Supplementary Material for:
Experimental Evidence on Inflation Expectation Formation

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Abstract

This supplementary material presents some additional analysis, robustness checks, and experimental instructions.

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1 Additional Analysis: Form of the ALM

We investigate which parameters are non-zero in the actual law of motion (ALM). Note that as long we do not omit any relevant variables from the ALM and allowing all the relevant variables to be non-zero, this constitutes a test of rational expectations. Table S1 below presents the results of estimating eq. (10) in the paper. As we can see in the table both first and second lags of inflation turn out to be significant in all treatments. The first lag of output gap is significant in Treatments 2-4, while the second lag is significant in treatment 1. Interest rate is also in Treatments 1-3 (in treatment 4 we cannot include it in the specification due to perfect multicollinearity that is due to contemporaneous Taylor rule). Next step is to compare the estimated form of ALM with REE. With respect to the first representation of the REE, see eq. (5), we can immediately conclude that the ALM includes also 2 lags of inflation, second lag of output gap and interest rate in Treatments 1-3. Thus, the representation is not the same as for REE representation 1. Regarding representation 2, which is detailed in eq. (6) and takes into account that subjects "are extracting" information of the shocks, we observe that our estimated ALMs are closer to this representation. Especially the strong significance of the lag of interest rates are suggestive of the existence of this equilibria. However, in this representation both the first and the second lag of output gap should be significant and the second lag of inflation should not be significant. Our estimations suggest that one of the lags of output gap is not significant while the second lag of inflation is always significant. Thus, according to the definition in the Appendix to this letter this equilibria is characterized by Misspecified Perception Equilibria level 2 (MPE2). This holds for treatment 2-4, while in Treatment 1 due to significance of lagged output gap one could define a higher order MPE. Indeed, this equilibria are very similar to the Behavioral Learning Equilibria of Hommes and Zhu (2014), but it includes also the second lag of inflation. PLMs that result in a MPE2 are trend extrapolation rules (M5 and M9). Properties of the MPE2, that are carefully studied in the companion paper, Pfajfar and Žakelj (2011), suggests that this equilibrium is both indeterminate (with the exception of treatment 3) and E-unstable (with complex roots).
We can conclude that our equilibrium has both elements of REE representation 2 and MPE2. However, REE representation 2 yields both a determinate and E-stable outcome (see the companion paper for details). If we compare the dynamic properties of both equilibria and those of the realized series from the experiments, we see that actually the dynamic properties of most sessions (see Figures in the Appendix) are closer to those expected under the MPE2 than under the REE representation 2. Therefore, we would characterize this (temporary) equilibrium as a more complex version of the MPE2 where (some) subjects also extract information about the shocks.

<table>
<thead>
<tr>
<th></th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
<th>Treatment 4</th>
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<td>$\pi_{t-1}$</td>
<td>1.6721***</td>
<td>1.7542***</td>
<td>1.3749***</td>
<td>1.6558***</td>
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<td>(0.0084)</td>
<td>(0.0098)</td>
<td>(0.0152)</td>
<td>(0.0029)</td>
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<td>$\pi_{t-2}$</td>
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<td>-0.9074***</td>
<td>-0.7191***</td>
<td>-0.8565***</td>
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<td>(0.0038)</td>
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<td>$y_{t-1}$</td>
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<td>0.1625***</td>
<td>0.2606***</td>
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<td>(0.0004)</td>
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<td>0.3793***</td>
<td>1.0089***</td>
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<td>3.672</td>
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<tr>
<td>Wald $\chi^2$</td>
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<td>$1.79 \cdot 10^7$</td>
<td>$6.15 \cdot 10^5$</td>
<td>$9.54 \cdot 10^5$</td>
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</tbody>
</table>

Table S1: Estimates of ALM for each treatment. Notes: Standard errors in parentheses are calculated using bootstrap procedures (1000 replications) that take into account potential presence of clusters in groups. */**/*** denotes significance at 10/5/1 percent level.

1In this letter, we focus on the discussion of the form of the solution, but also the coefficients could be non-optimal and the equilibrium could just be a temporary equilibrium. See our companion paper for more analysis on the effect of non-optimal coefficients.
2 Additional Analysis: Convergence

We designed both a dynamic panel data regression and one without lagged dependant variable as a regressor to study whether a time trend has an effect on the likelihood of forming expectations rationally \( (\text{RAT}_t) \). Regressions actually show that the likelihood of forming expectations with rational expectations is (significantly) negatively affected by a time trend, although the effect is relative small. Significant inertia is displayed in the dynamic panel data regressions as one would expect based on summary statistics (RE model does not switch to other models in 94.6% of the cases). We have also checked if there is a time trend in the inertia and found no evidence of a time-varying inertia. Thus, we can conclude that we failed to find any evidence of global convergence.

