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Publication date:
1997

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

de Jong, F. C. J. M., Drost, F. C., & Werker, B. J. M. (1997). *A Jump-Diffusion Model for Exchange-Rates in a Target Zone*. Institut de Statistique - University of Brussels.

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IS-P 1997-22 (n°80)

Octobre 1997

Série Probabilités et Statistique

INSTITUT DE STATISTIQUE ET DE RECHERCHE OPÉRATIONNELLE
CAMPUS DE LA PLAINE CP 210
BOULEVARD DU TRIOMPHE
B-1050 BRUXELLES
BELGIQUE

Executive editor : Claude Adan

Editor : F. Thomas Bruss

A Jump-Diffusion Model for Exchange Rates in a Target Zone

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September 1997

Abstract

We propose a simple jump-diffusion model for an exchange rate target zone. The model captures most stylized facts from the existing target zone models while remaining analytically tractable. The model is based on a modified two-limit version of the Cox, Ingersoll, and Ross (1985) model. In the model the exchange rate is kept within the band because the variance decreases as the exchange rate approaches the upper or lower limits of the band. We also consider an extension of the model with parity adjustments, which are modeled as Poisson jumps. Estimation of the model is by GMM based on conditional moments. We derive prices of currency options in our model, assuming that realignment jump risk is idiosyncratic. Throughout, we apply the theory to EMS exchange rate data. We show that, after the EMS crisis of 1993, currencies remain in an implicit target zone which is narrower than the officially announced target zones.

JEL codes: F31, G13

Keywords: exchange rate target zones, currency option pricing

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*De Jong and Drost: Tilburg University; Werker: Université Libre de Bruxelles and Tilburg University. Part of the work on this paper was performed while the second and third author were visiting the Université de Toulouse I (IDEI/GREMAQ). Joost Driessen and Manfred Bartels provided excellent computational assistance. We thank Jesper Lund, Nour Meddahi, Eric Renault, Paul Söderlind, and seminar participants at the Stockholm School of Economics, Tilburg, Toulouse, and the ESEM97 for useful comments. Special thanks to Antoon Pelsner for showing us how to derive the marginal distribution of the target zone process. A previous version of this paper was circulated under the title "Exchange Rate Target Zones: A New Approach".

1 Introduction

Following the seminal contribution by Krugman (1991), a lot of effort has been put into the modeling of exchange rates in a target zone.¹ In such a target zone, the exchange rate has a lower and upper bound, which the central banks defend by interventions in the foreign exchange markets. Krugman's important observation is that, due to the forward looking nature of exchange rate determination, these interventions have a stabilizing effect even when exchange rates are well inside the band and intervention is not currently effective. Obviously, for exchange rates in the European Monetary System (EMS) the assumption of a fully credible target zone is violated by historical exchange rate behaviour. Most exchange rates were prone to parity adjustments (realignments). Subsequent target zone models, e.g. Bertola and Caballero (1992), Bertola and Svensson (1993), and Dumas, Jennergren and Näslund (1995a), have therefore incorporated some form of realignment risk.

The empirical analysis of Krugman's model and its extensions has not been without problems. The estimation of these models is not trivial due to the use of a regulated Brownian motion as the driving process in most of these models. The likelihood function for this process is rather complicated (see Ball and Roma (1994)). An alternative is the method of simulated moments, but the experience with these estimators is not very satisfactory (see Smith and Spencer (1991), Lindberg and Söderlind (1994), and De Jong (1994)). Second, the pricing of derivatives, such as currency options, in those target zone models involves the solution of a non-trivial partial differential equation, see Dumas, Jennergren, and Näslund (1995a).

In this paper, we propose a relatively simple model for exchange rates in a target zone that captures most stylized facts from the existing target zone literature while remaining analytically tractable. In particular, estimation of the model and option pricing are simpler than in existing target zone models. The setup of our model deviates from the usual approach in the target zone literature because we do not start from a specification of economic fundamentals. Instead, we specify a stochastic process for the exchange rate itself. The model can be considered as the reduced form of a structural target zone model which involves economic fundamentals.² The stochastic process for the exchange rate exhibits mean reversion of the Ornstein-Uhlenbeck type. The instantaneous volatility depends on the level of the exchange rate and decreases if

¹For an overview of this literature see Svensson (1992a).

²A reduced form approach to target zone modeling is also taken by Vlaar and Palm (1993), Koedijk, Stork and De Vries (1996) and Bekaert and Gray (1996), who all specify their models in discrete time.

the exchange rate approaches the boundary of the target zone.³ Realignment jumps follow a Poisson process with constant intensity which is independent of the position of the exchange rate in the band. The size of the jump is not fixed but may depend on the current level of the exchange rate and the current central parity.

An advantage of our model over existing target zone models is that conditional expectations and variances of the exchange rates take simple analytical forms. Estimation of the model is by GMM based on these conditional moments. An application of the model is to derive estimates of implicit target zones, which are potentially different from officially announced target zones. In particular we estimate implicit target zones for EMS currencies. After the EMS crisis of 1993, the official target zone was widened from 2.25% to 15% for most currencies. Our empirical results show that an implicit target zone of around 6% is obeyed by the major EMS currencies.

The final part of the paper deals with currency option pricing in the target zone model. Here we distinguish two cases, models with and without realignment jumps. In the model without realignment jumps the market is complete and prices can be obtained as the expected payoff under the risk-neutral distribution of the exchange rate. Introducing realignment risk complicates the analysis because markets become incomplete in that case. Consequently, a single no-arbitrage condition is no longer sufficient to derive currency option prices. We solve this problem along the lines proposed by Melenberg and Werker (1996). In the first step of this method, option prices are calculated conditional on the jump process. In the second step the option price is obtained by taking the expectation of the conditional option prices with respect to the true distribution of the jumps.

The plan of this paper is as follows. In Section 2 we present the model and derive several important properties. In Section 3 we consider estimation of the model. We consider the small sample properties of these estimators using Monte Carlo simulations. In Section 4 we present an empirical application to EMS exchange rate data. In Section 5 we demonstrate how to use our model for the pricing of currency options. Finally, Section 6 concludes.

2 Model

In our model, we want to capture some well-known stylized facts of exchange rates in a target zone (see Vlaar and Palm (1993) for an overview). First, EMS exchange rates exhibit mean

³This model can be seen as two-limit version of the square root process, popularized in the financial literature by Cox, Ingersoll, and Ross (1985) in a model for the term structure of interest rates.

reversion within the band. Stated differently, exchange rates tend towards their central parity. However, there are infrequent adjustments of this central parity due to realignments. These adjustments may cause a long run upward or downward trend in the exchange rate. There is also evidence of strong heteroskedasticity due to a time-varying volatility of the exchange rates. Finally, EMS exchange rates exhibit jumps, not only due to realignments but also within the band. Our model intends to capture these stylized facts. We incorporate reversion towards the central parity, conditional heteroskedasticity and jumps in the model. To introduce the main features of our approach we start this section with a simple model that does not allow for parity adjustments. Realignments jumps will be introduced later.

2.1 A model without realignments

Let S_t be the exchange rate process under consideration and suppose that this exchange rate may not deviate more than a certain percentage, say z , from a central parity μ_t , i.e

$$\mu_t/(1+z) \leq S_t \leq \mu_t(1+z).$$

Put $X_t = \log S_t$, $M_t = \log \mu_t$, and $Z = \log(1+z)$, then the target zone condition on the exchange rate can be rewritten as

$$M_t - Z \leq X_t \leq M_t + Z. \quad (2.1)$$

The assumption that the band is symmetric is made mainly for notational simplicity; it is not difficult to introduce two distinct variables to describe the upper and lower margin of the band, say Z_- and Z_+ . The width of the target zone will be non-random in all our models, but could be treated as an unknown parameter.

In this section we discuss a model where the central parity is constant, $M_t = m$, which corresponds to a fully credible target zone. The exchange rate is assumed to satisfy the following stochastic differential equation,

$$dX_t = -\rho(X_t - m)dt + \sigma\sqrt{Z^2 - (X_t - m)^2}dW_t \quad (2.2)$$

where W_t is a Wiener process. In this model we have the following parameters and constants: $m, \rho > 0, Z > 0$, and $\sigma > 0$. If the continuous exchange rate process X_t approaches the boundaries $m \pm Z$, the volatility of the error term $\sigma\sqrt{Z^2 - (X_t - m)^2}dW_t$ decreases to zero, while the autoregressive part $-\rho(X_t - m)dt$ drives the exchange rate back to the central parity.

