market power, so that the flow externalities do not disappear ($\partial \pi_i / \partial u_j \neq 0$, for $j \neq i$).

extinction.¹

The non-cooperative behaviour of the firms can be analyzed as a N-player, continuous-time, infinite dynamic game (Başar and Olsder, 1982, Chapter 6, Appendix).

Definition 1: I_t^i is the information set of firm i at time t . The information structure is called:

- (i) open-loop if $I_t^i = \{x(0)\}$;
 - (ii) closed-loop if $I_t^i = \{x(s), 0 \leq s \leq t\}$;
 - (iii) memory-less if $I_t^i = \{x(0), x(t)\}$; and
 - (iv) feedback if $I_t^i = \{x(t)\}$,
- $0 \leq t \leq T, i = 1, \dots, N.$ □

Definition 2: The mappings $\theta^i = \{\theta_i(t), 0 \leq t \leq T\} \in \Theta^i$ where $\theta_i(t): I_t^i \rightarrow \mathbb{R}$ are the permissible strategies of firm i at time t , define a strategy for firm i ; the policy rule for firm i at time t is given by $u_i(t) = \theta_i(t, I_t^i)$ and is I_t^i -measurable. □

The normal form of the game consists of the permissible strategy spaces $\{\Theta^1, \dots, \Theta^N\}$ together with the objective functionals $\{J^1, \dots, J^N\}$.

Definition 3: The N-tuple of strategies $\{\theta^i \in \Theta^i, i = 1, \dots, N\}$ is a Nash Equilibrium Solution (NES) iff the inequalities $J^i(\theta^1, \dots, \theta^N) \geq J^i(\theta^1, \dots, \theta^{i-1}, \bar{\theta}^i, \theta^{i+1}, \dots, \theta^N)$, are satisfied for all $\bar{\theta}^i \in \Theta^i, i = 1, \dots, N.$ □

Under the open-loop, closed-loop, memory-less and feedback information structure, the NES is referred to as an OLNES, CLNES, MLNES and FBNES, respectively. The $J^i(\cdot)$ are, by Assumptions 1 and 2(a), continuous on $\mathbb{R}_+ \times \dots \times \mathbb{R}_+$ and are assumed to be strictly convex in $u^i, \forall u^j \in \mathbb{R}_+, j \neq i.$

¹ Since $\sigma(0) = 0$, stochastic fluctuations disappear after date T and the resource cannot be regenerated. Clemhout and Wan (1985a) discuss, for a deterministic model, whether extinction occurs asymptotically or in finite time.

We are also interested in socially optimal policies, which pertain when the government coordinates the actions of the firms and consumers in order to maximize the sum of profits and consumers' surplus and ignores issues of income distribution. Instead of (2.3), one maximizes the expected value of the social benefit function,

$$\text{Max}_Y B = \mathbb{E} \int_0^T [s(Y) - c(x)Y] e^{-rt} dt, \quad (2.4)$$

where $s(\cdot)$ denotes the gross consumers' surplus ($d(\cdot) = s'(\cdot)$).

3. Depletion under certainty

3.1 Efficient and collusive outcomes

We begin with examining the behaviour of oligopolistic resource markets under certainty, so that for the time being $\sigma(x) = 0$. To have an appropriate benchmark, we review the Pareto-efficient outcome that pertains under perfect competition or under a social planner. The efficient extraction plan follows from maximizing (2.4) subject to (2.1) with the aid of Pontryagin's Maximum Principle. The necessary conditions under Assumptions 1 and 2(a) are given by (e.g., Takayama, 1974, Theorem 8.C.3)

$$\left. \begin{array}{l} P(Y) \leq c(x) + \psi \\ Y \geq 0 \end{array} \right\} \text{c.s.}, \quad (3.1)$$

the adjoint equation

$$\frac{\dot{\psi}}{\psi} + H'(x) - \frac{c'(x)Y}{\psi} = r \quad (3.2)$$

and the terminal (or transversality) condition¹

¹ The other necessary conditions are $\lim_{t \rightarrow T} e^{-rt} [s(Y) - c(x)Y + \psi(H(x) - Y)] = 0$ and $e^{-rt}\psi(T) \geq 0$ for finite T or $\lim_{t \rightarrow T} e^{-rt}\psi(t) = 0$ for $T \rightarrow \infty$.

$$\lim_{t \rightarrow T} e^{-rt} \psi(t) [x(0) - x(t)] = 0, \quad (3.3)$$

where ψ is undiscounted shadow price of a unit of untapped reserves. If the price of the harvest does not cover the marginal cost of extraction plus the amortized marginal cost of resource depletion, there is no point in harvesting at all. Otherwise, (3.1) implies a positive catch and the corresponding price equals total marginal costs as under perfect competition. If Assumption 2(b) also holds, then the latter case holds.

(3.2) is an generalized revision of Hotelling's (1931) arbitrage rule for efficient extraction, which states that producers are indifferent between the return on alternative assets and the total return on holding a unit of stock consisting of capital gains plus the change in the natural growth rate and the reduction in harvesting costs from holding that unit. If the resource is not extinct in the steady-state (with $T \rightarrow \infty$),¹ then $H(x^*) = Y^*$, $x \leq x_c$ and

$$\left. \begin{aligned} \psi^* &= \frac{c'(x^*)Y^*}{H'(x^*) - r} \geq d(Y^*) - c(x^*) \\ Y^* &\geq 0 \end{aligned} \right\} \text{c.s.} \quad (3.4)$$

must hold. Hence, the shadow rent of a unit of the resource is, in equilibrium, equal to the amortized value of future reductions in cost arising from holding an extra unit where the market interest rate net of the marginal change in the natural growth rate is used as the net discount rate.

Under a cartel the firms act in a collusive manner to maximize joint profits, so that (3.1) is replaced by

$$\left. \begin{aligned} p(Y)(1 - \eta^{-1}) &\leq c(x) + \psi \\ Y &\geq 0 \end{aligned} \right\} \text{c.s.,} \quad (3.5)$$

¹ The possibility of extinction increases when prices are low and demand is high.

where $\eta = -P/(p'Y) \geq 0$ is the elasticity of market demand. The difference is that the price of the harvest is replaced by its marginal revenue, which typically means that a cartel under-extracts and allows the resource to replenish itself to a higher level, thus forcing down the shadow value of untapped reserves.¹ The reason is that in order to extract the consumers' surplus the sales of the resource must reduce, which typically leads to larger reserves.

