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Bos, Gerardus Gertrudus Johannes

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A LOGISTIC APPROACH TO THE DEMAND FOR PRIVATE CARS

G.G.J. BOS

A logistic approach to the demand for private cars

A logistic approach to the demand for private cars

Proefschrift

*Ter verkrijging van de graad van doctor in de economische wetenschappen
aan de Katholieke Hogeschool te Tilburg, op gezag van de rector magnificus
Dr. C.F. Scheffer, hoogleraar in de bedrijfshuishoudkunde, in het openbaar
te verdedigen op woensdag 24 juni 1970 des namiddags te 2.00 uur in de
aula van de Hogeschool te Tilburg*

door

Gerardus Gertrudus Johannes Bos

geboren te 's-Hertogenbosch

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Universitaire Pers Rotterdam

Wolters-Noordhoff, Groningen

PROMOTOR: PROF. DR. J.J.J. DALMULDER

*voor jou
voor jullie*

Voorwoord

...“What are you doing?”

‘Tracking something’ said Winnie the Pooh very mysteriously.

‘Tracking what?’ said Piglet coming closer.

‘That’s just what I ask myself. I ask myself. I ask myself, What?’

‘What do you think you’ll answer?’

‘I shall have to wait until I catch up with it’ said Winnie the Pooh.

A.A. Milne: ‘Winnie the Pooh’

Bij het begin van de studie welke resulteerde in dit proefschrift, stond mij een opzet voor ogen die aanmerkelijk verschilde van de aanpak waarvoor ik uiteindelijk heb gekozen. Met name had ik graag de structuur van en de prijsvorming op de markt voor gebruikte auto’s en de invloed van de op het fenomeen ‘auto’ betrekking hebbende belastingen in mijn beschouwing willen betrekken. Het bleek echter dat met het voorhanden zijnde statistische materiaal een dergelijke uitgebreide benadering van een zo gecompliceerd onderwerp niet mogelijk was.

In overleg met mijn promotor en met functionarissen van het Centraal Bureau voor de Statistiek is toen gekozen voor de bestudering van de groei van het wagenpark en de onderverdeling van de totale vraag in de te onderscheiden componenten. Deze probleemstelling werd met behulp van de logistische groeicurve nader geanalyseerd, waarbij tijdens het onderzoek meer de nadruk op de logistische curve als zodanig, dan op de toepassing daarvan op de groei van het wagenpark kwam te liggen.

Het is geen frase als ik stel dat ik zeer veel dank verschuldigd ben aan mijn Promotor Prof. Dr. J.J.J. Dalmulder. Gedurende mijn studie aan de Katholieke Hogeschool heeft hij steeds blijk gegeven van grote belangstelling voor mijn wetenschappelijke vorming en mijn persoonlijke omstandigheden. Ook mijn voortgezette opleiding aan de Universiteit van Chicago en de introductie bij Prof. Dr. Tj. Koopmans — toenmalig Director of Research of the Cowless Commission — is voor een groot deel door hem mogelijk gemaakt. Bij het schrijven van dit proefschrift heb ik wederom ervaren welke stimulansen er van hem uitgaan. Voor dit alles ben ik zeer dankbaar.

De heren A. van Reeken en J. Pompen ben ik bijzonder erkentelijk voor de stimulerende opmerkingen, het kritisch meedenken en ontwikkelen en het verzorgen van de uitvoerige computerberekeningen welke noodzakelijk waren voor de analyse van het gestelde probleem.

Ook Ir. R. Stobberingh die de moeite heeft genomen het manuscript enkele malen kritisch door te lezen en vele waardevolle opmerkingen heeft gemaakt en dr. J. Schilderincx die met mij de opzet van de testprocedure heeft doorgenomen, betuig ik mijn dank.

De toegang tot de dikwijls zeer moeilijk te verkrijgen statistische gegevens werd mij mogelijk gemaakt door de heren W. Jurg van het Centraal Bureau voor de Statistiek en D.J.J.C. Nengerman van de Rijksdienst voor het Wegverkeer waarvoor ik hen hier hartelijk dank zeg.

Ook de directie van de Financiële Afdeling van N.V. Philips' Gloeilampenfabrieken — met name Drs. J. Koning — wil ik danken voor de belangstelling welke zij steeds heeft getoond, ofschoon het onderwerp toch niet typisch in de 'Philips-sfeer' ligt.

Het uittypen van manuscript i ($i = 1 \dots n$) werd op uitstekende wijze verzorgd door Mej. I. Geluk. Het grote geduld en de grote toewijding waarmede zij de voor haar welhaast onleesbare formules op papier wist te zetten, heb ik ten eerste gewaardeerd. Haar nauwgezetheid heeft mij voor menige vergissing behoed.

Tenslotte dank ik mijn vrouw en kinderen, die ervoor hebben gewaakt, dat hun dissenterende echtgenoot en vader geen uit het gezinsleven deserterende huisgenoot is geworden en die mij na elke geestelijke afwezigheid weer op een charmante wijze terugvoerden in het leven van alle dag.

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I The approach to the problem

In paragraph 1 of this chapter we shall deal with some studies which in the last few years were devoted to the forecasting of the motor-car market. In paragraph 2 the data used in the calculations are given, while in paragraph 3 the logistic growth curve is dealt with in detail, while the reasons are stated for which this study is based on the logistic curve.

1. SOME WELL-KNOWN DUTCH STUDIES

The growth of the car population is a favourite subject of discussions in many ways. At congresses and in publications widely divergent estimates were made of the saturation level that would be attained in the near or remote future. The makers of these forecasts have started from one or more of the points stated below, or have taken them into account:

1. Extrapolation of Dutch growth curves in the past years.
2. Assumption of a particular car density (= number of cars per 100 inhabitants), mostly based on conditions prevailing in other, more or less comparable countries.
3. Correlation of the growth of motor traffic with a particular economic factor, like for instance the national product.
4. Consideration of some of the most important social and economic factors affecting the increase of the traffic, such as those of the consumption, prosperity, national income.
5. A more rapid increase of the national income than had originally been expected.
6. A change in the consumption pattern.
7. A reduction of the prices of means of transport resulting from increased productivity and the abolishment of import duties.
8. New and convenient financing conditions.

Among the publications which attracted great attention in the last years were no doubt the articles published in 'Economisch-Statistische Berichten' in the years 1960 and 1961.

The discussion started when an article by L. Becker was published in

which the relationship between the growth of the number of private cars and the car density was determined with the aid of a cross-section analysis.¹ By using this analysis he determined the above-mentioned relationship between the growth in the period 1952-1958 and the car density in 1952 for fourteen countries.

The research resulted in the following formula:

$$100 \frac{\Delta D}{D} = \alpha_0 - \alpha_1 \log D \quad (\text{I.1.1.})$$

in which

D = car density in 1952 in each of the fourteen countries.

$100 \frac{\Delta D}{D}$ = increase of the car density in per cent.

Besides the car density Becker also introduced a variable representing the development of prosperity. Therefore the model (I.1.1.) was changed into:

$$100 \frac{\Delta D}{D} = \alpha_0 - \alpha_1 \log D + \alpha_2 C \quad (\text{I.1.2.})$$

The result of a regression analysis was:

$$100 \frac{\Delta D}{D} = + 171,76 - 71,090 \log D + 2,4605 C$$

the correlation coefficient R being equal to 0,980. In this formula

D = car density in 1952 in each of the fourteen countries.

$100 \frac{\Delta D}{D}$ = increase of D in per cent.

C = increase of the consumption in the period 1950/51-1956/57 in per cent.

The correlation coefficient $R = 0,980$ is high (this means an unexplained part of 2%); it may be due to the fact that a few observations which 'did not fit into the pattern' were omitted.

In view of the high degree of conformity existing between the calculated and the actual values this model was used for forecasting the future number of passenger cars. With regard to the value of C Becker has calculated two alternatives, viz.: $C = 10$ (minimum estimate) and $C = 20$ (maximum estimate).

The results of the forecasts are given below.

1. BECKER L. 'De te verwachten groei van het Nederlandse personenautopark'. *Economisch-Statistische Berichten*, 24 februari 1960, pg. 212-216.

Expected number of passenger cars in:

		1964	1970
Minimum estimate	C = 10	790 000	895 000
Maximum estimate	C = 20	1 385 000	1 760 000

These results are much larger than those of forecasts made so far. Becker considers the most important causes to be:

a. A general underestimate of the effect which an increasing valuation on the consumer's preference scale exerts on the number of passenger cars....

It is a fact that a rise on the preference scale — called consumer acceptance by Becker — entails an increase of the number of cars sold, because the passenger car ranks higher on the list of desirable products.

b. 'The generally prevailing conservatism and lack of imagination, in result of which forecasts in many fields have been much too modest when the results were considered afterwards.'

In the early sixties Becker's calculations were regarded as being fairly revolutionary. They in fact meant that the total number of cars would be tripled within 10 years. Yet, in the period 1950-1960 this number was quadrupled. Furthermore the maximum estimate of more than 1 700 000 cars was already attained in 1967.

The model made by Becker can be transformed into a growth function if the following procedure is adopted:¹

Starting from the equation:

$$100 \frac{\Delta D}{D} = \alpha_0 - \alpha_1 \log D + \alpha_2 C \quad (\text{I.1.2})$$

the following substitutions are performed:

$$\text{a.} \quad 100 \frac{\Delta D}{D} = \frac{dD}{dt} \frac{1}{D}$$

$$\text{b.} \quad \alpha_1 \log D = \beta_1 \ln D$$

$$\text{c.} \quad \alpha_0 + \alpha_2 C = \beta_0$$

(I.1.2) can be replaced by:

$$\frac{dD}{dt} \frac{1}{D} = \beta_0 - \beta_1 \ln D \quad (\text{I.1.3})$$

1. KLAASENS L. *Verslagen van de collegediktaten*. 1961-1962. Rotterdam School of Economics.

Because $\frac{dD}{dt} \frac{1}{D}$ can also be written as $\frac{d \ln D}{dt}$, (I.1.3) can be written as

$$\frac{d \ln D}{dt} = \beta_0 - \beta_1 \ln D \quad (\text{I.1.4})$$

Because $\frac{d(\beta_0 - \beta_1 \ln D)}{d \ln D} = -\beta_1$, (I.1.3) can be converted to

$$\frac{1}{-\beta_1} \frac{d(\beta_0 - \beta_1 \ln D)}{dt} = \beta_0 - \beta_1 \ln D \quad (\text{I.1.5})$$

from which it follows that

$$\frac{d(\beta_0 - \beta_1 \ln D)}{\beta_0 - \beta_1 \ln D} = -\beta_1 dt \quad (\text{I.1.6})$$

or

$$d \ln (\beta_0 - \beta_1 \ln D) = -\beta_1 dt \quad (\text{I.1.7})$$

Integration of both terms yields:

$$\int d \ln (\beta_0 - \beta_1 \ln D) = -\int \beta_1 dt + \beta_2 \quad (\text{I.1.8})$$

or

$$\ln (\beta_0 - \beta_1 \ln D) = -\beta_1 t + \beta_2 \quad (\text{I.1.9})$$

in which β_2 is an integration constant.

It then follows that:

$$\beta_0 - \beta_1 \ln D = e^{\beta_2} e^{-\beta_1 t} \quad (\text{I.1.10})$$

or

$$\beta_0 - \beta_1 \ln D = \beta_3 e^{-\beta_1 t} \text{ where } \beta_3 = e^{\beta_2} \quad (\text{I.1.11})$$

or

$$\ln D = \frac{\beta_0}{\beta_1} - \frac{\beta_3}{\beta_1} e^{-\beta_1 t} \quad (\text{I.1.12})$$

The growth function thus has the following equation:

$$D = \frac{\beta_0}{e^{\beta_1}} - \frac{\beta_3}{\beta_1} e^{-\beta_1 t} \quad (\text{I.1.13})$$

The first derivative of (I.1.13) is:

$$\frac{dD}{dt} = D \cdot -\frac{\beta_3}{\beta_1} e^{-\beta_1 t} \cdot -\beta_1$$

or

$$\frac{dD}{dt} = \beta_3 D e^{-\beta_1 t} \quad (\text{I.1.14})$$

The second derivative is:

$$\frac{d^2 D}{dt^2} = \beta_3 e^{-\beta_1 t} \cdot \beta_3 D e^{-\beta_1 t} + \beta_3 D e^{-\beta_1 t} \cdot -\beta_1$$

or

$$\frac{d^2 D}{dt^2} = \beta_3 D e^{-\beta_1 t} (\beta_3 e^{-\beta_1 t} - \beta_1) \quad (I.1.15)$$

$$\frac{d^2 D}{dt^2} \text{ must be zero or for the point of inflexion holds } \beta_3 e^{-\beta_1 t} = \beta_1 \quad (I.1.16)$$

Substitution in (I.1.13) yields that

$$D = \frac{\beta_0}{e^{\beta_1}} - 1 \quad (I.1.17)$$

At the saturation level the car density is: $\frac{\beta_0}{e^{\beta_1}}$

This means that at the point of inflexion the density is: $\frac{\frac{\beta_0}{e^{\beta_1}} - 1}{\frac{\beta_0}{e^{\beta_1}}} = \frac{1}{e} = 36,76\%$

of the density at the saturation level.

In a note which Becker¹ published at a later date he compared his forecast with the actual situation. It may be concluded from the reasonably good conformity that the model serves its purpose well in the short run. In the long run, however, the densities calculated will deviate highly from the actual values.

Becker's forecast has been criticised extensively. It would carry us too far to subject all points criticised to an exhaustive treatise. May it therefore suffice to state the points criticised:

Becker bases the increase of the total number of passenger cars in the course of time on the differences existing from country to country at a given moment without indicating why the densities differ so much from country to country. A cross-section analysis would therefore be out of place, because the situation in all countries concerned has developed in its own individual way.

Becker states, however, that in all countries concerned the same urge to motorise the population exists. Differences in car densities at a given moment are connected with differences in prosperity and suchlike factors. Thus the increase of the car density will in particular be affected by the increase in prosperity.

1. BECKER L. 'De groei van het Nederlandse personenautopark'. *Economisch-Statistische Berichten*, 8 maart 1967.

Besides the car density Becker includes only one more explanatory factor in the analysis.

Geerlings^{1,2,3} mentions a number of factors which Becker has not dealt with. It is not difficult to increase this number with some imagination. Yet, this is no essential criticism. Becker's study must be interpreted within the framework in which it was written.

According to Geerlings Becker has overestimated the explanatory value of the formula he found. The proportion between the number of passenger cars calculated for 1958 and the actual number of cars counted in 1958 which Becker worked out ranges from 0.86 to 1.13.

This critical remark is justifiable, for Becker tried to explain the *increase* of the number of passenger cars. It is therefore evident to determine the proportion between the *calculated* and the *actual* increase. In that case the calculated average error of 5.7% is approximately three times larger, as appears from Table 1.

Becker stresses the importance of consumer acceptance, so that every country, even if the national prosperity would fail to increase, will still attain a density of 260 cars per 1000 inhabitants after a shorter or longer period. The increase is tied too strongly to the density already attained.

He probably means to say here that the point of inflexion of the curve lies farther than halfway the saturation level. The incorrectness of this statement has already been demonstrated in the foregoing.

The saturation level — resulting from the growth curve calculated by Becker — is much too high. Geerlings states that a 30% increase of the consumption in 6 years would mean a saturation level of 2850 cars per 1000 inhabitants. This objection is not sound either. Long-term extrapolations which from the point of view of calculation yield incorrect results, need not be incorrect in the short run.

The correctness of the model developed has not been compared with a pre-war development in the Netherlands. In this respect Geerlings has remarked that the changes in density for the 6-year periods 1927-1933, 1928-1934 have yielded fairly large differences between the actual increase and the one calculated according to the model. In his opinion these deviations may be ascribed to the unreliability of the coefficients. According to him the

1. GEERLINGS J.W.H. 'Verdiepvoudiging van ons autopark in de eerstkomende tien jaar?' *Economisch-Statistische Berichten*, 21 september 1960, pg. 927-929.

2. GEERLINGS J.W.H. 'De groei van ons autopark'. *Economisch-Statistische Berichten*, 23 november 1960, pg. 1130.

3. GEERLINGS J.W.H. 'De groei van het Nederlandse Autopark tot 1970'. *Economisch-Statistische Berichten*, 29 maart 1961, pg. 330-334.

factor C is underestimated, while the consumer acceptance is overestimated as

Table 1: differences between the actual and the calculated increase in car density (1952-1958).

Country	Actual increase in % $100 \frac{\Delta D}{D}$	Calculated increase in % $100 \frac{\Delta D}{D}$	Calculated number of passenger cars in 1958 as a percent- age of the actual number of passenger cars in 1958	Calculated growth percentage in proportion to the actual growth percentage
Netherlands	139	146	101	102
Greece	245	251	102	103
Italy	175	165	96	94
Western Germany	234	214	94	92
Norway	118	109	96	92
Switzerland	92	118	113	128
Luxembourg	131	132	100	101
Belgium	84	87	102	104
France	114	124	105	109
Great Britain	73	80	104	110
Australia	39	25	90	64
New Zealand	60	37	86	62
Canada	37	49	109	132
United States	19	26	106	137
average error			5,7%	15,9%

a result of a certain multicollinearity between the explanatory variables.

The above-mentioned criticism could have been avoided if the coefficients of the regression equation formulated by Becker had been provided with the relevant standard errors.

Buissink^{1,2} also passes considerable criticism on Becker's forecast.

He argues that it is inconsistent to derive the basic data for the extrapolation of the number of passenger cars in the Netherlands from the development in a number of different countries. He therefore agrees with the criticism passed by Geerlings that every country goes through a development of its own. According to Buissink Becker's refutation would be in sharp

1. BUISSINK J.D. 'Geen verdrievoudiging, doch verdubbeling van het aantal personenauto's'. *Economisch-Statistische Berichten*, 11 januari 1961, pg. 24-30.

2. BUISSINK J.D. 'Een voorbarige conclusie.' *Economisch-Statistische Berichten*, 15 maart 1961, pg. 285-286.

contrast to his own formula. A forecast of the total number of passenger cars used in the Netherlands should first and foremost be based on the preceding development in that country. The criticism which Buissink passed on Becker's model: 'Becker's error is that he has not involved three mutually independent factors x , y and z in his correlation calculus, but $f(x)$, y and z ' does not hold water.

Buissink also makes a new forecast in which he relates the number of passenger cars linearly to the number of persons. He then arrives at the following equations:

$$y_1^* = 2,70995 x - 26,338 \quad (\text{minimum}) \quad (\text{I.1.18})$$

$$y_2^* = 3,2724 x - 32,428 \quad (\text{maximum}) \quad (\text{I.1.19})$$

$$y_3^* = 2,9190 x - 28,619 \quad (\text{working average}) \quad (\text{I.1.20})$$

Nor does the introduction of a prosperity factor W open new vistas. The formula found now reads:

$$V^* = 0,717P + 1,324 W - 689 \quad r = 0,996 \quad (\text{I.1.21})$$

in which

V^* = calculated increase of the number of passenger cars

P = increase of population

W = increase of prosperity

After having passed considerable criticism on Buissink's forecast, Schotsman¹ also published a forecast. He uses the model:

$$\log y^* = 0,11374 + 0,06497 t \quad (\text{I.1.22})$$

or

$$y^* = e^{0,11374 + 0,06497 t} \quad (\text{I.1.23})$$

and then arrives at the conclusion that for 1970 the ceiling lies at approximately 1 900 000 cars, a figure which comes very near to the value of 1 800 000 found by Becker. 'For the time being he therefore ranges himself under Becker's banners – no duplication, but triplication'.

We shall not enter in detail into the discussion between Buissink and Schotsman which then developed, but briefly discuss Geerlings' model.

The starting point is a growth function, while an absolute maximum of one car per family is assumed. Geerlings, basing himself on an average family size of 4 persons, states quite rightly that this size will decrease as a result of

1. SCHOTSMAN C.J. 'Wel verdrievoudiging van het aantal personenauto's'. *Economisch-Statistische Berichten*, 22 februari 1961, pg. 206-207.

the ageing of the population and will tend to become 3,75. This means 267 cars per 1000 inhabitants.

Geerlings starts from the function:

$$E = a(P-D) \quad (I.1.24)$$

in which

E = relative increase of the car density as a result of the relative increase of prosperity, i.e. the elasticity of car density with respect to prosperity.

P = the saturation level

D = the car density

$P-D$ = the gap to be bridged

a = a constant

The elasticity E can also be written as:

$$\frac{\Delta D}{\Delta W} \quad \frac{W}{D}$$

in which

D = the car density

ΔD = the increase of the car density

W = the prosperity (= the average prosperity level in the last 4 years)

ΔW = the increase of prosperity.

It should be noted that the elasticity has a fixed relationship to the gap still to be bridged ($P-D$). Consequently E will decrease accordingly as the density approaches to the saturation level. The car density has moreover been related to all inhabitants. Geerlings does not start from international comparisons, nor does he think a limitation to the age category 15-64 to be necessary.

By virtue of the foregoing the model used by Geerlings can also be expressed by:

$$\frac{dD}{dW} \quad \frac{W}{D} = a(P-D) \quad (I.1.25)$$

or

$$\frac{dD}{D} = \frac{dW}{W} \quad a(P-D) \quad (I.1.26)$$

In the case of a constant increase of prosperity equation (I.1.26) changes into:

$$\frac{dD}{D} = a'(P-D) \quad (I.1.27)$$

in which $a' = a.c$, the latter representing the constant increase of prosperity.

Now the relationship found is:

$$100 \frac{\Delta D}{D} = 1,52 \frac{\Delta W}{W} (267-D) \quad (\text{I.1.28})$$

On the basis of this result Geerlings estimates the total number of passenger cars in 1970 to be more than 1 300 000 as a maximum estimate and slightly more than 1 000 000 as a minimum estimate. He arrives at the conclusion that the consumer acceptance is not an individual factor accounting for the car density, but a delayed effect of the increase of prosperity.

Summarising it may be said that Becker's, Buissink's and Geerlings' forecasts present the following picture:

	1970 minimum	1970 maximum
Becker	1 385 000	1 760 000
Buissink	approx. 1 000 000	
Geerlings	1 037 000	1 334 000

2. THE BASIC DATA

In this section the increase of the total number of passenger cars since 1950 is dealt with. The considerations are based on the data published every year on August 1st by the Central Bureau of Statistics. In the analysis the total number of passenger cars is considered in various ways; besides the absolute numbers of cars, the figures per 100 inhabitants and per 100 families are also dealt with. This tripartition is also applied to the moving averages of the cars. Table 2 first gives the variation of the population and of the families and of the total number of passenger cars for the various categories under consideration. In general the calculations will only be applied to the moving averages of the data taken from these tables; these averages are given in Table 3.

