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**STRATEGIC CAPACITY INVESTMENT UNDER
UNCERTAINTY WITH VOLUME FLEXIBILITY**

By

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Strategic Capacity Investment under Uncertainty with Volume Flexibility

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Abstract

This article considers investment decisions in an uncertain and competitive framework, with a first investor, the leader, always producing up to full capacity and a second investor, the follower, capable of adjusting output levels within the constraint of installed capacity. Both firms need to decide on the investment timing and the investment capacity levels. The main findings are as follows. Compared to a situation where the follower always produces up to full capacity, the leader has a larger incentive to accommodate a flexible follower. This is because the leader also benefits from the follower's volume flexibility. Due to the first mover advantage, the leader's value is higher than the follower's value, despite the follower's technological advantage in flexibility.

Keywords: Investment under Uncertainty, Volume Flexibility, Entry Deterrence/Accommodation, Capacity Choice, Duopoly

JEL classification: E22, C73, D81

1 Introduction

Uncertainty is a main characteristic of the business environment nowadays. The technology advancement has shortened product life cycles, increased product variety, and indulged more demanding consumers. This contributes to the uncertainty in consumer demand and poses challenges on the manufacturing firms. The ability to produce to the least cost is no longer enough. The capability to absorb demand fluctuations has become an important competitive issue. Flexibility is considered as an adaptive response to the environmental uncertainty (Gupta and Goyal, 1989). Browne et al. (1984) define eight different types of flexibilities, among which the volume flexibility is described as “the ability to operate an FMS (Flexible Manufacturing

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Systems) profitably at different production volumes.” Sethi and Sethi (1990) further describe volume flexibility of a manufacturing system as “its ability to be operated profitably at different overall output levels.” A key strategic objective of many manufacturing firms is to utilize flexibility (Beach et al., 2000) and the adoption of flexibility should be included in the strategic decision making process. One strategy is to “bank” flexibility, holding it in reserve to meet future needs. Then, the flexibility is treated as an investment which creates options for the firm (Gerwin, 1993). An important question for the decision makers is when and how much to invest, and how the flexibility influences the investment decisions in a competitive setting.

This article considers volume flexibility in a homogenous good market. Demand is linear and subject to stochastic shocks, which follow a geometric Brownian motion process. There are two firms that decide on entering the duopoly market by investing in a production plant. More specifically, they have to decide about the timing and the investment capacity. One firm, the leader, invests first in a dedicated technology. In the future, a more advanced technology such as volume flexibility might be available. If it happens, the other firm, i.e., the follower who invests secondly, has free access to such a technology. The leader has a first mover advantage but always has to produce up to capacity due to the technology being dedicated. The follower can adjust the output levels according to market demand. A surprising outcome of our research is that, since the market price is affected by the follower’s flexible output, the leader benefits from the follower’s flexibility when market demand is low. This is because the follower reduces the output quantity in such a case.

Our analysis starts with a market where no firms are active. Then two domains on market sizes are identified for the leader, with one domain where it is optimal to deter the entry of the flexible follower and the other one where it is optimal to accommodate the entry. For case of a dedicated follower, Huisman and Kort (2015) have shown that entry deterrence domain increases with uncertainty. We show that this still holds when the follower is flexible. Besides, we find that compared with a dedicated follower, the leader is less likely to deter a flexible follower. This is because when there is uncertainty about market demand, both the leader and the flexible follower tend to wait for more information about the market and invest later. For the entry deterrence strategy, the leader has an incentive to overinvest to deter the entry of the follower. Incapable of adjusting to the instant market demand, the leader is more vulnerable to the negative demand shocks. For the follower, the volume flexibility yields higher values and thus motivates to invest earlier compared with a dedicated follower. This results in a shorter monopoly period for the leader and diminishes the attractiveness of entry deterrence than in the case where the follower is dedicated. Furthermore, compared with a dedicated follower, it is more likely for the leader to accommodate a flexible follower. For the accommodation strategy, the two firms invest at the same time, so the incentive to overinvest in order to deter the follower’s entry disappears. The market price reacts to the follower’s output adjustment, and this diminishes the leader’s vulnerability to demand uncertainty. The incentive to overinvest in order to reduce the capacity size of the flexible follower and to benefit from the follower’s output adjustment is still strong. This makes accommodation of the flexible follower more attractive to the leader.

We also find that in a fast growing market, the flexible follower produces below capacity right after

investment. While in a slowly growing or shrinking market, the flexible follower produces up to capacity right after investment. In the intermediate case, the flexible follower produces up to capacity right after investment when uncertainty is low and below capacity when uncertainty is high. These findings are consistent with that for the flexible monopolist by Wen et al. (2017). The strategic interactions between the leader and the flexible follower do not influence these results. Moreover, there is free riding on the follower's flexibility since the volume flexibility affects market prices, and thus enlarges the profitability of the leader. So, the flexible follower cannot fully capture the innovative benefits from the technology advancement. However, this does not diminish the follower's incentive to invest in the volume flexibility technology, because it still generates a larger value for the follower whether the leader chooses and entry deterrence or entry accommodation strategy.

The duopoly model with volume flexibility first contributes to the research stream of monopolistic volume flexibility investment combining investment timing and capacity determination, by Dangl (1999), Hagspiel et al. (2016), and Wen et al. (2017). The general result is that flexibility leads to an increase in the optimal installed capacity and project value. The influence of flexibility on investment timing depends on two effects, with one effect that higher value motivates a flexible firm to invest earlier and the other effect that larger installed capacity motivates it to invest later. This article shows that flexibility affects the flexible follower in a similar way as it affects the flexible monopolist. Its influence on the leader depends on the leader's competition strategy, and the dedicated leader also gets higher values when playing accommodation strategy.

In this research, firms not only make decisions about capacities, but also about investment timings. It contributes to the literature of capacity choices with volume flexibility in a competitive framework. Gabszewicz and Poddar (1997) study a two-stage model with capacity choice in the first stage and capacity constrained quantity competition in the second stage and show that the firms choose the certainty-equivalent Cournot capacity. If the second stage is a capacity-constrained price competition instead of quantity competition, Reynolds and Wilson (2000) find that symmetric equilibrium does not exist in pure strategies for capacity choices if demand is sufficient volatile. Besanko and Doraszelski (2004) consider two models: quantity competition and price competition in each period of an infinite time horizon. Quantity competition results in an industry structure of equal-sized firms, while price competition results in unequal-sized firms. Besides the economics literature, volume flexibility is also studied in operations management. For example, Anupindi and Jiang (2008) consider the volume flexibility in a three-stage framework: capacity choice in the first stage, production decisions in the second stage and pricing decisions in the third stage. Flexible firms can make production decisions when demand is observed. Under competition, they find that firms choose to be inflexible for multiplicative demand shocks, while flexible for additive demand shocks. In a two-product setting, Goyal and Netessine (2011) introduce volume flexibility and find that volume flexibility combats aggregate demand uncertainty for the two products. Current research on volume flexibility focuses on the capacity choices and adopts discrete time models in the analysis. By using continuous time model in this research,

this article not only analyzes the decision of investment capacity, but also the decision of investment timing. More specifically, this research analyzes the influence of volume flexibility on the timing of market entry. In a competitive setting, the first investor has a larger incentive to accommodate than to deter the entry of the second investor, given the second investor has volume flexibility. This is due to the fact that volume flexibility combats demand uncertainty for both investors in the market, similarly as that proposed by Goyal and Netessine (2011) for two products.

The duopoly model with flexibility in this research also extends the literature on entry deterrence and entry accommodation investment. Spence (1977) and Dixit (1980) construct static investment models to show that entry can be deterred by installing excess capacity. Maskin (1999) introduces uncertainty and obtains the same conclusion. By discrete time models, Reynolds (1987) shows that the equilibrium capacity choice is a decreasing function of the current rival capacity. Besanko et al. (2010) argue that preemption is more likely when the products in the market have low heterogeneity and there is uncertainty about the entrant's exact cost/benefit of capacity addition/withdrawal. In this article, the products are homogeneous, time is continuous, and there is uncertainty about the market demand. It is shown that in the continuous time setting, excess capacity can also help the first investor to deter the entry of the second investor and the second investor's optimal capacity decreases with the first investor's capacity. However, the first investor's optimal investment timing and capacity are independent of the second investor's volume flexibility. Moreover, when the second investor has volume flexibility, the entry accommodation is more likely when the products are homogenous and there is market demand uncertainty. For the given incumbent's decisions, Yang and Zhou (2007) show that it is impossible for the incumbent with excess capacity to deter the potential entrant who holds the option to entry forever. This result is supported also by Huisman and Kort (2015), who not only consider about the deterrence of the potential entrant, but also the possibility of accommodation of the potential entrant. They construct the domains on market sizes of entry deterrence and accommodation strategy for the duopoly setting where the only difference between investors is the cost advantage for the first investor. In this article, the difference between the two investors is that the leader always producing up to full capacity, while the follower can adjust the output within the capacity constraint. Similar to Huisman and Kort (2015), this article also constructs the domains for the leader's entry strategies. By comparing situation of volume flexibility with situation of no volume flexibility, this article shows that the first investor has less incentive to deter the entry of the second investor if the second investor has volume flexibility.

This article is organized as follows. Section 2 describes the duopoly investment problem. Section 3 analyses the flexible follower's optimal investment decision. The dedicated leader's optimal investment decision is in Section 4. In Section 5, the influence of flexibility on the leader and the follower is analysed. Section 6 concludes.

2 Model Setup

Consider a framework where two firms can invest in production capacity to enter a market or serve a particular demand. Of the two firms, the follower (second investor) can access volume flexibility technology and adjust output levels up to the installed capacity after the investment. The leader (first investor) has no access to such technology and can only produce at full capacity level. Denote by $K_D \geq 0$ and $K_F \geq 0$ the capacity of the flexible follower and dedicated leader, respectively. For both firms, the unit cost for capacity investment and is $\delta > 0$ and the unit cost for production is $c > 0$. The price at time $t \geq 0$ is $p(t)$, given by the inverse demand function

$$p(t) = X(t) [1 - \gamma (q_D(t) + q_F(t))],$$

where $\gamma > 0$ is a constant, $q_D(t)$ and $q_F(t)$ denote the production output for the dedicated and flexible firm at time t , respectively, and the uncertainty in demand, $\{X(t)|t \geq 0\}$, follows a Geometric Brownian Motion (GBM) process

$$dX(t) = \alpha X(t)dt + \sigma X(t)dW_t,$$

in which $X(0) > 0$, α is the trend parameter, $\sigma > 0$ is the volatility parameter, and dW_t is the increment of a Wiener process. The inverse linear demand function has been adopted by Pindyck (1988) and Huisman and Kort (2015). Both firms are risk neutral and have a discount rate of r , which is assumed to be larger than α , the trend of GBM $X(t)$ (see Dixit and Pindyck, 1994), and larger than $\sigma^2 - \alpha$, the trend for GBM $\{1/X(t)\}$. This is to prevent that it is optimal for the firms to always delay the investment.

3 Flexible Follower's Optimal Investment Decision

The dedicated leader is assumed to be already in the market when the follower makes investment decisions. Given $X(t) = X$ and the leader's investment capacity K_D , denote $\pi_F(X, K_D, K_F)$ as the profit for the flexible follower after investing K_F . The flexible follower's investment decision is solved as an optimal stopping problem in dynamic programming, which can be formalised as

$$\max_{T \geq 0, K_F \geq 0} E \left[\int_T^\infty \pi_F(X(t), K_D, K_F) \exp(-rt) dt - \delta K_F \exp(-rT) \middle| X(0) \right],$$

conditional on the available information at time 0, where T is the time when the flexible follower invests, and K_F is the acquired capacity at time T . Afterwards, the follower can adjust its production between 0 and the invested capacity K_F . Denote by $V_F(X, K_D, K_F)$ the value for the flexible follower. The output by the flexible follower maximises the profit flow, which is

$$\pi_F(X, K_D, K_F) = \max_{0 \leq q_F \leq K_F} \{X [1 - \gamma (K_D + q_F)] - c\} q_F.$$

Given $0 \leq K_D < 1/\gamma$, the optimal production level for the follower is

$$q_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{X-c}{2\gamma X} - \frac{K_D}{2} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ K_F & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \quad (1)$$

The corresponding profit flow is

$$\pi_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{(X-c-\gamma X K_D)^2}{4\gamma X} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ (X-c-\gamma X K_D) K_F - K_F^2 \gamma X & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \quad (2)$$

The follower's value $V_F(X, K_D, K_F)$ satisfies the Bellman equation

$$rV_F = \pi_F + \frac{1}{dt} E[dV_F]. \quad (3)$$

Applying Ito's Lemma, substituting and rewriting lead to the following differential equation

$$\frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V_F}{\partial X^2} + \alpha X \frac{\partial V_F}{\partial X} - rV_F + \pi_F = 0. \quad (4)$$

Substituting (2) into (4) and employing value matching and smooth pasting for $X = c/(1 - \gamma K_D)$ and $X = c/(1 - \gamma K_D - 2\gamma K_F)$ yield the follower's value as

$$V_F(X, K_D, K_F) = \begin{cases} L(K_D, K_F) X^{\beta_1} & 0 < X < \frac{c}{1-\gamma K_D}, \\ M_1(K_D, K_F) X^{\beta_1} + M_2(K_D) X^{\beta_2} & \\ + \frac{(1-\gamma K_D)^2 X}{4\gamma(r-\alpha)} - \frac{c(1-\gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r+\alpha-\sigma^2)} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ N(K_D, K_F) X^{\beta_2} - \frac{cK_F}{r} + \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r-\alpha} X & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \end{cases} \quad (5)$$

in which

$$L(K_D, K_F) = \frac{c^{1-\beta_1} \left[(1-\gamma K_D)^{1+\beta_1} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_1} \right]}{4\gamma(\beta_1 - \beta_2)} F(\beta_2), \quad (6)$$

$$M_1(K_D, K_F) = -\frac{c^{1-\beta_1} (1-2\gamma K_F - \gamma K_D)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} F(\beta_2), \quad (7)$$

$$M_2(K_D) = \frac{c^{1-\beta_2} (1-\gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1 - \beta_2)} F(\beta_1), \quad (8)$$

$$N(K_D, K_F) = \frac{c^{1-\beta_2} \left[(1-\gamma K_D)^{1+\beta_2} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_2} \right]}{4\gamma(\beta_1 - \beta_2)} F(\beta_1), \quad (9)$$

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (10)$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (11)$$

with

$$F(\beta) = \frac{2\beta}{r} - \frac{\beta - 1}{r - \alpha} - \frac{\beta + 1}{r + \alpha - \sigma^2}. \quad (12)$$

The derivation of $L(K_D, K_F)$, $M_1(K_D, K_F)$, $M_2(K_D)$, $N(K_D, K_F)$ can be found in Appendix A.1. If $K_D = 0$, then the situation is just like that in the monopoly case. $L(K_D, K_F)X^{\beta_1}$ is positive and represents the option value to start producing in the future as soon as X reaches $c/(1 - \gamma K_D)$. $M_1(K_D, K_F)X^{\beta_1}$ is negative and corrects for the fact that if X reaches $c/(1 - \gamma K_D - 2\gamma K_F)$, the follower's output will be constrained by the installed capacity level. $M_2(K_D)X^{\beta_2}$ is negative and corrects for the positive quadratic form of cash flows such that when X drops below $c/(1 - \gamma K_D)$, the follower would temporarily suspend the production. $N(K_D, K_F)X^{\beta_2}$ is positive and describes the option value that if demand decreases, e.g. X drops below $c/(1 - \gamma K_D - 2\gamma K_D)$, the follower produces below full capacity. The optimal investment decision is found in two steps. First, given K_D and the level of X , the optimal value of K_F is found by maximising $V_F(X, K_D, K_F) - \delta K_F$, which yields $K_F(X, K_D)$. Second, the optimal investment threshold $X_F^*(K_D)$ for the follower can be derived. The two steps are summarised in the following proposition, where $\bar{\sigma} > 0$ is such that

$$\bar{\sigma}^2 = \frac{-2(\Lambda - \alpha^2)(2r - \alpha) - 4\sqrt{r\Lambda(\Lambda - \alpha^2)(r - \alpha)}}{\Lambda - (2r - \alpha)^2}, \quad (13)$$

with $\Lambda = \left(\frac{2\delta r(r - \alpha) - \alpha c}{c}\right)^2$. The proof can be found in Appendix A.2.

Proposition 1 *Given the dedicated firm has already invested in capacity $K_D \in [0, 1/\gamma)$, there are two possibilities regarding the follower's investment decisions:*

1. *Suppose $\alpha > \delta r^2/(c + \delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma > \bar{\sigma}$. Then the follower does not produce up to capacity right after investment. For any $X \geq c/(1 - \gamma K_D)$, the optimal capacity $K_F(X, K_D)$ that maximizes $V(X, K_D, K_F) - \delta K_F$ is*

$$K_F(X, K_D) = \frac{1}{2\gamma} \left\{ 1 - \gamma K_D - \frac{c}{X} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\}, \quad (14)$$

and the optimal investment threshold $X_F^(K_D)$ satisfies*

$$\begin{aligned} & \frac{c^{1-\beta_2}(1 - \gamma K_D)^{1+\beta_2}F(\beta_1)X^{\beta_2}}{4\gamma\beta_1} + \frac{1}{4\gamma} \left[\frac{\beta_1 - 1}{\beta_1} \frac{(1 - \gamma K_D)^2 X}{r - \alpha} - \frac{2c(1 - \gamma K_D)}{r} \right. \\ & \left. + \frac{\beta_1 + 1}{\beta_1} \frac{c^2}{X(r + \alpha - \sigma^2)} \right] - \delta K_F(X, K_D) = 0. \end{aligned} \quad (15)$$

If $X(0) < X_F^(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F(X_F^*(K_D), K_D)$. If $X(0) \geq X_F^*(K_D)$, then the follower invests immediately at $t = 0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.*

2. Suppose $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma \leq \bar{\sigma}$. Then the follower produces up to capacity right after the investment. For any $X \geq c/(1 - \gamma K_D)$, the optimal capacity $K_F(X, K_D)$ that maximizes $V(X, K_D, K_F) - \delta K_F$ satisfies

$$\frac{F(\beta_1) c^{1-\beta_2} (1 + \beta_2) (1 - 2\gamma K_F - \gamma K_D)^{\beta_2}}{2(\beta_1 - \beta_2)} X^{\beta_2} + \frac{1 - 2\gamma K_F - \gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta = 0, \quad (16)$$

and the optimal investment threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned} & \frac{c^{1-\beta_2} F(\beta_1) X^{\beta_2}}{4\gamma\beta_1} [(1 - \gamma K_D)^{1+\beta_2} - (1 - 2\gamma K_F - \gamma K_D)^{1+\beta_2}] \\ & + \frac{(\beta_1 - 1)X}{\beta_1} \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r - \alpha} - \frac{cK_F}{r} - \delta K_F = 0, \end{aligned} \quad (17)$$

with $K_F = K_F(X, K_D)$. If $X(0) < X_F^*(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F(X_F^*(K_D), K_D)$. If $X(0) \geq X_F^*(K_D)$, then the follower invests immediately at $t = 0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.

From Proposition 1, the influence of the leader's investment capacity on the follower's investment decision is concluded in Corollary 1. The proof is given in Appendix A.3.

Corollary 1 *Regardless of whether the flexible follower produces below or up to capacity right after investment, the dedicated leader's capacity level K_D influences the follower's investment decision such that*

$$\begin{aligned} \frac{dX_F^*(K_D)}{dK_D} &= \frac{\gamma X_F^*(K_D)}{1 - \gamma K_D} > 0; \\ \frac{dK_F^*(K_D)}{dK_D} &= -\frac{\gamma K_F^*(K_D)}{1 - \gamma K_D} < 0. \end{aligned}$$

This is understandable because the leader always produces up to capacity after investment, and the more the leader invests, the less market demand is left for the flexible follower. The shrink in the market share delays the follower's investment timing and decreases the investment capacity because the follower needs to wait for a larger market demand.

