Tilburg University

Transitional impacts of environmental policy in an endogenous growth model

Bovenberg, A.L.; Smulders, J.A.

Published in:
International Economic Review

Publication date:
1996

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):
Bovenberg, A. L., & Smulders, J. A. (1996). Transitional impacts of environmental policy in an endogenous growth model. *International Economic Review*, 37(4), 861-893.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

TRANSITIONAL IMPACTS OF ENVIRONMENTAL POLICY IN AN ENDOGENOUS GROWTH MODEL*

BY A. LANS BOVENBERG AND SJAK A. SMULDERS¹

To explore the link between a tighter environmental policy and economic growth, we employ an endogenous growth model with endogenous technological progress in abatement technologies. The environment, which is modelled as a renewable resource, acts both as a public consumption good and as a public input into production. The entire transition towards a new balanced-growth path after a tightening of environmental policy is computed analytically. We find sharp contrasts between short-run and long-run effects. Whereas the level and the growth rate of output may decline in the short run, income growth may improve in the long run.

1. INTRODUCTION

How environmental policy affects economic growth is a controversial issue. Industrialists typically argue that environmental policy hurts capital accumulation and growth by raising abatement costs.² Environmentalists, in contrast, maintain that environmental policy is needed in order to ensure that growth is sustainable.³ Four factors are crucial in understanding the link between growth and the environment: technological progress, ecological processes (and their interaction with the economy), time, and environmental policy. This paper presents an endogenous growth model that pays particular attention to these four key factors.

Most of the literature on environmental policy and economic growth (see, e.g., Jorgenson and Wilcoxon 1990, van der Ploeg and Withagen 1991, and Tahvonen and Kuuluvainen 1993) assumes that technological progress is exogenous and thus not affected by environmental policy. The experience of industrial countries, however, suggests that tighter environmental policies induce major technological advances in abatement technologies (see Ashford et al. 1985, and Lanjouw and Mody

* Manuscript received July 1994; final revision September 1995.

¹ We would like to thank two anonymous referees for their constructive comments on an earlier draft.

² Jorgenson and Wilcoxon (1990) and Hazilla and Kopp (1990) provide estimates of abatement costs in the United States and the impact of these costs on capital accumulation and growth. Jorgenson and Wilcoxon (1990) found that, over the period 1974–1985, abatement costs associated with environmental regulation reduced the average growth rate by 0.2 percentage points. In a general equilibrium context, the overall social costs of environmental regulation typically exceed direct abatement costs because regulations tend to reduce output, investment, and productivity (see also Schmalensee 1994, and Jaffe et al. 1995).

³ Empirical evidence suggests that pollution may seriously hurt growth by reducing productivity. For example, in an empirical study for Mexico, Margulis (1992) finds that pollution seriously harms labor productivity and the ability to learn (see also van Ewijk and van Wijnbergen 1995). Pearce and Warford (1993) provide a detailed account of productivity losses due to pollution in several major countries.

1993). Indeed, technological progress enhancing the productivity of natural resources plays a crucial role in reconciling sustained growth with a constant level of environmental quality. This paper, therefore, does model technological advances in environmentally friendly technologies by drawing on the recent literature on endogenous growth. In doing so, we assume that knowledge about abatement technologies is a pure public good because these anti-pollution technologies typically involve general principles (e.g., how to convert energy into motion). In the resulting model, the long-term growth rate depends on environmental policy. This is in contrast to models with exogenous technologies, in which environmental policy exerts only temporary effects on growth.

To assess the growth effects of environmental policy, we model the major economic and ecological functions performed by the natural environment. In particular, the environment acts as a sink for wastes generated by economic activities and as a source of natural inputs into production. Moreover, as a public consumption good, the environment yields amenities. Finally, it constitutes a life support system: environmental quality determines nature's capacity to grow and affects living and working conditions. In this latter capacity, environmental quality acts as a public production factor by enhancing the productivity of man-made inputs into production. Indeed, empirical evidence suggests that pollution causes serious productivity losses (see, Alfsen et al. 1992, Pearce and Warford 1993, Ballard and Medema 1993, Brendemoen and Vennemo 1994, and van Ewijk and van Wijnbergen 1995). Despite this evidence, most studies dealing with the impact of environmental policy on growth (e.g., Jorgenson and Wilcoxon 1990, Hazilla and Kopp 1990, Huang and Cai 1994) ignore the adverse effect of pollution on productivity. We show that the growth effects of a tighter environmental policy depend crucially on whether the benefits of a cleaner environment accrue in the form of amenities or higher productivity of nonenvironmental inputs. If the environment acts mainly as a public consumption good, a reduced pollution level inevitably hurts the productivity of man-made production factors, thereby depressing growth. However, if the environment plays an important role as a public input into production, the enhanced quality of the environment boosts productivity, thereby offsetting the adverse growth effect due to lower pollution.

Environmental policy faces severe informational and political constraints in setting the optimal level of environmental quality. In view of these difficulties, governments typically set ceilings (so-called standards) for the overall pollution level that are not necessarily optimal (see Baumol and Oates 1988, Chapter 11). In fact, it is widely believed that current environmental policies are still not sufficiently tight to fully internalize pollution externalities, in part because information about the serious nature of these externalities has become available only recently.⁴ Accordingly, we consider an economy that tightens its environmental standards because it starts off with an excessive level of pollution. Our analysis is thus closer to the literature on policy reform than to that on optimal policies (e.g., Ligthart and van der Ploeg 1994, and van Marrewijk, van der Ploeg and Verbeek 1993).

⁴ The arrival of new information explains why countries have tightened their environmental policy in recent years.

Another important aspect of the link between growth and environmental quality is the time factor. Whereas the costs of a tighter environmental policy occur in the short run, the benefits typically arise only in the long run because it takes time for nature to recover and for new abatement technologies to be developed. Several papers, including Ligthart and van der Ploeg (1994) and Gradus and Smulders (1993), abstract from stock-flow dynamics in both ecology and abatement technology by modelling both environmental quality and abatement as flow variables,⁵ implying that transitional dynamics are absent. We, in contrast, model environmental quality as a stock variable because the major environmental problems feature an important stock-flow dimension. By reducing pollution, society can invest in environmental quality, which can be interpreted as natural capital. In addition to environmental quality, abatement technology is modelled as a stock. Society accumulates this stock by investing in research and development (R&D).

Whereas most endogenous growth models dealing with environmental issues restrict the analysis to steady states (see Xepapadeas 1993, and Bovenberg and Smulders 1995), we analytically compute the entire dynamic adjustment path towards a balanced growth path. Our analysis of the transition reveals that short-run impacts may contrast quite sharply with long-run effects. For example, environmental policy may hurt short-run growth but enhance growth in the long run. Furthermore, the analysis of the dynamic adjustment path enables us to perform welfare analysis. In particular, we make explicit the trade-off between short-run costs and long-term benefits of environmental policy.

In analytically computing the transition, this paper contributes also to the literature on endogenous growth. Almost all this literature deals only with long-run impacts, thereby ignoring the adjustment path towards a new balanced-growth path. Exceptions are Chamley (1993), Mulligan and Sala-i-Martin (1993), and Cabbelé and Santos (1993), who analyze the transition in endogenous-growth models with two types of capital. We extend their analysis to models with three assets, namely, not only physical capital and knowledge, but also natural capital. We find that the incorporation of three types of capital gives rise to nontrivial dynamics. In particular, the adjustment path for particular variables may be nonmonotonic as a result of the interaction between differential adjustment speeds of the ecological and production systems. In contrast to Chamley (1993) and Cabbelé and Santos (1993), we explore shocks that affect the long-term growth rate. Furthermore, whereas Mulligan and Sala-i-Martin (1993) resort to numerical simulations, we solve the transition analytically by employing the linearization method for computing comparative dynamics developed by Judd (1982). We extend this method to endogenous growth models and to welfare functions with nonstationary arguments.

The rest of the paper is structured as follows. Following a discussion of the main features of the model in Section 2, Section 3 describes the initial balanced growth path. Section 4 explains how the comparative dynamics of an unanticipated tightening of environmental policy can be solved analytically. Section 5, the main body of the paper, provides the analytical solutions for welfare and the comparative dynamics of a number of major variables, including consumption, income, and the

⁵ This is in the tradition of Barro (1990), who models public infrastructure as a flow.

production share of the R&D sector. Finally, Section 6 concludes. An Appendix contains the main mathematical derivations.

2. THE MODEL

2.1. *Ecology.* The natural environment performs several important functions. In particular, environmental quality determines nature's capacity to grow, features an amenity value, and affects the living and working conditions in the economy. Moreover, it absorbs wastes from economic activity and provides natural inputs into production.

To model these various functions, the quality of the environment (N , e.g., the cleanliness of the air, and of the soil) is assumed to be a renewable resource:

$$(1) \quad \dot{N} = E(N) - P \quad \partial^2 E / \partial N^2 < 0$$

The term $E(N)$ represents natural growth through regeneration processes, which can be thought of as reproduction of natural resources. The services from ecosystems and solar energy inflow serve as inputs in regeneration processes. If the reproduction exceeds the resources that are needed to maintain the existing ecosystems (in the form of food and other energy), the ecosystem (or environmental quality) expands. Hence, $E(N)$ can be seen as the difference between natural resource reproduction and resource use for maintenance. Since the inflow of solar energy is fixed, the regeneration process features diminishing returns (which is an implication of the entropy law from physics). The resource needs for maintenance, in contrast, increase more or less proportionally with the stock of resources. As a result, the larger the stock of environmental quality becomes, the more difficult it is to regenerate the complete stock. Thus, natural capital accumulation features diminishing returns (i.e., $\partial^2 E / \partial N^2 < 0$).

The second term at the right-hand side of (1) indicates that environmental quality deteriorates through the extractive use of natural resources in production. In particular, production requires inputs that pollute the environment (e.g., pesticides in agriculture, fossil fuels resulting in emissions of carbon) or that directly harvest nature (e.g., water, wildlife, fish, wood). This extractive use of the environment is called pollution and will be denoted by P . The pollution-assimilating and self-generation capacities of the ecological system enable the environment to digest a certain flow of pollution without deteriorating in quality. Hence, the environment allows for a sustainable flow of pollution, which matches the biological regeneration rate of the ecological system. If the flow of pollution is relatively small (but constant), natural regeneration and growth processes outweigh the adverse effects of pollution on the ecological system so that environmental quality gradually rises over time. Due to the diminishing returns in natural regeneration, environmental quality eventually stabilizes at a long-run equilibrium level $\bar{N}(P)$. As a consequence, a long-run negative relationship exists between, on the one hand, the flow of pollution and, on the other hand, the stock of environment quality.⁶

⁶ Since $E(N)$ is concave, there may be two levels of environmental quality N for which $\dot{N} = 0$; one with $E_N > 0$ and low N and one with $E_N < 0$ and high N . However, with a constant level of pollution, only the equilibrium with $E_N < 0$ is stable. Hence, we focus on the latter equilibrium.

