MERGERS IN NONRENEWABLE RESOURCE OLIGOPOLIES
AND ENVIRONMENTAL POLICIES

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4 September 2018

ISSN 0924-7815
ISSN 2213-9532
Mergers in Nonrenewable Resource Oligopolies and Environmental Policies *

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Abstract

We study the profitability of horizontal mergers in nonrenewable resource industries, which account for a large proportion of merger activities worldwide. Each firm owns a private stock of the resource and uses open-loop strategies when choosing its extraction path. We analytically show that even a small merger (merger of 2 firms) is always profitable when the resource stock owned by each firm is small enough. In the case where pollution is generated by the industry’s activity, we show that an environmental policy that increases the firms’ production cost or reduces their selling price can deter a merger. This speeds up the industry’s extraction and thereby causes emissions to occur earlier than under a laissez-faire scenario.

JEL codes: Q39, L41, Q58
Keywords: exhaustible resources, horizontal mergers, environmental regulation, differential games

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*We are grateful to Ying Tung Chan and William Duan for their research assistance, and to Rick van der Ploeg, the editors and three anonymous referees for their insightful comments. We would also like to thank participants at the Tinbergen Institute conference “Combating Climate Change” (April 21–22, 2016). We thank the Canadian Social Sciences and Humanities Research Council (SSHRC) for financial support. Hassan Benchekroun also thanks the Fonds de recherche du Québec – Société et culture (FRQSC) for financial support.

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1 Introduction

This paper examines the incentive to merge in nonrenewable resource industries. This sector constitutes a large proportion of GDP in many economies,\(^1\) and also has a long history of mergers and acquisitions (M&A) activity, starting with Standard Oil’s acquisitions in the early 1900’s. The volume of M&A has been consistently higher in the exhaustible resource sector relative to many others. Moreover, this sector has experienced a spate of mega-mergers, starting in the late 1990’s, including the mergers of BP and Amoco (1998, $63 billion); Exxon and Mobil (1999, $74.2 billion); Total Fina and Elf Aquitaine (1999, $54.2 billion); Chevron and Texaco (2001, $45 billion); and Royal Dutch Petroleum and the Shell Group (2004).\(^2\) A first research question addressed in this paper is understanding why there is so much M&A activity in the exhaustible resource sector.

There exists a vast literature concerned with various aspects of horizontal mergers. Salant, Switzer & Reynolds (1983), henceforth referred to as SSR, is arguably one of the most influential papers in that literature. SSR’s important contribution is to show that horizontal mergers can be unprofitable, that is, the profits of the merged entity is smaller than the sum of the pre-merger profits of the individual firms that merge. In particular, in the case of a symmetric oligopoly with linear demand and constant marginal production cost where firms compete in quantity, a merger of two firms is never profitable unless it is a merger to form a monopoly. Moreover, the merged entity must be significant enough for the merger to be profitable. The basic intuition driving this result is that, in the case of strategic

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\(^1\) For example, exhaustible resource sectors, including oil, gas and minerals and mining, accounted for about 10% of Canadian GDP annually during 2008-2012, according to Statistics Canada.

\(^2\) The global value of M&A in the oil sector rose from $88.99 billion in 1997 (representing about 25% of global income from the oil sector in 1997) to $372 billion in 2007 (representing about 22% of global income from the oil sector in 2007) (see Kumar, 2012, for further details). In Canada, for instance, exhaustible resource extraction industries have seen rising volumes of M&A in recent years. According to the data provided by the Canadian Competition Bureau, during 2012 to 2013, about 20% of the 330 mergers that were reviewed by the Bureau were in this sector, with 16% of mergers being realized in oil and gas extraction industries. The highest value merger transactions in Canada in 2012 were realized in the oil and gas extraction industry in the form of cross-border acquisitions, according to Macleans and Blake Canadian Lawyers, including the C$15-billion acquisition of Nexen by China’s CNOOC and the C$5.5-billion acquisition of Progress Energy Resources by Malaysia’s Petronas.
substitutes such as in Cournot competition, when the merger participants decrease quantity, the non-merging firms respond by increasing their output levels, thereby mitigating the increase in market power of the merger participants. The increase in output of the outsiders more than offsets the benefit the merging firms can get from their reduction of output. SSR’s result has proven to be very robust to various modifications of the basic benchmark model (see, e.g., Stigler 1950; Kamien & Zang 1990, 1991, 1993; Gaudet & Salant 1991; Farrell & Shapiro 1990).

The nonrenewable resource sector requires a specific merger analysis to account for the fact that the output of firms, that is, their cumulative extraction over time, is limited by their stock. In that context, we investigate the profitability of mergers. We find that the result of SSR does not carry over to the case of nonrenewable resource industries: even a small merger (merger of 2 firms) is always profitable when the resource stock owned by each firm is small enough.3

We then analyze the impact, on the profitability of a merger, of an environmental policy that raises firms’ extraction costs (or reduces the price of the resource). This analysis is motivated by the fact that many important nonrenewable resources’ production and/or consumption generate a negative externality (e.g., oil or phosphate). The impact of an environmental tax on the resource has received a lot of attention recently, as a carbon tax on fossil fuels is often viewed as a natural instrument to slow down global warming. An important stream of that literature examines whether a carbon tax may result in the Green Paradox, that is, the unintended consequence of speeding up fossil fuel extraction and therefore increasing pollution (see Sinn, 2008, and, e.g., Pittel, van der Ploeg & Withagen, 2014, and Long, 2015).

Two papers that are closely related to ours are Benchekroun & Gaudet (2003), which examines the impact of an exogenous marginal production restriction in a nonrenewable resource duopoly, and Benchekroun & Gaudet (2015), which considers a renewable, common

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3It is possible to overturn SSR’s result if marginal cost is increasing (see, Perry and Porter, 1985). In this paper, we highlight a new mechanism through which this may occur, namely resource constraints.
pool resource. In the case considered in this paper, the production restriction is non-marginal and is determined endogenously in equilibrium, each firm owning a private stock of the non-renewable resource. We find that a tax on extraction may prevent a merger from happening. We show that a merger slows down the industry’s extraction rate, and therefore delays emissions. If a higher tax rate deters a merger, it follows that emissions occur earlier under the stricter environmental policy than under a laissez-faire scenario. This result clearly carries a similar flavor to a green paradox. However, the channel of the increase in pollution, i.e., the merger decision of the players, is novel.

In instances where resource owners are countries and not firms, coordination of interests among few resource owners is more likely to take the form of partial cartels rather than mergers. In the symmetric case where firms have identical constant marginal costs, all the results derived in our paper naturally extend to the case of a cartel.

We use a dynamic game model where firms compete in quantity in the output market while each firm faces a resource constraint (the cumulative extraction over time must not exceed its initial endowment of the resource). We use a continuous time framework with an endogenous time horizon. We follow much of the existing literature on oligopoly models of nonrenewable resource markets, and use open-loop strategies where firms choose a time path of extraction at the beginning of the game (see, e.g., Salant 1976, 1982; Lewis & Schmalensee 1980; Loury 1986; Gaudet & Long 1994; Benchekroun, Halsema & Withagen 2009, 2010).4,5

We also generalize our results to the case of a marginal cost function that is decreasing in the remaining resource stock.

4 None of these papers analyze the impact of mergers. Nor do they examine the interplay of an environmental policy on firms’ decisions to merge.