4 Dedicated Leader's Optimal Investment Decision

When deciding on the investment timing and capacity, the leader takes the follower's investment decision into account. For a given level of $X(t) = X$ at time t , suppose the leader invests at t with capacity size K_D . Because the follower's optimal threshold $X_F^*(K_D)$ increases with K_D , the leader can invest in a larger (smaller) capacity size such that the follower invests later (earlier). Assume there exists a critical capacity size for the leader, $\hat{K}_D(X)$, such that the follower's optimal threshold satisfies $X_F^*(\hat{K}_D) = X$. From Corollary 1, it can be concluded that if $K_D \leq \hat{K}_D(X)$, then $X \geq X_F^*(K_D)$, implying that the follower invests at the same time with the leader. If $K_D > \hat{K}_D(X)$, then $X < X_F^*(K_D)$, which implies that the follower invests later than the leader. The former case corresponds to the leader's entry accommodation strategy and the latter

corresponds to its entry deterrence strategy, as described by Huisman and Kort (2015). In the following analysis, the dedicated leader's value function is derived and the leader's entry accommodation and entry deterrence strategy will be analyzed.

To derive the leader's value function, first take a look at the leader's profit $\pi_D(X, K_D)$ for a given GBM level X and the invested capacity size K_D , when both firms are active in the market. Because of volume flexibility, the follower might not produce or produce below or up to capacity after investment. For each of these three cases, the leader's profit flow is

$$\pi_D(X, K_D) = \begin{cases} K_D(1 - \gamma K_D)X - cK_D & \text{if } 0 < X < \frac{c}{1 - \gamma K_D}; \\ \frac{K_D}{2}(X - c - \gamma X K_D) & \text{if } X \geq \frac{c}{1 - \gamma K_D} \text{ and } K_F^*(K_D) > \frac{X - c}{2\gamma X} - \frac{K_D}{2}; \\ XK_D[1 - \gamma(K_D + K_F^*(K_D))] - cK_D & \text{if } X \geq \frac{c}{1 - \gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X - c}{2\gamma X} - \frac{K_D}{2}. \end{cases}$$

Similar to the analysis for the follower, the leader's value $V_D(X, K_D)$ and profit $\pi_D(X, K_D)$ satisfy the following differential equation

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_D}{\partial X^2} + \alpha X \frac{\partial V_D}{\partial X} - rV_D + \pi_D = 0.$$

Substituting π_D into this differential equation and employing value matching and smooth pasting at $X = c/(1 - \gamma K_D)$ and $X = c/(1 - \gamma K_D - 2\gamma K_F^*(K_D))$ give the value of the leader as

$$V_D(X, K_D) = \begin{cases} \mathcal{L}(K_D)X^{\beta_1} + \frac{K_D(1 - \gamma K_D)}{r - \alpha}X - \frac{cK_D}{r} & \text{if } 0 \leq X < \frac{c}{1 - \gamma K_D}; \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} & \text{if } X \geq \frac{c}{1 - \gamma K_D} \text{ and } K_F^*(K_D) > \frac{X - c}{2\gamma X} - \frac{K_D}{2}; \\ + \frac{XK_D(1 - \gamma K_D)}{2(r - \alpha)} - \frac{cK_D}{2r} & \\ \mathcal{N}(K_D)X^{\beta_2} - \frac{cK_D}{r} & \text{if } X \geq \frac{c}{1 - \gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X - c}{2\gamma X} - \frac{K_D}{2}. \\ + \frac{K_D(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha}X & \end{cases} \quad (18)$$

with

$$\mathcal{L}(K_D) = \frac{c^{1 - \beta_1} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left[(1 - \gamma K_D)^{\beta_1} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_1} \right], \quad (19)$$

$$\mathcal{M}_1(K_D) = -\frac{c^{1 - \beta_1} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_1}, \quad (20)$$

$$\mathcal{M}_2(K_D) = \frac{c^{1 - \beta_2} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (1 - \gamma K_D)^{\beta_2}, \quad (21)$$

$$\mathcal{N}(K_D) = \frac{c^{1 - \beta_2} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2} \right]. \quad (22)$$

The derivation of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$ and $\mathcal{N}(K_D)$ and their signs can be found in Appendix A.4.

For $0 \leq X < c/(1 - \gamma K_D)$, the demand is very small and the follower's production is temporarily suspended. However, incapable of adjusting to the market demand, the leader still produces at full capacity.

In the leader's value function, $\mathcal{L}(K_D)X^{\beta_1}$ measures the decrease in the leader's value when the follower resumes production in the future. This happens as soon as X becomes larger than $c/(1-\gamma K_D)$. For $c/(1-\gamma K_D) \leq X < c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, e.g., $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) > (X-c)/(2\gamma X) - K_D/2$, the follower produces below capacity right after investment. $\mathcal{M}_1(K_D)X^{\beta_1}$ corrects for the fact that if X reaches $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, then the production of the follower is constrained by the follower's installed capacity, hence the value of the leader increases. The term $\mathcal{M}_2(K_D)X^{\beta_2}$ denotes the decrease in the leader's option value. This is due to the fact that when X falls below $c/(1-\gamma K_D)$, the market demand becomes so small that the follower suspends production, whereas the leader still produces at full capacity, which results in negative profit. For $X \geq c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, e.g., $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) \leq (X-c)/(2\gamma X) - K_D/2$, the follower produces up to capacity right after investment. The term $\mathcal{N}(K_D)X^{\beta_2}$ corrects for the fact that if X drops below $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, then the follower produces below capacity, and the value of the leader would increase.

Before the market entry of the follower, suppose the dedicated leader's value takes the following form

$$V_D(X, K_D) = \mathcal{B}(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r}.$$

After the follower's entry, the leader's value, for the cases that the follower produces below capacity and up to capacity right after investment, is shown in (18). Two leader's strategies are analysed as follows, namely the entry deterrence strategy and entry accommodation strategy.

- The flexible follower produces below capacity right after the investment when $\alpha > \delta r^2/(c + \delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma > \bar{\sigma}$.

In order to analyse the leader's entry deterrence and entry accommodation strategies, the leader's value function before and after the follower's entry is as follows

$$V_D(X, K_D) = \begin{cases} \mathcal{B}_1(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} & \text{before,} \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} & \text{after.} \end{cases} \quad (23)$$

with

$$\begin{aligned} \mathcal{B}_1(K_D) = & \mathcal{M}_1(K_D) + \mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) \\ & - \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) + \frac{cK_D}{2r}X_F^{*-\beta_1}(K_D), \end{aligned} \quad (24)$$

according to value matching at the follower's investment threshold $X_F^*(K_D)$, which is defined by (15). Intuitively, $\mathcal{B}_1(K_D)$ is negative (see Appendix A.5.1). It corrects for the fact that when $X(t)$ reaches $X_F^*(K_D)$, the follower enters the market, putting an end to the leader's monopolistic privilege. The leader can delay the follower's entry by investing an entry deterrence capacity $K_D^{det}(X) > \hat{K}_D(X)$. The leader can also invest with an entry accommodation capacity $K_D^{acc}(X) \leq \hat{K}_D(X)$, such that the two firms would invest at the same time. This critical capacity size $\hat{K}_D(X)$ can be derived from (15) as to satisfy

$$\frac{c^{1-\beta_2}X^{\beta_2}(1-\gamma K_D)^{1+\beta_2}F(\beta_1)}{2\beta_1} + \frac{\beta_1-1}{2\beta_1} \frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{c(1-\gamma K_D)}{r}$$

$$+ \frac{\beta_1 + 1}{2\beta_1} \frac{c^2}{X(r + \alpha - \sigma^2)} - \delta(1 - \gamma K_D) + \frac{c\delta}{X} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} = 0. \quad (25)$$

The leader's entry deterrence and accommodation strategies when the follower produces below capacity right after investment are described in the following proposition.

Proposition 2 Suppose $\alpha > \delta r^2/(c + \delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma > \bar{\sigma}$.

(a) *Entry Deterrence Strategy*

Entry deterrence strategy will be considered whenever $X \in (X_1^{det}, X_2^{det})$, where X_1^{det} satisfies

$$\left[-\frac{\delta}{(1 + \beta_1)F(\beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) + \frac{c^{1-\beta_2} X_F^{*\beta_2}(0)}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) - \frac{X_F^*(0)}{2(r - \alpha)} + \frac{c}{2r} \right] \left(\frac{X^{det}}{X_F^*(0)} \right)^{\beta_1} + \frac{X^{det}}{r - \alpha} - \frac{c}{r} - \delta = 0, \quad (26)$$

and X_2^{det} together with $K_D^{det}(X_2^{det})$ satisfy equations (25) and

$$\frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_1(K_D)(X^{det})^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X^{det} - \frac{c}{r} - \delta = 0. \quad (27)$$

The optimal investment capacity K_D^{det} and investment threshold X_D^{det} under the entry deterrence strategy are

$$\begin{aligned} K_D^{det} &= \frac{1}{(\beta_1 + 1)\gamma}, \\ X_D^{det} &\equiv X^{det}(K_D^{det}) = \frac{(\beta_1 + 1)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right), \end{aligned}$$

when $X < X_D^{det}$ and $X_D^{det} \in [X_1^{det}, X_2^{det}]$. If $X_D^{det} \leq X \leq X_2^{det}$, in order to implement the entry deterrence strategy, the leader invests immediately at X with capacity $K_D^{det}(X)$, which satisfies (27). The value of the entry deterrence strategy is

$$V_D^{det}(X) = \mathcal{B}_1(K_D^{det}(X))X^{\beta_1} + \frac{K_D^{det}(X)(1 - \gamma K_D^{det}(X))}{r - \alpha} X - \frac{cK_D^{det}(X)}{r}. \quad (28)$$

(b) *Entry Accommodation Strategy*

Entry accommodation strategy will be considered if $X \geq X_1^{acc}$, where X_1^{acc} and the corresponding $K_D^{acc}(X_1^{acc})$ satisfy (25) and

$$\begin{aligned} \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_1(K_D)(X^{acc})^{\beta_1} + \frac{1 - \gamma K_D - \beta_2 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_2(K_D)(X^{acc})^{\beta_2} \\ + \frac{1 - 2\gamma K_D}{2(r - \alpha)} X^{acc} - \frac{c}{2r} - \delta = 0. \end{aligned} \quad (29)$$

The optimal investment capacity for the entry accommodation strategy is

$$K_D^{acc} = \frac{1}{(\beta_1 + 1)\gamma},$$

and the optimal investment threshold $X_D^{acc} \equiv X^{acc}(K_D^{acc})$ satisfies

$$\frac{c}{2\beta_1} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{\beta_1 X^{acc}}{c(\beta_1 + 1)} \right)^{\beta_2} + \frac{(\beta_1 - 1)X^{acc}}{2(r - \alpha)(\beta_1 + 1)} - \frac{c}{2r} - \delta = 0. \quad (30)$$

when $X < X_D^{acc}$ and $X_D^{acc} \in [X_1^{acc}, +\infty)$. If $X \geq X_D^{acc}$, in order to implement the entry accommodation strategy, the leader invests immediately at X with capacity $K_D^{acc}(X)$ that satisfies (29). The value of the entry accommodation strategy is

$$V_D^{acc}(X) = \mathcal{M}_1(K_D^{acc}(X))X^{\beta_1} + \mathcal{M}_2(K_D^{acc}(X))X^{\beta_2} + \frac{K_D^{acc}(X)(1 - \gamma K_D^{acc}(X))}{2(r - \alpha)}X - \frac{cK_D^{acc}(X)}{2r}. \quad (31)$$

A numerical example is provided to demonstrate how the leader plays against a flexible follower when the follower produces below capacity right after investment. Figure 1 illustrates the capacity levels $K_D^{det}(X)$, $K_D^{acc}(X)$ and $\hat{K}_D(X)$ as functions of X . For the given parameter values, the leader would consider the entry deterrence strategy for $X \in [X_1^{det}, X_2^{det}]$, and the entry accommodation strategy for $X \geq X_1^{acc}$. When both strategies are applicable, the dedicated leader would choose the strategy that generates higher value.

Figure 1: Illustration of $\hat{K}_D(X)$, $K_D^{det}(X)$, and $K_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

More specifically, for the given parameter values in Figure 1, $X_1^{det} = 2.42$, $X_2^{det} = 11.73$ and the corresponding capacities are $K_D^{det}(X_1^{det}) = 6.04$ and $K_D^{det}(X_2^{det}) = 9.70$. The optimal threshold for entry deterrence strategy is $X_D^{det} = 6.30$. Suppose the current level of geometric Brownian motion is X . If $X < 6.30$, to delay the entry of the flexible follower, the leader waits until X reaches 6.30. For any X between 6.30 and 11.73, the leader needs to invest immediately to delay the flexible follower. For $X > 11.73$, the entry deterrence strategy is not possible because the market demand is large enough to hold both firms. Moreover, $X_D^{acc} = 8.50$ and $X_1^{acc} = 9.23$, which implies the accommodation strategy is to invest immediately when $X(t)$ reaches 9.23. $X_D^{acc} < X_1^{acc}$ would make X_D^{acc} meaningless for this numerical example.

Figure 2: Illustration of $V_D^{det}(X)$ and $V_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 2 shows the value of the entry deterrence strategy V_D^{det} and accommodation strategy V_D^{acc} as functions of X , when the flexible follower produces below capacity right after investment. For $X_1^{det} < X < \hat{X}$, the entry deterrence strategy is chosen by the leader and the leader would invest at $X_D^{det} = 6.30$ with capacity $K_D(X^{det}) = 6.67$. For $X \geq \hat{X}$, because $\hat{X} > X_1^{acc}$, the leader would choose entry accommodation strategy by investing immediately with capacity level $K_D^{acc}(X)$.

- The flexible follower produces up to capacity right after the investment when $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma^2 \leq \bar{\sigma}^2$.

Similar to the case where the flexible follower produces below capacity right after investment, the leader's value function before and after the follower's entry can be written as

$$V_D(X, K_D) = \begin{cases} \mathcal{B}_2(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} & \text{before,} \\ \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X - \frac{cK_D}{r} & \text{after,} \end{cases} \quad (32)$$

with $\mathcal{B}_2(K_D)$

$$\mathcal{B}_2(K_D) = \mathcal{N}(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r-\alpha}X_F^{*1-\beta_1}(K_D), \quad (33)$$

according to the value matching condition at the flexible follower's investment threshold $X_F^*(K_D)$, which is defined by (17).

Similar as $\mathcal{B}_1(K_D)$, $\mathcal{B}_2(K_D)$ corrects for fact that when the follower enters the market, i.e. X reaches $X_F^*(K_D)$, it would put an end to the leader's monopoly privilege. Thus, $\mathcal{B}_2(K_D)$ is negative, which proof can be found in Appendix A.6.1. Because $X_F^*(K_D)$ increases with K_D according to Corollary 1, it is possible for

the dedicated leader to delay the entry of flexible follower through the entry deterrence strategy by investing $K_D^{det}(X) > \hat{K}_D(X)$. Otherwise, the two firms invest at the same time, implying the entry accommodation strategy of the leader by investing $K_D^{acc} \leq \hat{K}_D(X)$. This critical size for the leader's capacity, $\hat{K}_D(X)$, can be derived from (17) with the follower's optimal investment capacity $K_F^*(X) \equiv K_F^*(K_D(X))$ satisfying (16).

The leader's investment decision under entry deterrence and accommodation strategies, when the follower produces up to capacity right after investment, are summarised in the following proposition.

Proposition 3 Suppose $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c + \delta r)$ and $\sigma^2 \leq \bar{\sigma}^2$.

(a) *Entry Deterrence Strategy*

The entry deterrence strategy is possible if $X \in (X_1^{det}, X_2^{det})$. X_2^{det} , $K_D^{det}(X_2^{det})$ and $K_F^*(X_2^{det})$ satisfy (16), (17) and

$$\frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_2(K_D)(X^{det})^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X^{det} - \frac{c}{r} - \delta = 0. \quad (34)$$

X_1^{det} satisfies

$$\begin{aligned} \frac{c}{2(\beta_1 - \beta_2)} \left(\frac{X^{det}}{X_F^*(0)} \right)^{\beta_1} \left\{ \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[\left(\frac{X_F^*(0)}{c} \right)^{\beta_2} - \left(\frac{X_F^*(0)(1 - 2\gamma K_F^*(0))}{c} \right)^{\beta_2} \right] \right. \\ \left. - \frac{\beta_1 - \beta_2}{r - \alpha} \frac{2\gamma X_F^*(0) K_F^*(0)}{c} \right\} + \frac{X^{det}}{r - \alpha} - \frac{c}{r} - \delta = 0, \end{aligned} \quad (35)$$

with $K_F^*(0)$ and $X_F^*(0)$ are such that

$$\frac{c(1 + \beta_2) F(\beta_1)}{2(\beta_1 - \beta_2)} \left(\frac{X_F^*(0)(1 - 2\gamma K_F^*(0))}{c} \right)^{\beta_2} + \frac{X_F^*(0)(1 - 2\gamma K_F^*(0))}{r - \alpha} - \frac{c}{r} - \delta = 0, \quad (36)$$

$$\begin{aligned} \frac{cF(\beta_1)}{4\gamma\beta_1} \left(\frac{X_F^*(0)}{c} \right)^{\beta_2} (1 - (1 - 2\gamma K_F^*(0))^{1+\beta_2}) + \frac{\beta_1 - 1}{\beta_1} \frac{X_F^*(0)(K_F^*(0) - \gamma K_F^{*2}(0))}{r - \alpha} \\ - \frac{cK_F^*(0)}{r} - \delta K_F^*(0) = 0. \end{aligned} \quad (37)$$

The optimal investment threshold X_D^{det} and the corresponding optimal capacity K_D^{det} for the entry deterrence strategy are equal to

$$\begin{aligned} K_D^{det} &= \frac{1}{(\beta_1 + 1)\gamma}, \\ X_D^{det} &\equiv X^{det}(K_D^{det}) = \frac{(\beta_1 + 1)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right), \end{aligned}$$

if $X < X_D^{det}$ and $X_D^{det} \in [X_1^{det}, X_2^{det}]$. If $X_D^{det} \leq X < X_2^{det}$, in order to implement the entry deterrence strategy, the leader invests immediately at X with capacity $K_D^{det}(X)$ that satisfies (34). The value of the entry deterrence strategy is

$$V_D^{det}(X) = \mathcal{B}_2(K_D^{det}(X))X^{\beta_1} + \frac{K_D^{det}(X)(1 - \gamma K_D^{det}(X))}{r - \alpha} X - \frac{cK_D^{det}(X)}{r}. \quad (38)$$

(b) *Entry Accommodation Strategy*

The entry accommodation strategy is possible if $X > X_1^{acc}$. X_1^{acc} , $K_D^{acc}(X_1^{acc})$ and $K_F^*(X_1^{acc})$ satisfy (16), (17) and

$$\frac{(1 - \gamma K_D - \beta_2 \gamma K_D)}{K_D(1 - \gamma K_D)} \mathcal{N}(K_D)(X^{acc})^{\beta_2} + \frac{X^{acc}(1 - \gamma K_D - \gamma K_F^*(K_D))(1 - 2\gamma K_D)}{(r - \alpha)(1 - \gamma K_D)} - \frac{c}{r} - \delta = 0. \quad (39)$$

The optimal investment capacity for the entry accommodation strategy is

$$K_D^{acc} = \frac{1}{(\beta_1 + 1)\gamma},$$

and the investment threshold $X_D^{acc} \equiv X^{acc}(K_D^{acc})$ satisfies

$$\begin{aligned} \frac{c(X^{acc})^{\beta_2}}{2\beta_1} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) & \left(\left(\frac{c}{1 - \gamma K_D^{acc}} \right)^{-\beta_2} - \left(\frac{c}{1 - \gamma K_D^{acc} - 2\gamma K_F^*(K_D^{acc})} \right)^{-\beta_2} \right) \\ & + \frac{(\beta_1 - 1)X^{acc}}{\beta_1(r - \alpha)} (1 - \gamma K_D^{acc} - \gamma K_F^*(K_D^{acc})) - \frac{c}{r} - \delta = 0, \end{aligned} \quad (40)$$

if $X < X_D^{acc}$ and $X_D^{acc} \in [X_1^{acc}, +\infty)$. If $X \geq X_D^{acc}$, in order to implement the entry accommodation strategy, the leader invests immediately at X and the corresponding capacity $K_D^{acc}(X)$ satisfies (39). The value of the entry accommodation strategy is

$$V_D^{acc}(X) = \mathcal{N}(K_D^{acc}(X))X^{\beta_2} + \frac{K_D^{acc}(X)(1 - \gamma K_D - \gamma K_F^*(K_D^{acc}(X)))}{r - \alpha}X - \frac{cK_D^{acc}(X)}{r}. \quad (41)$$

Similar to the previous case, a numerical example is provided to illustrate how the leader decides on investment when the follower produces up to capacity right after investment.