2.2. *Production.* Empirical evidence shows that environmental quality enhances the productivity of inputs into production by providing nonextractive services (see Margulis 1992, Alfson et al. 1992, Pearce and Warford 1993, Ballard and Medema 1993, Brendemoen and Vennemo 1994, and van Ewijk and van Wijnbergen 1995). Examples of these services are the carrier services of the environment, which provide physical and mental support to productive activities (e.g., the impact of soil and air quality on productivity in the agricultural sector, the effect of air quality on mental health, labor productivity, and physical depreciation of equipment). In providing these nonextractive services, the environment is a nonrival good. We capture these nonrival services by incorporating aggregate environmental quality N as an input into production.

The other inputs into production are rival, man-made production factors (i.e., private capital K) and rival inputs extracted from nature, which are measured in efficiency units (Z). Private capital should be interpreted broadly. It includes various kinds of private capital that can be produced. Accordingly, it includes not only physical but also human capital. No production is feasible without the extractive use of the environment (i.e., pollution P). However, pollution-augmenting technological progress is possible; the larger the stock of knowledge about abatement technologies, the more productive inputs extracted from nature become. In particular, by denoting the stock of pollution-augmenting technical knowledge by h and the flow of polluting inputs by P , we can represent the extractive use of the environment in efficiency units by $Z = hP$.

In addition to private capital (K), environmental technology capital (h) can be accumulated by devoting resources to investment. In this way, the model incorporates endogenous technological advances in abatement technologies. Private capital K is produced according to the same technology as consumption goods in the Y -sector (which we also call the 'final-goods sector'). A separate sector (the H -sector, which we also call the '(environmental) R&D sector') develops abatement technologies. Hence, the technology side of the model is described by two production functions corresponding to the two production sectors and two resource constraints corresponding to the two rival inputs:

$$(2) \quad Y = A(N) \cdot F(K_Y, Z_Y) = C + \dot{K}$$

$$(3) \quad H = A(N) \cdot G(K_H, Z_H) = \dot{h}$$

$$(4) \quad K_Y + K_H \leq K$$

$$(5) \quad Z_Y + Z_H \leq Z \equiv hP$$

The subscripts that are attached to the rival inputs denote the sector in which these inputs are employed. Total factor productivity A depends positively on N to capture nature's nonrival services. $F(\cdot)$ and $G(\cdot)$ feature constant returns with respect to the two rival inputs. The elasticities of substitution between capital and effective pollution in the two production sectors (denoted by σ_Y and σ_H , respectively) are constant and smaller than one. This implies that (effective) pollution is an essential input, as production cannot take place without pollution. Finally, we assume, rather

realistically, that the final-goods sector is pollution-intensive, so that the R&D sector is capital intensive.⁷

2.3. *Preferences.* The representative household cares about not only produced consumption commodities but also environmental amenities (denoted by c and N , respectively). The following utility function represents household preferences:

$$(6) \quad W = \int_0^{\infty} e^{-\theta t} U(c(t), N(t)) dt$$

$$(7) \quad U(c, N) = \left(\frac{\sigma_c}{\sigma_c - 1} \right) (c \cdot N^{\phi})^{1-1/\sigma_c}$$

The relative weight of environmental amenities in utility, $(\partial U / \partial N)N / (\partial U / \partial c)c$, equals the constant ϕ , which enables us to parameterize environmental preferences. The constant elasticity of intertemporal substitution σ_c is assumed to be smaller than one, as commonly found in empirical studies.

2.4. *Saving and Investment.* Producers maximize profits by equalizing the marginal return on capital K to the rate of return on capital markets, r

$$(8) \quad A \frac{\partial F}{\partial K_Y} = r$$

Arbitrage in the capital market ensures that capital earns the same return in both sectors:

$$(9) \quad \frac{\partial F}{\partial K_Y} = q_h \frac{\partial G}{\partial K_H}$$

Consumers optimize intertemporal utility by choosing a consumption path such that the return on an additional unit of postponed consumption coincides with the marginal utility of current consumption. This yields the well-known Keynes-Ramsey rule:

$$(10) \quad \vartheta - \frac{\partial \dot{U}}{\partial c} \bigg/ \frac{\partial U}{\partial c} = r$$

⁷ Hence, $\beta > \alpha$ where β (α) denotes the (variable) output share of private capital in the H-sector (α , Y-sector): $\alpha \equiv (\partial Y / \partial K_Y) \cdot K_Y / Y$, $\beta \equiv (\partial H / \partial K_H) \cdot K_H / H$.

Note that, in contrast to the standard Ramsey model, the change in marginal consumption depends not only on the change in consumption, but also on the change in amenities over time. In particular, the last term on the left-hand side of (10) can be written as $-(1/\sigma_c)\dot{c}/c + \phi(1 - 1/\sigma_c)\dot{N}/N$.

2.5. Public Policy. The environment acts both as a public input into the two production sectors (N enters the production functions, thereby enhancing the productivity of man-made inputs) and a nonrival consumption good (N enters utility functions). Also knowledge about pollution-augmenting technologies (h) is nonrival, because the abatement technologies can be applied by several firms at the same time. The nonrival nature of both technology and the environmental quality, which contrasts with the rival nature of private capital K and (effective) pollution Z , gives rise to externalities and hence calls for government intervention.

Without government intervention, pollution would grow without bound because producers would fail to take into account the adverse effects of pollution on the ecological system. To prevent this, the government limits aggregate pollution. In practice, governments face serious difficulties in setting the optimal level of economy-wide pollution because they lack sufficient information on environmental benefits from pollution reduction. We acknowledge these difficulties by assuming that policymakers set a pollution standard in a suboptimal manner. In particular, rather than being determined in an optimal fashion, the time path of the pollution standard is constant. This is observed in practice as emissions trading, which typically specifies a fixed pollution level for a number of years (see Hahn 1989). Moreover, in the initial equilibrium, the standard allows for an excessive level of pollution from a social welfare point of view. This reflects the widely shared belief that pollution externalities are currently not fully internalized. Moreover, it provides a rationale for the recent tightening of environmental policy in many countries, in part in response to new information about the serious nature of pollution damages. The fact that pollution is excessive in the initial equilibrium is formalized by the following inequality:⁸

$$(11) \quad \left(\frac{\partial U}{\partial N} \bigg/ \frac{\partial U}{\partial c} + (F + q_h G) \frac{\partial A}{\partial N} \right) \frac{1}{\tau_P} + \frac{\dot{\tau}_P}{\tau_P} + \frac{\partial E}{\partial N} > r$$

where q_h and τ_P denote, respectively, the relative price of abatement technologies supplied by the R&D sector and the marginal productivity of pollution in the final goods sector (i.e., $\tau_P \equiv A(\partial F/\partial Z_Y)h$).⁹ The left-hand side represents the benefits

⁸ This inequality is derived from the expression for the optimal level of environmental quality N . We derive this latter expression by maximizing welfare (6) subject to (1)–(5) and (7). Denoting by θ_K , θ_h and θ_N the costate variables associated to K , h and N , we derive the optimality condition for N : $\partial U/\partial N + (\partial A/\partial N)(\theta_K F + \theta_h G) + \theta_N \partial E/\partial N = \vartheta \theta_N - \dot{\theta}_N$. Combining the other optimality conditions with (10) and the definition $\tau_P \equiv A(\partial F/\partial Z_Y)h$, we find $\theta_N/\theta_K = \tau_P$, $\theta_K = \partial U/\partial c$, $\vartheta - \dot{\theta}_N/\theta_N = r - \dot{\tau}_P/\tau_P$. Substituting these results and defining $q_h = \theta_h/\theta_K$, we arrive at (11).

⁹ We take the price of the output from the Y-sector as the numeraire. Hence, τ_P and q_h are measured in terms of consumption goods (or private capital goods).

from an additional unit of natural capital (N) in terms of final goods. These benefits consist of additional amenities ($\partial U/\partial N$), additional nonrival productive services in both sectors ($\partial A/\partial N$), a scarcity rent ($\dot{\tau}_P/\tau_P$), and improvements in nature's absorption capacity ($\partial E/\partial N$). Amenity and productivity gains are evaluated relative to the cost of foregone production due to the change in pollution, that is, the marginal product of a polluting input τ_P . Inequality (11) thus indicates that the social returns from investing in environmental quality (through reducing pollution) exceed the social returns on alternative investments (i.e., the right-hand side of (11)). Accordingly, society would gain from investing in natural capital by marginally reducing pollution.

In implementing the exogenous pollution ceiling, the government employs cost-effective environmental policy instruments so that the pollution standard is attained with minimum costs.¹⁰ This implies that the marginal value of pollution, τ_P , is the same in both production sectors. Depending on the nature of the adopted environmental policy instrument, this marginal value can be interpreted as, alternatively, the price of pollution permits, the level of a pollution tax, or the shadow value of efficient regulation.¹¹

$$(12) \quad A \frac{\partial F}{\partial Z_Y} h = q_h A \frac{\partial G}{\partial Z_H} h = \tau_P$$

In addition to environmental quality, also knowledge is a nonrival good: a particular idea can be applied by various firms at the same time. Specific knowledge, while nonrival, may be excludable by means of patents so that it can be provided through private incentives. General knowledge, in contrast, is much more difficult to appropriate. Knowledge about environmentally friendly technologies typically belongs to this latter category because it generally involves general principles (e.g., energy-efficiency improvements, new principles how to convert energy into motion). Accordingly, new abatement technologies are assumed to be nonexcludable. In order to induce the R&D sector to produce these technologies, the government provides a subsidy.¹² To ensure that pollution abatement is achieved in a cost-effective way, the government sets the level of the subsidy (i.e., price at which

¹⁰ This is in the tradition of Baumol and Oates (1988, Chapter 11).

¹¹ Public revenues from these environmental policy instruments are returned to households in a lump-sum fashion.

¹² Romer (1990) and Grossman and Helpman (1991) allow technological change to arise from market incentives by assuming that knowledge is in part excludable through patents. In particular, R&D yields patentable new ideas that are sold to monopolistic producers. Knowledge spillovers imply that in these models research should be subsidized, albeit less than fully. For an application of this approach to environmental innovation, see Hung et al. (1992), and Verdier (1993).

An advantage of our approach is that the assumption of perfect competition can be maintained. Hence, we can abstract from distortions in the goods market and thus focus on environmental distortions. The interaction between the two types of externalities is left for future research.

the output of the R&D sector is sold), q_h , so that the social return on investment in abatement technology equals the return on alternative investments r .¹³

$$(13) \quad A \frac{\partial G}{\partial Z_H} P + \frac{\dot{q}_h}{q_h} = r$$

The left-hand side of (10) shows that the return on abatement technology consists of the current return in production and a capital gains term.

2.6. *Aggregation.* In order to explore economy-wide economic growth, we define the aggregate values (in terms of consumption goods) of man-made assets, national product, and national income as follows:

$$(14) \quad q_M M = K + q_h h$$

$$(15) \quad q_J J = Y + q_h H = C + \dot{K} + q_h \dot{h}$$

$$(16) \quad Y + q_h H = rK + \tau_P P$$

where M and J denote aggregate man-made assets and aggregate output, respectively, and q_M and q_J represent the corresponding price deflators in terms of consumption goods. From the supply side, national product amounts to the aggregate output produced by the two production sectors (see the first equality in (15)). From the demand side, it corresponds to the three spending categories, namely consumption, investment in private capital, and investment in environmental R&D (see the second equality in (15)). Equation (16) follows from constant returns in both production sectors.

