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Einmahl, Jesson; Einmahl, John; de Haan, L.F.M.

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**LIMITS TO HUMAN LIFE SPAN THROUGH
EXTREME VALUE THEORY**

By

Jesson J. Einmahl, John H.J. Einmahl,
Laurens de Haan

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Limits to human life span through extreme value theory

Jesson J. Einmahl

Tilburg University

John H.J. Einmahl

Department of Econometrics & OR and CentER, Tilburg University
and

Laurens de Haan

Erasmus University Rotterdam and University of Lisbon

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Abstract

There is no scientific consensus on the fundamental question whether the probability distribution of the human life span has a finite endpoint or not and, if so, whether this upper limit changes over time. Our study uses a unique dataset of the ages at death - in days - of all (about 285,000) Dutch residents, born in the Netherlands, who died in the years 1986-2015 at a minimum age of 92 years and is based on extreme value theory, the coherent approach to research problems of this type. Unlike some other studies we base our analysis on the configuration of thousands of mortality data of old people, not just the few oldest old. We find compelling statistical evidence that there is indeed an upper limit to the life span of men and to that of women for all the 30 years we consider and, moreover, that there are no indications of trends in these upper limits over the last 30 years, despite the fact that the number of people reaching high age (say 95 years) was almost tripling. We also present estimates for the endpoints, for the force of mortality at very high age, and for the so-called perseverance parameter.

Keywords: aging, endpoint, extreme value index, oldest, statistics of extremes

JEL codes: C12, C13, C14

1 Introduction

Consider the life span of a human being as a random variable. We consider the question whether the support of the probability distribution of this random variable has a finite right endpoint. There is no scientific agreement on this question at present. Let us look at the problem purely from a statistical point of view ignoring medical and demographical considerations. If the endpoint turns out to be finite, there are further questions: how to estimate this endpoint and does it change over the years?

The present study is based on a dataset provided by Statistics Netherlands (CBS) of the ages at death *in days* of all (about 285,000) Dutch residents, born in the Netherlands, who died in the years 1986-2015 at a minimum age of 92 years. The oldest person in these 30 years, a woman, reached 115.2 years; the oldest man, reached 111.4 years. Tables 1 and 2 provide some insight in the age build-up.

Table 1: Women

Age at death						
113 – 116	0	0	0	2	0	0
110 – 113	2	2	3	2	6	3
107 – 110	20	17	25	27	28	53
104 – 107	97	148	178	198	286	393
101 – 104	604	945	1038	1251	1506	1961
98 – 101	2383	3313	3992	4650	5487	6914
95 – 98	6914	9122	11013	12560	13924	16635
92 – 95	11647	14538	17623	19210	20819	24730
Year of death	'86- '90	'91- '95	'96- '00	'01- '05	'06- '10	'11- '15

A cell contains the number of women in a certain age category who died in a given 5-year period

The proper context for research of this type is extreme value theory since one wants to look beyond the largest observation. Suppose that the distribution F of a human life span X is in the domain of attraction of an extreme value distribution (this is the extreme value condition) or equivalently that the corresponding residual life time stabilizes at great age

Table 2: Men

Age at death						
110 – 113	0	0	0	1	0	1
107 – 110	6	8	6	7	3	7
104 – 107	47	50	32	38	31	57
101 – 104	246	265	254	247	270	359
98 – 101	1022	1154	1040	1107	1196	1554
95 – 98	3060	3196	3275	3349	3752	4793
92 – 95	5369	5684	5914	6374	7268	9634
Year of death	'86- '90	'91- '95	'96- '00	'01- '05	'06- '10	'11- '15

A cell contains the number of men in a certain age category who died in a given 5-year period

after normalization, i.e., for some positive function a and all $x > 0$

$$\lim_{t \rightarrow \omega} P((X - t)/a(t) \geq x \mid X \geq t) \text{ exists and is positive.} \quad (1)$$

Here $\omega := \sup\{x \in \mathbb{R} : F(x) < 1\} \leq \infty$ denotes the right endpoint or upper limit of the distribution. Then, for a proper choice of the function a , the limit is (excluding a degenerate distribution) the survival function of a so-called generalized Pareto distribution: $(1 + \gamma x)^{-1/\gamma}$ for some real $\gamma \in \mathbb{R}$, the extreme value index. It is remarkable that this parametric limiting model comes out of a simple continuation principle. A crucial property for our purposes is that if $\gamma < 0$, the endpoint ω of F is finite, see Theorem 1.2.1 in de Haan and Ferreira (2006). Estimators in extreme value theory, hence also those for the endpoint of the distribution, are based only on a set of large order statistics of the life spans.