3.2 Oligopoly outcomes with static information patterns

Perhaps the most frequently used solution concept for equilibrium of a dynamic oligopoly with restricted entry and exit is the open-loop Nash equilibrium (e.g., Başar and Olsder, 1982, Section 6.5.1). For example, it has been used by Dasgupta and Heal (1979, Chapter 11), Salant (1976), Khalatbari (1977) and Bolle (1980) in their studies of oligopolistic resource markets. Each firm is assumed to take the harvests of rival firms (and the total number of firms in the industry) as given and to condition its strategy on the information available at the start of the planning period, $x(0)$, that is the information of firm i at time t is given by $I_t^i = \{x(0)\}$. This solution concept relies upon static information patterns, since firms are assumed to not update their information sets even though there might be an individual incentive to use incoming data on the stock of reserves.

Proposition 1: Under Assumptions 1 and 2(a), the necessary conditions for a symmetric OLNES with $\sigma(x) = 0$ are: (2.1),

¹ Collusion may therefore be more desirable from an environmental point of view than perfect competition, although it clearly is inefficient from an economic point of view.

$$\left. \begin{aligned} p(Y)(1 - \eta^{-1}N^{-1}) &\leq c(x) + \psi_i \\ u_i &\geq 0 \end{aligned} \right\} \text{c.s.}, \quad (3.6)$$

$$\frac{\dot{\psi}_i}{\psi_i} + H'(x) - \frac{c'(x)Y}{N\psi_i} = r, \quad (3.7)$$

and similar terminal (or transversality) conditions as in Section 3.1, where $Y = Nu_i$ and ψ_i is the value (rent) of an extra unit of reserves to firm i .

Proof: The i -th inequality of Definition 3 says that $\theta^i(x_0) = u^i$ maximizes $J^i(u^1, \dots, u^{i-1}, \bar{u}^i, u^{i+1}, \dots, u^N)$ s.t. $\dot{x} = H(x) - \bar{u}_i - \sum_{j \neq i} u_j$. Since the $\{u^j, j \neq i\}$ are open-loop policies, they only depend on x_0 and not on u^i and therefore firm i faces a standard optimal control problem. Hence, the result follows directly from Pontryagin's Maximum Principle: $\{\partial h^i / \partial u_i \leq 0, u_i \geq 0, \text{c.s.}\}$ and $\{r\psi_i - \dot{\psi}_i = \partial h^i / \partial x\}$, where the Hamiltonian of firm i is defined as $h^i \equiv p(\bar{u}_i + \sum_{j \neq i} u_j) \bar{u}_i - c(x) \bar{u}_i + \psi_i [H(x) - \bar{u}_i - \sum_{j \neq i} u_j]$. Upon imposing symmetry, one obtains (3.6)-(3.7). $\square$

The steady-state rent to the industry, $\psi = N\psi_i$, satisfies¹

$$\psi^* = \frac{c'(x^*)Y^*}{H'(x^*) - r} \geq N [p(Y^*)(1 - \eta^{-1}N^{-1}) - c(x^*)], \quad (3.8)$$

whilst $x^* \leq x_c$ and $H(x^*) = Y^*$ must also hold in equilibrium. The marginal revenue of the harvest of each firm matches the marginal cost of its extraction plus the rent of untapped reserves, where the latter item equals the capitalized value (using the net rate of discount, $r - H'(x^*)$) of the reduction in future operating costs for each firm due to a marginal increase in the current harvest.

¹ It can be shown (cf. Ferschtman and Muller, 1984) that under certain conditions the OLNES exists and has the property of conditional global asymptotic stability, so that the game converges to (3.8) regardless of the initial condition, x_0 .

Corollary 1: If $c(x) = 0$ and $\partial\eta/\partial Y = 0$ and Assumptions 1 and 2 hold, the symmetric OLNES for $T \rightarrow \infty$ is efficient.

Proof: It follows from (3.6) that $\psi_i = p(Y)(1 - \eta^{-1}N^{-1})$, so that for fixed nN $\dot{\psi}_i/\psi_i = \dot{P}/P$. Hence, (3.7) becomes $\dot{P}/P = r - H'(x)$. Since this yields the same arbitrage equation as (3.1)-(3.2), the OLNES is efficient. Note that when T is finite the transversality conditions differ for the efficient solution and the OLNES, so that the result does not hold for finite-horizon problems. $\square$

The open-loop oligopoly outcome typically results in inefficient extraction. However, for the special case of zero extraction costs and iso-elastic demand, the OLNES gives rise to Hotelling's arbitrage rule and is therefore socially optimal (cf. Khalatbari, 1977; Dasgupta and Heal, 1979, Section 11.5 for exhaustible resources.)¹ When demand is not iso-elastic, r in Corollary 1 becomes $r - (\dot{\gamma}/\gamma)$ where $\gamma \equiv N - \eta^{-1}(Y)$ and $\text{sign}(\dot{\gamma}) = \text{sign}(-\eta')$ (assuming $\dot{Y} < 0$ for rising rents). Hence, if a fall in supply increases competition with substitutes of the resource ($\eta' > 0$), one has too rapid extraction rates. On the other hand, if a rise in the price increases the incentive for firms to invent substitutes that did not exist before ($\eta' < 0$), one has excessive conservation. In general, the presence of extraction costs gives rise to stock-externalities and leads to further inefficiencies.

¹ In fact, for this case a cartel also extracts efficiently. However, if each firm owns a part of the pool and seepage is sluggish, the outcome results in too rapid extraction. This inefficiency arises from an unclear definition of private property rights (Khalatbari, 1977).

3.3 A plethora of Nash equilibria

The conventional open-loop Nash equilibrium is only one of many Nash equilibria and is not necessarily the most sensible solution concept under all circumstances. Due to its passive information structure, the differential game consists of one sub-game only and is really a static game.¹ Much more general is the class of closed-loop, no-memory Nash equilibria (e.g., Başar and Olsder, 1982, Section 6.5.2), which postulates for each firm a conjecture or reaction hypothesis about the harvest strategies of its rivals of the following form

$$u_j(t) = \theta_j(t, x(0), x(t)), j = 1, \dots, N. \quad (3.9)$$

The memory-less information structure is more realistic, because firms are assumed to have access to the current, as well as the initial, stock of reserves.