This may also entail a danger, as a fluctuating trend may thus be introduced into the series. On the other hand, however, there is the purely practical argument that in the course of years the effect of certain influences, for instance measures taken by the government — an increase of the excise on petrol, an increase of the road tax, etc. — has been found to remain perceptible after the year in which such measures were taken, though in the course of time this effect diminishes gradually. This means that the number of cars present in any year will still be affected by measures taken in previous years. To reduce such effects as much as possible the five-year moving average is always used in the calculations. In consequence the moving average of the

population figures must also be determined.

The data taken from Table 3 are graphically represented in Graphs 1 to 5.

Table 2: *basic data.*

Year	Population in millions	Number of families in millions	Passenger cars		
			Absolute number in thousands	per 100 families	per 100 inhabitants
1950	10,1145	2,5875	138,625	5,357	1,370
1951	10,2640	2,6425	157,002	5,941	1,530
1952	10,3820	2,6900	172,712	6,421	1,664
1953	10,4935	2,7325	187,608	6,866	1,788
1954	10,6155	2,7800	219,428	7,893	2,067
1955	10,7510	2,8350	267,888	9,449	2,492
1956	10,8895	2,8950	327,938	11,328	3,012
1957	11,0265	2,9550	376,433	12,739	3,414
1958	11,1870	3,0150	421,000	13,964	3,763
1959	11,3475	3,0800	456,509	14,822	4,023
1960	11,4865	3,1475	522,200	16,591	4,546
1961	11,6385	3,2150	615,500	19,145	5,288
1962	11,8055	3,2875	729,651	22,195	6,181
1963	11,9660	3,3550	865,516	25,798	7,233
1964	12,1270	3,4225	1059,066	30,944	8,733
1965	12,2945	3,5000	1272,898	36,369	10,353
1966	12,4560	3,5825	1502,226	41,932	12,060
1967	12,5980	3,6650	1725,000	47,067	13,693

A consideration of these graphs gives rise to the surmise that the growth process has an exponential character. This could then be approached, for instance, by the model:

$$y(t) = me^{bt} \quad (I.2.1)$$

in which:

$y(t)$ = calculated number of passenger cars at moment t .

e = base of the natural logarithm

t = time

m, b = coefficients

Table 3: the moving averages of the population in millions and families and of the cars in thousands, per 100 families and per 100 inhabitants.

Year	Population in millions	Number of families in millions	Passenger cars		
			Absolute number in thousands	per 100 families	per 100 inhabitants
1950	10,1145	2,5875	138,625	5,357	1,370
1951	10,2535	2,6400	156,113	5,906	1,521
1952	10,3739	2,6865	175,075	6,495	1,684
1953	10,5012	2,7360	200,928	7,314	1,908
1954	10,6263	2,7865	235,115	8,391	2,204
1955	10,7552	2,8395	275,859	9,655	2,554
1956	10,8939	2,8960	322,537	11,074	2,949
1957	11,0403	2,9560	369,954	12,460	3,341
1958	11,1874	3,0185	420,816	13,889	3,752
1959	11,3372	3,0825	478,328	15,452	4,207
1960	11,4930	3,1490	548,972	17,343	4,760
1961	11,6488	3,2170	637,875	19,710	5,454
1962	11,8047	3,2855	758,387	22,934	6,396
1963	11,9663	3,3560	908,526	26,890	7,558
1964	12,1298	3,4295	1085,871	31,448	8,912
1965	12,2883	3,5050	1284,941	36,422	10,415
1966	12,4495	3,5825	1500,041	41,789	12,035
1967	12,5980	3,6650	1725,000	47,067	13,693

From (I.2.1) it follows that:

$$\ln y(t) = \ln m + bt \quad (\text{I.2.2})$$

In other words: if the exponential character does exist, the logarithms of the data will display a linear relationship.

An exponential growth means, however, that the number of passenger cars will increase infinitely. This is 'highly doubtful'. It is obvious that at some time a saturation level will be attained. The exact height of this level and the time at which it will be attained are problems that will have to be dealt with in the following chapters. They will be based on the use of the logistic growth curve. The why's and wherefore's of this method will be explained in the following paragraph.

3. THE CHOICE OF THE GROWTH FUNCTION

A study of graphs 3, 4 and 5 gives the impression that the number of passenger cars increases ever more rapidly in time. As was stated before, it was immediately clear that this is impossible. It would in fact mean that within a limited number of years the total number of passenger cars would be a multiple of the population figure. This induces us to investigate whether it would be possible to approximate the time series by a growth function, like the logistic curve or Gompertz' curve. To enable a choice to be made from these two curves the following series are derived from the given set of observations $y(t)$

$$\ln \frac{d \ln y(t)}{dt} \quad (I.3.1)$$

and

$$\ln \frac{d \frac{1}{y(t)}}{dt} \quad (I.3.2)$$

It is supposed that the linear or non-linear behaviour of the derived series serves as a standard for the choice of the adaptation function. The standard for the linear behaviour of such a series used here is the correlation coefficient in the case of single regression of the derived series.

It is therefore supposed that:

$$\ln \frac{d \ln y(t)}{dt} = A + Bt \quad (I.3.3)$$

or

$$\frac{d \ln y(t)}{dt} = e^{A + Bt} \quad (I.3.4)$$

Integration now yields:

$$\int d \ln y(t) = \int e^{A + Bt} dt \quad (I.3.5)$$

It applies to $B = 0$ that:

$$\ln y(t) = e^A t + C \quad (I.3.6)$$

or

$$y(t) = e^{e^A t + C} \quad (I.3.7)$$

If $B \neq 0$, it applies that:

$$\ln y(t) = \frac{e^A}{B} e^{Bt} + C \quad (I.3.8)$$

or

$$y(t) = \frac{e^A}{e^B} e^{Bt + C} \quad (I.3.9)$$

If $B = 0$, $y(t)$ is exponential and if $B \neq 0$, $y(t)$ can be described by the Gompertz' curve.

In the same way the derived series (I.3.2) is analysed. According to the standard it applies that:

$$\ln \frac{d \frac{1}{y(t)}}{dt} = A + Bt \quad (I.3.10)$$

$$\frac{d \frac{1}{y(t)}}{dt} = e^{A + Bt} \quad (I.3.11)$$

Integration of both terms yields

$$\int d \frac{1}{y(t)} = \int e^{A + Bt} dt \quad (I.3.12)$$

If $B = 0$, (I.3.12) becomes:

$$\frac{1}{y(t)} = e^{At + C} \quad (I.3.13)$$

or

$$y(t) = \frac{1}{e^{At + C}} \quad (I.3.14)$$

If $B \neq 0$, (I.3.12) changes into:

$$\frac{1}{y(t)} = \frac{e^A}{B} e^{Bt} + C$$

or

$$y(t) = \frac{1}{\frac{e^A}{B} e^{Bt} + C} \quad (I.3.15)$$

If $A = B = 0$, $y(t)$ is constant ($= 0$).

If $A \neq 0$ and $B = 0$, $y(t)$ is hyperbolic.

If $B \neq 0$, $y(t)$ is logistic.

If $A \neq 0$ and $B \neq 0$, $y(t)$ is logistic.

The first derivative of $y(t)$ at point $t_{i+1/2} (= \frac{t_i + t_{i+1}}{2})$ is linear approximated

so that in fact only approximated values are given for the first derivative at points situated between two successive moments t_i

The two derived series dealt with above are given in Table 4, both for passenger cars in thousands and for the moving averages. Below each series the values of A and B are given, as well as the correlation coefficient r.

Table 4: the derived series of $y(t)$.

Year	y(t) (passenger cars in thousands) ¹	$\ln \frac{d \ln y(t)}{dt}$	$\ln \frac{d \frac{1}{y(t)}}{dt}$	Moving average		
				y(t) ¹	$\ln \frac{d \ln y(t)}{dt}$	$\ln \frac{d \frac{1}{y(t)}}{dt}$
1950	139	-2,08356	-7,07693	139	-2,13024	-7,12083
1951	157	-2,35002	-7,45358	156	-2,16600	-7,27335
1952	173	-2,49218	-7,69499	175	-1,98244	-7,21573
1953	188	-1,85361	-7,16528	201	-1,85069	-7,23117
1954	219	-1,61171	-7,10085	235	-1,83373	-7,37265
1955	268	-1,59822	-7,28821	276	-1,85579	-7,55283
1956	328	-1,98111	-7,84210	323	-1,98660	-7,83061
1957	376	-2,19021	-8,17637	370	-2,04936	-8,02645
1958	421	-2,51360	-8,59645	421	-2,05493	-8,16049
1959	457	-2,00662	-8,19669	478	-1,98230	-8,22069
1960	522	-1,80554	-8,14466	549	-1,89648	-8,27864
1961	616	-1,77118	-8,27747	688	-1,75416	-8,29758
1962	730	-1,76749	-8,44423	758	-1,71130	-8,43145
1963	866	-1,60039	-8,46293	909	-1,72420	-8,62386
1964	1059	-1,69331	-8,74900	1086	-1,78182	-8,85494
1965	1273	-1,79786	-9,02860	1285	-1,86575	-9,10061
1966	1502	-1,97848	-9,36162	1500	-1,96801	-9,35031
1967	1725			1725		
		A = -2,17084	-6,93362		-2,05576	-6,84251
		B = 0,02489	-0,12534		0,01538	-0,13476
		r = 0,42549	-0,91659		0,56517	-0,98357

1. In rounded numbers. The calculations are made in actual figures.

The comparison of the absolute value of the correlation coefficients reveals that the derived series (I.3.2), the one yielding the logistic growth curve, can be accepted as being the best approximation. This is also shown by Graphs 6 and 7.

What now does such a logistic growth curve look like and what are its characteristics.

The above curve is generally regarded as a good description of the trend-wise development of a growth process. Originally it was only used in biology, but later it was found that in economics and psychology also problems occur that can be described with the aid of a logistic growth curve. Various leading economists and statisticians are of the opinion that, particularly with respect to the introduction of new products, developments occur that correspond to the shape of this curve. Biological experiments have shown that the growth of animal populations – under otherwise constant conditions – can be described by the logistic growth curve. Thus it was empirically proved that the increase of an animal population is proportional to the population figure attained and to the extent to which that population could still be increased at this moment. In other words: it was proved that the increase of a time series $y(t)$ at a particular moment is proportional to the value of the time series proper at this moment and to the extent to which the time series is capable of growing before the saturation level k is attained.

This means that:

$$\frac{dy(t)}{dt} = \alpha [k - y(t)] y(t) \quad (I.3.16)$$

which can be derived from (I.3.15) by taking $k = \frac{1}{c}$ and $\alpha = -Bc$.

The economic consequence of the foregoing would be that the increase of a certain type of product in use by the consumers is proportional to:

- a. the number of products in use at that moment;
- b. the difference between the number of products in use and the maximum possible number of products in use.

If these assumptions are fulfilled, it is possible to determine the increase of the number of products in use at any moment. As this increase faithfully reflects the initial sales, this means that the total of the initial sales is predictable. It should be borne in mind, however, that the development of the degree of saturation of a new product needs not be based on economic laws, but that is in all cases based on practical experiences. These experiences made it clear that we may speak of a certain psychological law according to which there are always people who assume an attitude of expectation, while others react very spontaneously to the introduction of a new consumer product. Consequently the logistic growth curve is based on the frequency distribution of the psychological attitude of the prospective buyers varying from 'ultra spontaneously' to 'ultra hesitant'.

The theory of the logistic growth curve is based on the assumption that the circumstances under which this growth takes place do not change. In the

economic field this assumption will never prove to be completely correct. Factors that will probably also affect the growth are:

- a. the population
- b. the average income
- c. the distribution of incomes
- d. price of the product
- e. range of applications of the product.

Usually the greatest changes will occur in the average income of the consumer and the price of the product. Consequently the necessary corrections should be made in time for the factual analysis of the growth process.

From (I.3.16) it follows that:

$$dt = \frac{dy(t)}{\alpha y(t) [k - y(t)]}$$

Integration yields

$$\begin{aligned} t &= \int \frac{1}{\alpha y(t) [k - y(t)]} dy(t) + C = \int \frac{k}{\alpha k y(t) [k - y(t)]} dy(t) + C = \\ &= \frac{1}{\alpha k} \left[\int \frac{1}{y(t)} dy(t) + \int \frac{1}{k - y(t)} dy(t) \right] + C \end{aligned} \quad (I.3.17)$$

Now αk is assumed to be equal to β

$$\beta t = \ln y(t) - \ln [k - y(t)] + C = -\ln \frac{k - y(t)}{y(t)} + C$$

$$me^{-\beta t} = \frac{k - y(t)}{y(t)}, \text{ m being equal to } e^{-C}$$

It is now assumed that $-\beta = b$:

$$my(t)e^{bt} = k - y(t)$$

$$y(t) [1 + me^{bt}] = k$$

$$y(t) = \frac{k}{1 + me^{bt}} \quad (I.3.18)$$

The parameters k , m and b will be determined with the aid of the time series.

In literature the following expression also occurs:

$$y(t) = \frac{k}{1 + ae^{\beta t + \gamma}} \quad (I.3.19)$$

This may be written in the form:

$$y(t) = \frac{k}{1 + \alpha e^{\gamma} e^{\beta t}} \quad (I.3.20)$$

In our notation αe^{γ} is replaced by m , while β is replaced by b .

The first derivative of $y(t)$ is:

$$\frac{dy(t)}{dt} = by(t) \left[\frac{y(t)}{k} - 1 \right] \quad (I.3.21)$$

the second derivative of $y(t)$ is:

$$\frac{d^2 y(t)}{dt^2} = b^2 \left[2y(t) - k \right] \frac{y(t)}{k} \left[\frac{y(t)}{k} - 1 \right] \quad (I.3.22)$$

From (I.3.18) it also follows that

$$\lim_{t \rightarrow -\infty} y(t) = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} y(t) = k$$

In other words: $y(t)$ has the lines $y = 0$ and $y = k$ as horizontal asymptotes.

From (I.3.22) it follows that the coordinates of the point of inflexion of the logistic curve are $t_0 = \frac{1}{b} \ln m$ en $y(t_0) = \frac{1}{2} k$.

It follows from (I.3.21) that if b is negative, $\frac{dy(t)}{dt} > 0$, that means that $y(t)$

increases from the asymptote $y = 0$ via the point of inflexion to asymptote $y = k$. Consequently $y(t)$ is a monotonously increasing function approaching to the maximum k if $t \rightarrow \infty$. This is illustrated in Graph 8.

It is assumed that the observations are equidistant, in other words:

$$t_{i+1} - t_i = 1.$$

From the foregoing it then follows that: $y(t_{i+1}) \geq y(t_i)$.

It is therefore difficult to approximate time series in which no increasing trend is perceptible or in which no saturation level is expected by a logistic curve. Some methods of calculating the parameters of the logistic curve are dealt with in the following chapters.

Graph 1: moving average of the population in millions.

Graph 2: moving average of the number of families in millions.

Graph 3: moving average of the number of passenger cars in thousands.

Graph 4: moving average of the number of passenger cars per 100 families.

Graph 5: moving average of the number of passenger cars per 100 inhabitants.

Graph 6: the derived series $\ln \frac{d \ln y(t)}{dt}$ (moving average).

Graph 7: the derived series $\ln \frac{d}{dt} \frac{1}{y(t)}$ (moving average).

Graph 8: the general shape of the logistic function.

II Some methods of calculating the parameters of the logistic growth curve

In this chapter we shall carry out a critical investigation of some of the well known methods of calculating the parameters of the logistic growth curve. The purpose of this investigation is to find the weak points of these methods and, where possible, to elaborate further the calculation technique.

In paragraph 1 we shall discuss the Raymond Pearl method, known as the three point method.

Paragraph 2 will deal with the approximation given by Hotelling, while paragraph 3 will be devoted to the modification used by Erkelens.

Finally, in paragraph 4 we shall present different methods of calculating the parameter m .

1. THE THREE POINT METHOD

This method, developed by Raymond Pearl, shows how a logistic growth curve can be determined by means of three equidistant points of which the coordinates are known.

In order to simplify matters we shall, wherever the three different points are mentioned in this paragraph, substitute y_i for $y(t_i)$, i having the values 1, 2 or 3.

Starting from (I.3.18)

$$y(t) = \frac{k}{1 + me^{bt}}$$

it follows that

$$\text{or } me^{bt} = \left[\frac{k}{y(t)} - 1 \right]$$

$$bt = \ln \left[\frac{k}{y(t)} - 1 \right] - \ln m \quad (\text{II.1.1})$$

For the three points (t_1, y_1) , (t_2, y_2) and (t_3, y_3) the following holds:

$$bt_1 = \ln \left(\frac{k}{y_1} - 1 \right) - \ln m \quad (\text{II.1.2})$$

$$bt_2 = \ln \left(\frac{k}{y_2} - 1 \right) - \ln m \quad (\text{II.1.3})$$

$$bt_3 = \ln \left(\frac{k}{y_3} - 1 \right) - \ln m \quad (\text{II.1.4})$$

Assuming the three t-values to be equidistant it follows that:

$$b(t_2 - t_1) = b(t_3 - t_2) \quad (\text{II.1.5})$$

If we insert the equations (II.1.2), (II.1.3) and (II.1.4) in (II.1.5) we get

$$\ln \left(\frac{k}{y_2} - 1 \right) - \ln \left(\frac{k}{y_1} - 1 \right) = \ln \left(\frac{k}{y_3} - 1 \right) - \ln \left(\frac{k}{y_2} - 1 \right)$$

or

$$\ln \frac{\left(\frac{k}{y_2} - 1 \right)}{\left(\frac{k}{y_1} - 1 \right)} = \ln \frac{\left(\frac{k}{y_3} - 1 \right)}{\left(\frac{k}{y_2} - 1 \right)} \quad (\text{II.1.6})$$

from which it follows that:

$$\frac{\frac{k}{y_2} - 1}{\frac{k}{y_1} - 1} = \frac{\frac{k}{y_3} - 1}{\frac{k}{y_2} - 1} \quad (\text{II.1.7})$$

or

$$\frac{y_1 (k - y_2)}{y_2 (k - y_1)} = \frac{y_2 (k - y_3)}{y_3 (k - y_2)} \quad (\text{II.1.8})$$

From (II.1.2) and (II.1.3) we derive that

$$b = \ln \frac{y_1 (k - y_2)}{y_2 (k - y_1)} / (t_2 - t_1) \quad (\text{II.1.9})$$

Dividing by y_2 and multiplying by $(k - y_2)$ equation (II.1.8) becomes

$$\frac{y_1 (k - y_2)^2}{y_2^2 (k - y_1)} = \frac{k - y_3}{y_3} \quad (\text{II.1.10})$$

Working out this quadratic equation we get

$$k_1 = \frac{2y_1y_2y_3 - y_2^2(y_1 + y_3)}{y_1y_3 - y_2^2} \quad (\text{II.1.11})$$

$$k_2 = 0.$$

It is clear that the second root is not relevant.

The method mentioned above has, however, certain shortcomings. It follows from the above that the value of b and k is determined by the difference in the t -values, viz. by $(t_3 - t_2)$ and $(t_2 - t_1)$. In view of the equidistance of t_1 , t_2 and t_3 , the true value of t_i is therefore of no importance. However, it can be seen from the methods of calculating m that a true value of t_i is definitely required in order to determine m .

A second drawback becomes evident when selecting the three equidistant points. The question of which points must be chosen and according to what criterion cannot be answered exactly. If a particular choice leads to a result which is not acceptable for one reason or another – e.g. negative k -values or positive b -values – then other years are chosen, and consequently other t -values. This is repeated until the results of the calculations are found to be acceptable.

Rather more important are the limitations which must be imposed on the value of the central point y_2 in order to obtain an acceptable asymptote k .

It was derived above that

$$k = \frac{2y_1y_2y_3 - y_2^2(y_1 + y_3)}{y_1y_3 - y_2^2}$$

Now assume that

$$y_1 = y_2 - \eta$$

$$y_2 = y_2$$

$$y_3 = y_2 + \epsilon$$

where both η and ϵ are greater than zero.

Substituting this in the above k -value we get

$$\begin{aligned} k &= \frac{2(y_2 - \eta)y_2(y_2 + \epsilon) - y_2^2(y_2 - \eta + y_2 + \epsilon)}{(y_2 - \eta)(y_2 + \epsilon) - y_2^2} \\ &= \frac{\epsilon y_2^2 - \eta y_2^2 - 2\eta \epsilon y_2}{\epsilon y_2 - \eta y_2 - \eta \epsilon} \end{aligned} \quad (\text{II.1.12})$$

$$\begin{aligned}
&= \frac{y_2 (\epsilon y_2 - \eta y_2 - \eta \epsilon) - \eta \epsilon y_2}{\epsilon y_2 - \eta y_2 - \eta \epsilon} \\
&= y_2 - \frac{\eta \epsilon}{(\epsilon - \eta) - \frac{\eta \epsilon}{y_2}} \quad (\text{II.1.13})
\end{aligned}$$

We are faced with three possibilities with respect to the relation between ϵ and η

1. $\epsilon < \eta$

$(\epsilon - \eta)$ is then negative, from which it follows that the denominator of the second expression is negative. The second expression is then negative and k is therefore greater than y_2 .

2. $\epsilon = \eta$

If this premise is followed through in (I.1.13) it will be found that $k = 2y_2$ and that (t_2, y_2) is the point of inflexion.

3. $\epsilon > \eta$

or $(\epsilon - \eta) > 0$

Here there are two possibilities

a. $(\epsilon - \eta) > \frac{\eta \epsilon}{y_2}$

or

$$y_2 > \frac{\eta \epsilon}{\epsilon - \eta}$$

From this it follows that $k < y_2$

b. $(\epsilon - \eta) < \frac{\eta \epsilon}{y_2}$,

or

$$y_2 < \frac{\eta \epsilon}{\epsilon - \eta}$$

from which it follows that $k > y_2$.

3a yields k -values which are lower than the starting values which is very likely and unuseful.

Calculating experiences have shown that an arbitrary selection of three points can lead to negative k -values. Thus there are y_2 -values which lead to

k-values which are smaller than zero. It is therefore advisable to investigate the relation between y_2 and k.

Let us assume that

$$k = y_2 - \frac{\eta\epsilon}{(\epsilon-\eta) - \frac{\eta\epsilon}{y_2}} < 0 \quad (\text{II.1.14})$$

If $(\epsilon-\eta) > \frac{\eta\epsilon}{y_2}$ it follows from (II.1.14) that

$$y_2(\epsilon-\eta) - 2\eta\epsilon < 0$$

or

$$y_2 < \frac{2\eta\epsilon}{\epsilon-\eta}$$

from which it follows that

$$\frac{\eta\epsilon}{\epsilon-\eta} < y_2 < \frac{2\eta\epsilon}{\epsilon-\eta} \quad (\text{II.1.15})$$

In order to present k graphically as a function of y_2 a few characteristic points must be specified more precisely. It must first be ascertained that the area of definition is determined by the values of y_2 for which it holds that

$$y_2 \geq 0.$$

From

$$\frac{dk}{dy_2} = 1 + \frac{\eta\epsilon}{[(\epsilon-\eta) - \frac{\eta\epsilon}{y_2}]^2} \frac{\eta\epsilon}{y_2^2} > 0 \quad (\text{II.1.16})$$

it follows that k is an increasing function of y_2 .