As is clear from the square-root in (2.2), this model is strongly inspired by the model of Cox, Ingersoll, and Ross (1985) and the shifted CIR model of Duffie and Kan (1996). On the other hand, the instantaneous variance is, like in the geometric Brownian motion, a polynomial of degree two. In contrast to the CIR type models with one boundary at the bottom, we have two boundaries here. To keep the exchange rate in the target zone we do not need to specify an explicit intervention process as is usual in the target zone literature. Existence of a solution to this process follows from Proposition 2.13 in Chapter 5 of Karatzas and Shreve (1991), see also Yamada and Watanabe (1971).

One of the interesting features of our model is the marginal distribution of the exchange rate. The Krugman (1991) model predicts a U-shaped marginal distribution, which is generally not found empirically. There is ample evidence that the marginal distribution has most mass near the middle of the band [see, e.g., Bertola and Caballero (1992), Flood, Rose, and Mathieson (1991), and Lindberg and Söderlind (1994)]. Let

$$Y_t = X_t - m$$

denote the deviation of the log exchange rate from the fixed central parity. From the Kolmogorov equations it follows that the marginal distribution of Y_t is a shifted Beta, with density function

$$p(y) = \frac{1}{Z} \frac{\Gamma(\beta + \frac{1}{2})}{\Gamma(\beta)\sqrt{\pi}} (1 - (y/Z)^2)^{\beta-1}, \quad (2.3)$$

where $\beta \equiv \rho/\sigma^2$. This distribution can take various shapes,⁴ depending on the sign of β . If $0 < \beta < 1$, the distribution is U-shaped with peaks at the boundaries. For $\beta = 1$, the marginal distribution is uniform. If $\beta > 1$ the pull towards the central parity is sufficiently strong to generate a unimodal, hump-shaped marginal density. This density is empirically more plausible than the marginal distribution in the model of Lindberg and Söderlind (1994). In that model, the marginal distribution has most mass in the middle of the band but also has two peaks at the boundaries.

To be able to estimate the unknown parameters, it would be convenient to have an explicit form of the discrete time model. Efficient econometric analysis of the model must be based on the conditional distribution of the exchange rate given the history of the process. However, the conditional distribution is rather complicated and unknown for the model with jumps. Therefore, we decided to base our econometric analysis on the conditional moments of the

⁴Several possibilities are graphed in Figure 1.

process, which have simple analytical forms. Let $h > 0$ denote the time interval between two observations. In Appendix A we derive the following conditional mean, variance, and skewness⁵

$$E_t(Y_{t+h}) = e^{-\rho h} Y_t, \quad (2.4a)$$

$$Var_t(Y_{t+h}) = \{e^{-\sigma^2 h} - 1\}e^{-2\rho h} Y_t^2 + \{1 - e^{-(\sigma^2 + 2\rho)h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}, \quad (2.4b)$$

$$Sk_t(Y_{t+h}) = \{e^{-3\sigma^2 h} - 1\}e^{-3\rho h} Y_t^3 + \{1 - e^{-(3\sigma^2 + 2\rho)h}\}e^{-\rho h} Y_t \frac{3\sigma^2 Z^2}{3\sigma^2 + 2\rho}. \quad (2.4c)$$

From these expressions the unconditional moments of Y_t follow immediately:

$$\begin{aligned} E(Y_t) &= 0, \\ Var(Y_t) &= \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}, \\ Sk(Y_t) &= 0. \end{aligned}$$

The discrete observations of the logarithm of the exchange rate in the target zone model have an AR(1)-ARCH(1) structure. This can be made more explicit by writing the conditional variance as

$$Var_t(Y_{t+h}) = \omega_h + \alpha_h Y_t^2, \quad (2.5)$$

with

$$\omega_h = \{1 - e^{-(\sigma^2 + 2\rho)h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho} > 0,$$

and

$$\alpha_h = \{e^{-\sigma^2 h} - 1\}e^{-2\rho h} < 0.$$

Obviously, the volatility decreases if the process approaches one of the boundaries $\pm Z$. The conditional skewness of the exchange rate also depends on the current position of the exchange rate in the band. We will exploit these analytical expressions for the conditional moments to derive estimators for the parameters in the model, see Section 3.

2.2 Models with realignments

The main drawback of the model discussed above is the absence of parity changes. We now generalize the ideas of the previous subsection to a class of models with realignments. We model

⁵The conditional skewness, $Sk_t(X_{t+h})$, is defined as the centralized third moment $E_t[(X_{t+h} - E_t(X_{t+h}))^3]$

the central parity as a separate stochastic process, M_t , which exhibits occasional jumps. Following Svensson (1992b), Bertola and Svensson (1993), and Dumas, Jennergren, and Näslund (1995a) the jumps are modeled as a Poisson process with constant jump intensity. The size of the jumps is predetermined and depends on the current exchange rate and the current central parity. The model for the exchange rate itself is similar to the model presented in the previous section. The drift of the exchange rate is of the Ornstein-Uhlenbeck type, but with the current central parity as reference point. The instantaneous variance follows the parameterization in (2.2). This leads to the following model⁶

$$dX_t = -\rho(X_{t-} - M_{t-})dt + \sigma\sqrt{Z^2 - (X_{t-} - M_{t-})^2}dW_t, \quad (2.6a)$$

$$dM_t = (X_{t-} - M_{t-})dN_t. \quad (2.6b)$$

where N_t is a Poisson process with jump intensity θ . The effect of the realignment jump in this model is to reset the central parity, M_t , to the current value of the exchange rate, X_{t-} ,

$$M_t = M_{t-} + (X_{t-} - M_{t-})\Delta N_t = \begin{cases} M_{t-} & \text{if } \Delta N_t = 0 \\ X_{t-} & \text{if } \Delta N_t = 1 \end{cases}.$$

This realignment structure does not allow for jumps outside the current target zone and the exchange rate itself is a continuous variable.⁷ Of course, a different assumption on the realignment process could be made.

To show that the system (2.6a)–(2.6b) allows a solution (X_t, M_t) , let $Y_t = X_t - M_t$ be the exchange-rate-within-the-band, defined as the deviation between the exchange rate X_t and the central parity M_t . The stochastic differential equation for Y_t is given by

$$dY_t = -\rho Y_{t-}dt + \sigma\sqrt{Z^2 - Y_{t-}^2}dW_t - Y_{t-}dN_t. \quad (2.7)$$

This determines a unique solution Y_t . Plugging in this solution into (2.6b) yields M_t and, hence, also X_t . This proves the existence of a unique solution of the system of stochastic differential equations (2.6a)–(2.6b). As before, it is clear that $|Y_t| = |X_t - M_t| \leq Z$, so that the exchange rate X_t remains in the band $[M_t - Z, M_t + Z]$.

The conditional first and second moments (variances and covariances) of the exchange rate X_t and the central parity M_t in model (2.6a)–(2.6b) can be calculated using the methods

⁶The expression X_{t-} denotes the exchange rate just before time t .

⁷A similar assumption is made in the model of Dumas, Jennergren, and Näslund (1995a).

explained in Appendix A. To simplify the exposition we present the conditional moments of the central parity process M_t and the difference process $Y_t = X_t - M_t$.

$$E_t Y_{t+h} = e^{-h(\rho+\theta)} Y_t, \quad (2.8a)$$

$$E_t M_{t+h} = M_t + \frac{\theta}{\rho + \theta} \{1 - e^{-h(\rho+\theta)}\} Y_t, \quad (2.8b)$$

$$\begin{aligned} \text{Var}_t Y_{t+h} &= \{e^{-h(\sigma^2+2\rho+\theta)} - e^{-2h(\rho+\theta)}\} Y_t^2 \\ &+ \{1 - e^{-h(\sigma^2+2\rho+\theta)}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho + \theta}, \end{aligned} \quad (2.8c)$$

$$\text{Cov}_t(Y_{t+h}, M_{t+h}) = \frac{\theta}{\rho + \theta} \{e^{-2h(\rho+\theta)} - e^{-h(\rho+\theta)}\} Y_t^2, \quad (2.8d)$$

$$\begin{aligned} \text{Var}_t M_{t+h} &= \frac{\theta}{\sigma^2 + 2\rho + \theta} \{1 - e^{-h(\sigma^2+2\rho+\theta)}\} (Y_t^2 - \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho + \theta}) \\ &+ \frac{\theta^2}{(\rho + \theta)^2} \{2e^{-h(\rho+\theta)} - e^{-2h(\rho+\theta)} - 1\} Y_t^2 + (\frac{\sigma^2 Z^2}{\sigma^2 + 2\rho + \theta}) \theta h. \end{aligned} \quad (2.8e)$$

These formulas show that the difference process Y_t has again the AR-ARCH property, similar to the exchange rate in the fixed parity model. The mean reversion is determined by the sum $\rho + \theta$. The variance of the central parity M_t increases with the horizon h and therefore X_t and M_t are integrated of order 1, using the terminology of Engle and Granger (1987). Moreover, the central parity M_t and the exchange rate X_t are co-integrated. The direction of the drift is determined by the sign of Y_t : if the exchange rate is above the central parity the expected drift is upward, otherwise it is downward.