3. BALANCED GROWTH

The particular specification of the utility and production functions discussed in Section 2 guarantees that the long-run equilibrium is a balanced-growth path.¹⁴ On a balanced-growth path, the resource shares allocated to both production sectors are

¹³ The subsidies are financed through lump-sum taxation.

¹⁴ The sufficient conditions for balanced endogenous growth are the following: (i) the production functions in both production sectors exhibit constant returns to scale with respect to the augmentable factors capital and effective pollution; (ii) intertemporal utility features a constant elasticity of intertemporal substitution, σ_c ; and (iii) intratemporal utility features a unitary elasticity of substitution between environmental amenities and consumption of produced commodities. See Bovenberg and Smulders (1995) for an extensive discussion of these conditions.

constant (i.e., K_H/K and Z_H/Z are constant over time). The man-made capital stocks (i.e., K, h) and the consumption of produced goods, c , grow at a common positive growth rate (denoted by g). In contrast, the stock of natural capital, N , and the extractive use of the environment, P , are constant over time. Hence, long-run growth in output is ecologically sustainable. Moreover, the long-run growth rate depends on preferences, technology, and environmental policy, and is thus endogenous.

Compared to man-made capital, natural capital and polluting inputs become increasingly scarce as time elapses. In particular, on a balanced-growth path, the shadow price (in terms of the consumption good) of pollution, τ_P , increases at the growth rate g .

National income, man-made wealth, and aggregate consumption (C) are related in the following way on a balanced-growth path:¹⁵

$$(17) \quad q_J J / q_M M = r$$

$$(18) \quad C / q_M M = r - g$$

Given the arbitrage conditions for abatement capital (10) and private capital (11), the current return on man-made assets in terms of output amounts to the rate of interest (see (17)). Expression (18) reveals that the part of permanent income that remains after investment, $(r - g)q_M M$, is consumed.

The share of pollution-augmenting knowledge (h) in the aggregate value of man-made assets, denoted by ω , and the share of national product generated in the environmental R&D sector are given by:

$$(19) \quad \frac{q_h h}{q_M M} = \frac{\tau_P P}{q_J J} = \left(\frac{(1 - \alpha)r}{r + (\beta - \alpha)g} \right) \equiv \omega$$

$$(20) \quad \frac{q_h H}{q_J J} = \left(\frac{(1 - \alpha)g}{r + (\beta - \alpha)g} \right) \equiv \frac{\omega g}{r}$$

According to the first equality in (19), the asset share of polluting-augmenting knowledge capital equals the production share of pollution because both P and h raise effective pollution, Z , and thus share the same productivity.¹⁶ Since the final-goods sector is pollution-intensive (i.e., $\beta > \alpha$), the production share of pollu-

¹⁵ Appendix A derives the steady-state ratios in (17)–(20).

¹⁶ $\tau_P P / q_J J$ is labeled the share of pollution in production. If the government uses revenue-raising market instruments for environmental policy (pollution charges or auctioned permits), it equals the expenditure share on pollution. If nonrevenue-raising regulation is applied, however, it represents a rent share as part of production accounted for by pollution accrues to firms as rent.

tion in that sector, $1 - \alpha$, exceeds the corresponding aggregate share, ω . This is especially so if the rate of growth is large compared with the rate of return. In that case, investment is important compared with consumption. Hence, the capital-intensive R&D sector, which is a pure investment sector, becomes more important compared with the pollution-intensive final-goods sector, which also produces consumer goods.

Expression (20) indicates that the income share of the R&D sector is large if pollution is an important input into production (i.e., $1 - \alpha$ large and β small). Comparison of (19) and (20) reveals that the share of environmental knowledge in man-made assets (i.e., $q_h h / q_M M$) exceeds the output share of the environmental R&D sector (i.e., $q_h H / q_J J$). The reason for the relatively small output share of the R&D sector is that consumption goods are produced only in the final-goods sector.

As described in Section 2.5, the initial balanced-growth path features excessive pollution. Accordingly, the social costs of pollution exceed the social benefits. This difference between costs and benefits is measured by the gap between the social rate of return on investment in environmental quality (r^N) and the return on private capital (r). In the steady state, this gap can be expressed as:

$$(21) \quad r^N - r = \left[\frac{P}{N} \left(\phi \frac{r - g}{r} + a_N \right) \frac{1}{\omega} + g + \frac{\partial E}{\partial N} \right] - r > 0$$

where $a_N \equiv (\partial A / \partial N) \cdot N / A$ denotes the elasticity of factor productivity with respect to natural capital. The left-hand side of (21) can be interpreted as an implicit, distortionary subsidy wedge on pollution: pollution is provided at too low a price.

Expression (21) reveals that the welfare distortions due to excessive pollution are serious if the benefits from additional environmental capital are large compared with the benefits of a larger flow of polluting inputs. In particular, more investment in natural capital yields substantial net benefits if natural capital is valuable in utility and production (i.e., ϕ and/or a_N are large) and if pollution yields only a small contribution to production (i.e., the share of pollution in output, ω , is low).

4. THE LINEARIZED DYNAMIC SYSTEM

We employ the model to examine the effects of narrowing the gap between the social costs and benefits of pollution through an unanticipated, permanent reduction in the ceiling for economy-wide pollution. This tightening of the pollution standard allows for an analytical solution of the entire transition path by log-linearizing the model around the initial balanced-growth path. The log-linearized model can be reduced to the four dynamic equations displayed in Table 1.¹⁷ Tildes denote relative changes from the initial steady state, so $\tilde{x} \equiv dx/x$. From now on, non-tilded variables refer to initial steady-state values. The associated elasticities are explained in the note to the table. The policy shock we consider is an exogenous reduction in pollution at time $t = 0$: $\tilde{P}(t) = \tilde{P} < 0$ for $t > 0$.

The linearized model is saddle-point stable. Two stable roots determine how rapidly the economy converges to a new long-run equilibrium with balanced growth.

¹⁷ The Appendix provides the derivation and solution of the log-linearized model.

TABLE 1
THE DYNAMIC SYSTEM*

$$\begin{aligned}
 & \begin{bmatrix} \dot{N} \\ \dot{\bar{q}} \\ \dot{\bar{C}/M} \\ \dot{\bar{K}/h} \end{bmatrix} \\
 &= \begin{bmatrix} -\delta & 0 & 0 & 0 \\ 0 & \left(\frac{1-\alpha+\beta}{\beta-\alpha}\right)r & 0 & 0 \\ (1-\sigma_c)(\phi\delta-a_Nr) & \sigma_cr\left(\frac{1-\alpha}{\beta-\alpha}\right) & r-g & 0 \\ \left(\frac{r-g}{1-\omega}\right)a_N & \frac{1}{\beta-\alpha}\left(\frac{\beta g}{1-\omega}\sigma_H - \frac{r+(\beta-\alpha)g}{\beta-\alpha}\sigma_u\right) & -\left(\frac{r-g}{1-\omega}\right) & -\left(\frac{r+(\beta-\alpha)g}{\beta-\alpha}\right) \end{bmatrix} \\
 &\times \begin{bmatrix} \bar{N} \\ \bar{q}_h \\ \bar{C}/M \\ \bar{K}/h \end{bmatrix} + \begin{bmatrix} -\delta\varepsilon \\ -r \\ -\omega r + (1-\sigma_c)\phi\delta\varepsilon \\ \frac{r+(\beta-\alpha)g}{\beta-\alpha} + \frac{\omega r - g}{1-\omega} \end{bmatrix} \bar{P}
 \end{aligned}$$

* $a_N = (\partial A/\partial N) \cdot N/A$, $\alpha = (\partial Y/\partial K_Y) \cdot K_Y/Y$, $\beta = (\partial H/\partial K_H) \cdot K_H/Y$, $\delta = -\partial E/\partial N$, $\varepsilon = -P/(\partial E/\partial N)N$, $\sigma_u = (K_Y/K)\sigma_Y + (K_H/K)\sigma_H$, ω see (19), ϕ and σ_c see (7).

We can solve the model recursively. First, we solve for natural capital and the so-called aggregate variables, which describe the behavior of the aggregate economy. These latter variables include the rate of return, the growth rates of consumption, man-made assets, and aggregate output. Finally, we determine the solution for the relative variables. These variables involve the allocation of economic activity over the two production sectors and include the capital-knowledge ratio, K/h , the shares of capital and effective pollution allocated to the final-goods sector, and the output share of the R&D sector. The behavior of the aggregate variables during the transition is characterized by only one of the stable roots, $\delta \equiv -\partial E/\partial N$, which can be interpreted as the adjustment speed of the ecological system. In particular, the ecological system converges slowly to a new long-run equilibrium if a better environmental quality exerts only small adverse effects on the absorption capacity of nature (see also Section 5.1).

The aggregate variables evolve monotonically over time, as their behavior is described by only three elements: the short-term (or initial) effect, the long-term

effect, and the rate of convergence, δ :

$$(22) \quad \tilde{x}_m(t) = \tilde{x}_m(0)e^{-\delta t} + \tilde{x}_m(\infty)(1 - e^{-\delta t})$$

where x_m can be read as $N(t)$, $r(t)$, $q_h(t)$, $g_c(t)$, $g_M(t)$, or $g_J(t)$, where g_x denotes the growth rate of variable x .

In contrast to the aggregate variables, the relative variables (including K/h , K_Y/K , Z_Y/Z) may feature nonmonotonic behavior; their behavior during the transition depends not only on the speed of adjustment of the ecological system, but also on that of the two-sector production block. In particular, these variables develop according to:

$$(23) \quad \tilde{x}_n(t) = \tilde{x}_n(0)e^{-\lambda t} + \tilde{x}_n(\infty)(1 - e^{-\lambda t}) + T_{\tilde{x}_n} \left(\frac{e^{-\delta t} - e^{-\lambda t}}{\lambda - \delta} \right)$$

where

$$(24) \quad \lambda \equiv \frac{r + (\beta - \alpha)g}{\beta - \alpha}$$

Just like the initial effect $\tilde{x}_n(0)$ and the long-run effect $\tilde{x}_n(\infty)$, $T_{\tilde{x}_n}$ is solved as a function of the exogenous shock to the system. If $T_{\tilde{x}_n}$ is zero (or if $\lambda = \delta$), $\tilde{x}_n$ evolves monotonically at rate λ . The parameter λ , which is the second stable root of the linearized system, amounts to the speed of adjustment of the two-sector production system. The production system adjusts more quickly if the outputs of the two production sectors are more 'similar' (as reflected in a smaller difference between β and α).

The term $T_{\tilde{x}_n}$ introduces the possibility of nonmonotonic behavior. It reflects temporary effects due to the interaction between the economic system (with adjustment speed λ) and the ecological system (with adjustment speed δ). If $T_{\tilde{x}_n}$ is positive, $\tilde{x}_n$ is temporarily higher compared with the situation in which it converges monotonically to its long-run value at rate λ . If $T_{\tilde{x}_n}$ is negative, it is temporarily lower.