Figure 3: Illustration of $\hat{K}_D(X)$, $K_D^{det}(X)$, and $K_D^{acc}(X)$ when the flexible follower produces up to capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.02$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 3 gives a numerical illustration for $\hat{K}_D(X)$, $K_D^{det}(X)$, and $K_D^{acc}(X)$ as functions of X when the flexible follower produces up to capacity right after investment. For the given parameter values, the entry deterrence strategy will be considered if $X \in (X_1^{det}, X_2^{det})$ with $X_1^{det} = 2.70$ and $X_2^{det} = 11.51$. The corresponding capacities are $K_D^{det}(X_1^{det}) = 0$ and $K_D^{det}(X_2^{det}) = 9.57$. The optimal investment threshold is $X_D^{det} = 6.28$ and the optimal capacity size is $K_D^{det} = 6.18$ if $X < 6.28$. For the entry accommodation strategy, $X_D^{acc} = 8.47$ and $K_D^{acc} = 6.18$ if $X < 8.47$. However, for this numerical example, $X_1^{acc} = 8.76$ and it holds that $X_D^{acc} < X_1^{acc}$, implying X_D^{acc} is still meaningless. Thus, in order to play the entry accommodation strategy, the leader waits until X reaches 8.76 and invests immediately.

Figure 4: Illustration of $V_D^{det}(X)$ and $V_D^{acc}(X)$ when the flexible follower produces up to capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.02$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 4 demonstrates the value of the entry deterrence strategy $V_D^{det}(X)$ and the entry accommodation strategy $V_D^{acc}(X)$ changing with X when the flexible follower produces up to capacity right after investment. Again, the dedicated leader will consider the entry deterrence strategy for $X_1^{det} < X < \hat{X}$ and the entry accommodation strategy for $X \geq \hat{X}$. So to implement the entry deterrence strategy, the leader invests at $X_D^{det} = 6.28$ with capacity $K_D^{det} = 6.18$ if $X < X_D^{det}$, and invests immediately at X with capacity $K_D^{det}(X)$ if $X_D^{det} \leq X < \hat{X}$. To implement the entry accommodation strategy, the leader invests immediately at X when $X \geq \hat{X}$ with capacity $K_D^{acc}(X)$.

It can be concluded from Proposition 2 and 3 that the entry accommodation strategy is not possible for $X < X_1^{acc}$, and the entry deterrence strategy is not possible for $X > X_2^{det}$. When $X_1^{acc} < X < X_2^{det}$, the strategy that gives higher value will be chosen by the leader. Huisman and Kort (2015) has shown analytically that $X_1^{acc} < X_2^{det}$ when there is no volume flexibility for the follower. We check numerically whether this still holds for a flexible follower in Figure 5. It is shown that departing from the default

parameter values $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$ and $\gamma = 0.05$, when changing σ , α , r , c , δ and γ , X_2^{det} is always larger than X_1^{acc} . Thus, we can assume that $X_2^{det} > X_1^{acc}$ also holds for the setting with follower's volume flexibility. However, different from Huisman and Kort (2015), where $X_D^{acc} < X_1^{acc}$ always holds, the numerical analysis in Figure 5 shows that for significantly small α and δ , it actually holds that $X_D^{acc} > X_1^{acc}$. Note that X_D^{acc} implies that the market demand should be large enough to accommodate both firms' entry at the same time. When α is small/negative, implying the follower produces up to full capacity right after investment, a large demand is required and it leads to $X_D^{acc} > X_1^{acc}$. When δ is small, i.e., the investment costs are small, both firms could install large capacities and a large X_D^{acc} is resulted. The above results are summarized in the following proposition.

Proposition 4 Denote $\hat{X}$ as

$$\hat{X} = \min\{X | X_1^{acc} < X < X_2^{det} \text{ and } V_D^{acc}(X) = V_D^{det}(X)\}.$$

For a given level X , the optimal investment capacity for the leader is

$$K_D^*(X) = \begin{cases} K_D^{det}(X_D^{det}) & \text{if } 0 \leq X < X_D^{det}, \\ K_D^{det}(X) & \text{if } X_D^{det} \leq X < \hat{X}, \\ K_D^{acc}(X_D^{acc}) & \text{if } \hat{X} \leq X < X_D^{acc}, \\ K_D^{acc}(X) & \text{if } X \geq \max\{\hat{X}, X_D^{acc}\}. \end{cases} \quad (42)$$

The optimal investment threshold for the leader is

$$X_D^* = \begin{cases} X_D^{det} & \text{if } 0 \leq X < X_D^{det}, \\ X & \text{if } X_D^{det} \leq X < \hat{X}, \\ X_D^{acc} & \text{if } \hat{X} \leq X < X_D^{acc}, \\ X & \text{if } X \geq \max\{\hat{X}, X_D^{acc}\}. \end{cases} \quad (43)$$

The value of the leader is given by

$$V_D^*(X) = \begin{cases} \left(\frac{X}{X_D^{det}}\right)^{\beta_1} V_D^{det}(X_D^{det}) & \text{if } 0 \leq X < X_D^{det}, \\ V_D^{det}(X) & \text{if } X_D^{det} \leq X < \hat{X}, \\ \left(\frac{X}{X_D^{acc}}\right)^{\beta_1} V_D^{acc}(X_D^{acc}) & \text{if } \hat{X} \leq X < X_D^{acc}, \\ V_D^{acc}(X) & \text{if } X \geq \max\{\hat{X}, X_D^{acc}\}. \end{cases} \quad (44)$$

The optimal capacity level for the leader $K_D^*(X)$ and for the flexible follower $K_F^*(X)$ are demonstrated in Figure 6. For the given parameter values, $X_D^{det} = 6.3$, $X_1^{acc} = 8.3671 < X_D^{acc} = 8.3710 < \hat{X} = 9.2809$, and the flexible follower produces up to capacity right after investment. According to Proposition 4, if $X < \hat{X}$, the leader chooses entry deterrence strategy. Note that for $X < X_D^{det}$, the leader is waiting to

Figure 5: Illustration of X_1^{acc} , X_2^{det} , and X_D^{acc} . Default parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$

invest. Once level X_D^{det} is reached, the leader invests with capacity $K_D^{det}(X_D^{det}) = 5.7143$. Then the follower would wait until $X_F^*(K_D^{det}) = 8.3682$ is reached and invest with $K_F^*(K_D^{det}) = 4.1746$. If $X_D^{det} < X \leq \hat{X}$, the leader invests immediately at level X with $K_D^{det}(X)$, and the follower invests later at $X_F^*(K_D^{det}(X))$ with $K_F^*(K_D^{det}(X))$. If $X \geq \hat{X}$, the leader applies entry accommodation strategy. Because $X \geq \hat{X} > X_D^{acc}$, the leader invests immediately at X with capacity level $K_D^{acc}(X)$. The follower invests at the same time with capacity $K_F^*(K_D^{acc}(X))$. Figure 6 shows that for the entry deterrence strategy, the leader's optimal investment capacity increases with X when $X < \hat{X}$. This is because as the demand becomes larger, in order to postpone the follower's entry and to prolong the monopoly privilege, the leader needs to install more capacity. According to Huisman and Kort (2015), when $X = \hat{X}$, this over-investment can be seen as the difference in K_D^* , because the entry accommodation strategy capacity corresponds to the Stackelberg leader's capacity level. The increase of K_D^* for $X \geq \hat{X}$ is less dramatic than that for $X < \hat{X}$. This increase is only to reduce the follower's investment capacity rather than to postpone the follower's entry. Correspondingly to the increase in the leader's optimal capacity levels, the follower's optimal investment capacity decreases with X . More specifically, $K_F^*(X)$ decreases faster for $X < \hat{X}$ because of the over-investment effect and much slower for $X \geq \hat{X}$.

Figure 7 demonstrates the value for the dedicated leader $V_D^*(X)$ and the flexible follower $V_F^*(X)$. Note that if $X < X_D^{det}$, the value of the leader is the value of holding the option to invest, not the immediate value at the moment of investment as in Huisman and Kort (2015), thus it is not equal 0. If $X \geq X_D^{det}$,

Figure 6: Illustration of $K_D^*(X)$ and $K_F^*(X)$ when the flexible follower produces up to capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.01$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

then the leader's value is the value of immediate investment. The leader switches from the entry deterrence to the entry accommodation strategy at $\hat{X}$. When $X < X_D^{det}$, the follower's value also comes from holding the option to invest. When $X_D^{det} \leq X < \hat{X}$, the follower expects the leader to play entry deterrence strategy and to invest at X . Then the follower would adjust the investment timing accordingly. For the given parameter values in Figure 7, the flexible follower does not invest for $X \leq \hat{X}$ and thus holds an option to invest. The kink in the follower's value function is because the leader switches from entry deterrence to entry accommodation strategy, where the follower invests immediately at the same time with the leader.

Figure 8 demonstrates the optimal capacity levels for the two firms when the follower produces below capacity right after investment. For given parameter values $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, and $\delta = 0.5$, if $X < X_D^{det}$, the leader waits until X reaches $X_D^{det} = 4.3050$ to implement the entry deterrence strategy. The entry deterrence strategy will be chosen when $X < \hat{X} = 4.4779$. If $X \geq \hat{X}$, then the leader chooses the entry accommodation strategy. However, different from the previous example where $X_D^{acc} < \hat{X}$, here $X_1^{acc} = 4.4072 < \hat{X} < X_D^{acc} = 4.8238$, implying that the leader chooses the accommodation strategy for $\hat{X} \leq X < X_D^{acc}$ but waits to invest until X_D^{acc} is reached. So the leader is holding an option to invest in the accommodation strategy. This is shown in Figure 8 as the void area when $\hat{X} \leq X < X_D^{acc}$.

Figure 9 demonstrates the values of the leader and the follower as functions of X when the follower produces below capacity right after investment. When $X < X_D^{det}$, the leader waits to invest with entry deterrence strategy capacity. The follower is also waiting to invest, and expects the leader to invest at X_D^{det} with capacity $K_D^{det}(X_D^{det})$. For $X_D^{det} \leq X < \hat{X}$, the leader invests immediately at level X with deterrence capacity $K_D^{det}(X)$. The follower invests later than the leader. When $\hat{X} \leq X < X_D^{acc}$, the leader switches to

Figure 7: Illustration of $V_D^*(X)$ and $V_F^*(X)$ when the flexible follower produces up to capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.01$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 8: Illustration of $K_D^*(X)$ and $K_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 0.5$.

entry accommodation strategy and waits to invest at X_D^{acc} with capacity $K_D^{acc}(X_D^{acc})$. The follower invests at the same time with the leader with capacity $K_F^*(K_D^{acc}(X_D^{acc}))$. Because of the switch of strategies, both value functions are shown to jump at $\hat{X}$. When $X \geq X_D^{acc}$, the leader invests immediately with the entry accommodation strategy capacity $K_D^{acc}(X)$. The follower also invests at the same time as the leader with capacity $K_F^*(K_D^{acc}(X))$.

Figure 9: Illustration of $V_D^*(X)$ and $V_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

5 Influence of Flexibility

In order to analyse the influence of the follower's volume flexibility, the optimal investment decisions without flexibility are derived in Appendix B. By comparing the leader's investment decisions with a flexible and with a dedicated follower, we can get the following proposition.

Proposition 5 *Volume flexibility does not influence the leader's entry deterrence strategy. Moreover, it also does not influence the leader's optimal capacity under entry accommodation strategy.*

In this section, numerical analysis is carried out to check how flexibility influences the leader and follower's investment decisions. More specifically, it considers the possible occurrence of each strategy by comparing X_1^{det} , X_2^{det} , and X_1^{acc} . Because the flexibility influences the investment threshold for accommodation strategy, the focus is put on the accommodation strategy, or rather on the switch between the entry deterrence and accommodation strategy. The impact of flexibility on the leader's accommodation strategy capacity and option values at this switch is also analyzed. Moreover, this section looks further at the follower's optimal investment decisions under leader's entry deterrence and accommodation strategies. The follower's investment thresholds and capacities are compared according to whether the production flexibility is available. The influence of the flexibility on the follower's values at the moment of investment is also analyzed.

5.1 Flexibility Influences Dedicated Leader

Though the follower's flexibility influences neither the leader's deterrence strategy, nor the investment capacity under accommodation strategy, it does influence the possibility to implement these two strategies. The analysis is focused on the interval $[X_1^{det}, X_2^{det}]$, where the entry deterrence strategy is considered; and region that $X \geq X_1^{acc}$, where the accommodation strategy is considered.

Figure 10: Illustration of X_1^{det} , X_2^{det} , and X_1^{acc} with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 10 demonstrates that under entry deterrence strategy, X_1^{det} with flexibility is not smaller than that without flexibility and X_2^{det} with flexibility is not larger than that without flexibility. Thus the interval to implement entry deterrence strategy shrinks when the follower is flexible. For the entry accommodation strategy, X_1^{acc} with flexibility is smaller than that without flexibility, so the interval to implement entry accommodation strategy enlarges when the follower is flexible. The changes in the intervals reflect the tendency for the leader to implement the corresponding strategy. It holds that for the given parameters, the leader tends to delay the flexible follower's entry less and is more likely to choose accommodation strategy in case of a flexible follower.

Figure 11 shows that the dedicated leader's accommodation strategy threshold is higher than that without flexibility, possibly because a flexible follower invests more than an inflexible follower at X_D^{acc} and a higher demand is required to choose the accommodation strategy. For the given parameter values, we have $X_1^{acc} > X_D^{acc}$, which makes the optimal threshold X_D^{acc} meaningless as in the case of without flexibility. From Proposition 4, the leader does not necessarily invest at threshold X_D^{acc} to apply the entry accommodation.

Figure 11: Illustration of X_D^{acc} , $\hat{X}$, and $K_D^{acc}(\hat{X})$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

If $\hat{X} \geq X_D^{acc}$, the leader invests at $\hat{X}$, the switch from entry deterrence to accommodation. So for the accommodation strategy, we further analyse the influence of follower's flexibility on $\hat{X}$. In Figure 11, it is shown that $\hat{X} > X_1^{acc}$, implying that $\hat{X}$ is meaningful. Moreover, $\hat{X}$ increases with market uncertainty σ . This means that the leader switches to the accommodation strategy later in a more volatile market. The intuition is that both the leader and the follower invest more in case of a larger uncertainty, see also Figure 15. Therefore, a higher demand is required to accommodate the two firms. However, $\hat{X}$ with flexibility is smaller than without flexibility, implying that the dedicated leader switches to accommodation strategy earlier when the follower is flexible. This is consistent with the findings above that accommodation strategy is more likely with a flexible follower. Furthermore, Figure 11 also demonstrates that when switching to accommodation strategy, the leader invests less if the follower is flexible. This will be explained in the following subsection.

Next, we check how follower's flexibility affects the leader's value. Figure 12 demonstrates the dedicated leader's value under entry deterrence strategy at investment threshold X_D^{det} and under accommodation strategy at $\hat{X}$, with and without flexibility. For the entry deterrence strategy, Figure 12a shows that the leader's value at the investment threshold X_D^{det} with flexibility is no larger than that without flexibility. This is because flexibility does not change the dedicated leader's investment threshold and capacity under entry deterrence strategy, but it makes the follower enter the market earlier (see Figure 13), which puts an earlier end to the leader's monopoly privilege. Similar analysis can be done for the accommodation strategy. The follower's flexibility makes the leader invest earlier and less under accommodation strategy. However, comparing the leader's values at the moment of investment, it is larger than that without flexibility under accommodation strategy, as shown in Figure 12. This implies that the follower's flexibility also is good for the leader when accommodating the flexible follower's entry. If the leader deters the flexible follower's entry, the flexibility decreases leader's value.

(a) Entry deterrence strategy.

(b) Switch between deterrence and accommodation.

Figure 12: Illustration of $V_D^{det}(X_D^{det}, K_D^{det})$ when investing at the optimal threshold X_D^{det} , and $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ when investing at level $\hat{X}$. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

5.2 Flexibility Influences Flexible Follower

In this subsection, we compare how the flexibility influences the follower's investment threshold, capacity, and value when the dedicated leader takes entry deterrence and accommodation strategies.

Figure 13: Illustration of $X_F(K_D^{det})$, $K_F(K_D^{det})$, and $V_F(X_F^*(K_D^{det}))$ under entry deterrence strategy. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

When the leader chooses the entry deterrence strategy, the flexibility allows a flexible follower to invest with more capacity, as shown in Figure 13. This is because the follower can adjust output levels according to market demand, and invests more in case of upward demand shocks in the future. According to the analysis of the monopoly case as in Wen et al. (2017), larger capacity means more investment costs, and the firm invests later so that the market demand is higher to compensate for the investment costs. However, as shown in Figure 13, this is not the case in the duopoly model. Here, the flexible follower invests more and invests earlier than the inflexible follower. Twofold reasons are provided. On one hand, the inflexible follower always produces up to full capacity, even when the profit is negative for low levels of X . This decreases the optimal

investment capacity, and delays inflexible follower's investment because of the preference for large market demand. On the other hand, the flexible follower can invest more because the future output quantity can adjust to market demand. The flexibility also gives higher values to the follower as illustrated in Figure 13. The higher value is due to the flexibility to avoid overproduction in case of low demands, and motivates the follower to invest earlier. Besides, the difference between with and without flexibility increases with σ . This is because for smaller σ , market uncertainty is low and the flexible follower produces up to capacity right after investment, so the difference in $X_F(K_D^{det})$ and $K_F(K_D^{det})$ is relatively small. However, with more market uncertainty, i.e., larger σ , the flexible follower produces below capacity right after investment and more capacity is put on hold for future positive demand shocks, so the difference is relatively large.

Figure 14: Illustration of $p(X_F(K_D^{det}))$, $q_F(X_F(K_D^{det}))$, $\pi(X_F(K_D^{det}))$, and $q_F(X_F(K_D^{det})) / K_F(K_D^{det})$ with and without flexibility under the entry deterrence strategy. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

We can also compare the market price $p(X_F(K_D^{det}))$, the follower's output levels $q_F(X_F(K_D^{det}))$, the profit $\pi(X_F(K_D^{det}))$, and the utilization rate $q_F(X_F(K_D^{det})) / K_F(K_D^{det})$, i.e., how much of the capacity is used for production at the moment of investment. As demonstrated in Figure 14, the market price is higher when the inflexible follower invests. The market price is influenced by both the leader and follower's output levels and the market demand. Figure 14 shows that when the flexible follower produces up to capacity for small σ , the output is close to or slightly larger than that of the inflexible follower at the moment of investment.

Because inflexible follower invests later, the market demand is larger. These two effects lead to higher instant market prices for the inflexible follower at $X_F^*(K_D^{det})$, given the same entry deterrence strategy of the leader. When the flexible follower produces below capacity, the flexible follower's instant output gradually falls below the inflexible follower's output as σ increases. Given the same output levels from the leader, there is less instant output from the follower. However, the inflexible follower invests much later than the flexible follower, so the market demand is much higher. Thus, the instant market price when the inflexible follower invests is still higher. This also leads to higher instant profit flows of the inflexible follower. However, if the flexible follower's value at investment threshold $X_F(K_D^{det})$ is "discounted" to the inflexible follower's investment threshold $X_F^*(K_D^{det})$, the flexible follower has higher values, as shown in Figure 13. Figure 14 also demonstrates that the utilization rate is 1 when the flexible follower produces up to capacity, and it decreases with σ when the flexible follower produces below capacity at threshold $X_F(K_D^{det})$. The latter result is consistent with the findings in Hagspiel et al. (2016). The reason is that a higher σ implies larger market uncertainty, and for a positive market trend α , the firm invests later with more capacity as shown in Figure 13. Although for the output decisions, only current market demand matters. The investment being delayed implies larger market demand, and the output also increases with σ . However, as shown in Figure 14, the invested capacity $K_F(K_D^{det})$ increases faster, thus the utilization rate decreases with σ .