2.3 Conclusion

To conclude this section, we discuss the merits of the models above. All the formulations capture the usual properties of credible exchange rate target zones: decreased volatility near the boundaries and mean reverting processes. Moreover, the conditional moments have an AR-ARCH property: conditional expectations are autoregressive and conditional variances depend on past squared observations. The model without realignments (2.2) is just a starting point and could be used for descriptive purposes for periods when it is known that there are no parity changes. To include the possibility of parity changes we extend (2.2) to the system of stochastic differential equations (2.6a)–(2.6b). Of course, one may criticize the simplicity of the realignment process since the probability of jumps is constant over time and the central parity jumps exactly to the prevailing exchange rate at the time of the jump.⁸ However, some

⁸This assumption is of course empirically easily rejected but is maintained to facilitate the analysis of the model.

important properties of EMS exchange rates are captured by this realignment model. Of course, the proof of the pudding is in the eating and in Section 4 we will see how these models perform for several currencies in the EMS.

3 GMM estimation of the target zone model

Target zone models of exchange rates have proved to be hard to estimate. The original model of Krugman (1991) and most of its extensions (Lindberg and Söderlind (1994), Bertola and Caballero (1992), and Bertola and Svensson (1993), among others) are based on the regulated Brownian motion or a regulated Ornstein-Uhlenbeck process, sometimes augmented with a jump process. This stochastic structure makes it hard to analyze discretely sampled data since the likelihood function and conditional moments do not take simple analytical forms. In response, many authors (Smith and Spencer (1991), Lindberg and Söderlind (1994), and others) have relied on method of simulated moments estimation, which is feasible because the underlying processes are relatively easy to simulate. The experience with this type of estimators is not generally positive, however. The problem is that unconditional expectation and variance of the exchange rate provide relatively little information about the parameters of interest, see De Jong (1994) for an empirical assessment of the performance of these estimators in a target zone model. The best estimator available so far is the Maximum Likelihood estimator of Ball and Roma (1994), which is specific for the model with exogenous mean reversion and a fully credible target zone (originally proposed by Lindberg and Söderlind (1994)). The estimator works well, but is fairly complicated and cannot easily be extended to other target zone models.

In our models, the likelihood function of the exchange rate is not known, except in some special cases. However, we do have exact analytical expressions for the conditional moments of at least second order. The information in these moments can be utilized using the GMM framework. The GMM literature is vast; a paper which focusses especially on conditionally heteroskedastic models is Meddahi and Renault (1996). The general idea is as follows: let m_t denote a vector which is composed of deviations between powers of the innovations, ε_t , and their conditional expectations. By construction, the conditional expectation of this vector is zero: $E_{t-1}m_t(\theta) = 0$. In general, this moment depends on a vector of unknown parameters, θ . To estimate θ , we can exploit moment restrictions of the form

$$E[z_{t-1}m_t(\theta)] = 0, \tag{3.1}$$

where the instrument z_{t-1} is known at $t - 1$. The GMM estimator is found by solving the sample analog of the moment equation

$$\sum_{t=1}^n z_{t-1} m_t(\hat{\theta}) = 0. \quad (3.2)$$

It is well known (see, e.g. Godambe (1985) and Newey (1990)) that the optimal instrument given the choice of the moment vector m_t is

$$z_{t-1} = E_{t-1}\left(\frac{\partial m_t'}{\partial \theta}\right) Var_{t-1}(m_t)^{-1}. \quad (3.3)$$

In general, including more elements in the moment vector m_t will enhance the efficiency of the GMM estimator, provided that the optimal instrument is used to generate the moment restrictions (3.1).

If the true $Var_{t-1}(m_t)$ is very complicated, the optimal instrument can be difficult to construct. Following Meddahi and Renault (1996), in such a case we can specify some approximation V_{t-1} and use the following instrument

$$z_{t-1} = E_{t-1}\left(\frac{\partial m_t'}{\partial \theta}\right) V_{t-1}^{-1}. \quad (3.4)$$

The GMM estimator based on this instrument will be less efficient, but may be more tractable than the estimator based on the optimal instrument (3.3). Alternatively, if an initial consistent estimator is available, we could construct the optimal instrument by non-parametric estimates of $E_{t-1}(\frac{\partial m_t'}{\partial \theta})$ and $Var_{t-1}(m_t)$, where m_t is evaluated in the initial consistent estimate (see Wefelmeyer (1996)).

The particular versions of the GMM estimator that we employ are Quasi Maximum Likelihood (QML) and an extension of QML based on the first three conditional moments of the exchange rate. Both estimators do not use the optimal instruments based on the true variance-covariance matrix of the moments. Instead, the approximations of this matrix based on the normal distribution are used. Appendix B gives more details on the implementation of this estimator.

We tested the performance of the GMM estimators in the target zone models on simulated data. In the experiments, we simulate data from the simple model (2.2) without realignments. In our simulations, the midpoint of the band is normalized at zero, $m = 0$, and the bandwidth is equal to the EMS bandwidth, $Z = 2.25$, expressed as a percentage. The values of the parameters ρ and σ^2 are chosen to resemble actual EMS exchange rate data. From the estimates

in Section 4 it appears that the choice $(\rho, \sigma^2) = (0.05, 0.02)$ for weekly data is reasonable.⁹ However, we also conducted experiments for data with a higher variance, $\sigma^2 = 0.04$. Note that both sets of parameters imply a hump-shaped marginal distribution function.

To simulate from the continuous time process we used a refined Euler discretisation scheme. More specifically, we simulated daily data from repeated drawings from a normal distribution with conditional mean and variance given by the expressions in equations (2.4a)–(2.4b). We sampled one in five simulated values, which corresponds to using weekly observations. Using these simulated time series we estimated the parameters by QML and with the three moment GMM estimator. To summarize, the experiments differ in the following aspects:

- The true parameter values: $(\rho, \sigma^2) = (0.01, 0.02)$ or $(\rho, \sigma^2) = (0.01, 0.04)$;
- Whether to treat Z as given or as an additional parameter to estimate;
- The number of simulated observations, 300 or 1200.

All experiments were repeated 100 times.

In the left panels of Table 1 we report descriptive statistics of the parameter estimates in the case where the bandwidth Z is fixed. The results are encouraging. The mean of the estimator is quite close to the true value and the standard errors are small. Only in the small sample (300 observations) there seems to be a slight overestimation of ρ ; the estimates for σ^2 are about right on average. The GMM standard errors are in line with the Monte Carlo standard deviation of the parameter estimates. The gain in efficiency by adding the third moment in the estimation is marginal for this model with a fixed bandwidth.

The right panels of Table 1 reports the results for the experiments where the bandwidth Z was treated as an additional unknown parameter. Again, there is a slight upward bias in the estimates of ρ . The estimates of σ^2 are also slightly upward biased and have much bigger standard errors than in the case where Z is fixed. There is now a clear gain in efficiency of the three moment GMM estimator over the QML estimator. Especially the QML estimates of Z in the case of 300 weekly observations have very large standard errors and show a large upward bias. The GMM estimates of Z are much more precise. The GMM estimator for the other parameters (ρ and σ) seems also more efficient than the QML estimator.

⁹The parameter values are given on a weekly basis. Hence, the half-life of the process in weeks is $\ln(2)/\rho$, which equals around 14 for $\rho = 0.05$.

The final simulation results concern the model with realignments. Table 2 presents statistics of the GMM estimator that uses the conditional mean and variance of the exchange rate within the band, as well as the conditional mean of the central parity, (2.8a)–(2.8c). The weighting matrix is again based on a normal approximation along the lines of Appendix B. We report simulation evidence for one set of true values, $(\rho, \sigma^2, \theta) = (0.05, 0.02, 0.02)$, and a sample size of 700 weekly observations. The 3 moment GMM estimator performs rather well, although there is a slight downward bias in θ , which is compensated by an upward bias in ρ . We also experimented with a ‘5 moment GMM’ estimator which uses, in addition to those three moments, the conditional variance of the central parity and the conditional covariance of the exchange rate with the band and the central parity, (2.8a)–(2.8e). The performance of this ‘5 moment’ GMM estimator is worse than the performance of the three moment GMM¹⁰ and is therefore not used in the empirical applications.