Moreover it holds that: $\lim_{y_2 \rightarrow 0} k = 0$

Assume that: $y_2 = \frac{\eta\epsilon}{\epsilon-\eta}$

From (II.1.13) it then follows that:

$$\begin{array}{ll} \lim_{y_2 \downarrow \frac{\eta\epsilon}{\epsilon-\eta}} k \rightarrow -\infty & \text{and} \quad \lim_{y_2 \uparrow \frac{\eta\epsilon}{\epsilon-\eta}} k \rightarrow +\infty \end{array}$$

from which it follows that the line

$$y_2 = \frac{\eta\epsilon}{\epsilon-\eta} \text{ is a vertical asymptote.}$$

It has already been shown that values of k which are greater than y_2 belong to values of y_2 which are smaller than

$$\frac{\eta\epsilon}{\epsilon-\eta}$$

$$\text{If } y_2 = \frac{2\eta\epsilon}{\epsilon-\eta} \text{ then } k \text{ becomes zero.}$$

It also holds that $\lim k$ for large values of y_2 tends to y_2 .

We can summarize the foregoing as follows:

$$\lim_{y_2 \rightarrow 0} k = 0$$

$$\text{If } y_2 < \frac{\eta\epsilon}{\epsilon-\eta} \text{ then } k > y_2$$

$$\lim_{y_2 \downarrow \frac{\eta\epsilon}{\epsilon-\eta}} k \rightarrow -\infty$$

$$\lim_{y_2 \uparrow \frac{\eta\epsilon}{\epsilon-\eta}} k \rightarrow +\infty$$

$$\text{If } y_2 < \frac{2\eta\epsilon}{\epsilon-\eta} \text{ then } k < 0$$

$$\text{If } y_2 = \frac{2\eta\epsilon}{\epsilon-\eta} \text{ then } k = 0$$

$$\lim_{y_2 \rightarrow \infty} k \rightarrow y_2$$

The relation between k and y_2 is shown in graph 9.

In the foregoing it was determined that $k > y_2$ if $\epsilon < \eta$. From the graph it

can be seen that this applies for every value of y_2 which is smaller than

$$\frac{\eta\epsilon}{\epsilon-\eta}$$

[An economically useful value of k must, however, satisfy $k > y_3$ or $k > y_2 + \epsilon$.

For $k = y_2 + \epsilon$ it holds that

$$y_2 + \epsilon = y_2 - \frac{\eta\epsilon}{(\epsilon-\eta) - \frac{\eta\epsilon}{y_2}} \quad (\text{II.1.17})$$

or

$$\epsilon(\epsilon-\eta) - \frac{\eta\epsilon^2}{y_2} = -\eta\epsilon,$$

from which it follows that

$$\epsilon - \frac{\eta\epsilon}{y_2} = 0 \quad \text{or, in other words:} \quad \epsilon = \frac{\eta\epsilon}{y_2}$$

from which it follows that $y_2 = \eta$.

The area of definition of y_2 containing k -values which can be used in the 3-point method is thus determined between

$$\eta \leq y_2 < \frac{\eta\epsilon}{\epsilon-\eta} \quad (\text{II.1.18})$$

Only y_2 -values from this area yield k -values which are positive and larger than $y_2 + \epsilon$.

2. THE HOTELLING METHOD

Hotelling also uses a sort of 3-point method, the difference being that he always repeats the procedure for three consecutive points. Consequently a large amount of calculation has to be done for a reasonable number of points.

The equation

$$y(t) = \frac{k}{1 + me^{bt}} \quad (\text{I.3.18})$$

satisfies the differential equation

$$\frac{dy(t)}{dt} = \alpha [k - y(t)] y(t) \quad (I.3.16)$$

From this it follows that

$$\frac{1}{y(t)} \frac{dy(t)}{dt} = \alpha k - \alpha y(t)$$

Assuming that $\alpha = -\frac{b}{k}$, it follows that $\alpha k = -b$

Substitute this in (I.3.16) and we have

$$\frac{1}{y(t)} \frac{dy(t)}{dt} = -b + \frac{b}{k} y(t) \quad (II.2.1)$$

from which b and k can be calculated.

As we already mentioned above, one of the drawbacks of Hotelling's method is that it entails a great deal of calculation. In addition it also brings with it the difficulties inherent in numerical differentiation. Numerical integration on the other hand involves these difficulties to a lesser extent. In the following we shall attempt to determine the logistic growth curve by means of integration.

We shall start with the differential equation (II.2.1)

$$\begin{aligned} \frac{1}{y(t)} \frac{dy(t)}{dt} &= -b + \frac{b}{k} y(t) \\ &= \frac{d \ln y(t)}{dt} \end{aligned} \quad (II.2.2)$$

from which it follows that

$$y(t) = \frac{k}{b} \frac{d \ln y(t)}{dt} + k \quad (II.2.3)$$

Integration between the limits u_1 and u_2 yields

$$\int_{u_1}^{u_2} y(t) dt = \frac{k}{b} \int_{u_1}^{u_2} \frac{d \ln y(t)}{dt} dt + k \int_{u_1}^{u_2} dt \quad (II.2.4)$$

$$= \frac{k}{b} [\ln y(t)]_{u_1}^{u_2} + k (u_2 - u_1) \quad (II.2.5)$$

or

$$\int_{u_1}^{u_2} y(t)dt = \frac{k}{b} \ln \left[\frac{y(u_2)}{y(u_1)} \right] + k(u_2 - u_1) \quad (\text{II.2.6})$$

This equation can be resolved by means of the trapezoidal or with Simpson's rule.

It should be noted that the trapezoidal rule interpolates linearly and the Simpson rule parabolically.

In the foregoing we have already mentioned some of the advantages of the integration approximation over the Hotelling method.

One advantage from the statistical point of view should not pass unmentioned. In regression analysis the independent variable should if possible offer fewer possibilities of error than the dependent variable.

In the Hotelling method errors can arise in the $\frac{1}{y(t)}$ values and in the $\frac{dy(t)}{dt}$ values.

In the integration method errors can only arise in $\ln y$.

3. THE ERKELENS METHOD

Erkelens starts from the formula

$$y(t) = \frac{k}{1 + e^{a+bt}} \quad (\text{II.3.1})$$

$$= \frac{k}{1 + e^a e^{bt}}$$

$$= \frac{k}{1 + m e^{bt}} \quad (\text{I.3.18})$$

If $z(t)$ is substituted for $\frac{1}{y(t)}$

then the above equation becomes

$$z(t) = \frac{1 + m e^{bt}}{k} \quad (\text{II.3.2})$$

$$= \frac{1}{k} + \frac{m}{k} e^{bt} \quad (\text{II.3.3})$$

Thus

$$z(t+1) = \frac{1}{k} + \frac{m}{k} e^{bt} e^b \quad (\text{II.3.4})$$

$$z(t) - z(t+1) = \frac{m}{k} e^{bt} (1 - e^b), \quad (\text{II.3.5})$$

from which it follows that

$$\ln [z(t) - z(t+1)] = bt + \ln \frac{m}{k} (1 - e^b) \quad (\text{II.3.6})$$

from which b and $\ln \frac{m}{k} (1 - e^b)$ can be determined, and from these two also $\frac{m}{k}$.

One drawback of this method is that although the ratio between m and k is determined, the actual values of these parameters are not.

It is impossible to calculate m with the help of the method developed by Rhodes.

Erkelens seeks to avoid this difficulty by transforming (II.3.3) into

$$e^{bt} = \frac{k}{m} z(t) - \frac{1}{m} \quad (\text{II.3.7})$$

From the arithmetical averages of the sequences $\frac{1}{y(t)} = z(t)$ and $e^{bt}, \frac{1}{m}$ and $\frac{1}{k}$ can now be determined, and therefore also m and k .

Erkelens now assumes that $\frac{1}{k}$ can be calculated with the aid of the formula

$$\frac{1}{k} = \frac{1}{n} \left[\sum_{i=1}^n \frac{1}{y(t_i)} - \frac{m}{k} \sum_{i=1}^n e^{bt_i} \right] \quad (\text{II.3.8})$$

In addition m can be resolved by means of linear regression from equation (II.3.7). The ratio between k and m , however, is now estimated again.

In the event of over-determination of k or m and otherwise plausible results, a choice can be made from the different curves based on a comparison of the residual variants.

4. CALCULATION OF M

The best known methods of calculating m are those of Rhodes and Davis. In

this paragraph we shall first explain these methods and then calculate m by means of the method of least squares.

The Rhodes method

From the equation of the logistic growth curve it follows that

$$1 + me^{bt} = \frac{k}{y(t)} \quad \text{or} \quad me^{bt} = \frac{k}{y(t)} - 1$$

from which it follows that

$$\ln m = \ln \left[\frac{k}{y(t)} - 1 \right] - bt \quad (\text{II.4.1})$$

If the number of $y(t_i)$ -values which is available is equal to n , it follows from (II.4.1) that

$$n \ln m = \sum_{i=1}^n \ln \left[\frac{k}{y(t_i)} - 1 \right] - b \sum_{i=1}^n t_i \quad (\text{II.4.2})$$

Because the difference in time between the consecutive t -values is always one,

$\sum_{i=1}^n t_i$ is the sum of a finite arithmetical series, from which it follows that, if

we assume $t_1 = 1$

$$\sum_{i=1}^n t_i = 0,5 n(n+1)$$

From the above it then follows that

$$m = e^{\frac{1}{n} \sum_{i=1}^n \ln \left[\frac{k}{y(t_i)} - 1 \right] - 0,5 b(n+1)} \quad (\text{II.4.3})$$

The Davis method

In determining m Davis starts from the premise that the logistic growth curve passes through the point $(\bar{t}, \bar{y})$,

where

$$\bar{t} = \frac{1}{n} \sum_{i=1}^n t_i \quad \text{and} \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

and therefore

$$\ln m = \ln \left(\frac{k}{y} - 1 \right) - b\bar{t} \quad (\text{II.4.4})$$

Under the hypotheses made earlier in the Rhodes method regarding the t_i -values it holds that

$$\begin{aligned} \bar{t} &= \frac{1}{n} [0,5 n (n+1)] \\ &= 0,5 (n+1) \end{aligned} \quad (\text{II.4.5})$$

From this it follows that

$$\begin{aligned} &[\ln \left(\frac{k}{y} - 1 \right) - 0,5 b (n+1)] \\ m &= e \end{aligned} \quad (\text{II.4.6})$$

The method of least squares

This method can be described as follows

$$\sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1}{y^*(t_i)} \right]^2 \text{ must be minimised by variation of } m,$$

$y^*(t_i)$ being the calculated value for $y(t_i)$

$$\sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1 + me^{bt_i}}{k} \right]^2 \text{ reaches an extreme with regard to } m \text{ if}$$

$$\frac{d}{dm} \sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1 + me^{bt_i}}{k} \right]^2 = 0 \quad (\text{II.4.7})$$

$$\frac{d}{dm} \sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1 + me^{bt_i}}{k} \right]^2 =$$

$$-2 \sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1 + me^{bt_i}}{k} \right] \frac{e^{bt_i}}{k}$$

In other words

$$\sum_{i=1}^n \frac{e^{bt_i}}{y(t_i)} - \sum_{i=1}^n \frac{e^{bt_i}}{k} - \sum_{i=1}^n \frac{me^{2bt_i}}{k} = 0 \quad (\text{II.4.8})$$

If (II.4.8) is multiplied by k , it becomes

$$m \sum_{i=1}^n e^{2bt_i} = k \sum_{i=1}^n \frac{e^{bt_i}}{y(t_i)} - \sum_{i=1}^n e^{bt_i} \quad (\text{II.4.9})$$

From this it follows that

$$m = \frac{k \sum_{i=1}^n \frac{e^{bt_i}}{y(t_i)} - \sum_{i=1}^n e^{bt_i}}{\sum_{i=1}^n e^{2bt_i}} \quad (\text{II.4.10})$$

The second derivative of $\sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1}{y^*(t_i)} \right]^2$ is

$$2 \sum_{i=1}^n \frac{e^{2bt_i}}{k^2} \quad (\text{II.4.11})$$

which is positive. This means that the extreme of $\sum_{i=1}^n \left[\frac{1}{y(t_i)} - \frac{1 + me^{bt_i}}{k} \right]^2$ is a minimum.

The last method is used in our calculations.

The influence on the curve of a change in m is shown in the following graph. 10. If m is changed only the shape of the curve will alter, the asymptotes will hold the same values.

Graph 10: the influence of a change in m on the shape of the curve.

III The regression method used to determine the parameters of the logistic growth curve

In order to determine whether a growth process can be described by the logistic growth curve, double reciprocal paper is generally used. $\frac{1}{y(t)} = z(t)$ is plotted on the horizontal axis and the $\frac{1}{y(t+1)} = z(t+1)$ values on the vertical axis. If the scatter diagram can be approximated by a straight line, the time series can be illustrated by a logistic growth curve.

In most cases the three point method described in the previous chapter is used to determine the parameters of the curve, and only the equations (II.1.9) and (II.1.11) are used as starting points.

In this chapter we shall investigate the mathematical background of the regression method. We start in paragraph 1 by determining the linear regression between $z(t)$ and $z(t+1)$. In paragraph 2, k and b will then be estimated by means of graphs. In paragraph 3 the so-called fixed kernel (y_0) is introduced.

1. DETERMINING THE LINEAR REGRESSION

Starting from (I.3.18)

$$y(t) = \frac{k}{1 + me^{bt}}$$

it follows that

$$me^{bt} = \frac{k}{y(t)} - 1 \quad (\text{III.1.1})$$

If we assume that

$$\frac{1}{y(t)} = z(t)$$

equation (III.1.1) can then be written as

$$me^{bt} = k z(t) - 1$$

or

$$e^{bt} = \frac{k \cdot z(t) - 1}{m} \quad (\text{III.1.2})$$

From the equidistance of the $y(t)$ -values it follows that

$$e^{b(t+1)} = \frac{k \cdot z(t+1) - 1}{m} \quad (\text{III.1.3})$$

It also holds that

$$e^{b(t+1)} = e^b \cdot e^{bt} = e^b \cdot \frac{k \cdot z(t) - 1}{m} \quad (\text{III.1.4})$$

From (III.1.3) and (III.1.4) it follows that

$$k \cdot z(t+1) - 1 = e^b [k \cdot z(t) - 1]$$

or

$$z(t+1) = e^b z(t) + \frac{(1-e^b)}{k} \quad (\text{III.1.5})$$

Thus we have a linear relation between $z(t+1)$ and $z(t)$. This relation can be tested by:

a. plotting the pairs of observations $z(t)$ and $z(t+1)$ on a graph.

b. fitting by simple regression a straight line to the scatter; the correlation coefficient r can be used as a standard for the goodness of the relationship.

Because all the $z(t)$ values, except the first and the last, are both dependent and independent variables, there is little point, in view of the auto-correlation related thereto, in checking the simple regression.

Once it has been ascertained that an approximation, by means of a logistic growth curve, of the time series to be investigated would serve a purpose, k , b and m can be determined.

2. GRAPHICAL ESTIMATION OF K AND B

Write equation (III.1.5) as

$$z(t+1) = P z(t) + Q \quad (\text{III.2.1})$$

The regression analysis can only lead to a useful estimation of b if P in (III.2.1) is positive. Regression analysis yields for P :

$$P = \frac{\sum_{i=1}^n [z(t_i) - \bar{z}(t)] [z(t_i + 1) - \bar{z}(t + 1)]}{\sum_{i=1}^n [z(t_i) - \bar{z}(t)]^2} \quad (\text{III.2.2})$$

The denominator of this fraction is always positive. Therefore we must find out whether the numerator is also positive.

Substitution of

$$\bar{z}(t) = \frac{1}{n} \sum_{i=1}^n z(t_i) \quad \text{and} \quad \bar{z}(t+1) = \frac{1}{n} \sum_{i=1}^n z(t_i + 1)$$

transforms the numerator of (III.2.2) into:

$$n \sum_{i=1}^n z(t_i) z(t_i + 1) - \sum_{i=1}^n z(t_i) \sum_{i=1}^n z(t_i + 1) \quad (\text{III.2.3})$$

If $[y(t_i)]$ is a monotonic increasing sequence, $[z(t_i)]$ is monotonic decreasing. If $[z(t_i)]$ and consequently also $[z(t_i + 1)]$ are monotonic decreasing sequences which satisfy the conditions that $z(t_i) > 0$ and $z(t_i + 1) > 0$, because the smallest $y(t_i)$ value is greater than zero, then the following inequality of Tchebycheff holds:

$$n \sum_{i=1}^n z(t_i) z(t_i + 1) > \sum_{i=1}^n z(t_i) \sum_{i=1}^n z(t_i + 1) \quad (\text{III.2.4})$$

From this it follows that also the numerator of (III.2.2) is positive. P is therefore positive, from which it follows that b exists.

In the equation of the logistic growth curve b must also be negative. This is, however, only the case if $p < 1$. In order to prove this we start from

$$z(t_i + 1) = z(t_i) - \delta(t_i) \quad (\text{III.2.5})$$

(III.2.2) can in the following way be written:

$$P = \frac{n \sum_{i=1}^n z(t_i) [z(t_i) - \delta(t_i)] - \sum_{i=1}^n z(t_i) \sum_{i=1}^n [z(t_i) - \delta(t_i)]}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2} \quad (\text{III.2.6})$$

1. Handbook of mathematical functions pg. 11, formula 3.2.7

$$\begin{aligned}
&= \frac{n \sum_{i=1}^n z^2(t_i) - n \sum_{i=1}^n z(t_i) \delta(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2 + \sum_{i=1}^n z(t_i) \sum_{i=1}^n \delta(t_i)}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2} \\
&= \frac{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2} - \frac{n \sum_{i=1}^n z(t_i) \delta(t_i) - \sum_{i=1}^n z(t_i) \sum_{i=1}^n \delta(t_i)}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2}
\end{aligned}$$

$$P = 1 - \frac{\sum_{i=1}^n z(t_i) \delta(t_i) - \sum_{i=1}^n z(t_i) \sum_{i=1}^n \delta(t_i)}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2} \quad (\text{III.2.7})$$

It has already been shown that P is positive. P will therefore be smaller than one if the fraction on the right side of (III.2.7) is positive, or

$$0 < \frac{n \sum_{i=1}^n z(t_i) \delta(t_i) - \sum_{i=1}^n z(t_i) \sum_{i=1}^n \delta(t_i)}{n \sum_{i=1}^n z^2(t_i) - \left[\sum_{i=1}^n z(t_i) \right]^2} < 1$$

According to the inequality of Tchebycheff it holds that

$$n \sum_{i=1}^n z(t_i) \delta(t_i) - \sum_{i=1}^n z(t_i) \sum_{i=1}^n \delta(t_i) > 0, \text{ if } \delta(t_i)$$

is monotonic decreasing.

This is valid if the differences between the $z(t_i)$ -values decrease.

That means:

$$z(t_i + 1) - z(t_i + 2) < z(t_i) - z(t_i + 1) \quad (\text{III.2.8})$$

or

$$\frac{1}{y(t_i+1)} - \frac{1}{y(t_i+2)} < \frac{1}{y(t_i)} - \frac{1}{y(t_i+1)} \quad (\text{III.2.9})$$

From the last column of table 4 of chapter I it follows that except for the y -value of 1951,

$$\ln \left[\frac{1}{y(t_i+1)} - \frac{1}{y(t_i+2)} \right] < \ln \left[\frac{1}{y(t_i)} + \frac{1}{y(t_i+1)} \right]$$

from which the inequality above can be deduced.

Therefore $\delta(t)$ is monotonic decreasing and positive and the numerator of the second expression of the right side of (III.2.7) is positive. Since the denominator, as has already been shown, is positive, the fraction on the right side of (III.2.7) is positive and it can be concluded that $0 < P < 1$.

Starting from the requirement that b must be negative, it follows from (III.2.1) that the sign of k is dictated by the sign of Q .

If it holds that

$$Q = \bar{z}(t+1) - P \bar{z}(t) > 0$$

or

$$P < \frac{\bar{z}(t+1)}{\bar{z}(t)}$$

then $k > 0$.

Thus the interval of P for a positive k -value is smaller than the interval for a negative value of b . This is shown in the following graph.

If $0 < P < 1$, b is negative

If $0 < P < \frac{\bar{z}(t+1)}{\bar{z}(t)}$, k is positive.

That is why a method is being sought which will make the requirements for P less stringent, in connection with a positive k -value, in other words a method to make the fraction $\frac{\bar{z}(t+1)}{\bar{z}(t)}$ larger.

We start from the inequality $\frac{\bar{z}(t+1) + \alpha}{\bar{z}(t) + \alpha} > \frac{\bar{z}(t+1)}{\bar{z}(t)}$ where $\alpha > 0$.

If all the $z(t)$ -values are increased by α it will be seen from substitution in (III.2.3) that the value of P is not changed, while the area within which k is positive and is limited by $\frac{\bar{z}(t+1)}{\bar{z}(t)}$ becomes larger.

3. INTRODUCTION OF A FIXED KERNEL

In paragraph 2 we mentioned that, in order to calculate k , it seems expedient to increase all the $z(t_i)$ -values by α . It is, however, difficult to find an economic reason for this. A reduction of all $y(t_i)$ -values by a certain number has, however, an analogous effect.

The growth analysis is then applied to the series formed by the reduced $y(t_i)$ -values.

In practice this means that a certain part of each member of the original series of $y(t_i)$ -values does not increase, or at any rate not logistically. For the rest of this chapter this part will be known as 'the fixed kernel' and is indicated by y_0 .

What is the influence of the fixed kernel on k and on b ?