5 We note that the equilibrium derived using open-loop strategies may not be subgame perfect (see, e.g., Karp & Newberry 1991; Groot, Withagen & de Zeeuw, 1992, 2003). A set of papers use stationary Markovian strategies, that is, strategies that depend on the vector of stocks. The equilibrium within this class of strategies is by construction subgame perfect, but much more challenging to characterize. In an effort to derive analytical solutions, these papers rely on specific functional forms or assumptions, such as isoelastic demand and zero extraction cost (Eswaran & Lewis 1985; Reinganum & Stokey 1985; Benchekroun & Long 2006), economic abandonment where the resource is not exhausted in full (Salo & Tahvonen 2001), an exogenously fixed time horizon (Hartwick & Brolley 2008; Polasky 1996; Wan & Boyce 2014). For instance, Wan & Boyce 2014 offers, in a two-period model, a full characterization of the duopolistic equilibrium in the case of Cournot and Stackelberg games.
We proceed as follows. Section 2 presents the model. Section 3 presents the analysis of the profitability of mergers. Section 4 analyzes the impact of a tax on extraction. Section 5 analyses the case where the marginal cost function is decreasing in the resource stock remaining. Section 6 concludes.

2 The Model and Preliminary Analysis

2.1 The Model

We consider an exhaustible resource industry involving \( n \) firms. A number \( n_S \) of these firms initially each own a stock \( S_{0S} \) while \( n_L \) firms each initially own a stock \( S_{0L} \), with \( n_L+n_S = n \). Without loss of generality, we shall consider that \( S_{0S} \leq S_{0L} \) and that the firms with an initial endowment of \( S_{0S} \) are the first \( n_S \) firms, that is, \( S_{0i} = S_{0S} \) for \( i = 1, \ldots, n_S \) and \( S_{0i} = S_{0L} \) for \( i = n_S + 1, \ldots, n \). Marginal extraction costs are constant and identical across all firms, and given by \( c \). Firms therefore may only differ with respect to their initial stock of the resource.

Let \( q_i(t) \geq 0 \) denote the extraction rate at time \( t \geq 0 \) of firm \( i \). Define \( Q_S(t) = \sum_{i=1}^{n_S} q_i(t) \) to be the aggregate supply of the firms initially endowed with a stock \( S_{0S} \), and \( Q_L(t) = \sum_{i=n_S+1}^{n} q_i(t) \) to be the aggregate supply of the firms initially endowed with a stock \( S_{0L} \). Demand for the resource is stationary and linear with a choke price \( a>c \), so that the inverse demand at time \( t \geq 0 \) for the extracted resource is given by \( p(t) = a - b [Q_S(t) + Q_L(t)] \).\(^6\) An admissible extraction path \( q_i(t) \) for firm \( i \) is such that \( q_i(t) \geq 0 \) for all \( t \geq 0 \) and

\[
\int_0^\infty q_i(t) \, dt \leq S_{0i}. \tag{1}
\]

Firms are oligopolists in the resource market where they compete à la Cournot, and the

\(^6\)More precisely, \( p(t) = \max \{a - bQ(t), 0\} \). Throughout the paper, we will focus on cases where the outcome is such that \( a - bQ(t) > 0 \) for all \( t \). This is true as long as the choke price \( a \) is large enough.
objective of a firm $i$ is to maximize the discounted sum of its profits

$$\int_0^\infty e^{-rt} [a - b (Q_S (t) + Q_L (t)) - c] q_i (t) \, dt. \quad (2)$$

Each firm takes the extraction paths of its competitors as given and chooses an extraction path that maximizes its discounted sum of profits (2), subject to the resource constraint (1).

We characterize an open-loop Nash-Cournot equilibrium (OLNE) of this game. More precisely:

**Definition 1** *Open-loop Nash-Cournot equilibrium (OLNE)*

A $n$-tuple vector of extraction paths $q (\cdot) \equiv (q_1 (\cdot), \ldots, q_n (\cdot))$ with $q (t) \geq 0$ for all $t \geq 0$ is an open-loop Nash-Cournot equilibrium if:

(i) the resource constraint is satisfied for all $i = 1, \ldots, n$, and

(ii) for all $i = 1, \ldots, n$ we have

$$\int_0^\infty e^{-rt} [a - b Q_i (t) - c] q_i (t) \, dt \geq \int_0^\infty e^{-rt} [a - b (\hat{q}_i (t) + Q_{-i} (t)) - c] \hat{q}_i (t) \, dt$$

for all $\hat{q}_i$ satisfying the resource constraint, where $Q_{-i} (t) = \sum_{j=1, j\neq i}^n q_j (t)$.

Remark: note that by using the open-loop solution concept, we are assuming that firms have the ability to commit to the whole extraction path at the beginning of the game. A Nash equilibrium results in an outcome that is time-consistent. An alternative is to consider Markovian strategies, that is, extraction strategies that are stock dependent. The resulting equilibrium is then subgame perfect (see Dockner et al. 2000 Ch. 4). The use of Markovian strategies assumes that firms are able to adjust their production plans. This flexibility may not be a realistic assumption in resources such as oil, for technical reasons (see e.g., Anderson et al. 2017). Another drawback of using such strategies is that the model becomes analytically untractable and one must rely on numerical methods. Moreover, such
numerical methods suffer from the curse of dimensionality: they fail to deliver a solution in cases such as ours where the number of state variables can be large. We opt for a solution belonging to the set of open-loop strategies for analytical tractability. Indeed, the possibility of a profitable merger can be characterized analytically when stock levels are small enough, while for arbitrary stock levels, a numerical analysis can be handled with built-in functions available in formal computation softwares such as Mathematica or Maple.

2.2 The Pre-Merger Equilibrium

We now proceed to characterize an OLNE of the above-defined game. We first show that all firms exhaust their stocks in finite time. Let $T_S$ and $T_L$ denote the time at which firms with initial stocks $S_{0S}$ and $S_{0L}$ respectively exhaust their stocks. We will show that when $S_{0S} < S_{0L}$, we have $T_S < T_L$. The equilibrium then consists of two phases: phase I from date $0$ to $T_S$, and phase II from $T_S$ to $T_L$. During phase I, the extraction of both types of firms is positive until $T_S$ where the extraction and the stock of firms $i = 1, \ldots, n_S$ vanish. During phase II, only firms $i = n_S + 1, \ldots, n$ still own a positive stock, until $T_L$ where the extraction and the stock of these remaining firms vanish. We denote by $q_S$ and $q_L$ the extraction paths of firms initially endowed with stocks $S_{0S}$ and $S_{0L}$ respectively.

**Proposition 1:** Let

$$q_S(t) \equiv \begin{cases} \frac{(a-c)}{b(1+n)} \left(1 - e^{-r(T_S-t)}\right) & \text{for } t \in [0,T_S] \\ 0 & \text{for } t \geq T_S \end{cases},$$

$$q_L(t) \equiv \begin{cases} \frac{(a-c)}{b(1+n)} \left(1 - \frac{1+n_{S}}{1+n_{L}} e^{-r(T_L-t)} + \frac{n_S}{1+n_L} e^{-r(T_S-t)}\right) & t \in [0,T_S] \\ \frac{(a-c)}{b(1+n_{L})} \left(1 - e^{-r(T_L-t)}\right) & t \in [T_S,T_L] \\ 0 & t \geq T_L \end{cases}$$
where $T_S$ and $T_L$ are the unique solutions to:

$$
\int_0^{T_S} q_S(t) \, dt = S_{0S}
$$

and

$$
\int_0^{T_L} q_L(t) \, dt = S_{0L}.
$$

Then the $n$-tuple vector $q^{eq}$ where $q^{eq}_j = q_S$ when $j = 1, \ldots, n_S$ and $q^{eq}_k = q_L$ when $k = n_S + 1, \ldots, n$ constitutes an OLNE.