In the present study, the analysis has been performed separately for each of the 30 years and for women and men. Note that we sort the data according to the year of *death* not birth. In this way, we can compare recent years instead of (birth) years in the 19th century. We see the women/men who died in such a given year as a random sample from the imaginary population of all women/men who could have (been born and who) died in that given year. For women the number k of selected upper order statistics is fixed at 1500 each year and for men at 1000, small fractions of the total. The fractions have been chosen

differently in order to ensure that the ages are comparable, cf. Tables 1 and 2 displaying the age configuration. Such large subsample sizes will lead to rather precise results. The age of the 1500th oldest woman increases from 95.3 to 98.7 years during the 30 years. This corresponds to a three-fold increase in women dying at age 95 or more in 30 years. For men the age of the 1000th oldest increases from 94.5 to 96.0 years.

We develop a test for the null hypothesis that at least one of the 30 extreme value indices is non-negative against the alternative that they are all negative (see Section 3). As we shall see, the null hypothesis is rejected, hence there is clear evidence for an upper limit of the human life span distribution for each of the years considered. In order to answer the last question above about the changes in endpoints over the years we develop a test for equality of the upper limits over the years.

As said, we consider the life spans $X_1, X_2, \dots$ as i.i.d. random variables taken from F (satisfying (1) with a nondegenerate limit). Let the sequence $k = k(n)$ satisfy $k(n) \rightarrow \infty$, $k(n)/n \rightarrow 0$, as $n \rightarrow \infty$. Then the order statistic $X_{n-k(n),n} \xrightarrow{P} \omega$ and, as (1) suggests - given $X_{n-k,n}$ - the random variables $(X_{n-k+1,n} - X_{n-k,n}, X_{n-k+1,n} - X_{n-k,n}, \dots, X_{n,n} - X_{n-k,n})$ behave for large n approximately as the k -th order statistics from the generalized Pareto distribution when scaled. This opens the way to develop pseudo-maximum likelihood estimators $\hat{\gamma}$ for γ and $\hat{a}$ for the function a . Under somewhat stricter conditions it can be proved for these estimators that $\sqrt{k}(\hat{\gamma} - \gamma, \hat{a}(\frac{n}{k})/a(\frac{n}{k}) - 1)$ has asymptotically a normal distribution (Drees et al. (2004)). Moreover, if $\gamma < 0$, the estimator

$$\hat{\omega} := X_{n-k,n} - \frac{\hat{a}(\frac{n}{k})}{\hat{\gamma}}$$

satisfies: $\sqrt{k}(\hat{\omega} - \omega)/a(\frac{n}{k})$ is asymptotically normal (Section 4.5 of de Haan and Ferreira (2006)). These are the estimators we use here.

Several researchers have followed more or less this path using various datasets: Aarssen and de Haan (1994), Watts et al. (2006), Hanayama and Sibuya (2016) and Gbari et al. (2017). Generally the conclusion is: there is a finite limit which is in the range 117-123 years. There are also statistical papers not explicitly using extreme value theory: Weon and Je (2009), finite endpoint, and Gampe (2010), infinite endpoint. Rootzén and Zholud

(2017) is a case in between; there no finite upper limit is found. The latter two papers are based on a recently constructed database of world-wide supercentenarians.

The main contributions of the present paper are

- (Finite Maximum Life Span) that we consider more homogeneous populations (considering only one year of death, one gender and one country) and that we prove through *one* test that the endpoints of the life span distributions for *all* 30 years are finite (both for women and men) based on reliable and precise data, and
- (No Trend) that we find no significant upward, downward, or parabolic trend in these 30 endpoints, although the *average* life span increased rapidly, with about 5 years.

Assuming that F has a density F' , the *force of mortality* at time t is defined as $F'(t)/(1 - F(t))$ (which is in fact the failure or hazard rate). There is a simple connection between the force of mortality and the scale function a from (1):

$$\lim_{t \rightarrow \omega} \frac{F'(t)}{1 - F(t)} \cdot a \left(\frac{1}{1 - F(t)} \right) = 1 \quad \text{for all } \gamma \in \mathbb{R}.$$

Since the function a is regularly varying, this gives an opportunity to extrapolate the force of mortality from high ages to even higher ages, even beyond the observations.

Finally we shall discuss the so-called *perseverance parameter*, which is the percentage of the total possible residual life time at great age that a person is realizing on average. This parameter turns out to be equal to $-\gamma/(1 - \gamma)$.

