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Aging and political decision making on public pensions

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Abstract. In this paper decision making on public pensions in a representative democracy is modeled within the framework of the well-known two-overlapping-generations (OLG) general-equilibrium model with rational expectations. The model is used to analyze the effects of aging on the economy in general and on the evolution of public pension schemes in particular, where aging is interpreted as a combination of a decrease in the rate of population growth and an increase in the political influence of pensioners. Analytical results are derived for the long run as well as for the short run by the method of comparative statics and comparative dynamics respectively. It appears that an increase in transfers to the old is not guaranteed if due to aging their political power increases.

JEL classification: J14, H55

Key words: Aging, overlapping generations, pensions

1. Introduction

In most developed countries, a substantial part of the elderly's income is provided for by means of public pension schemes. Generally, these public pension plans are financed by a Pay-As-You-Go (or PAYG) scheme where current pension payments are financed out of current tax payments that are collectively decided upon. These PAYG-schemes have drawn a great deal of attention recently due to the aging of the population and the increasing tax rates this might entail. It is of interest to consider what could be the reactions of policy makers to the demographic changes that are taking place. In doing so, we build upon earlier public-choice

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literature.¹ The central theme in this literature is the dynamic interaction between public pension schemes and individual savings for old age.² As the savings decision has to be taken when young while the public pension benefit is actually decided upon when old, anticipation plays an important role in this interaction. It is often assumed that expectations are not rational (e.g. Boadway and Wildasin 1989) or that future decision making can somehow be precommitted (e.g. Hansson and Stuart 1989).³ Here we assume perfect foresight and uncommitted decision making, however. Another problem with most of the public-choice literature on public pensions is that it takes the economy to be exogenous with a given interest rate and a given endowment. This has the drawback that it leads to a bang-bang solution in the long run: either there is a public pension system but then there is no private saving or there is private saving but no transfers.⁴ In this paper we obviate this problem by using a closed-economy general-equilibrium framework. This framework is presented in Sect. 2.

In Sect. 3 decision making on public pensions is modeled. In line with the public-choice literature it is assumed that the decisions on public pensions are made by politicians who merely represent the currently living generations but who, in making their decisions, take anticipated decisions by future politicians into account.

In Sect. 4 we investigate how a PAYG-pension system reacts to the aging of the population which can be expected to occur in the next century in most countries of the Western world. Aging can have several effects. Firstly, due to the dependency-ratio effect, aging leads to higher contribution rates. This effect has drawn the most attention and concern expressed e.g. in policy circles.⁵ However, aging also implies that the political power of the old, measured by their sheer number, increases, which may lead to higher benefits. Moreover, the per capita political power of the old may change, e.g. because the situation of the old draws more (political) attention. Finally, aging implies that less saving is necessary to maintain a certain capital-labor ratio. In this paper all these effects of aging are taken into account and it is derived under what circumstances aging will lead to an increasing or a decreasing relative importance of the PAYG-scheme. It should be stressed that the short-run effects can differ considerably from the effects in the long run. Therefore, we not only perform a comparative statics analysis in order to trace the long-run effects, but we also study the short-run consequences of aging. We derive analytical results for the short run for fairly general specifications for utility and production functions by the method of comparative dynamics. Moreover, we distinguish between expected and unexpected shocks.

It turns out that the effect of a decrease in the rate of population growth on long-run consumption is ambiguous if the political power of an old individual increases due to the demographic change. Interestingly, an increase in transfers to the old is not guaranteed in this case: transfer payments may fall in the long run due to general-equilibrium effects. Short-run gains and losses resulting from an unexpected or an expected decrease in population growth, on the other hand, appear to be determined by the size of the PAYG-financed public pension system together with the endogenous change in political power. In particular, if the existing size of the PAYG-system is large, aging does not necessarily imply an instantaneous extension of the PAYG-system and an increase in old-age consumption, even if aging goes along with an increasing political power of old individuals. This contradicts the Hansson and Stuart (1989) result where a rise in the political power of the old is predicted to lead to an evolution of increasing tax rates. It also

contradicts the result derived by Cutler et al. (1990) that short-run gains in consumption are likely if aging can be expected to occur a few decades ahead.

The paper winds up with some concluding remarks in Sect. 5.

2. The model

2.1 The individuals

The analysis is based on the well-known two-overlapping-generations model where there is a large number of individuals living for two periods and every one is identical except for differences in age. Since the young's savings decisions are assumed to be decentralized and uncoordinated, the young take aggregate savings and tax rates as given when deciding on their savings. It is assumed that every individual born at time t works a fixed amount of time at wage w_t during the first period of his life. A part of this wage is taxed away by the government by a lump-sum tax τ_t to be transferred to the old. The remainder is used for savings for old age (s_t) and for consumption (c_t^y). So we get:

$$c_t^y = w_t - \tau_t - s_t . \quad (1)$$

When old, the individual consumes the return on his savings and the transfer payment η from the government. So, the consumption at time $t+1$ of an old individual born at time t (c_t^r) is equal to:

$$c_t^r = (1 + r_{t+1})s_t + \eta_{t+1} . \quad (2)$$

Individuals are assumed not to be altruistic: they do not care about the levels of utility of individuals born in another time period. It is assumed that lifetime utility of an individual born at time t can be represented by the following separable utility function:

$$U_t = u(c_t^y) + \frac{1}{1+\theta} u(c_t^r) , \quad (3)$$

where the felicity functions obey the Inada-conditions. The parameter θ indicates the rate of time preference. The first-order condition for maximization of lifetime utility reads:

$$u'(c_t^y) = \frac{1+r_{t+1}}{1+\theta} u'(c_t^r) , \quad (4)$$

where $u'(c) \equiv \frac{du}{dc}$. This condition states that the young use savings to equate the marginal rate of substitution of consumption in the first and the second part of their lives to the rate of return on savings.⁶