<table>
<thead>
<tr>
<th>( R\text{AT}_t )</th>
<th>Probit, PA</th>
<th>Probit, RE</th>
<th>Logit, PA</th>
<th>Logit, RE</th>
<th>Logit, FE</th>
<th>dyn panel</th>
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<td>( t-1 )</td>
<td>-0.0081</td>
<td>-0.0096</td>
<td>-0.0129</td>
<td>-0.0172</td>
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<td>(0.0747)</td>
<td>(0.0761)</td>
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<td>( y_{t-1} )</td>
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<td>0.0940</td>
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<td>(0.0650)</td>
<td>(0.0726)</td>
<td>(0.1108)</td>
<td>(0.0345)</td>
<td>(0.0053)</td>
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<tr>
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<td>-0.0066</td>
<td>-0.0109</td>
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<td>-0.0033</td>
<td>-0.0054</td>
<td>-0.0053***</td>
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<td>(0.0065)</td>
<td>(0.0108)</td>
<td>(0.0013)</td>
<td>(0.0004)</td>
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<tr>
<td>( \left( \text{RAT}<em>{t-1} - \text{RAT}</em>{t-2} \right)^2 )</td>
<td>-0.0008</td>
<td>-0.0007</td>
<td>-0.0013</td>
<td>-0.0011</td>
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<td>(0.0190)</td>
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<tr>
<td>( \text{RAT}_{t-1} )</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Wald ( \chi^2 )</td>
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<td>3.134</td>
<td>4.014</td>
<td>3.497</td>
<td>65.388</td>
<td>2647.896</td>
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</tbody>
</table>

Table S2: Determinants of using Rational Expectations. Notes: RE stands for random effects, PA population averagesand FE for Fixed effects. Dyn panel is conducted using the system GMM estimator of Blundell and Bond (1998) for dynamic panels. Standard errors in parentheses are calculated using bootstrap procedures (1000 replications) that take into account potential presence of clusters in groups. */**/*** denotes significance at 10/5/1 percent level.
3 Results for Specific Models

In this subsection we present some general patterns of estimated models. Estimations of the sticky information type model (M2) suggest that about 97% of agents display a significantly positive $\lambda_1$, with the average $\lambda_1$ 0.20.

All the participants have $\vartheta$ positive and significant at a 5 percent level in the estimation of M3. 13.4% of participants have a constant gain parameter significantly lower than 1, while 53.7% of them update their forecasts with an error correction term significantly greater than 1. This means that the latter agents possibly overreact to their past errors. Their prevalence might imply problems with dynamic stability in certain treatments. If the estimated parameter ($\tau$ in this version) is significantly different from 0, we conclude that agents actually learn from their past mistakes with a decreasing gain over time. Our tests do not support the hypothesis that the coefficient decreases over time as the $R^2$ is always greater (for all subjects) for a constant gain model.

In the case of the trend extrapolation model (M5) we find that the constant is significant at the 5% level in 28.7% of cases while the $\tau_1$ is significant in 78.2% of cases at the same level. Most of the times $\tau_1$ is between 0 and 1, but there are a few cases when $\tau_1$ is significantly below 0 (6.9%) and for 15.3% of subjects it is significantly higher than 1. We refer to these rules as strong trend extrapolation.

By estimating the General model (M6) we find that 81.9% of agents take inflation into account when making their predictions. These results give an indication of which variables are most commonly used for forecasting in our experiment. Thus, inflation is the most commonly used as an explanatory variable for inflation forecasts, while only a small proportion of subjects implement lags of output gap in their forecasts. About 56.0% of subjects take the interest rate into account. This suggests that at least for some subjects it will be difficult to not to reject the RE of the representation 1. $^2$$^3$

$^2$Estimation of the extended M6 that included also the subjects' previous forecasts suggest that 66.7% subjects also consider their own forecast from the previous period for forecasting (together with the estimation of M2-M4). This implies that a high proportion of subjects based their forecasts on private information (previous period forecasts of other subjects were not directly observable).