Figure 15: Illustration of $X_F(K_D^{acc}(\hat{X}))$, $K_F(K_D^{acc}(\hat{X}))$, and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

The dedicated leader switches from deterrence to accommodation strategy at $\hat{X}$. Note that for the accommodation strategy, the follower invests at the same time with the leader, thus in Figure 15 $X_F(K_D^{acc}(\hat{X}))$ is the same as $\hat{X}$ in Figure 11. Figure 15 shows that under the leader's accommodation strategy, the flexible follower invests earlier and more than the inflexible follower. The story here is similar to that in the deterrence strategy, where the flexible follower also invests earlier and more than the inflexible follower, and has higher value.

Figure 16 demonstrates the market price, the follower's output levels, profit flows and flexible follower's utilization rate at the moment of investment when the leader takes entry accommodation strategy. As shown in Figure 16 and unlike that for the entry deterrence strategy, the market price at the moment of investment when the follower is flexible is not always lower than when the follower is inflexible. There are

three factors that affect market prices: the market demand at the moment of investment, the follower's output quantity, and the leader's installed capacity. Because the flexible follower invests earlier as shown in Figure 15, the market demand is smaller. The output $q_F(K_D^{acc}(X))$ in Figure 16 shows that the flexible follower produces more than an inflexible follower. Thus, if the leader produces the same output level as in the entry deterrence strategy, the market price with a flexible follower would be lower than that with an inflexible follower. However, Figure 11 shows that the leader installs smaller capacity when the follower is flexible. Because the leader always produces up to capacity, this might result in a market price higher than that with an inflexible follower. For smaller σ , the follower's profit at the moment of investment, $\pi_F(K_D^{acc}(\hat{X}))$ is larger when the follower is flexible. Whereas for larger σ , $\pi_F(K_D^{acc}(\hat{X}))$ is larger when the follower is inflexible. The reasons are as follows. For smaller σ , the price levels at the moment of investment are higher, and the flexible follower is producing more than the inflexible follower. Thus the flexible follower has larger profit at the moment of investment. However, when σ is large, the price level at the moment of investment is much higher for the inflexible follower. Though the flexible follower still produces more than an inflexible follower, the later has higher profit flows at the moment of investment. Figure 16 also demonstrates that when the flexible follower produces below capacity right after investment, the utilization rate decreases with σ . The intuition is the same as for the deterrence strategy case.

Figure 16: Illustration of $p(K_D^{acc}(\hat{X}))$, $q_F(K_D^{acc}(\hat{X}))$, $\pi_F(K_D^{acc}(\hat{X}))$, and $q_F(K_D^{acc}(\hat{X}))/K_F(K_D^{acc}(\hat{X}))$ with and without flexibility under the entry accommodation strategy. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

5.3 First Mover Advantage v.s. Technological Advantage

In this subsection, we investigate whether the leader's first mover advantage can be overcome by the flexible follower's technological advantage.

Figure 17: Comparison of $V_D^{det}(X_F^*, K_D^{det}(X_F^*))$ and $V_F(X_F^*, K_D^{det}(X_F^*))$ under the entry deterrence strategy, $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy, with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 17 compares the leader and the follower's values for the leader's entry deterrence and accommodation strategy with and without flexibility. The leader always has higher values than the follower, implying the first mover advantage cannot be leapfrogged by the volume flexibility advantage. For the duopoly model in this paper, according to Corollary 1, the optimal installed capacity by the follower decreases with the leader's installed capacity size. So this result is in a way consistent with Gal-Or (1985) that the leader has higher payoff than the follower if the reaction function of the follower is downward-sloping. The players are symmetric in Gal-Or's model. In our model, the players are asymmetric and time is continuous. The possible reason, why it is more difficult for the technological advantage to take over the first mover advantage, is that there is spill over of the technological benefit. The flexible follower cannot fully capture the benefit of the technological advancement.

6 Conclusion

This article introduces volume flexibility into strategic capacity investment problem under uncertainty. In the duopoly framework, the follower has technological advantage over the leader in that the follower can adjust output quantity within the constraint of installed production capacity, and the leader always produces up to capacity. When making decisions about investment timing and investment capacity, the leader not only takes into account the incentives to preempt, but also the influence of follower's volume flexibility on the market price. This is because the flexible follower competes against the dedicated leader on one hand, and on the other hand makes the market price fluctuate less when there is demand uncertainty. We show that compared with a dedicated follower, the dedicated leader is more likely to accommodate the entry of the flexible follower. This is due to the fact that entry deterrence strategy decreases the leader's value when the follower is flexible, and the entry accommodation strategy increases the leader's value. The leader does not like to play entry deterrence because volume flexibility enables the follower to enter the market earlier and thus shortens the leader's monopoly period. Whereas when playing accommodation strategy, two firms enter the market later than that under the deterrence strategy, so the market demand is larger. In a way, the leader benefits more from the less fluctuating market prices due to follower's volume flexibility. Dixit (1980) proves that in a static setting, entry deterrence is largely ineffective if the leader cannot commit to producing at full capacity, because the leader facing irrevocable entry finds it best to make an accommodating output reduction. We prove that in a stochastic dynamic setting, deterring the entry of a flexible follower does not necessarily make the leader better off, and the leader's commitment to an output quantity is still ineffective to deter the follower's entry. In fact, the leader commits to the same output level under deterrence and accommodation strategy, but invests at different timings. The establishment of the role for uncertainty is also an attempt to answer to Huisman and Kort (2015).

Our model assumes exogenous firm roles and takes the dedicated firm as the leader in the market, this is in a way reasonable because technology advancement takes time. We have shown that technology advancement cannot overtake the first mover advantage in the sense that the dedicated leader has higher value than the flexible follower. So it might be interesting to see how asymmetric firms interact with each other strategically in an endogenous firm role setting. Another possible extension of this article is to investigate other demand structures. Our model assumes multiplicative demand function. We find the dedicated leader installs monopolistic capacity size for the entry deterrence and accommodation strategy. This is a strong result. It is worthwhile to do a robustness check with a different demand function.

Appendix

A

A.1 Derivation of $L(K_D, K_F)$, $M_1(K_D, K_F)$, $M_2(K_D)$, $N(K_D, K_F)$

Let $X_1 = c/(1 - \gamma K_D)$ and $X_2 = c/(1 - \gamma K_D - 2\gamma K_F)$. Employing value matching and smooth pasting conditions gives the following equations:

$$\begin{aligned}
 L(K_D, K_F) &= M_1(K_D, K_F) + \frac{X_1^{-\beta_1}}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2c\beta_2(1 - \gamma K_D)}{r} \right. \\
 &\quad \left. - \frac{X_1(1 - \gamma K_D)^2(\beta_2 - 1)}{r - \alpha} - \frac{c^2(\beta_2 + 1)}{(r + \alpha - \sigma^2)X_1} \right] \\
 &= \frac{c^{1-\beta_1}(1 - 2\gamma K_F - \gamma K_D)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} \left[-\frac{2\beta_2}{r} + \frac{\beta_2 - 1}{r - \alpha} + \frac{\beta_2 + 1}{r + \alpha - \sigma^2} \right] \\
 &\quad + \frac{c^{1-\beta_1}(1 - \gamma K_D)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2\beta_2}{r} - \frac{\beta_2 - 1}{r - \alpha} - \frac{1 + \beta_2}{r + \alpha - \sigma^2} \right] \\
 &= \frac{c^{1-\beta_1} \left[(1 - \gamma K_D)^{1+\beta_1} - (1 - 2\gamma K_F - \gamma K_D)^{1+\beta_1} \right]}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2\beta_2}{r} - \frac{\beta_2 - 1}{r - \alpha} - \frac{1 + \beta_2}{r + \alpha - \sigma^2} \right], \\
 M_1(K_D, K_F) &= \frac{X_2^{-\beta_1}}{\beta_1 - \beta_2} \left\{ \frac{1}{4\gamma} \left[-\frac{2c\beta_2(1 - \gamma K_D)}{r} + \frac{X_2(\beta_2 - 1)(1 - \gamma K_D)^2}{r - \alpha} + \frac{c^2(\beta_2 + 1)}{(r + \alpha - \sigma^2)X_2} \right] \right. \\
 &\quad \left. + \frac{cK_F\beta_2}{r} + \frac{(1 - \beta_2)(1 - \gamma K_D - \gamma K_F)K_F X_2}{r - \alpha} \right\} \\
 &= \frac{X_2^{-\beta_1}(1 - 2\gamma K_F - \gamma K_D)}{4\gamma(\beta_1 - \beta_2)} \left[-\frac{2c\beta_2}{r} + \frac{(\beta_2 - 1)c}{r - \alpha} + \frac{c(\beta_2 + 1)}{r + \alpha - \sigma^2} \right] \\
 &= \frac{c^{1-\beta_1}(1 - 2\gamma K_F - \gamma K_D)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} \left[-\frac{2\beta_2}{r} + \frac{\beta_2 - 1}{r - \alpha} + \frac{\beta_2 + 1}{r + \alpha - \sigma^2} \right], \\
 M_2(K_D) &= \frac{X_1^{-\beta_2}}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2c\beta_1(1 - \gamma K_D)}{r} - \frac{X_1(\beta_1 - 1)(1 - \gamma K_D)^2}{r - \alpha} - \frac{c^2(\beta_1 + 1)}{(r + \alpha - \sigma^2)X_1} \right] \\
 &= \frac{c^{1-\beta_2}(1 - \gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2\beta_1}{r} - \frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1 + 1}{r + \alpha - \sigma^2} \right], \\
 N(K_D, K_F) &= M_2(K_D) + \frac{X_2^{-\beta_2}}{\beta_1 - \beta_2} \left\{ \frac{1}{4\gamma} \left[-\frac{2c\beta_1(1 - \gamma K_D)}{r} + \frac{X_2(\beta_1 - 1)(1 - \gamma K_D)^2}{r - \alpha} \right. \right. \\
 &\quad \left. \left. + \frac{c^2(\beta_1 + 1)}{(r + \alpha - \sigma^2)X_2} \right] + \frac{cK_F\beta_1}{r} + \frac{(1 - \beta_1)(1 - \gamma K_D - \gamma K_F)K_F X_2}{r - \alpha} \right\} \\
 &= M_2(K_D) + \frac{X_2^{-\beta_2}(1 - \gamma K_D - 2\gamma K_F)}{4\gamma(\beta_1 - \beta_2)} \left[-\frac{2c\beta_1}{r} + \frac{c(\beta_1 - 1)}{r - \alpha} + \frac{c(\beta_1 + 1)}{r + \alpha - \sigma^2} \right] \\
 &= \frac{c^{1-\beta_2} \left[(1 - \gamma K_D)^{1+\beta_2} - (1 - \gamma K_D - 2\gamma K_F)^{1+\beta_2} \right]}{4\gamma(\beta_1 - \beta_2)} \left[\frac{2\beta_1}{r} - \frac{\beta_1 - 1}{r - \alpha} - \frac{1 + \beta_1}{r + \alpha - \sigma^2} \right].
 \end{aligned}$$

Let

$$F(\beta) = \frac{2\beta}{r} - \frac{\beta - 1}{r - \alpha} - \frac{\beta + 1}{r + \alpha - \sigma^2}$$

$$= \frac{\beta (2\alpha\sigma^2 - r\sigma^2 - 2\alpha^2) + r (2\alpha - \sigma^2)}{r (r - \alpha) (r + \alpha - \sigma^2)}.$$

From $r > \alpha$, $r > \sigma^2 - \alpha$, it follows that $\beta_2 < -1$, $F(\beta_1) < 0$, and $F(\beta_2) > 0$. Thus, we conclude that $L(K_D, K_F) > 0$, $M_1(K_D, K_F) < 0$, $N(K_D, K_F) > 0$, and $M_2(K_D) < 0$ for the duopoly model.

A.2 Proof of Proposition 1

The optimal investment capacity $K_F(X, K_D)$ of the follower maximizes $V_F(X, K_D, K_F) - \delta K_F$. The analysis is carried out for three different regions.

- Region 1: $0 < X < c/(1 - \gamma K_D)$.

In this region, we have

$$\begin{aligned} V_F(X, K_D, K) - \delta K_F &= L_1(K_D, K_F) X^{\beta_1} - \delta K_F \\ &= \frac{c^{1-\beta_1} \left[(1 - \gamma K_D)^{1+\beta_1} - (1 - 2\gamma K_F - \gamma K_D)^{1+\beta_1} \right] F(\beta_2)}{4\gamma(\beta_1 - \beta_2)} X^{\beta_1} - \delta K_F. \end{aligned}$$

Taking the first order condition with respect to K_F gives

$$\frac{c^{1-\beta_1}(1 + \beta_1)F(\beta_2)X^{\beta_1}}{2(\beta_1 - \beta_2)} (1 - 2\gamma K_F - \gamma K_D)^{\beta_1} - \delta = 0. \quad (45)$$

Thus,

$$K_F(X, K_D) = \frac{1}{2\gamma} \left\{ 1 - \gamma K_D - \frac{c}{X} \left[\frac{2\delta(\beta_1 - \beta_2)}{cF(\beta_2)(1 + \beta_1)} \right]^{\frac{1}{\beta_1}} \right\}. \quad (46)$$

The second order partial derivative of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F is

$$-\frac{\beta_1 \gamma F(\beta_2)(1 + \beta_1)}{\beta_1 - \beta_2} \left(\frac{1 - 2\gamma K_F - \gamma K_D}{c} \right)^{\beta_1 - 1} X^{\beta_1} < 0.$$

Thus, $K_F(X, K_D)$ maximizes $V_F(X, K_F, K_D) - \delta K_F$.

- Region 2: $c/(1 - \gamma K_D) \leq X < c/(1 - \gamma K_D - 2\gamma K_D)$.

In this region, we have

$$\begin{aligned} &V_F(X, K_D, K_F) - \delta K_F \\ &= M_1(K_D, K_F) X^{\beta_1} + M_2(K_D) X^{\beta_2} + \frac{(1 - \gamma K_D)^2 X}{4\gamma(r - \alpha)} - \frac{c(1 - \gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r + \alpha - \sigma^2)} - \delta K_F \\ &= -\frac{c^{1-\beta_1}(1 - 2\gamma K_F - \gamma K_D)^{1+\beta_1}}{4\gamma(\beta_1 - \beta_2)} F(\beta_2) X^{\beta_1} + \frac{c^{1-\beta_2}(1 - \gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1 - \beta_2)} F(\beta_1) X^{\beta_2} + \frac{(1 - \gamma K_D)^2 X}{4\gamma(r - \alpha)} \\ &\quad - \frac{c(1 - \gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r + \alpha - \sigma^2)} - \delta K_F. \end{aligned}$$

Taking the first order condition with respect to K_F gives

$$K_F(X, K_D) = \frac{1}{2\gamma} \left\{ 1 - \gamma K_D - \frac{c}{X} \left[\frac{2\delta(\beta_1 - \beta_2)}{cF(\beta_2)(1 + \beta_1)} \right]^{\frac{1}{\beta_1}} \right\}. \quad (47)$$

The second order partial derivative with respect to K_F is

$$-\frac{\beta_1 \gamma F(\beta_2)(1+\beta_1)}{\beta_1 - \beta_2} \left(\frac{1 - 2\gamma K_F - \gamma K_D}{c} \right)^{\beta_1 - 1} X^{\beta_1} < 0.$$

- Region 3: $X \geq c/(1 - \gamma K_D - 2\gamma K_F)$.

In this region, we have

$$\begin{aligned} & V_F(X, K_D, K_F) - \delta K_F \\ &= N_2(K_D, K_F) X^{\beta_2} + \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r - \alpha} X - \frac{c K_F}{r} - \delta K_F \\ &= \frac{c^{1-\beta_2} [(1 - \gamma K_D)^{1+\beta_2} - (1 - 2\gamma K_F - \gamma K_D)^{1+\beta_2}] F(\beta_1)}{4\gamma(\beta_1 - \beta_2)} X^{\beta_2} \\ &+ \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r - \alpha} X - \frac{c K_F}{r} - \delta K_F. \end{aligned}$$

Taking the first order condition with respect to K_F yields $K_F(X, K_D)$ must satisfy

$$\frac{F(\beta_1) c^{1-\beta_2} (1 + \beta_2) (1 - 2\gamma K_F - \gamma K_D)^{\beta_2}}{2(\beta_1 - \beta_2)} X^{\beta_2} + \frac{1 - 2\gamma K_F - \gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta = 0. \quad (48)$$

The second order partial derivative with respect to K_F is

$$\begin{aligned} & -\frac{2\gamma\beta_2(1+\beta_2)c^{1-\beta_2}F(\beta_1)(1-2\gamma K_F-\gamma K_D)^{\beta_2-1}}{2(\beta_1-\beta_2)} X^{\beta_2} - \frac{2\gamma X}{r-\alpha} \\ & \leq -\frac{\gamma\beta_2(1+\beta_2)c^{1-\beta_2}F(\beta_1)(1-2\gamma K_F-\gamma K_D)^{\beta_2-1}}{\beta_1-\beta_2} \left(\frac{c}{1-\gamma K_D-2\gamma K_F} \right)^{\beta_2} \\ & - \frac{2\gamma}{r-\alpha} \frac{c}{1-\gamma K_D-2\gamma K_F} \\ & = -\frac{\gamma\beta_2(1+\beta_2)F(\beta_1)}{\beta_1-\beta_2} \frac{c}{1-\gamma K_D-2\gamma K_F} - \frac{2\gamma}{r-\alpha} \frac{c}{1-\gamma K_D-2\gamma K_F} \\ & = \frac{\gamma c}{1-\gamma K_D-2\gamma K_F} \left[-\frac{\beta_2(1+\beta_2)F(\beta_1)}{\beta_1-\beta_2} - \frac{2}{r-\alpha} \right] \\ & < 0, \end{aligned}$$

with the last step being concluded from the Appendix B by Wen et al. (2017).