2.2 The markets

We close the model by conventional equilibrium conditions on the factor markets and the goods market. Production per worker at time t is described by a standard neoclassical production function $f(k_t)$, $\left(f'(k) \equiv \frac{df}{dk} > 0, f'' \equiv \frac{d^2f}{dk^2} < 0\right)$ where k_t is the capital-labor ratio. Assuming that firms act competitively and given exogenous labor supply, factor market equilibrium is described by the marginal productivity conditions:

$$f(k_t) - k_t f'(k_t) = w_t, \quad (5)$$

$$f'(k_t) = r_t. \quad (6)$$

Equilibrium on the goods market implies for our closed-economy model that investment be equal to saving or in per worker terms:

$$(1 + n_t)k_{t+1} = s_t, \quad (7)$$

where n is the rate of population growth which is assumed to be determined exogenously. Eqs. (1), (2), (5)–(7) imply the following resource constraint for the economy as a whole:

$$f(k_t) + k_t = (1 + n_t)k_{t+1} + c_t^y + \frac{1}{1+n} c_{t-1}^r, \quad (8)$$

which simply says that total supply of goods can be allocated to investment or to consumption by the currently living generations.

3. Decision making on public pensions

In general, there are two possible ways to model the institutional form of the political process: either as a direct democracy, as is assumed e.g. in a seminal article on decision making on public pension schemes by Browning (1975), or as a representative democracy, as is assumed here. In the case of a representative democracy there is not one infinitely long lived government that optimizes social welfare, but the government consists of a sequence of drafts of politicians. Each draft of politicians is under the influence only of both current generations, the old and the young. The relative political power of these generations depends upon the support they can provide for the government's survival. This implies that, in contrast to a social planner, politicians do not take account of future generations' interests because the support from current generations does not depend upon their offspring's utility as intergenerational altruism is assumed to be absent.⁷ So, the government at time t is assumed to maximize the target function:⁸

$$W_t = (1 + \rho(n_t)) U_{t-1} + (1 + n_t) U_t. \quad (9)$$

In (9) $\rho(n)$ can be interpreted as a measure for the political power of an old individual relative to a young individual. Notice that in this interpretation it is in principle possible for $\rho(n)$ to be negative. However, as we are interested in public pensions, i.e. transfers from the young to the old, we can exclude this possibility without loss of generality.⁹

It is assumed that this relative political power is a negative function of n , i.e. $\frac{d\rho(n)}{dn} < 0$. So aging, i.e. a decrease in n , implies an increase in the relative political influence of old individuals. This assumption is in line with a model by Kristov et al. (1992) which endogenizes the relative political power. In that model the relative political power is the result of expenditures by old and young individuals on political activity. The more old individuals spend compared to young ones, the higher their utility is weighted by politicians. If a generation increases relatively in size, it is becoming more important for the re-election chances of the politicians so that a given level of pressure-group expenditures becomes more effective. Because in this paper we are only interested in the effects of aging, the parameter ρ is not endogenized as in Kristov et al. but merely a negative function of n . It is, however, also possible that aging decreases the relative political influence of an old individual. This is, for example, the case if the increased scarcity of labor due to a drop in population growth gives the young more economic power which in turn leads to an increased political influence. A decrease in the relative political influence of an old individual in case of aging may also be caused by the fact that the relatively small number of young can organize themselves more easily into a pressure group. This is of course due to the free-rider problem of organizing pressure groups (see Olson (1965)). So, in principle, it may also be the case that $\frac{d\rho(n)}{dn} > 0$. Here this possibility is excluded for the sake of clearness. The results

for this case can, however, easily be derived from the equations below.¹⁰

The government runs the public pension system as a PAYG-scheme. Abstracting from administrative costs the budget restriction of this scheme reads:

$$\eta_t = (1 + n_t) \tau_t . \quad (10)$$

Now assume that each generation of politicians adopts Nash-behavior towards the private sector.¹¹ As the link between the current and the future tax rate goes via current savings, this immediately implies that politicians take future tax rates as given. In that case the first-order condition for the tax rate at a given time t that maximizes target function (9) is:

$$u'(c_t^y) = \frac{1 + \theta}{1 + \rho(n_t)} u'(c_t^o) . \quad (11)$$

It follows directly from (4) and (11) that the steady state of the model is characterized by:¹²

$$r = \rho(n) . \quad (12)$$

This is the well-known modified golden rule with $\rho(n)$ playing the role of the social discount factor. The higher this factor, the lower the capital stock per

worker in the steady state will be, which is of course accomplished by a higher transfer which has a negative effect on savings. Notice that the economy is dynamically efficient in the steady state ($r > n$) if $\rho(n) > n$. The economy may also be dynamically inefficient in the long run, however. So, the existence of intergenerational transfers in itself is not sufficient to prevent dynamic inefficiency from occurring.¹³ This is due to the fact that politicians only have the current tax rate as an instrument. For a Pareto improvement current *and* future tax rates should rise which cannot be implemented by a finitely lived government.