5. COMPARATIVE DYNAMICS

This section investigates the analytical solutions for the comparative dynamics of a number of major economic variables. To identify the economic intuition behind our analytical results, we analyze two specific cases in more detail. In the first case, besides as a source and sink, the environment acts only as a public consumption good so that the environmental benefits accrue entirely in the form of environmental amenities (i.e., $a_N = 0$). In the second case, the environment is a pure public production factor so that benefits take the form of only beneficial productivity effects (i.e., $\phi = 0$). Figure 1 compares these two special cases by providing the time path of the main variables corresponding to particular numerical examples.

FIGURE 1

ENVIRONMENT AS CONSUMPTION GOOD VERSUS ENVIRONMENT AS CAPITAL GOOD.

5.1. *The Quality of the Environment.* To find the solution path for the quality of the environment, we linearize (1) (see also the first row of Table 1):

$$(25) \quad \dot{\tilde{N}} = (\partial E / \partial N) \tilde{N} - (P/N) \tilde{P}$$

Solving this equation, we find that a lower flow of pollution gradually enhances the stock of natural capital:

$$(26) \quad \tilde{N}(t) = (1 - e^{-\delta t}) \tilde{N}(\infty)$$

$$(27) \quad \tilde{N}(\infty) = -\varepsilon \tilde{P}$$

where $\varepsilon \equiv -P/(\partial E/\partial N)N$. Figure 1a illustrates the time path of environmental quality N . The stock of natural capital monotonically approaches its steady-state value at the rate $\delta \equiv -\partial E/\partial N$. Adjustment is relatively slow if a higher stock of natural capital exerts a relatively small adverse impact on the growth of natural capital. This is likely to be so if the economy starts off with a relatively low level of environmental quality. In that case, the long-run beneficial impact is large, whereas the approach towards this new long-run equilibrium with improved environmental quality is slow.

5.2. *Welfare.* The welfare effect is given by (see Appendix Section F):

$$(28) \quad \tilde{W}(0) = \left(\frac{r}{r-g} \right) \omega \tilde{P} + \left(\frac{r}{r-g} \right) \eta a_N \tilde{N}(\infty) + \eta \phi \tilde{N}(\infty)$$

where

$$(29) \quad \eta \equiv \delta / (r - g + \delta)$$

The first term represents the costs of investing in the environment, namely the decline in production associated with a lower flow of polluting inputs. We call this first term the *crowding-out effect*: production is crowded out by the additional investment in natural capital in the form of a lower pollution flow. The adverse crowding-out effect becomes more important, the larger is the input share of effective pollution (ω , see (19)).

The other two terms at the right-hand side of (28) represent the benefits of investing in the environment. The second term stands for the benefits of a cleaner environment as a public production factor, that is, the favorable productivity effects. Since these productivity gains accrue only gradually, the long-run productivity term is weighted by η , which can be interpreted as the weight of long-run effects for overall-welfare. In particular, the steady-state effects weigh heavily if the adjustment speed of the ecological system, δ , is large relative to the net discount rate, $(r - g)$. In that case, the long run approaches rapidly and is thus not discounted heavily.

The third term at the right-hand side of (28) represents the benefits of the environment as a public consumption good. In particular, it measures the increase in

permanent imputed income from additional environmental amenities. This term increases in ϕ , which measures the importance of environmental amenities in overall utility. Also environmental amenities accrue only gradually. Hence, like the second term, the third term involves the factor η .

Substituting (26) and (27) into (28) to eliminate the relative change in environmental quality, we find:

$$(30) \quad \tilde{W}(0) = -(r^N - r) \left(\frac{r}{r - g} \right) \left(\frac{\omega}{r - g + \delta} \right) \tilde{P}$$

The gain in welfare is thus proportional to the initial distortionary subsidy to pollution, $r - r^N$, defined in (21).

Welfare can be decomposed into two components: first, discounted consumption of produced commodities and, second, discounted environmental amenities. Whereas a tighter environmental policy unambiguously boosts the second component, it may reduce the first one so that society may face a trade-off between the two welfare components. It reaps a double dividend (i.e., not only more amenities but also more discounted consumption from produced commodities) if

$$(31) \quad \eta a_N \varepsilon > \omega.$$

This condition for a double dividend states that the beneficial productivity effects of a larger stock of natural capital should exceed the crowding out effects of investing in natural capital. For this condition to be met, the input share of pollution in production (ω) should be small, the productivity effects of environmental quality in production (a_N) should be large, environmental quality should be rather sensitive to a reduction in pollution (i.e., ε large, see (27)), and, finally, the long-run effects should approach rapidly and should not be discounted heavily (i.e., η large, see (29)).

If the environment is a pure production factor, environmental amenities are absent. In that case, overall welfare consists only of discounted consumption of produced goods. Accordingly, inequality (31) must be met for welfare to improve. Productivity effects are zero if the environment is a pure consumption good ($a_N = 0$). In that case, a tighter environmental policy yields only a single dividend because discounted consumption declines.

5.3. Consumption. The effect of environmental policy on the path of consumption, which is a nonstationary variable, is fully described by the initial jump in the level of consumption and the impact on the growth rate of consumption. As an aggregate variable, the growth rate of consumption develops monotonically through time (see Section 4 and, in particular, expression (22)). Hence, its behavior is captured by the long-run and short-run growth effects and the rate of convergence δ (discussed in Section 5.1). This sub-section first investigates the effects on, respectively, the long-run growth rate, the short-term growth rate of consumption, and the initial jump in the level of consumption. At the end of this sub-section, we use these results to explore the entire consumption path.

A. *Long-term growth.* In the long run, consumption, income, and man-made assets grow at the same rate. The (steady-state) Keynes-Ramsey rule (10) directly relates the long-term growth impact to the long-term effect on the rate of return (see Appendix Section B):

$$(32) \quad \left(\frac{g}{\sigma_c r} \right) \bar{g}(\infty) = \bar{r}(\infty) = \left(\frac{1 - \alpha}{1 - \alpha + \beta} - a_N \varepsilon \right) \bar{P}.$$

The long-run effect on growth is ambiguous. Growth rises in the long run if the productivity effect is important (i.e., a_N is large) and if pollution plays only a minor role in production (i.e., β and α are large). In that case, the positive productivity impact of a higher stock of natural capital dominates the crowding-out effect.

If the environment is a pure consumption good (i.e., $a_N = 0$), growth unambiguously declines. If the environment is a pure production factor, however, long-run growth improves.¹⁸ This contrast illustrates the importance of distinguishing between the consumption-good and capital-good features of the environment.

B. *Short-run consumption growth.* The short-run growth rate of consumption follows from the (out-of-steady state) Keynes-Ramsey rule (10), which reads in linearized form

$$(33) \quad g\bar{g}_c = -\phi\dot{N} + \sigma_c \left[r\bar{r} + \phi\dot{N} \right].$$

Environment as a Pure Production Factor: If the environment is a pure *production factor* ($\phi = 0$), the rate of return on capital markets is the only determinant of the growth rate of consumption. The initial impact on the rate of return is given by (see Appendix Section C):

$$(34) \quad \bar{r}(0) = \left(\frac{1 - \alpha}{1 - \alpha + \beta} \right) \bar{P}$$

The rate of return declines in the short run due to the crowding-out effect. The return declines substantially if pollution is an important input into production (i.e., α and β are small). Due to the drop in the initial rate of return, consumption growth slows down in the short run. However, after its initial downward jump, the rate of return and thus the growth rate of consumption recover. In the long run, both the rate of return and the growth rate exceed their initial steady-state values (see Section 5.3.a, c.f. Figure 1b). Thus, we find a sharp contrast between the short-run and long-run effects on consumption growth. Hence, steady-state results are a poor indicator for short-term effects.

Environment as a Pure Consumption Good: If the environment is a pure *consumption good* (i.e., $a_N = 0$), the short-run growth rate of consumption depends not only

¹⁸ We can write (32) as: $\bar{r}(\infty) = -(\bar{P}/\eta)[\eta a_N \varepsilon - \omega \eta \lambda / (r - g + \lambda)]$, where we have used $(1 - \alpha)/(1 - \alpha + \beta) = \omega \lambda / (r - g + \lambda)$, and $q_h H / q_J J$ in (20). The expression in square brackets is positive if the inequality in (31) is satisfied (since $\lambda > 0$, $\eta < 1$, see (24) and (29)).

on the rate of return but also on the change in environmental quality (see (33)). The first term in (33) represents utility smoothing. With rising environmental quality, consumption of produced goods can decline if consumers desire to maintain a constant stream of utility over time. Indeed, the first term stands for *intra*temporal substitution between private commodity consumption and environmental amenities. The second term stands for *inter*temporal substitution. If $\sigma_c > 0$, households accept lower current streams of utility in return for (sufficiently) higher future streams of utility. The terms in square brackets represent the overall return on postponing utility governing this trade-off: the physical rate of return on saving (r) enables households to consume more produced consumption goods in the future, while the rise in environmental quality allows consumers to benefit from a higher marginal utility on consumption of produced commodities.

With the intertemporal substitution elasticity being smaller than unity, a rising level of environmental quality reduces the growth rate of consumption. The reason is that *inter*temporal substitution is more difficult than *intra*temporal substitution.¹⁹ Intuitively, as a result of utility smoothing, households find it attractive to move their consumption of commodities forward in anticipation of a higher future quality of the environment. Hence, amenity effects reinforce the drop in short-run consumption growth due to the fall in the rate of return.

C. The initial consumption level. The short-run jump in the level of consumption is given by (see Appendix Section E):

$$(35) \quad \tilde{c}(0) = \tilde{W}(0) - \sigma_c \left[\left(\frac{r}{r-g} \right) [(1-\eta)\tilde{r}(0) + \eta\tilde{r}(\infty)] + \eta\phi\tilde{N}(\infty) \right].$$

The first term represents the positive income effect due to the gain in welfare. The second term stands for intertemporal substitution. If the future rate of return is expected to be high, households shift their consumption away from the present towards the future. The intertemporal substitution elasticity governs the sensitivity of intertemporal consumption decisions with respect to movements in the return.

The term in long square brackets summarizes the rate of return on postponing consumption. Since the capital-market return, r , develops monotonically, its time path is characterized by three elements, the short-run effect, the long-run effect, and the speed of adjustment from the short run to the long run. The weight of the long-run impact is given by η (defined in (29)). The last term in square brackets represents the intertemporal substitution effects associated with higher environmental amenities in the future. In particular, the anticipation of more future environmental amenities raises the marginal utility of future consumption, thereby raising the overall return on delaying consumption.

¹⁹ Whereas the intertemporal substitution elasticity is smaller than one, the intra-temporal elasticity of substitution equals unity (see footnote 14).