4 Empirical results for the EMS

In this section we present empirical results of our target zone model on the exchange rates of EMS currencies to the Deutsche mark. The setup of the section is as follows. First, we present estimates of the target zone model (2.2) with fixed central parity and fully credible bands. Our specification gives the opportunity to treat the bandwidth as a free parameter, so that we can estimate implicit bands. These may be wider or narrower than the officially announced band, indicating a weaker or stronger commitment to the exchange rate parity than specified by the official target zone. After that, we show the evidence on the model containing realignment jumps, (2.6a)–(2.6b), which is more suited for the EMS which showed several realignments. The data used are weekly log exchange rates of the Deutsche mark expressed in local currency, multiplied by 100 to allow an interpretation as a percentage deviation from the central parity.

4.1 Estimation results for model without realignments

In Table 3 we report the estimates of the model with a fixed central parity and no jumps, which was presented in equation (2.2). To estimate this model we use data only from the most recent stable period in the EMS, which runs from January 1987 to August 1992 and contains 297

¹⁰This may be due to the use of an inefficient weighting matrix, or a result of small sample problems.

weekly observations.¹¹ The officially allowed deviation from the central parity in this period was 2.25%. The model is estimated using the GMM estimator presented in Section 3, based on the conditional mean, variance, and skewness given in (2.4a)–(2.4c). The first part of Table 3 reports estimates for a fixed bandwidth parameter $Z = 2.25$. The mean reversion is quite strong for the Dutch guilder, which is not very surprising as the guilder is always very close to the central parity. For the other currencies the mean reversion is less strong but significantly different from zero, and always larger than the estimated variance parameter σ^2 . The latter result implies a hump-shaped marginal distribution of the exchange rate. The results for the Danish krone are somewhat different as the estimate of ρ is not significantly different from zero and smaller than the estimated σ^2 .

The second part of Table 3 presents estimates over the same period where the bandwidth Z was treated as an additional free parameter. For the Belgian franc and the Danish krone, the implicit band is somewhat wider than the official band, but the difference is not significant. For the French franc the implicit band is significantly narrower than the official target zone. For reasons mentioned earlier, no reasonable estimate for the bandwidth Z of the Dutch guilder could be obtained.

Table 4 extends the sample period to the full period in which currencies participated in the EMS exchange rate mechanism with narrow bands. This covers the period February 1980 to June 1993.¹² The weeks with realignments were omitted from the estimation, which was achieved by skipping the weeks with a realignment in computing the GMM criterion. The point estimates are of course different from the ones obtained over the shorter period, but qualitatively the conclusions are similar. If the bandwidth is estimated, the estimated mean reversion is (in most cases) somewhat weaker and the estimated variance parameter is smaller. These effects are probably due to the result that the implicit bandwidth is wider than the official bandwidth. The differences are not statistically significant, however.

Next, we extend the data to cover the period after the EMS crisis in August 1993. The subsequent widening of the bands on most currencies to 15% deviation from the central parity is often viewed as a *de facto* reversion to floating exchange rates, see Obstfeld and Rogoff (1995). On the other hand, Campa and Chang (1996) present evidence that observed prices of

¹¹The Irish pound and the Italian lira were subject to realignment in this period and are therefore excluded from the analysis. Other currencies, such as the Spanish peseta and the Portuguese escudo, have participated in the EMS for too short a period for a meaningful econometric analysis.

¹²The Italian lira dropped out of the EMS in 1992; the last observation for the lira is therefore 7 July 1992.

options on the Deutsche mark/Italian Lira and the Deutsche mark/French franc exchange rates over this period are consistent with much narrower bands. We try to assess this hypothesis by estimating implicit bands and testing whether these are narrower or wider than the posted ones. Table 5 presents estimates of this implicit bandwidth. The model assumes that before the 1993 realignment the relevant bandwidth is equal to the official bandwidth of 2.25% (6% for the lira). The bandwidth after that realignment is treated as an unknown parameter. The other parameters (ρ and σ^2) are constant over the whole sample period. The results show that the implicit bandwidth for the period after the 1993 realignments, estimated around 6% for most currencies, is narrower than the officially announced bandwidth. The estimated implicit bandwidth for the Italian lira, which wasn't in the EMS in this period, is much larger than the estimate for the other currencies. The Dutch guilder was virtually unaffected by the realignments. Notice that the estimates of the implicit bandwidth for the French franc and the Italian lira are wider than the minimal bandwidth consistent with option prices, as derived in Campa and Chang (1996). Cavaliere (1997) finds that the implicit band before the 1993 realignments was also narrower than the official bandwidth.

4.2 Estimation results for model with realignments

In Table 6 the estimates of the target zone model with realignments (2.6a)–(2.6b) are reported. We use a GMM estimator based on the conditional mean and variance of the exchange rate within the band ($Y_t = X_t - M_t$) and the conditional mean of the central parity, given in (2.8a)–(2.8c). The most surprising feature of the results are the large estimates for the jump intensity θ . The estimated number of jumps per year is around 2, which is much greater than the actual number of realignments. This means that some large changes within the band could be confused for realignments. The estimates of ρ are typically negative, but the sum of ρ and θ is always positive, which ensures that the exchange rate within the band, Y_t , is mean reverting.

The somewhat disappointing results of the model with realignments lead us to an alternative estimator for the jump intensity, θ . A simple frequency estimator (the actual number of realignments divided by the total number of observations) was calculated. The remaining parameters were estimated by GMM taking the estimated value of θ as given. Standard errors were calculated as if θ were known. The empirical results of this procedure are reported in the right-hand-side panel of Table 6. The estimates for ρ and σ^2 make much more sense now. Compared with the estimates obtained from the model without realignments, the estimated

variance parameter σ^2 is smaller, especially if the jump probability is high. Apparently, for several currencies the jump process accounts for a substantial part of the variance in exchange rates. This will have important ramifications for option pricing, as we demonstrate in Section 5.

5 Option pricing

One of the most important reasons to consider continuous time models in finance is the pricing of financial derivatives. Since the seminal work of Black and Scholes (1973) it is well-known that continuous time models often allow for explicit calculation of option prices under a no-arbitrage assumption only. In this section we will derive prices of European-type options written on exchange rates that stay within a target zone. More specifically we will consider the exchange rate processes discussed in Section 2. The first model does not allow for realignments in the central parity. It will turn out that the market described in this case is complete, so that option prices can be calculated under an absence of arbitrage possibilities assumption only. Model (2.6a)–(2.6b) does allow for realignments. We will see that these realignments cannot be hedged perfectly when only trading the underlying exchange rate (and a riskless asset). In this case the market is incomplete, and implicit assumptions on agents' preferences are necessary to obtain option prices. We will follow the approach initiated by Hull and White (1987), by assuming that the realignment risk is not priced for the domestic investor. More specifically we will follow the framework outlined in Melenberg and Werker (1996).

It will turn out that analytical formulas are hard to obtain. Therefore we will calculate option prices using risk-neutral simulation techniques. Moreover it will turn out that the absence of arbitrage opportunities in a target zone model imposes restrictions on the domestic and foreign instantaneous interest rates. One of the viable restrictions is Uncovered Interest Parity, which we will assume in this section. It must be stressed however that other possibilities exist and can be handled with our technique.

5.1 Option pricing in models without realignments

We consider the pricing of European call options in the target zone model without realignments (2.2). To do so (and in fact obtain the price of any derivative claim on the currency) we derive the risk-neutral distribution of the exchange rate. Recall that, with X_t denoting the log

exchange rate, we have

$$dX_t = -\rho(X_t - m)dt + \sigma\sqrt{Z^2 - (X_t - m)^2}dW_t.$$

We let r denote the domestic instantaneous interest rate and r^f the foreign interest rate. In principle we could allow both interest rates to be arbitrary functions of X_t . Later it will turn out that a no-arbitrage assumption induces restrictions on the functional form of r and r^f .

Proposition 5.1 *Under the above assumptions the risk-neutral distribution of the exchange rate is described by*

$$d\exp(X_t) = [r_t - r_t^f] \exp(X_t)dt + \sigma\sqrt{Z^2 - (X_t - m)^2} \exp(X_t)d\tilde{W}_t, \quad (5.1)$$

with $\tilde{W}$ a risk-neutral Brownian motion.