The results will be discussed in Section 2. Section 3 contains the mathematical details.

2 Results

2.1 Are the endpoints of the life span distributions finite?

We performed a formal (simultaneous) test for the null hypothesis that at least one of the 30 extreme value indices is non-negative against the alternative hypothesis that they are all are negative, with significance level 5%. Recall that a negative extreme value index

implies a finite upper endpoint of the distribution. Therefore we first need to estimate the extreme value indices γ_j , $j = 1, \dots, 30$, for the 30 years and for women and for men. For these (pseudo-maximum likelihood) estimates $\hat{\gamma}_j$, we will choose the sample fraction $k = 1500$ for women and $k = 1000$ for men. Recall that for the asymptotic theory it is required that $k(n) \rightarrow \infty$, $k(n)/n \rightarrow 0$, as $n \rightarrow \infty$. The present values of k are indeed large, whereas the fractions $k(n)/n$ are small (n is about 70,000). We also made plots of $\hat{\gamma}$ as a function of k for various cases and they show that for our choices there is a good balance between variance (small for large k) and squared bias (small for small k). Moreover choosing somewhat different k leads to similar results as below. The estimated extreme value indices are exhibited in Figure 1. Clearly they are all negative, hinting at a finite upper endpoint, i.e., a finite maximum life span.

Figure 1: Estimated extreme values indices

The aforementioned test is based on the maximum of the estimated extreme value indices. Here, for the women this is the estimated value corresponding to the year 2001 and it is equal to -0.089 . The test leads to an asymptotic p -value of $0.0003 (= \Phi(-0.089\sqrt{1500})$ with Φ the standard normal distribution function) establishing that the upper limit of the probability distribution of the human life span of Dutch women is finite for all of the 30 years under consideration. For men, the asymptotic p -value of this test is $0.0055 (= \Phi(-0.080\sqrt{1000}))$ leading to the same conclusion. The mathematical details of the test are discussed in Section 3.

2.2 What is the upper endpoint of the life span distribution?

The results on the endpoints for women and men are displayed separately in Figure 2. There are 4 graphs in each display. The blue graph depicts the estimated upper endpoints $\hat{\omega}_j, j = 1, \dots, 30$, in years. The red graph shows the 95% one-sided upper confidence bound for each year thus indicating the level of uncertainty in the estimation. The average estimated upper endpoint is 115.7 years and the maximum estimated upper endpoint, corresponding to the year 2001, is 123.7 years; the average of the upper confidence bounds is 120.3 years. In addition, the black graph shows the age of the oldest person to die in each year and the grey graph shows the age of the 1500th (for women) oldest person to die in each year. For men we show the 1000th oldest at death. In contrast to the estimated limits of the life span and the annual oldest women (average is 110.0), the ages of the 1500th oldest gradually *increase* from 95.3 to 98.7 years, an increase of about 0.11 year per calendar year. In other words, we see that the probability distribution at high age shifts towards the endpoint, over the years, but that the endpoints themselves do not increase, see the next subsection. The picture for men is similar, but the estimation results are a bit less precise than for women, since the sample fraction k is smaller. For men, the average endpoint estimate is 114.1 and the maximum endpoint estimate is 124.7; the average upper confidence bound is 119.6. The age of the 1000th oldest gradually increases from 94.5 to 96.0 years; the average oldest is 107.6.

Figure 2: Estimated maximum human life span

Observe that the average endpoint estimates for women and men are relatively close (1.6 years in difference only). In contrast, the average age of death during the 30 years that we consider shows a gender difference of more than 5 years.

2.3 Is there a trend in the maximum human life span?

Visual inspection does not reveal any trend in the endpoint estimates for both genders. A “no trend” conclusion would more or less contradict the findings in Dong et al. (2016) where it is claimed that the endpoints increase at first and decrease later. That paper and its method are much contested so we performed classical linear and quadratic regression analyses to investigate this. We found no significant upward, downward, or parabolic trend, for both genders. Hence there are no clear global changes in the maximum human lifespan over the years.

We also developed and applied an asymptotic χ^2 -test for testing the null hypothesis of equality of the upper limits of the distribution (the details are given in Section 3). For Dutch women, the asymptotic p -value is 0.0081 and hence the null hypothesis is rejected at the 5% level: the upper endpoints ω_j , $j = 1, \dots, 30$, are not all equal (but since this inequality of endpoints is not due to a trend, it is more of the oscillations type). For Dutch men the null hypothesis of equal endpoints is not rejected (asymptotic p -value is 0.4270), confirming that there is no trend in the endpoints.