Expression (35) for the initial jump in consumption can be written as:

$$(36) \quad \tilde{c}(0) = (1 - \sigma_c)\tilde{W}(0) + \sigma_c \left(\frac{(\beta - \alpha)r}{r + (\beta - \alpha)g} \right) \tilde{r}(0).$$

Consumption increases in most cases because income effects dominate intertemporal substitution effects (note that $\sigma_c < 1$). Only if the intertemporal substitution elasticity is close to one and the final-goods sector is much more pollution-intensive than the environmental R&D sector (i.e., $\beta - \alpha$ is large), may short-run consumption decline. Intuitively, if consumption is relatively pollution-intensive, the production of investment goods is relatively pollution-extensive. Hence, compared with income, the rate of return benefits substantially from a tighter environmental policy. Therefore, the intertemporal substitution effect associated with a higher return may offset the income effect, especially if the intertemporal allocation of consumption is quite sensitive to the rate of return.

D. The overall consumption path. Figure 1c contains typical consumption paths following a tightening of environmental policy. The dotted line represents the consumption path in case the environment is a pure consumption good (i.e., $a_N = 0$). In that case, consumption typically jumps up on impact, reflecting the gain in overall welfare. However, the growth rate of consumption drops sharply as the quality of the environment gradually improves. Although the growth rate recovers as the growth rate of environmental amenities declines, the long-run growth rate remains below its initial level.

The unbroken line stands for a typical consumption path if the environment is a pure production factor (i.e., $\phi = 0$). Also in this case consumption rises on impact, reflecting the increase in welfare. Moreover, just as in the consumption-good case, the growth rate declines on impact and gradually recovers. However, the long-run growth rate exceeds its initial steady-state value. In this case, the level of consumption may exceed its level on the initial growth path at each point in time.

5.4. Income and Saving. Income is a nonstationary variable. Like the consumption path, the development of income is thus described by four elements: the initial-level effect, the short-term growth effect, the long-term growth effect (see (32)), and the adjustment speed δ (see Section 5.1). Since the latter two elements have been discussed above, this sub-section explores the short-run effects on the level and growth of income before it investigates the overall income path.

A. The initial income level. The stocks of man-made and natural assets are fixed in the short run. Despite the fixed capital base, national income declines initially because of the crowding-out effect, due to less extractive use of the environment (i.e., a lower flow of pollution) (see Appendix C):

$$(37) \quad \tilde{J}(0) = \omega \tilde{P}.$$

The crowding-out effect becomes more important if pollution accounts for a large input share (i.e., ω large).

B. *Short-run income growth.* The growth rate of national product is determined by the growth of both man-made and natural assets (see Appendix Section D):

$$(38) \quad \tilde{g}_J(0) = \tilde{g}_M(0) + \left(\frac{a_N}{g} \right) \dot{N}(0).$$

The right-hand side of (38) can be interpreted as a broad measure for aggregate saving because it includes saving in the form of not only man-made capital but also natural assets.

The short-run growth rate of man-made assets, which can be viewed as saving as conventionally defined, is affected by the initial changes in national product and consumption (see Appendix Section D):

$$(39) \quad \begin{aligned} g\tilde{g}_M(0) &= r\tilde{J}(0) - (r - g)\tilde{c}(0) \\ &= -\eta[ra_N + (r - g)\phi]\tilde{N}(\infty) \\ &\quad + \sigma_c[(1 - \eta)r\tilde{r}(0) + \eta r\tilde{r}(\infty) + \eta(r - g)\phi\tilde{N}(\infty)]. \end{aligned}$$

The first term at the last right-hand side of expression (39) indicates that consumption smoothing hurts saving in the form of man-made assets; in anticipation of future productivity and amenity gains, households consume more, thereby reducing the accumulation of man-made assets. Hence, large welfare gains as a result of a higher stock of environmental capital widen the contrast between reduced growth of man-made assets in the *short* run and increased growth of these assets in the *long* run. The second term in square brackets at the last right-hand side of (39) represents intertemporal substitution.

By substituting (34) and (32) into (39) to eliminate, respectively, the short-run and long-run effects on the rate of return, we arrive at:

$$(40) \quad g\tilde{g}_M(0) = -(1 - \sigma_c)\eta[ra_N + (r - g)\phi]\tilde{N}(\infty) + \sigma_c r \left(\frac{1 - \alpha}{1 - \alpha + \beta} \right) \tilde{P}.$$

A tighter environmental policy harms the accumulation of produced assets in the short run. Intuitively, the economy adjusts its asset portfolio away from man-made capital towards natural assets. In particular, the accumulation of man-made assets is hurt by the consumption boom on account of both intertemporal utility smoothing and intertemporal substitution associated with a lower initial rate of return.

Substituting the growth rates of man-made and natural assets (from (40) and (26), respectively) into (38), we find for the initial growth rate of income:

$$(41) \quad \begin{aligned} g\tilde{g}_J(0) &= \eta(\delta - g + \sigma_c r)a_N\tilde{N}(\infty) \\ &\quad + \sigma_c r \left(\frac{1 - \alpha}{1 - \alpha - \beta} \right) \tilde{P} - (r - g)\phi(1 - \sigma_c)\eta\tilde{N}(\infty). \end{aligned}$$

C. *The overall income path.* Together with long-run income growth (see (32)), the adjustment speed δ , and the initial level effect (see (37)), expression (41) characterizes the adjustment path for income. Figure 1d contains typical adjustment paths for income following a tightening of environmental policy.

Environment as a Pure Consumption Good: Long-run growth declines if the environment is a public consumption good (i.e., $a_N = 0$, see Section 5.3.a). Short-run growth declines even more because households raise consumption and hence reduce saving in anticipation of future amenity gains. Hence, following the initial drop in income, the gap between income on the initial balanced-growth path and income on the new path widens. Intuitively, households rely more on environmental amenities and less on consumption of produced goods as a source of welfare.

Environment as a Pure Production Factor: If the environment is a pure production factor (i.e., $\phi = 0$), income growth improves not only in the long run but also in the short run if the rate with which natural capital accumulates exceeds the income growth rate on the initial balanced-growth path (i.e., $\delta > g$).²⁰ Intuitively, the accumulation of natural capital more than offsets slower accumulation of man-made capital.

Growth may decline in the short run, however, if the improvement in environmental quality occurs only slowly.²¹ In particular, if intertemporal substitution is small and the initial growth rate exceeds the rate of convergence of the ecological system (i.e., $g > \delta$), growth declines initially. Intuitively, natural capital accumulates slowly compared with man-made capital, while a small intertemporal substitution elasticity reduces the intertemporal substitution effects associated with a higher long-run return. The case with a decline in short-run growth is especially interesting because of the sharp contrast between long-run and short-run impacts on growth. In particular, although a tighter environmental policy may raise welfare and long-run growth, such a policy harms short-run growth.

5.5. *The Knowledge Intensity of Production.* The overall capital-intensity of production, K/h , indicates the direction of technological progress. Alternatively, it can be viewed as an indicator for the composition of man-made assets and, in particular, for the relative size of private versus public capital. The solution for the capital-knowledge ratio can be written as (see Appendix G):

$$(42) \quad \bar{K}(t) - \bar{h}(t) = (\bar{K}(\infty) - \bar{h}(\infty))(1 - e^{-\lambda t}) + T_{\bar{K}-\bar{h}} \left(\frac{e^{-\delta t} - e^{-\lambda t}}{\lambda - \delta} \right)$$

²⁰ With $\phi = 0$, we may write (41) as (use (26)):

$$g\bar{g}_J(0) = \eta(\delta - g)a_N\bar{N}(\infty) - \sigma_c r[\eta a_N \varepsilon - \omega]\bar{P} - \sigma_c r[\omega - (1 - \alpha)/(1 - \alpha + \beta)]\bar{P}.$$

The terms in square brackets are positive (see (31) and (19)). Hence, the entire expression is positive if $\delta > g$.

²¹ This is likely to be the case if the initial quality of the environment is small (see also the discussion at the end of Section 5.1). An alternative interpretation of a small value for δ is that the time scale of ecological systems is large compared with that of economic systems.

where

$$(43) \quad \bar{K}(\infty) - \bar{h}(\infty) = \bar{P} + \sigma_u \left(\frac{-\bar{P}}{1 - \alpha + \beta} \right) + \left(\frac{(\beta - \alpha)g}{\alpha r + (\beta - \alpha)g} \right) \left[\beta(1 - \sigma_H) \left(\frac{-\bar{P}}{1 - \alpha + \beta} \right) + \left(\frac{\vartheta}{r - \vartheta} \right) \bar{r}(\infty) \right]$$

$$(44) \quad T_{\bar{K}-\bar{h}} = \frac{r - g}{1 - \omega} \left[\left(\frac{\delta + \sigma_c \vartheta}{r - g + \delta} \right) a_N + \eta(1 - \sigma_c) \phi \right] \varepsilon \bar{P}$$

$$(45) \quad \sigma_u \equiv u \sigma_Y + (1 - u) \sigma_H$$

where $u \equiv K_Y/K = \alpha[r - (1 - \beta)g]/[\alpha r + (\beta - \alpha)g]$.

Environment as a Pure Consumption Good: If the environment does not improve productivity, the long-run capital-knowledge ratio declines for three reasons. First, compared with capital, polluting-saving knowledge is a better substitute for pollution because the substitution elasticities between capital and effective pollution are smaller than unity. Accordingly, abatement technologies rather than private capital act as the most important substitute for pollution. Second, the reduction in the economy-wide flow of pollution harms output in the pollution-intensive sector—the final-goods sector producing private capital—the most. Hence, the accumulation of private capital rather than that of abatement technologies is most heavily damaged by a more ambitious environmental policy. Third, without productivity effects, the rate of return and hence the growth rate falls. This harms activity in the pure ‘investment’ sector—the environmental R&D sector. Since this sector is capital-intensive, the aggregate demand for capital and hence the capital-knowledge ratio declines.

The beneficial effects on amenities impact the transition path but leave the long-run equilibrium unaffected. In particular, amenity effects imply a more rapid initial accumulation of pollution-augmenting technologies relative to private capital. The reason is that amenity effects cause households to consume more initially. This provides a boost to the consumption-goods industry and, since this sector is pollution-intensive, also to the development of pollution-augmenting technologies. Later on, as these effects reverse themselves, consumption becomes less important compared with investment. The associated move of resources towards the pure investment sector (i.e., the environmental R&D sector), which is capital-intensive, stimulates the accumulation of private capital relative to that of pollution-augmenting knowledge and hence slows down the upward trend in the knowledge-capital ratio. In fact, the knowledge-intensity of production may develop nonmonotonically if powerful amenity effects produce a large enough temporary consumption boom (i.e., σ_c small and ϕ large so that $T_{\bar{K}-\bar{h}}$ has a large (negative) value), see Figure 1e.

Environment as a Pure Production Factor: If a better environmental quality improves productivity, the long-run decline in the capital-knowledge ratio is reduced.

In fact, the capital–knowledge ratio may rise rather than fall on the new balanced-growth path. Intuitively, productivity effects raise the rate of return and the growth rate. This draws activity away from the consumption-goods sector towards the pure ‘investment’ sector (the R&D sector). Since this latter sector is capital-intensive, the overall economy relies more on capital, as reflected in a higher capital–knowledge ratio.