4. The effects of aging

In this section we consider the effects of aging. Aging is, of course, primarily a decrease in population growth (n). Aging, however, also changes the political influence of an old individual relative to that of a young one. So both weights $\rho(n)$ and n in the target function (9) change.¹⁴ The short-run as well as the long-run consequences of once-and-for-all changes in $\rho(n)$ and n are derived analytically.¹⁵ The long-run effects follow from comparing steady states, while the short-run consequences are traced by comparative dynamics.¹⁶ Regarding the short run, the effects of unexpected and expected shocks can be distinguished. The analysis learns that the short-run effects of aging can be (but need not be) opposite to the effects in the long run.

The comparative statics as well as the comparative dynamics analysis is based on linearization of the model presented above around the steady state. It is assumed that the economy is dynamically efficient in this initial steady state, i.e. $\rho > n$. Let $n_t = n + \gamma h_t$ where h_t describes the time pattern of the perturbation of the steady-state value of the parameter n and γ the magnitude. It is assumed that at some $t = 0$ the shock unexpectedly occurs (i.e. $h_0 = h_1 = \dots = h < 0$) or is expected to occur at time $z > 0$ (i.e. $h_0 = h_1 = \dots = h_{z-1} = 0$, $h_z = h_{z+1} = \dots = h < 0$). It will be convenient in the sequel to introduce total consumption per worker which is defined as:

$$c_t = c_t^y + \frac{c_{t-1}^r}{1+n_t}. \quad (13)$$

The linearized model can then be condensed to a system describing the changes in this variable and in the capital-labor ratio k_t :

$$\begin{bmatrix} \frac{\partial k_{t+1}}{\partial \gamma} \\ \frac{\partial c_{t+1}}{\partial \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{\partial k_t}{\partial \gamma} \\ \frac{\partial c_t}{\partial \gamma} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \quad t = 0, 1, \dots \quad (14)$$

where M is a matrix describing the effects of the current and next-period change in n on the state variables and J is the Jacobian matrix (see the Appendix for details). As the system in (14) is saddlepoint stable it can be solved to find the

initial effect of changes in n on total consumption i.e. $\frac{\partial c_0}{\partial \gamma}$ (see Blanchard and Kahn 1980). The evolution of the state variables c and k can then be traced by using the same system again. The changes in all other variables can be derived from the changes of these state variables. In particular, we calculate the changes in consumption of the young and the old and the effect on the tax rate, respectively.

The short-run effects of an unexpected decrease in n

An unexpected demographic change leads to changes in intergenerational transfers and savings which affect total consumption per worker (c) and the capital-labor ratio at the time of the shock and in all periods after the shock. These effects can be found from solving the system in (14). For this particular case, the initial change in total consumption is described by (vi) in the Appendix which is rewritten here for convenience¹⁷:

$$\begin{aligned} \frac{\partial c_0}{\partial \gamma} = & (1+n)(\lambda_2 - 1 - \delta) \frac{\partial k_0}{\partial \gamma} + (1-\lambda_2)k \sum_{j=1}^{\infty} h_j \lambda_2^{-j} \\ & + \frac{c^r}{(1+n)^2} \left[\frac{\lambda_2 - 1}{\lambda_2} \sum_{j=0}^{\infty} h_j \lambda_2^{-j} - h_0 \right] \\ & + \frac{\rho'}{1+\rho} \left[\frac{1}{(1+n)\alpha^r \lambda_2} h_0 + \left(\frac{1}{\alpha^y} + \frac{1}{(1+n)\alpha^r \lambda_2} \right) \sum_{j=1}^{\infty} h_j \lambda_2^{-j} \right] \end{aligned} \quad (15)$$

Note that $\frac{\partial c_0}{\partial \gamma}$ may be dubbed an initial macroeconomic effect as it gives the additional (positive or negative) consumption (expressed per worker) available for the whole economy at $t = 0$. This additional consumption has to be allocated to the currently living young and old individuals.

Eq. (15) reflects the forward-looking behavior of the current generation of politicians that is able to take account of all current (h_0 terms) and all future (terms h_j , $j > 0$) of aging in its own decision. These effects can be discerned in political power effects that spring off from the increase in the relative power of the old $\left(\text{all terms containing } \rho' = \frac{d\rho(n)}{dn} \right)$ and effects that are merely the result

of the demographic change (all other terms). The demographic effect consists of three elements. Firstly, a decrease in n causes an upward jump in the capital-labor ratio, which implies a higher production and thus a higher consumption per worker. This is reflected by the first term in (15), $\left(\frac{\partial k_0}{\partial \gamma} = -\frac{k}{1+n} h_0 > 0 \right)$.

Secondly, there is a *capital thickening* effect (k , see the second term in (15)).¹⁸ This effect stems from the fact that a decrease in n implies that a certain level of saving per worker ceteris paribus leads to a higher capital-labor ratio next period. This is anticipated by immediately decreasing the saving rate, so that consumption increases. A third effect comes from the fact that an increase in the so-called

dependency ratio, which is defined as the number of pensioners per worker, implies that total consumption per worker has to be shared with a larger number of pensioners. This *dependency-ratio* effect $\left(\frac{c^r}{(1+n)^2}\right)$ may influence savings and thus total consumption. Of course, in case of an unexpected decrease in n , the dependency-ratio effect not only affects future consumption but initial consumption as well. However, if the change in population growth is once-and-for-all, so that $h_j = h \forall j \geq 0$, then the initial dependency-ratio effect is exactly compensated by the future effect. Summing up, these 'pure' aging effects of a once-and-for-all decrease in n have a non-negative effect on initial macroeconomic consumption per worker.