The optimal investment threshold $X_F^*(K_D)$ in each region can be derived by the value matching and smooth pasting conditions at $X_F^*(K_D)$:

$$\begin{cases} AX_F^{*\beta_1} &= V_F(X_F^*, K_D, K_F(X_F^*, K_D)) - \delta K_F(X_F^*, K_D), \\ \beta_1 AX_F^{*\beta_1-1} &= \frac{d}{dX} [V_F(X_F^*, K_D, K_F(X_F^*, K_D)) - \delta K_F(X_F^*, K_D)]. \end{cases}$$

Thus, $X_F^*(K_D)$ satisfies the implicit equation,

$$\begin{aligned} & V_F(X_F, K_D, K_F(X_F, K_D)) - \delta K_F(X_F, K_D) \\ &= \frac{X_F(K_D)}{\beta_1} \frac{d[V_F(X_F, K_D, K_F(X_F, K_D)) - \delta K_F(X_F, K_D)]}{dX}. \end{aligned}$$

- Region 1

The optimal investment threshold $X_F^*(K_D)$ satisfies the following equation:

$$\begin{aligned} & \frac{c^{1-\beta_1} \left[(1-\gamma K_D)^{1+\beta_1} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_1} \right] F(\beta_2)}{4\gamma(\beta_1 - \beta_2)} X^{\beta_1} - \delta K_F \\ &= \frac{X}{\beta_1} \beta_1 X^{\beta_1-1} \frac{c^{1-\beta_1} \left[(1-\gamma K_D)^{1+\beta_1} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_1} \right] F(\beta_2)}{4\gamma(\beta_1 - \beta_2)}, \end{aligned}$$

which is equivalent to

$$\delta K_F = 0. \quad (49)$$

- Region 2

The optimal threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned} & - \frac{F(\beta_2) c^{1-\beta_1} (1-2\gamma K_F - \gamma K_D)^{1+\beta_1} X^{\beta_1}}{4\gamma(\beta_1 - \beta_2)} + \frac{c^{1-\beta_2} (1-\gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1 - \beta_2)} F(\beta_1) X^{\beta_2} \\ & + \frac{(1-\gamma K_D)^2 X}{4\gamma(r-\alpha)} - \frac{c(1-\gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r+\alpha-\sigma^2)} - \delta K_F \\ &= - \frac{F(\beta_2) c^{1-\beta_1} (1-\gamma K_D - 2\gamma K_F)^{1+\beta_1} X^{\beta_1}}{4\gamma(\beta_1 - \beta_2)} \\ & + \frac{X}{\beta_1} \left[\frac{\beta_2 F(\beta_1) c^{1-\beta_2} (1-\gamma K_D)^{1+\beta_2} X^{\beta_2-1}}{4\gamma(\beta_1 - \beta_2)} + \frac{(1-\gamma K_D)^2}{4\gamma(r-\alpha)} - \frac{c^2}{4\gamma X^2(r+\alpha-\sigma^2)} \right], \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \frac{c^{1-\beta_2} (1-\gamma K_D)^{1+\beta_2} F(\beta_1) X^{\beta_2}}{4\gamma\beta_1} + \frac{1}{4\gamma} \left[\frac{\beta_1 - 1}{\beta_1} \frac{(1-\gamma K_D)^2 X}{r-\alpha} - \frac{2c(1-\gamma K_D)}{r} \right. \\ & \left. + \frac{\beta_1 + 1}{\beta_1} \frac{c^2}{X(r+\alpha-\sigma^2)} \right] - \delta K_F = 0. \end{aligned} \quad (50)$$

- Region 3

The optimal investment threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned} & \frac{c^{1-\beta_2} \left[(1-\gamma K_D)^{1+\beta_2} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_2} \right] F(\beta_1)}{4\gamma(\beta_1 - \beta_2)} X^{\beta_2} \\ & + \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r-\alpha} X - \frac{c K_F}{r} - \delta K_F \\ &= \frac{\beta_2 X^{\beta_2}}{\beta_1} \frac{c^{1-\beta_2} \left[(1-\gamma K_D)^{1+\beta_2} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_2} \right] F(\beta_1)}{4\gamma(\beta_1 - \beta_2)} \\ & + \frac{X}{\beta_1} \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r-\alpha}. \end{aligned}$$

Rearranging terms yields

$$\begin{aligned} & \frac{X^{\beta_2} c^{1-\beta_2} F(\beta_1)}{4\gamma\beta_1} \left[(1-\gamma K_D)^{1+\beta_2} - (1-2\gamma K_F - \gamma K_D)^{1+\beta_2} \right] \\ & + \frac{(\beta_1 - 1) X}{\beta_1} \frac{K_F - \gamma K_D K_F - \gamma K_F^2}{r-\alpha} - \frac{c K_F}{r} - \delta K_F = 0. \end{aligned} \quad (51)$$

Note that in the monopoly case, whether the flexible firm produces up to capacity depends on the economic setting. Next, we are going to examine the conditions for the flexible follower to produce below and up to capacity.

If the firm produces below capacity right after investment, then $K_F(X, K_D) > q_F(X, K_D, K_F)$, i.e.,

$$\frac{1}{2\gamma} \left\{ 1 - \gamma K_D - \frac{c}{X} \left[\frac{2\delta(\beta_1 - \beta_2)}{cF(\beta_2)(1 + \beta_1)} \right]^{\frac{1}{\beta_1}} \right\} > \frac{X(1 - \gamma K_D) - c}{2\gamma X}.$$

It is equivalent to

$$2\delta(\beta_1 - \beta_2) < cF(\beta_2)(1 + \beta_1), \quad (52)$$

which is the same as in the monopoly case. Furthermore, we can deduce that

$$2\delta(\beta_1 - \beta_2) \geq cF(\beta_2)(1 + \beta_1) \quad (53)$$

would define Region 3, where the firm produces up to capacity right after investment. The definitions of Region 2, equation (52), and Region 3, equation (53), for the flexible follower firm are the same as that for the monopoly flexible firm in Wen et al. (2017).

A.3 Proof of Corollary 1

- Region 2

Derive $dX_F^*(K_D)/dK_D$ and check whether the leader's installed capacity level would delay the flexible follower's investment. Dividing (15) by $1 - \gamma K_D$, we get

$$\begin{aligned} & \frac{c^{1-\beta_2} F(\beta_1)}{4\gamma\beta_1} [X(1 - \gamma K_D)]^{\beta_2} - \frac{\delta}{2\gamma} \left\{ 1 - \frac{c}{X(1 - \gamma K_D)} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ & + \frac{1}{4\gamma} \left[\frac{\beta_1 - 1}{\beta_1} \frac{X(1 - \gamma K_D)}{r - \alpha} - \frac{2c}{r} + \frac{\beta_1 + 1}{\beta_1} \frac{c^2}{r + \alpha - \sigma^2} \frac{1}{X(1 - \gamma K_D)} \right] = 0. \end{aligned} \quad (54)$$

Comparing (54) with the implicit equation that determines the optimal investment threshold in the corresponding monopoly model (see (Wen et al., 2017)), we find that $X(1 - \gamma K_D)$ replaces X^* in the corresponding monopoly case. Denote $x(K_D) = X_F^*(K_D)(1 - \gamma K_D)$, which is not smaller than c according to the follower's value function, and (54) can be rewritten as

$$\begin{aligned} & \frac{c^{1-\beta_2} F(\beta_1)}{\beta_1} [x(K_D)]^{\beta_2} - 2\delta \left\{ 1 - \frac{c}{x(K_D)} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ & + \frac{\beta_1 - 1}{\beta_1} \frac{x(K_D)}{r - \alpha} - \frac{2c}{r} + \frac{\beta_1 + 1}{\beta_1} \frac{c^2}{r + \alpha - \sigma^2} \frac{1}{x(K_D)} = 0. \end{aligned}$$

Taking the derivative with respect to K_D yields

$$\left\{ \frac{c^{1-\beta_2} \beta_2 F(\beta_1)}{\beta_1} [x(K_D)]^{\beta_2-1} - \frac{2c\delta}{x^2(K_D)} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\}$$

$$+\frac{\beta_1-1}{\beta_1(r-\alpha)}-\frac{\beta_1+1}{\beta_1}\frac{c^2}{r+\alpha-\sigma^2}\frac{1}{x^2(K_D)}\Big\}\frac{dx(K_D)}{dK_D}=0. \quad (55)$$

First check that whether the coefficient of $dX(K_D)/dK_D$ equals to 0. This coefficient can be rewritten as

$$\begin{aligned} Y(x(K_D)) &= \frac{\beta_2}{x(K_D)} \left\{ 2\delta - \frac{2c\delta}{x(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} - \frac{\beta_1-1}{\beta_1} \frac{x(K_D)}{r-\alpha} \right. \\ &\quad \left. + \frac{2c}{r} - \frac{\beta_1+1}{\beta_1} \frac{c^2}{r+\alpha-\sigma^2} \frac{1}{x(K_D)} \right\} - \frac{2c\delta}{x^2(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \\ &\quad + \frac{\beta_1-1}{\beta_1(r-\alpha)} - \frac{\beta_1+1}{\beta_1} \frac{c^2}{r+\alpha-\sigma^2} \frac{1}{x^2(K_D)} \\ &= \frac{2\delta\beta_2}{x(K_D)} - \frac{2c\delta\beta_2}{x^2(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} - \frac{2c\delta}{x^2(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \\ &\quad - \frac{(\beta_1-1)(\beta_2-1)}{\beta_1(r-\alpha)} + \frac{2c\beta_2}{rx(K_D)} - \frac{(\beta_1+1)(\beta_2+1)}{\beta_1} \frac{c^2}{r+\alpha-\sigma^2} \frac{1}{x^2(K_D)} \\ &= \frac{2\delta\beta_2}{x(K_D)} - \frac{2c\delta(\beta_2+1)}{x^2(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} - \frac{1}{\beta_1(r-\alpha)} \frac{2(\alpha-r)}{\sigma^2} + \frac{2c\beta_2}{rx(K_D)} \\ &\quad - \frac{1}{\beta_1} \frac{c^2}{r+\alpha-\sigma^2} \frac{2(\sigma^2-r-\alpha)}{\sigma^2} \frac{1}{x^2(K_D)} \\ &= \frac{2\delta\beta_2}{x(K_D)} - \frac{2c\delta(\beta_2+1)}{x^2(K_D)} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + \frac{2}{\beta_1\sigma^2} + \frac{2c\beta_2}{rx(K_D)} + \frac{2}{\beta_1\sigma^2} \frac{c^2}{x^2(K_D)}, \end{aligned} \quad (56)$$

with $x(K_D) \geq c$. (56) is a quadratic form of $1/x(K_D)$. The discriminant is

$$\begin{aligned} &\left(\frac{2c\beta_2}{r} + 2\delta\beta_2 \right)^2 - \frac{8}{\beta_1\sigma^2} \left\{ \frac{2c^2}{\beta_1\sigma^2} - 2c\delta(1+\beta_2) \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ &= 4 \left\{ \beta_2^2 \left(\delta + \frac{c}{r} \right)^2 - \frac{4c^2}{\beta_1^2\sigma^4} + \frac{4c\delta(1+\beta_2)}{\beta_1\sigma^2} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ &= \frac{4}{\beta_1^2\sigma^4} \left\{ \beta_1^2\beta_2^2\sigma^4 \left(\delta + \frac{c}{r} \right)^2 - 4c^2 + 4c\delta\sigma^2(\beta_1+\beta_1\beta_2) \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ &= \frac{4}{\beta_1^2\sigma^4} \left\{ \frac{4r^2}{\sigma^4} \sigma^4 \left(\delta^2 + \frac{2c\delta}{r} + \frac{c^2}{r^2} \right) - 4c^2 + \left(4c\delta\sigma^2\beta_1 - 4c\delta\sigma^2\frac{2r}{\sigma^2} \right) \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ &= \frac{4}{\beta_1^2\sigma^4} \left\{ 4c\delta\sigma^2\beta_1 \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + 8rc\delta - 8rc\delta \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + 4r^2\delta^2 \right\} \\ &> 0. \end{aligned}$$

Besides,

$$\begin{aligned} Y(c) &= \frac{2\delta\beta_2}{c} - \frac{2\delta\beta_2}{c} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} - \frac{2\delta}{c} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + \frac{4}{\beta_1\sigma^2} + \frac{2\beta_2}{r} \\ &= \frac{2\delta\beta_2}{c} \left\{ 1 - \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} - \frac{2\delta}{c} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + \frac{4r+2\sigma^2\beta_1\beta_2}{r\beta_1\sigma^2} \\ &= \frac{2\delta\beta_2}{c} \left\{ 1 - \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} - \frac{2\delta}{c} \left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} + \frac{4r-4r}{r\beta_1\sigma^2} \end{aligned}$$

< 0 .

There are at most two positive values of x bigger than c that make $Y(x(K_D)) = 0$. If $Y(x(K_D)) \neq 0$ and for (55) to hold, it must be $dx(K_D)/dK_D = 0$. If $Y(x(K_D)) = 0$, then $x(K_D)$ is a constant, which also means that $dx(K_D)/dK_D = 0$. Thus, it can be concluded that

$$\frac{dx(K_D)}{dK_D} = \frac{dX_F^*(K_D)}{dK_D}(1 - \gamma K_D) - \gamma X_F^*(K_D) = 0,$$

which leads to

$$\frac{dX_F^*(K_D)}{dK_D} = \frac{\gamma X_F^*(K_D)}{1 - \gamma K_D} > 0, \quad (57)$$

implying that investing in more capacity by the dedicated leader would delay the investment of the flexible follower. According to (14), taking the derivative of $K_F^*(K_D)$ with respect to K_D , we get

$$\begin{aligned} \frac{dK_F^*(K_D)}{dK_D} &= \frac{1}{2\gamma} \left\{ -\gamma + \frac{c}{X_F^{*2}(K_D)} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \frac{dX_F^*(K_D)}{dK_D} \right\} \\ &= \frac{1}{2} \left\{ -1 + \frac{c}{(1 - \gamma K_D)X_F^*(K_D)} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right\} \\ &= -\frac{\gamma K_F^*(K_D)}{1 - \gamma K_D} \leq 0. \end{aligned} \quad (58)$$

This implies that an increase in the inflexible leader's investment capacity decreases the flexible follower's optimal capacity to invest with.

Figure 18: Illustration of $X_F^*(K_D)$, $K_F^*(K_D)$, $q_F^*(K_D)$, and $V_F^*(K_D)$ when the follower produces below capacity right after the investment. Parameter values are $\alpha = 0.04$, $r = 0.1$, $\sigma = 0.3$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 18 illustrates how the flexible follower's optimal investment threshold $X_F^*(K_D)$, the optimal investment capacity $K_F^*(K_D)$, the output level right after the investment $q_F^*(K_D) = q_F(X_F^*(K_D), K_D, K_F^*(K_D))$, and the project value when the firm invests $V_F^*(K_D) = V_F(X_F^*(K_D), K_D, K_F^*(K_D))$, change with the inflexible leader's investment capacity K_D when the firm produces below capacity right after the investment. It illustrates that the increase in the leader's investment capacity indeed delays the flexible follower's investment and decreases the capacity with which the follower enters the market. In fact, if the leader invests

with a capacity that is very close to the market size, then the flexible follower can be almost kept out of the market.

- Region 3

Check whether the leader's capacity influences the the investment decision of the flexible follower. The investment timing $X_F^*(K_D)$ and investment capacity $K_F^*(K_D)$ are determined by (16) and (17) when the follower produces up to capacity right after the investment. Rewriting these two equations yields

$$\frac{F(\beta_1)c^{1-\beta_2}(1+\beta_2)}{2(\beta_1-\beta_2)}[z(K_D)]^{\beta_2} + \frac{1}{r-\alpha}z(K_D) - \frac{c}{r} - \delta = 0, \quad (59)$$

and

$$\begin{aligned} & \frac{c^{1-\beta_2}F(\beta_1)}{4\gamma\beta_1X_F^*(K_D)} \{[w(K_D)]^{1+\beta_2} - [z(K_D)]^{1+\beta_2}\} - \frac{cK_F^*(K_D)}{r} - \delta K_F^*(K_D) \\ & + \frac{\beta_1-1}{4\gamma\beta_1(r-\alpha)X_F^*(K_D)} [w^2(K_D) - z^2(K_D)] = 0. \end{aligned} \quad (60)$$

respectively, and

$$\begin{aligned} w(K_D) &= X_F^*(K_D)(1-\gamma K_D), \\ z(K_D) &= X_F^*(K_D)(1-\gamma K_D - 2\gamma K_F^*(K_D)). \end{aligned}$$

Taking the derivative of (59) with respect to K_D for all $K_D \geq 0$ yields

$$\left[\frac{F(\beta_1)c^{1-\beta_2}\beta_2(1+\beta_2)[z(K_D)]^{\beta_2-1}}{2(\beta_1-\beta_2)} + \frac{1}{r-\alpha} \right] \frac{dz(K_D)}{dK_D} = 0. \quad (61)$$

For (61) to hold, it should be that $dz(K_D)/dK_D = 0$, implying $z(K_D)$ is a constant. If $dz(K_D)/dK_D \neq 0$, then $z(K_D)$ changes with K_D , and (61) cannot hold for all K_D , a contradiction. From $dz(K_D)/dK_D = 0$, it holds that

$$\frac{dX_F^*(K_D)}{dK_D} = \frac{\gamma X_F^*(K_D)}{1-\gamma K_D - 2\gamma K_F^*(K_D)} \left(1 + 2 \frac{dK_F^*(K_D)}{dK_D} \right). \quad (62)$$

Taking the derivative of (60) with respect of K_D gives

$$\begin{aligned} & - \frac{c^{1-\beta_2}F(\beta_1)}{4\gamma\beta_1X_F^{*2}(K_D)} \{[w(K_D)]^{1+\beta_2} - [z(K_D)]^{1+\beta_2}\} \frac{dX_F^*(K_D)}{dK_D} \\ & + \frac{(1+\beta_2)c^{1-\beta_2}F(\beta_1)}{4\gamma\beta_1X_F^*(K_D)} [w(K_D)]^{\beta_2} \frac{dw(K_D)}{dK_D} \\ & - \frac{\beta_1-1}{4\gamma(r-\alpha)\beta_1X_F^{*2}(K_D)} [w^2(K_D) - z^2(K_D)] \frac{dX_F^*(K_D)}{dK_D} \\ & + \frac{(\beta_1-1)w(K_D)}{2\gamma(r-\alpha)\beta_1X_F^*(K_D)} \frac{dw(K_D)}{dK_D} - \left(\frac{c}{r} + \delta \right) \frac{dK_F^*(K_D)}{dK_D} \\ & = - \left(\frac{c}{r} + \delta \right) \left(\frac{K_F^*(K_D)}{X_F^*(K_F)} \frac{dX_F^*(K_D)}{dK_D} + \frac{dK_F^*(K_D)}{dK_D} \right) \\ & + \left\{ \frac{(1+\beta_2)c^{1-\beta_2}F(\beta_1)[w(K_D)]^{\beta_2}}{4\gamma\beta_1X_F^*(K_D)} + \frac{(\beta_1-1)w(K_D)}{2\gamma(r-\alpha)\beta_1X_F^*(K_D)} \right\} \frac{dw(K_D)}{dK_D} \end{aligned}$$

$$\begin{aligned}
&= -\left(\frac{c}{r} + \delta\right) \left[\frac{\gamma K_F^*(K_D)}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} \left(1 + 2 \frac{dK_F^*(K_D)}{dK_D}\right) + \frac{dK_F^*(K_D)}{dK_D} \right] \\
&\quad + \left[(1 - \gamma K_D) \frac{dX_F^*(K_D)}{dK_D} - \gamma X_F^*(K_D) \right] \left\{ \frac{(1 + \beta_2)c^{1-\beta_2} F(\beta_1)[w(K_D)]^{\beta_2}}{4\gamma\beta_1 X_F^*(K_D)} + \frac{(\beta_1 - 1)w(K_D)}{2\gamma(r - \alpha)\beta_1 X_F^*(K_D)} \right\} \\
&= -\left(\frac{c}{r} + \delta\right) \frac{\gamma K_F^*(K_D) + (1 - \gamma K_D) \frac{dK_F^*(K_D)}{dK_D}}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} + \gamma X_F^*(K_D) \left[\frac{1 - \gamma K_D}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} (1 + \right. \\
&\quad \left. 2 \frac{dK_F^*(K_D)}{dK_D}) - 1 \right] \left\{ \frac{(1 + \beta_2)c^{1-\beta_2} F(\beta_1)[w(K_D)]^{\beta_2}}{4\gamma\beta_1 X_F^*(K_D)} + \frac{(\beta_1 - 1)w(K_D)}{2\gamma(r - \alpha)\beta_1 X_F^*(K_D)} \right\} \\
&= -\left(\frac{c}{r} + \delta\right) \frac{\gamma K_F^*(K_D) + (1 - \gamma K_D) \frac{dK_F^*(K_D)}{dK_D}}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} \\
&\quad + \frac{\gamma K_F^*(K_D) + (1 - \gamma K_D) \frac{dK_F^*(K_D)}{dK_D}}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} \left\{ \frac{(1 + \beta_2)c^{1-\beta_2} F(\beta_1)[w(K_D)]^{\beta_2}}{2\beta_1} + \frac{(\beta_1 - 1)w(K_D)}{\beta_1(r - \alpha)} \right\} \\
&= 0.
\end{aligned}$$

Thus,

$$\frac{c}{r} + \delta = \frac{(1 + \beta_2)c^{1-\beta_2} F(\beta_1)[w(K_D)]^{\beta_2}}{2\beta_1} + \frac{(\beta_1 - 1)w(K_D)}{\beta_1(r - \alpha)}. \quad (63)$$

Taking the derivative of (63) with respect to K_D yields

$$\left\{ \frac{(1 + \beta_2)\beta_2 c^{1-\beta_2} F(\beta_1)[w(K_D)]^{\beta_2-1}}{2\beta_1} + \frac{\beta_1 - 1}{\beta_1(r - \alpha)} \right\} \frac{dw(K_D)}{dK_D} = 0. \quad (64)$$

Similar to that (61) implies $z(K_D)$ is a constant, the fact that (64) holds implies $w(K_D)$ is also a constant and satisfies

$$\frac{dw(K_D)}{dK_D} = -\gamma X_F^*(K_D) + (1 - \gamma K_D) \frac{dX_F^*(K_D)}{dK_D} = 0.$$

It can be further derived that

$$\frac{dX_F^*(K_D)}{dK_D} = \frac{\gamma X_F^*(K_D)}{1 - \gamma K_D} > 0. \quad (65)$$

Moreover, from (62) and (65), we get

$$\frac{dK_F^*(K_D)}{dK_D} = -\frac{\gamma K_F^*(K_D)}{1 - \gamma K_D} < 0. \quad (66)$$

Thus, for the case that the flexible follower produces up to capacity right after the investment, the inflexible leader can delay and decrease the investment of the follower by investing in larger capacity.