If all $y(t_i)$ -values contain a fixed kernel y_0 , the logistic growth curve can be written as follows:

$$y(t) = y_0 + \frac{k}{1 + me^{bt}} \quad (\text{III.3.1})$$

If equation (III.1.1) is substituted for $\frac{y(t)}{y(t+1)}$ in $\frac{1}{y(t+1)} = \frac{1}{y(t)} + \frac{y(t)}{y(t+1)}$ we have

$$\frac{1}{y(t+1)} = \frac{1}{y(t)} \frac{\frac{y_0 (1 + me^{bt}) + k}{1 + me^{bt}}}{\frac{y_0 [1 + me^{b(t+1)}] + k}{1 + me^{b(t+1)}}}$$

$$= \frac{1}{y(t)} \frac{y_0 (1 + me^{bt}) + k}{y_0 [1 + me^{b(t+1)}] + k} \frac{1 + me^{bt} e^b}{1 + me^{bt}} \quad (\text{III.3.2})$$

If the second factor of the right side of (III.3.2) is expressed by $S(t)$, (III.3.2), becomes

$$\frac{1}{y(t+1)} = \frac{1}{y(t)} S(t) \frac{1 + me^{bt} e^b}{1 + me^{bt}}$$

or

$$\begin{aligned} \frac{1}{y(t+1)} &= \frac{1}{y(t)} S(t) \frac{1 + me^{bt} e^b + e^b - e^b}{1 + me^{bt}} \\ &= \frac{1}{y(t)} S(t) \frac{1 + e^b (1 + me^{bt}) - e^b}{1 + me^{bt}} \\ &= \frac{1}{y(t)} S(t) \left[\frac{1 - e^b}{1 + me^{bt}} + e^b \right] \end{aligned} \quad (\text{III.3.3})$$

From (III.3.1) it follows that

$$1 + me^{bt} = \frac{k}{y(t) - y_0}$$

Substituting this in (III.3.3) we get

$$\begin{aligned} \frac{1}{y(t+1)} &= \frac{1}{y(t)} S(t) \left[e^b + \frac{1 - e^b}{k} [y(t) - y_0] \right] \\ &= S(t) \frac{e^b}{y(t)} + \frac{1}{y(t)} \frac{1 - e^b}{k} [y(t) - y_0] \end{aligned} \quad (\text{III.3.4})$$

According to (III.2.1) it holds that $\frac{1 - e^b}{k} = Q$.

If this is substituted in equation (III.3.4), the latter can be written as follows:

$$\begin{aligned} \frac{1}{y(t+1)} &= S(t) \left[\frac{e^b}{y(t)} + \frac{1}{y(t)} Q [y(t) - y_0] \right] \\ &= S(t) \left[\frac{e^b}{y(t)} + \frac{1}{y(t)} y(t) Q - \frac{1}{y(t)} y_0 Q \right] \end{aligned}$$

$$\begin{aligned}
&= S(t) \left[\frac{e^b}{y(t)} + Q - \frac{y_0}{y(t)} Q \right] \\
&= S(t) \left[\frac{1}{y(t)} (e^b - y_0 Q) + Q \right] \quad \text{(III.3.5)}
\end{aligned}$$

$$\begin{aligned}
&\text{or} \quad \frac{1}{y(t+1)} = Q S(t) + \frac{1}{y(t)} S(t) (e^b - y_0 Q) \quad \text{(III.3.6)} \\
&\frac{1}{y(t+1)} = V(t) + \frac{1}{y(t)} U(t) \quad \text{(III.3.7)}
\end{aligned}$$

where

$$\begin{aligned}
U(t) &= S(t) [e^b - y_0 Q] \\
V(t) &= Q S(t)
\end{aligned}$$

From the above we obtain the estimates b' and k' of b and k .

$$b' = \ln (e^b - y_0 Q) + \ln S(t) = \ln U(t) \quad \text{(III.3.8)}$$

and

$$\begin{aligned}
k' &= \frac{1 - e^b S(t) + y_0 V(t)}{V(t)} \quad \text{(III.3.9)} \\
&= y_0 + \frac{1 - e^b S(t)}{V(t)} \\
&= y_0 + \frac{\frac{1}{S(t)} - e^b}{Q}
\end{aligned}$$

Substitution of $Q = \frac{1 - e^b}{k}$ from (III.2.1) yields

$$k' = y_0 + \frac{\frac{1}{S(t)} - e^b}{1 - e^b} k \quad \text{(III.3.10)}$$

If we substitute $y_0 = 0$, we come back to the original values for k and b , for now it holds that $S(t) = 1$.

Starting from

$$S(t) = \frac{y_0(1 + me^{bt}) + k}{y_0 [1 + me^{b(t+1)}] + k}$$

and substituting $\frac{k}{y(t) - y_0}$ for $1 + me^{bt}$ we can write

$$S(t) = \frac{y_0 \left[\frac{k}{y(t) - y_0} \right] + k}{y_0 \left[\frac{k}{y(t+1) - y_0} \right] + k} \quad (\text{III.3.11})$$

$$\begin{aligned} &= \frac{y_0 k + y(t) k - y_0 k}{y(t) - y_0} \cdot \frac{y(t+1) - y_0}{y_0 k + y(t+1) k - y_0 k} \\ &= \frac{y(t+1) - y_0}{y(t) - y_0} \cdot \frac{y(t)}{y(t+1)} \end{aligned} \quad (\text{III.3.12})$$

If $0 < y_0 < y(t)$, it holds that $S(t) > 1$, for

$$\frac{y(t+1) - y_0}{y(t) - y_0} \cdot \frac{y(t)}{y(t+1)} > 1$$

Thus $[y(t+1) - y_0] y(t) > [y(t) - y_0] y(t+1)$,

in other words

$$y(t) y(t+1) - y_0 y(t) > y(t) y(t+1) - y_0 y(t+1)$$

from which it follows that

$$-y(t) > -y(t+1)$$

or

$$y(t+1) > y(t)$$

It can be seen from (III.3.12) that $S(t)$ is dependent on y_0 , $y(t)$ and $y(t+1)$. Calculation of $S(t)$ with the aid of n observations $y(t)$ is difficult if not impossible.

Conclusion

Let us assume that we dispose of a number of $y(t)$ values, without a fixed kernel, which satisfy exactly the requirements for a logistic growth curve.

Now if all the $y(t)$ -values are increased by y_0 , the logistic growth curve will in fact be shifted up by y_0 . We could now reasonably expect that the k -value

would also increase by the same value y_0 . The following proves that this is not the case.

For if we calculate the saturation level k' belonging to a set of $y(t)$ -values 'encumbered with a fixed kernel', we find according to (III.3.10) that

$$k' = y_0 + \frac{\frac{1}{S(t)} - e^b}{1 - e^b} k$$

Since $S(t) \geq 1$, the numerator of $\frac{\frac{1}{S(t)} - e^b}{1 - e^b}$ is smaller than or equal to the denominator, the fraction is consequently smaller than or equal to 1 and therefore $k' \leq y_0 + k$. This means that whenever y_0 is not removed from the time series, the calculated saturation level is too low. It is therefore necessary to first reduce the $y(t)$ -values by y_0 , calculate k and then increase k by y_0 . The saturation level thus obtained is then higher than the saturation level k' .

IV Calculations of the parameters according to the various methods

In this chapter the results will be given of the calculations for which the methods discussed in the preceding chapters were adopted.

In paragraph 1 attention will be paid to the results of the three-point, the differential, the integration and the Erkelens method. In paragraph 2 the results are stated that were achieved with the aid of the regression method, while m will be determined with the method of least squares.

In paragraph 3 an exponential estimate will be made for the years 1968-1969-1970-1971, while finally in paragraph 4 the sensitivity of $y(t)$ to the parameter will be determined.

All techniques will be applied to the moving averages of:

- a. cars in thousands,¹
- b. cars per 100 families,²
- c. cars per 100 inhabitants.²

The relevant data are given in Table 3 and graphs 3, 4 and 5.

1. CALCULATION RESULTS OF THE VARIOUS METHODS

A. Three-point method

The three points chosen to form the basis of the calculation for cars in thousands are:

$$t_1 = 1951 \rightarrow y_1 = 156$$

$$t_2 = 1959 \rightarrow y_2 = 478$$

$$t_3 = 1967 \rightarrow y_3 = 1725$$

If we start from the original model:

1. For all calculations concerning 'cars in thousands' the thousands were not rounded off. For simplicity's sake the results are stated in rounded-off thousands.

2. The calculations concerning the number of cars per 100 families and per 100 inhabitants were carried out 3 decimal. The results, however, were rounded off to one decimal.

$$y = \frac{k}{1 + me^{bt}} \quad (I.3.18)$$

it follows that $k = -4266$ and $b = -0.13117$. It is clear that the value calculated for k is not acceptable; it is therefore pointless to discuss the value of b . As can be seen from Table 5, k assumes a different value for different combinations (t_i, y_i) .

Table 5: values of k and b at various (t_i, y_i) combinations.

years	k	b
1951 - 1959 - 1967	-4266	-0,13117
1953 - 1960 - 1967	-4418	-0,13321
1955 - 1961 - 1967	-3002	-0,12225
1957 - 1962 - 1967	-3760	-0,12559
1959 - 1963 - 1967	1 051 724	-0,16048
1961 - 1964 - 1967	5032	-0,21317
1963 - 1965 - 1967	3823	-0,24247
1965 - 1966 - 1967	3660	-0,24970

Starting from the y_i -values belonging to the years 1951, 1959 and 1967 it can be calculated that:

$$\begin{aligned} \eta &= y_2 - y_1 = 478 - 156 = 322 \\ \epsilon &= y_3 - y_2 = 1\,725 - 478 = 1\,247 \end{aligned}$$

from which it follows that:

$$\frac{\eta\epsilon}{\epsilon-\eta} = 435 \quad \text{and} \quad 2 \frac{\eta\epsilon}{\epsilon-\eta} = 870.$$

This proves that $\frac{\eta\epsilon}{\epsilon-\eta} < y_2 < 869$; k must therefore be negative, a fact also

revealed by the calculations. The relationship between k and y_2 can be determined and is represented in Graph 11. The vertical asymptote lies at $y_2 = 435$. According to the theory developed in Chapter II the y_2 -values lying between 322 and 435 might yield a suitable value for k .

y_2 is, however, equal to 478; Graph 11 consequently shows that k is negative.

Since in Chapter III it was found to be desirable to introduce a fixed kernel, this fixed kernel will be used in the following calculations.

In the case discussed here this means that the given value of y_2 ($= 478$) is

reduced by y_0 , so that $(y_2 - y_0)$ will lie in the relevant area, in which k consequently assumes a positive and acceptable value. In Table 6 y_0 is determined by varying the value 478 of y_2 in steps of 50.

Table 6: *three point method.*
Moving average: cars in thousands.

y_0	$y_2 - y_0$	k	b
-750	1228	556	0,04352
-700	1178	490	0,03898
-650	1128	421	0,03406
-600	1078	351	0,02872
-550	1028	276	0,02288
-500	978	197	0,01648
-450	928	111	0,00941
-400	878	18	0,00156
-350	828	-86	-0,00724
-300	778	-205	-0,01720
-250	728	-349	-0,02858
-200	678	-531	-0,04180
-150	628	-780	-0,05742
-100	578	-1169	-0,07634
-50	528	-1919	-0,10001
0	478	-4266	-0,13117
50	428	30479	-0,17550
100	378	3304	-0,24419
150	328	1672	-0,52482
200	278	1053	
250	228	709	
300	178	481	
350	128	310	
400	78	174	
450	28	59	

b is calculated according to (II.1.9); if the y -values are corrected for y_0 it is quite possible that

$\frac{y_1 (k - y_2)}{y_2 (k - y_1)}$ becomes negative.

Hence in these cases the value of b cannot be calculated.

This table clearly shows that there are only two values of y_0 that can yield acceptable values for k and b , viz.:

y_0	k	b	m
100	3304	-0,24419	88,08443
50	30479	-0,17550	400,17221

The other values of k and b are negative and positive respectively, while some k -values are smaller than y_3 , which means that the saturation level lies below a y -value already attained. m is calculated by means of least squares as is indicated in chapter II.

It should be noted that the two values of y_0 given are not the only ones which correct y_2 so that the appurtenant values of k and b are acceptable. y_0 may assume all values ranging from:

$$y_2 - \eta = y_1 \text{ to } y_2 - \frac{\eta\epsilon}{\epsilon - \eta}, \text{ consequently } 43 < y_0 < 156.$$

Similar conclusions can be drawn concerning the number of cars per 100 families and the number of cars per 100 inhabitants.

If we start again from the years 1951, 1959 and 1967 the corresponding y -values for the number of cars per 100 families are:

$$t_1 = 1951 \rightarrow y_1 = 5,9$$

$$t_2 = 1959 \rightarrow y_2 = 15,5$$

$$t_3 = 1967 \rightarrow y_3 = 47,0$$

from which it follows that:

$$\eta = y_2 - y_1 = 15,5 - 5,9 = 9,6$$

$$\epsilon = y_3 - y_2 = 47,0 - 15,5 = 31,5$$

$$\frac{\eta\epsilon}{\epsilon - \eta} = 13,7 \quad \text{and} \quad \frac{2\eta\epsilon}{\epsilon - \eta} = 27,4$$

$$k = -103,4 \quad \text{and} \quad b = -0,10974$$

k is negative and hence fundamentally unsuitable.

Table 7 shows how k changes if different (t_i, y_i) -combinations are chosen for the three points.

The relationship between k and y_2 is given in Graph 12. From the calculations and from the graph it follows that: $y_2 > \frac{\eta\epsilon}{\epsilon - \eta}$ and that k will therefore

Table 7: values of k and b for different (t_i, y_i) -combinations.

years	k	b
1951 – 1959 – 1967	-103,4	-0,10974
1953 – 1960 – 1967	-101,6	-0,11076
1955 – 1961 – 1967	-62,5	-0,09720
1957 – 1962 – 1967	-72,9	-0,09888
1959 – 1963 – 1967	-1449,6	-0,13656
1961 – 1964 – 1967	125,6	-0,19491
1963 – 1965 – 1967	97,1	-0,22469
1965 – 1966 – 1967	78,7	-0,27325

be negative. If k is to be acceptable at all, y_0 should range from 1,8 to 5,9. The value of y_2 corrected with y_0 is shown in Table 8, the step being equal to one car.

Table 8 shows that only four possibilities are acceptable, viz.:

y_0	k	b	m
5	54,8	-0,31108	97,00123
4	81,9	-0,23233	62,21997
3	151,8	-0,18756	72,08204
2	841,6	-0,15568	288,30299

The first possibility is hardly acceptable. It means, that, since y_3 is already 47,0, the maximum has now almost been attained, which is not very likely.

The last possibility would mean that at the saturation level more than 841 cars are available per 100 families, i.e. more than 8 cars per family, which is highly improbable. Again m is calculated by means of least squares.

As regards the number of cars per 100 inhabitants it applies that:

$$\begin{aligned} t_1 = 1951 \rightarrow y_1 &= 1,5 \\ t_2 = 1959 \rightarrow y_2 &= 4,2 \\ t_3 = 1967 \rightarrow y_3 &= 13,7 \end{aligned}$$

From this it follows that:

$$\begin{aligned} \eta = y_2 - y_1 &= 2,7 \quad \text{and} \quad \epsilon = y_3 - y_2 = 9,5 \\ \frac{\eta\epsilon}{\epsilon - \eta} &= 3,7 \quad \text{and} \quad \frac{2\eta\epsilon}{\epsilon - \eta} = 7,4 \\ k &= -30,0 \quad \text{and} \quad b = -0,11694 \end{aligned}$$

Table 8: *three-point method.*¹⁾
Moving average: cars per 100 families.

y_0	$y_2 - y_0$	k	b
-20	35,5	13,2	0,03079
-19	34,5	11,8	0,02775
-18	33,5	10,3	0,02453
-17	32,5	8,8	0,02113
-16	31,5	7,3	0,01751
-15	30,5	5,6	0,01367
-14	29,5	3,9	0,00958
-13	28,5	2,1	0,00520
-12	27,5	0,2	0,00050
-11	26,5	-1,9	-0,00454
-10	25,5	-4,1	-0,00999
-9	24,5	-6,6	-0,01589
-8	23,5	-9,3	-0,02232
-7	22,5	-12,5	-0,02936
-6	21,5	-16,3	-0,03711
-5	20,5	-20,8	-0,04570
-4	19,5	-26,6	-0,05529
-3	18,5	-34,4	-0,06612
-2	17,5	-45,7	-0,07849
-1	16,5	-64,5	-0,09282
0	15,5	-103,4	-0,10974
1	14,5	-239,6	-0,13024
2	13,5	841,6	-0,15568
3	12,5	151,8	-0,18756
4	11,5	81,9	-0,23233
5	10,5	54,8	-0,31108
6	9,5	40,1	
7	8,5	30,6	
8	7,5	23,8	
9	6,5	18,7	
10	5,5	14,5	
11	4,5	11,1	
12	3,5	8,1	
13	2,5	5,4	
14	1,5	3,1	
15	0,5	0,9	

¹⁾ see remark on page 52.

In this case as well k is negative and therefore unsuitable.

Table 9 shows how k varies if a different (t_i, y_i) combination is chosen.

Table 9: values of k and b for various (t_i, y_i) combinations.

years	k	b
1951 – 1959 – 1967	-30,0	-0,11694
1953 – 1960 – 1967	-30,2	-0,11847
1955 – 1961 – 1967	-19,5	-0,10582
1957 – 1962 – 1967	-23,1	-0,10806
1959 – 1963 – 1967	-312,7	-0,14383
1961 – 1964 – 1967	40,0	-0,19885
1963 – 1965 – 1967	30,0	-0,22835
1965 – 1966 – 1967	26,4	-0,25152

It can also be read from Graph 13 – the relationship between k and y_2 – that k will be negative. In Table 10 the value selected for y_2 has been corrected with variable values of y_0 .

Acceptable values of k are obtained if

$$0,5 < y_0 < 1,5$$

The only value from the table that could be accepted is:

y_0	k	b	m
1,0	25,5	-0,23390	71,46718

This saturation level shows an increase in the number of cars so that every fourth inhabitant is a carowner. This situation may be confidently anticipated. m is calculated in the usual manner.

Naturally it is also possible to calculate different values of k in the above examples by varying the steps in which y_0 decreases. Yet, y_0 will at any rate be a value between the limits we have indicated, as otherwise the calculated value of k will be incorrect.

Summarising it may be stated that the foregoing analysis has proved that the three point-method is actually unserviceable. It is also possible to change the saturation level by varying the three points to be selected, hence a variation of the moments selected. Thus k -values that are too small or negative can be eliminated. This makes the method eminently suitable for wishful thinkers.

Table 10: *three-point method.*¹
Moving average: cars per 100 inhabitants.

y_0	$y_2 - y_0$	k	b
-8,0	12,2	6,8	0,05480
-7,5	11,7	6,2	0,05097
-7,0	11,2	5,6	0,04683
-6,5	10,7	4,9	0,04233
-6,0	10,2	4,3	0,03746
-5,5	9,7	3,6	0,03203
-5,0	9,2	2,9	0,02610
-4,5	8,7	2,1	0,01951
-4,0	8,2	1,3	0,01216
-3,5	7,7	0,4	0,00388
-3,0	7,2	-0,6	-0,00554
-2,5	6,7	-1,8	-0,01639
-2,0	6,2	-3,2	-0,02907
-1,5	5,7	-5,2	-0,04416
-1,0	5,2	-8,2	-0,06259
-0,5	4,7	-13,7	-0,08590
0,0	4,2	-30,0	-0,11694
0,5	3,7	355,4	-0,16210
1,0	3,2	25,5	-0,23390
1,5	2,7	12,5	-0,63624
2,0	2,2	7,6	
2,5	1,7	4,8	
3,0	1,2	3,0	
3,5	0,7	1,6	
4,0	0,2	0,4	

B. Hotelling's method

Hotelling's method is essentially an application of the three-point method on a series of three consecutive points. Calculations carried out according to Hotelling's differential method also show that an adaptation of the logistic curve yields negative values of k for the time series 1950-1967. Correction by means of a fixed kernel y_0 may also result in acceptable values of k in this case. The steps by which y_0 will be varied are 10 for cars in thousands and 1 for cars per 100 families or 100 inhabitants. The measure of adaptation used is a correlation coefficient r . Tables 12, 13 and 14 show that r is maximum at the values of y_0 , k and b given in Table 11.

1. See remark on page 52.

Table 11: maximum value of r at calculated values of y_0 , k and b .

	Moving average		
	cars in thousands	cars per 100 families	cars per 100 inhabitants
y_0	100	4,0	1,1
k	2995	74,3	18,8
b	-0,24895	-0,23783	-0,26551
m	81,50406	57,75382	69,59270
r	-0,65906	-0,68293	-0,67134

Refinement of y_0 may yield a slightly higher correlation coefficient in the vicinity of the value of y_0 calculated above. m is calculated by means of least squares.

Though with the method used above all combinations (t_i, y_i) are involved in the calculation — whereas in the three-point method only three selected combinations (t_i, y_i) are used — the differential method remains an application of the three-point method, albeit a continuous one.

The drawback of this method — negative values of k — is evident again. Consequently the differential method has the same flaws as the three-point method.

Table 12: Hotelling's differential Method.
Moving average: cars in thousands.

y_0	k	b	r
0	-5569	-0,13508	0,53305
10	-7391	-0,14223	0,44828
20	-11056	-0,14500	0,33280
30	-22350	-0,15848	0,17950
40	479000	-0,16782	-0,00878
50	19851	-0,17821	-0,20925
60	9947	-0,18989	-0,38620
70	6532	-0,20320	-0,51603
80	4790	0,21865	-0,59780
90	3724	-0,23700	-0,64202
100	2995	-0,25953	-0,65906
110	2455	-0,28858	-0,65494
120	2024	-0,32932	-0,62980
130	1644	-0,39831	-0,57162

Table 13: *Hotelling's differential Method.*
Moving average: cars per 100 families.

y_0	k	b	r
0,0	-124,0	-0,11251	0,54072
1,0	-300,1	-0,13217	0,28723
2,0	628,9	-0,15746	-0,15339
3,0	142,9	-0,19244	-0,54621
4,0	74,3	-0,24792	-0,68294
5,0	43,5	-0,37814	-0,62145

Table 14: *Hotelling's differential Method.*
Moving average: cars per 100 inhabitants.

y_0	k	b	r
0,0	-36,6	-0,11999	0,56133
0,1	-47,3	-0,12720	0,48365
0,2	-66,9	-0,13504	0,37902
0,3	-115,8	-0,14364	0,24008
0,4	-453,5	-0,15314	0,06523
0,5	227,0	-0,16375	-0,13204
0,6	88,9	-0,17571	-0,32129
0,7	54,3	-0,18943	-0,47339
0,8	38,5	-0,20545	-0,57753
0,9	29,3	-0,22465	-0,63920
1,0	23,2	-0,24847	-0,66821
1,1	18,8	-0,27968	-0,67134
1,2	15,3	-0,32461	-0,64880
1,3	12,3	-0,40577	-0,58390

C. Integration method.

The results yielded by the integration method are none too good. Tables 16, 17 and 18 show low correlation coefficients for values of y_0 , k and b that with respect to sign are acceptable. The values of y_0 , k and b at which r is maximum are given in Table 15, though k in all cases is smaller than the number of cars already reached in 1967.

Why now are these results so disappointing?