**Proof:**

Each firm $i$ takes the supply paths of the other firms as given and maximizes (2) subject to (1). The current value Hamiltonian associated with the problem of firm $i$ is given by:

$$
H_i(q_i, q_{-i}, \lambda_i, t) = [a - b (Q_S + Q_L) - c] q_i + \lambda_i (-q_i).
$$

where $(q_i, q_{-i})$ is an $n$-tuple vector obtained from the vector $q$ by deleting its $i$th component and replacing it by $q_i$.

Exploiting symmetry, the maximum principle yields the interior solution

$$
a - b ((1 + n_S) q_j + n_L q_k) - c - \lambda_j = 0
$$

$$
a - b (n_S q_j + (1 + n_L) q_k) - c - \lambda_k = 0
$$

for $j = 1, \ldots, n_S$ and $k = n_S + 1, \ldots, n$, with

$$
\dot{\lambda}_j = r \lambda_j
$$

$$
\dot{\lambda}_k = r \lambda_k
$$

During the second phase where only firms $k = n_S + 1, \ldots, n$ extract a positive quantity,
the maximum principle yields:

\[
a - b \left(1 + n_L\right) q_k - c - \lambda_k = 0
\]

(11)

with

\[
\dot{\lambda}_k = r\lambda_k.
\]

(12)

The terminal times \(T_S\) and \(T_L\) are endogenous and determined by

\[
H_j(q_j(T_S), q_{-j}(T_S), \lambda_j(T_S), T_S) = 0
\]

for \(j = 1, \ldots, n_S\) and

\[
H_k(q_k(T_L), q_{-k}(T_L), \lambda_k(T_L), T_L) = 0
\]

for \(k = 1 + n_S, \ldots, n\). These last conditions along with the maximum principle imply that

\[
q_j(T_S) = 0 \text{ and } q_k(T_L) = 0.
\]

(13)

Solving for \((q_j, q_k)\), from (7) and (8), we obtain the following:

\[
\frac{a - c - \lambda_j}{b} = (1 + n_S) q_j + n_L q_k
\]

(14)

\[
\frac{a - c - \lambda_k}{b} = n_S q_j + (1 + n_L) q_k,
\]

(15)

or

\[
q_j = \frac{a - c - \lambda_j}{b} \left(1 + n_L\right) - \frac{a - c - \lambda_k}{b} \left(n_L\right)
\]

which yields

\[
q_j = \frac{(a - c - \lambda_j) (1 + n_L) - (a - c - \lambda_k) n_L}{b (1 + n)}.
\]
Therefore, by symmetry, we obtain the following:

\[ q_j = \frac{a - c - \lambda_j (1 + n_L) + \lambda_k n_L}{b (1 + n)} \]  
\[ q_k = \frac{a - c - \lambda_k (1 + n_S) + \lambda_j n_S}{b (1 + n)} \]  

From (9), (10), (12) and continuity of the costate variable \( \lambda_k \) at \( T_S \), we have

\[ \lambda_j = \lambda_j e^{rt} \text{ for all } t \in [0, T_S] \]  
\[ \lambda_k = \lambda_k e^{rt} \text{ for all } t \in [0, T_L] . \]

The costate variables, \( \lambda_{j0} \) and \( \lambda_{k0} \), are determined using conditions (13), along with (16) and (11). From (11), we have:

\[ q_k (T_L) = \frac{a - c - \lambda_{k0} e^{rT_L}}{b (1 + n_L)} = 0 , \]

that is,

\[ (a - c) e^{-rT_L} = \lambda_{k0} . \]

From (16), we have:

\[ q_j (T_S) = \frac{a - c - \lambda_{j0} e^{rT_S} (1 + n_L) + \lambda_{k0} e^{rT_S} n_L}{b (1 + n)} = 0 \]

and

\[ \frac{(a - c) e^{-rT_S} + \lambda_{k0} n_L}{(1 + n_L)} = \lambda_{j0} , \]

or

\[ \frac{(a - c) (e^{-rT_S} + e^{-rT_L} n_L)}{(1 + n_L)} = \lambda_{j0} . \]

Using \( \lambda_{j0} \) and \( \lambda_{k0} \) and substituting (18) and (19) into (16), (17) and (11) yields (3) and (4).
for $t \leq T_S$.

For $t \geq T_S$, we have a symmetric equilibrium among $n_L$ firms that exhaust a stock during a time interval of length $T_L - T_S$. We can therefore use the above analysis to obtain the equilibrium path.

The equilibrium paths (3) and (4) are determined as functions of the terminal times $T_L$ and $T_S$. These dates are determined from the resource constraint conditions, i.e., (5) and (6). It can be shown that such a non-linear system in $(T_L, T_S)$ admits a unique solution, with $T_L \geq T_S$. 

We can now compute the value function of each player, which constitute a building block to conduct the analysis of the profitability of a merger. The equilibrium discounted sum of profits for an individual firm of each type is given by:

$$V_S = \int_0^{T_S} \left[ a - b \left( n_S q_i^e (t) + n_L q_j^e (t) \right) - c \right] q_i^e (t) e^{-rt} dt$$

(20)

and

$$V_L = \int_0^{T_L} \left[ a - b \left( n_S q_i^e (t) + n_L q_j^e (t) \right) - c \right] q_j^e (t) e^{-rt} dt$$

(21)

for $i = 1, \ldots, n_S$ and $j = n_S + 1, \ldots, n$. It will be useful to explicitly write the equilibrium discounted sum of profits as functions of $(n_S, n_L, S_{0S}, S_{0L})$. Substitution of $q^e$ and integrating gives respectively $V_S(n_S, n_L, S_{0S}, S_{0L})$ and $V_L(n_S, n_L, S_{0S}, S_{0L})$. The expressions of these functions for any $(n_S, n_L, S_{0S}, S_{0L})$ are too cumbersome to report here. Instead we report these value functions for two special cases that will be instrumental in the analysis of a horizontal merger in the following section. First, we have

$$V_S(n - 1, 1, S_0, S_0) = \frac{a^2 e^{-rT} (rT (n - 1) + e^{rT} + ne^{-rT} - n - 1)}{br (n + 1)^2},$$

(22)

where $V_S(n - 1, 1, S_0, S_0)$ is the discounted sum of profits of a single firm in a symmetric
n-firm oligopoly, where all firms initially own a stock \( S_0 \). Second, we have

\[
V_L(n - m, 1, S_0, mS_0) = \frac{(a - c)^2}{4br} \left( e^{r(T_S - 2T_L)} \left( 1 - e^{r(T_L - T_S)} \right)^2 + \frac{\Psi}{(2 - m + n)^2} \right)
\]

with

\[
\Psi \equiv 4 - e^{-2rT_S} (n - m)^2 + 4e^{-rT_S} (-2 + (m - n) (1 - rT_S))
+ (e^{-2rT_L} + e^{-rT_S} - e^{r(T_S - 2T_L)}) (2 - m + n)^2,
\]

where \( V_L(n - m, 1, S_0, mS_0) \) represents, in an \( n - m + 1 \) firms asymmetric oligopoly, the value function of a single firm that owns a stock \( mS_0 \) where \( m \in \{2, \ldots, n\} \) while each of the remaining \( n - m \) firms owns a stock \( S_0 \).

Note that the equilibrium discounted sum of profits, reported in (22) and (23), are expressed in terms of terminal times, \( T \) in the case of (22) and \( T_S \) and \( T_L \) in the case of (23). The stocks do not appear in their expressions. However, the terminal times themselves depend on \( S_0 \). The relationship between the terminal times \( T, T_S \) and \( T_L \) and the stock \( S_0 \) will be discussed in more detail in the next section.