On top of this, $\frac{\partial c_0}{\partial \gamma}$ is determined by the political power effect, which in turn consists of two elements. Firstly, there is the effect of the immediate increase in the political power of the old which increases transfers. This decreases the room for old-age savings by the young. Secondly, there is an effect which stems from the fact that in future periods redistribution to the old will be engendered by the then changed political power. This decreases the need for old-age savings by the young and lowers the capital-labor ratio. So, the increased political power of the old in case of a once-and-for-all decrease in n triggers a decrease in savings and thus leads to an immediate increase of total consumption possibilities. This leads us to the following proposition:

Proposition 1. *An unexpected once-and-for-all decrease in population growth initially increases total consumption per worker.*

In order to find the effect of aging on the consumption of old and young individuals, we differentiate (11). Using (13) we then find:

$$\frac{\partial c_t^y}{\partial \gamma} = (1-\varepsilon) \left[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1+n)^2} h_t \right] - \varepsilon \frac{\rho'}{(1+\rho)\alpha^y} h_t \quad (t = 0, 1, \dots), \quad (16)$$

$$\frac{\partial c_{t-1}^r}{\partial \gamma} = (1+n)\varepsilon \left[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1+n)^2} h_t \right] + (1+n)\varepsilon \frac{\rho'}{(1+\rho)\alpha^y} h_t \quad (t = 0, 1, \dots). \quad (17)$$

The parameter ε indicates the share of the young in the change of total consumption. This share is defined as $\varepsilon = \frac{\alpha^y}{\alpha^y + (1+n)\alpha^r}$, where α^y is the absolute rate of risk aversion of a young individual which is defined as $\alpha^y = \frac{-u''(c^y)}{u'(c^y)}$. Analogously, α^r stands for the risk aversion of an old individual. The equations show that the larger the absolute rate of risk aversion of a generation relative to the risk aversion of the other generation, the smaller its part in the change of total consumption.¹⁹

As indicated above the macroeconomic effect of aging at a particular time t , i.e. $\frac{dc_t}{d\gamma}$, is the result of all current and future aging effects. Eqs. (16) and (17) show

that in order to find the effect on the consumption of old and young individuals, two current effects of a change in n should be added to this macroeconomic effect. The first is the dependency-ratio effect, which affects consumption of both young and old individuals negatively. The second effect stems from the changed individual political power which leads to redistribution from young to old. Upon inspection of (16) and (17) it is clear that the effects of demographic change on individual consumption are, in general, ambiguous. The following proposition can, however, easily be derived by using (15), (16) and (17):

Proposition 2. *An unexpected once-and-for-all decrease in population growth initially leads to*²⁰

- *higher old-age consumption if $\tau < (1 - \lambda_1)s$*
- *lower young-age consumption if $\tau > (1 - \lambda_1)s$*

For other cases the effects are ambiguous.

The condition $\tau < (>) (1 - \lambda_1)s$ in Proposition 2 states that the PAYG-system is relatively small (large) at the time of the shock. If the PAYG-system is small then the consumption possibilities for both young and old can increase for demographic reasons. Moreover, redistribution from young to old will arise for political reasons. As a result old-age consumption will unambiguously increase. The other result in the proposition can be explained analogously.

It is interesting to observe that aging need not increase initial old-age consumption even though the individual political power of the old increases. This will typically be the case if the political power of elderly individuals is already relatively large resulting in an extensive PAYG-scheme. Then aging leads to a large dependency-ratio effect. In particular, if $\tau > (1 - \lambda_1)s$ the adverse demographic effect on individual consumption can dominate the positive political power effect. It immediately follows that transfer payments may decrease in spite of the increased political power of the old.

The effect on the tax rate is given by:

$$\frac{\partial \tau_t}{\partial \gamma} = [r\nu - 1 - r] \frac{\partial k_t}{\partial \gamma} + \varepsilon \frac{\partial c_t}{\partial \gamma} - (1 - \varepsilon) \frac{c^r}{(1 + n)^2} h_t + \varepsilon \frac{\rho'}{(1 + \rho)\alpha^y} h_t \quad (18)$$

The first term in this equation indicates the effect of a change in the capital-labor ratio. At a given interest rate an increase in the capital stock per worker implies an increase of the consumption by pensioners of $(1 + r) \frac{\partial k_t}{\partial \gamma}$ thus decreasing the

need for redistribution from young to old through τ . An increase in the capital-labor ratio also decreases the interest rate, thus increasing the need for redistribution. This effect is measured by ν , which is the (positively defined) elasticity of the production function $\nu = \frac{-f''(k)k}{f'(k)}$. The larger this elasticity, the larger the

effect of a change in the capital-labor ratio on the interest rate. The second term in (18) represents the effect of a change in total consumption per worker. As a part ε of this consumption goes to the old, more total consumption ceteris paribus increases the tax rate. In the third term we recognize the dependency-ratio effect again. An increase in the dependency ratio ceteris paribus increases the tax

rate. Finally, there is a political-power effect on the tax rate: aging increases the political influence of old individuals which has a positive effect on transfers from the young to the old. In general the total effect on the tax rate that results in the case of an unexpected change in the rate of population growth is ambiguous. It immediately follows, however, that the tax rate increases initially if $\nu > \frac{1+r}{r}$.