$^3$We also investigate the nature of the forecast error in more depth. We estimate a model where we regress the forecast errors on past observed forecast error and changes of other macroeconomic variables. Subjects often do not exploit the informational content of the output gap and most importantly subjects
We find that 56.5% of participants learn according to the first setup with lagged inflation, as in model (M7). The gain parameter $\vartheta$ is in the range between 0.0001 and 0.1000, with a mean value of 0.02900 and the median is 0.01125. We also estimate adaptive learning with the PLMs that include the lagged output gap (M8) and the AR(1) form (M10). However, these models rarely outperform the other models studied here. In the learning version of the trend extrapolation model (M9) 31.5% of our subjects have positive gains. The optimal gains are on average slightly higher than before as they range between 0.0003 and 0.7900 with a mean value of 0.0654 (the median is 0.0310). This version of the PLM (M9) often performs better than previous versions of learning in terms of root mean square error (RMSE). In Section 5.4 we compare different models and find that this version of constant gain learning indeed best represents the behavior of a significant proportion of our subjects. As an alternative approach to determine the average gain, we exclude from our sample all subjects for whom learning does not represent the best model.\footnote{We will consider Comparison 1 from Table 5 and exclude model (M8) as it is generally associated with extremely high values of the gain parameter.} In this case, we find that the average gain of these subjects is 0.0447 with a standard deviation of 0.0537 (the median is 0.0260). The standard deviation is quite high as there are a few very high values, but most of the gains fall in the range between 0.01 and 0.07.

\footnotetext[4]{We will consider Comparison 1 from Table 5 and exclude model (M8) as it is generally associated with extremely high values of the gain parameter.}
4 Experimental Instructions

Thank you for participating in this experiment, a project of economic investigation. Your earnings depend on your decisions and the decisions of the other participants. There is a show up fee of the 4 Euros assured. From now on until the end of the experiment you are not allowed to communicate with each other. If you have some question raise your hand and one of the instructors will answer the question in private. Please do not ask aloud.

The Experiment

All participants receive exactly the same instructions. You and 8 other subjects all participate as agents in the same fictitious economy. You will have to predict future values of given economic variables. The experiment consists of 70 periods. The rules are the same in all the periods. You will interact with the same 8 subjects during the whole experiment.

Imagine that you work in a firm where you have to predict inflation for the next period. Your profit depends on the accuracy of your inflation expectation.

Information in Each Period

The economy will be described with 3 variables in this experiment: the inflation rate, the output gap, and the interest rate.

- **Inflation** measures general rise in prices in the economy. Each period it depends on the inflation expectations of the agents in economy (you and other 8 participants in this experiment), output gap and small random shocks.

- The **output gap** measures for how much (in %) the actual Gross Domestic Product differs from the potential one. If the output gap is greater than 0, it means that the economy is producing more than the potential level, if negative, less than potential

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5Instructions used for experiments at Universitat Pompeu Fabra are in the Spanish language. In experimental sessions, they were accompanied with the screenshots of the experimental interface and the profit table with earnings for various combinations of estimation error and confidence interval.
level. It depends each period on inflation expectations of the agents in economy, past output gap, interest rate and small random shocks.

- The **interest rate** is (in this experiment) the price of borrowing the money (in %) for one period. The interest rate is set by the monetary authority. Their decision mostly depends on inflation (expectations) of the agents in economy.

All given variables might be relevant for inflation forecast, but it is up to you to work out their relation and possible benefit of knowing them. The evolution of variables will partly depend on the inputs of you and other subjects and also different random shocks influencing the economy.

- You enter the economy in period 1. In this period you will be given computer generated past values of inflation, output gap and interest rate for 10 periods back (Called: -9, -8, ... -1, 0)

- In period 2 you will be given all past values as seen in period 1 plus the value from period 1 (Periods: -9, -8, ... 0, 1).

- In period 3 you will see all past values as in period 2 (Periods: -9, -8, ... 1, 2) plus YOUR prediction about inflation in period 2 that you made in period 1.

- In period t you will see all past values of actual inflation up to period (Periods: -9, -8, ... , ) and your predictions up to period (Periods: 2, 3, ... , ).

**What Do You Have to Decide?**

Your payoff will depend on the accuracy of your prediction of the inflation in the future period. In each period your prediction will consist of two parts:

1. *Expected inflation*, (in %) that you expect to be in the NEXT period (*Exp.Inf.*)

2. The *Confidence Interval* (*Conf.Int.*) around your prediction for which you think there is 95% probability that the actual inflation will fall into. The interval is determined as the number of percentage points for which the actual inflation can be higher or lower.
**Example 1** Let’s say you think that inflation in the next period will be 3.7%. And you also think there is most likely (95% probability) that the actual inflation will not differ from that value for more than 0.7 percentage points. Therefore, you expect that there is 95% probability that actual inflation in the next period will be between 3.0% and 4.4% (3.7% ± 0.7%). Your inputs in the experiment will be 3.7 under 1) and 0.7 under 2).