2.4 Force of mortality

The force of mortality $\lambda(t) := F'(t)/(1 - F(t))$, where t represents time, is an instantaneous measure of mortality. It is approximately (for small $\varepsilon > 0$) the probability of dying before time $t + \varepsilon$ given survival until time t , standardized by dividing by ε . This provides a way of estimating λ . Extreme value theory offers another (here more relevant) way to estimate λ using the asymptotic relation between λ and the scale function a from (1):

$$\lim_{t \rightarrow \omega} \lambda(t) \cdot a\left(\frac{1}{1 - F(t)}\right) = 1 \quad \text{for all } \gamma \in \mathbb{R}.$$

(This relation follows for negative γ immediately from combining (1.1.37) and (1.2.14) in de Haan and Ferreira (2006).) In particular, if $k(n) \rightarrow \infty$, $k(n)/n \rightarrow 0$ as $n \rightarrow \infty$,

$$\lambda\left(F^{-1}\left(1 - \frac{k}{n}\right)\right) \cdot \hat{a}\left(\frac{n}{k}\right) \xrightarrow{P} 1 \quad \text{and} \quad \lambda(X_{n-k,n}) \cdot \hat{a}\left(\frac{n}{k}\right) \xrightarrow{P} 1 \quad \text{as } n \rightarrow \infty,$$

i.e., the force of mortality at the time of death of the $(k + 1)$ st oldest, can be estimated by $1/\hat{a}(\frac{n}{k})$ where $\hat{a}(\frac{n}{k})$ is provided by the pseudo-maximum likelihood procedure. Note that the scale function a is regularly varying with index γ (follows from Lemma 1.2.9 in de Haan and Ferreira (2006)). Hence for $s > 0$ we define $\hat{\lambda}(F^{-1}(1 - \frac{ks}{n})) = s^{\hat{\gamma}}/\hat{a}(\frac{n}{k})$. For example when taking $s = 1/k$ we obtain the “theoretical” time of death of the oldest and find

$$\hat{\lambda}\left(F^{-1}\left(1 - \frac{1}{n}\right)\right) = \frac{k^{-\hat{\gamma}}}{\hat{a}(\frac{n}{k})}.$$

This leads to *average* (over the 30 years) values of $\hat{\lambda}(F^{-1}(1 - \frac{1}{n}))$, in *days*, of 0.0031 for women and 0.0029 for men. Observe that when measuring time in days, the force of mortality is approximately the probability of dying the coming day.

Alternatively, we can use relation (1.1.37) of de Haan and Ferreira (2006) directly: for $\gamma < 0$ $\lim_{t \rightarrow \omega} \lambda(t)(\omega - t) = -1/\gamma$, showing that the force of mortality increases quickly when approaching ω . We can estimate $\lambda(t)$ with

$$\hat{\lambda}(t) = \frac{1}{-\hat{\gamma}(\hat{\omega} - t)} \quad \text{for any age } t < \hat{\omega}.$$

When taking $t = \hat{\omega} - 1$ years, this yields an *average* estimated force of mortality, in *days* again, of 0.0206 for women and 0.0197 for men.

2.5 Perseverance parameter

If the extreme value index is negative, the maximum life span is finite. For a person of high age t there is a maximum residual life time $\omega - t$. We are interested in the percentage of this maximum residual life time a person is living on average. The extreme value condition implies that this percentage stabilizes at high age (cf. Lemma A.4 in Aarssen and de Haan (1994)):

$$\lim_{t \rightarrow \omega} E\left(\frac{X - t}{\omega - t} \mid X > t\right) = \frac{-\gamma}{1 - \gamma}.$$

We call this limit α , the perseverance parameter. Using $\hat{\alpha} = -\hat{\gamma}/(1 - \hat{\gamma})$ we find an *average* $\hat{\alpha}$ of 0.1208 for women and 0.1271 for men. Observe that α can also be seen as an approximation of the mean residual life time at the very high age of $\omega - 1$ years. The just

obtained average estimates lead to estimated mean residual life times at this age of about 44 and 46 *days*, respectively.

3 Mathematical details of the two tests

3.1 Testing whether all extreme value indices are negative

Here we assume that the life spans for each year of death j are independent and identically distributed.