Proposition 2: Under Assumptions 1 and 2(a) and the policy rules (3.9), the necessary conditions for a symmetric MLNES are: (2.1), (3.6) and

$$\frac{\dot{\psi}_i}{\psi_i} + H'(x) - \frac{c'(x)u_i}{\psi_i} + \left\{ \frac{u_i p'(Y)}{\psi_i} - 1 \right\} \sum_{j \neq i} \frac{\partial \theta_j}{\partial x} = r \quad (3.10)$$

Proof: The procedure is as in Proposition 1, but now one must take into account the effects of the current stock, $x(t)$, upon the actions of the rival firms and consequently upon the Hamiltonian of firm i :

$r\psi_i - \dot{\psi}_i = \partial h^i / \partial x(t) + \sum_{j \neq i} (\partial h^i / \partial u_j) (\partial \theta_j / \partial x(t))$ where h^i is defined as before. The above arbitrage equation yields (3.10). □

¹ For infinite discrete-time differential games, one could convert the dynamic game to a static game by stacking the policy instruments for all periods of the planning horizon (e.g., Kydland, 1975).

It is clear from (3.10) that this class of Nash equilibria suffers from conjectural non-uniqueness, since equilibria can be sustained for almost any set of conjectures (cf. "bootstrap" equilibria). For example, if each firm assumes that its rivals do not condition their harvest on the current amount of untapped reserves, $\partial \theta_j / \partial x(t) = 0$, $j \neq i$, then the OLNES results as a special case (see (3.7)). However, in general, each firm might take account of the fact that, if it depletes an extra unit of the resource, then its rivals will react and typically reduce their catch somewhat as their cost of extraction and rent of the resource have gone up. This means that the price the firm under consideration can fetch for its harvest does not fall as much, the marginal cost of extraction does not rise as much and the natural replenishment rate does not change as much as in the absence of such a response from its rivals. It follows that, with the MLNES (or CLNES), firms tend to initially have a larger harvest at a lower price and therefore deplete the stock of reserves at a faster rate.

Using (3.6) and assuming a non-zero harvest, it follows from (3.10) that the steady-state rent to the industry is given by

$$\psi^* = \frac{c'(x^*)Y^* + N(N-1)\{p(Y^*) - c(x^*)\} \frac{\partial \theta_j}{\partial x}}{H'(x^*) - r} = N [p(Y^*)(1 - n^{-1}N^{-1}) - c(x^*)] \quad (3.11)$$

which, together with $H(x^*) = Y^*$, can be solved for x^* , Y^* and ψ^* . Hence, if firms are profitable and $(\partial \theta_j / \partial x) > 0$, the steady-state stock of untapped reserves is lower than for the OLNES. If the stock is forced below the stock that yields the maximum harvest, the steady-state rent and price of the harvest are higher and the harvest smaller than for the OLNES. Otherwise, the steady-state rent and price are lower and the harvest bigger than with static information patterns.

Corollary 2: For an open-access fishery (cf. Berck and Perloff, 1984) the symmetric OLNES and MLNES coincide.

Proof: For $u_i = 0$, the result is obvious. Otherwise, (3.6) gives
 $(u_i p' / \psi_i) - 1 = u_i p' / [P(1 - \eta^{-1} N^{-1}) - c(x)] - 1$. Free entry and exit ensures that $P = c(x)$ holds, so that the above expression becomes $(Nu_i / Y) - 1$ or, upon imposing symmetry, zero. Hence, (3.10) reduces to (3.7). □

This result suggests that dynamic information patterns are a disequilibrium issue, since in the long-run there is no difference between the OLNES and MLNES. Fudenberg and Levine (1985) show, in a different context, that under certain conditions as the influence of individual players diminishes the OLNES and CLNES grow closer together, but they also provide counter-examples to this result.

Ferschtman and Kamien show that the stationary, symmetric OLNES to a dynamic, linear-quadratic duopoly problem with sluggish price adjustment yields exactly the same price as the counter-part static Cournot equilibrium price, but that the corresponding FBNES yields a price below the static Cournot equilibrium price. It is interesting that the present model gives a qualitatively similar result.

Definition 4: With memory-less or closed-loop information patterns, the strategies $\{\theta^i \in \Theta^i, i = 1, \dots, N\}$ constitute a Sub-game Perfect Nash Equilibrium Solution (SPNES) if there exist value functions $V^i(\dots)$ defined on $[0, T] \times R$ satisfying $V^i(T, x) = 0$ and $V^i(t, x) = E_t \int_t^T \pi_i(x(s), \theta_1(s, I_s^1), \dots, \theta_N(s, I_s^N)) e^{-r(s-t)} ds \geq E_t \int_t^T \pi_i(\bar{x}_i(s), \theta_1(s, I_s^1), \dots, \theta_{i-1}(s, I_s^{i-1}), \bar{\theta}_i(s, I_s^i), \theta_{i+1}(s, I_s^{i+1}), \dots, \theta_N(s, I_s^N)) e^{-r(s-t)} ds,$

$$\begin{aligned} \text{where } dx &= [H(x) - \sum_{i=1}^N \theta_i(s, I_s^i)]dt + \sigma(x)dw, \quad x(0) = x_0 \quad \text{and} \\ d\bar{x}_i &= [H(\bar{x}_i) - \bar{\theta}_i(t, I_s^i) - \sum_{j \neq i} \theta_j(t, I_s^j)]dt + \sigma(\bar{x}_i)dw, \quad \bar{x}_i(0) = x_0, \\ \forall \bar{\theta}_i(s, I_s^i) &\in \Theta^i. \end{aligned} \quad \square$$

Hence, the SPNES¹ is a FBNES and restricts the class of closed-loop, no-memory Nash equilibrium solutions to satisfy sub-game perfectness (Selten, 1975), so that a restriction of the SPNES for $[0, T]$ to $[t, T]$ is also a SPNES for $[t, T]$. Hence, the SPNES is history-independent and there is no gain for any firm to re-optimize its extraction plans after any arbitrary numbers of periods. It therefore yields credible strategies and is, by construction, time-consistent. Note, however, that even for the OLNES the problem of time-inconsistency (Kydland and Prescott, 1977) does not occur and there is no need for firms to commit and bind themselves in advance of the planning period. This follows directly from the assumptions that all firms in the industry have equal market power and that there is no dominant firm which can by simply announcing its strategies, if believed, affect the behaviour of the other firms and therefore has an incentive to cheat. It is important to realize that different information patterns imply very different outcomes for the Nash equilibrium (see Sections 4.2 and 5), even though the time-inconsistency problem is absent.