Your goal is to maximize your payoff, given with the equation:

\[
W = \max \left\{ \frac{100}{1 + |\text{Inflation} - \text{Exp.Inf.}|} - 20, 0 \right\} + \max \left\{ \frac{100x}{1 + \text{Conf.Int.}} - 20, 0 \right\}
\]

where \(\text{Exp.Inf.}\) is your expectation about the inflation in the NEXT period, \(\text{Conf.Int.}\) is the confidence interval you have chosen, Inflation is the actual inflation in the next period, and \(x\) is a variable with value 1 if

\[
\text{Exp.Inf.} - \text{Conf.Int.} \leq \text{Inflation} \leq \text{Exp.Inf.} + \text{Conf.Int.}
\]

and 0 otherwise.

This expression tells you, that \(x\) will be 1, if actual inflation falls between \(\text{Exp.Inf.} - \text{Conf.Int.}\) (3.0% in our example) and \(\text{Exp.Inf.} + \text{Conf.Int.}\) (4.4% in our example).

The first part of the payoff function states that you will receive some payoff if the actual value in the next period will differ from your prediction in this period for less than 4 percentage points. The smaller this difference will be, the higher the payoff you receive. With a zero forecast error (\(|\text{Inflation} - \text{Exp.Inf.}| = 0\)), you would receive 80 units. However, if your forecast is 1 percentage point higher or lower than the actual inflation rate, you will get only 30 units (100/2 − 20). If your forecast error is 4 percentage points or more, you will receive 0 units (100/5 − 20).

The second part of the payoff function simply states that you will get some extra payoff if the actual inflation is within your expected interval and if that interval is not be larger than ±4 percentage point. The more certain of the actual value you are, the smaller interval you give, and the higher will be your payoff if the actual inflation indeed is in the given interval but there will also be higher chances that actual value will fall
outside your interval. In our example this interval is ±0.7 percentage points. If the actual inflation falls in this interval you would receive 38.8 units \((100/(1 + 0.7) - 20)\) in addition to the payoff from the first part of the payoff function. If the actual values is outside your interval, your receive 0.

In the attached sheet you can find table which shows various combinations of forecast error and confidence interval needed to earn a given number of points. See also figure on the next page.

**Information After Each Period**

Your payoff depends on your predictions for the next periods and actual realization in next period. Because the actual inflation will be only known in the next period, you will also be informed about you current period \((t)\) prediction and earnings after the end of NEXT period \((t + 1)\). Therefore:

- After Period 1 you will not receive any earnings, since you did not make any prediction for the period 1.

- In any other period, you will receive the information about the actual inflation rate in this period and your inflation and confidence interval prediction from previous period. You will also be informed if the actual inflation value is in your expected interval and what are your earnings for this period.

The units in the experiment are fictitious. Your actual payoff will be the sum of profits from all the periods converted to euros in 1/500 conversion.

If you have any questions please ask them now!

**Questionnaire\(^6\)**

1. If you believe that inflation in the next period will be \(\ldots\) \(4.2\%\) \(\ldots\), and you are quite sure that it will be higher than \(\ldots\) \(3.5\%\) \(\ldots\) and lower than \(\ldots\) \(4.9\%\) \(\ldots\), you will type:

---

\(^6\)Options (1) and (2) are pointing to the different fields on the screenshot of the experimental interface.
Under (1) _ _ _ _ _ _ _ _ _ _ for inflation, and
Under (2) _ _ _ _ _ _ _ _ _ _ for confidence interval.

2. If you are now in period _ _ _ _ _ _ _ _ _, you have information about past inflation, output gap and interest rate up to period _ _ _ _ _ _ _ _ _ _ and you have to predict the inflation for period _ _ _ _ _ _ _ _ _ _.
5 Additional Tables and Figures

![Graphs showing inflation under different expectation formation rules]

Figure S1: Simulation of inflation under alternative expectation formation rules
Rational expectations
PLM of RPE, $\xi = 0.05$

Naive expectations

Adaptive expectations, $\varphi = 0.75$

Adaptive expectations, $\varphi = 1.4$

Trend extrapolation, $\tau = 0.3$

Trend extrapolation, $\tau = 0.7$

Figure S2: Simulation of inflation under alternative expectation formation rules
Figure S3: Switching behavior, no rational expectations, Part 1.
Figure S4: Switching behavior, no rational expectations, Part 2.
Figure S5: Switching behavior, including rational expectations, Part 1.
Figure S6: Switching behavior, including rational expectations, Part 2.
Figure S7: Histogram of inflation forecasts for all treatments.

References