Figure 19 illustrates how the flexible follower's optimal investment threshold $X_F^*(K_D)$, the optimal investment capacity $K_F^*(K_D)$, and the project value at the moment of investment $V_F^*(K_D)$, as functions of K_D . It confirms that $X_F^*(K_D)$ increases with K_D , while $K_F^*(K_D)$ and $V_F^*(K_D)$ decrease with K_D . Thus, the leader can delay the investment of the flexible follower. The leader can even prevent the follower entering the market by investing with a capacity close to the market boundary.

Figure 19: Illustration of $X_F^*(K_D)$, $K_F^*(K_D)$, and $V_F^*(K_D)$ when the follower produces up to capacity right after investment. Parameter values are $\alpha = 0.03$, $r = 0.1$, $\sigma = 0.14$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

A.4 Derivation of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, $\mathcal{N}(K_D)$

Employing value matching and smooth pasting at $X_1 = \frac{c}{1-\gamma K_D}$ and $X_2 = \frac{c}{1-\gamma K_D - 2\gamma K_F^*(K_D)}$, then given K_D ($0 \leq K_D < 1/\gamma$),

$$\mathcal{L}X_1^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X_1 - \frac{cK_D}{r} = \mathcal{M}_1X_1^{\beta_1} + \mathcal{M}_2X_1^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_1 - \frac{cK_D}{2r} \quad (67)$$

and

$$\beta_1 \mathcal{L}X_1^{\beta_1-1} + \frac{K_D(1-\gamma K_D)}{r-\alpha} = \beta_1 \mathcal{M}_1X_1^{\beta_1-1} + \beta_2 \mathcal{M}_2X_1^{\beta_2-1} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}, \quad (68)$$

$$\mathcal{M}_1X_2^{\beta_1} + \mathcal{M}_2X_2^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_2 - \frac{cK_D}{2r} = \mathcal{N}X_2^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X_2 - \frac{cK_D}{r} \quad (69)$$

and

$$\beta_1 \mathcal{M}_1X_2^{\beta_1-1} + \beta_2 \mathcal{M}_2X_2^{\beta_2-1} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)} = \beta_2 \mathcal{N}X_2^{\beta_2-1} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}. \quad (70)$$

From (67), it holds that

$$\mathcal{L}X_1^{\beta_1} - \mathcal{M}_1X_1^{\beta_1} = \mathcal{M}_2X_1^{\beta_2} - \frac{K_D(1-\gamma K_D)X_1}{2(r-\alpha)} + \frac{cK_D}{2r}. \quad (71)$$

From (68), it stands that

$$\beta_1 \left(\mathcal{L}X_1^{\beta_1-1} - \mathcal{M}_1X_1^{\beta_1-1} \right) = \beta_2 \mathcal{M}_2X_1^{\beta_2-1} - \frac{K_D(1-\gamma K_D)X_1}{2(r-\alpha)}.$$

Thus, it can be derived that

$$\begin{aligned} \mathcal{M}_2 &= \frac{X_1^{-\beta_2}}{\beta_1 - \beta_2} \left[\frac{K_D(\beta_1 - 1)(1-\gamma K_D)X_1}{2(r-\alpha)} - \frac{\beta_1 cK_D}{2r} \right] \\ &= \frac{cK_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r} \right) \left(\frac{c}{1-\gamma K_D} \right)^{-\beta_2}. \end{aligned} \quad (72)$$

From (69), it holds that

$$\mathcal{N}X_2^{\beta_2} - \mathcal{M}_2X_2^{\beta_2} = \mathcal{M}_1X_2^{\beta_1} - \frac{K_D(1-\gamma K_D - 2\gamma K_F^*(K_D))X_2}{2(r-\alpha)} + \frac{cK_D}{2r}. \quad (73)$$

From (70), it stands that

$$\beta_2 \mathcal{N} X_2^{\beta_2} - \beta_2 \mathcal{M}_2 X_2^{\beta_2} = \beta_1 \mathcal{M}_1 X_2^{\beta_1} - \frac{K_D(1 - \gamma K_D - 2\gamma K_F^*(K_D))X_2}{2(r - \alpha)}.$$

Thus, it can be derived that

$$\begin{aligned} \mathcal{M}_1 &= \frac{X_2^{-\beta_1}}{\beta_1 - \beta_2} \left[\frac{(1 - \beta_2)K_D(1 - \gamma K_D - 2\gamma K_F^*(K_D))X_2}{2(r - \alpha)} + \frac{\beta_2 c K_D}{2r} \right] \\ &= \frac{X_2^{-\beta_1}}{\beta_1 - \beta_2} \left[\frac{(1 - \beta_2)c K_D}{2(r - \alpha)} + \frac{\beta_2 c K_D}{2r} \right] \\ &= \frac{c K_D}{2(\beta_1 - \beta_2)} \left(\frac{1 - \beta_2}{r - \alpha} + \frac{\beta_2}{r} \right) \left(\frac{c}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} \right)^{-\beta_1} \\ &= -\frac{c K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left(\frac{c}{1 - \gamma K_D - 2\gamma K_F^*(K_D)} \right)^{-\beta_1}. \end{aligned} \quad (74)$$

From (71), it can be derived that

$$\begin{aligned} \mathcal{L} &= \mathcal{M}_1 + \mathcal{M}_2 X_1^{\beta_2 - \beta_1} - \frac{c K_D X_1^{-\beta_1}}{2} \left(\frac{1}{r - \alpha} - \frac{1}{r} \right) \\ &= \mathcal{M}_1 + \frac{c K_D X_1^{-\beta_1}}{2} \left[\frac{\beta_1 - 1}{(r - \alpha)(\beta_1 - \beta_2)} - \frac{\beta_1}{r(\beta_1 - \beta_2)} - \frac{1}{r - \alpha} + \frac{1}{r} \right] \\ &= \mathcal{M}_1 + \frac{c K_D X_1^{-\beta_1}}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \\ &= \frac{c K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) (X_1^{-\beta_1} - X_2^{-\beta_1}) \\ &= \frac{c^{1-\beta_1} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left[(1 - \gamma K_D)^{\beta_1} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_1} \right]. \end{aligned} \quad (75)$$

From (73), it can be derived that

$$\begin{aligned} \mathcal{N} &= \mathcal{M}_1 X_2^{\beta_1 - \beta_2} + \mathcal{M}_2 + X_2^{-\beta_2} \left(\frac{c K_D}{2r} - \frac{c K_D}{2(r - \alpha)} \right) \\ &= \mathcal{M}_2 + \frac{c K_D X_2^{-\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{1 - \beta_2}{r - \alpha} + \frac{\beta_2}{r} \right) + \frac{c K_D X_2^{-\beta_2}}{2} \left(\frac{1}{r} - \frac{1}{r - \alpha} \right) \\ &= \mathcal{M}_2 + \frac{c K_D X_2^{-\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{1 - \beta_1}{r - \alpha} + \frac{\beta_1}{r} \right) \\ &= \frac{c K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (X_1^{-\beta_2} - X_2^{-\beta_2}) \\ &= \frac{c^{1-\beta_2} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2} \right]. \end{aligned} \quad (76)$$

Next, we look at the signs for $\mathcal{L}$, $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{N}$. In order to do this, we first check the signs for $(\beta - 1)/(r - \alpha) - \beta/r = \frac{\alpha\beta - r}{r(r - \alpha)}$ with $\beta = \beta_1$ and $\beta = \beta_2$. If $\alpha \geq 0$, then $\alpha\beta_2 - r < 0$ because $\beta_2 < 0$. If $\alpha < 0$, then

$$\alpha\beta_2 - r = \alpha \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r}{\alpha} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \right),$$

with $\frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r}{\alpha} > 0$. From

$$\begin{aligned} & \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r}{\alpha} \right)^2 - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 - \frac{2r}{\sigma^2} \\ &= -\frac{2r}{\alpha} \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right) + \frac{r^2}{\alpha^2} - \frac{2r}{\sigma^2} \\ &= -\frac{r}{\alpha} + \frac{r^2}{\alpha^2} > 0, \end{aligned}$$

we get $\alpha\beta_2 - r < 0$. So, $\frac{\beta_2-1}{r-\alpha} - \frac{\beta_2}{r} < 0$.

If $\alpha \leq 0$, then $\alpha\beta_1 - r < 0$. If $\alpha > 0$, then

$$\alpha\beta_1 - r = \alpha \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r}{\alpha} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \right),$$

with $\frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r}{\alpha} < 0$, because $r > \alpha$. From

$$\begin{aligned} & \left(\frac{r}{\alpha} + \frac{\alpha}{\sigma^2} - \frac{1}{2} \right)^2 - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 - \frac{2r}{\sigma^2} \\ &= \frac{r^2}{\alpha^2} + \frac{2r}{\alpha} \left(\frac{\alpha}{\sigma^2} - \frac{1}{2} \right) - \frac{2r}{\sigma^2} \\ &= \frac{r^2}{\alpha^2} - \frac{r}{\alpha} > 0, \end{aligned}$$

we get $\alpha\beta_1 - r < 0$. So, $\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r} < 0$.

Thus, it can be concluded that for $0 \leq K_D < 1/\gamma$

$$\mathcal{L}(K_D) < 0,$$

$$\mathcal{M}_1(K_D) > 0,$$

$$\mathcal{M}_2(K_D) < 0,$$

$$\mathcal{N}(K_D) > 0.$$

A.5 Proof of Proposition 2

A.5.1 Negative $\mathcal{B}_1(K_D)$

Before the derivation of the dedicated leader's optimal investment capacity in the entry deterrence and accommodation strategies, first look at the sign of $\mathcal{B}_1(K_D)$. We have

$$\begin{aligned} \mathcal{B}_1(K_D) &= \mathcal{M}_1(K_D) + \mathcal{M}_2(K_D) X_F^{*\beta_2 - \beta_1}(K_D) \\ &\quad - \frac{K_D(1 - \gamma K_D)}{2(r - \alpha)} X_F^{*1 - \beta_1}(K_D) + \frac{cK_D}{2r} X_F^{* - \beta_1}(K_D) \\ &= \frac{1}{X_F^{*\beta_1}(K_D)} \left[-\frac{cK_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left(\frac{X_F^*(K_D)}{X_2(K_D)} \right)^{\beta_1} \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{cK_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{X_F^*(K_D)}{X_1(K_D)} \right)^{\beta_2} - \frac{cK_D}{2(r - \alpha)} \frac{X_F^*(K_D)}{X_1(K_D)} + \frac{cK_D}{2r} \Big] \\
& = \frac{cK_D}{2X_F^{*\beta_1}(K_D)} \left[-\frac{1}{\beta_1 - \beta_2} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left(\frac{X_F^*(K_D)}{X_2(K_D)} \right)^{\beta_1} \right. \\
& \quad \left. + \frac{1}{\beta_1 - \beta_2} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{X_F^*(K_D)}{X_1(K_D)} \right)^{\beta_2} - \frac{1}{r - \alpha} \frac{X_F^*(K_D)}{X_1(K_D)} + \frac{1}{r} \right].
\end{aligned}$$

For $X_F^*(K_D)/X_1(K_D)$ and $X_F^*(K_D)/X_2(K_D)$, it holds that

$$\frac{d}{dK_D} \frac{X_F^*(K_D)}{X_1(K_D)} = \frac{1}{X_1^2(K_D)} \left(\frac{\gamma K_F^*(K_D) X_1(K_D)}{1 - \gamma K_D} - X_F^*(K_D) \frac{\gamma X_1(K_D)}{1 - \gamma K_D} \right) = 0$$

and

$$\frac{d}{dK_D} \frac{X_F^*(K_D)}{X_2(K_D)} = \frac{1}{X_2^2(K_D)} \left(\frac{\gamma K_F^*(K_D) X_2(K_D)}{1 - \gamma K_D} - X_F^*(K_D) \frac{\gamma X_2(K_D)}{1 - \gamma K_D} \right) = 0.$$

This implies that $X_F^*(K_D)/X_1(K_D)$ and $X_F^*(K_D)/X_2(K_D)$ are constants and do not change with K_D . So we can set $K_D = 0$, and then

$$\frac{X_F^*(K_D)}{X_1(K_D)} = \frac{X_F^*(0)}{c}$$

and

$$\frac{X_F^*(K_D)}{X_2(K_D)} = \frac{X_F^*(0)}{c} (1 - 2\gamma K_F^*(0)) = \left(\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right)^{\frac{1}{\beta_1}}.$$

Equation (15) just becomes the corresponding implicit equation to determine X^* for the monopoly case:

$$F(\beta_1) \left(\frac{X^*}{c} \right)^{\beta_2} + \frac{\beta_1 - 1}{r - \alpha} \frac{X^*}{c} - \frac{2\beta_1}{r} + \frac{\beta_1 + 1}{r + \alpha - \sigma^2} \frac{c}{X^*} - \frac{2\beta_1\delta}{c} \left(1 - \frac{c}{X^*} \left[\frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right]^{\frac{1}{\beta_1}} \right) = 0. \quad (77)$$

Recall

$$\begin{aligned}
\mathcal{B}_1(K_D) &= \frac{cK_D}{2X_F^{*\beta_1}(K_D)} \left[-\frac{1}{\beta_1 - \beta_2} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \left(\frac{X_F^*(K_D)}{X_2(K_D)} \right)^{\beta_1} \right. \\
&\quad \left. + \frac{1}{\beta_1 - \beta_2} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{X_F^*(K_D)}{X_1(K_D)} \right)^{\beta_2} - \frac{1}{r - \alpha} \frac{X_F^*(K_D)}{X_1(K_D)} + \frac{1}{r} \right] \\
&= \frac{cK_D}{2(\beta_1 - \beta_2)X_F^{*\beta_1}(K_D)} \left[-\left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) \frac{2\delta(\beta_1 - \beta_2)}{c(1 + \beta_1)F(\beta_2)} \right. \\
&\quad \left. + \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{X^*}{c} \right)^{\beta_2} - (\beta_1 - \beta_2) \left(\frac{1}{r - \alpha} \frac{X^*}{c} - \frac{1}{r} \right) \right] \\
&= \frac{cK_D}{2X_F^{*\beta_1}(K_D)} \mathcal{F}(X^*),
\end{aligned}$$

where X^* satisfies (77). Next, we show $\mathcal{F}(X^*)$ is negative numerically. The demonstration is shown in Figure 20. Note that γ does not influence $\mathcal{F}(X^*)$, so the numerical analysis is just about the influence of α , σ , r , c and δ . The default parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$. Some combination of parameter values does not make the flexible follower produce below capacity right after investment. After

ruling out these combinations, $\mathcal{F}(X^*)$ changing with parameters is illustrated in Figure 20. The numerical analysis confirms that $\mathcal{B}_1(K_D)$ is negative when the flexible follower produces below capacity right after investment. In the following analysis, we will take that $\mathcal{B}_1(K_D)$ as negative.

Figure 20: Illustration of negative $\mathcal{F}(X^*)$ changing with α , σ , r , c and δ . Parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$.

A.5.2 Proof of Proposition 2

In order to get the optimal investment decisions for the dedicated leader, we first calculate the first derivative of $\mathcal{B}_1(K_D)$ with respect of K_D . First, $\mathcal{M}_1(K_D)$ can be rewritten as

$$\begin{aligned}\mathcal{M}_1(K_D) &= -\frac{c^{1-\beta_1}K_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left[\frac{c}{X_F^*(K_D)}\left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}\right]^{\beta_1} \\ &= -\frac{cK_D}{2(\beta_1-\beta_2)X_F^{*\beta_1}(K_D)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)} \\ &= -\frac{K_D}{X_F^{*\beta_1}(K_D)}\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right).\end{aligned}$$

With $dK_F^*(K_D)/dK_D$ and $dX_F^*(K_D)/dK_D$ being defined by (57) and (58), it can be calculated that

$$\begin{aligned}\frac{d\mathcal{M}_1(K_D)}{dK_D} &= -\frac{1-\gamma K_D-\beta_1\gamma K_D}{1-\gamma K_D}\left(\frac{1}{X_F^*(K_D)}\right)^{\beta_1}\frac{\delta}{F(\beta_2)(1+\beta_1)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right) \\ &= -\frac{c(1-\gamma K_D-\beta_1\gamma K_D)}{(1-\gamma K_D)c^{\beta_1}}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\frac{1}{2(\beta_1-\beta_2)}\left(\frac{c}{X_F^*}\right)^{\beta_1}\frac{2(\beta_1-\beta_2)\delta}{c(1+\beta_1)F(\beta_2)} \\ &= -\frac{c(1-\gamma K_D-\beta_1\gamma K_D)}{(1-\gamma K_D)c^{\beta_1}}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\frac{1}{2(\beta_1-\beta_2)}(1-\gamma K_D-2\gamma K_F^*)^{\beta_1} \\ &= \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\left[-\frac{c^{1-\beta_1}K_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)(1-\gamma K_D-2\gamma K_F^*)^{\beta_1}\right] \\ &= \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D).\end{aligned}$$

Furthermore, it is such that

$$\begin{aligned}&\frac{d}{dK_D}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) \\ &= \frac{c^{1-\beta_2}X_F^{*\beta_2-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)[(1-\gamma K_D)^{\beta_2}-\beta_2\gamma K_D(1-\gamma K_D)^{\beta_2-1}] \\ &\quad + \mathcal{M}_2(K_D)\frac{\gamma(\beta_2-\beta_1)X_F^{*\beta_2-\beta_1}(K_D)}{1-\gamma K_D} \\ &= \frac{(1-\gamma K_D-\beta_2\gamma K_D)X_F^{*\beta_2-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\left(\frac{c}{1-\gamma K_D}\right)^{1-\beta_2}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right) \\ &\quad + \mathcal{M}_2(K_D)\frac{\gamma(\beta_2-\beta_1)X_F^{*\beta_2-\beta_1}(K_D)}{1-\gamma K_D} \\ &= \frac{(1-\gamma K_D-\beta_2\gamma K_D)X_F^{*\beta_2-\beta_1}(K_D)}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D) + \frac{(\beta_2-\beta_1)\gamma K_DX_F^{*\beta_2-\beta_1}(K_D)}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D) \\ &= \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D).\end{aligned}$$

We also have

$$\begin{aligned}&\frac{d}{dK_D}\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) \\ &= \frac{1}{2(r-\alpha)}\left[(1-2\gamma K_D)X_F^{*1-\beta_1}(K_D) + K_D(1-\gamma K_D)\frac{(1-\beta_1)\gamma X_F^{*1-\beta_1}(K_D)}{1-\gamma K_D}\right]\end{aligned}$$

$$= \frac{(1 - \gamma K_D - \beta_1 \gamma K_D) X_F^{*1-\beta_1}(K_D)}{2(r - \alpha)}$$

and

$$\begin{aligned} \frac{d}{dK_D} \frac{cK_D}{2r} X_F^{*- \beta_1}(K_D) &= \frac{c}{2r} \left(X_F^{*- \beta_1}(K_D) - \beta_1 K_D \frac{\gamma X_F^{*- \beta_1}(K_D)}{1 - \gamma K_D} \right) \\ &= \frac{c X_F^{*- \beta_1}(K_D) (1 - \gamma K_D - \beta_1 \gamma K_D)}{2r(1 - \gamma K_D)}. \end{aligned}$$

Thus, according to (24), we get that

$$\begin{aligned} \frac{d\mathcal{B}_1(K_D)}{dK_D} &= \frac{d\mathcal{M}_1(K_D)}{dK_D} + \frac{d}{dK_D} \mathcal{M}_2(K_D) X_F^{*\beta_2-\beta_1}(K_D) \\ &\quad - \frac{d}{dK_D} \frac{K_D(1 - \gamma K_D)}{2(r - \alpha)} X_F^{*1-\beta_1}(K_D) + \frac{d}{dK_D} \frac{cK_D}{2r} X_F^{*- \beta_1}(K_D) \\ &= \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_1(K_D) + \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_2(K_D) X_F^{*\beta_2-\beta_1}(K_D) \\ &\quad - \frac{(1 - \gamma K_D - \beta_1 \gamma K_D) X_F^{*1-\beta_1}(K_D)}{2(r - \alpha)} + \frac{c(1 - \gamma K_D - \beta_1 \gamma K_D) X_F^{*- \beta_1}(K_D)}{2r(1 - \gamma K_D)} \\ &= \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_1(K_D). \end{aligned}$$

Next, we analyse the entry deterrence and accommodation strategies for the dedicated leader, which include the optimal investment capacities and optimal investment thresholds. The proof for the second order derivative can be found in the supplement file.