Reinganum and Stokey (1985) use a deterministic oligopoly model for exhaustible resources with iso-elastic demand and zero extraction costs to examine the choice of strategy space. The OLNES corresponds to path strategies with commitment over the entire length of the planning period and the SPNES corresponds to decision rule strategies with no commitment at all. Intermediate cases depend on the length of the period over which firms can make

¹ Both the OLNES and the SPNES (and FBNES) were originally discussed in Starr and Ho (1969a, b). The SPNES to the problem of oligopolistic, renewable resource markets is discussed for the general stochastic case in Section 4.2.

commitment, which can vary from zero to T . Reinganum and Stokey (1985) find that shortening the period of commitment leads to more rapid depletion rates, which fits in with the above analysis.

It follows from Definitions 3 and 4 that every SPNES is a MLNES, but not necessarily vice versa. Although restricting the solution to sub-game perfectness usually gives uniqueness within the class of CLNES's as long as firms adopt finite horizons and can perfectly observe the stock of untapped reserves, multiple SPNES's can arise for the case of an infinite planning horizon (e.g., as shown by Papavassilopoulos and Olsder, 1982, for a linear-quadratic differential game) or the case of imperfect or noisy observations.¹ Even with finite horizons and perfect observations, it may be advantageous for an individual firm to condition its current harvest on past, as well as current, stocks of the resource. Başar and Olsder (1982, Section 6.3) and de Zeeuw (1984, Chapter 4) give linear-quadratic examples (in a discrete-time context) of an infinite number of memory strategies that are Pareto-superior to both the OLNES and the SPNES.² Similarly, within the context of oligopolistic resource depletion, firms will find it (even in deterministic environments) optimal to condition their harvest on the expected growth in the stock level as well as on current and initial stock levels.³ For example,

¹ These are related to the "folk" theorems obtained in the literature on repeated games (e.g., Fudenberg and Maskin, 1983). The extension of differential game theory to stochastic observations poses tremendous problems (some of these are discussed for the linear-quadratic, zero-sum case by Bagchi and Olsder, 1981).

² This informational non-uniqueness arises mainly from the existence of uncountably many representations of the same trajectory under dynamic information patterns.

³ This is in sharp contrast to a social planner or cartel, who have no incentive to do this.

$$u_j(t) = \theta_j(t, x(0), x(t), \dot{x}^e(t)), \quad (3.9')$$

and

$$\dot{x}^e(t) = \xi \int_{-\infty}^t \dot{x}(\tau) \exp\{-\xi(t - \tau)\} d\tau, \quad \xi \geq 0 \quad (3.12)$$

where $\dot{x}^e(t)$ is a weighted average of past stock growth rates.¹ However, it will be argued in Section 4.2 that, when the resource dynamics are stochastic and the shocks are serially uncorrelated, the SPNES is as good as any other CLNES with memory strategies and the problem of informational non-uniqueness is then resolved (cf. Başar and Ho, 1974; Başar, 1979; Başar and Olsder, 1982, Section 6.4).

4. Depletion under uncertainty

4.1 Efficient and collusive outcomes

Section 3.3 showed the large number of Nash equilibria that can result for oligopolistic resource extraction in deterministic environments. However, for stochastic environments of the form (2.1) with $\sigma(x) \neq 0$, it turns out that the credible (or feedback) Nash equilibrium is the only sensible solution concept. Before we discuss this, the efficient and collusive outcomes under uncertainty are analyzed when both Assumptions 1 and 2 hold.

For the case of a social planner who maximizes (2.4) subject to (2.1), the stochastic version of the Jacobi-Hamilton-Bellman equation of dynamic programming (e.g., Davis, 1977, Section 6.1; Chow, 1979) is given by

¹ Brito (1972) analyzes this special type of memory strategy for two countries in Nash equilibrium deciding whether to consume or invest in arms and finds that a more sophisticated information pattern of the form (3.9') and (3.12) destabilizes the arms race.

$$rV - V_t = \frac{1}{2}\sigma^2(x)V_{xx} + \max_Y \{s(Y) - c(x)Y + V_x(H(x) - Y)\} \quad (4.1)$$

where the value function, $V(t, x)$, is the optimal discounted stream of expected social benefits from time t onwards. It follows that

$$Y(x, V_x) = p^{-1} [c(x) + V_x(t, x)] \quad (4.2)$$

holds, so that the harvest declines when the marginal cost of extraction or the shadow price of the untapped resource increases. The Markovian nature of (2.1) ensures that the catch strategies need only be conditioned on the current and not on past stock levels. Upon differentiation of (4.1) with respect to x and application of Ito's differential rule, one obtains the efficient arbitrage rule (Pindyck, 1984)

$$\frac{E_t dV_x}{V_x dt} + H'(x) - \frac{c'(x)Y(x, V_x)}{V_x} = r + \delta \quad (4.3)$$

where $\delta = -\sigma'(x)\sigma(x)V_{xx}/V_x \geq 0$ is a risk-premium factor that the planner is prepared to pay to eliminate stock growth uncertainty. Stochastic variations reduce the value of the untapped resource and lead to bigger harvests, because fluctuations reduce the value of the stock and, as the variance increases with the stock, there is an incentive to harvest more and because fluctuations increase expected extraction costs over time (due to the convexity of $c(x)$). On the other hand, stochastic fluctuations reduce the expected natural growth rate (due to the concavity of $H(x)$) and therefore retard extraction. Hence, the net effect of stochastic fluctuations on the harvest rate is ambiguous. Obviously, in the absence of stochastic shocks (4.3) reduces to (3.2) ($\delta = 0$ and $V_x = \psi$).

A cartel maximizes the expected value of the discounted stream of total profits, which leads to (4.3) and, instead of (4.2),

$$Y(x, V_x) = p^{-1} \left[\frac{c(x) + V_x(t, x)}{1 - \frac{1}{\eta}} \right] \quad (4.4)$$

Hence, a cartel employs the same arbitrage equation (4.3) and an increase in the stock of untapped reserves still reduces its rent and the marginal cost of extraction. The main difference is that the cartel restricts the harvest below the efficient level.