PROOF: Consider a foreign money market account, whose value (in the foreign currency) satisfies

$$dV_t^f = r_t^f V_t^f dt.$$

In the domestic currency the value, $V_t = \exp(X_t)V_t^f$, consequently satisfies

$$\begin{aligned} dV_t &= \exp(X_t)dV_t^f + V_t^f d\exp(X_t) \\ &= \left[r_t^f - \rho(X_t - m) + \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2) \right] V_t dt + \sigma V_t \sqrt{Z^2 - (X_t - m)^2} dW_t. \end{aligned}$$

The discounted value $\exp(-\int_0^t r_s ds)V_t$ therefore follows the SDE

$$\begin{aligned} d\exp(-\int_0^t r_s ds)V_t &= \exp(-\int_0^t r_s ds)dV_t - \exp(-\int_0^t r_s ds)r_t V_t dt \\ &= \left[r_t^f - r_t - \rho(X_t - m) + \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2) \right] \exp(-\int_0^t r_s ds)V_t dt \\ &\quad + \sigma \exp(-\int_0^t r_s ds)V_t \sqrt{Z^2 - (X_t - m)^2} dW_t \end{aligned} \quad (5.2)$$

$$= \sigma \exp(-\int_0^t r_s ds)V_t \sqrt{Z^2 - (X_t - m)^2} d\tilde{W}_t, \quad (5.3)$$

where equality (5.3) signifies the no-arbitrage assumption, i.e. $\tilde{W}_t$ is a risk-neutral Brownian motion¹³. This in turn implies that

$$\begin{aligned} dV_t &= \exp(\int_0^t r_s ds)d\exp(-\int_0^t r_s ds)V_t + r_t V_t dt \\ &= r_t V_t dt + \sigma V_t \sqrt{Z^2 - (X_t - m)^2} d\tilde{W}_t. \end{aligned}$$

¹³Formally, we know that discounted values of attainable payoffs are martingales under the risk-neutral distribution. Therefore (5.3) holds for some martingale $\tilde{W}$. Since its quadratic variation must be t , because quadratic variations are invariant under absolutely continuous changes of measure, Levy's characterization theorem implies that $\tilde{W}$ is a Brownian motion.

To obtain the risk-neutral distribution of the exchange rate note that $X_t = \log(V_t/V_t^f)$ which implies

$$\begin{aligned} dX_t &= V_t^{-1}dV_t - \frac{1}{2}V_t^{-2}d\langle V, V \rangle_t - (1/V_t^f)dV_t^f \\ &= \left[r_t - r_t^f - \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2) \right] dt + \sigma\sqrt{Z^2 - (X_t - m)^2}d\tilde{W}_t. \end{aligned} \quad (5.4)$$

Using Itô's Lemma to rewrite (5.4) in terms of the exchange rate $\exp(X_t)$ yields the desired result. $\square$

We have shown that, under the no-arbitrage assumption, the exchange rate $\exp(X_t)$ satisfies (5.1) where $\tilde{W}$ is a risk-neutral Brownian motion. If $r_t - r_t^f$ would be constant, (5.1) does not have a solution, which suggests that arbitrage opportunities should exist. It is not difficult to see that these arbitrage opportunities occur if the exchange rate is near the target zone boundaries. In case, e.g., $r_t - r_t^f > 0$ and the exchange rate is close to the upper boundary, domestic instantaneous risk-free investments yield higher than foreign risk-free returns while the exchange rate is known not to increase by much because it remains within the target zone. Similar reasoning applies in case $r_t - r_t^f < 0$.

To obtain a viable model, we will impose Uncovered Interest Parity (UIP). This means that we suppose that

$$r_t - r_t^f = -\rho(X_t - m). \quad (5.5)$$

It must be stressed that the UIP assumption is only one of possible choices that gives a viable model. The UIP assumption is made here for convenience.

By (5.4) and (5.5) we obtain

$$d(X_t - m) = -\left[\rho(X_t - m) + \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2) \right] dt + \sigma\sqrt{Z^2 - (X_t - m)^2}d\tilde{W}_t. \quad (5.6)$$

Notice that now the log exchange rate is mean reverting and therefore will remain inside the target zone under the risk neutral distribution. Given such parametric assumptions, derivation of prices of European type call options on the exchange rate is straightforward. Consider a European call option with maturity T and exercise price $K \equiv \exp(m + k)$. A value of $k = 0.02$ implies that the exercise price of the option is 2 percent above the long-term value of the exchange rate. Under the no-arbitrage assumption, the domestic option price is given by

$$\tilde{E}_0 \exp\left(-\int_0^T r_s ds\right) \max\{\exp(X_T) - K, 0\},$$

where $\tilde{E}$ signifies that the expectation is taken under the risk-neutral probability measure. Particular option prices are easily obtained using risk-neutral simulation from the stochastic differential equation (5.6).

5.2 Option pricing in models with realignments

In this section we consider option pricing in the model with realignments, (2.6a)–(2.6b). As mentioned above this model describes an incomplete market so that option prices are not solely determined by a no-arbitrage assumption. Consequently we will have to make assumptions on how to deal with the realignment risk. The method we use builds on the work of Melenberg and Werker (1996). Essentially, this method integrates out all possible jump processes. Conditionally on a specific jump process, the market is complete and option pricing follows the approach outlined in the previous subsection. Option prices are obtained taking the expectation of these prices (expressed in domestic currency) with respect to the distribution of the jump process. The fact that going to the Equivalent Martingale Measure does not affect the distribution of realignments, implies that its risk is not priced. Note that this holds for the domestic investor. The Equivalent Martingale Measure concept is not invariant under changes of currencies. Hence the realignment risk probably is priced for the foreign investor. See also Dumas, Jennergren, and Näslund (1995b).

The ideas used in this section are quite straightforward. We consider the model

$$\begin{aligned} dX_t &= -\rho(X_{t-} - M_{t-})dt + \sigma\sqrt{Z^2 - (X_{t-} - M_{t-})^2}dW_t, \\ dM_t &= (X_{t-} - M_{t-})dN_t. \end{aligned}$$

There are two sources of uncertainty in this model, W and N . Together these define an incomplete market. However, if we condition on N , we are left with a single source of uncertainty and the market is complete. This allows for the computation of option prices conditional on the behaviour of N along the same lines as in Section 5.1. Intuitively we say that the option can perfectly be hedged if the realignments were known beforehand. Of course, they are not known and to obtain the actual derivative prices we must take the expectation over N of this conditional option pricing formula. This expectation will be taken under the true probability measure. Hence, we make the assumption that the jump intensity under the Equivalent Martingale Measure that describes the option prices, equals the objective one (i.e. under the true probability). It is not difficult to see that a change of the jump intensity yields another possible

Equivalent Martingale Measure. This verifies that indeed the market is incomplete. Requiring that the jump intensity remains the same is simply one of the possible restrictions on agents' preferences that may be imposed. It implies that no risk-premium is paid on realignment risk, i.e. one might say that this risk is idiosyncratic. For rigorous proofs of the validity and an interpretation of the procedure we refer to Melenberg and Werker (1996).

To summarize, we have the

Proposition 5.2 *In the model with idiosyncratic realignment risk the risk-neutral distribution of the exchange rate is described by*

$$d\exp(X_t) = \left[r_t - r_t^f\right] \exp(X_t)dt + \sigma\sqrt{Z^2 - (X_t - M_t)^2} \exp(X_t)d\tilde{W}_t, \quad (5.7a)$$

$$dM_t = (X_{t-} - M_{t-})dN_t. \quad (5.7b)$$

PROOF: We can completely copy the proof of Proposition 5.1, working conditional on N . This immediately yields (5.7a). Under the stated assumptions the risk-neutral distribution with respect to N simply equals the true one. This yields (5.7b). $\square$

As mentioned above, the concept of an Equivalent Martingale Measure is *not* invariant under a change of currency. This is a consequence of the simple fact that domestic investors should discount with the domestic interest rate, while foreign investors discount with the foreign interest rate. This implies that the assumption made is also particular to the domestic investor. Her foreign colleague might very well face a risk premium on realignment risk. For further discussion we refer to Dumas, Jennergren, and Näslund (1995b).

5.3 Numerical examples

In this section we give some numerical examples of our currency option pricing model. We compare the option prices generated by our model with option prices based on the models of Garman and Kohlhagen (1983) and Grabbe (1983). These models are essentially adaptations of the standard Black-Scholes model for the pricing of currency options and assume that the exchange rate follows a geometric Brownian motion and that interest rates are constant. We will show that our model sometimes can generate prices which are very different from the Black-Scholes values.

In the first experiment we calculate prices of European call options on exchange rates with maturity 52 weeks. The initial value of the exchange rate is equal to the central parity. Option

prices are calculated from the target zone model without realignment jumps (2.2), the target zone model with realignments, and the Black-Scholes model. For the target zone models, the interest rate differential follows specification (5.5), where M_{t-} replaces $\bar{m}$ in the model with realignments. The numerical results for the target zone models are based on 10000 simulations where an Euler approximation with 1000 steps is used. The Black-Scholes prices are obtained by using the instantaneous standard deviation $\sigma\sqrt{Z^2 - X_0^2}$ as the volatility parameter and a constant interest rate equal to the initial interest rate.¹⁴

The prices and implied volatilities of call options for various exercise prices are graphed in Figure 2A.¹⁵ The figure clearly shows that, in a credible target zone, the Black-Scholes model overprices currency options relative to our model. The main reason for this overpricing is that the Black-Scholes model ignores the mean reversion and the decreasing volatility of the exchange rate close to the boundaries of the target zone. The overpricing is most pronounced for deep in-the-money and out-of-the-money options. Option prices in the model with realignments are higher than the prices in the model without realignments. This is not surprising since the realignments increase the dispersion of the exchange rate at maturity, which increases option prices. Despite the rather high probability of a realignment jump, the Black-Scholes model still overprices currency options. This result is specific for this parameter setting, however.