1. Entry Deterrence Strategy

The investment capacity $K_D^{det}(X)$ for a given level of X satisfies the following equation

$$\begin{aligned} \frac{\partial V_D(X, K_D) - \delta K_D}{\partial K_D} &= \frac{d\mathcal{B}_1(K_D)}{dK_D} X^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta \\ &= \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_1(K_D) X^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta = 0. \end{aligned} \quad (78)$$

The entry deterrence strategy cannot happen when $K_D^{det}(X) < \hat{K}_D(X)$, which yields $X > X_2^{det}$ with X_2^{det} and $K_D^{det}(X_2^{det})$ satisfying (25) and (78). This is because the demand is high enough for the follower to invest immediately to enter the market. The entry deterrence strategy also does not happen when $K_D^{det}(X) < 0$, yielding $X < X_1^{det}$ with X_1^{det} satisfying

$$\begin{aligned} \psi(X_1^{det}) &= \left[-\frac{\delta}{(1 + \beta_1)F(\beta_2)} \left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r} \right) + \frac{c^{1-\beta_2} X_F^{*\beta_2}(0)}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \right. \\ &\quad \left. - \frac{X_F^*(0)}{2(r - \alpha)} + \frac{c}{2r} \right] \left(\frac{X_1^{det}}{X_F^*(0)} \right)^{\beta_1} + \frac{X_1^{det}}{r - \alpha} - \frac{c}{r} - \delta = 0. \end{aligned} \quad (79)$$

Thus, the entry deterrence strategy is only possible when $X \in (X_1^{det}, X_2^{det})$. Suppose the investment threshold of the dedicated leader is $X^{det}(K_D)$ if the follower invests with capacity K_D in the entry

deterrence strategy. The leader's value function before and after the investment is as follows

$$V_D(X, K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{det}(K_D), \\ \mathcal{B}_1(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} & X^{det}(K_D) \leq X < X_F^*(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} & X \geq X_F^*(K_D). \end{cases} \quad (80)$$

The value matching and smooth pasting conditions to determine $X^{det}(K_D)$ are

$$\begin{aligned} \mathcal{A}(K_D)X^{\beta_1} &= \mathcal{B}_1(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} - \delta K_D, \\ \beta_1 \mathcal{A}(K_D)X^{\beta_1-1} &= \beta_1 \mathcal{B}_1(K_D)X^{\beta_1-1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}. \end{aligned}$$

Thus, the threshold of the entry deterrence strategy $X^{det}(K_D)$ is

$$X^{det}(K_D) = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{1 - \gamma K_D} \left(\frac{c}{r} + \delta \right). \quad (81)$$

Substituting $X^{det}(K_D)$ into equation (78), the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ can be derived such that

$$\begin{aligned} K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det})) = \frac{1}{(\beta_1 + 1)\gamma}, \\ X^{det}(K_D^{det}) &= \frac{(\beta_1 + 1)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right). \end{aligned}$$

2. Entry Accommodation Strategy

Note that from (72), we can get

$$\begin{aligned} \frac{\partial \mathcal{M}_2(K_D)}{\partial K_D} &= \frac{c}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{c}{1 - \gamma K_D} \right)^{-\beta_2} \\ &\quad + \frac{c}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{c}{1 - \gamma K_D} \right)^{-\beta_2 - 1} \frac{-c\beta_2\gamma K_D}{(1 - \gamma K_D)^2} \\ &= \frac{c}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{c}{1 - \gamma K_D} \right)^{-\beta_2} \left(1 - \frac{\beta_2\gamma K_D}{1 - \gamma K_D} \right) \\ &= \frac{cK_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{c}{1 - \gamma K_D} \right)^{-\beta_2} \frac{1 - \gamma K_D - \beta_2\gamma K_D}{K_D(1 - \gamma K_D)} \\ &= \frac{1 - \gamma K_D - \beta_2\gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_2(K_D). \end{aligned}$$

The optimal capacity $K_D^{acc}(X)$ satisfies the following first order condition

$$\begin{aligned} &\frac{\partial V(X, K_D) - \delta K_D}{\partial K_D} \\ &= \frac{d\mathcal{M}_1(K_D)}{dK_D} X^{\beta_1} + \frac{d\mathcal{M}_2(K_D)}{dK_D} X^{\beta_2} + \frac{1 - 2\gamma K_D}{2(r - \alpha)} X - \frac{c}{2r} - \delta \\ &= \frac{1 - \gamma K_D - \beta_1\gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_1(K_D) X^{\beta_1} + \frac{1 - \gamma K_D - \beta_2\gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{M}_2(K_D) X^{\beta_2} \end{aligned}$$

$$+ \frac{1-2\gamma K_D}{2(r-\alpha)}X - \frac{c}{2r} - \delta = 0. \quad (82)$$

The entry accommodation strategy only happens when $X \geq X_F^*(K_D)$, implying that the market demand is large enough to allow both the dedicated leader and the flexible follower to invest at the same time. Let X_1^{acc} be such that $X_1^{acc} = X_F^*(K_D^{acc}(X_1^{acc}))$, then X_1^{acc} and the corresponding $K_D^{acc}(X_1^{acc})$ satisfy (82) and (25). Suppose the dedicated leader invests at $X^{acc}(K_D)$ when capacity level is K_D in the entry accommodation strategy, the leader's value function before and after investment is

$$V_D(X, K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{acc}(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} & X \geq X_F^*(K_D) \geq X^{acc}(K_D). \end{cases} \quad (83)$$

The value matching and smooth pasting conditions to determine $X^{acc}(K_D)$ are

$$\begin{aligned} \mathcal{A}(K_D)X^{\beta_1} &= \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{cK_D}{2r} - \delta K_D, \\ \beta_1 \mathcal{A}(K_D)X^{\beta_1-1} &= \beta_1 \mathcal{M}_1(K_D)X^{\beta_1-1} + \beta_2 \mathcal{M}_2(K_D)X^{\beta_2-1} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}. \end{aligned}$$

Thus, the investment capacity $K_D^{acc}(X^{acc})$ and investment threshold $X^{acc}(K_D^{acc})$ satisfy equation (82) and

$$(\beta_1 - \beta_2)\mathcal{M}_2(K_D)X^{\beta_2} + \frac{(\beta_1 - 1)K_D(1-\gamma K_D)}{2(r-\alpha)}X - \frac{c\beta_1 K_D}{2r} - \beta_1 \delta K_D = 0. \quad (84)$$

Rewrite these two equations, then

$$\begin{aligned} \frac{1-\gamma K_D - \beta_1 \gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)X^{\beta_2} + \frac{1-\gamma K_D - \beta_2 \gamma K_D}{1-\gamma K_D} \frac{c}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r} \right) \left(\frac{X}{X_1} \right)^{\beta_2} \\ + \frac{1-2\gamma K_D}{2(r-\alpha)}X - \frac{c}{2r} - \delta = 0, \end{aligned}$$

$$\frac{c}{2\beta_1} \left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r} \right) \left(\frac{X}{X_1} \right)^{\beta_2} + \frac{(\beta_1 - 1)(1-\gamma K_D)}{2\beta_1(r-\alpha)}X - \frac{c}{2r} - \delta = 0.$$

Solving these equations, we can get

$$K_D^{acc} \equiv K_D^{acc}(X^{acc}(K_D^{acc})) = \frac{1}{(\beta_1 + 1)\gamma}.$$

Let

$$Z(X) = \frac{c}{2\beta_1} \left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r} \right) \left(\frac{\beta_1}{(\beta_1 + 1)c} \right)^{\beta_2} X^{\beta_2} + \frac{\beta_1 - 1}{2(\beta_1 + 1)(r-\alpha)}X - \frac{c}{2r} - \delta.$$

Then

$$Z\left(\frac{(\beta_1 + 1)(r-\alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right)\right) = \frac{c}{2\beta_1} \left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r} \right) \left(\frac{\beta_1(r-\alpha)}{(\beta_1 - 1)c} \left(\frac{c}{r} + \delta \right) \right)^{\beta_2} + \frac{1}{2} \left(\frac{c}{r} + \delta \right) - \frac{c}{2r} - \delta$$

$$= \frac{c}{2\beta_1} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{\beta_1(r - \alpha)}{(\beta_1 - 1)c} \left(\frac{c}{r} + \delta \right) \right)^{\beta_2} - \frac{\delta}{2} < 0.$$

Because

$$\frac{dZ(X)}{dX} = \frac{c\beta_2}{2\beta_1} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\frac{\beta_1}{(\beta_1 + 1)c} \right)^{\beta_2} X^{\beta_2 - 1} + \frac{\beta_1 - 1}{2(\beta_1 + 1)(r - \alpha)} > 0,$$

it can be concluded that

$$X^{acc}(K_D^{acc}) > \frac{(\beta_1 + 1)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right).$$

A.6 Proof of Proposition 3

A.6.1 Negative $\mathcal{B}_2(K_D)$

When the flexible follower produces up to capacity right after investment, we have

$$\begin{aligned} \mathcal{B}_2(K_D) &= \mathcal{N}(K_D) X_F^{*\beta_2 - \beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r - \alpha} X_F^{*1 - \beta_1}(K_D) \\ &= \frac{c^{1 - \beta_2} K_D}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) [(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F(K_D))^{\beta_2}] X_F^{*\beta_2 - \beta_1}(K_D) \\ &\quad - \frac{\gamma K_D K_F^*(K_D)}{r - \alpha} X_F^{*1 - \beta_1}(K_D) \\ &= \frac{c K_D X_F^{* - \beta_1}(K_D)}{2(\beta_1 - \beta_2)} \left\{ \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left(\left(\frac{X_F^*(K_D)}{X_1(K_D)} \right)^{\beta_2} - \left(\frac{X_F^*(K_D)}{X_2(K_D)} \right)^{\beta_2} \right) \right. \\ &\quad \left. + \frac{\beta_1 - \beta_2}{r - \alpha} \left(\frac{X_F^*(K_D)}{X_2(K_D)} - \frac{X_F^*(K_D)}{X_1(K_D)} \right) \right\}. \end{aligned}$$

Note that

$$\begin{aligned} \frac{dX_F^*(K_D)}{dK_D} &= \frac{\gamma X_F^*(K_D)}{1 - \gamma K_D}, \\ \frac{dX_1(K_D)}{dK_D} &= \frac{\gamma X_1(K_D)}{1 - \gamma K_D}, \\ \frac{dX_2(K_D)}{dK_D} &= \frac{\gamma X_2(K_D)}{1 - \gamma K_D}. \end{aligned}$$

Thus, for the terms in $\mathcal{B}_2(K_D)$, we have

$$\begin{aligned} \frac{d}{dK_D} \frac{X_F^*(K_D)}{X_1(K_D)} &= \frac{1}{X_1^2(K_D)} \left(\frac{\gamma X_F^*(K_D) X_1(K_D)}{1 - \gamma K_D} - \frac{\gamma X_F^*(K_D) X_1(K_D)}{1 - \gamma K_D} \right) = 0, \\ \frac{d}{dK_D} \frac{X_F^*(K_D)}{X_2(K_D)} &= \frac{1}{X_2^2(K_D)} \left(\frac{\gamma X_F^*(K_D) X_2(K_D)}{1 - \gamma K_D} - \frac{\gamma X_F^*(K_D) X_2(K_D)}{1 - \gamma K_D} \right) = 0. \end{aligned}$$

Similar to the case that flexible follower produces below capacity right after investment, $X_F^*(K_D)/X_1(K_D)$ and $X_F^*(K_D)/X_2(K_D)$ are constants and do not change with K_D . Thus

$$\begin{aligned} \frac{X_F^*(K_D)}{X_1(K_D)} &= \frac{X_F^*(0)}{c}, \\ \frac{X_F^*(K_D)}{X_2(K_D)} &= \frac{X_F^*(0)}{X_2(0)}. \end{aligned}$$

Let $X_F^*(0) = X^*$ and $X_2(0) = 1 - 2\gamma K^*$, with X^* as the optimal investment threshold and K^* as the optimal capacity in the monopoly case where the firm produces up to capacity right after investment. We rewrite $\mathcal{B}_2(K_D)$ as

$$\mathcal{B}_2(K_D) = \frac{cK_D X_F^{*- \beta_1}(K_D)}{2(\beta_1 - \beta_2)} \mathcal{G}(X^*, K^*),$$

where X^* and K^* satisfy

$$\frac{c(1 + \beta_2)F(\beta_1)}{2(\beta_1 - \beta_2)} \left(\frac{(1 - 2\gamma K^*)X^*}{c} \right)^{\beta_2} + \frac{c}{r - \alpha} \frac{(1 - 2\gamma K^*)X^*}{c} - \frac{c}{r} - \delta = 0$$

and

$$\frac{cF(\beta_1)}{4\gamma\beta_1} \left(\left(\frac{X^*}{c} \right)^{\beta_2} - (1 - 2\gamma K^*) \left(\frac{(1 - 2\gamma K^*)X^*}{c} \right)^{\beta_2} \right) + \frac{\beta_1 - 1}{\beta_1} \frac{(1 - \gamma K^*)X^* K^*}{r - \alpha} - \frac{cK^*}{r} - \delta K^* = 0.$$

$\mathcal{B}_2(K_D)$ is intuitively negative. However, it is too complicated to show this analytically. So we try to show it is negative numerically to verify the conjecture. Figure 21 demonstrates $\mathcal{G}(X^*, K^*)$ changing with parameters. The default parameter values are given as $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, and $\gamma = 0.05$. Some combination of parameter values does not define the case that the follower produces up to capacity right after investment. After ruling out such combinations, the negative $\mathcal{G}(X^*, K^*)$ is illustrated in Figure 21. This confirms the conjecture that $\mathcal{B}_2(K_D)$ is negative. So in the following analysis, we assume negative $\mathcal{B}_2(K_D)$.

A.6.2 Proof of Proposition 3

We start with the derivative of $\mathcal{B}_2(K_D)$ with respect to K_D , where $K_F^*(K_D)$ and $X_F^*(K_D)$ are defined by (16) and (17), and $dK_F^*(K_D)/dK_D$ and $dX_F^*(K_D)/dK_D$ are defined by (65) and (66).

$$\begin{aligned} \frac{d\mathcal{N}(K_D)}{dK_D} &= \frac{c^{1-\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2} \right. \\ &\quad \left. - \gamma\beta_2 K_D \left((1 - \gamma K_D)^{\beta_2-1} - \left(1 + 2\frac{dK_F^*(K_D)}{dK_D} \right) (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2-1} \right) \right] \\ &= \frac{c^{1-\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2} \right. \\ &\quad \left. - \gamma\beta_2 K_D \left((1 - \gamma K_D)^{\beta_2-1} - \frac{(1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2}}{1 - \gamma K_D} \right) \right] \\ &= \frac{c^{1-\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[(1 - \gamma K_D)^{\beta_2} - (1 - \gamma K_D - 2\gamma K_F^*(K_D))^{\beta_2} \right] \cdot \\ &\quad \frac{1 - \gamma K_D - \beta_2 \gamma K_D}{1 - \gamma K_D} \\ &= \frac{X_1(1 - \gamma K_D - \beta_2 \gamma K_D)}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (X_1^{-\beta_2} - X_2^{-\beta_2}) \\ &= \frac{\mathcal{N}(K_D)(1 - \gamma K_D - \beta_2 \gamma K_D)}{K_D(1 - \gamma K_D)}, \end{aligned}$$

$$\frac{d}{dK_D} \mathcal{N}(K_D) X_F^{*\beta_2 - \beta_1} = \frac{d\mathcal{N}(K_D)}{dK_D} X_F^{*\beta_2 - \beta_1} + \mathcal{N}(K_D) X_F^{*\beta_2 - \beta_1} \frac{\gamma(\beta_2 - \beta_1)}{1 - \gamma K_D}$$

Figure 21: Illustration of negative $\mathcal{G}(X^*, K^*)$ changing with α , σ , r , c , δ and γ . Parameter values are $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

$$\begin{aligned}
&= \frac{(1 - \gamma K_D - \beta_2 \gamma K_D) \mathcal{N}(K_D)}{K_D(1 - \gamma K_D)} X_F^{*\beta_2 - \beta_1} + \frac{(\beta_2 \gamma K_D - \beta_1 \gamma K_D) \mathcal{N}(K_D)}{K_D(1 - \gamma K_D)} X_F^{*\beta_2 - \beta_1} \\
&= \frac{(1 - \gamma K_D - \beta_1 \gamma K_D) \mathcal{N}(K_D)}{K_D(1 - \gamma K_D)} X_F^{*\beta_2 - \beta_1},
\end{aligned}$$

and

$$\begin{aligned}
\frac{d}{dK_D} K_D K_F^* X_F^{*1 - \beta_1} &= K_F^* X_F^{*1 - \beta_1} + K_D \frac{-\gamma K_F^*}{1 - \gamma K_D} X_F^{*1 - \beta_1} + K_D K_F^* (1 - \beta_1) X_F^{* - \beta_1} \frac{\gamma X_F^*}{1 - \gamma K_D} \\
&= K_F^* X_F^{*1 - \beta_1} - \frac{\gamma K_D K_F^* X_F^{*1 - \beta_1}}{1 - \gamma K_D} + \frac{\gamma(1 - \beta_1) K_D K_F^* X_F^{*1 - \beta_1}}{1 - \gamma K_D} \\
&= K_F^* X_F^{*1 - \beta_1} - \frac{\beta_1 \gamma}{1 - \gamma K_D} K_D K_F^* X_F^{*1 - \beta_1} \\
&= \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} K_D K_F^* X_F^{*1 - \beta_1}.
\end{aligned}$$

Thus,

$$\frac{d\mathcal{B}_2(K_D)}{dK_D} = \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_2(K_D).$$

1. Entry Deterrence Strategy

The optimal capacity by the dedicated leader $K_D^{det}(X)$ satisfies the following first order condition

$$\begin{aligned}
\frac{\partial V_D(X, K_D) - \delta K_D}{\partial K_D} &= \frac{d\mathcal{B}_2(K_D)}{dK_D} X^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta \\
&= \frac{1 - \gamma K_D - \beta_1 \gamma K_D}{K_D(1 - \gamma K_D)} \mathcal{B}_2(K_D) X^{\beta_1} + \frac{1 - 2\gamma K_D}{r - \alpha} X - \frac{c}{r} - \delta = 0. \quad (85)
\end{aligned}$$