4.2 Credible extraction strategies

This section characterizes the SPNES (e.g., Başar and Olsder, 1982, Section 6.6) or credible extraction strategies for a dynamic oligopoly where firms have access to closed-loop or memory-less information patterns.

Proposition 3: When Assumptions 1 and 2 hold and firms have access to closed-loop or memory-less information patterns, the symmetric SPNES gives rise to (2.1), the industry harvest rate

$$Y = \theta(t, x) = p^{-1} \left(\frac{c(x) + V_x N^{-1}}{1 - \eta^{-1} N^{-1}} \right), \quad (4.5)$$

and the arbitrage equation (for the industry as a whole)

$$\frac{E_t dV_x}{V_x dt} + H'(x) - \frac{c'(x)Y}{V_x} = r + \delta + \frac{(N-1)\{p(Y) - c(x)\}\theta_x(t, x)}{V_x} \quad (4.6)$$

which may together be solved for the stock of untapped reserves, x , its rent, V_x , and the harvest rate, Y .

Proof: Definition 4 suggests the method of dynamic programming for obtaining the SPNES. This yields the Jacobi-Hamilton-Bellman equation for firm i :

$$rV^i - V_t^i = \frac{1}{2}\sigma^2(x)V_{xx}^i + \max_{u_i} \{p(Y)u_i - c(x)u_i + V_x^i(H(x) - Y)\} \quad (4.7)$$

where $V^i(t, x)$ denotes the optimal discounted stream of expected profits accruing to firm i from time t onwards. Upon performing

the maximization, imposing symmetry and aggregating, one obtains (4.5). (Assumption 2(b) ensures an interior solution). Upon aggregation of (4.7), differentiation with respect to x and application of Ito's differential rule ($E_t dV_x = V_{tx} dt + V_{xx}(H(x) - Y)dt + \frac{1}{2}\sigma^2(x)V_{xxx}dt$), one obtains (4.6). □

The Appendix shows, for a discrete-time version of the model, that when firms have closed-loop or memory-less information patterns, every NES consists only of feedback strategies and consequently $\{x(t)\}$ is a sufficient statistic for each firm, even though their information sets include $\{x(s), 0 \leq s < t\}$ or $\{x(0)\}$. In other words, firms do not have an incentive to condition their strategies on past stock levels (i.e., use memory strategies) and therefore the problem of informational non-uniqueness (see Section 3.3) is eliminated. This result depends crucially on the stochastic shocks to resource growth being serially uncorrelated and implies a unique symmetric NES for the stochastic game. As $\sigma(x) \rightarrow 0$, the SPNES corresponds to a robust NES of the deterministic game as it is insensitive to zero-mean, serially uncorrelated random shocks to resource growth.

In the long-run firms enter and exit the industry until all unexploited profit opportunities are wiped out ($p = c$). The bias due to the ability of firms to take account of dynamic information patterns then disappears and the open-loop extraction plans with static information patterns will also be credible. In the short-run firms are able to make oligopoly profits (or losses), so that use of information on past and current stock levels in the formulation of extraction plans introduces an upward (downward) bias of the market rate of interest over and above the risk-premium. This follows from the concavity of the value functions, so that at the optimum

$$\theta_x = \frac{c'(x) + V_{xx}N^{-1}}{(1 - \eta^{-1}N^{-1})d'(Y)} > 0 \quad (4.8)$$

and the information bias on the right-hand-side of (4.6) is positive (unless firms make losses). Hence, as closed-loop, memory or feedback information patterns take account of the fact that the depletion of an additional unit of the resource leads rivals to harvest less, the extraction occurs on average at a faster rate than with static information patterns and the stock of untapped resource is depleted to a lower level.

4.3 Isolated markets

Corollary 1 shows that when there are no stock externalities (and demand is iso-elastic) the OLNES is efficient. These conditions are, however, not sufficient for the efficiency of the SPNES. This Section shows that when there are no flow externalities either, which occurs when there are isolated markets ($P_i = p(u_i)$ instead of $P = p(Y)$), the SPNES is efficient.

Assumption 2': $p(u_i)$ is continuously differentiable on R_+ ; $p'(u_i) < 0$,
 $\forall u_i \geq 0$; $\lim_{u_i \rightarrow 0} p(u_i) = \infty$.

Proposition 4: If markets are isolated, $\partial \eta / \partial u_i = 0$ (where $\eta \equiv -P_i / (p'(u_i))$),
 $c(x) = 0$ and Assumptions 1 and 2' hold, the SPNES for
 $T \rightarrow \infty$ is efficient.

Proof: The principle of sub-game perfection gives rise to the following Jacobi-Hamilton-Bellman equation:

$$rV^i - V_t^i = \frac{1}{2}\sigma^2(x)V_{xx}^i + \text{Max}_{u_i} \{p(u_i)u_i + V_x^i(H(x) - Y)\} \quad (4.9)$$

which yields $Y = Np^{-1}(V_x/N(1 - \eta^{-1}))$ and therefore

$E_t dY/Ydt = -\eta E_t dV_x/V_x dt$. Upon aggregation, differentiation with respect to x and application of Ito's rule, the arbitrage equation becomes

$$E_t dY/Y = - \eta [r + \delta - H'(x)], \quad (4.10)$$

which can be solved together with (2.1) to give the SPNES. The Jacobi-Hamilton-Bellman equation for a social planner is given by:

$$rV - V_t = \frac{1}{2} \sigma^2(x) V_{xx} + \max_Y \{Ns(Y/N) + V_x (H(x) - Y)\}, \quad s'(u_i) = p(u_i), \quad (4.11)$$

which yields $Y = Np^{-1}(V_x)$ and therefore $E_t dY/Ydt = - \eta E_t dV_x/V_x dt$. Upon differentiation with respect to x and application of Ito's rule, the efficient arbitrage equation reduces to (4.10). Hence, the SPNES is efficient. □

For the general case of isolated markets with stock-dependent extraction costs and not necessarily iso-elastic demand, the CLNES, MLNES, FBNES and SPNES coincide with the OLNES.