### 3 Profitability of Mergers

We exploit the characterization of the OLNE in the above-defined game to investigate the profitability of mergers of a subset of firms in the industry. We first focus on the symmetric case, where all firms have equal initial stocks, and provide an analytical proof that a merger is always profitable when the stock of the resource is small enough. We then examine, through numerical simulations, the impact of a merger on the speed of extraction of the resource. Finally, we investigate an asymmetric case, where the initial stock of merging firms differ from the initial stock of an outsider.
3.1 Mergers in a symmetric oligopoly

We focus here on the case where the $n$ firms own the same initial stock $S_0 \equiv S_{0S} = S_{0L}$. We consider a merger of a subset $m$ of the $n$ firms. Since the marginal cost of extraction is constant, the market structure after a merger corresponds to asymmetric oligopoly competition between $n - m + 1$ firms, where the merged entity owns an initial stock equal to $mS_0$ while the $n - m$ outsiders each own an initial stock of $S_0$. For simplicity, henceforth we set $c = 0$, which does not qualitatively affect our results. The OLNNE for the post merger oligopoly is readily obtained from Proposition 1 by setting $n_L = 1$, $S_{0L} = mS_0$, $n_S = n - m$ and $S_{0S} = S_0$.

A merger is profitable when

$$G \equiv V_L (n - m, 1, S_0, mS_0) - mV_S (n - 1, 1, S_0, S_0) > 0.$$  

From (5), it follows that the exhaustion time $T$ in the pre-merger game is given by:

$$\frac{a}{br(1 + n)} (e^{-rT} - 1 + rT) = S_0.$$  

It is easy to show that, for a given $S_0$, this condition defines a unique $T$. We denote by $T_{pre}(S_0)$ the terminal time at which the stock is exhausted in the pre-merger game. It can be shown that $T_{pre}$ is a strictly increasing function of $S_0$ with $T_{pre}(0) = 0$. For values of $S_0$ close to 0 we have the following Lemma.

**Lemma 1:** When $S_0 \to 0$, $T_{pre} \to 0$ with

$$\frac{arT_{pre}^2}{2b(1 + n)} + O[T_{pre}]^3 = S_0$$

where $O[T_{pre}]^3$ is an expression such that $\lim_{T_{pre} \to 0} \frac{O[T_{pre}]^3}{T_{pre}^2} = 0$.

**Proof:** This follows from total differentiation of (24) and using the second order Taylor series expansion of $e^{-rT}$ around 0.\hfill \blacksquare
Remark 1: Lemma 1 allows us to establish that 
\[
\lim_{S_0 \to 0} \frac{S_0}{T_{pre}} = \frac{ar}{2b(1+n)} \quad \text{and that} \quad \lim_{S_0 \to 0} \frac{O[T_{pre}]^3}{S_0} = 0.
\]

In the case of a merger, let \( T_S \) and \( T_L \) represent the times at which the \( n_S \) outsider firms and the \( n_L \) merging firms respectively exhaust their resources, so that \( T_S \) and \( T_L \) solve (5) and (6). It can be shown that \( T_S \) and \( T_L \) are the solutions to the system of equations

\[
S_0 = \frac{(a - c) \left( e^{-rT_S} + rT_S - 1 \right)}{b(2 + n - m)}
\]

(26)

\[
mS_0 = \frac{(a - c) \left( -e^{-rT_S} (n - m) + e^{-rT_L} (2 + n - m) - 2 + T_L (2 + n - m) - (n - m) rT_S \right)}{2b (2 + n - m) r}.
\]

(27)

**Lemma 2:** When \( S_0 \to 0 \), we have \( T_S \to 0 \) and \( T_L \to 0 \) with

\[
\frac{arT_S^2}{2b (2 + n_S)} + O[T_S]^3 = S_0
\]

(28)

and

\[
\frac{arT_L^2}{2b} - \frac{anSr}{2b (2 + n_S)} T_S^2 + O[T_L]^3 + O[T_S]^3 = 2n_L^2 S_0
\]

(29)

where \( O[T_S]^3 \) and \( O[T_L]^3 \) are expressions such that \( \lim_{T_S \to 0} \frac{O[T_S]^3}{T_S} = 0 \) and \( \lim_{T_L \to 0} \frac{O[T_L]^3}{T_L} = 0 \).

**Proof:** This follows from total differentiation of (5), (6) and using the second order Taylor series expansion of \( e^{-rT_S} \) and \( e^{-rT_L} \) around 0. \( \blacksquare \)

Remark 2: Lemma 2 allows us to establish that 
\[
\lim_{S_0 \to 0} \frac{S_0}{T_{pre}} = \frac{ar}{2b(2+n_S)} \quad \text{and that} \quad \lim_{S_0 \to 0} \frac{O[T_{pre}]^3}{S_0} = 0
\]

along with 
\[
\lim_{S_0 \to 0} \left( \frac{arT_L^2}{2bS_0} - \frac{anSr}{2b(2+n_S)} T_S^2 \right) = 2n_L^2,
\]

which yield the following:

\[
\lim_{S_0 \to 0} \left( \frac{arT_L^2}{2bS_0} \right) - \lim_{S_0 \to 0} \left( \frac{anSr}{2b(2+n_S)} \frac{T_S^2}{S_0} \right) = 2n_L^2
\]

or

\[
\lim_{S_0 \to 0} \left( \frac{arT_L^2}{2bS_0} \right) - \frac{anSr}{2b(2+n_S)} \frac{2b(2+n_S)}{ar} = 2n_L^2
\]
and, after simplification,
\[
\lim_{S_0 \to 0} \left( \frac{a r T^2_{L}}{2 b S_0} \right) = 2 n^2_L + n_S
\]
with \( \lim_{S_0 \to 0} \frac{O[T_L]}{S_0} = 0 \).

In order to analyze the gain \( G \) from a merger when \( S_0 \) is close to 0, we first note that \( G \) can be expressed as a function of the dates of exhaustion of the stocks \( (T_{pre}, T_S, T_L) \), and that a change in the stock \( S_0 \) affects \( G \) through its impact on \( (T_{pre}, T_S, T_L) \). Note that when \( S_0 = 0 \), we have \( G = 0 \) and when \( S_0 \to 0 \), we have \( (T_{pre}, T_S, T_L) \to 0 \). We have the following lemmata:

**Lemma 3:** When \( T_S \to 0 \) and \( T_L \to 0 \), we have

(i)
\[
V_S(n - 1, 1, S_0, S_0) = \frac{a^2 r}{2 b (1 + n)} T^2_{pre} + O [T_{pre}]^3
\]

(ii)
\[
V_L(n - m, 1, S_0, m S_0) = \frac{a^2 r}{4 b} T^2_{L} - \frac{a^2 (n - m) r}{8 b + 4 b (n - m)} T^2_S + O [T_L]^3 + O [T_S]^3.
\]

**Proof:** This follows from total differentiation of \( V_S(n - 1, 1, S_0, S_0) \) with respect to \( T_{pre} \) and \( V_L(n - m, 1, S_0, m S_0) \) with respect to \( (T_S, T_L) \) and using the second order Taylor series expansion of \( e^{-rT} \), \( e^{-rT_S} \) and \( e^{-rT_L} \) around 0.\footnote{We are now ready to determine the sign of the gains from a merger when \( S_0 \) is close to 0.}