The second experiment is the same as the first, except for the initial value of the exchange rate, which is chosen to be below the central parity, $X_0 = M_0 - 0.01$. The prices and implied volatilities of the model prices are graphed in Figure 2B. Like in the first experiment, the target zone model prices are lower than the Black-Scholes prices. Unlike in the first experiment, option prices in the model with realignments are *lower* than prices in the model without realignments. The explanation for this results is that, due to the low initial value of the exchange rate, downward jumps of the central parity are more likely than upward jumps, which causes a lower expected payoff of the call option.

The third experiment is again the same as the first, but with an initial value of the exchange rate above the central parity, $X_0 = M_0 + 0.01$. The numerical results are graphed in Figure 2C. We first discuss the results for the model without realignments. In contrast with the previous results, the model prices for some options are now larger than the Black-Scholes prices. In

¹⁴The other parameter settings are as follows: the bandwidth Z equals 0.0225; the volatility parameter $\sigma^2 = 0.014$; the mean-reversion parameter $\rho = 0.01$; the foreign interest rate is fixed at $r_f' = 0.0008$ per week. In the model with realignments the jump intensity is either $\theta = 0.045$ or $\theta = 0.09$ on a weekly basis.

¹⁵In the figures the exercise price is expressed as a fraction of the initial central parity, $\exp(M_0)$

the model with realignments, upward jumps are now much more likely than downward jumps because the initial exchange rate is above the central parity. Hence, call prices in the model with realignments are larger than both the Black-Scholes prices and the prices in the model with a credible target zone.

We repeated the preceding analysis for options with a shorter maturity, 13 weeks. The deviations between the target zone model prices and the Black-Scholes prices are now less pronounced, but the same general conclusions can be drawn. These results are different from the usual results on volatility smiles. For example, Renault (1995), who investigates option pricing for stochastic volatility models, shows that deviations from the Black-Scholes model are more pronounced for close-to-maturity options.

To summarize, the target zone model yields call option prices which are very different from Black-Scholes prices. In the model without realignments, the Black-Scholes model overestimates (in most cases) option prices because it ignores the decreasing volatility of the exchange rate in the target zone. Another difference between the Black-Scholes model and the target zone model is the mean reversion under the risk-neutral distribution induced by the time-varying interest rate differential in (5.5). Realignments tend to increase option prices because of the increased dispersion of the option payoff. On the other hand, if there is a relatively large probability of downward jumps the model with realignments may generate lower option prices than the model without realignments.

6 Conclusions

In this paper we develop new models for exchange rates in a target zone. These models are based on the idea that the volatility of the exchange rate should decrease near the band, while mean-reversion stays in effect. Both models with a fully credible band and models with realignments are considered. Since the likelihood function for discrete time observations of the exchange rates is not known, we develop a GMM estimator based on exact conditional moments. Simulation experiments show that the GMM estimator works quite well for parameter values typically found for EMS currencies. In the empirical part we estimate the model on EMS data. One of the interesting results is that we estimate implicit bandwidths for the period after the 1993 realignment. It turns out that these implicit bands, though wider than the previous 2.25% bandwidth, are narrower than the official target zone which admits 15% deviations to

either side of the central parity. The final section of the paper develops a method to price currency options in target zone models with and without realignments. We conclude the paper by calculating option prices in our models. The results show that naive application of the Black-Scholes formula can significantly overprice (if there are no realignments) or underprice (if there is a high probability of devaluation) European type call options.

A potential further application of the model is an assesment of target zone credibility. Following Bertola and Svensson (1993) and Svensson (1993), one may adjust foreign-domestic interest rate differentials for the expected rate of devaluation within the target zone to get an estimate of the expected rate of realignment. Alternatively, the intensity of jumps in the model could be estimated using observed option prices, for example along the lines of Campa and Chang (1996). Such applications are left for further research.

A Conditional moments

In this appendix we show how to derive the conditional moments of the stochastic process (2.2), which we repeat here for convenience

$$dY_u = -\rho Y_u du + \sigma \sqrt{Z^2 - Y_u^2} dW_u.$$

With Itô's theorem we obtain

$$de^{\rho u} Y_u = e^{\rho u} \sigma \sqrt{Z^2 - Y_u^2} dW_u.$$

By integrating both sides over the interval, $(t, t+h]$,

$$\begin{aligned} Y_{t+h} &= e^{-\rho h} Y_t + e^{-\rho h} \sigma \int_{(t, t+h]} e^{\rho(u-t)} \sqrt{Z^2 - Y_u^2} dW_u \\ &= e^{-\rho h} Y_t + \varepsilon_{t+h}. \end{aligned}$$

Observe that the innovations,

$$\varepsilon_{t+h} = e^{-\rho h} \sigma \int_{(t, t+h]} e^{\rho(u-t)} \sqrt{Z^2 - Y_u^2} dW_u,$$

form a martingale difference sequence. The conditional expectation therefore equals

$$E_t(Y_{t+h}) = e^{-\rho h} Y_t.$$

The conditional variance of the innovations can be derived as follows,

$$\text{Var}_t Y_{t+h} = E_t \varepsilon_{t+h}^2$$

$$\begin{aligned}
&= e^{-2\rho h} \sigma^2 \int_{(t,t+h]} e^{2\rho(u-t)} \{Z^2 - E_t Y_u^2\} du \\
&= e^{-2\rho h} \sigma^2 \int_{(t,t+h]} e^{2\rho(u-t)} \{Z^2 - e^{-2\rho(u-t)} Y_t^2 - \text{Var}_t Y_u\} du.
\end{aligned} \tag{A.1}$$

Define $f(h) = e^{2\rho h} \text{Var}_t Y_{t+h}$ and obtain from (A.1) the following ordinary differential equation,

$$\frac{d}{dh} f(h) = -\sigma^2 f(h) + \sigma^2 \{e^{2\rho h} Z^2 - Y_t^2\},$$

with boundary condition $f(0) = 0$. This equation is solved by

$$\begin{aligned}
f(h) &= \{e^{-\sigma^2 h} - 1\} Y_t^2 + \{e^{2\rho h} - e^{-\sigma^2 h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho} \\
&= \{e^{-\sigma^2 h} - 1\} \{Y_t^2 - \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}\} + \{e^{2\rho h} - 1\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}.
\end{aligned}$$

Thus, the conditional variance is given by

$$\begin{aligned}
\text{Var}_t Y_{t+h} &= e^{-2\rho h} f(h) \\
&= \{e^{-\sigma^2 h} - 1\} e^{-2\rho h} Y_t^2 + \{1 - e^{-(\sigma^2 + 2\rho)h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}.
\end{aligned}$$

In a similar way one can derive all higher conditional moments. Using Itô's formula, one easily verifies that the drift for Y_t^p is given by a polynomial in lower powers of Y_t . Analogously to the derivation of the conditional expectation of Y_t , one may derive from the SDE for Y_t^p an ordinary differential equation for $E_t Y_{t+h}^p$. Solving this equation for $p = 3$ yields (2.4c).

The conditional moments of the model with realignment jumps (2.6a)–(2.6b) are derived as follows. The conditional expectations¹⁶ of X_{t+h} and M_{t+h} given information available at time t are obtained by integrating over the interval $(t, t+h]$. This yields

$$\begin{aligned}
E_t X_{t+h} &= X_t + E_t \int_{(t,t+h]} dX_u = X_t - \rho \int_{(t,t+h]} (E_t X_u - E_t M_u) du, \\
E_t M_{t+h} &= M_t + E_t \int_{(t,t+h]} dM_u = M_t + \theta \int_{(t,t+h]} (E_t X_u - E_t M_u) du.
\end{aligned}$$

Solving the corresponding differential equations,

$$\begin{aligned}
\frac{d}{dh} E_t X_{t+h} &= -\rho (E_t X_{t+h} - E_t M_{t+h}), \\
\frac{d}{dh} E_t M_{t+h} &= \theta (E_t X_{t+h} - E_t M_{t+h}),
\end{aligned}$$

¹⁶When using conditional expectations, denoted by E_t , we condition only on the observable variables $X_h, X_{2h}, \dots, X_t$ and not on $\{W_s, 0 \leq s \leq t\}$, the complete past of the Brownian motion driving the price process.

with boundary conditions $E_t X_t = X_t$ and $E_t M_t = M_t$, one obtains,

$$\begin{aligned} E_t X_{t+h} &= \left\{ \frac{\theta}{\theta + \rho} + \frac{\rho}{\theta + \rho} e^{-(\theta + \rho)h} \right\} X_t + \frac{\rho}{\theta + \rho} \{1 - e^{-(\theta + \rho)h}\} M_t, \\ E_t M_{t+h} &= \frac{\theta}{\theta + \rho} \{1 - e^{-(\theta + \rho)h}\} X_t + \left\{ \frac{\rho}{\theta + \rho} + \frac{\theta}{\theta + \rho} e^{-(\theta + \rho)h} \right\} M_t. \end{aligned}$$

Hence, the conditional expectations are weighted averages of the current price and the current reference price. Jointly, they form a VAR(1), compare the situation with the model of subsection 2.1. The martingale implications of these two conditional moments allow for estimation of the mean parameters ρ and θ . Similar but tedious calculations yield the conditional second moments.