5. Example: Zero extraction costs and iso-elastic demand

We consider the special case of zero extraction costs and iso-elastic demand, $Y = bP^{-\eta}$, because then the open-loop oligopoly is efficient¹ and serves as a useful benchmark for comparison with an oligopoly with more dynamic information patterns. We also assume a logistic growth curve,

$$H(x) = gx \left(1 - \frac{x}{x_c}\right) \quad (5.1)$$

where g is the intrinsic growth rate and $\sigma(x) = \sigma x$, so that the variance

¹ Strictly speaking, the OLNES does not give contingent extraction rules to ensure correction for stochastic shocks and therefore firms are worse off than in the efficient outcome. However, if the firms revise their strategies at every point of time to allow for stochastic shocks, the OLNES of this example coincides with the efficient solution.

of the shocks increases with the size of untapped reserves. In that case, Pindyck (1984) shows that, for $\eta = \frac{1}{2}$, the efficient outcome yields

$$V^E(t, x) = -\frac{\phi_1^E}{x} - \frac{g\phi_1^E}{rx_c}, \quad \phi_1^E = \left(\frac{2b}{r + g - \sigma^2} \right)^2 \quad (5.2)$$

as the solution to (4.1), $V_x^E = \phi_1^E/x^2$, $V_{xx}^E = -2\phi_1^E/x^3 < 0$, the harvest rate is given by

$$Y = \theta^E x, \quad \theta^E = (r + g - \sigma^2)/2, \quad (5.3)$$

for $r < g$ $x(\infty)$ has a gamma distribution with shape parameter $\alpha^E = (g - r)/\sigma^2$ and scale parameter $\beta^E = (\sigma^2 x_c/2g)$,

$$\mathbb{E}^E x(\infty) = \beta^E \alpha^E = \frac{x_c}{2} \left(1 - \frac{r}{g}\right), \quad \text{for } r < g, \quad (5.4)$$

and

$$\mathbb{E}^E Y(\infty) = \frac{x_c}{4} \left(1 - \frac{r}{g}\right) (r + g - \sigma^2), \quad \text{for } r < g. \quad (5.5)$$

Stochastic variations in the natural replenishment rate (or reductions in the rate of impatience or the intrinsic growth rate) reduce the social value of the resource (due to $V_{xx}^E < 0$), increase the shadow price of reserves and consequently reduce the harvest rate.¹ The reduction in the harvest rate, due to stochastic fluctuations is just sufficient to leave the expected stock unchanged. If the discount rate exceeds the intrinsic growth rate, extinction of the resource occurs with probability 1.

The value function for the industry under a feedback oligopoly, say $V^F(t, x)$, follows from (4.7) and must satisfy

1

When η is increased or $H(x)$ is more skewed to the left (as with the Gompertz growth curve), it is possible that stochastic fluctuations increase rent and the harvest rate (Pindyck, 1984).

$$rV^F - V_t^F = \frac{1}{2} \sigma^2 x^2 V_{xx}^F + b \left(\frac{V_x^F}{N - \eta^{-1}} \right)^{1-\eta} + V_x^F \left[gx \left(1 - \frac{x}{x_c} \right) - b \left(\frac{V_x^F}{N - \frac{1}{\eta}} \right)^{-\eta} \right] \quad (5.6)$$

For $\eta = \frac{1}{2}$ and $N > 3$, it can be shown that

$$V^F(t, x) = -\frac{\phi_1^F}{x} - \frac{g\phi_1^F}{rx_c}, \quad \phi_1^F = \left(\frac{b(N-3)}{r+g-\sigma^2} \right)^2 / (N-2) \quad (5.7)$$

is a solution of (5.6), so that the rent is given by $V_x^F = \phi_1^F/x^2$ and the harvest rate follows from (4.5),

$$Y = \theta(t, x) = \theta^F x, \quad \theta^F = (r + g - \sigma^2)(N-2)/(N-3). \quad (5.8)$$

It is clear that the SPNES for an oligopoly with dynamic information patterns is inefficient and leads to too rapid extraction rates ($\theta^F > \theta^E$). For a smaller number of firms in the industry, competition is reduced, profit margins and the industry's value, $V^F(t, x)$, are higher. the information bias is larger (cf. (4.6)) and extraction rates are more excessive. The extraction rate for a feedback oligopoly is more than twice the efficient harvest rate.¹ If extinction does not occur with probability 1, the steady-state stock of untapped reserves, $x(\infty)$, has a gamma distribution with shape parameter $\alpha^F = (2(g - \theta^F)/\sigma^2) - 1$ and scale parameter $\beta^F = (\sigma^2 x_c / 2g)$, so that (using $\theta^F > \theta^E$)

¹ For the case of exhaustible resources and general η , it can be shown that

$$\theta^F = \theta^E (1 - \eta)^{-1} (N - \eta^{-1}) / (N - \eta^{-1} - 1)$$

where $\theta^E = r$ (Hotelling's rule), holds. If $(N-1)^{-1} < \eta < 1$ holds, then $\theta^F > (1 - \eta)^{-1} \theta^E > \theta^E$ and for $\eta = \frac{1}{2}$ one has that $\theta^F > 2\theta^E$ and $N > 3$ must hold. Hence, in general, a high elasticity of demand implies more excessive extraction rates.

$$\mathbb{E}^F x(\infty) = \beta^F \alpha^F = \frac{x_c}{2} \left[1 - \left(\frac{r + 2(\theta^F - \theta^E)}{g} \right) \right] < \mathbb{E}^E x(\infty), \quad (5.9)$$

$$\mathbb{E}^F Y(\infty) = \frac{x_c}{2} \left[1 - \left(\frac{r + 2(\theta^F - \theta^E)}{g} \right) \right] (r + g - \sigma^2) (N - 2) / (N - 3), \quad (5.10)$$

and

$$\text{skew}^F(x(\infty)) = 2/\sqrt{\alpha^F} = \frac{2\sigma}{\sqrt{2(g - \theta^F) - \sigma^2}} > \text{skew}^E(x(\infty)). \quad (5.11)$$

Hence, the excessive extraction rates that occur with an oligopoly with dynamic information patterns lead to a lower expected value (and variance) of the steady-state stock of untapped reserves. The expected steady-state harvest increases due to a higher extraction rate and decreases due to the lower steady-state stock. The distribution of the steady-state stock is also more skewed for the credible oligopoly than for the efficient outcome.