The positive impact of the productivity effects on capital intensity can cause nonmonotonic behavior of the capital–knowledge ratio during the transition (see Figure 1e). With a low initial rate of return, the forces stimulating the development of pollution-augmenting technologies (discussed above in the case that the environment is a consumption good) dominate. As time elapses, however, the accumulation of natural capital causes the rate of return and the level of output to recover. This stimulates investment and saving and hence the environmental R&D sector. Since this sector is capital-intensive, the capital–knowledge ratio increases during the later phases of the transition.

5.6. The Production Structure. A tighter environmental policy changes the production structure in terms of the relative size of the two production sectors. This section explores the income share of the environmental R&D sector, $q_h H/q_J J$, which can be interpreted as the public sector, since it is financed with public subsidies. If the substitution elasticities in both sectors are equal (i.e., $\sigma_Y = \sigma_H = \sigma_u$), we can write the impact on this share as follows (see Appendix Section H):

$$(46) \quad \overline{\left(\frac{q_h H}{q_J J}\right)} = \left(1 - \frac{\omega g}{r}\right) \left[\left(\frac{1}{\nu - u}\right) (\tilde{K} - \tilde{h}) + \left((1 - \sigma_u) \left(\frac{1}{\nu - u} - (\beta - \alpha) \right) + \frac{\beta - \alpha}{\nu - u} \right) \left(\frac{-\tilde{P}}{1 - \alpha + \beta} \right) \right]$$

where $\nu \equiv Z_Y/hP = 1 - (1 - \beta)g/r$, and u is defined in (45).

The initial impacts are found by setting the relative change in the capital–knowledge ratio equal to zero ($\tilde{K} - \tilde{h} = 0$). Hence, environmental policy expands the size of the public sector in the short run. The reason is that the decline in pollution makes effective pollution scarcer compared with the other input into production—private capital. This reduces the size of the pollution-intensive sector (the final-goods sector) but boosts the pollution-extensive R&D sector.

The development of the sectoral allocation of resources through time reflects the adjustment of the aggregate capital–knowledge ratio (see 46)). By substituting the long-run solution for the capital–knowledge ratio, we find for the long-run impact:

$$(47) \quad \overline{\frac{q_h H}{q_J J}}(\infty) = \left(1 - \frac{\omega g}{r}\right) \left[\left(\alpha(1 - \sigma_Y) + \beta \frac{1 - \nu}{\nu} (1 - \sigma_H) \right) \frac{-\tilde{P}}{1 - \alpha + \beta} + \sigma_c \left(\frac{\vartheta}{\nu g} \right) \tilde{r}(\infty) \right].$$

Environment as a Pure Consumption Good: Following the initial move towards the environmental R&D sector, resources start to return to the final-goods sector. The reason can be found in the stock-flow dynamics of investments in pollution-augmenting technologies: the investments in R&D gradually expand the stock of pollution-augmenting technologies and thus the economy-wide ratio of effective pollution to capital. The larger relative supply of effective pollution boosts the pollution-intensive sector (final-goods sector). Moreover, in the absence of productivity effects, the rate of return declines. This harms the environmental R&D sector, which is a pure investment sector, and pulls resources towards the consumption-goods sector.

Expression (47) reveals that, despite the recovery of the share of resources allocated to final-goods sector, the R&D sector remains relatively more important on the new balanced-growth path compared with the situation in the initial equilibrium if the intertemporal substitution elasticity and the substitution elasticity in production between effective pollution and capital are small. Intuitively, to substitute for pollution, the economy invests mainly in pollution-augmenting technologies instead of either investing in private capital (which would happen if intrasectoral substitution would be easy) or consuming (which would happen if intertemporal substitution would be difficult).

Although amenities affect neither short-run nor long-run effects, they may result in a temporary shift of resources into the consumption-goods sector. The reason is that households shift their consumption forward in anticipation of improved future amenities. When the consumption boom peters out, resources move back into the pure investment sector—the R&D sector. Hence, the relative size of this sector may develop in nonmonotonic way (see Figure 1f).

Environment as a Pure Production Factor: If a better environmental quality enhances long-run productivity, investment becomes more attractive in the new steady state. Hence, in the long run, more resources are drawn into the environmental R&D sector, which is the pure investment sector, thereby boosting the income share of the public sector.

During the transition, the income share of the public sector may develop in a nonmonotonic way, mirroring the nonmonotonic behavior of the capital-knowledge ratio. In particular, in the short run, more resources are allocated to the pollution-extensive R&D sector. However, the share of resources allocated to environmental R&D gradually declines, as the accumulation of pollution-augmenting technologies favors the pollution-intensive final-goods sector. Eventually, however, the accumulation of natural capital boosts the rate of return. This stimulates saving and investment, thereby increasing the income share of the pure investment sector, the R&D sector.

6. CONCLUSIONS

Using an endogenous growth model, this paper analytically computed the transition from an initial equilibrium with excessive pollution towards a new steady state with a tighter environmental policy. The results indicate that the production sector may adjust in a nonmonotonic fashion. Initially, more resources are devoted to

developing abatement technologies. As more of these technologies become available, resources move out of the environmental R&D sector towards the pollution-intensive consumption-goods sector. However, over time, the economy becomes less oriented towards consumption and more towards investment. This pushes resources back into the R&D sector.

We found that the growth effects of a tighter environmental policy depend crucially on whether the environmental benefits accrue in the form of amenities (in the form of a public consumption good) or in the form of higher productivity of the production processes (in the form of a public production factor). If the environment acts mainly as a public consumption good, a tighter environmental policy reduces growth in both the long and the short run. In this case, society faces a trade-off between more produced consumption goods and a higher quality of the environment.

If the environment acts mainly as a public investment good, long-run growth rises as a result of a more ambitious environmental policy. Anticipating long-run productivity gains, households raise short-run consumption—despite a decline in initial output. Accordingly, saving declines, thereby harming the accumulation of produced assets. If the accumulation of natural capital occurs relatively slowly, the reduced accumulation of man-made assets hurts short-run income growth. However, if the ecological system adjusts rapidly, short-run growth rises as a result of rapid accumulation of natural capital. In that case, following an initial drop, output quickly recovers and relatively quickly exceeds its initial steady-state path.

*CentER, Tilburg University CPB, Netherlands Bureau for Economic Policy Analysis, and OCFEB, Erasmus University, The Netherlands
Tilburg University, The Netherlands*

APPENDIX

A. *Steady-State Ratios* (Section 3). First we derive the steady-state ratios (in terms of r , g , ϑ , σ_c , ϕ , β , α , and δ) given in Section 3. Note that (4) and (5) apply with an equality sign. In the steady state, $\dot{q}_h = 0$, $\dot{K} = gK$, and $H = gh$.

Equation (13) implies $(1 - \beta)ghP/Z_H = r$ in the steady state, which yields after substitution of (5):

$$(A.1) \quad \frac{Z_Y}{Z} = \frac{r - (1 - \beta)g}{r} \equiv \nu.$$

Condition (12) reads in the steady state $(1 - \alpha)(Y/Z_Y)h = q_h(1 - \beta)(H/Z_H)h = \tau_P$ which yields after substitution of (5) and (A.1):

$$(A.2) \quad \frac{\tau_P P}{Y} = \frac{1 - \alpha}{\nu} = \frac{(1 - \alpha)r}{r - (1 - \beta)g},$$

$$(A.3) \quad \frac{q_h H}{Y} = \frac{1 - \alpha}{1 - \beta} \frac{1 - \nu}{\nu} = \frac{(1 - \alpha)g}{r - (1 - \beta)g}.$$

After substitution of (4), the steady state versions of conditions (8) and (9) can be written as $\alpha(Y/K)(K/K_Y) = q_h \beta(H/K)/(1 - K_Y/K) = r$. This implies the following steady state ratios:

$$(A.4) \quad \frac{K_Y}{K} = \alpha \left(\frac{r - (1 - \beta)g}{\alpha r + (\beta - \alpha)g} \right) \equiv u,$$

$$(A.5) \quad \frac{Y}{K} = r \left(\frac{r - (1 - \beta)g}{\alpha r + (\beta - \alpha)g} \right).$$

The steady-state share of knowledge in man-made assets follows from $H = gh$ and (14):

$$(A.6) \quad \frac{q_h h}{q_M M} = \frac{(q_h H/Y)}{(q_h H/Y) + g(K/Y)}.$$

Substitution of (A.3) and (A.5) into (A.6) yields the second equality in (19).

Using (14) and $Y = C + gK$ (see (2)), we can write the steady-state consumption to man-made asset ratio as:

$$(A.7) \quad \frac{C}{q_M M} = \left(\frac{C}{K} \right) \left(\frac{K}{q_M M} \right) = \left(\frac{Y}{K} - g \right) \left(1 - \frac{q_h h}{q_M M} \right).$$

Substitution of (A.5) and (19) into (A.7) yields (18).

Using (15), $H = gh$, and (14), we can find the income to man-made asset ratio:

$$(A.8) \quad \frac{q_J J}{q_M M} = \frac{q_h H}{q_M M} + \left(\frac{Y}{K} \right) \left(\frac{K}{q_M M} \right) = \left(\frac{q_h h}{q_M M} \right) g + \frac{Y}{K} \left(1 - \frac{q_h h}{q_M M} \right).$$

Substitution of (A.5) and (19) yields (17).

The share of the environmental R&D sector in total production in (20) follows straightforwardly from $H = gh$, (17) and (19).

The share of pollution in production can be written as the ratio in (A.2) multiplied by one minus the ratio in (20) (see (15)), so that we find that $\tau_P P/q_J J = \omega$ as stated in (19).

B. The Rate of Return ((32), (34)). Loglinearizing (9), the left-hand side equality in (12), (8), and (13), we arrive at, respectively:

$$(B.1) \quad \frac{1 - \alpha}{\sigma_Y} (\tilde{Z}_Y - \tilde{K}_Y) = \tilde{q}_h + \frac{1 - \beta}{\sigma_H} (\tilde{Z}_H - \tilde{K}_H),$$

$$(B.2) \quad -\frac{\alpha}{\sigma_Y} (\tilde{Z}_Y - \tilde{K}_Y) = \tilde{q}_h - \frac{\beta}{\sigma_H} (\tilde{Z}_H - \tilde{K}_H),$$

$$(B.3) \quad \tilde{r} = a_N \tilde{N} + \left(\frac{1 - \alpha}{\sigma_Y} \right) (\tilde{Z}_Y - \tilde{K}_Y),$$

$$(B.4) \quad \dot{\tilde{q}}_h = r\tilde{r} - r \left[a_N \tilde{N} - \frac{\beta}{\sigma_H} (\tilde{Z}_H - \tilde{K}_H) + \tilde{P} \right].$$

From (B.1) and (B.2), we derive:

$$(B.5) \quad \frac{1}{\sigma_Y} (\tilde{K}_Y - \tilde{Z}_Y) = \frac{1}{\sigma_H} (\tilde{K}_H - \tilde{Z}_H),$$

$$(B.6) \quad \frac{1}{\sigma_Y} (\tilde{K}_Y - \tilde{Z}_Y) = - \left(\frac{1}{\beta - \alpha} \right) \tilde{q}_h.$$

Substituting (B.3), (B.5), and (B.6) into (B.4), we find the second dynamic equation in Table 1:

$$(B.7) \quad \dot{\tilde{q}}_h = r \left(\frac{1 - \alpha + \beta}{\beta - \alpha} \tilde{q}_h - \tilde{P} \right).$$

Under our assumption that $\beta > \alpha$, the equation is unstable. Consequently, q_h jumps immediately to its steady-state value:

$$(B.8) \quad \tilde{q}_h(t) = \left(\frac{\beta - \alpha}{1 - \alpha + \beta} \right) \tilde{P}.$$

The rate of return follows from (B.3) and (B.6):

$$(B.9) \quad \tilde{r} = a_N \tilde{N} + \left(\frac{1 - \alpha}{\beta - \alpha} \right) \tilde{q}_h.$$

Substituting (25) and (B.8) we arrive at the right-hand side equality in (32) and at (34). The first equality in (32) follows from (33) and (26).