The null hypothesis to be tested is: for at least one $j = 1, \dots, 30$, $\gamma_j \geq 0$ against the alternative that all $\gamma_j < 0$; the significance level is 0.05, say. The test statistic is $\max_{1 \leq j \leq 30} \hat{\gamma}_j$. Assume the null hypothesis holds and $\gamma_{j_0} \geq 0$. From Drees et al. (2004): if $k(n) \rightarrow \infty$ and $\sqrt{k}A_{j_0}(\frac{n}{k}) \rightarrow 0$ as $n \rightarrow \infty$ (where A_{j_0} is the second order norming function), then $\sqrt{k}(\hat{\gamma}_{j_0} - \gamma_{j_0})/(1 + \gamma_{j_0})$ is asymptotically standard normal. Now, for $c = \Phi^{-1}(0.05) = -1.645$,

$$\begin{aligned} P(\sqrt{k} \max_{1 \leq j \leq 30} \hat{\gamma}_j \leq c) &= P(\bigcap_{j=1}^{30} \{\sqrt{k} \hat{\gamma}_j \leq c\}) \\ &\leq P(\sqrt{k} \hat{\gamma}_{j_0} \leq c) \leq P(\sqrt{k}(\hat{\gamma}_{j_0} - \gamma_{j_0})/(1 + \gamma_{j_0}) \leq c), \end{aligned} \quad (2)$$

for $\sqrt{k} + c \geq 0$, which is the case for $k \geq 3$. The right-hand-side of (2) converges to $\Phi(-1.645) = 0.05$ as $n \rightarrow \infty$. Hence the critical region is $(-\infty, -1.645/\sqrt{k}]$. Similarly the asymptotic p -value is equal to $\Phi(v\sqrt{k})$ where v is the value of the test statistic.

3.2 Testing whether all endpoints are equal

Here we assume, in addition, that the life spans from different years of death are also independent of each other, i.e., that we have independent random samples.

In general for $\gamma < 0$: if $k(n) \rightarrow \infty$ and $\sqrt{k}A(\frac{n}{k}) \rightarrow 0$ (with A the second order norming function) as $n \rightarrow \infty$, then $\sqrt{k}(\hat{\omega} - \omega)/\hat{a}(\frac{n}{k}) \xrightarrow{d} N(0, \sigma^2(\gamma))$, as $n \rightarrow \infty$, with $\sigma^2(\gamma) = 2 + 2\gamma^{-1} + 5\gamma^{-2} + 4\gamma^{-3} + \gamma^{-4}$, see pp. 146 and 147 in de Haan and Ferreira (2006).

For our setup this implies

$$\left(\frac{\sqrt{k}(\hat{\omega}_j - \omega_j)}{\hat{a}_j(\frac{n}{k})\sigma(\hat{\gamma}_j)} \right)_{j=1}^{30} \xrightarrow{d} (Z_j)_{j=1}^{30}, \quad \text{as } n \rightarrow \infty,$$

with $Z_1, \dots, Z_{30}$ independent standard normal random variables. Writing $d_j = k/(\hat{a}_j(\frac{n}{k})\sigma(\hat{\gamma}_j))^2$, $d = \sum_{j=1}^{30} d_j$, and $r_j = d_j/d$, this can be written as

$$\left(\sqrt{dr_j}(\hat{\omega}_j - \omega_j) \right)_{j=1}^{30} \xrightarrow{d} (Z_j)_{j=1}^{30}. \quad (3)$$

The null hypothesis to be tested is $\omega_1 = \dots = \omega_{30}$, against the alternative that the ω_j are not all equal. We use the test statistic

$$T = d \cdot \sum_{j=1}^{30} r_j (\hat{\omega}_j - \tilde{\omega})^2,$$

with $\tilde{\omega} = \sum_{j=1}^{30} r_j \hat{\omega}_j$. Now assume $r_j \xrightarrow{P} \rho_j$, for certain constants $\rho_j, j = 1, \dots, 30$. Under H_0 , we have $T = d \sum_{j=1}^{30} r_j (\hat{\omega}_j - \omega)^2 - d(\tilde{\omega} - \omega)^2$, with ω the common value of the ω_j . Then by (3) the limit in distribution of T is $\sum_{j=1}^{30} Z_j^2 - \left(\sum_{j=1}^{30} \sqrt{\rho_j} Z_j \right)^2$. Since $\sum_{j=1}^{30} \rho_j = \sum_{j=1}^{30} r_j = 1$, i.e., $(\sqrt{\rho_1}, \dots, \sqrt{\rho_{30}})$ is a unit vector, this limiting random variable is well-known to have a χ_{29}^2 -distribution. [In fact the condition $r_j \xrightarrow{P} \rho_j$ can be weakened to $r_j - \rho_{j,n} \xrightarrow{P} 0$ for deterministic sequences $\rho_{j,n}$.]

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