The above relied on the inequalities $g > r$ and $g > \frac{1}{2}\sigma^2 + \theta^F$ being satisfied, so that the steady-state gamma distributions of reserves are non-degenerate for the efficient and credible oligopoly outcomes respectively. In general, it is possible to have extinction with probability 1 for the credible oligopoly outcome and not for the efficient (or open-loop oligopoly) outcome if

$$g > r > \frac{\sigma^2}{2} \left(\frac{N-1}{N-2} \right) - \left(\frac{g}{N-2} \right) \quad (5.12)$$

holds. For example, if there are no stochastic variations in the replenishment rate ($\sigma = 0$), then extinction with probability 1 occurs *always* with a credible oligopoly, even though the efficient or open-loop oligopoly (see Section 3.2) outcome need not lead to extinction. This example highlights the huge inefficiencies of credible strategies compared with strategies that

restrict themselves to static information patterns. It also illustrates that to sustain a credible oligopoly without definite extinction, the variance of the stochastic fluctuations in the replenishment rate should be relatively large compared with the discount rate. If this is the case and the discount rate exceeds the intrinsic growth rate, it is actually possible to have extinction in the efficient outcome even though this does not occur with probability 1 for a credible oligopoly.

6. Economic policy

It has been established that credible extraction strategies are inefficient even when demand is iso-elastic and extraction costs are zero. One way to correct for these externalities is to employ Pigouvian taxes. If each firm pays a stock-dependent tax of $\rho(x)$ per unit of time, the credible arbitrage equation is

$$\frac{E_t dV_x}{V_x dt} + H'(x) - \frac{\rho'(x)N}{V_x} = r + \delta + \frac{(N-1)\{p(Y) - c(x)\}\theta_x}{V_x} \quad (6.1)$$

which becomes the efficient (or open-loop) arbitrage equation when

$$\rho'(x) = - (N-1)\{p(Y) - c(x)\}\theta_x / N \quad (6.2)$$

holds. For the example of Section 5, the tax function $\rho(x) = \rho_1/x - \rho_0$ yields, instead of (5.7), the value function

$$V^F(t, x) = \phi_0^F - \frac{\phi_1^F}{x} - \frac{g\phi_1^F}{rx_c}, \quad \phi_0^F = N\rho_0/r, \quad (6.3)$$

where ϕ_1^F follows from solving

$$(r + g - \sigma^2)\phi_1^F - b(N - 3)(N - 2)^{-\frac{1}{2}} \sqrt{\phi_1^F} - N\rho_1 = 0. \quad (6.4)$$

When the parameter ρ_1 is chosen to ensure efficiency, $\phi_1^F = \phi_1^E$, one obtains

$$\rho_1 = \frac{2b^2[2 - (N - 3)(N - 2)^{-\frac{1}{2}}]}{(r + g - \sigma^2)N}. \quad (6.5)$$

The distortionary tax, ρ_1/x , is complemented with a lump-sum subsidy, ρ_0 , to leave the government's budget unaffected, $\rho(x) = 0$. Note that there is no incentive for any individual firm to deviate from the efficient equilibrium, even though the tax levied on each firm depends on a common stock. Since the tax each firm pays diminishes with the stock on untapped reserves (for $N < 8$), there is an incentive for firms to harvest less and allow the stock to replenish itself to the efficient level. When there is a large number of firms ($N \geq 8$), a distortionary subsidy is required ($\rho_1 < 0$) to induce firms to harvest efficiently.

An alternative way to ensure efficient harvesting is to use the legal system to introduce private property rights, but this will be difficult with seepage of the resource across the common. Yet another way to obtain efficiency is to encourage free entry and exit until all profits and consequently the information bias and associated inefficiencies (cf. (6.2)) are eliminated, but this may be politically infeasible when there is a forceful lobby of existing firms.

7. Concluding remarks

The conditioning of harvest strategies on the initial stock of reserves gives, even in deterministic environments, very misleading results for the behaviour of an oligopoly, although not for a cartel, competitive industry or

a social planner. For example, with zero extraction costs and iso-elastic demand, an oligopoly with static information patterns is efficient. However, if firms are allowed to have access to past and current stock levels, inefficiencies occur and more rapid extraction rates (or even extinction) are possible. This follows from each firm taking account of the fact that, if it harvests an extra unit at any time in the future, the increased cost of extraction and rent of the resource will induce its rivals to harvest somewhat less and therefore the price it can fetch for its harvest and its marginal cost of extraction will not fall as much. This illustrates that an increase in information can be detrimental to firms in the industry, although, typically, it would benefit a cartel or social planner. In general it pays each firm to condition its harvest on past, as well as current, stock levels, but if the stochastic shocks to the natural replenishment rate are serially uncorrelated pure feedback strategies suffice. Although the feedback and open-loop Nash equilibrium solutions give very different outcomes for an oligopoly extracting from a common pool, it can be easily shown that for separate pools with no seepage or for isolated markets the two equilibria coincide. It is clear that, apart from this special case, extraction strategies should be based upon realistic information sets.

It is important to stress that the inefficiencies resulting from dynamic information patterns are quite independent of the problem of time-inconsistency, because there is no incentive in either the open-loop or the closed-loop Nash equilibrium for any firm to renege on announced strategies. The principle of sub-game perfectness is therefore not necessary for ruling out time-inconsistency. However, when there is unequal market power due to the presence of a dominant firm, the feedback strategies do rule out equilibria that are based on expectations of the actions of the market leader that would not be rational to carry out if called upon to do so. For example, in a model

of one dominant producer (OPEC) and a competitive fringe of oil producers, binding commitments on pricing strategies are required to avoid the problem of time-inconsistency in the open-loop Stackelberg equilibrium. Also, the market leader is typically worse off in the feedback Stackelberg than in the open-loop Stackelberg (or even feedback Nash) equilibrium, due to his inability to convince the fringe that he is renouncing his market power (Maskin and Newbery, 1978; Newbery, 1981; Karp, 1984). These types of credibility problems did not occur in the present paper, because all firms were assumed to have equal market power.

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Appendix

This Appendix derives the NES for a discrete-time version of the game considered in the main body of the paper:

$$\max_{u^i} J^i(u^1, \dots, u^N) = E_{\varepsilon} \Pi_i(u^1, \dots, u^N, \varepsilon), \quad \Pi_i = \sum_{t=0}^T \pi_i(x_t, u_{1t}, \dots, u_{Nt}) \rho^t \quad (A.1)$$

subject to

$$x_{t+1} = H(x_t) - \left(\sum_{i=1}^N u_{it} \right) + \sigma(x_t) \varepsilon_t, \quad x_0 = \bar{x}_0, \quad \varepsilon_t \sim \text{IN}(0, 1), \quad (A.2)$$

where $\rho \equiv 1/(1+r)$, $u^i = \{u_{i0}, \dots, u_{iT}\}$ and $\varepsilon = \{\varepsilon_0, \dots, \varepsilon_T\}$.