**Proposition 2:** For \( S_0 \) positive and sufficiently small, the profitability of a merger is always positive.

**Proof:** Using Lemma 3, we can write the gain from a merger, \( G \equiv V_L(n - m + 1, 1, m S_0, S_0) - m V_S(n - 1, 1, S_0, S_0) \) when \( S_0 \to 0 \) as

\[
G = \frac{a^2 r}{4 b} T^2_{L} - \frac{a^2 (n - m) r}{8 b + 4 b (n - m)} T^2_S + O [T_L]^3 + O [T_S]^3 - m \left( \frac{a^2 r}{2 b (1 + n)} T^2_{pre} + O [T_{pre}]^3 \right),
\]
which, using Lemma 1 and Lemma 2, yields the following:

\[ G = a (m^2 S_0 - mS_0) + O [T_{\text{pre}}]^3 + O [T_L]^3 + O [T_S]^3 \]

or

\[ G = am (m - 1) S_0 + O [T_{\text{pre}}]^3 + O [T_L]^3 + O [T_S]^3 \]

where \( O [T_{\text{pre}}]^3, O [T_L]^3, O [T_S]^3 \) are expressions such that \( \lim_{S_0 \to 0} \frac{O[T_{\text{pre}}]}{S_0} = 0 \), \( \lim_{S_0 \to 0} \frac{O[T_L]}{S_0} = 0 \), and \( \lim_{S_0 \to 0} \frac{O[T_S]}{S_0} = 0 \) (see Remarks 1 and 2 above). Therefore, when \( S_0 \) is positive and close enough to 0 we have

\[ G \simeq am (m - 1) S_0 > 0 \text{ when } m > 1. \]

Proposition 2 provides a sharp contrast with the case of a standard Cournot model without resource stock contraints. Within a similar setting to ours, that is, with linear demand and constant marginal cost, Salant et al (1983) show that a merger cannot be profitable unless 80% of the industry participates in the merger. As \( S_0 \to \infty \), our model converges to the standard Cournot setting. However, in the presence of stock constraints, there exists a range of stock levels for which even the smallest merger (\( n_L \geq 2 \) merger participants, where \( n_S \) may be arbitrarily large) is profitable. This is because, unlike in the standard Cournot model, where outsiders respond to the merger by increasing output and mitigating the merger participants’ gain in market power, here the outsiders are restricted in their response, due to their resource constraints. Within our context, the \( n_S \) outsiders exhaust their stocks earlier than the merger participants, that is, \( T_L > T_S \) in the post-merger equilibrium with symmetric firms. This results in greater merger-induced market power than in the standard Cournot model.\(^7\)

\(^7\)The mechanism captured by our model, driving the profitability of mergers, also applies to cases of partial cartelizations which have been an important feature of nonrenewable resource markets. In the case of partial cartelization, in response to the cartel’s reduction in output, the non-members would be hindered in their attempt to expand output due to limited resource stocks, similar to the outsiders to the merger in our model, making the cartel more profitable.
3.2 Mergers and resource extraction

In this section, we examine the extraction paths and exhaustion dates of firms participating or not in a merger.

Figure 1 below provides a plot of the dates of exhaustion of the stocks \((T_{\text{pre}}, T_S, T_L)\), of a firm in the premerger case, a firm not part of a merger and a firm that is part of the merger respectively, for \(n = 3\) and \(m = 2\). Following Kagan et al. (2015), we choose the following parameter values: \(a = 0.855\), \(b = a^2\), and \(r = 0.1\). Exhaustion dates are plotted against the stock initially owned by the firms, where \(S_0 \equiv S_{0S} = S_{0L}\), that is, firms are symmetric in the premerger game.

![Figure 1: Terminal dates as a function of initial stock](image)

As Figure 1 illustrates, \(T_S < T_{\text{pre}} < T_L\) for any \(S_0\). When a merger occurs, the outsider firm tends to extract from its resource stock faster than in the premerger scenario, whereas each of the merging firms extracts the resource more slowly than in the premerger case. This is in line with standard oligopoly theory where, for strategic substitutes, a merger induces the merging firms to reduce their output, which results in outsider firms expanding theirs. The
specificity of a nonrenewable industry is that the overall cumulated extraction is a constant.

Our simulations indicate that a merger results in a slower extraction rate for the industry. This is illustrated in Figure 2, which plots the total extraction by the industry at date 0 under a post-merger \( (Q_{post}) \) and a pre-merger \( (Q_{pre}) \) scenario as a function of the initial resource stock, using the same parameter values as in Figure 1.

Figure 2: Initial industry extraction as a function of initial stock

Figure 2 illustrates that a merger results in an initial overall extraction rate that is lower than when there is no merger. Simulations using combinations of \( n, m \) with \( n \leq 10 \) and \( m \in \{2, n - 1\} \) and various values for the parameters \( a, b \) and \( r \) show that this result is qualitatively robust: a merger results in a slower extraction rate for the industry at any resource stock level.
3.3 The case of an asymmetric oligopoly

We now use a numerical experiment to illustrate that Proposition 2 carries over when firms have different initial endowments of the resource. Consider three firms, two with an initial stock $S_{0L}$ and one with an initial stock $S_{0S} = fS_{0L}$, where $f \in (0, 1]$. Figure 3 illustrates, as a function of $S_{0L}$, the gains resulting from a merger of the two firms with initial stock $S_{0L}$, using the same parameter values as in Figures 1 and 2. In line with Proposition 2, Figure 3 shows that the merger is profitable when the value of $S_{0L}$ is sufficiently small.

![Figure 3: Gains from merger as a function of $S_{0L}$](image)

The following result is robust to changes in parameter values.

**Result 1:** A given merger is more likely to be profitable the smaller is the stock of the outsider and the larger are the stocks of the merger participants.

Result 1 is illustrated in Figure 3, where the smaller is $f$, the greater is the range of $S_{0L}$ for which the merger is profitable. This may explain why some of the largest firms in the oil extraction sector have merged since the 1990’s. This result is intuitive: we know that a merger of two firms in a duopoly is always profitable. The smaller the outsider firm’s stock relative to the stock of the merged firms when two firms merge, the closer the profit of the
merged firm to the profit of a merger of firms in a duopoly. Therefore, a merger of two firms in the case of a triopoly, when the outsider firm’s stock is small relative to the stock of the merged firms, is more likely to be profitable than when the outsider firm’s stock is large relative to the stock of the merged firms.

4 Impact of a tax on extraction

Non renewable resource industries can be important sources of pollution. A natural policy instrument that is often considered is the imposition of a unit tax on the resource produced. In this section, we examine the interplay of such policy on firms’ incentives to merge and, thus, on the industry’s market structure.

More precisely, we examine the effect of stricter environmental policies where, for simplicity, we assume that each unit extracted generates a unit of polluting emissions. The policy maker sets a constant tax per unit of extraction, $\tau$. The objective function of a firm $i$ is then given by:

$$\max_{q_i} \int_0^\infty e^{-rt} [a - b (Q_S(t) + Q_L(t)) - c - \tau] q_i(t) dt$$

subject to the resource constraint. The OLNE and the profitability analysis can then be directly exploited by assuming that the marginal cost of extraction is increased by $\tau$. Our numerical simulations yield the following result, which is robust to changes in the parameter values.

**Result 2:** A given merger is less likely to be profitable the higher the emission tax.

Result 2 is illustrated by Figure 4, where the higher is $\tau$, the smaller is the range of $S_{0L}$ for which the merger is profitable.
An implication of Result 2 is that the emission of pollutants may be affected in a perverse manner due to the imposition of a higher emission tax, resulting in a green paradox. Note that in Hotelling models in which the long-run cumulative extraction is not impacted by the tax, a constant unit tax induces markets to postpone the extraction of the resource. Indeed, a constant tax means that the present value of the tax decreases over time, encouraging markets to delay extraction, irrespective of the market structure. Thus, the first effect of the tax policy under study is opposite to a green paradox. On the other hand, Result 2 indicates that a tax might deter a merger, which increases the speed of extraction. This is the second effect, which drives the green paradox. In what follows, we illustrate that, when a merger is deterred due to a higher emission tax, the second effect dominates within our context.