B GMM Estimation

In this appendix we provide a detailed description of the GMM estimators used in this paper. We use the following notation for the first three conditional moments (mean, variance and skewness): $\mu_t = E_{t-1}(X_t)$, $h_t = Var_t(X_t)$ and $s_t = E_{t-1}(X_t - \mu_t)^3$, all of which are functions of the unknown parameters θ .

The Quasi Maximum Likelihood estimator uses the information in the conditional mean and variance. Defining $\varepsilon_t \equiv X_t - \mu_t$, the QML estimator can be written in GMM form (see also Meddahi and Renault, 1996) by choosing the vector of moments

$$m_t = \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^2 - h_t \end{pmatrix},$$

and the approximation to the variance matrix

$$V_{t-1} = \begin{pmatrix} h_t & 0 \\ 0 & 2h_t^2 \end{pmatrix}.$$

The resulting moment equations

$$-\sum_{t=1}^n \left[\frac{\partial \mu_t}{\partial \theta} \frac{\varepsilon_t}{h_t} + \frac{\partial h_t}{\partial \theta} \frac{(\varepsilon_t^2 - h_t)}{2h_t^2} \right] = 0$$

are identical to the first order conditions of the QML estimator.

We also employ a GMM estimator which uses, in addition to the conditional mean and variance, the conditional skewness of the data. The vector of moments therefore is

$$m_t = \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^2 - h_t \\ \varepsilon_t^3 - s_t \end{pmatrix}.$$

An approximation to the variance matrix of the moments is obtained by calculation higher moments as if the conditional distribution were normal, which yields

$$V_{t-1} = \begin{pmatrix} h_t & 0 & 3h_t^2 \\ 0 & 2h_t^2 & 0 \\ 3h_t^2 & 0 & 15h_t^3 \end{pmatrix},$$

and the instrument

$$z_{t-1} = E_{t-1}\left(\frac{\partial m'_t}{\partial \theta}\right)V_{t-1}^{-1} = -\left(\frac{\partial \mu_t}{\partial \theta} \frac{1}{h_t} - \frac{\partial s_t}{\partial \theta} \frac{1}{2h_t^2}, \frac{\partial h_t}{\partial \theta} \frac{1}{2h_t^2}, \frac{\partial s_t}{\partial \theta} \frac{1}{6h_t^3}\right)$$

The asymptotic variance of $\hat{\theta}$ takes the familiar form $A^{-1}BA^{-1}$, where A and B are the limits of

$$\begin{aligned} A &= \frac{1}{n} \sum_t z_{t-1} E_{t-1}\left(\frac{\partial m_t}{\partial \theta'}\right), \\ B &= \frac{1}{n} \sum_t z_{t-1} Var_{t-1}(m_t) z'_{t-1}. \end{aligned}$$

References

- BALL, C. AND A. ROMA (1994), Target zone modelling and estimation for European Monetary System exchange rates, *Journal of Empirical Finance* **1**, 385-420.
- BEKAERT, G. AND S.F. GRAY (1996), Target zones and exchange rates: An empirical investigation, working paper Stanford University.
- BERTOLA, G. AND R. CABALLERO (1992), Target zones and realignments, *American Economic Review* **82**, 520-536.
- BERTOLA, G. AND L.E.O. SVENSSON (1993), Stochastic devaluation risk and the empirical fit of target zone models, *Review of Economic Studies* **60**, 689-712.
- BLACK, F. AND M. SCHOLES (1973), The pricing of options and corporate liabilities, *Journal of Political Economy* **81**, 637-659.
- CAMPA, J.M. AND P.H.K. CHANG (1996), Arbitrage-based tests of target-zone credibility: Evidence from ERM cross-rate options, *American Economic Review* **86**, 726-740.
- CAVALIERE, G. (1997), Detecting undeclared target zones within the European Monetary System, paper presented at ESEM97.
- COX, J., J. INGERSOLL AND S. ROSS (1985), A theory of the term structure of interest rates, *Econometrica* **53**, 385-407.
- DE JONG, F. (1994), A univariate analysis of EMS exchange rates in a target zone, *Journal of Applied Econometrics* **9**, 31-45.
- DUFFIE, D. AND R. KAN (1996), A yield factor model of interest rates, *Mathematical Finance* **6**, 379-406.
- DUMAS, B., L.P. JENNERGREN AND B. NÄSLUND (1995a), Realignment risk and currency option pricing in target zones, *European Economic Review* **39**, 1523-1544.
- DUMAS, B., L.P. JENNERGREN AND B. NÄSLUND (1995b), Siegel's paradox and the pricing of currency options, *Journal of International Money and Finance* **14**, 213-223.

- ENGLE, R.F. AND C.W. GRANGER (1987), Co-integration and error correction: Representaion, estimation and testing, *Econometrica* **55**, 251-276.
- FLOOD, R.P., A. ROSE AND D. MATHIESON (1991), An empirical exploration of exchange rate tarhet zones, *Carnegie-Rochester Series on Public Policy* **35**, 7-65.
- GARMAN, M.B. AND S.W. KOHLHAGEN (1983), Foreign currency option values, *Journal of International Money and Finance* **2**, 231-237.
- GODAMBE, V.P. (1985), The foundations of finite sample estimation in stochastic processes, *Biometrika* **72**, 419-428.
- GOURIÉROUX, C., A. MONTFORT AND A. TROGNON (1984), Pseudo Maximum Likelihood methods: theory, *Econometrica* **52**, 681-700.
- GRABBE, J.O. (1983), The pricing of call and put options on foreign exchange, *Journal of International Money and Finance* **2**, 239-253.
- HULL, J. AND A. WHITE (1987), The pricing of options on assets with stochastic volatility, *Journal of Finance* **42**, 281-300.
- KARATZAS, I. AND S. SHREVE (1991), *Brownian Motion and Stochastic Calculus*, Springer-Verlag, Berlin.
- KOEDIJK, C., PH. STORK AND C.G. DE VRIES (1996), An EMS target zone model in discrete time, forthcoming in *Journal of Applied Econometrics*.
- KRUGMAN, P. (1991), Target zones and exchange rate dynamics, *Quarterly Journal of Economics* **106**, 669-682.
- LINDBERG, H. AND P. SÖDERLIND (1994), Target Zone models and Intervention Policy: the Swedish case, *Scandinavian Journal of Economics* **96**, 499-514.
- MEDDAHI, N. AND E. RENAULT (1996), Linear statistical inference for ARCH-type processes, working paper Université de Toulouse.
- MELENBERG, B. AND B.J.M. WERKER (1996), On the pricing of options in incomplete markets, discussion paper, Tilburg University.
- MERTON, R.C. (1976), Option pricing when the underlying stock returns are discontinuous, *Journal of Financial Economics* **3**, 125-144.

- NEWKEY, W. (1990), Semiparametric efficiency bounds, *Journal of Applied Econometrics* 5, 99-135.
- OBSTFELD, M. AND K. ROGOFF (1995), The Mirage of Fixed Exchange Rates, *Journal of economic perspectives* 9, 73-96.
- RENAULT, E. (1995), Econometric models of option pricing errors, paper presented at the Econometric Society World Meeting, Tokio, August 1995.
- SMITH, G.W. AND M.G. SPENCER (1991), Estimation and testing in models of exchange rate target zones and process switching, In: P. Krugman and M. Miller (eds.), *Exchange Rate Targets and Currency Bands*, Cambridge University Press.
- SVENSSON, L.E.O. (1992a), An interpretation of recent research on exchange rate target zones, *Journal of Economic Perspectives* 6, 119-144.
- SVENSSON, L.E.O. (1992b), The foreign exchange risk premium in a target zone with devaluation risk, *Journal of International Economics* 33, 21-40.
- SVENSSON, L.E.O. (1993), Assessing target zone credibility: Mean reversion and devaluation expectations in the ERM, 1979-1992, *European Economic Review* 37, 763-802.
- VLAAR, P. AND F. PALM (1993), The message in weekly exchange rates in the European Monetary System: Mean reversion, heteroskedasticity and jumps, *Journal of Business and Economics Statistics* 11, 351-360.
- WEFELMEYER, W. (1996), Quasi-likelihood models and optimal inference, *The Annals of Statistics* 24, 405-422.
- YAMADA, T. AND S. WANATABE (1971), On the uniqueness of solutions of stochastic differential equations, *Journal of Mathematics Kyoto University* 11, 155-167.