As regards the Hotelling method, it was already stated that the calculation is based on a series of three consecutive points y_{t-1} , y_t , y_{t+1} ;, while the relationship between these three points may be fairly intricate. An error made in one

Table 15. maximum value of r at calculated values of y_0 , k and b

	Moving average		
	cars in thousands	cars per 100 families	cars per 100 inhabitants
y_0	100	4,0	1,1
k	1589	42,4	10,7
b	-0,34777	-0,34185	-0,35539
m	72,46688	56,69694	60,61726
r	-0,66082	-0,68449	-0,67298

Table 16: integration method.
Moving average: cars in thousands.

y_0	k	b	r
0	-1162	-0,09926	0,52899
10	-1017	-0,09789	0,44324
20	-704	-0,08738	0,32671
30	-156	-0,03546	0,17264
40	614	-1,56730	-0,01558
50	1409	-0,28078	-0,21494
60	1952	-0,24677	-0,39030
70	2134	-0,24732	-0,51886
80	2052	-0,26005	-0,59994
90	1841	-0,28172	-0,64386
100	1589	-0,31376	-0,66082
110	1337	-0,36188	-0,65670
120	1095	-0,44227	-0,63157
130	852	-0,62030	-0,57341

Table 17: *integration method.*
Moving average: cars per 100 families.

y_0	k	b	r
0,0	-23,6	-0,07316	0,53798
0,5	-19,0	-0,06986	0,43207
1,0	-8,8	-0,04777	0,28200
1,5	8,9	0,16984	0,07957
2,0	31,0	-0,31778	-0,15915
2,5	48,0	-0,23972	-0,38428
3,0	53,5	-0,23988	-0,54881
3,5	49,9	-0,26108	-0,64411
4,0	42,4	-0,30098	-0,68449
4,5	33,9	-0,37374	-0,68121
5,0	25,0	-0,55374	-0,62305

Table 18: *integration method.*
Moving average: cars per 100 inhabitants.

y_0	k	b	r
0,0	-8,1	-0,08453	0,55778
0,1	-7,3	-0,08401	0,47912
0,2	-5,4	-0,07688	0,37338
0,3	-2,1	-0,04610	0,23340
0,4	2,9	0,25373	0,05810
0,5	8,6	-0,33793	-0,13851
0,6	13,3	-0,25011	-0,32638
0,7	15,6	-0,24103	-0,47690
0,8	15,7	-0,25009	-0,57996
0,9	14,5	-0,26990	-0,64108
1,0	12,7	-0,30010	-0,66989
1,1	10,7	-0,34925	-0,67298
1,2	8,7	-0,43290	-0,65044
1,3	6,7	-0,63612	-0,58554

of the observations used may therefore have a strong effect on the eventual result.

As regards the integration method it may be noted that the approximation of the integral with the aid of a trapezium rule will often be a coarse one. It has moreover been found that an increase of y_0 may lead to a change of sign of k . As a result of this the sign of $\frac{b}{k}$ or $\frac{k}{b}$ used in the Hotelling and integration

method respectively may also change. Thus it becomes possible that at a given moment — at a particular value of y_0 — the above ratios become equal to 0.

D. Erkelens' exponential method

It can be seen from Tables 20, 21 and 22 that the best adaptation with Erkelens' method is obtained if the value stated in Table 19 is selected for y_0 .

Table 19. maximum value of r at calculated values of y_0 , m/k and b

	Moving average		
	cars in thousands	cars per 100 families	cars per 100 inhabitants
y_0	60	2,5	0,6
m/k	0,00048	0,01751	0,06268
b	-0,19100	-0,17438	-0,17676
r	-0,99242	-0,98824	-0,99006

As was already stated in Chapter II, this method only yields the ratio between m and k and not the separate parameters.

*Table 20: Erkelens' exponential method.
Moving average: cars in thousands.*

y_0	m/k	b	r
0	0,00075	-0,13477	-0,98360
10	0,00070	-0,14245	-0,98633
20	0,00066	-0,15068	-0,98858
30	0,00061	-0,15954	-0,99034
40	0,00057	-0,16913	-0,99159
50	0,00052	-0,17956	-0,99231
60	0,00048	-0,19100	-0,99243
70	0,00044	-0,20365	-0,99187
80	0,00040	-0,21780	-0,99051
90	0,00035	-0,23385	-0,98811
100	0,00031	-0,25243	-0,98430
110	0,00026	-0,27457	-0,97836
120	0,00022	-0,30235	-0,96863
130	0,00016	-0,34148	-0,94958

Table 21: *Erkelens' exponential method.*
Moving average: cars per 100 families.

y_0	m/k	b	r
0,0	0,02884	-0,11206	-0,96967
0,5	0,02645	-0,12199	-0,97579
1,0	0,02414	-0,13294	-0,98068
1,5	0,02189	-0,14512	-0,98440
2,0	0,01968	-0,15881	-0,98695
2,5	0,01751	-0,17438	-0,98824
3,0	0,01534	-0,19240	-0,98811
3,5	0,01315	-0,21375	-0,98617
4,0	0,01091	-0,23991	-0,98163
4,5	0,00857	-0,27392	-0,97252
5,0	0,00596	-0,32453	-0,95154

Table 22: *Erkelens' exponential method.*
Moving average: cars per 100 inhabitants.

y_0	m/k	b	r
0,0	0,09925	-0,11938	-0,97596
0,1	0,09282	-0,12712	-0,97991
0,2	0,08657	-0,13545	-0,98322
0,3	0,08045	-0,14446	-0,98590
0,4	0,07445	-0,15425	-0,98795
0,5	0,06854	-0,16495	-0,98935
0,6	0,06268	-0,17676	-0,99006
0,7	0,05685	-0,18990	-0,98999
0,8	0,05102	-0,20470	-0,98902
0,9	0,04514	-0,22162	-0,98691
1,0	0,03917	-0,24141	-0,98324
1,1	0,03303	-0,26532	-0,97719
1,2	0,02654	-0,29588	-0,96682
1,3	0,01917	-0,34072	-0,94484

2. CALCULATIONS OF THE PARAMETERS ACCORDING TO THE REGRESSION METHOD

Like in the foregoing, the calculations according to the regression method have again been carried out with respect to the moving average of the three series dealt with.

The results of these calculations are given in Tables 23, 24 and 25.

As regards cars in thousands it was found that maximum correlation is obtained if $y_0 = 60$. In that case the saturation level k lies at 4142.

The step by which y_0 is varied has been reduced after the value $y_0 = 65$, so as to obtain the best possible approximation of the maximum correlation coefficient.

As regards cards per 100 families, it can be seen from Table 24 that the

Table 23: *regression method.*
Moving average: cars in thousands.¹

y_0	k	b	r
135	227	-1,77466	0,92968
130	295	-1,07781	0,96598
125	370	-0,78668	0,98218
120	453	-0,62482	0,99006
115	544	-0,52165	0,99417
110	645	-0,45001	0,99645
105	759	-0,39720	0,99776
100	889	-0,35652	0,99855
95	1040	-0,32411	0,99903
90	1219	-0,29758	0,99932
85	1434	-0,27541	0,99950
80	1702	-0,25654	0,99961
75	2045	-0,24024	0,99967
70	2503	-0,22598	0,99971
65	3151	-0,21337	0,99973
64,5	3231	-0,21219	0,99973
64,0	3324	-0,21102	0,99973
63,5	3401	-0,20986	0,99973
63,0	3492	-0,20872	0,99973
62,5	3588	-0,20759	0,99973
62,0	3688	-0,20647	0,99973
61,5	3793	-0,20537	0,99973
61,0	3904	-0,20427	0,99973
60,5	4020	-0,20319	0,99973
60,0	4142	-0,20387	0,99973

1. Though the correlation coefficients after the y_0 -value of 65 are equal to the decimal places shown in the table, the last coefficient at $y_0 = 60$ is still the largest, as computer calculations to eight decimal places revealed that this correlation coefficient at $y_0 = 60$ actually was the largest.

Table 24: *regression method.*
Moving average: cars per 100 families.

y_0	k	b	r
5,0	10,125	-0,89619	0,97651
4,5	17,915	-0,46965	0,99596
4,0	28,172	-0,32948	0,99893
3,5	43,306	-0,25777	0,99953
3,4	47,349	-0,24728	0,99957
3,3	51,895	-0,23766	0,99960
3,2	57,056	-0,22881	0,99961
3,1	62,979	-0,22061	0,99963
3,0	69,860	-0,21301	0,99963
2,9	77,967	-0,20592	0,99963
2,9 ¹	80,228	-0,20680	0,99963

¹ This value of $y_0 = 2,9$ is in fact the rounded-off value of $y_0 = 2,875$.

Table 25: *regression method.*
Moving average: cars per 100 inhabitants.

y_0	k	b	r
1,3	2,4	-1,11640	0,96402
1,2	3,9	-0,59869	0,99131
1,1	5,8	-0,42085	0,99726
1,0	8,1	-0,32975	0,99894
0,9	11,3	-0,27344	0,99947
0,8	16,3	-0,23459	0,99964
0,7	25,0	-0,20579	0,99967

correlation coefficient is maximum if $y_0 = 2,875$. In that case the saturation level lies at 80. Here, too, the step applied to y_0 has been varied.

As regards cars per 100 inhabitants Table 25 shows that maximum correlation is obtained if $y_0 = 0,7$. In that case the saturation level k is 25.

3. EXPONENTIAL ESTIMATE OF THE NUMBER OF CARS FOR 1968, 1969, 1970 and 1971

In Chapter I it was stated that an exponential growth for an unlimited number of years should be rejected for reasons of common sense. This does

not mean, however, that an exponential growth cannot be accepted as an approximation for a limited number of years. One is therefore justified in asking how the saturation level will be affected if the exponential growth found for the years 1950-1967 is continued to 1971.

From the model

$$y(t) = y_0 + me^{bt} \quad (\text{IV.4.1})$$

it follows that

$$y(t) - y_0 = me^{bt}$$

or

$$\ln [y(t) - y_0] = \ln m + bt \quad (\text{IV.4.2})$$

The best fitting of the time series $y(t)$ to the model (IV.4.2) follows from the tables 26, 27 and 28.

Table 26: *exponential adaptation: $y(t) = y_0 + me^{bt}$.
Moving average: cars in thousands.*

y_0	m	b	r
50,0	76,79196	0,17262	0,99950
49,5	77,13730	0,17234	0,99951
49,0	77,48264	0,17207	0,99952
48,5	77,82821	0,17179	0,99952
48,0	78,17383	0,17152	0,99954
47,5	78,51942	0,17124	0,99954
47,0	78,86535	0,17097	0,99955
46,5	79,21146	0,17070	0,99956
46,0	79,55757	0,17043	0,99957
45,5	79,90384	0,17016	0,99957
45,0	80,25009	0,16989	0,99958
44,5	80,59672	0,16963	0,99958
44,0	80,94322	0,16936	0,99959
43,5	81,29008	0,16910	0,99959
43,0	81,63670	0,16884	0,99959
42,5	81,98366	0,16858	0,99960
42,0	82,33085	0,16832	0,99960
41,5	82,67820	0,16806	0,99961

Table 27: *exponential adaptation: $y(t) = y_0 + me^{bt}$.
Moving average: cars per 100 families.*

y_0	m	b	r
5,0	0,75476	0,24283	0,96781
4,9	0,83806	0,23530	0,97347
4,8	0,91752	0,22886	0,97769
4,7	0,99446	0,22319	0,98099
4,6	1,06969	0,21812	0,98364
4,5	1,14370	0,21350	0,98582
4,4	1,21681	0,20927	0,98765
4,3	1,28928	0,20534	0,98920
4,2	1,36127	0,20168	0,99053
4,1	1,43290	0,19825	0,99168
4,0	1,50428	0,19503	0,99268
3,9	1,57549	0,19198	0,99355
3,8	1,64659	0,18908	0,99432
3,7	1,71762	0,18633	0,99499
3,6	1,78863	0,18371	0,99559
3,5	1,85965	0,18120	0,99611
3,4	1,93071	0,17880	0,99658
3,3	2,00182	0,17650	0,99699
3,2	2,07302	0,17429	0,99736
3,1	2,14430	0,17216	0,99768
3,0	2,21569	0,17011	0,99796
2,9	2,28720	0,16813	0,99821
2,8	2,35884	0,16622	0,99843
2,7	2,43061	0,16437	0,99863
2,6	2,50251	0,16258	0,99879
2,5	2,57457	0,16084	0,99894
2,4	2,64678	0,15916	0,99906
2,3	2,71914	0,15752	0,99917
2,2	2,79166	0,15593	0,99925
2,1	2,86434	0,15439	0,99933
2,0	2,93719	0,15289	0,99938
1,9	3,01020	0,15143	0,99943
1,8	3,08337	0,15000	0,99946
1,7	3,15670	0,14861	0,99948
1,6	3,23021	0,14726	0,99949

Table 28: *exponential adaptation: $y(t) = y_0 + me^{bt}$
Moving average: cars per 100 inhabitants.*

y_0	m	b	r
1,3	0,17930	0,25662	0,96198
1,2	0,26085	0,22962	0,98156
1,1	0,33378	0,21254	0,98939
1,0	0,40437	0,19962	0,99361
0,9	0,47426	0,18915	0,99612
0,8	0,54412	0,18030	0,99767
0,7	0,61427	0,17264	0,99862
0,6	0,68488	0,16589	0,99918
0,5	0,75603	0,15986	0,99946
0,4	0,82778	0,15441	0,99954

Consequently, for cars in thousands the model (IV.4.1) changes into:

$$y(t) = 41,5 + 82,67820 e^{0,16806t} \quad (\text{IV.4.3})$$

for cars per 100 families into:

$$y(t) = 1,6 + 3,23021 e^{0,14726t} \quad (\text{IV.4.4})$$

for cars per 100 inhabitants into:

$$y(t) = 0,4 + 0,82778 e^{0,15441t} \quad (\text{IV.4.5})$$

Extrapolation on the basis of the above models yields the following values for 1968, 1969, 1970 and 1971.

Table 29: *moving average: cars in thousands.*

$b = 0,16806$		$m = 82,67820$		$y_0 = 41,5$	
Year	t	bt	e^{bt}	me^{bt}	calc. $y(t)$
1968	19	3,19314	24,3613	2014,148	2056
1969	20	3,36120	28,8179	2382,612	2424
1970	21	3,52926	34,0898	2818,483	2860
1971	22	3,69732	40,3261	3334,089	3376

Table 30: moving average: cars per 100 families.

$$b = 0,14726$$

$$m = 3,23021$$

$$y_0 = 1,6$$

Year	t	bt	e ^{bt}	me ^{bt}	calc. y(t)
1968	19	2,79794	16,412	53,013	54,6
1969	20	2,94520	19,010	61,408	63,0
1970	21	3,09246	22,020	71,132	72,7
1971	22	3,23972	25,518	82,396	84,0

Table 31: moving average: cars per 100 inhabitants.

$$b = 0,15441$$

$$m = 0,82778$$

$$y_0 = 0,4$$

Year	t	bt	e ^{bt}	me ^{bt}	calc. y(t)
1968	19	2,93379	18,80262	15,564	16,0
1969	20	3,08820	21,93320	18,156	18,6
1970	21	3,24261	25,58475	21,179	21,6
1971	22	3,39702	29,84453	24,705	25,1

On the basis of the above values of the three different categories estimated exponentially for the years 1968, 1969, 1970 and 1971 k , b , m and r are determined for the series 1950-1968, 1950-1969, 1950-1970 and 1950-1971.

The values of b and m are determined with the aid of the method of least squares – as already stated in chapter II – which results in a refinement of the two coefficients. For, if we start from:

$$y(t) = y_0 + \frac{k}{1 + me^{bt}}$$

we may also write:

$$\begin{aligned} y(t) &= y_0 + \frac{k}{1 + e^a \cdot e^{bt}} \\ &= y_0 + \frac{k}{1 + e^{a+bt}} \end{aligned}$$

For simplicity's sake Tables 32, 33 and 34 only give the calculated coefficients.

Table 32: moving average: cars in thousands.

Observation period prolonged by:	y ₀	k	b	m	r
1968	59	4843	-0,20155	74,49926	0,99873
1968 and 1969	57	5943	-0,19736	89,55231	0,99885
1968, 1969 and 1970	56	6861	-0,19577	102,68201	0,99881
1968, 1969, 1970 and 1971	55	8108	-0,19363	119,94856	0,99882

Table 33: moving average: cars per 100 families.

Observation period prolonged by:	y ₀	k	b	m	r
1968	2,8	91,4	-0,20330	44,38219	0,99759
1968 and 1969	2,8	106,9	-0,19842	50,50745	0,99767
1968, 1969 and 1970	2,7	125,5	-0,19381	57,74027	0,99775
1968, 1969, 1970 and 1971	2,6	142,9	-0,19131	64,95491	0,99761

Table 34: moving average: cars per 100 inhabitants.

Observation period prolonged by:	y ₀	k	b	m	r
1968	0,5	30,5	-0,20159	53,76876	0,99819
1968 and 1969	0,7	35,3	-0,19819	61,23740	0,99821
1968, 1969 and 1970	0,7	39,9	-0,19642	68,65826	0,99807
1968, 1969, 1970 and 1971	0,6	45,0	-0,19481	77,07791	0,99788

Starting from the series: 1950-1967.

1950-1968

1950-1969

1950-1970

1950-1971

the forecasts of cars in thousands, cars per 100 families and cars per 100 inhabitants are given in Tables 35, 36, 37, 38 and 39 and graphs 14, 15 and 16.

Table 35: *forecast of cars in thousands, cars per 100 families and cars per 100 inhabitants on the basis of the coefficients calculated for the series 1950-1967.*

	cars in thousands	cars per 100 families	cars per 100 inhabitants
y ₀	60	2,9	0,7
k	4142	80,2	25,0
b	-0,20387	-0,20680	-0,20579
m	64,50050	39,51653	45,83807
r	0,99872	0,99759	0,99785
Year			
1968	1830	48,0	14,0
1969	2039	52,0	15,3
1970	2250	55,9	16,6
1971	2458	59,5	17,7
1972	2660	62,8	18,8
1973	2852	65,7	19,8
1974	3030	68,4	20,7
1975	3194	70,7	21,5
1976	3341	72,7	22,2
1977	3472	74,5	22,7
1978	3586	75,9	23,2
1979	3686	77,2	23,7
1980	3771	78,2	24,0
1981	3844	79,1	24,3
1982	3905	79,8	24,6

Table 36: *forecast of cars in thousands, cars per 100 families and cars per 100 inhabitants on the basis of the coefficients calculated for the series 1950-1968.*

	cars in thousands	cars per 100 families	cars per 100 inhabitants
y ₀	59	2,8	0,5
k	4843	91,4	30,5
b	-0,20155	-0,20330	-0,20159
m	74,49926	44,38219	53,76876
r	0,99873	0,99759	0,99819
Year			
1969	2136	54,7	16,3
1970	2377	59,2	17,8
1971	2620	63,5	19,3
1972	2860	67,5	20,7
1973	3092	71,2	22,1
1974	3313	74,5	23,3
1975	3518	77,5	24,4
1976	3707	80,1	25,4
1977	3877	82,3	26,3
1978	4028	84,3	27,1
1979	4161	86,0	27,7
1980	4276	87,4	28,3
1981	4375	88,6	28,8
1982	4460	89,5	29,2
1983	4532	90,4	29,5

Table 37: *forecast of cars in thousands, cars per 100 families and cars per 100 inhabitants on the basis of the coefficients calculated for the series 1950-1969.*

	cars in thousands	cars per 100 families	cars per 100 inhabitants
y ₀	57	2,8	0,7
k	5943	106,9	35,3
b	-0,19736	-0,19842	-0,19819
m	89,55231	50,50745	61,23740
r	0,99885	0,99767	0,99821
Year			
1970	2513	62,7	18,7
1971	2802	67,8	20,5
1972	3095	77,8	22,2
1973	3386	77,4	23,8
1974	3671	81,7	25,3
1975	3944	85,6	26,8
1976	4200	89,1	28,0
1977	4438	92,2	29,2
1978	4654	94,9	30,2
1979	4849	97,2	31,1
1980	5021	99,2	31,9
1981	5172	101,0	32,6
1982	5303	102,4	33,1
1983	4515	103,6	33,6
1984	5511	104,7	34,0

Table 38: *forecast of cars in thousands, cars per 100 families and cars per 100 inhabitants on the basis of the coefficients calculated for the series 1950-1970.*

	cars in thousands	cars per 100 families	cars per 100 inhabitants
y ₀	56	2,7	0,7
k	6861	125,5	39,9
b	-0,19577	-0,19381	-0,19642
m	102,68201	57,74027	68,65826
r	0,99881	0,99775	0,99807
Year			
1971	2934	71,9	21,5
1972	3265	77,9	23,4
1973	3601	83,6	25,3
1974	3934	89,0	27,1
1975	4259	94,0	28,8
1976	4570	98,6	30,4
1977	4862	102,7	31,8
1978	5132	106,5	33,0
1979	5379	109,7	34,2
1980	5600	112,6	35,1
1981	5795	115,0	36,0
1982	5967	117,1	36,7
1983	6116	118,9	37,4
1984	6245	120,5	37,9
1985	6354	121,7	38,3

Table 39: *forecast of cars in thousands, cars per 100 families and cars per 100 inhabitants on the basis of the coefficients calculated for the series 1950-1971.*

	cars in thousands	cars per 100 families	cars per 100 inhabitants
y_0	55	2,6	0,6
k	8108	142,9	45,0
b	-0,19363	-0,19131	-0,19481
m	119,94856	64,95491	77,07791
r	0,99882	0,99761	0,99788
Year			
1972	3438	82,1	24,7
1973	3825	88,8	26,8
1974	4217	95,2	28,9
1975	4607	101,2	30,9
1976	4988	106,8	32,8
1977	5353	112,0	34,5
1978	5697	116,7	36,1
1979	6016	120,8	37,4
1980	6308	124,5	38,7
1981	6370	127,7	39,7
1982	6308	130,5	40,7
1983	7009	132,8	41,5
1984	7187	134,9	42,2
1985	7341	136,6	42,7
1986	7473	138,1	43,2
1987	7658	139,3	43,6

Calculations made by the A.N.W.B. show that in 1980 there will be approximately 6 000 000 cars. This possibility is employed in the saturation level of 8 108 000 calculated by us.

4. DETERMINING THE SENSITIVITY OF $Y(T)$ TO THE PARAMETERS Y_0 , K, B AND M IN THE REGRESSION METHOD USED

The relationship dealt with in the previous chapters are behaviouristic relationships dependent on the technological development and on the priorities of the consumer. As these factors may vary in time, the coefficients also governing these relationships may vary in time as well. If, however, the coefficients change in time, the forecasts will also change in time. It is

therefore useful to investigate the sensitivities of the forecasts made to variations in the parameters y_0 , k , b and m . This investigation will be based on the function $f(x_1, \dots, x_n)$.