Indeed, as noted in Section 3.2, a merger results in a lower initial extraction rate of the resource than in the premerger scenario. If an increase in $\tau$ prevents a merger from being realized that would otherwise have occurred, the path of emissions in equilibrium is altered, so that more emissions occur earlier than would have been the case with a lower tax level. This is illustrated with the following example of a symmetric triopoly where we set again $a = 0.8555$, $b = 0.8555^2$, $r = 0.1$ and examine the impact of a unit tax $\tau = 0.05$. Figure
5 plots the industry’s initial extraction rate as a function of the stock in four cases: (i) symmetric triopoly ($Q_{\text{pre}}$) and $\tau = 0$, (ii) symmetric triopoly and $\tau = 0.05$, (iii) merger of 2 firms among 3 ($Q_{\text{post}}$) and $\tau = 0$ and (iv) merger of 2 firms among 3 and $\tau = 0.05$. We note again that, for both $\tau = 0$ and $\tau = 0.05$, a merger results in a decrease of the initial industry’s extraction rate, for all initial stock levels.

To illustrate the implication of Result 2, we consider the case where $S_0 = S_{0L} = S_{0S} = 7.5$. The gains from a merger when $\tau = 0$ (resp. $\tau = 0.05$) are $0.0154$ (resp. $-0.0006$). That is, a merger is profitable when $\tau = 0$ and unprofitable when $\tau = 0.05$. Comparing the extraction rates under case (ii) and case (iii), we find from Figure 5 that the industry’s extraction rate are higher under case (ii) than under case (iii). Figure 6 plots the path of the industry’s cumulative extraction rate under case (ii) and case (iii): when the profitability of a merger is taken into account, a tax of $\tau = 0.05$ results in a faster depletion of the resource than the case of $\tau = 0$.

![Figure 5: Initial industry extraction as a function of initial stock](image-url)
In cases where the damage from pollution, which we have not explicitly modeled in this paper, is convex in the emission level and/or where pollution is accumulative, the prevention of the merger due to a higher tax could ultimately adversely affect the environment.

5 The case of economic exhaustion

Thus far, we have assumed the physical exhaustion of the resource. For some resources, the marginal extraction cost is a decreasing function of the level of the stock. In this case, it may happen that the extraction cost increases to levels that render the exploitation of the remaining stock unprofitable, so that some resource is left in the ground. We now revisit the profitability of a merger in an oligopolistic industry in this case.

Assume that the marginal cost of extraction from each mine\(^8\) is given by:

\[ c_0 - c_1 S, \]

\(^8\)We use the term "mine" to represent more generally a single facility from which the resource is extracted.
where $c_0 > 0$ and $c_1 > 0$ are such that the choke price $a < c_0$, which implies that a mine will be shut down when the stock reaches the level

$$S \equiv \frac{c_0 - a}{c_1}.$$ 

For comparison purposes, we limit our attention to the case of a symmetric triopoly where each firm owns an initial stock $S_0$, with $S_0 > \bar{S}$. Let $x \equiv S - \bar{S}$, it is straightforward to rewrite the discounted sum of profits of firm $i = 1, 2, 3$ as

$$\int_0^\infty (-bQ + c_1 x_i) q_i e^{-rt} dt.$$

(31)

Firm $i$ maximizes (31) subject to

$$\dot{x}_i = -q_i$$

(32)

and

$$x_i (0) = x_0 \equiv S_0 - \bar{S}.$$  

(33)

**Proposition 3:** Let

$$q(t) = -\sigma e^{\sigma t} x_0$$

(34)

where

$$\sigma \equiv \frac{1}{2} \left( r - \sqrt{r(c_1 + r)} \right) < 0,$$

then the vector $(q, q, q)$ constitutes an OLNE of the symmetric triopoly game.

**Proof:** See Appendix.

From Proposition 3, it follows that the equilibrium discounted sum of profits of a single firm is given by:

$$\int_0^\infty - (3b\sigma x + c_1 x) \sigma x e^{-rt} dt = -x_0^2 (3b\sigma + c_1) \sigma \int_0^\infty e^{-(2\sigma + r)t} dt$$

$$= -\sigma \frac{3b\sigma + c_1}{-2\sigma + r} x_0^2.$$
We now consider a merger of 2 firms. Since the marginal cost of extraction is no longer constant, we can no longer use the pre-merger equilibrium to deduce the post-merger equilibrium. Consider an industry that consists of two firms, an outsider firm that owns a single mine and a merged entity that owns two mines, mine 1 and mine 2. Let $S_S(\cdot), S_{L1}(\cdot)$ and $S_{L2}(\cdot)$ respectively denote the stock path of the outsider firm and of mine 1 and mine 2 of the merged entity with $S_S(0) = S_{0S}, S_{L1}(0) = S_{0L1}$ and $S_{L2}(0) = S_{0L2}$. Denote by $q_S(\cdot)$ the extraction path of the outsider firm and by $q_{L1}(\cdot)$ and $q_{L2}(\cdot)$ the extraction paths of the merged entity from mine 1 and mine 2 respectively. We drop the argument of the paths whenever the explicit reference to time is not necessary. While the outsider firm chooses $q_S$, the merged entity chooses a pair of extraction paths $\{q_{L1}, q_{L2}\}$.

**Proposition 4:** Suppose that $S_{0S} = S_{0L1} = S_{0L2} = S_0$ and let $x_0 = S_0 - \bar{S}$, then the vector $(q_o, \{q_m, q_m\})$ where

$$q_o(t) = -\frac{1}{6}((3 + \sqrt{3})\mu_1 e^{\mu_1 t} - (-3 + \sqrt{3})\mu_3 e^{\mu_3 t})x_0 \quad (35)$$

and

$$q_m(t) = \frac{1}{6}(-(3 + 2\sqrt{3})\mu_1 e^{\mu_1 t} + (-3 + 2\sqrt{3})\mu_3 e^{\mu_3 t})x_0 \quad (36)$$

with

$$\mu_1 = \frac{1}{2}r - \frac{1}{2}\sqrt{\frac{1}{3b} r \left(6c_1 - 2\sqrt{3}c_1 + 3br \right)} < 0 \quad (37)$$

and

$$\mu_3 = \frac{1}{2}r - \frac{1}{2}\sqrt{\frac{1}{3b} r \left(6c_1 + 2\sqrt{3}c_1 + 3br \right)} < 0 \quad (38)$$

constitutes an OLNE between the merged entity and the outsider firm.