C. *Income* ((37), (38)). The production functions in the left-hand side equalities in (2) and (3) are linearized as:

$$(C.1) \quad \tilde{Y} = a_N \tilde{N} + \alpha (\tilde{K}_Y - \tilde{Z}_Y) + \tilde{Z}_Y,$$

$$(C.2) \quad \tilde{H} = a_N \tilde{N} + \beta (\tilde{K}_H - \tilde{Z}_H) + \tilde{Z}_H.$$

Aggregate income is a weighted average of the output of both sectors; see (15) and (20):

$$(C.3) \quad \tilde{J} = \left(\frac{Y}{q_J J} \right) \tilde{Y} + \left(\frac{q_h H}{q_J J} \right) \tilde{H} = \left(1 - \frac{\omega g}{r} \right) \tilde{Y} + \frac{\omega g}{r} \tilde{H}.$$

Aggregate private capital and effective pollution are weighted averages of the sectoral input levels, see (4) and (5) with equality sign, (A.1) and (A.4):

$$(C.4) \quad \tilde{K} = u \tilde{K}_Y + (1 - u) \tilde{K}_H,$$

$$(C.5) \quad \tilde{h} + \tilde{P} = \nu \tilde{Z}_Y + (1 - \nu) \tilde{Z}_H.$$

Knowledge capital and private capital are aggregated to man-made assets M , see (14) and (19):

$$(C.6) \quad \tilde{M} = \left(\frac{K}{q_M M} \right) \tilde{K} + \left(\frac{q_h h}{q_M M} \right) \tilde{h} = (1 - \omega) \tilde{K} + \omega \tilde{h}.$$

Substituting (C.1) and (C.2) into (C.3), using (C.4) and (C.5) to eliminate $\tilde{K}_H$ and $\tilde{Z}_H$, substituting (A.1) and (A.4) to eliminate u and ν (so that $\tilde{K}_Y$ and $\tilde{Z}_Y$ cancel out), (19) and (20) to introduce ω , and (C.6) to introduce $\tilde{M}$, we find:

$$(C.7) \quad \tilde{J} = a_N \tilde{N} + \omega \tilde{P} + \tilde{M}.$$

Substituting $\tilde{M}(0) = \tilde{N}(0) = 0$, we arrive at (37). Differentiating (C.7) with respect to time, taking into account that pollution is constant (i.e. $\dot{\tilde{P}} = 0$), we find (38).

D. *The Growth Rate of Man-Made Assets* ((39)). From the right-hand side equalities in (2) and (3), we find:

$$(D.1) \quad g\tilde{g}_K = \frac{Y}{K}(\tilde{Y} - \tilde{K}) - \frac{C}{K}(\tilde{C} - \tilde{K}),$$

$$(D.2) \quad g\tilde{g}_h = g(\tilde{H} - \tilde{h}).$$

The growth rate of aggregate man-made assets is a weighted average of the growth rates of two man-made assets; see (14) and (19):

$$(D.3) \quad \tilde{g}_M = \left(\frac{K}{q_M M} \right) \tilde{g}_K + \left(\frac{q_h h}{q_M M} \right) \tilde{g}_h = (1 - \omega) \tilde{g}_K + \omega \tilde{g}_h.$$

First substitute (D.1) and (D.2) into (D.3), then aggregate income and assets according to (C.3) and (C.6), respectively, and finally substitute (17) and (18). This yields:

$$(D.4) \quad g\tilde{g}_M = r(\tilde{J} - \tilde{M}) - (r - g)(\tilde{c} - \tilde{M}).$$

For $t = 0$, (D.4) yields the first equality in (39) (as $\tilde{M}(0) = 0$). We find the second equality in (39) by substituting (37) and (E.11).

E. *Consumption* ((33), (35)). As a result of our linearization procedure, the consumption to man-made capital ratio evolves over time according to:

$$(E.1) \quad \dot{\tilde{c}/\tilde{M}} = g(\tilde{g}_c - \tilde{g}_M).$$

Substitution of (33), (D.4), (25), (C.7) and (B.9) into (E.1) yields the third differential equation in Table 1. To solve this differential equation in the unknown variable $(\tilde{c} - \tilde{M})$, we use Laplace transforms (cf. Judd 1982). In particular, the Laplace

transform of a variable $\tilde{x}$ is defined as:

$$(E.2) \quad L_{\tilde{x}}(s) = \int_0^{\infty} \tilde{x}(t) e^{-st} dt.$$

The Laplace transform of the time derivative of a variable $\tilde{x}$ is:

$$(E.3) \quad L_{\dot{\tilde{x}}}(s) = \int_0^{\infty} \dot{\tilde{x}}(t) e^{-st} dt = sL_{\tilde{x}}(s) - \tilde{x}(0).$$

To simplify notation, rewrite the third dynamic equation by collecting all terms not containing $\tilde{c} - \tilde{M}$ in $\tilde{x}_c$:

$$(E.4) \quad \dot{\tilde{c}/\tilde{M}} = (r - g)(\tilde{c} - \tilde{M}) + \tilde{x}_c,$$

where

$$(E.5) \quad x_c = \sigma_c r \tilde{r} + (\sigma_c - 1) \phi \dot{N} - r [\omega \tilde{P} + a_N \tilde{N}].$$

The Laplace transform of (E.4) is:

$$(E.6) \quad (s - (r - g)) L_{(\tilde{c} - \tilde{M})}(s) = L_{\tilde{x}_c}(s) + \tilde{c}(0) - \tilde{M}(0).$$

Since M is a predetermined variable, $\tilde{M}(0) = 0$. To find the initial change $\tilde{c}(0)$, we use the condition that $L_{\tilde{c} - \tilde{M}}(s)$ is bounded for $s > 0$. This implies that the RHS of (E.6) should be zero for $s = r - g > 0$. Hence:

$$(E.7) \quad \tilde{c}(0) = -L_{\tilde{x}_c}(r - g) = -\int_0^{\infty} \tilde{x}_c e^{-(r-g)t} dt.$$

Since $\tilde{x}_c$ is a combination of variables that develop monotonically at rate δ (cf. (22)), also $\tilde{x}_c$ does. Hence, we can write its Laplace transform as:

$$(E.8) \quad L_{\tilde{x}_c}(s) = \int_0^{\infty} \tilde{x}_c(t) e^{-st} dt = \frac{\tilde{x}_c(0) - \tilde{x}_c(\infty)}{s + \delta} + \frac{\tilde{x}_c(\infty)}{s}.$$

Substituting (E.7) into (E.6), setting $\tilde{M}(0)$ to zero, and using (E.8) to rewrite the Laplace transforms, we derive:

$$(E.9) \quad L_{\tilde{c} - \tilde{M}}(s) = -\frac{1}{s + \delta} \left(\frac{\tilde{x}_c(0) - \tilde{x}_c(\infty)}{\delta + r - g} \right) - \frac{1}{s} \left(\frac{\tilde{x}_c(\infty)}{r - g} \right).$$

Inverting the Laplace transform, we arrive at:

$$(E.10) \quad \begin{aligned} \tilde{c}(t) - \tilde{M}(t) = & \left[-\left(\frac{\tilde{x}_c(0) - \tilde{x}_c(\infty)}{\delta + r - g} \right) - \left(\frac{\tilde{x}_c(\infty)}{r - g} \right) \right] e^{-\delta t} \\ & + \left[\frac{-\tilde{x}_c(\infty)}{r - g} \right] (1 - e^{-\delta t}). \end{aligned}$$

Hence, $\tilde{c} - \tilde{M}$ develops monotonically at rate δ (cf. (22)). The first (second) term in square brackets in (E.10) represents $\tilde{c}(0)$ ($\tilde{c}(\infty) - \tilde{M}(\infty)$). Substituting (E.5) for $\tilde{x}_c$ and noting that $\dot{N}(0) = \delta\tilde{N}(\infty)$ (from (26)), that $\eta \equiv \delta/(r - g + \delta)$ and $\delta(1 - \eta) = \eta(r - g)$ (from (29)), we arrive at:

$$(E.11) \quad \tilde{c}(0) = \left(\frac{r}{r - g} \right) \left[\omega\tilde{P} + \eta a_N \tilde{N}(\infty) \right] + \eta\phi\tilde{N}(\infty) \\ - \sigma_c \left[\left(\frac{r}{r - g} \right) [(1 - \eta)\tilde{r}(0) + \eta\tilde{r}(\infty)] + \eta\phi\tilde{N}(\infty) \right],$$

$$(E.12) \quad \tilde{c}(\infty) - \tilde{M}(\infty) = \frac{-x_c(\infty)}{r - g} = \left(\frac{r}{r - g} \right) \left[\omega\tilde{P} + a_N \tilde{N}(\infty) - \sigma_c \tilde{r}(\infty) \right].$$

(In section F below, we will derive that the first three terms in (E.11) amount to welfare $\tilde{W}(0)$, so that (E.11) can be written as in (35).)

F. *Welfare* ((28), (35), (36)). Welfare at time $t = 0$ [see (6) and (7)] can be written as:

$$(F.1) \quad W(0) = \left(\frac{1}{1 - 1/\sigma_c} \right) c(0)^{1 - 1/\sigma_c} \int_0^\infty \left[N(t)^{\phi(1 - 1/\sigma_c)} e^{-R(t)} \right] dt,$$

where $R(t)$ is the cumulative growth-corrected discount rate, defined as:

$$(F.2) \quad R(t) \equiv \int_0^t [\vartheta + (1/\sigma_c - 1)g_c(\tau)] d\tau.$$

Hence, welfare depends on the time path of N and g_c . Using the fact that both variables develop monotonically (see (33), (B.9) and (26)), we derive for welfare at $t = 0$ ²²:

$$(F.3) \quad \left(\frac{1}{1 - 1/\sigma_c} \right) \frac{dW(0)}{W(0)} \\ \equiv \tilde{W}(0) = \tilde{c}(0) + \left(\frac{g}{r - g} \right) [(1 - \eta)\tilde{g}_c(0) + \eta\tilde{g}_c(\infty)] + \phi[\eta\tilde{N}(\infty)].$$

Eliminating $\tilde{g}_c(0)$ and $\tilde{g}_c(\infty)$, using (33) and taking into account from (26) that $\dot{N}(0) = \delta\tilde{N}(\infty)$ and from (29) that $\delta(1 - \eta) = \eta(r - g)$, we find (35). Substituting (E.11) into (35) to eliminate $\tilde{c}(0)$, we find (28). Equation (36) is derived from (28), (32), (34), and (E.11).