If in this function we make the independent variable x_i ($i = 1, \dots, n$) vary by Δx_i , the variation Δf of f will be equal to:

$$\Delta f = \sum_{i=1}^n \frac{\delta f}{\delta x_i} \Delta x_i \quad (\text{IV.5.1})$$

This formula applied to the variable $y(t)$ – which in fact is determined by y_0 , k , b and m – yields:

$$\Delta y(t) = \frac{\delta y(t)}{\delta y_0} \Delta y_0 + \frac{\delta y(t)}{\delta k} \Delta k + \frac{\delta y(t)}{\delta b} \Delta b + \frac{\delta y(t)}{\delta m} \Delta m \quad (\text{IV.5.2})$$

To be able to determine $\Delta y(t)$, the value of the factors in the right-hand term of (IV.5.2) must therefore be found.

The value of the derivatives can be simply found.

$$\frac{\delta y(t)}{\delta y_0} = \frac{\delta}{\delta y_0} \left[y_0 + \frac{k}{1 + me^{bt}} \right] = 1 \quad (\text{IV.5.3})$$

$$\frac{\delta y(t)}{\delta k} = \frac{\delta}{\delta k} \left[y_0 + \frac{k}{1 + me^{bt}} \right] = \frac{1}{1 + me^{bt}} \quad (\text{IV.5.4})$$

$$\frac{\delta y(t)}{\delta b} = \frac{\delta}{\delta b} \left[y_0 + \frac{k}{1 + me^{bt}} \right] = \frac{-t k me^{bt}}{(1 + me^{bt})^2} \quad (\text{IV.5.5})$$

$$\frac{\delta y(t)}{\delta m} = \frac{\delta}{\delta m} \left[y_0 + \frac{k}{1 + me^{bt}} \right] = \frac{-k e^{bt}}{(1 + me^{bt})^2} \quad (\text{IV.5.6})$$

If now the equations (IV.5.3), (IV.5.4), (IV.5.5) and (IV.5.6) are substituted in (IV.5.2), the latter assumes the following form:

$$\Delta y(t) = \Delta y_0 + \frac{1}{1 + me^{bt}} \Delta k - \frac{t k me^{bt}}{(1 + me^{bt})^2} \Delta b - \frac{k e^{bt}}{(1 + me^{bt})^2} \Delta m \quad (\text{IV.5.7})$$

Δy_0 and Δk can be determined with the information supplied by the regression method.

In paragraph 1 of Chapter III a linear relationship between $z(t+1)$ and $z(t)$ is demonstrated. In (III.2.1) this relationship is represented by:

$$z(t+1) = Pz(t) + Q$$

If now the fixed kernel is taken into account, this equation can be written as follows:

$$\left[\frac{1}{y(t+1) - y_0} \right] = C_1 \left[\frac{1}{y(t) - y_0} \right] + C_2 \quad (\text{IV.5.8})$$

from which C_1 and C_2 can be determined by linear regression while it also applies that:

$$k = \frac{1-C_1}{C_2} \quad (\text{IV.5.9})$$

so that k is also known.

The variation of $\frac{1}{y(t+1) - y_0}$ in (IV.5.8) can be expressed as:

$$\Delta \left[\frac{1}{y(t+1) - y_0} \right] = \Delta C_1 \left[\frac{1}{y(t) - y_0} \right] + C_1 \Delta \left[\frac{1}{y(t) - y_0} \right] + \Delta C_2 \quad (\text{IV.5.10})$$

or connected with (IV.5.8)

$$\begin{aligned} \left[\frac{1}{y(t+1) - y_0} \right] - C_1 \left[\frac{1}{y(t) - y_0} \right] - C_2 &= \\ \Delta C_1 \left[\frac{1}{y(t) - y_0} \right] + C_1 \left[\frac{1}{y(t) - y_0} \right]^2 (\Delta y_0) + \Delta C_2 & \quad (\text{IV.5.11}) \end{aligned}$$

or

$$\begin{aligned} \left[\frac{1}{y(t+1) - y_0} \right] &= \\ (C_1 + \Delta C_1) \left[\frac{1}{y(t) - y_0} \right] + \Delta y_0 \left[\frac{1}{y(t) - y_0} \right]^2 C_1 + (C_2 + \Delta C_2) & \quad (\text{IV.5.12}) \end{aligned}$$

With the aid of regression calculations $(C_2 + \Delta C_2)$, $(C_1 + \Delta C_1)$ and Δy_0 can now be determined. C_1 and C_2 were found from (IV.5.8) and consequently the values of ΔC_1 and ΔC_2 are also known.

Δk can be calculated, as according to (IV.5.9) it applies that:

$$k = \frac{1-C_1}{C_2}$$

from which it follows that

$$\Delta k = \frac{-1}{C_2} \Delta C_1 - \frac{(1-C_1)}{C_2^2} \Delta C_2 \quad (\text{IV.5.13})$$

If the values of C_1 , C_2 , ΔC_1 and ΔC_2 are substituted in this equation, Δk is also known.

We finally have to determine Δm and Δb .

The equation $y(t) = y_0 + \frac{k}{1 + mebt}$

can be rewritten as:

$$\ln \left[\frac{k}{y(t) - y_0} - 1 \right] = \ln m + bt \quad (\text{IV.5.14})$$

The variability in $\ln m$ and b therefore depends on the variability in k , y_0 and on errors in the regression.

So

$$\Delta \left[\ln \left[\frac{k}{y(t) - y_0} - 1 \right] \right] = \Delta(b)t + \Delta(\ln m) \quad (\text{IV.5.15})$$

The left-hand side of (IV.5.15) can be approximated as follows:

$$\begin{aligned} \Delta \left[\ln \left[\frac{k}{y(t) - y_0} - 1 \right] \right] &= \frac{\delta \ln \left[\frac{k}{y(t) - y_0} - 1 \right]}{\delta k} \Delta k + \\ &+ \frac{\delta \ln \left[\frac{k}{y(t) - y_0} - 1 \right]}{\delta y_0} \Delta y_0 \end{aligned} \quad (\text{IV.5.16})$$

The first term of the right-hand side of (IV.5.16) is elaborated to:

$$\frac{\delta \ln \left[\frac{k}{y(t) - y_0} - 1 \right]}{\delta k} = \left[\frac{1}{\frac{k}{y(t) - y_0} - 1} \right] \left[\frac{1}{y(t) - y_0} \right] =$$

$$= \frac{1}{k-y(t)+y_0} \quad (\text{IV.5.17})$$

and the second term to:

$$\begin{aligned} \frac{\delta \ln \left[\frac{k}{y(t)-y_0} - 1 \right]}{\delta y_0} &= \left[\frac{1}{\frac{k}{y(t)-y_0} - 1} \right] \left[\frac{k}{[y(t)-y_0]^2} \right] = \\ &= \frac{k}{[k-y(t)+y_0][y(t)-y_0]} \quad (\text{IV.5.18}) \end{aligned}$$

If this result is substituted in (IV.5.16), this expression changes into:

$$\begin{aligned} \Delta \left[\ln \left[\frac{k}{y(t)-y_0} - 1 \right] \right] &= \left[\frac{1}{k-y(t)+y_0} \right] \Delta k + \\ &+ \frac{k}{[k-y(t)+y_0][y(t)-y_0]} \Delta y_0 \quad (\text{IV.5.19}) \end{aligned}$$

Since Δk and Δy_0 are known, the left-hand side of (IV.5.19) can be calculated. If these values are substituted in (IV.5.15), it is possible to determine the values of Δb and $\Delta(\ln m)$ by means of regression, and Δm , because $\Delta(\ln m)$ is equal to $\frac{1}{m} \Delta m$.

All factors of the right-hand side of (IV.5.2) are now known, so $\Delta y(t)$ can be determined.

The above method of calculation yields the results given in Tables 40, 41 and 42.

Table 40: values of the parameters and the calculated variation for cars in thousands in 1967 and for the extrapolated years 1968, 1969, 1970 and 1971.

Year	calculated value	Variation
1967	$y_0 = 60$	$\Delta y_0 = 0,2$
	$k = 4242$	$\Delta k = 305$
	$b = -0,20387$	$\Delta b = 0,00222$
	$m = 64,50050$	$\Delta m = 4,29121$
1968	$y_0 = 59$	$\Delta y_0 = 0,108$
	$k = 4843$	$\Delta k = 243$
	$b = -0,20155$	$\Delta b = 0,00146$
	$m = 74,49926$	$\Delta m = 3,96332$
1969	$y_0 = 57$	$\Delta y_0 = 0,3$
	$k = 5943$	$\Delta k = 840$
	$b = -0,19736$	$\Delta b = 0,00362$
	$m = 89,55231$	$\Delta m = 11,38962$
1970	$y_0 = 56$	$\Delta y_0 = 0,3$
	$k = 6861$	$\Delta k = 1113$
	$b = -0,19577$	$\Delta b = 0,00406$
	$m = 102,68201$	$\Delta m = 14,84893$
1971	$y_0 = 55$	$\Delta y_0 = 0,4$
	$k = 8108$	$\Delta k = 1801$
	$b = -0,19363$	$\Delta b = 0,00525$
	$m = 119,94856$	$\Delta m = 23,67635$

Table 41: values of the parameters and the calculated variation for cars per 100 families in 1967 and for the extrapolated years 1968, 1969, 1970 and 1971.

Year	Calculated value	Variation
1967	$y_0 = 2,9$	$\Delta y_0 = 0,0$
	$k = 80,2$	$\Delta k = 2,7$
	$b = -0,20680$	$\Delta b = 0,00180$
	$m = 39,51653$	$\Delta m = 1,08601$
1968	$y_0 = 2,8$	$\Delta y_0 = 0,0$
	$k = 91,4$	$\Delta k = 1,5$
	$b = -0,20330$	$\Delta b = 0,00086$
	$m = 44,38219$	$\Delta m = 0,58790$
1969	$y_0 = 2,8$	$\Delta y_0 = 0,0$
	$k = 106,9$	$\Delta k = 5,9$
	$b = -0,19842$	$\Delta b = 0,00265$
	$m = 50,87224$	$\Delta m = 2,23625$
1970	$y_0 = 2,7$	$\Delta y_0 = 0,0$
	$k = 125,5$	$\Delta k = 13,2$
	$b = -0,19381$	$\Delta b = 0,00463$
	$m = 57,74027$	$\Delta m = 4,86762$
1971	$y_0 = 2,6$	$\Delta y_0 = 0,0$
	$k = 142,9$	$\Delta k = 17,6$
	$b = -0,19131$	$\Delta b = 0,00531$
	$m = 64,95491$	$\Delta m = 6,33285$

Table 42: values of the parameters and the calculated variations for cars per 100 inhabitants in 1967 and for the extrapolated years 1968, 1969, 1970 and 1971.

Year	Calculated value	Variation
1967	$y_0 = 0,7$	$\Delta y_0 = 0,0$
	$k = 25,0$	$\Delta k = 0,0$
	$b = -0,20845$	$\Delta b = 0,00004$
	$m = 45,83807$	$\Delta m = 0,03717$
1968	$y_0 = 0,5$	$\Delta y_0 = 0,0$
	$k = 30,5$	$\Delta k = 2,3$
	$b = -0,20159$	$\Delta b = 0,00304$
	$m = 53,76876$	$\Delta m = 3,39403$
1969	$y_0 = 0,7$	$\Delta y_0 = 0,0$
	$k = 35,3$	$\Delta k = 3,6$
	$b = -0,19819$	$\Delta b = 0,00392$
	$m = 61,23740$	$\Delta m = 5,19197$
1970	$y_0 = 0,7$	$\Delta y_0 = 0,0$
	$k = 39,9$	$\Delta k = 4,0$
	$b = -0,19642$	$\Delta b = 0,00395$
	$m = 68,65826$	$\Delta m = 5,75734$
1971	$y_0 = 0,6$	$\Delta y_0 = 0,0$
	$k = 45,0$	$\Delta k = 5,0$
	$b = -0,19481$	$\Delta b = 0,00430$
	$m = 77,07791$	$\Delta m = 6,90587$

On the basis of the values of y_0 , k , b and m given in Tables 40, 41 and 42, and of the relevant variations extrapolations based on the years:

1950-1967

1950-1968

1950-1969

1950-1970

1950-1971

have been made. They again apply to the categories:

cars in thousands, cars per 100 families and cars per 100 inhabitants.

The results are given in Tables 43, 44, 45, 46 and 47.

Table 43: extrapolation of the series 1950-1967 with relevant sensitivity limits.

Year	cars in thousands		cars per 100 families		cars per 100 inhabitants	
	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$
1968	1830	130	48,0	1,5	14,0	0,0
1969	2039	146	52,0	1,6	15,3	0,0
1970	2250	161	55,9	1,8	16,6	0,0
1971	2458	177	59,5	1,9	17,7	0,0
1972	2660	191	62,8	2,0	18,8	0,0
1973	2852	206	65,7	2,1	19,8	0,0
1974	3030	219	68,4	2,2	20,7	0,0
1975	3194	231	70,7	2,3	21,5	0,0
1976	3341	242	72,7	2,3	22,2	0,0
1977	3472	251	74,5	2,4	22,7	0,0
1978	3586	260	75,9	2,5	23,2	0,0
1979	3686	267	77,2	2,5	23,7	0,0
1980	3771	273	78,2	2,5	24,0	0,0
1981	3844	279	79,1	2,6	24,3	0,0
1982	3905	283	79,8	2,6	24,6	0,0
1983	3957	287	80,4	2,6	24,8	0,0
1984	4000	290	80,9	2,6	24,9	0,0
1985	4035	293	81,3	2,6	25,1	0,0
1986	4065	295	81,6	2,7	25,2	0,0
1987	4090	297	81,9	2,7	25,3	0,0
1988	4110	298	82,1	2,7	25,4	0,0
1989	4127	299	82,3	2,7	25,4	0,0
1990	4140	300	82,4	2,7	25,5	0,0
1991	4152	301	82,6	2,7	25,5	0,0
1992	4161	302	82,7	2,7	25,5	0,0
1993	4168	303	82,8	2,7	25,6	0,0
1994	4174	303	82,8	2,7	25,6	0,0

Table 44: *extrapolation of the series 1950-1968 with relevant sensitivity limits.*

Year	cars in thousands		cars per 100 families		cars per 100 inhabitants	
	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$
1969	2136	105	54,7	0,9	16,3	1,1
1970	2377	117	59,2	0,9	17,8	1,2
1971	2620	129	63,5	1,0	19,3	1,4
1972	2860	141	67,5	1,0	20,7	1,5
1973	3092	153	71,2	1,1	22,1	1,6
1974	3313	164	74,5	1,2	23,3	1,7
1975	3518	174	77,5	1,2	24,4	1,7
1976	3707	184	80,1	1,3	25,4	1,8
1977	3877	192	82,3	1,3	26,3	1,9
1978	4028	200	84,3	1,3	27,1	1,9
1979	4161	207	86,0	1,4	27,7	2,0
1980	4276	213	87,4	1,4	28,3	2,0
1981	4375	218	88,6	1,4	28,8	2,1
1982	4460	222	89,5	1,4	29,2	2,1
1983	4532	225	90,4	1,4	29,5	2,1
1984	4592	229	91,1	1,5	29,8	2,2
1985	4643	231	91,6	1,5	30,0	2,2
1986	4685	233	92,1	1,5	30,2	2,2
1987	4720	235	92,5	1,5	30,4	2,2
1988	4749	236	92,8	1,5	30,5	2,2
1989	4773	238	93,1	1,5	30,7	2,2
1990	4793	239	93,3	1,5	30,7	2,2
1991	4809	239	93,5	1,5	30,8	2,2
1992	4823	240	93,6	1,5	30,9	2,2
1993	4834	241	93,7	1,5	30,9	2,2
1994	4843	241	93,8	1,5	31,0	2,3

Table 45: *extrapolation of the series 1950-1969 with relevant sensitivity limits.*

Year	cars in thousands		cars per 100 families		cars per 100 inhabitants	
	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$
1970	2513	347	62,7	3,3	18,7	1,8
1971	2802	388	67,8	3,6	20,5	2,0
1972	3095	430	72,8	3,8	22,2	2,1
1973	3386	471	77,4	4,1	23,8	2,3
1974	3671	511	81,7	4,3	25,3	2,4
1975	3944	550	85,6	4,6	26,8	2,6
1976	4200	586	89,1	4,8	28,0	2,7
1977	4438	620	92,2	4,9	29,2	2,8
1978	4654	650	94,9	5,0	30,2	2,9
1979	4849	678	97,2	5,2	31,1	3,0
1980	5021	702	99,2	5,3	31,9	3,1
1981	5172	723	101,0	5,4	32,6	3,2
1982	5303	742	102,4	5,5	33,1	3,3
1983	5415	758	103,6	5,6	33,6	3,3
1984	5511	771	104,7	5,6	34,0	3,3
1985	5593	783	105,5	5,7	34,3	3,4
1986	5662	793	106,2	5,7	34,6	3,4
1987	5719	801	106,8	5,7	34,9	3,4
1988	5768	808	107,3	5,8	35,1	3,5
1989	5808	813	107,7	5,8	35,2	3,5
1990	5841	818	108,0	5,8	35,4	3,5
1991	5869	822	108,3	5,8	35,5	3,5
1992	5892	825	108,6	5,8	35,6	3,5
1993	5911	828	108,8	5,9	35,6	3,5
1994	5927	830	108,9	5,9	35,7	3,5

Table 46: *extrapolation of the series 1950-1970 with relevant sensitivity limits.*

Year	cars in thousands		cars per 100 families		cars per 100 inhabitants	
	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$
1971	2934	467	71,9	7,2	21,5	2,1
1972	3265	521	77,9	7,8	23,4	2,3
1973	3601	575	83,6	8,4	25,3	2,5
1974	3934	629	89,0	9,0	27,1	2,6
1975	4259	682	94,0	9,5	28,8	2,8
1976	4570	732	98,6	10,0	30,4	3,0
1977	4862	780	102,7	10,5	31,8	3,1
1978	5132	824	106,5	10,9	33,0	3,3
1979	5379	864	109,7	11,2	34,2	3,4
1980	5600	900	112,6	11,5	35,1	3,5
1981	5795	931	115,0	11,8	36,0	3,6
1982	5967	959	117,1	12,0	36,7	3,6
1983	6116	983	118,9	12,2	37,4	3,7
1984	6245	1004	120,5	12,3	37,9	3,8
1985	6354	1022	121,7	12,5	38,3	3,8
1986	6447	1037	122,8	12,6	38,7	3,9
1987	6526	1050	123,7	12,7	39,0	3,9
1988	6592	1061	124,5	12,8	39,3	3,9
1989	6648	1070	125,1	12,8	39,5	3,9
1990	6694	1077	125,6	12,9	39,7	4,0
1991	6732	1083	126,1	12,9	39,8	4,0
1992	6764	1089	126,4	13,0	39,9	4,0
1993	6791	1093	126,7	13,0	40,0	4,0
1994	6813	1096	127,0	13,0	40,1	4,0

Table 47: *extrapolation of the series 1950-1971 with relevant sensitivity limits.*

Year	cars in thousands		cars per 100 families		cars per 100 inhabitants	
	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$	calc. y(t)	$\Delta y(t)$
1972	3439	125	82,1	2,2	24,7	0,6
1973	3825	186	88,8	2,9	26,8	0,8
1974	4217	260	95,2	3,9	28,9	1,1
1975	4607	345	101,2	5,0	30,9	1,4
1976	4988	441	106,8	6,1	32,8	1,7
1977	5353	545	112,0	7,2	34,5	2,0
1978	5697	654	116,7	8,3	36,1	2,3
1979	6016	765	120,8	9,3	37,4	2,6
1980	6308	875	124,5	10,3	38,7	2,9
1981	6570	981	127,7	11,2	39,7	3,2
1982	6803	1081	130,5	12,1	40,7	3,4
1983	7008	1174	132,8	12,8	41,5	3,6
1984	7187	1258	134,9	13,5	42,2	3,8
1985	7341	1334	136,6	14,1	42,7	4,0
1986	7474	1402	138,1	14,6	43,2	4,1
1987	7586	1462	139,3	15,1	43,6	4,3
1988	7682	1513	140,4	15,5	44,0	4,4
1989	7762	1558	141,2	15,8	44,3	4,5
1990	7830	1596	142,0	16,1	44,5	4,6
1991	7886	1629	142,6	16,3	44,7	4,6
1992	7933	1657	143,1	16,5	44,9	4,7
1993	7973	1680	143,5	16,7	45,0	4,7
1994	8005	1700	143,8	16,8	45,1	4,7

Graph 11: relationship between k and y_2 for the moving averages of the cars in thousands.

Graph 12: relationship between k and y_2 for the moving average of the number of cars per 100 families.

Graph 13: relationship between k and y_2 for the moving average of the number of cars per 100 inhabitants.

Graph 14: growth of the number of cars in thousands by exponential extrapolation for the years 1968 – 1969 – 1970 – 1971.¹

1. The saturationlevel is inclusive y_0 .

Graph 15: growth of the number of cars per 100 families by exponential extrapolation for the years 1968 – 1969 – 1970 – 1971.¹

1. The saturationlevel is inclusive y_0 .

Graph 16: growth of number of cars per 100 inhabitants by exponential extrapolation for the years 1968 – 1969 – 1970 – 1971.¹

1. The saturationlevel is inclusive y_0 .

V Influence of income and Price

In paragraph 1 of this chapter the current opinion on the effect of income and price on the increase of the total number of cars will be dealt with. On the basis of the reasons stated in section 1 the income and price will then be introduced into the logistic curve in paragraph 2. Finally it will be investigated in paragraph 3 whether the approximation given in sections 1 and 2 is acceptable.

1. A FEW THEORETICAL NOTES ON THE IMPORTANCE OF INCOME AND PRICE

In the previous chapters it was shown that in the long run the total number of cars tends to reach a saturation level. In the calculation of this level the time was only employed as explaining variable. It is also possible, however, to hold the view that, if seen from a macro-economic angle, price and income, besides time, will exert an influence. The passenger car is the most expensive durable consumer good and it may therefore be expected that the level or the variation of the income will have the greatest effect on the increase of the total number. Another point is that — once people have acquired a car — they must also be able to 'run' it, which means that the income is used for covering the running costs.

How now will in general be the influence — if any — exerted on the increase of the total number of cars by the income.

Investigations made before the Second World War in America suggest that a higher income per capita gives rise to a larger number of cars per 1000 inhabitants. As is shown by graph 17 this trend is also discernible in the Netherlands.

It is clear, both in the Netherlands and in the United States, that the number of cars decreased in the notorious slump period.

Before the Second World War the possession of a car was, however, a

luxury which only few could afford. This luxury character was strongly stressed and it is therefore reasonable that in that period – on the basis of the graphs – the income indeed affected the total number of registered cars.