**Proof:** See Appendix.

It is now straightforward, using Propositions 3 and 4, to compute the equilibrium discounted sum of profits for each firm under both the pre-merger scenario and when a merger occurs, from which we can then infer the profitability of a merger. Let $W_{ot}(x_0)$ denote the
equilibrium discounted sum of profits of a merged entity and $V_{ol}(x_0)$ denote the equilibrium discounted sum of profits for the two firms that merge under the pre-merger scenario, when all firms own identical stocks $x_0$. It can be shown that

$$V_{ol}(x_0) = \frac{2\sigma (c_1 + 3\sigma)}{2\sigma - r} x_0^2$$

and that

$$W_{ol}(x_0) = -\frac{1}{6} \left( \Omega c_1 - \frac{(19 + 11\sqrt{3}) \mu_1^2}{2\mu_1 - r} + \frac{(-19 + 11\sqrt{3}) \mu_3^2}{2d_3 - r} + \frac{2\mu_1\mu_3}{\mu_1 + \mu_3 - r} \right) x_0^2$$

where

$$\Omega \equiv \mu_1 \left( -\frac{7}{(2\mu_1 - r)} - \frac{4\sqrt{3}}{(2\mu_1 - r)} + \frac{1}{\mu_1 + \mu_3 - r} \right)$$

$$+ \mu_3 \left( -\frac{7}{(2\mu_3 - r)} + \frac{4\sqrt{3}}{(2\mu_3 - r)} + \frac{1}{\mu_1 + \mu_3 - r} \right).$$

**Result 3:** A merger of two firms can be profitable.

Result 3 is illustrated in Figure 7 using a numerical example, where we set $r = 0.05, b = 1$ and $x_0 = 100$, and plot the gains from a merger $W_{ol}(x_0) - V_{ol}(x_0)$ as a function of $c_1$. 

26
We highlight two important qualitative differences with respect to the case of physical scarcity: (i) when the firms’ initial stocks are identical, whether a merger is profitable or not does not depend on the initial stock level, (ii) when positive, the gains from a merger are typically relatively very small.

Figure 8 illustrates how the relative gains from a merger, i.e. \( \frac{W_{0d}(x_0) - V_{0d}(x_0)}{V_{0d}(x_0)} \), depend on the parameter \( c_1 \). The maximum gain from a merger is approximately 0.3% of the total payoff when there is no merger. The larger is \( c_1 \), the more important is the role of the cost effect in the firm’s payoffs, and the less important is the market interaction between firms. In this case, the extraction and payoff of a firm when a given merger occurs and when it does not occur are very close to each other.
When $c_1$ is sufficiently small, then the market interaction is relatively more important than the stock effect on costs, so that we retrieve the standard result from static oligopoly theory that a merger of two firms is not profitable. These qualitative results are robust to changes in the values of the parameters $b$ and $r$.

6 Conclusion

This paper showed that, if resource stock levels are small enough, small mergers (mergers of two firms) are profitable under constant marginal costs and may also be profitable when the marginal cost is decreasing in the resource stock level. The profitability of mergers arises because outsiders are limited in their response in terms of increased output due to their finite resource stocks. Mergers allow the merger participants to raise prices more than in industries without stock constraints. Therefore, antitrust authorities should be cautious when ruling on mergers in non-renewable resource industries. At the same time, mergers in these industries reduce environmental damage by delaying extraction.

The interplay of environmental regulations and incentives to merge deliver interesting insights: an environmental tax can affect merger profitability. In the case of a polluting
resource, a unit tax on extraction may deter mergers and, therefore, may result in causing emissions earlier than under laissez-faire. A small increase of the tax may result in a non-marginal jump upwards of the industry’s pollution.

In our analysis, we ignored the possibility that a perfect substitute to the resource may exist. This simplifies the analysis and, at the same time, allows to directly contrast our results with those obtained in the SSR framework. However, e.g. for fossil fuels, the existence of a backstop technology that can provide a substitute to the resource at a given fixed price, and that is supplied by a perfectly competitive market, has received a significant amount of attention. In this case, the resource owner, e.g., OPEC, can resort to limit pricing: charging a price that is just enough to undercut the backstop substitute. Recent important contributions on limit pricing arising in fossil fuel extraction include Andrade de Sa and Daubane (2016) and Van der Meijden, Ryszka and Withagen (2018)\(^9\), or Salant (1977) and Hoel (1978, 1984) for earlier contributions. A natural and relevant follow-up research question to our analysis is the profitability of a merger or of cartel formation\(^10\) in the presence of a backstop substitute. In that case, if, for example, initial endowments of the nonrenewable resource and/or demand elasticity are such that limit pricing takes place from the initial date until the extinction of the resource, then a merger may not affect the price path of the resource\(^11\) and a specific analysis of cartel or merger profitability is in order. This is left for future research.

\(^9\)Andrade de Sa and Daubane (2016) considers a nonrenewable resource monopoly facing a backstop substitute. They show that, when the price elasticity of demand is smaller than one, the monopolist choses, at each moment, a corner solution, i.e., induces the limit price which deters the backstop-substitute production. Van der Meijden, Ryszka and Withagen (2018) consider a non-renewable resource supplier who faces demand from two regions, one of which employs a tax on the imported resource and a subsidy on the available backstop technology, and one that has no environmental policy in place. They show that the resource extraction path possibly contains two limit pricing phases.

\(^10\)As noted above, in instances where resource owners are countries and not firms, coordination of interests among the resource owners takes the form of cartels rather than mergers.

\(^11\)We thank an anonymous referee for pointing to this possibility.
References


Appendix:

Proof of Proposition 3

Consider the symmetric triopoly game described by (31-33), given extraction paths \( q_1 \) and \( q_2 \) of firm 1 and firm 2; the necessary conditions for a positive extraction \( q_3 \) of the third firm are given by

\[
H_{q_3} = -2bq_3 - q_1 - q_2 + c_1 x - \lambda_3 = 0 \tag{39}
\]

\[
\dot{\lambda}_3 = r\lambda_3 - c_1 q_3 \tag{40}
\]

\[
\dot{x} = -q_3 \tag{41}
\]

\[
x (0) = x_0
\]

with

\[
\lim_{t \to \infty} x (t) = 0
\]

For a symmetric equilibrium and interior solutions, we have \( q_3 = q_1 = q_2 = q \), and \( \lambda_3 = \lambda_1 = \lambda_2 = \lambda \). From (39), we obtain the following:

\[
\lambda = -4bq + c_1 x
\]

and therefore

\[
\dot{\lambda} = -4b\dot{q} + c_1 \dot{x}. \tag{42}
\]

From (40) and (42), we obtain:

\[
-4b\dot{q} + c_1 \dot{x} = r (-4bq + c_1 x) - c_1 q \tag{43}
\]

From (41) and (43), it follows that the symmetric OLNE when firms are identical is given
by the solution to the following system:

\[
\dot{q} = rq - \frac{c_1}{4b} rx
\]  

(44)

\[
\dot{x} = -q
\]  

(45)

\[x(0) = x_0\]

with

\[\lim_{t \to \infty} x(t) = 0.\]

Moreover, (44) and (45), yield:

\[
\begin{bmatrix}
\dot{q} \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
r & -\frac{c_1}{4b} \\
-1 & 0
\end{bmatrix}
\begin{bmatrix}
q \\
x
\end{bmatrix}.
\]

The two eigenvalues of

\[
\begin{bmatrix}
r & -\frac{c_1}{4b} \\
-1 & 0
\end{bmatrix}
\]

are given by \(\sigma_1 = \frac{1}{2} \left( r - \sqrt{\frac{1}{b} (br^2 + c_1 r)} \right) < 0\) and \(\sigma_2 = \frac{1}{2} \left( r + \sqrt{\frac{1}{b} (c_1 + br)} \right) > 0\), and the associated eigenvectors are \(\left\{ -\sigma_1, 1 \right\} \) and \(\left\{ -\sigma_2, 1 \right\} \) respectively.

Given that the trajectory cannot oscillate around the steady state, the solution is given by:

\[
\begin{bmatrix}
q \\
x
\end{bmatrix} = A \begin{bmatrix}
\sigma e^{\sigma t} \\
e^{\sigma t}
\end{bmatrix}
\]

with \(A = x_0\) and \(\sigma = \sigma_1\).