So, when the elasticity of the production function is relatively large an unanticipated decrease in population growth that increases the political influence of elderly individuals will lead to an increase in the tax rate. So in this case, aging may lead to the initiation of a PAYG-scheme in an economy where such a scheme did previously not exist.

The short-run effects of an expected decrease in n

Notice first that in the case of an expected once-and-for-all change at time $t = z$ (i.e. $h_0 = \dots = h_{z-1} = 0$, $h_z = h_{z+1} = \dots < 0$) there is no dependency-ratio effect at time $t = 0$. As a result, from (16) and (17) it follows that $\frac{\partial c_0^y}{\partial \gamma}$ and $\frac{\partial c_{-1}^r}{\partial \gamma}$ are proportional to the change in total consumption. Moreover, the capital-labor ratio does not jump in the case of an expected change, i.e. $\frac{\partial k_0}{\partial \gamma} = 0$.

The effect of an expected future decrease in n on c_0 is then given by the first line of (vi) in the Appendix:

$$\frac{\partial c_0}{\partial \gamma} = \left[\frac{\lambda_2 - 1}{\lambda_2} \frac{c'}{(1+n)^2} - (\lambda_2 - 1)k + \frac{\rho'}{1+\rho} \left(\frac{1}{\alpha^y} + \frac{1}{(1+n)\alpha^r\lambda_2} \right) \right] \sum_{j=z}^{\infty} h_j \lambda_2^{-j} \quad (19)$$

So the initial macroeconomic effect is determined by the anticipated dependency-ratio effects, capital-thickening effects²¹ and political-power effects. The difference between the dependency-ratio effect and the capital-thickening effect depends on s and $\frac{c'}{1+n}$ again. The following proposition can easily be derived from (19):

Proposition 3. *An anticipated once-and-for-all decrease in n initially increases individual consumption if $\tau < (1 - \lambda_1 + \delta)s$. Otherwise the effect is ambiguous.*

Notice that expected demographic shocks affect the consumption possibilities for both current young and current old individuals in the same direction. This is due to the fact that the relative political influence of both generations changes only in the future. Comparing with Proposition 2 it appears that the size of the PAYG-scheme is again a determining variable for the effects of aging. So, once again, it can be concluded that the fact that aging increases the political influence of old individuals does not imply that it leads to short-run gains in consumption. However, as the shock is further away in time, for this case larger values of τ are allowed for the effect on individual consumption to be positive. In particular, in contrast to an unexpected shock, for $(1 - \lambda_1)s < \tau < (1 - \lambda_1 + \delta)s$ individual con-

sumption unambiguously increases in the case of an anticipated decrease in n . This leads to the following corollary:

Corollary 1. *An anticipated once-and-for-all decrease in n initially increases the tax rate if $\tau < (1 - \lambda_1 + \delta)s$.*

Notice that it follows from this corollary that an anticipated decrease in population growth will lead to the initiation of a PAYG-scheme in an economy where such a scheme previously did not exist. The reason is that all young individuals living before the time of the shock are able to increase their consumption due to the anticipation of the increased consumption possibilities for the elderly which are engendered by the future change in political power. As a result, the tax rate will gradually increase and savings will gradually decrease. This leads to a lower future capital stock. Therefore, all generations born after the shock may lose while those born before the shock will gain. In that case, the fruits of the gain in political power are only reaped by those who are old on the time of or before the shock. All future generations, however, are locked in a disadvantageous situation which they cannot alter, even though they have the option to change the system. Actually, the situation that arises here is the preferred one given the new values of the parameters.

Corollary 1 only deals with the case of a relatively low tax rate. If the PAYG-scheme is larger ($\tau > (1 - \lambda_1 + \delta)s$), consumption possibilities may initially fall in the case of an expected aging. This is due to the fact that in that case the dependency ratio has a dominant negative effect on old-age consumption in the future. This decrease in old-age consumption is anticipated by the young living in the period before the shock. They react by raising their savings. As in the period before the shock the political power has not yet changed, this higher savings level will, through the political process, lead to a decrease in transfers to the old in that period. This in turn incites a savings reaction of previous generations. Notice that it follows from (18) that in the case of an anticipated shock the change in the tax rate is proportional to the change in total consumption per worker. So, indeed, a decreasing level of individual consumption can be associated with a decreasing tax rate.

The long-run effects

By differentiating (8) and (12) we find for the long-run effect of a change in n :

$$\frac{\partial k}{\partial \gamma} = -(1 + \theta) \frac{k}{r\nu} \rho' h, \quad (20)$$

$$\frac{\partial c}{\partial \gamma} = (r - n) \frac{\partial k}{\partial \gamma} - \frac{s}{1 + n} h = \frac{(n - r)(1 + \theta)k}{r\nu} \rho' h - \frac{s}{1 + n} h. \quad (21)$$

Notice that there is no pure demographic effect on the capital-labor ratio in the long run. The sole effect on this variable is a political-power effect. The increase in the political power of old individuals decreases the capital-labor ratio in the long run. This affects long-run consumption possibilities negatively. The scale of this political-power effect on consumption depends on the change in the capital-

labor ratio that is necessary in order to re-establish the equality between the interest rate and $\rho(n)$. In other words, it depends on the concavity of the production function as measured by the elasticity ν . The larger this elasticity, the smaller the change in the capital-labor ratio necessary to produce the required change in r and the smaller the effect on total consumption. On top of this political-power effect long-run consumption is affected by a demographic effect. Capital thickening increases consumption possibilities. Together with (16) and (17), this leads to the following proposition:

Proposition 4. *A decrease in population growth decreases the capital-labor ratio in the long run. The effect on long-run consumption is ambiguous.*

By substituting $\frac{\partial c}{\partial \gamma}$ from (21), we can rewrite Eq. (18) for the long run as:

$$\frac{\partial \tau}{\partial \gamma} = [(r\nu - 1 - n) + (\varepsilon - 1)(r - n)] \frac{\partial k}{\partial \gamma} - (1 - \varepsilon) \frac{c^r}{(1 + n)^2} h - \varepsilon k h + \varepsilon \frac{\rho'}{(1 + \rho)\alpha^\gamma} h. \quad (22)$$

From this equation in combination with Eq. (20) it immediately follows that $\frac{\partial \tau}{\partial \gamma} > 0$ if $\nu < \frac{1+n}{r}$. So, a relatively low elasticity of the production function implies that the tax rate rises in the long run. It should be noted that this does not necessarily imply that the benefit rate rises too. As:

$$\frac{\partial \eta}{\partial \gamma} = (1 + n) \frac{\partial \tau}{\partial \gamma} + \tau h \quad (23)$$

this is only guaranteed if $\tau = 0$.

When the elasticity of the production function is relatively high, the tax rate may decrease in the long run, even though the political influence of old individuals rises. This is due to the general-equilibrium effects. The initial increase in τ decreases savings which raises the interest rate and depresses the wage rate. In the case of a high elasticity of the production function, this change in factor payments is large relative to the fall in savings. Consequently, the consumption of the old rises relatively strong and young-age consumption falls relatively strong. Therefore, purchasing power should be transferred from the old to the young by a decrease in taxes in order to realize the preferred ratio of utilities as described by $\rho(n)$.

As a qualifying note, it can be observed here that in our analysis we abstract from the possibility of endogenous fertility. As is well known from the literature on this subject (see Cigno (1992)), an increase in the benefit rate will lead fertile individuals to take less children, i.e. $dn < 0$. Through the political-power effect τ may increase even more according to our analysis. This may lead to a new round of decreasing fertility and increasing benefit rates. This process can lead to a vanishing population but, on the other hand, there are countervailing powers in the model that can stop the process of ever increasing tax rates. Firstly, the larger the tax rate, the larger is (ceteris paribus) the negative effect of a decrease in population growth on the benefit rate (see (23)). Secondly, the general-equilibrium part

of the model can stop the process. The point is that the decrease in n , which results from a higher benefit rate, and the higher rate of interest, which is due to the increased political power of the old, will both decrease $\frac{1+n}{r}$. This will possibly lead to $v > \frac{1+n}{r}$ so that the increase in the tax rate is no longer guaranteed.

5. Concluding remarks

In this paper we analyzed the effects of aging in a positive general-equilibrium model. Aging was interpreted as a combination of two effects: not only a decrease in the rate of population growth but also a change in the relative political power of elderly individuals. It may be interesting to compare the results of our analysis to those in the existing literature.

Apart from the political-power effect, the long-run consequences of aging are well known: the capital-labor ratio does not change and consumption possibilities rise due to capital thickening. The effects are different if aging increases the political influence of old individuals, however. In that case, the capital-labor ratio falls and the effect on consumption is ambiguous.

The short-run effects of aging are, even if we abstract from a political-power effect, less well known. From Propositions 2 and 3 it has become clear that for unexpected as well as for expected shocks the size of the PAYG-financed public pension scheme is a determining variable for the pure demographic effects in the short-run. In particular, for an extensive public pension system both types of shocks may lead to initial losses in individual consumption. This contradicts the result by Cutler et al. (1990) who suggest that expected decreases in the rate of population growth will lead to short-run gains while such demographic changes lead to losses if they are unexpected. On the other hand, if the PAYG-taxes are relatively low, gains in consumption will be enabled by a decrease in savings. In an OLG-model these gains in consumption possibilities are partly transferred to the old through an increase in the tax rate. So it follows that if a PAYG-scheme is non-existent, it can come about because of an expected decrease in the rate of population growth. If the political influence of old individuals rises in case of aging, this of course has an additional positive effect on the transfers to the old.

Hansson and Stuart (1989) analyzed the case of an unexpected increase in political power of the old. In fact, they consider the initiation of a public pension scheme as arising from a (permanent) change in the target function in favor of current and future old individuals under the assumption of dynamic efficiency. According to their results, such a change would imply a windfall gain for the old individuals at the time of the initial change. Our Proposition 2 indicates, however, that if the change in political power is due to aging processes such a windfall gain need not occur. Moreover, even if such a windfall gain occurs, this does not have to go along with the initiation of a PAYG-scheme if such a scheme did previously not exist. If general-equilibrium effects raise the interest rate relatively strong, the consumption of the old can rise in spite of a decrease in the tax rate. Another striking difference with Hansson-Stuart is the resulting evolution of the public pension scheme. Their model generates a sequence of monotonously increasing tax rates. In the general-equilibrium framework adopted here, however, future tax rates may decrease even if the initial effect on the tax rate is positive. The point

is that the effect of increased political power is counteracted by the decreasing wage rate and the rising interest rate that result from the decrease in savings. In particular, if the elasticity of the production function (ν) is relatively high, the countervailing power of the changed factor payments may outweigh the positive effects on the tax rate. This demonstrates that the "over-insurance" result that is often found in (public-choice) models with exogenous interest rates may be mitigated if account is taken of general-equilibrium effects.