²² $\tilde{W}$ is defined as $\tilde{W} \equiv dW(0)/[(1 - 1/\sigma_c)W(0)]$ so that $\tilde{W} > 0$ if welfare rises [note that $W < (>) 0$ if $\sigma_c < (>) 1$].

G. *Capital Knowledge Ratio* ((42)–(45)). Linearizing $(d(K/h)/dt)/(K/h)$ and substituting (D.3), we find that the ratio K/h evolves over time according to:

$$(G.1) \quad \dot{\bar{K}}/\bar{h} = g(\tilde{g}_K - \tilde{g}_h) = \left(\frac{g}{1 - \omega} \right) (\tilde{g}_M - \tilde{g}_h).$$

The derivation of the last dynamic equation in Table 1 involves the following steps. First, substitute (D.4) and (D.2) into (G.1) and substitute subsequently (C.7) and (C.2). Next, substitute (B.5) and (B.6) to eliminate $\tilde{K}_H - \tilde{Z}_H$, and substitute (B.5), (B.6), (C.4), and (C.5) to eliminate $\tilde{Z}_H - \tilde{h}$. Finally, substitute (A.1), (A.4), and (19) to eliminate ν , u , and ω . Let $\tilde{x}_k$ collect all terms other than the terms with $(\tilde{K} - \tilde{h})$ in the differential equation for K/h in Table 1:

$$(G.2) \quad \tilde{x}_k = \left(\frac{1}{1 - \omega} \right) \left[r(a_N \tilde{N} + \omega \tilde{P}) - g(a_N \tilde{N} + \tilde{P}) - (r - g)(\tilde{c} - \tilde{M}) \right] \\ + \lambda \tilde{P} + \left(\frac{\beta g \sigma_H}{1 - \omega} - \lambda \sigma_u \right) \left(\frac{\tilde{q}_h}{\beta - \alpha} \right),$$

so that $d(\tilde{K} - \tilde{h})/dt = -\lambda(\tilde{K} - \tilde{h}) + \tilde{x}_k$, where λ is defined in (24). The Laplace transform of this differential equation reads:

$$(G.3) \quad (s + \lambda) L_{(\tilde{K} - \tilde{h})}(s) = L_{\tilde{x}_k}(s) + \tilde{K}(0) - \tilde{h}(0).$$

Both $\tilde{K}$ and $\tilde{h}$ are predetermined variables. Accordingly, $\tilde{K}(0) - \tilde{h}(0) = 0$. $\tilde{x}_k$ is a combination of variables that develop monotonically at rate δ . Hence, also $\tilde{x}_k$ does. Consequently, we can write the Laplace transform of $\tilde{x}_k$ in (G.3) in terms of $\tilde{x}_k(0)$, $\tilde{x}_k(\infty)$, s , and δ (use (E.8)):

$$(G.4) \quad L_{\tilde{K} - \tilde{h}}(s) = \left(\frac{1}{s + \lambda} \right) \frac{\tilde{x}_k(\infty)}{s} + \left(\frac{1}{s + \lambda} \right) \frac{\tilde{x}_k(0) - \tilde{x}_k(\infty)}{s + \delta}.$$

Inverting the Laplace transform in (G.4), we arrive at:

$$(G.5) \quad \tilde{K} - \tilde{h} = \left[\frac{\tilde{x}_k(\infty)}{\lambda} \right] (1 - e^{-\lambda t}) + [\tilde{x}_k(0) - \tilde{x}_k(\infty)] \frac{e^{-\lambda t} - e^{-\delta t}}{\delta - \lambda}.$$

Hence, the first and second terms in square brackets represent the long-run solution for $\tilde{K} - \tilde{h}$ and its temporary part, $T_{\tilde{K} - \tilde{h}}$, respectively (cf. (23) and (42)). We find (43) by evaluating $\tilde{x}_k$ in (G.2) for $t = \infty$, dividing by λ , and substituting (E.12) to eliminate $(\tilde{c} - \tilde{M})$, (B.9) to rewrite $\tilde{r}(\infty)$, and (B.8) to eliminate $\tilde{q}_h(\infty)$. To find the temporary part $T_{\tilde{K} - \tilde{h}}$ in (44), we subtract $\tilde{x}_k(\infty)$ from $\tilde{x}_k(0)$ [see (G.2)], and substitute (B.8), (B.9), (E.11), (E.12) and (29).

H. *The Size of the Public Sector* ((46), (47)). From (14), (15), (C.3) and (20) we find $\tilde{q}_J = (\omega g/r) \tilde{q}_h$. Using this relationship and (C.3), we find:

$$(H.1) \quad \tilde{q}_h + \tilde{H} - \tilde{q}_J - \tilde{J} = \left(1 - \frac{\omega g}{r} \right) (\tilde{q}_h + \tilde{H} - \tilde{Y}).$$

Substitution of (C.1), (C.2), (B.5), (B.6), (C.5), and (B.8), and setting $\sigma_Y = \sigma_H = \sigma_u$ (see (45)), one finds (46). Substituting (43) into (46), we arrive at (47).

REFERENCES

- ALFSEN, K., A. BRENDAMOEN AND S. GLOMSRØD, "Benefits of Climate Policies: Some Tentative Calculations," Discussion paper 69, Statistics Norway, 1992.
- ASHFORD, N. A., C. AYERS, AND R. F. STONE, "Using Regulation to Change the Market for Innovation," *Harvard Environmental Law Review* 9 (1985), 419-466.
- BALLARD, C. L. AND S. G. MEDEMA, "The Marginal Efficiency Effects of Taxes and Subsidies in the Presence of Externalities," *Journal of Public Economics* 52 (1993), 199-216.
- BARRO, R. J., "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98 (1990), 103-125.
- BAUMOL, W. J. AND W. E. OATES, *The Theory of Environmental Policy* (Cambridge: Cambridge University Press, 1988).
- BOVENBERG, A. L. AND S. SMULDERS, "Environmental Quality and Pollution-augmenting Technological Change in a Two-sector Endogenous Growth Model," *Journal of Public Economics* 57 (1995), 369-391.
- BRENDAMOEN, A. AND H. VENNEMO, "A Climate Treaty and the Norwegian Economy: A CGE Assessment," *The Energy Journal* 15 (1994), 77-93.
- CABBALÉ, J. AND M. S. SANTOS, "On Endogenous Growth with Physical and Human Capital," *Journal of Political Economy* 101 (1993), 1042-1067.
- CHAMLEY, C., "Externalities and Dynamics in Models of 'Learning or Doing'," *International Economic Review* 34 (1993), 3, 583-609.
- EWIJK, C. VAN, AND S. VAN WIJNBERGEN, "Can Abatement Overcome the Conflict between the Environment and Economic Growth?," *De Economist* 143 (1995), 197-216.
- GRADUS, R. AND S. SMULDERS, "The Trade-off between Environmental Care and Long-term Growth; Pollution in Three Proto-type Growth Models," *Journal of Economics* 58 (1993) 25-51.
- GROSSMAN, G. M. AND E. HELPMAN, *Innovation and Growth in the Global Economy*, (Cambridge: MIT Press, 1991).
- HAHN, R. W., "Economic Prescriptions for Environmental Problems: How the Patient Followed the Doctor's Orders," *Journal of Economic Perspectives* 3 (1989), 95-114.
- HAZILLA, M. AND R. J. KOPP, "Social Cost of Environmental Quality Regulations: A General Equilibrium Analysis," *Journal of Political Economy* 98 (1990), 853-873.
- HUANG, C.-H., AND D. CAI, "Constant Returns Endogenous Growth with Pollution Control," *Environmental and Resource Economics* 4 (1994), 383-400.
- HUNG, V. T. Y., P. CHANG AND K. BLACKBURN, "Endogenous Growth, Environment and R&D," in C. Carraro (ed.), *Trade Innovation and Environment* (Dordrecht: Kluwer Academic Press, 1994).
- JAFFE, A. B., S. R. PETERSON, P. R. PORTNEY, R. N. STAVINS, "Environmental Regulation and the Competitiveness of U.S. Manufacturing: What Does the Evidence Tell Us?" *Journal of Economic Literature* 33 (1995), 132-163.
- JORGENSEN, D. W. AND P. J. WILCOXEN, "Environmental Regulation and U.S. Economic Growth," *Rand Journal of Economics* 21 (1990), 314-340.
- JUDD, K. L., "An Alternative to Steady State Comparisons in Perfect Foresight Models," *Economics Letters* 10 (1982), 55-59.
- LANJOUW, J. AND A. MODY, "Stimulating Innovation and the International Diffusion of Environmentally Responsive Technology: The Role of Expenditures and Institutions," mimeo, World Bank, 1993.
- LIGTHART, J. E. AND F. VAN DER PLOEG, "Pollution, the Cost of Public Funds and Endogenous Growth," *Economics Letters* 46 (1994), 351-361.
- MARGULIS, S. "Back-of-the-Envelope Estimates of Environmental Damage Costs in Mexico," Policy Research Working Papers, The World Bank, Washington, January 1992.

- MARREWIJK, C. VAN, F. VAN DER PLOEG AND J. VERBEEK, "Pollution, Abatement and Endogenous Growth: Is Growth Bad for the Environment?" working paper, The World Bank, Washington, February 1994.
- MULLIGAN, C. AND X. SALA-I-MARTIN, "Transitional Dynamics in Two-sector Models of Endogenous Growth," *Quarterly Journal of Economics* 108 (1993), 739-773.
- PEARCE, D. W. AND J. J. WARFORD, *A World Without End: Economics, Environment, and Sustainable Development* (Oxford: Oxford University Press, 1993).
- PLOEG, F. VAN DER AND C. WITHAGEN, "Pollution Control and the Ramsey Problem," *Environmental and Resource Economics* 1 (1991), 215-230.
- ROMER, P. M. "Endogenous technological change," *Journal of Political Economy* 98 (1990), S71-S102.
- SCHMALENSEE, R., "The Costs of Environmental Protection," in M. Kotowski (ed.), *Balancing Economic Growth and Environmental Goals* (Washington DC: American Council for Capital Formation Center for Policy Research, 1994), 55-75.
- TAHVONEN, O. AND J. KUULUVAINEN, "Economic Growth, Pollution and Renewable Resources," *Journal of Environmental Economics and Management* 24 (1993), 101-118.
- VERDIER, T. "Environmental Pollution and Endogenous Growth: A Comparison between Emission Taxes and Technological Standards," Working Paper 57.93, Foundatione Eni Enrico Mattei, 1993.
- XEPAPADEAS, A., "Long-run Growth, Environmental Pollution and Increasing Returns," mimeo, University of Crete, 1993.