Therefore, the OLNE path is given by:

\[
\begin{bmatrix}
q \\
x
\end{bmatrix} = x_0 \begin{bmatrix}
-\sigma e^{\sigma t} \\
e^{\sigma t}
\end{bmatrix}.
\]
Proof of Proposition 4

The merged entity, denoted firm $M$, controls two stocks, denoted $S_{L1}$ and $S_{L2}$:

\[ \dot{S}_{L1} = -q_{L1} \]

\[ \dot{S}_{L2} = -q_{L2} \]

where $q_{Li}$ is the extraction from stock $S_{Li}$ with $i = 1, 2$. Firm $M$ maximizes the joint profit from both stocks and the outsider controls stock $S_S$. We therefore have the following:

\[ H^S (q_S, \lambda_S, t) = [a - b (Q_S + Q_L) - c_0 + c_1 S_S] q_S + \lambda_S (-q_S). \]

\[ H^M (q_{L1}, q_{L2}, \lambda_{L1}, \lambda_{L2}, t) = [a - b (Q_S + Q_L) - c_0 + c_1 S_{L1}] q_{L1} \]

\[ + [a - b (Q_S + Q_L) - c_0 + c_1 S_{L2}] q_{L2} \]

\[ + \lambda_{L1} (-q_{L1}) + \lambda_{L2} (-q_{L2}). \]

The maximum principle yields

\[ H^S_{q_S} = a - c_0 - 2b q_S - b Q_L + c_1 S_S - \lambda_S \]

(46)

\[ H^M_{q_{L1}} = a - c_0 - 2b q_{L1} - 2b q_{L2} - b q_S + c_1 S_{L1} - \lambda_{L1} \]

(47)

\[ H^M_{q_{L2}} = a - c_0 - 2b q_{L2} - 2b q_{L1} - b q_S + c_1 S_{L2} - \lambda_{L2} \]

(48)

where $H_z$ denotes the partial derivative of $H$ with respect to variable $z$.

The necessary conditions are

\[ H^S_{q_S} \leq 0 \text{ and } H^S_{q_S} q_S = 0 \]
The other Maximum Principle conditions are

\[
H_{q_{L1}}^M \leq 0 \quad \text{and} \quad H_{q_{L1}}^M q_{L1} = 0
\]

\[
H_{q_{L2}}^M \leq 0 \quad \text{and} \quad H_{q_{L2}}^M q_{L2} = 0
\]

\[
\dot{\lambda}_S = r\lambda_S - c_1 q_S
\]

\[
\lambda_{L1} = r\lambda_{L1} - c_1 q_{L1}
\]

\[
\lambda_{L2} = r\lambda_{L2} - c_1 q_{L2}
\]

and

\[
\dot{S}_S = -q_S
\]

\[
\dot{S}_{L1} = -q_{L1}
\]

\[
\dot{S}_{L2} = -q_{L2}
\]

with

\[
S_{S}(0) = S_{0S}
\]

\[
S_{L1}(0) = S_{0L1}
\]

\[
S_{L2}(0) = S_{0L2}
\]

When \( S_{0L1} = S_{0L2} = S_0 \), the Merged entity operates both mines at \( q_{L1} = q_{L2} \) and, therefore,

\[
H_{q_S}^S = -2bq_S - bQ_L + c_1 S_S - \lambda_S \tag{49}
\]

\[
H_{q_{L1}}^M = -4bq_{L1} - bq_S + c_1 S_{L1} - \lambda_{L1}. \tag{50}
\]

The necessary conditions are

\[
H_{q_S}^S \leq 0 \quad \text{and} \quad H_{q_S}^S q_S = 0
\]
\[ H^M_{qL1} \leq 0 \text{ and } H^M_{qL1} q_{L1} = 0. \]

The other Maximum Principle conditions are

\[ \dot{\lambda}_S = r\lambda_S - c_1 q_S \]
\[ \dot{\lambda}_{L1} = r\lambda_{L1} - c_1 q_{L1} \]

and

\[ \dot{S}_S = -q_S \]
\[ \dot{S}_{L1} = -q_{L1}, \]

with

\[ S_S(0) = S_0S \]
\[ S_{L1}(0) = S_0L_1. \]

Three different regimes can occur: Simultaneous (\( \Phi_S \)), Merged entity produces alone (\( \Phi_M \)), and Outsider produces alone (\( \Phi_O \)). We focus on the regime where the solution is interior, i.e. regime \( \Phi_S \):

\[ -2bq_S - 2bq_{L1} + c_1 S_S = \lambda_S \]  
(51)
\[ -4bq_{L1} - bq_S + c_1 S_{L1} = \lambda_{L1}. \]  
(52)

Taking the time derivative of the conditions above yields

\[ -2b\dot{q}_S - 2b\dot{q}_{L1} + c_1 \dot{S}_S = \dot{\lambda}_S \]  
(53)
\[ -4b\dot{q}_{L1} - b\dot{q}_S + c_1 \dot{S}_{L1} = \dot{\lambda}_{L1}, \]  
(54)
and, therefore, the other Maximum Principle conditions yield

$$-2b\dot{q}_S - 2b\dot{q}_{L1} + c_1 \dot{S}_S = r (-2bq_S - 2bq_{L1} + c_1 S_S) - c_1 q_S$$

$$-4b\dot{q}_{L1} - b\dot{q}_S + c_1 \dot{S}_{L1} = r (-4bq_{L1} - bq_S + c_1 S_{L1}) - c_1 q_{L1}$$

or

$$-2b\dot{q}_S - 2b\dot{q}_{L1} = r (-2bq_S - 2bq_{L1} + c_1 S_S) \quad (55)$$

$$-4b\dot{q}_{L1} - b\dot{q}_S = r (-4bq_{L1} - bq_S + c_1 S_{L1}) \quad (56)$$

and

$$\dot{S}_S = -q_S$$

$$\dot{S}_{L1} = -q_{L1},$$

with

$$S_S (0) = S_{0S}$$

$$S_{L1} (0) = S_{0L1},$$

along with

$$\lim_{t \to \infty} S_S (t) = \frac{c_0 - a}{c_1} \quad \text{and} \quad \lim_{t \to \infty} S_{L1} (t) = \frac{c_0 - a}{c_1}.$$
\[ 6bq_{L1} = -2r \left( -4bq_{L1} - bq_S + c_1S_{L1} \right) + r \left( -2bq_S - 2bq_{L1} + c_1S_S \right) \]

or

\[ -3bq_S = r \left( -3bq_S + 2c_1S_S - c_1S_{L1} \right) \]

\[ 6bq_{L1} = r \left( 6bq_{L1} - 2c_1S_{L1} + c_1S_S \right). \]

Equivalently

\[ \dot{q}_S = rq_S - r \frac{2c_1}{3b} S_S + r \frac{c_1}{3b} S_{L1} \quad (57) \]

\[ \dot{q}_{L1} = rq_{L1} - r \frac{2c_1}{6b} S_{L1} + r \frac{c_1}{6b} S_S \quad (58) \]

with

\[ \dot{S}_S = -q_S \quad (59) \]

\[ \dot{S}_{L1} = -q_{L1}, \quad (60) \]

and the initial and transversality conditions

\[ S_S (0) = S_{0S} \quad (61) \]

\[ S_{L1} (0) = S_{0L1} \quad (62) \]

\[ \lim_{t \to \infty} S_S (t) = 0 \quad \text{and} \quad \lim_{t \to \infty} S_{L1} (t) = 0. \quad (63) \]

Solving the system of differential equation (57) – (60) with the conditions (61) – (63) yields (35) and (36) ■