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Appendix

Comparative dynamics

The method of comparative dynamics works as follows. Let h_t describe the time pattern of the perturbation of the steady-state value of the parameter n and γ the magnitude. So $n_t = n + \gamma h_t$. The effects of a marginal change in n can then be traced by differentiation of the first-order Eqs. (4) and (12) and the resource constraint (8) with respect to γ . This yields the following linear system in the state variables k (capital stock per worker) and $c = c^y + \frac{c^r}{1+n}$ (total consumption per worker):

$$\begin{bmatrix} \frac{\partial k_{t+1}}{\partial \gamma} \\ \frac{\partial c_{t+1}}{\partial \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{\partial k_t}{\partial \gamma} \\ \frac{\partial c_t}{\partial \gamma} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \quad t = 0, 1, \dots \quad (i)$$

where M is a matrix describing the effects of the current and next-period change in the exogenous parameter on the state variables:

$$M = \begin{bmatrix} 0 & -\frac{k}{1+n} \\ \frac{c^r}{(1+n)^2} + \frac{\rho'}{1+\rho} \frac{1}{(1+n)\alpha^r} & \delta k - \frac{c^r}{(1+n)^2} + \frac{\rho'}{1+\rho} \frac{1}{\alpha^y} \end{bmatrix}, \quad (ii)$$

and J is the Jacobian matrix:

$$J = \begin{bmatrix} \frac{1+r}{1+n} & -\frac{1}{1+n} \\ -\delta(1+r) & 1+\delta \end{bmatrix}, \quad (iii)$$

where $\nu = \frac{-f''(k)k}{f'(k)}$ is the (positively defined) elasticity of the production function, $\delta = \frac{\nu r}{\alpha^y k(1+r)(1+n)(1-\varepsilon)} > 0$ and $\varepsilon = \frac{\alpha^y}{\alpha^y + (1+n)\alpha^r}$. The parameter α^y stands for the absolute rate of risk aversion of a young individual which is defined as $-\frac{u''(c^y)}{u'(c^y)}$. Analogously, α^r indicates the risk aversion of an old individual.

This system comprises one predetermined variable (k) and one forward-looking variable (c). The eigenvalues of the Jacobian matrix J are:

$$\begin{aligned}\lambda_1 &= \frac{1}{2} \left[2 + \frac{r-n}{1+n} + \delta - \sqrt{\left(\frac{r-n}{1+n} + \delta \right)^2 + 4\delta} \right] \\ \lambda_2 &= \frac{1}{2} \left[2 + \frac{r-n}{1+n} + \delta + \sqrt{\left(\frac{r-n}{1+n} + \delta \right)^2 + 4\delta} \right].\end{aligned}\quad (\text{iv})$$

As easily can be checked, one of these eigenvalues lies inside while the other lies outside the unit circle. So the system is saddlepoint stable and can be solved to find the initial effect of changes in n on total consumption (see Blanchard and Kahn 1980):

$$\begin{aligned}\frac{\partial c_0}{\partial \gamma} &= \frac{\lambda_1 - j_{11}}{j_{12}} \left[\frac{\partial k_0}{\partial \gamma} + m_{11} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_i + m_{12} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_{i+1} \right] \\ &\quad - m_{21} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_i - m_{22} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_{i+1},\end{aligned}\quad (\text{v})$$

where j_{ij} and m_{ij} ($i, j = 1, 2$) denote elements of J and M . The evolution of both k and c can then be derived from the system equations. Given the evolution of total consumption we can calculate the time paths of consumption by the young or the old using the definition of c . The effect on the pension system then follows.

Using (ii), (iii) and (v) it is straightforward to derive the initial effect of a change in population growth:²²

$$\begin{aligned}\frac{\partial c_0}{\partial \gamma} &= \left[\frac{\lambda_2 - 1}{\lambda_2} \left(\frac{c'}{(1+n)^2} - \lambda_2 k \right) + \frac{\rho'}{1+\rho} \left(\frac{1}{\alpha^y} + \frac{1}{(1+n)\alpha^r \lambda_2} \right) \right] \sum_{j=1}^{\infty} h_j \lambda_2^{-j} \\ &\quad - (1+n)(\delta + 1 - \lambda_2) \frac{\partial k_0}{\partial \gamma} + \left[\frac{\rho'}{1+\rho} \frac{1}{(1+n)\alpha^r} - \left(\frac{c'}{(1+n)^2} \right) \right] \frac{h_0}{\lambda_2}.\end{aligned}\quad (\text{vi})$$

The first term on the right hand side of this equation describes the initial effect on total consumption of an expected change in population growth while the second term and third term indicate the additional effect in case of an unexpected decrease in n ($h_0 < 0$). The latter effect is, of course, positive.²³ The effect of an unexpected once-and-for-all decrease in n is the sum of both terms with $h_0 = h_1 = \dots = h < 0$.

Endnotes

¹ See Breyer (1994) and Verbon (1993) for a survey of this literature.