Table 1: Simulation results target zone model without realignments.

Weekly data (300 or 1200 observations) were simulated from process (2.2) using an Euler discretisation scheme with five intermediate steps. The Euler scheme is refined by drawing from a normal distribution with conditional mean and variance given by the expressions in equations (2.4a)–(2.4b). The table reports the average over 100 simulations of the point estimate and the estimated standard error. The Monte Carlo standard deviation of the point estimates is reported in [].

$$(\rho, \sigma^2, Z) = (0.05, 0.02, 2.25)$$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
300 observations:					
mean QML estimate	0.0566	0.0202	0.0556	0.0211	9.4357
mean st. error	(0.0169)	(0.0018)	(0.0246)	(0.0132)	(1.2439)
st. deviation	[0.0175]	[0.0016]	[0.0185]	[0.0062]	[71.7937]
mean GMM estimate	0.0541	0.0202	0.0578	0.0226	2.1828
mean st. error	(0.0160)	(0.0018)	(0.0154)	(0.0046)	(0.1758)
st. deviation	[0.0160]	[0.0018]	[0.0183]	[0.0066]	[0.2283]
1200 observations:					
mean QML estimate	0.0505	0.0201	0.0526	0.0208	2.2524
mean st. error	(0.0076)	(0.0009)	(0.0084)	(0.0036)	(0.1489)
st. deviation	[0.0088]	[0.0009]	[0.0080]	[0.0027]	[0.1213]
mean GMM estimate	0.0504	0.0199	0.0524	0.0208	2.2346
mean st. error	(0.0076)	(0.0009)	(0.0077)	(0.0020)	(0.0734)
st. deviation	[0.0072]	[0.0009]	[0.0080]	[0.0027]	[0.0997]

$$(\rho, \sigma^2, Z) = (0.05, 0.04, 2.25)$$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
300 observations:					
mean QML estimate	0.0539	0.0402	0.0549	0.0415	2.2411
mean st. error	(0.0134)	(0.0034)	(0.0202)	(0.0163)	(0.2219)
st. deviation	[0.0147]	[0.0038]	[0.0145]	[0.0047]	[0.3254]
mean GMM estimate	0.0519	0.0385	0.0551	0.0415	2.2214
mean st. error	(0.0132)	(0.0037)	(0.0123)	(0.0052)	(0.0430)
st. deviation	[0.0161]	[0.0040]	[0.0153]	[0.0061]	[0.0670]
1200 observations:					
mean QML estimate	0.0517	0.0404	0.0501	0.0401	2.2520
mean st. error	(0.0067)	(0.0018)	(0.0064)	(0.0052)	(0.0743)
st. deviation	[0.0072]	[0.0016]	[0.0065]	[0.0034]	[0.0498]
mean GMM estimate	0.0492	0.0389	0.0526	0.0411	2.2267
mean st. error	(0.0066)	(0.0020)	(0.0061)	(0.0026)	(0.0208)
st. deviation	[0.0073]	[0.0021]	[0.0068]	[0.0030]	[0.0344]

Table 2: Simulation results target zone model with realignments.

Weekly data (700 observations) were simulated from process (2.6a)–(2.6b) using an Euler discretisation scheme with five intermediate steps. The Euler scheme is refined by drawing from a normal distribution with conditional mean and variance given by the expressions in equations (2.8a)–(2.8c). The table reports the average over 100 simulations of the point estimate and the estimated standard error. The Monte Carlo standard deviation of the point estimates is reported in [].

$$(\rho, \sigma^2, \theta) = (0.05, 0.02, 0.02)$$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$
3 moment GMM:			
mean estimate	0.0611	0.0222	0.0093
mean st. error	(0.0161)	(0.0018)	(0.0073)
st. deviation	[0.0296]	[0.0025]	[0.0118]

Table 3: Estimation results for the fixed parity model

The model is given in equation (2.2). The sample period is January 1987 to August 1992 (297 weekly observations). Estimation by GMM based on conditional mean, variance and skewness (2.4a)–(2.4c).

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
Bfr/DM	0.053 (0.017)	0.02058 (0.00266)	0.044 (0.014)	0.01384 (0.00644)	2.594 (0.456)
Dfl/DM	0.212 (0.054)	0.00235 (0.00033)			
Dkr/DM	0.020 (0.081)	0.05178 (0.02619)	0.032 (0.013)	0.00903 (0.00757)	3.272 (1.222)
Ffr/DM	0.019 (0.009)	0.01150 (0.00172)	0.023 (0.008)	0.01631 (0.00331)	2.022 (0.058)

Table 4: Estimation results for fixed parity model until last realignment.

The model is given in equation (2.2). The sample period is 1 February 1980 to 25 June 1993 except for the Lira: 1 February 1980 to 10 July 1992 . GMM estimation based on conditional mean, variance and skewness (2.4a)–(2.4c). Weeks with realignments were omitted from the GMM criterion. Due to these exclusions the number of observations is different for each series.

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
Bfr/DM	0.075 (0.028)	0.03705 (0.00896)	0.010 (0.007)	0.00450 (0.00644)	3.420 (1.000)
Dfl/DM	0.052 (0.020)	0.00640 (0.00118)			
Dkr/DM	0.061 (0.014)	0.03236 (0.00449)	0.034 (0.008)	0.01455 (0.00484)	2.678 (0.326)
Ffr/DM	0.117 (0.059)	0.04743 (0.01452)	0.031 (0.009)	0.00536 (0.00585)	3.750 (1.810)
Irp/DM	0.028 (0.003)	0.02342 (0.00345)	0.027 (0.008)	0.00885 (0.00288)	2.896 (0.350)
Lire/DM	0.011 (0.006)	0.00387 (0.00046)	0.012 (0.005)	0.00317 (0.00081)	6.498 (0.601)

Table 5: Estimation results for fixed parity model with implicit bandwidth after last realignment.

The model is given by equation (2.2) Sample period: 1 February 1980 to 3 March 1995 (788 weekly observations). Bandwidth is fixed at official value before 25 June 1993 (for the Lira: 10 July 1992); estimation of implicit bandwidth Z_{impl} after this date. GMM estimator based on conditional mean, variance and skewness (2.4a)–(2.4c).

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}_{impl}$
Bfr/DM	0.076 (0.028)	0.03381 (0.00878)	6.918 (1.342)
Dfl/DM	0.044 (0.017)	0.00654 (0.00123)	1.014 (0.087)
Dkr/DM	0.062 (0.014)	0.03009 (0.00439)	8.581 (1.025)
Ffr/DM	0.117 (0.056)	0.04486 (0.01432)	5.596 (0.590)
Irp/DM	0.030 (0.003)	0.02325 (0.00334)	5.322 (0.537)
Lire/DM	0.010 (0.006)	0.00389 (0.00046)	24.275 (3.200)

Table 6: Estimates of target zone model with realignments

The model estimated is given by equations (2.6a)–(2.6b). The sample period is 1 February 1980 to 25 June 1993 (700 weekly observations), except for the Lira: 1 February 1980 to 10 July 1992 (650 weekly observations). GMM estimation based on conditional moments (2.8a)–(2.8c). In the second panel θ is estimated by a frequency estimator.

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$
Bfr/DM	-0.020 (0.011)	0.00771 (0.00227)	0.046 (0.015)	0.012 (0.009)	0.00941 (0.00305)	0.009
Dfl/DM	-0.023 (0.021)	0.00213 (0.00030)	0.072 (0.019)	0.046 (0.019)	0.00561 (0.00086)	0.001
Dkr/DM	-0.005 (0.019)	0.01822 (0.00486)	0.051 (0.026)	0.059 (0.025)	0.01616 (0.01337)	0.009
Ffr/DM	-0.025 (0.023)	0.01394 (0.00569)	0.065 (0.030)	0.083 (0.037)	0.01456 (0.01475)	0.007
Irp/DM	-0.032 (0.022)	0.01110 (0.00282)	0.079 (0.029)	0.075 (0.027)	0.01752 (0.00885)	0.010
Lire/DM	-0.027 (0.023)	0.00390 (0.00119)	0.061 (0.028)	0.016 (0.012)	0.00757 (0.00757)	0.012

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