How now is the situation after the Second World War as regards the increase of the number of cars.

The study made by Becker¹ shows that countries having a different level of prosperity and growth of prosperity also display differences in the numbers of cars per 1000 inhabitants. According as the prosperity level is higher, the number of cars will be larger. Such a development cannot be equally marked in every situation, however. When the income level is so low that an increase of this level will first lead to the purchase of other – necessary – goods, as is the case in developing countries, this increase will contribute little to an increase of the total number of cars.

The situation is different again in countries where the income per head of population is higher, a situation which particularly applies to the Netherlands. Here a higher income level may indeed result in a relatively large increase of the total number of cars, because then the additional income can indeed be spent on other – less necessary – consumer goods. If this assumption is true, it will have to be proved by calculations. If a high income per head of population and a high degree of motorisation should already have been attained, the effect of the income on the increase of the total number of cars will decrease again. In such a case the saturation level is near and a further increase of the income will contribute little to a growth of the number of cars. The effect of the income or the change of income on the total number of registered cars is fully governed, however, by the attitude of the consumers. In the past the car was far more widely regarded as a luxury than at present. Now the possession of a car may be considered a must if the owner sets store by social prestige.

On the basis of the foregoing it may be said that the motorisation process has become a penetration process² progressing with more or less intensity. Consequently the car penetrates into more income brackets or, as Becker once put it: the consumer acceptance increases.

As a rule the purchase of a car is preceded by a period of saving. Here, however, a distinction should be made between saving as 'saving' and saving as 'payment by instalments'. In the first case – saving as 'saving' – this will lead to a relationship with a time lag between the gaining of the income and the purchase of a car. The existence of this time lag cannot be proved statistically

1. See chapter I, paragraph 1.

2. VERMETTEN, *Statistica Neerlandica*, 1964 no. 4, pg. 463-471.

if it is shorter than the length of the period between two successive statistical data. The fact is that if the statistical data, like annual figures, are published and if the time lag between saving and purchase is shorter than one year, it will not be possible to prove the existence of this lag.

Nor is it possible to ascertain the duration of the lag statistically if there is question of saving as 'paying by instalments'. As a rule the lag will then consist of the maximum term of the credit obtained. This term varies, however, from 18 to 36 months according to the institution giving the credit, while the distribution over these terms is unknown.

The data published about the credits given by the financing companies do not provide a suitable basis for forming an opinion about the distribution.

The data are distorted by the personal loans and loans given for the sake of good relations — now and in the future — and which can be used for the purchase of cars.

In the majority of cases the car will, however, be purchased by a combination of the above-mentioned possibilities. Saving as 'saving' for the down payment and saving as 'payment by instalments' for the payment of the various instalments.

It was not possible to obtain data about this possibility and the use to which the credit obtained was put, or about the size and class of the cars.

The height of the amount saved is governed by many factors. The amount that can be saved in the case of saving as 'saving' must be at least as large as the total amount required for the payment of the credit instalments and the costs connected with owning a car. This means in fact that the consumer must be able to bear these costs without cutting down considerably on the other expenses. Only in that case is it possible to 'run' a car — at least in financial respect.

It may therefore be concluded that for every consumer a lower income limit can be assumed, above which he can decide to buy a car. It is not possible to determine this threshold value statistically on the basis of real figures, as this would require a very deep knowledge of the income distribution and the consumptive habits of the individuals. Intuitively it may be stated, however, that this threshold is constantly decreasing, as the possession of a car is given an ever-growing priority.

2. CALCULATING THE GROWTH CURVE WHEN INCOME AND PRICE ARE ALSO INCLUDED IN THE MODEL

As was already stated at the beginning of this chapter, the increase of the number of cars was so far only affected by time in this study. Since, however,

income and price exert some influence, it is obvious that these variables should also be included in the growth function.

This function can then be written as follows:

$$y(t) = y_0 + \frac{k}{1 + me^{bt} + c I(t) + d P(t)} \quad (V. 2.1)$$

in which $I(t)$ and $P(t)$ represent the income and price sequences. As was argued in Chapter I, the increase in the number of cars is proportional to the difference between the saturation level and the number of cars already attained. The absolute height of the saturation level and the fixed kernel, both in thousands and in number per 100 families or 100 inhabitants is assumed to be constant. If consequently price and income should exert an influence, this will only be an influence on the 'rate of growth'.

The income sequence used is the real, net national income per head of population, in constant prices, as published by the Central Bureau of Statistics. The basis of that sequence is 1950.

Table 48: income data for the years 1950-1967

Year	Index of the Real Net National Income per capita I	Moving average of the index of the Real Net National Income per capita I'
1950	100,000	100,000
1951	99,300	99,967
1952	100,600	105,000
1953	109,100	109,820
1954	116,000	115,420
1955	124,100	120,920
1956	127,300	124,160
1957	128,100	127,320
1958	125,300	130,940
1959	131,800	134,480
1960	142,200	138,580
1961	145,000	144,200
1962	148,600	150,800
1963	153,400	156,880
1964	164,800	162,720
1965	172,600	168,580
1966	174,200	174,540
1967	177,900	178,433

The price sequence is the sequence published by prof. J.S. Cramer¹ in his study of the price fluctuations of new cars.

The price index is determined as a weighted average of the catalogue prices of 35 types of car weighted with the sales per type in the basic year. This index figure was corrected for quality changes of the car. Thus a price index for cars of constant quality was obtained. If this index is corrected again with the price index for the cost of living, a relative price index for cars of constant quality is obtained. Both series are stated in Table 49.

Table 49: price data for the years 1950-1967

Year	Price index of cars of constant quality P_1	Moving average of price indices of cars of constant quality P'_1	Relative price index for cars of constant quality P_2	Moving average of relative price indices of cars of constant quality P'_2
1950	100,000	100,000	100,000	100,000
1951	102,900	102,300	111,300	109,567
1952	104,000	100,200	117,400	110,240
1953	101,800	97,620	114,900	111,280
1954	92,300	94,260	107,600	109,640
1955	87,100	89,600	105,200	107,060
1956	86,100	85,780	103,100	107,040
1957	80,700	83,820	104,500	108,120
1958	82,700	82,320	114,800	109,680
1959	82,500	80,500	113,000	110,940
1960	79,600	78,720	113,000	110,840
1961	77,000	75,780	109,400	108,160
1962	71,800	72,240	104,000	105,560
1963	68,600	68,600	101,400	102,820
1964	64,800	65,300	100,000	101,580
1965	61,400	62,660	99,300	101,320
1966	60,500	60,380	103,200	101,620
1967	58,600	58,567	102,700	102,933

For the sake of simplicity the tables only state the relevant results of calculations based on the series:

Cars in thousands	Income index $I' (t)$
Cars per 100 families	Price index P'_1
Cars per 100 inhabitants	Price index P'_2

1. CRAMER J.S. 'A price index of new passenger cars 1950-1965'. *Statistica Neerlandica*, no. 2, 1966.

The calculations are consequently based on the opinion expressed in paragraph 1 that the effect of price and income also determines the number of registered cars.

The results of the calculations will have to show whether this is true.

Table 50: relationship between cars in thousands and time, income and price.

$y_0 = 60$

$k = 4142$

Year	t	Actual number of cars (x 1000)	Calculated number of cars at explaining variables (x 1000)				
			t, I	t, P ₁	t, P ₂	t, I, P ₁	t, I, P ₂
1950	1	139	144	142	143	143	143
1951	2	156	153	154	154	154	154
1952	3	175	175	172	174	176	177
1953	4	201	201	197	198	201	205
1954	5	235	234	228	229	234	238
1955	6	276	274	270	268	273	276
1956	7	323	313	318	312	310	312
1957	8	370	358	369	362	355	355
1958	9	421	413	427	420	411	412
1959	10	478	476	496	488	475	477
1960	11	549	554	576	572	533	553
1961	12	638	658	678	679	657	649
1962	13	758	790	800	804	789	779
1963	14	909	938	942	949	935	919
1964	15	1086	1103	1099	1105	1099	1085
1965	16	1285	1287	1265	1271	1284	1281
1966	17	1500	1497	1444	1448	1496	1519
1967	18	1725	1663	1622	1632	1664	1679

Table 51: calculated parameters for different sets of explaining variables

Explaining variables	Calculated parameters			
	m	b	c	d
t, I	327,34019	-0,11942	-0,01788	
t, P ₁	13,05145	-0,16363		-0,01499
t, P ₂	29,13091	-0,20125		0,00722
t, I, P ₁	546,75455	-0,12272	-0,01924	-0,00364
t, I, P ₂	4048,08810	-0,05969	-0,03142	-0,01165

Table 52: *relationship between cars per 100 families and time, income and price.*

$$y_0 = 2,9 (= 2,875)$$

$$k = 80,0$$

Year	t	Actual number of cars	Calculated number of cars at explaining variables				
			t, I	t, P ₁	t, P ₂	t, I, P ₁	t, I, P ₂
1950	1	5,4	5,6	5,5	5,5	5,5	5,4
1951	2	5,9	5,8	5,8	5,8	5,8	5,8
1952	3	6,5	6,5	6,4	6,4	6,5	6,6
1953	4	7,3	7,3	7,1	7,2	7,3	7,5
1954	5	8,4	8,3	8,1	8,1	8,4	8,6
1955	6	9,7	9,6	9,4	9,4	9,6	9,7
1956	7	11,1	10,7	10,9	10,7	10,6	10,7
1957	8	12,5	12,0	12,4	12,2	11,9	11,9
1958	9	13,9	13,6	14,1	13,9	13,5	13,6
1959	10	15,5	15,3	16,1	15,8	15,3	15,4
1960	11	17,3	17,5	18,3	18,2	17,5	17,5
1961	12	19,7	20,4	21,1	21,2	20,4	20,1
1962	13	22,9	24,1	24,5	24,6	24,1	23,7
1963	14	26,9	28,0	28,2	28,4	28,0	27,4
1964	15	31,4	32,3	32,1	32,3	32,2	31,7
1965	16	36,4	36,7	36,0	36,2	36,7	36,6
1966	17	41,8	41,6	40,0	40,1	41,7	42,4
1967	18	47,1	44,9	43,8	44,1	45,1	45,5

Table 53: *calculated parameters for different sets of explaining variables.*

Explaining variables	Calculated parameters			
	m	b	c	d
t, I	348,31247	-0,09364	-0,02396	
t, P ₁	5,71217	-0,15810		0,01815
t, P ₂	14,59092	-0,20351		0,00905
t, I, P ₁	1062,09640	-0,10086	-0,02693	-0,00795
t, I, P ₂	15819,52100	-0,00292	-0,04453	-0,01770

Table 54: *relationship between cars per 100 inhabitants and time, income and price.*

$$y_0 = 0,7$$

$$k = 25$$

Year	t	Actual number of cars (x 1000)	Calculated number of cars at explaining variables				
			t, I	t, P ₁	t, P ₂	t, I, P ₁	t, I, P ₂
1950	1	1,4	1,4	1,4	1,4	1,4	1,4
1951	2	1,5	1,5	1,5	1,5	1,5	1,5
1952	3	1,7	1,7	1,7	1,7	1,7	1,7
1953	4	1,9	1,9	1,9	1,9	1,9	1,9
1954	5	2,2	2,2	2,1	2,1	2,2	2,2
1955	6	2,5	2,5	2,5	2,5	2,5	2,5
1956	7	2,9	2,9	2,9	2,9	2,8	2,9
1957	8	3,3	3,2	3,3	3,3	3,2	3,2
1958	9	3,8	3,7	3,8	3,7	3,6	3,7
1959	10	4,2	4,2	4,4	4,3	4,2	4,2
1960	11	4,8	4,8	5,0	5,0	4,8	4,8
1961	12	5,5	5,6	5,9	5,9	5,6	5,6
1962	13	6,4	6,7	6,8	6,9	6,7	6,7
1963	14	7,6	7,9	7,9	8,0	7,9	7,9
1964	15	8,9	9,1	9,1	9,1	9,1	9,1
1965	16	10,4	10,5	10,3	10,3	10,5	10,5
1966	17	12,0	12,0	11,5	11,6	12,0	12,0
1967	18	13,7	13,0	12,7	12,8	13,1	13,0

Table 55: *calculated parameters for different sets of explaining variables.*

Calculated parameters

Explaining variables	m	b	c	d
t, I	372,78431	-0,09947	-0,02307	
t, P ₁	7,25942	-0,16205		0,01729
t, P ₂	17,38834	-0,20525		0,00881
t, I, P ₁	1137,98650	-0,10668	-0,02604	-0,00794
t, I, P ₂	344,91568	-0,10101	-0,02272	0,00042

A consideration of the signs reveals that the sign of the income variable is correct. An increase of the income here gives rise to an increase of the demand. It may be assumed that the price must have the opposite effect. An increase of the price normally brakes the growth. In other words: the signs of

the coefficients b and c must be the same and the signs of the coefficients c and d must be opposed.

A comparison of tables 51, 53 and 55 yields the following survey:

cars in thousands			cars per 100 families			cars per 100 inhabitants		
sign of the coefficient in table 51 of			sign of the coefficient in table 55 of			sign of the coefficient in table 53 of		
t	I	P	t	I	P	t	I	P
+	+		+	+		+	+	
+		-	+		+	+		+
+		+	+		+	+		+
+	+	-	+	+	-	+	+	-
+	+	-	+	+	-	+	+	+

+ = correct

- = incorrect

This shows that the price has no unambiguous effect on the demand for cars.

3. TESTING THE CALCULATED RELATIONSHIPS

It is therefore necessary to check the calculated relationships for multicollinearity. This check is made in three stages.

A. First it is investigated whether multicollinearity exists indeed. For this purpose the correlation matrix $[r_{ij}]$ is determined which is constituted by the correlation coefficients of the explaining variables.

Because all coefficients lie between zero and one, the determinant $|r_{ij}|$ will also be limited by

$$0 \leq |r_{ij}| \leq 1$$

In the case of complete independence of the explaining variables the value of the determinant will be equal to one. In the case of complete dependence $|r_{ij}|$ is equal to zero.

Bartlett¹ has shown that after a certain transformation the determinant

1. BARTLETT M.S. 'Test of significance in Factor Analysis'. *British Journal of psychology*, Statistical section 3, 1950.

$|r_{ij}|$ has a chi-square distribution with a number of degrees of freedom equal to $\nu = \frac{1}{2}n(n-1)$.

If now $|r_{ij}|$ approaches to zero (one) χ^2 increases (decreases), so that this χ^2 -value is also a measure of the presence or absence of multicollinearity. The critical value used in practice is $|r_{ij}| = \text{approx. } 0,5$.

B. If the presence of multicollinearity has been ascertained, it must be investigated to which explaining variables it should be attributed. For this purpose the squared multiple correlation coefficients are calculated for each separate explaining variable, in combination with the others. These coefficients are then tested for significance with the aid of a $F(R^2 x_i, \hat{x})$ test, with $(T-n)$ and $(n-1)$ degrees of freedom. If a squared correlation coefficient is found to be statistically significant, the relative explaining variable — which was the basic variable to be explained in the determination of the multiple correlation coefficient — is the variable to which the multicollinearity must be attributed.

C. Now it must still be indicated which of the other explaining variables are affected by the variable causing the multicollinearity. To achieve this the partial correlation coefficients between the explaining variables are calculated, which coefficients are then checked for significance with the aid of a t-test. The multicollinearity pattern is now complete.

The autocorrelation is tested with the aid of the Durban Watson Statistic — $\hat{d}$ — and the value of $\hat{\rho}$, the first order correlation coefficient of the residuals.

According to Malinvaud¹ this value should not exceed 0,50.

Below only one of the relationships calculated in paragraph 2 is analysed. The other combinations of variables are considered in a similar way. These analyses are given in Appendix A.

$$\begin{array}{lll}
 z' & = & -1,74 - 0,54t + 0,20p_1 \quad (V.3.1) \\
 (t) & & (-1,89) \quad (-6,16) \quad (1,48) \quad (q = 13) \\
 (e) & & 81\% \quad 19\% \\
 F(R^2_{x_i}, \hat{x}) & & 0,22 \quad 0,22 \quad (q_1 = 1, q_2 = 14)
 \end{array}$$

1. MALINVAUD E. 'Méthodes Statistiques de l'économétrie, 1964'.

$$\begin{aligned}
s_e^2 &= 2,59 \\
R^2 &= 0,77 \\
\bar{R}^2 &= 0,73 \\
F &= 21,50 \quad (q_1 = 2, \quad q_2 = 13) \\
\hat{d} &= 0,941 \quad (N = 16, \quad K = 2) \\
\hat{\rho} &= 0,52 \\
|r_{ij}| &= 0,984 \\
\chi^2 &= 0,091 \quad (q = 1)
\end{aligned}$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	p ₁
t	1,00	-0,12
p ₁	-0,47	1,00

The critical value of $_{95}t^{13}$ is 1,771. This means that it is possible to distinguish the value of the coefficient of the explaining variable t significantly from zero, while it is impossible to do so for p₁. In other words: the time is significant as an explaining variable, whereas the price is not significant. The contribution of p₁ to the variance explained is 19%. The critical value of the F interdependence test — $_{95}t^{14}$ is 4,60. In view of the value of the t and p₁ coefficients it may be concluded that no multicollinearity is present. This is also indicated by the value of $|r_{ij}|$. It appears from the multicollinearity pattern¹ that the partial correlation coefficient between t and p₁ is only -0,12. The relevant t-value is -0,47; hence it is considerably smaller than the critical $_{95}t^{14}$ -value which is equal to 1,761. The ρ -value 0,52 indicates that a slight degree of autocorrelation exists. Malinvaud states that the value of ρ should not exceed 0,50.

The tests used² clearly show that both price and income do not contribute in any way to the increase of the total number of cars, in other words: in our model the growth is exclusively a function of time.

Consequently there is no point in including prices and income in the model as explaining variables.

1. The elements above the main diagonal are the partial correlation coefficients. The values of the t-test are given below the main diagonal.

2. For the explanation of these tests: see J. Schilderincx's thesis: Een econometrisch model van de Nederlandse economie.

*Graph 17: income per capita in
Holland and the United States.*

VI Determination of demand categories

In paragraph 1 of this chapter a description is given of the data and the changes in the counting definition. In paragraph 2 the import of second-hand cars is described. In paragraph 3 the service life is determined and the replacement demand, the expansion demand, the new car demand and the used car demand are calculated.

1. DESCRIPTION OF THE DATA AND CHANGES OF THE DEFINITION OF COUNTING

Every year on August 1st the total number of passenger cars is published in the statistics of motor vehicles issued by the Central Bureau of Statistics. This figure is based on the registration cards issued by the Government Inspectorate of Road Traffic, with the restriction, however, that the list only comprises the data of motor vehicles for which a valid road tax card exists, or a voucher for the payment of the road tax is available at the central registration office on August 1st, besides the registration card.

These totals published by the Central Bureau of Statistics for the years 1954-1966 are given in Table 56.

For 1967 the only total figure known is a revised estimate, while for 1968 only a course estimate was made.

Up to 1952 the statistics did not provide any definition of counting. The description given in 1952 was:

'The count includes . . . passenger cars comprising a group of . . . motor vehicles for the transport of maximum 9 persons including the driver. In so far as a licence has been issued for passenger cars in accordance with the Passenger Transport in Cars Act proving that these motor vehicles are intended for paid passenger transport, a special form is used for the tax declaration'.

This renders it possible to publish separate figures for taxis included in this

category (both the taxis in the proper sense of the word — with fixed stand, serving as rented cars for traffic with driver). These taxis have also been included in Table 56.

This definition was used till 1957. In 1958 the statistics concerning motor vehicles were not published. In 1959 the above definition was modified as follows:

‘. passenger cars including motor vehicles (also the three-wheeled vehicles) suitable for the transport of maximum 9 persons including the driver. This category also comprises stationcars, i.e. the dual-purpose cars with a registration card for passenger cars, for which the Government Inspectorate for Road Traffic has not recorded any loading capacity’.

Following this the category lorries and delivery vans was made to include all dual-purpose cars for which the Government Inspectorate for Road Traffic has recorded a loading capacity irrespective of whether these vehicles are provided with a registration number for a passenger car or a lorry.

This definition of counting was maintained until 1962. Only then an end was put to the situation in which a car with a registration number for a passenger car was counted among the lorries. All dual-purpose cars with a registration number for passenger cars have since that time been included in the category passenger cars.

This course of affairs can be schematically represented as follows:

To make the data published by the Central Bureau of Statistics since August 1st, 1962, comparable with those published before August 1st, 1962, all numbers published were corrected with the cooperation of the Central Bureau of Statistics.

Dual-purpose cars included in the category lorries in the period 1957-1962 were transferred to the category passenger cars. The correct numbers, in so far as correction was still possible, are given in Table 57. This table also shows an estimate for 1958 of the distribution according to age. For that year the Central Bureau of Statistics only published the total number of cars: 421 000.

Table 56: number of passenger cars on August 1st according to figures published every year by the Central Bureau of Statistics.

	1967	1966	1965	1964	1963	1962	1961	1960	1959	1958	1957	1956	1955	Before 1955	Total
1967															1725000
1966		126432	301027	228429	189171	143039	118733	99915	77127	53359	50352			114642 ¹	1502226
1965			167509	224418	185721	141172	118129	99453	78429	55133	57161	45743		100030 ²	1272898
1964				152706	182497	139513	114565	94811	74581	53145	59517	51915	45699	90117	1059066
1963					111053	136808	112025	91631	71445	51483	60687	58179	53708	118497	865516
1962						92718	114991	93056	71682	50156	59715	57519	54368	135446	729651
1961							75707	91412	69353	46469	58277	56869	54148	150258	602493
1960								60662	70166	46384	58213	56810	54198	165879	512312
1959									44344	48132	60589	57988	55659	183679	450391
1958										—	—	—	—		—
1957											40111	61207	59657	214701	375676
1956												42199	60134	225133	327466
1955													35360	232528	267888
1954															

1. Before 1957

2. Before 1956

Table 57: number of cars corrected for a change in the definition of counting on August 1st.

[illegible]

With the aid of information on production, imports, exports, both of new and second-hand cars for the years 1956, 1957, 1958 and 1959 the distribution according to age was built up for the year 1958.

2. IMPORT OF SECOND-HAND CARS

The import of second-hand cars distorts the statistical information that must be used to determine the mortality rate and hence the average service life of the total number of cars on the road.

If the numbers stated in Table 57 should not be corrected for this import the incredible phenomenon would occur that — in spite of the mortality — there would be more cars of a certain age in a specific year than in the previous year and this cannot be reconciled with the results of a mortality process.

The number of second-hand cars imported in the years 1954-1967 and published in the statistics concerning the import, export and transit is shown in Table 58. In this table the calendar year has intentionally been divided into two periods, viz.: January — July and August — December, as the total number of cars in use is published for the situation on August 1st. Correction

Table 58: imported second-hand cars per calendar year and per shifted year.