Definition 5: The strategies $\{\theta^i, i = 1, \dots, N\}$ satisfying

$$J^i(\theta^1, \dots, \theta^N) \geq J^i(\theta^1, \dots, \theta^{i-1}, \theta_{i0}, \dots, \theta_{it-1}, \bar{\theta}_{it}, \theta_{it+1}, \dots, \theta_{iT}, \theta^{i+1}, \dots, \theta^N), \quad \forall \bar{\theta}_{it} \in \Theta_{it}; \quad i = 1, \dots, N; \quad t = 0, \dots, T$$

constitute a Stagewise Nash Equilibrium Solution (SWNES). $\square$

Obviously, every SWNES is a NES but not vice versa.

Proposition 5: With closed-loop or memory-less information patterns, the NES comprises only feedback strategies. For $\{\theta_{it}(x_t), i = 1, \dots, N, t = 0, \dots, T\}$ to constitute such a NES, it is necessary and sufficient that there exist value functions $V_{it}(\cdot) : \mathbb{R} \rightarrow \mathbb{R}, i = 1, \dots, N, t = 0, \dots, T$ such that the following recursions are satisfied:

$$\begin{aligned} V_{it}(x_t) &= \max_{\bar{u}_{it}} E_{\varepsilon_t} [\pi_i(x_t, \theta_{1t}(x_t), \dots, \theta_{i-1t}(x_t), \bar{u}_{it}, \theta_{i+1t}(x_t), \dots, \theta_{Nt}(x_t)) + \rho V^i(H(x_t) - \bar{u}_{it} - \sum_{j \neq i} \theta_{jt}(x_t) + \sigma(x_t) \varepsilon_t)] \\ &= E_{\varepsilon_t} [\pi_i(x_t, \theta_{1t}(x_t), \dots, \theta_{Nt}(x_t)) + \rho V^i(H(x_t) - \sum_{i=1}^N \theta_{it}(x_t) + \sigma(x_t) \varepsilon_t)] \end{aligned}$$

and $V_{iT+1}(x_{T+1}) = 0$.

Proof: A special case of the general proof presented in Basar and Olsder (1982, Section 6.4, Theorem 9) is presented. The i -th inequality of Definition 5 describes an N -firm static game:

$$\begin{aligned} \text{Max}_{\bar{u}_{iT}} \mathbb{E}_{\epsilon^T} \pi_i(x_T, \theta_{1T}(\cdot), \dots, \theta_{i-1T}(\cdot), \bar{u}_{iT}, \theta_{i+1T}(\cdot), \dots, \theta_{NT}(\cdot)) = \\ \mathbb{E}_{\epsilon^T} \pi_i(x_T, \theta_{1T}(\cdot), \dots, \theta_{NT}(\cdot))], \text{ where } \epsilon^t = \{\epsilon_0, \dots, \epsilon_t\} \text{ and} \\ x_{t+1} = H(x_t) - \sum_{i=1}^N \theta_{it}(x_t) + \sigma(x_t)\epsilon_t, x_0 = \bar{x}_0, t = 1, \dots, T. \end{aligned}$$

Since ϵ_T is statistically uncorrelated with ϵ^{T-1} , one has

$$\begin{aligned} \mathbb{E}_{\epsilon^{T-1}} \text{Max}_{\bar{u}_{iT}} \mathbb{E}_{\epsilon^T} \pi_i(x_T, \theta_{1T}(\cdot), \dots, \theta_{i-1T}(\cdot), \bar{u}_{iT}, \theta_{i+1T}(\cdot), \dots, \theta_{NT}(\cdot)) \\ = \mathbb{E}_{\epsilon^T} \pi_i(x_T, \theta_{1T}(\cdot), \dots, \theta_{NT}(\cdot)) \end{aligned}$$

which implies that the maximizing $\bar{u}_{iT}^i$ is a function of x_T . Due to noise, x_T cannot be expressed in terms of $\{x_0, \dots, x_{T-1}\}$, hence under the closed-loop or memory-less information patterns feedback strategies result ($\bar{u}_{iT} = \theta_{iT}(x_T)$). Furthermore, the choice of such a feedback strategy is independent of the strategies employed by any of the firms at any previous stage.

Now consider stage $t = T - 1$ and use the property that ϵ^{T-2} , ϵ_{T-1} and ϵ_T are statistical independent:

$$\begin{aligned} \mathbb{E}_{\epsilon^{T-2}} \text{Max}_{\bar{u}_{iT-1}} \mathbb{E}_{\epsilon_{T-1}} [\pi_i(x_{T-1}, \theta_{1T-1}(\cdot), \dots, \theta_{i-1T-1}(\cdot), \bar{u}_{iT-1}, \\ \theta_{i+1T-1}(\cdot), \dots, \theta_{NT-1}(\cdot)) + \rho V_{iT}(H(x_{T-1}) - \bar{u}_{iT-1} - \sum_{j \neq i} \theta_{jT-1}(\cdot) + \\ \sigma(x_{T-1})\epsilon_{T-1})] = \mathbb{E}_{\epsilon_{T-1}} [\pi_i(x_{T-1}, \theta_{1T-1}(\cdot), \dots, \theta_{NT-1}(\cdot)) + \\ \rho V_{iT}(H(x_{T-1}) - \sum_{i=1}^N \theta_{iT-1}(\cdot) + \sigma(x_{T-1})\epsilon_T)]. \end{aligned}$$

Hence, any $\theta_{iT-1}(\cdot)$ has to be a feedback strategy independent of all past strategies and past values of the state ($\bar{u}_{iT-1} = \theta_{iT-1}(x_{T-1})$).

The other stages follow from backward recursion.

Note that every SWNES is a SPNES, since the strategies of the SWNES at $t = s$ did not depend on the strategies at stages $t < s$. Since every SPNES (under closed-loop or memory-less information patterns) is a NES, every SWNES is also a NES. Also every NES is a SWNES, so that this Proposition holds for the NES. □

Hence, the introduction of serially uncorrelated shocks ensures that $I_t^i = \{t, x_t\}$ is a sufficient statistic for each firm and thus eliminates the problem of informational non-uniqueness. The reason for this is that NES's with memory strategies do not exist when there is white noise.

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