² In addition to public pension schemes and savings, the elderly's consumption is often financed by private occupational pension schemes that are typically fully funded. In this paper we assume that private occupational pension schemes can be identified with individual savings for old age.

- ³ Hansson and Stuart use a two-overlapping-generations model where the decisions on public pension schemes are inherently constitutional, which implies that any living generation can block legislation that would leave it worse off. The initiation of a public pension scheme is then explained as a switch from private to collective decision making that favors current generations. This switch implies that a larger weight is given to the welfare of the old in determining the size of the transfers and results in a sequence of rising transfer payments and decreasing saving rates.
- ⁴ See e.g. Verbon and Verhoeven (1992).
- ⁵ See e.g. Van den Noord and Herd (1993).
- ⁶ Notice that an internal solution has been assumed. Excluding perverse solutions, the existence of such a solution will be guaranteed due to the requirement of equilibrium on the goods market to be discussed below.
- ⁷ As a consequence, the resulting policy satisfies stationary dynamic consistency by construction. Nonstationary dynamic consistency is not a relevant concept in this case (see Bernheim 1989).
- ⁸ Bernheim (1989) calls this "the most natural class of welfare functions for a representative government" (p. 124). An analogous approach to the political decision making process has been applied by Verbon (1988) and Hettich and Winer (1989).
- ⁹ As will be shown below, it is not even guaranteed that public pensions exist in the long run if $\rho(n) > 0$. It should also be noticed that the model has no stationary state in case $\rho(n) \leq 0$. This can be overcome by allowing for capital depreciation, however.
- ¹⁰ The results for this case can also be found in Meijdam and Verbon (1995a).
- ¹¹ Alternatively one could assume that the government acts as a Stackelberg-leader towards the private sector, i.e. that first the tax rate is fixed and then savings react to it. However, as observed in note 2, savings may be organized in pension funds. Within one period (one generation) these pension funds may react to the tax rate, but also the tax rate may react to the level of savings set by the pension funds. So neither of the players is a natural leader in this game. Therefore, the assumption of Nash-behavior seems most appropriate.
- ¹² When no time subscript is used the steady-state value is meant.
- ¹³ More strongly, if the economy is characterized by dynamic inefficiency in the steady state, it is not even guaranteed that a public pension scheme will exist in the steady state. So collective decision making will not necessarily lead to a transfer from the young to the old even if old individuals have the larger relative political power ($\rho(n) > 0$) and the economy is dynamically inefficient ($r = \rho(n) < n$). To demonstrate this, take the relation between old-age and young-age consumption in the steady state equal to ϕ , so that $c^y = \phi c^r = \frac{f(k) - nk}{\beta}$ where $\beta = 1 + \frac{\phi}{1+n}$ and the resource constraint (8) has been used. The tax rate can then be written, using Eqs. (1), (5), (6) and (7), as $\tau = f(k) - rk - \frac{f(k) - nk}{\beta} = (1+n)k$, which is positive if and only if $\frac{f(k)}{k} \geq \frac{(1+n+\phi)f'(k) + (1+n)(1+\phi+2n)}{\phi}$. For realistic values of the parameters of the utility function and the production function this inequality does not hold.
- ¹⁴ The effect of a change in n separately is analyzed in Meijdam and Verbon (1995b). The model used there is analogous to the one presented here, except for the way the decision making on pensions is modeled. There it is assumed that the government maximizes a Benthamite social welfare function. The results of a change in n for a given value of ρ carry over to the positive model presented here, however.
- ¹⁵ Most results can directly be generalized to temporary changes in these parameters.
- ¹⁶ This method is described by Judd (1982) for a continuous-time model. It can easily be generalized to discrete time, however. See Meijdam and Verhoeven (1994), who also show that the approximation of the short-run effects of a parameter change by this method is as reliable as a standard comparative-statics analysis. The method is explained in the Appendix.
- ¹⁷ δ and λ_2 are defined in the Appendix. There it is also shown that $\delta > 0$ and $\lambda_2 - 1 - \delta > 0$.
- ¹⁸ This is exactly the opposite of the capital-dilution effect that occurs in case of an increase in n (see e.g. Blanchet and Kessler 1991, p. 140).
- ¹⁹ Notice that this implies that in case the felicity function is of the CRRA type, i.e., the relative rate of risk aversion is constant, the generation with the largest level of per capita consumption gets the largest part of the change in total consumption.

- ²⁰ The stable root of the linearized system (λ_1) as well as the unstable root λ_2 is defined in the Appendix. We have assumed that $(1-\varepsilon)\varepsilon(\lambda_2-1)<1$. If this condition is not fulfilled the effects on young-age consumption are ambiguous.
- ²¹ The capital-thickening effect is reinforced by the unstable eigenvalue, however. This is due to the additional decrease in savings that is necessary in order to decrease the capital-labor ratio in anticipation of a future decrease in population growth.
- ²² We have used the fact that $(1+n)(\lambda_2-1) = \frac{\lambda_1-j_{11}}{j_{12}} + \delta(1+n)$.
- ²³ Note that $\frac{\partial k_0}{\partial \gamma} = \frac{-k}{1+n} h_0 > 0$. Moreover, given the fact that $\frac{\lambda_1-j_{11}}{j_{12}} = -\frac{j_{11}}{\lambda_2-j_{11}} > 0$ it immediately follows from footnote (22) that $\delta+1-\lambda_2 < 0$.