The entry deterrence strategy cannot happen if $K_D^{det}(X) < \hat{K}_D(X)$. If we assume that the dedicated leader invests at X , then the deterrence strategy is only possible when $X < X_2^{det}$. X_2^{det} , $K_D^{det}(X_2^{det})$ and $K_F^*(K_D^{det})$ satisfy (85), (16) and (17), with $X_F^*(K_D^{det}) = X_2^{det}$. Similar to the case that the flexible follower produces below capacity right after investment, the deterrence strategy is not possible if $K_D^{det} < 0$, which results that $X > X_1^{det}$ with X_1^{det} satisfying

$$\begin{aligned}
\frac{c}{2(\beta_1 - \beta_2)} \left(\frac{X_1^{det}}{X_F^*(0)} \right)^{\beta_1} \left\{ \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) \left[\left(\frac{X_F^*(0)}{c} \right)^{\beta_2} - \left(\frac{X_F^*(0)(1 - 2\gamma K_F^*(0))}{c} \right)^{\beta_2} \right] \right. \\
\left. - \frac{\beta_1 - \beta_2}{r - \alpha} \frac{2\gamma X_F^*(0) K_F^*(0)}{c} \right\} + \frac{1}{r - \alpha} - \frac{c}{r} - \delta = 0, \quad (86)
\end{aligned}$$

where $K_F^*(0)$ and $X_F^*(0)$ satisfy (16) and (17). Thus, the entry deterrence strategy is only possible if $X \in (X_1^{det}, X_2^{det})$. If the leader applies entry deterrence strategy and invests at $X^{det}(K_D)$ with capacity level K_D , then the value function before and after investment is

$$V_D(X, K_D) = \begin{cases} \mathcal{A}(K_D) X^{\beta_1} & X < X^{det}(K_D), \\ \mathcal{B}_2(K_D) X^{\beta_1} + \frac{K_D(1 - \gamma K_D)}{r - \alpha} X - \frac{c K_D}{r} & X^{det}(K_D) \leq X < X_F^*(K_D), \\ \mathcal{N}(K_D) X^{\beta_2} + \frac{K_D(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha} X - \frac{c K_D}{r} & X \geq X_F^*(K_D). \end{cases} \quad (87)$$

For a given capacity level K_D , from value matching and smooth pasting at $X^{det}(K_D)$, $X^{det}(K_D)$ must satisfy

$$\begin{aligned}\mathcal{A}(K_D)X^{\beta_1} &= \mathcal{B}_2(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{cK_D}{r} - \delta K_D, \\ \beta_1 \mathcal{A}(K_D)X^{\beta_1-1} &= \beta_1 \mathcal{B}_2(K_D)X^{\beta_1-1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}.\end{aligned}$$

It can be derived that

$$X^{det}(K_D) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)} \left(\frac{c}{r} + \delta \right). \quad (88)$$

$K_D^{det}(X^{det})$ and $X^{det}(K_D^{det})$ satisfy (85), thus the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ are

$$\begin{aligned}K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det})) = \frac{1}{(\beta_1+1)\gamma}, \\ X^{det}(K_D^{det}) &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1} \left(\frac{c}{r} + \delta \right).\end{aligned}$$

2. Entry Accommodation Strategy

We first derive the optimal capacity under the accommodation strategy. Note that

$$\mathcal{N}(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)} \left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r} \right) \left[X_1^{-\beta_2}(K_D) - X_2^{-\beta_2}(K_D) \right],$$

with

$$\begin{aligned}X_1(K_D) &= \frac{c}{1-\gamma K_D}, \\ X_2(K_D) &= \frac{c}{1-\gamma K_D - 2\gamma K_F^*(K_D)}.\end{aligned}$$

Because

$$\frac{d}{dK_D} X_1^{-\beta_2}(K_D) = -\frac{\gamma\beta_2 X_1^{-\beta_2}(K_D)}{1-\gamma K_D}$$

and

$$\frac{d}{dK_D} X_2^{-\beta_2}(K_D) = -\frac{\gamma\beta_2 X_2^{-\beta_2}(K_D)}{1-\gamma K_D},$$

then

$$\begin{aligned}\frac{d\mathcal{N}(K_D)}{dK_D} &= \frac{c}{2(\beta_1-\beta_2)} \left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r} \right) \left[X_1^{-\beta_2}(K_D) - X_2^{-\beta_2}(K_D) - \frac{\gamma\beta_2 K_D}{1-\gamma K_D} (X_1^{-\beta_2}(K_D) - X_2^{-\beta_2}(K_D)) \right] \\ &= \frac{c}{2(\beta_1-\beta_2)} \left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r} \right) (X_1^{-\beta_2}(K_D) - X_2^{-\beta_2}(K_D)) \frac{1-\gamma K_D - \beta_2\gamma K_D}{1-\gamma K_D} \\ &= \frac{1-\gamma K_D - \beta_2\gamma K_D}{K_D(1-\gamma K_D)} \mathcal{N}(K_D).\end{aligned}$$

Thus, the investment capacity by the dedicated leader $K_D^{acc}(X)$ for a given level of X satisfies the first order condition

$$\frac{\partial V_D(X, K_D) - \delta K_D}{\partial K_D} = \frac{d\mathcal{N}(K_D)}{dK_D} X^{\beta_2} + \frac{X}{r-\alpha} [(1-\gamma K_D - \gamma K_F^*(K_D))$$

$$\begin{aligned}
& -\gamma K_D \left(1 + \frac{dK_F^*(K_D)}{dK_D} \right) \Big] - \frac{c}{r} - \delta \\
& = \frac{(1 - \gamma K_D - \beta_2 \gamma K_D) \mathcal{N}(K_D)}{K_D(1 - \gamma K_D)} X^{\beta_2} - \frac{c}{r} - \delta \\
& + \frac{X(1 - \gamma K_D - \gamma K_F^*(K_D))(1 - 2\gamma K_D)}{(r - \alpha)(1 - \gamma K_D)} = 0
\end{aligned} \tag{89}$$

Entry accommodation strategy only happens when the market has grown large enough to hold the two firms, i.e., $X \geq X_F^*(K_D)$. Define $X_1^{acc} = X_F^*(K_D^{acc}(X_1^{acc}))$, then X_1^{acc} , $K_D^{acc}(X_1^{acc})$, and $K_F^*(K_D^{acc})$ satisfy (89), (16), and (17). Suppose the dedicated leader uses the entry accommodation strategy and invests at $X^{acc}(K_D)$ with capacity K_D , then the leader's value function is

$$V_D(X, K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{acc}(K_D), \\ \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha} X - \frac{cK_D}{r} & X \geq X_F^*(K_D) \geq X^{acc}(K_D). \end{cases} \tag{90}$$

From value matching and smooth pasting, we get that the investment threshold $X^{acc}(K_D)$ satisfies

$$\begin{aligned}
\mathcal{A}(K_D)X^{\beta_1} &= \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha} X - \frac{cK_D}{r} - \delta K_D, \\
\beta_1 \mathcal{A}(K_D)X^{\beta_1-1} &= \beta_2 \mathcal{N}(K_D)X^{\beta_2-1} + \frac{K_D(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha}.
\end{aligned}$$

Thus, it holds that $X^{acc}(K_D)$ must satisfy

$$\frac{\beta_1 - \beta_2}{\beta_1} \mathcal{N}(K_D)X^{\beta_2} + \frac{\beta_1 - 1}{\beta_1(r - \alpha)} X K_D (1 - \gamma K_D - \gamma K_F^*(K_D)) - \frac{cK_D}{r} - \delta K_D = 0. \tag{91}$$

Rewrite (89) and (91), then $X^{acc}(K_D^{acc})$ and K_D^{acc} satisfy

$$\begin{aligned}
& \frac{1 - \gamma K_D - \beta_2 \gamma K_D}{1 - \gamma K_D} \frac{cX^{\beta_2}}{2(\beta_1 - \beta_2)} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (X_1^{-\beta_2} - X_2^{-\beta_2}) \\
& + \frac{X(1 - \gamma K_D - \gamma K_F^*(K_D))}{r - \alpha} \frac{1 - 2\gamma K_D}{1 - \gamma K_D} - \frac{c}{r} - \delta = 0, \\
& \frac{cX^{\beta_2}}{2\beta_1} \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (X_1^{-\beta_2} - X_2^{-\beta_2}) + \frac{X(1 - \gamma K_D - \gamma F^*(K_D))}{r - \alpha} \frac{\beta_1 - 1}{\beta_1} \\
& - \frac{c}{r} - \delta = 0.
\end{aligned}$$

From

$$\begin{aligned}
\frac{1 - \gamma K_D - \beta_2 \gamma K_D}{(\beta_1 - \beta_2)(1 - \gamma K_D)} &= \frac{1}{\beta_1}, \\
\frac{1 - 2\gamma K_D}{1 - \gamma K_D} &= \frac{\beta_1 - 1}{\beta_1},
\end{aligned}$$

it holds that the optimal investment capacity is

$$K_D^{acc} \equiv K_D^{acc}(X^{acc}(K_D^{acc})) = \frac{1}{(\beta_1 + 1)\gamma}.$$

B No Flexibility

This section analyzes what the follower and the leader's decisions are when there is no flexibility. It means that both firms would always produce up to full capacity. For the follower, given that the leader invests and produces K_D and the follower invests and produces K_F , the profit flow at time t equals

$$\pi_F(t) = (X(t) (1 - \gamma(K_D + K_F)) - c) K_F.$$

Here, we do not allow production suspension. So for a low level X , i.e., $X(1 - \gamma(K_D + K_F)) < c$, the firms may have negative profit flows. Given the initial geometric Brownian motion level X , the value of the follower is

$$\begin{aligned} V_F(X, K_D, K_F) &= E \left[\int_{t=0}^{\infty} K_F (X(t) (1 - \gamma(K_D + K_F)) - c) \exp(-rt) dt \mid X(0) = X \right] \\ &= \frac{X K_F (1 - \gamma(K_D + K_F))}{r - \alpha} - \frac{c K_F}{r}. \end{aligned}$$

The follower's investment capacity maximizes

$$\max_{K_F > 0} V_F(X, K_D, K_F) - \delta K_F,$$

thus, given X and K_D ,

$$K_F(X, K_D) = \frac{1}{2\gamma} \left(1 - \gamma K_D - \frac{r - \alpha}{X} \left(\frac{c}{r} + \delta \right) \right). \quad (92)$$

Before the investment, the follower holds an option to invest. Suppose the option value is

$$V_F(X, K_D) = A_F(K_D) X^{\beta_1}.$$

According to value matching and smooth pasting, the investment threshold $X_F(K_D, K_F)$ when investing with K_F satisfies the following

$$\begin{aligned} A_F X_F^{\beta_1} &= \frac{X_F^* K_F (1 - \gamma(K_D + K_F))}{r - \alpha} - \frac{c K_F}{r} - \delta K_F, \\ \beta_1 A_F X_F^{\beta_1 - 1} &= \frac{K_F (1 - \gamma(K_D + K_F))}{r - \alpha}. \end{aligned}$$

Thus

$$X_F(K_D, K_F) = \frac{\beta_1 (r - \alpha)}{(\beta_1 - 1)(1 - \gamma K_D - \gamma K_F)} \left(\frac{c}{r} + \delta \right). \quad (93)$$

Combining (92) and (93), the follower's optimal investment capacity and threshold are

$$K_F^*(K_D) = \frac{1 - \gamma K_D}{(1 + \beta_1)\gamma}, \quad (94)$$

$$X_F^*(K_D) = \frac{(\beta_1 + 1)(r - \alpha)}{(\beta_1 - 1)(1 - \gamma K_D)} \left(\frac{c}{r} + \delta \right). \quad (95)$$

If $X_F^*(K_D) \leq X(0)$, then the follower would invest immediately at $t = 0$ with capacity $K_F^*(X(0), K_D)$.

For the leader, to deter or accommodate the entry of the follower would be dependent on the leader's critical capacity level

$$\hat{K}_D(X) = \frac{1}{\gamma} \left(1 - \frac{(\beta_1 + 1)(r - \alpha)}{(\beta_1 - 1)X} \left(\frac{c}{r} + \delta \right) \right). \quad (96)$$

B.1 Entry Deterrence Strategy

If the leader invests a capacity larger than $\hat{K}_D(X)$, then the follower invests later. However, if the leader invests a capacity not larger than $\hat{K}_D(X)$, then the follower invests at the same time with the leader. Suppose the investment threshold is $X_D^{det}(K_D)$ when investing capacity K_D , then the leader's value under entry deterrence strategy is assumed to be

$$V_D(X, K_D) = \begin{cases} A_D(K_D)X^{\beta_1} & \text{if } X < X_D^{det}(K_D), \\ B_D(K_D)X^{\beta_1} + \frac{XK_D(1-\gamma K_D)}{r-\alpha} - \frac{cK_D}{r} & \text{if } X_D^{det}(K_D) \leq X < X_F^*(K_D), \\ \frac{\beta_1 XK_D(1-\gamma K_D)}{(1+\beta_1)(1-\alpha)} - \frac{cK_D}{r} & \text{if } X \geq X_F^*(K_D). \end{cases}$$

By value matching at $X_F^*(K_D)$, we get

$$B_D(K_D)X_F^{*\beta_1} + \frac{X_F^*K_D(1-\gamma K_D)}{r-\alpha} = \frac{\beta_1 X_F^*K_D(1-\gamma K_D)}{(\beta_1+1)(r-\alpha)}.$$

Thus,

$$\begin{aligned} B_D(K_D) &= -\frac{K_D(1-\gamma K_D)X_F^*}{(\beta_1+1)(r-\alpha)}X_F^{*-\beta_1} \\ &= -\frac{K_D}{\beta_1-1} \left(\frac{c}{r} + \delta\right) \left(\frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)} \left(\frac{c}{r} + \delta\right)\right)^{-\beta_1}. \end{aligned}$$

Suppose the leader invests at X , then the investment capacity under deterrence strategy $K_D^{det}(X)$ satisfies

$$-\frac{1-(\beta_1+1)\gamma K_D}{(\beta_1-1)(1-\gamma K_D)} \left(\frac{c}{r} + \delta\right) \left(\frac{X(\beta_1-1)(1-\gamma K_D)}{(\beta_1+1)(r-\alpha) \left(\frac{c}{r} + \delta\right)}\right)^{\beta_1} + \frac{X(1-2\gamma K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0. \quad (97)$$

The corresponding value for the leader's entry deterrence strategy is

$$\begin{aligned} V_D^{det}(X) &= -\frac{K_D^{det}(X)}{\beta_1-1} \left(\frac{c}{r} + \delta\right) \left(\frac{X(\beta_1-1)(1-\gamma K_D^{det}(X))}{(\beta_1+1)(r-\alpha) \left(\frac{c}{r} + \delta\right)}\right)^{\beta_1} \\ &\quad + \frac{XK_D^{det}(X)(1-\gamma K_D^{det}(X))}{r-\alpha} - \frac{cK_D^{det}(X)}{r} - \delta K_D^{det}(X). \end{aligned} \quad (98)$$

If X is sufficiently small, then the optimal investment threshold is

$$X_D^{det}(K_D^{det}) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D^{det})} \left(\frac{c}{r} + \delta\right). \quad (99)$$

Substitute (99) into (97) gives

$$1 - (\beta_1+1)\gamma K_D^{det} = (1 - (\beta_1+1)\gamma K_D^{det}) \left(\frac{\beta_1}{\beta_1+1}\right)^{\beta_1}.$$

Thus, we have

$$\begin{aligned} K_D^{det} &= \frac{1}{(\beta_1+1)\gamma}, \\ X_D^{det} &\equiv X_D^{det}(K_D^{det}) = \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1} \left(\frac{c}{r} + \delta\right). \end{aligned}$$

The corresponding follower's investment decisions are

$$\begin{aligned} K_F^*(K_D^{det}) &= \frac{\beta_1}{(\beta_1 + 1)^2 \gamma}, \\ X_F^*(K_D^{det}) &= \frac{(\beta_1 + 1)^2 (r - \alpha)}{\beta_1 (\beta_1 - 1)} \left(\frac{c}{r} + \delta \right). \end{aligned}$$

Moreover, the entry deterrence strategy can not happen for

$$0 \leq \hat{K}_D(X) < K_D^{det},$$

i.e.,

$$X_1^{det} \leq X \leq X_2^{det},$$

where X_2^{det} satisfies (96) and (97) such that

$$X_2^{det} = \frac{2(\beta_1 + 1)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right),$$

and X_1^{det} satisfies

$$-\frac{1}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right) \left(\frac{X(\beta_1 - 1)}{(\beta_1 + 1)(r - \alpha) \left(\frac{c}{r} + \delta \right)} \right)^{\beta_1} + \frac{X}{r - \alpha} - \frac{c}{r} - \delta = 0. \quad (100)$$

If $X_D^{det} \leq X$, then the deterrence strategy is implemented immediately with $K_D^{det}(X)$ satisfying (97).

B.2 Entry Accommodation Strategy

Under the entry accommodation strategy, the follower invests at the same time as the leader. Suppose the investment threshold is $X_D^{acc}(K_D)$ when investing capacity K_D , then the leader's value under entry accommodation strategy is assumed to be

$$V_D(X, K_D) = \begin{cases} A_D(K_D) X^{\beta_1} & \text{if } X < X_D^{acc}(K_D), \\ \frac{X K_D (1 - \gamma K_D)}{2(r - \alpha)} - \frac{c K_D}{2r} + \frac{\delta K_D}{2} & \text{if } X \geq X_D^{acc}(K_D). \end{cases}$$

For a given level of X , the investment capacity under the entry accommodation strategy is

$$K_D^{acc}(X) = \frac{1}{2\gamma} \left(1 - \frac{r - \alpha}{X} \left(\frac{c}{r} + \delta \right) \right).$$

The accommodation strategy can only be chosen when $K_D^{acc}(X) \leq \hat{K}_D(X)$, which means that it is only possible when

$$X \geq X_1^{acc} = \frac{(\beta_1 + 3)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right).$$

Moreover, the value matching and smoothing pasting conditions yield that for the given capacity K_D , the investment threshold $X_D^{acc}(K_D)$ satisfies

$$A_D(K_D) X^{\beta_1} = \frac{X K_D (1 - \gamma K_D)}{2(r - \alpha)} - \frac{c K_D}{2r} - \frac{\delta K_D}{2},$$

$$\beta_1 A_D(K_D) X^{\beta-1} = \frac{K_D(1 - \gamma K_D)}{2(r - \alpha)}.$$

Thus, it holds that

$$X_D^{acc}(K_D) = \frac{\beta_1(r - \alpha)}{(\beta_1 - 1)(1 - \gamma K_D)} \left(\frac{c}{r} + \delta \right).$$

Then the optimal investment capacity K_D^{acc} and the optimal investment threshold X_D^{acc} are

$$\begin{aligned} K_D^{acc} &= \frac{1}{(\beta_1 + 1)\gamma}, \\ X_D^{acc} &= \frac{(\beta_1 + 1)(r - \alpha)}{(\beta_1 - 1)} \left(\frac{c}{r} + \delta \right). \end{aligned}$$

If $X_D^{acc} \leq X$, then the leader invests immediately at X with capacity

$$K_D(X) = \frac{1}{2\gamma} \left(1 - \frac{r - \alpha}{X} \left(\frac{c}{r} + \delta \right) \right).$$

Note that $X_1^{acc} > X_D^{acc}$. This means that the leader implements the accommodation strategy only when X reaches X_1^{acc} . Then the leader invests at X_1^{acc} with capacity

$$K_D(X_1^{acc}) = \frac{2}{(\beta_1 + 3)\gamma}.$$

The leader's value at X_1^{acc} is

$$V_D(X_1^{acc}, K_D(X_1^{acc})) = \frac{2}{(\beta_1 - 1)(\beta_1 + 3)\gamma} \left(\frac{c}{r} + \delta \right).$$

The corresponding follower's investment decisions under the leader's accommodation strategy are

$$\begin{aligned} K_F^*(K_1^{acc}) &= \frac{\beta_1 + 1}{(\beta_1 + 3)\gamma}, \\ X_F^*(K_1^{acc}) &= \frac{(\beta_1 + 3)(r - \alpha)}{\beta_1 - 1} \left(\frac{c}{r} + \delta \right). \end{aligned}$$

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