Year	Jan.(t) — July(t)	Aug.(t) — Dec.(t)	Jan.(t) — Dec.(t)	Aug.(t) — July(t + 1) ¹
1954	1317	807	2124	
1955	1734	1255	2989	2541
1956	2954	1504	4458	4209
1957	2506	1679	4185	4010
1958	2732	1705	4437	4411
1959	4256	3036	7292	5961
1960	6929	4448	11377	9965
1961	8425	6002	14428	12873
1962	14791	11776	26567	20793
1963	22479	15890	38369	34255
1964	30347	18573	48920	46237
1965	29635	20088	49723	48208
1966	35084	24115	59199	55172
1967	33966	22410	56376	55605
1968	34510	18092	52602	56920

1. This period relates to August of the preceeding year to Juli of the year after which the figures are stated.

of this number with the number of imported second-hand cars on the last day of the year would introduce an unnecessary inaccuracy.

Consequently, Table 58 also concerns the vehicles which crossed the border — however battered they may have been — and the vehicles intended for the scrap heap or for the supply of parts.

The imported second-hand cars included in the total number present on August 1st published by the Central Bureau of Statistics also received a registration number, however, from the Government Inspectorate for Road Traffic. No statistics had been published up to 1965 for this category.

Yet, the Central Bureau of Statistics had old file cards which had never been processed nor published — on which the life history of the imported, second-hand, registered cars was recorded. By courtesy of the Central Bureau of Statistics these cards could be used for this study. The analysis yields the distribution given in Table 59. (pagina 105)

With the aid of Table 59 it can now be calculated how large the number of imported second-hand, registered cars was in the shifted years (August-July).

Table 60 also gives these numbers as a percentage of the second-hand cars imported in the years 1960-1968.

Table 60: imported second-hand registered cars in 1960-1968.

	Imported second-hand registered cars		As a percentage of the imported second-hand cars	
	Jan.(t) – Dec.(t)	Aug. (t-1) – July (t)	Calendar years	Shifted years
1960	8676		76,2	
1961	13374	10177	92,0	79,0
1962	19962	16290	75,1	78,2
1963	28310	24435	73,7	71,3
1964	39437	37342	80,6	80,7
1965	41781	40275	84,0	83,8
1966	48603	45960	82,1	83,3
1967	49383	49138	86,7	88,4
1968	48842	48953	92,9	86,0

Table 59: distribution according to age of the imported, second-hand, registered cars imported during the period 1960-1968.

Period ¹	Total	Age																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	rest
601	6605	0	121	710	688	880	854	814	687	661	586	304	177	58	17	4						44
602	2071	0	215	179	160	209	278	227	211	219	166	109	65	19	2	5						7
611	8106	0	1071	816	706	797	982	957	927	613	531	312	232	99	34	8	0					21
612	5268	0	645	471	477	545	682	661	624	382	350	212	132	50	7	7	4					19
621	11022	0	916	980	852	1032	1223	1435	1502	1128	688	594	317	216	88	24	6	2				19
622	8940	0	708	762	734	1005	1126	1328	1178	790	518	318	275	130	35	7	4	1				21
631	15495	0	1	1186	1404	1552	1992	2175	2364	1813	1224	779	465	273	158	72	12	4	0			21
632	12815	0	213	956	1256	1569	1951	1911	1762	1299	901	442	281	139	79	30	8	3	1			14
641	24527	1	1790	1986	1957	2863	3158	3733	3111	2320	1581	1000	494	250	150	64	30	8	4	1		26
642	14910	1	1463	1308	1101	2092	2220	2300	1725	1206	705	386	201	92	38	25	7	4	1	0		35
651	25365	0	2213	2570	2538	3210	3761	3436	3414	1947	1075	550	335	146	80	28	24	8	1	1	0	28
652	16416	1032	1855	2032	1861	2094	2340	1914	1469	843	440	242	154	62	27	14	7	2	0	0	0	28
661	29544	0	2445	3629	3991	3893	4226	4304	2855	2240	977	450	242	143	149							0
662	19059	0	2324	2351	2712	2654	2734	2675	1455	1166	495	210	144	139								0
671	30079	0	2681	3745	4021	4477	4045	4213	3192													3705
672	19304	0	3080	2677	2761	3017	2476	2399	1417													1477
681	29649	0	2374	4617	4451	4603	4663	3529	2690	1404												1318
682	19193	0	2992	3317	3127	3066	2354	1814	1036	620												867

1. The first two figures of this column relate to the year in question, the third figure relates to the months Jan.—July(1) and Aug.—Dec.(2).

The percentage for the calendar year 1961 (viz. 92%) is very high. One might think that this maverick is the consequence of imperfect statistical information. A simple calculation shows the origin of these 92%.

	1960		1961		1962
	Jan. 1st	Aug. 1st	Jan. 1st	Aug. 1st	Jan. 1st
a. imported second-hand	6929	4448	8425	6002	
b. of which registered	6605	2071	8106	5260	
b/a	95,3%	46,6%	96,2%	87,8%	
a. imported second-hand cars per calendar year		11377		14428	
b. of which registered		8676		13374	
b/a		76,2%		92,0%	
a. imported second-hand cars per shifted year			12873		
b. of which registered			10177		
b/a			79,0%		

An informal statement issued by the Government Inspectorate for Road Traffic mentioned a total number of 8 672 registration numbers issued for imported second-hand cars in the year 1960, so that our count of 8 676 tallies well.

Consequently the obvious remark that a number of cars relating to the second period of 1960 would no longer be present for some reason or other does not hold. It should rather be assumed that government measures published at the end of 1961 which were to take effect on January 1st, 1962, have been the cause of the relatively high number of imported second-hand registered cars in the second period of 1961.

Let us assume that 75% of the second-hand cars imported before 1960 was registered. If this percentage is applied to the years 1955-1959, the number of imported, second-hand, registered cars can also be reasonably estimated for these years.

Table 61: numbers of second-hand cars imported in 1954-1959 and the numbers of imported, second-hand, registered cars calculated from these figures.

Year	Imported second-hand cars		imported second-hand registered cars	
	Jan. - July	Aug. - Dec.	75% of column 1	75% of column 2
1954	1317	807	988	605
1955	1734	1255	1301	941
1956	2954	1504	2216	1128
1957	2506	1679	1880	1259
1958	2732	1705	2049	1279
1959	4256	3036	3192	2277

How now are the imported, second-hand, registered cars distributed over the years 1954-1959 according to age.

To obtain a rough impression the percentage distribution of the age groups is first determined for the years 1960-1966. These percentages are given in Table 62.

It is now possible to derive estimates of the number of cars imported in the years 1954-1960 from Table 62. For this purpose the average is taken of the percentages belonging to an equal age; this is done for the three years' period starting in 1960. In this case as well the year is divided into two periods. The three-year period is an arbitrary one. It could also have been five years. The results might have been slightly different in that case.

It should now be noted that the sum of the average percentages calculated in this way for equal ages need not be equal to 100.

For simplicity's sake, however, the sum thus obtained has been used as conversion factor with respect to 100 to correct the percentages. The calculated percentages are given in Table 63.

On the basis of these percentages the distribution according to age of the imported cars is determined for the years 1954-1959, also distributed over the months January-July and August-December. The calculated distribution according to age is given in Table 64.

Table 62: *percentage distribution of imported, second-hand, registered cars according to age.*

Period ¹	Total	Age																					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	rest	
601	6605	0	1,83	10,74	10,38	13,32	12,92	12,32	10,40	10,00	8,87	4,60	2,67	0,87	0,25	0,06							0,77
602	2071	0	10,38	8,64	7,72	10,09	13,42	10,96	10,18	10,57	8,01	5,26	3,13	0,91	0,10	0,24							0,63
611	8106	0	13,21	10,06	8,70	9,83	12,11	11,80	11,43	7,56	6,55	3,84	2,86	1,22	0,41	0,09	0						0,33
612	5268	0	12,24	8,94	9,55	10,35	12,95	12,55	11,85	7,25	6,64	4,02	2,51	0,95	0,13	0,13	0,08						0,36
621	11022	0	8,31	8,89	7,73	9,36	11,10	13,02	13,63	10,23	6,24	5,39	2,88	1,96	0,80	0,22	0,05	0,02					0,17
622	8940	0	7,92	8,52	8,21	11,24	12,60	14,85	13,18	8,84	5,79	3,56	3,08	1,45	0,39	0,08	0,04	0,01					0,24
631	15495	0	0,01	7,65	9,06	10,02	12,86	14,04	15,26	11,70	7,90	5,03	3,00	1,76	1,02	0,46	0,08	0,03	0				0,12
632	12815	0	1,66	7,46	9,80	12,24	15,22	14,91	13,75	10,14	7,03	3,45	2,19	1,08	0,62	0,23	0,06	0,02	0,01				0,13
641	24527	0	7,30	8,10	7,98	11,67	12,88	15,22	12,68	9,46	6,45	4,08	2,01	1,02	0,61	0,26	0,12	0,03	0,02	0			0,11
642	14910	0,01	9,81	8,77	7,38	14,03	14,89	15,43	11,57	8,09	4,73	2,59	1,35	0,62	0,25	0,17	0,05	0,03	0,01	0			0,23
651	25365	0	8,72	10,13	10,00	12,66	14,83	13,55	13,46	7,68	4,24	2,17	1,32	0,58	0,32	0,11	0,09	0,03	0	0	0	0	0,11
651	16416	6,29	11,30	12,38	11,34	12,76	14,25	11,66	8,95	5,14	1,68	1,47	0,94	0,38	0,16	0,09	0,04	0,01	0	0	0	0	0,16
661	29544	0	8,28	12,28	13,51	13,18	14,30	14,57	9,66	7,58	3,31	1,52	0,82	0,48	0,50								0,17
662	19059	0	12,19	12,34	14,23	13,93	14,34	14,04	7,63	6,12	2,60	1,10	0,76	0,73									0,01
671	30079	0	8,91	12,45	13,37	14,88	13,45	14,01	10,61														12,32
672	19304	0	15,96	13,87	14,30	15,63	12,83	12,43	7,34														7,55
681	29649	0	8,01	15,57	15,01	15,52	15,73	11,90	9,07	4,73													4,46
682	19193	0	15,59	17,28	16,29	15,97	12,26	9,45	5,40	3,23													4,53

1. The first two figures of this column relate to the relevant year; the third figure relates to the months Jan. — July (1) and Aug. — Dec. (2).

Table 63: average triennial percentages of imported, second-hand, registered cars according to age.

Year	Calculated		Corrected	
	Import Jan. – July	Import Aug. – Dec.	Import Jan. – July	Import Aug. – Dec.
1	0,00	0,00	0,00	0,00
2	7,78	10,18	7,79	9,63
3	9,90	14,56	9,92	13,74
4	8,94	8,49	8,96	8,03
5	10,84	10,50	10,86	9,98
6	12,04	12,99	12,06	12,28
7	12,38	12,78	12,40	12,08
8	11,82	11,73	11,84	11,09
9	9,26	8,88	9,28	8,40
10	7,22	6,81	7,23	6,44
11	4,61	4,28	4,62	4,05
12	2,80	2,90	2,81	2,74
13	1,35	1,10	1,35	1,04
14	0,49	0,21	0,49	0,20
15	0,12	0,07	0,12	0,07
<15	0,27	0,22	0,27	0,21
Total	99,82	105,76	100,00	100,00

Table 64: age structure of the imported, used, registered cars imported in the period 1954-1959.

Period ¹	Total	Age															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
541	988	0	77	98	89	107	119	122	117	92	71	46	28	13	5	1	3
542	605	0	58	84	49	60	74	73	67	51	39	25	17	6	1	0	1
551	1301	0	101	129	117	141	157	160	154	121	94	60	37	18	6	2	4
552	941	0	91	129	76	94	115	113	104	79	61	38	26	10	2	1	2
561	2216	0	173	220	199	241	267	274	262	206	160	102	62	30	11	3	6
562	1128	0	109	154	91	112	139	136	125	95	73	46	31	12	2	1	2
571	1880	0	146	186	168	204	228	234	223	174	136	87	53	25	9	2	5
572	1259	0	121	172	101	126	155	152	140	106	81	51	34	13	3	1	3
581	2049	0	160	103	184	223	246	253	243	190	148	95	58	28	10	2	6
582	1279	0	123	176	102	128	157	155	142	107	82	52	35	13	3	1	3
591	3192	0	249	317	286	347	385	395	377	296	231	147	90	43	16	4	9
592	2277	0	219	313	183	227	280	275	252	191	147	92	62	24	5	2	5

1. See remark table 62.

Table 59 already gives the age groups of the imported, used, registered cars as from 1960.

Table 64 gives the age structure of these cars for the years 1954-1959.

A combination of these tables thus yields the age structure of imported, used, registered cars from 1954 up to and including 1968.

3. DETERMINATION OF THE MORTALITY RATES AND THE CALCULATION OF THE SUBDIVISION OF TOTAL DEMAND

For the calculation of the "mortality" percentages use is made of the data made available both by the Central Bureau of Statistics and by the Government Inspectorate for Road Traffic and the Car and Bicycle Manufacturers' Association; they are given in Tables 65 and 66 together with previously processed data, both for calendar years and for shifted years.

Table 65: data for calculating the mortality rates (per calendar year).

Year	January – December									
	A	B	C	D	E	F	G	H	I	J
1955	18249	3503	49243	63989	63500	2242	—	240	2242	2989
1956	21361	4249	56474	73586	68896	3344	—	166	3344	4458
1957	15814	3485	52538	61926	65243	15793	197706	223	3139	4185
1958	15811	8030	43703	51516	56188	8149	220804	122	3328	4437
1959	28339	8894	59074	78590	76847	10886	264904	89	5469	7292
1960	46565	16264	70816	98862	100154	13068	322991	94	8676	11377
1961	40337	12785	91775	120790	120195	15580	378867	78	13374	14428
1962	43111	13423	122594	148008	146031	17871	452665	147	19962	26567
1963	42226	13012	152174	187911	181853	33085	582443	238	28310	38369
1964	60925	29409	194716	229817	226090	42293	705718	291	39437	48920
1965	59801	27304	264030	307062	303789	43287	833676	297	41781	49723
1966	54499	26474	197275	223063	213389	52030	850821	320	48603	59199
1967	75036	34427	223930	266784	263678	58619	1016250	1169	49383	56376
1968	92000	51670	292239	333303	334675	58430	1251395	1835	48842	52602

Column A = Production

Column B = Export of new cars

Column C = Import of new cars

Column D = Registration card, part I,
accelerated procedure

Column E = Additions to the total
number of cars

Column F = Registration card, part I,
on application

Column G = Registration under a name

Column H = Export of second-hand cars

Column I = Import of second-hand,
registered cars

Column J = Import of second-hand cars

Table 66: data for calculating the mortality rates.

Year	January – July							Shifted year (Aug. – July)		
	A	B	C	D	E	F	G	H	I	J
1955	11029	2664	29611	37976	35717	1906	—	272	1906	2541
1956	10603	2620	36982	44965	43961	3157	—	202	3157	4209
1957	10531	2471	32035	37200	41299	11778	118424	217	3008	4010
1958	10640	5289	26502	32017	35276	5137	136821	142	3308	4411
1959	15117	5348	37969	47803	47019	6394	161010	100	4471	5961
1960	30612	10935	44530	63515	63015	8272	199786	89	7474	9965
1961	25543	8435	61630	81204	79074	10207	242339	84	10177	12873
1962	25661	9027	79795	96258	95697	13633	283347	108	16290	20793
1963	24197	7131	95500	120897	116719	19326	351849	207	24435	34255
1964	35346	15399	136508	159535	154811	27159	453396	263	37342	46237
1965	35954	16565	150421	177226	173381	27330	494834	327	40275	48208
1966	31733	15302	120289	135973	130002	32435	530050	260	45960	55172
1967	40175	18540	139785	168392	167386	35790	626154	776	49138	55605
1968	49167	29539	190390	221959	214985	35968	799541	1477	48953	56920
1969	—	26117	204232	231597	226152	21318	641324	1658	—	28783

Column A = Production

Column B = Export of new cars

Column C = Import of new cars

Column D = Registration card, part I,
accelerated procedure

Column E = Additions to the total
number of cars

Column F = Registration card, part I,
on application

Column G = Registration under a name

Column H = Export of second-hand cars

Column I = Import of second-hand,
registered cars

Column J = Import of second-hand cars

In Appendix (B) the calculation is given by means of which the mortality rates have been determined. The following remarks could be made concerning this point.

All passenger cars manufactured or assembled in the Netherlands are newly produced, i.e. they leave the factory or the assembler's premises as new cars.

If this number is reduced by the exported new or assembled cars, the remainder is available for the home market.

If the number of imported new cars is added to this remainder, the sum is the total number of cars available for the home market.

The manufacturer and importer receive the so-called part I of the registration card for every manufactured or imported new car.

Consequently the registration card in principle relates to a specific car and is only issued if in the view of the Government Inspectorate for Road Traffic

— the authority issuing the registration cards — the relevant car has proved to comply with the legal regulations and if it has been proved that the customs duties have been paid.

The registration card consists of part I and part II. In the first part the registration number and the identification data of the car are stated and this part is issued shortly after the car has been manufactured in the Netherlands, or has been imported into that country. This means that the car is already provided with a registration number before it is sold to the eventual owner.

In part II the registration number, the name and the address of the owner of the car are stated. This part is issued on behalf of the Government Inspectorate by the Commissioners of Customs and Excise. In case of change of ownership the new owner applies for a new part II with one of the Commissioners of Customs and Excise and then submits part I. The inside of part II is used as an application form, so that its validity as part II is then cancelled. Consequently the car retains its original registration number.

The registration of cars appearing in the records is laid down in Section 93 of the Highway Code. This section only stipulates that the cars must be entered in a register. This registration takes place in a central card file. A copy of part I (technical description) and the application form used to obtain part II — the so-termed registration under a name — are arranged together in the sequence of the registration numbers in the central card file.

Car manufacturers and importers receive registration cards for their manufacturer's and commercial stocks. Consequently such cards are not issued for a specific motor vehicle. Their validity expires after five years.

The issue of part II to new cars — in which case the new car is registered under a name for the first time — thus means that the car is added to the total number of cars in use.

The "scrap" percentages calculated in Appendix (B) are shown in Table 67.

Table 67: "scrap" percentages per year

Year	Age										
	1	2	3	4	5	6	7	8	9	10	11
1955	5,55	1,27	3,35	4,77	4,49	2,28	3,46	6,36	19,24	21,05	22,08
1956	11,34	3,30	3,28	3,80	1,94	2,48	4,95	16,61	15,59	16,69	
1957	4,43	2,92	5,21	1,24	1,85	3,69	9,42	9,53	14,63		
1958	8,86	5,20	1,49	2,17	3,14	7,03	6,33	9,31			
1959	4,29	2,58	3,24	3,41	2,08	2,25	7,29				
1960	5,84	4,00	3,77	0,98	1,20	5,82					
1961	5,05	4,15	0,32	0,62	4,59						
1962	6,31	-0,36	1,52	2,64							
1963	0,62	0,43	1,34								
1964	1,70	0,64									
1965	2,03										

If now the triennial average of these percentages is again determined, a reasonable picture is obtained of the "mortality" from year to year. As unfortunately the available data do not permit of a calculation of the mortality percentages for more than 11 years, they are adapted in "stylized" form, in which way a certain arbitrary element is of course introduced. But on the basis of the percentages calculated for the first 11 years the percentages for the second period of 10-11 years can be estimated by graph with a reasonable degree of certainty. It is in this case assumed that the first maverick per year — for 1955 this is for instance 5,55% — is a statistical error.

These "mortality" percentages are given in Table 68 and Graph 18.

Table 68: mortality rates.

Age	Mortality rates
0	0,020300
1	0,026500
2	0,039600
3	0,064900
4	0,107800
5	0,159800
6	0,221000
7	0,293500
8	0,382000
9	0,484000
10	0,587500
11	0,688000
12	0,777000
13	0,849500
14	0,905500
15	0,945000
16	0,970500
17	0,985500
18	0,993500
19	0,997450
20	0,999150
21	0,999770
22	0,999951
23	0,999993
24	0,999999
25	1,000000

These mortality rates can now be used for calculating:

- a. the total demand
- b. the expansion demand
- c. the replacement demand
- d. the demand for new cars
- e. the demand for used cars.

It is clear that, according as the saturation level is approached more closely, the expansion demand will decrease, while the replacement demand will increase.

The components a, b, c, d and e are – for the time series 1950-1967 and for the extrapolated series 1950-1968, 1950-1969, 1950-1970 and 1950-1971 – given in Tables 69, 70, 71, 72 and 73.

The forecasts up to 1994 are only made for the category cars in thousands.

The graphical representation of Table 73 is given in graph 19.

Table 69: subdivision of the total demand based on the time series 1950-1967.

Year	Total demand	Expansion demand	Replacement demand	Demand for new cars	Demand for used cars
1972	396 503	201 893	185 610	307 707	79 796
1973	400 312	191 683	208 629	314 765	85 547
1974	413 452	178 595	234 857	322 547	90 904
1975	422 581	163 506	259 075	326 772	95 810
1976	428 208	147 305	281 903	328 980	100 229
1977	433 015	130 799	302 216	328 862	104 153
1978	435 437	114 656	320 781	327 844	107 592
1979	438 488	99 376	339 112	327 914	110 574
1980	440 779	85 292	355 487	327 646	113 133
1981	444 064	72 591	371 473	328 753	115 310
1982	447 419	61 339	386 080	330 268	117 150
1983	450 651	51 518	399 133	331 955	118 696
1984	453 874	43 050	410 824	333 886	119 988
1985	456 479	35 821	420 658	335 417	121 062
1986	458 443	29 700	428 743	336 490	121 953
1987	459 690	24 553	435 137	337 000	122 690
1988	460 355	20 249	440 106	337 057	123 297
1989	460 661	16 666	443 995	336 863	123 797
1990	460 743	13 693	447 050	336 535	124 208
1991	460 787	11 236	449 551	336 242	124 545
1992	460 873	9 210	451 663	336 051	124 821
1993	461 065	7 542	453 523	336 016	125 048
1994	461 358	6 172	455 186	336 124	125 233

Table 70: subdivision of the total demand based on the time series 1950-1968.

Year	Total demand	Expansion demand	Replacement demand	Demand for new cars	Demand for used cars
1972	429 903	239 888	190 015	344 108	85 795
1973	448 341	232 253	216 088	355 578	92 762
1974	465 190	220 552	244 638	365 810	99 379
1975	477 941	205 625	272 316	372 394	105 548
1976	487 498	188 450	299 048	376 297	111 201
1977	494 449	170 014	324 435	378 148	116 302
1978	499 491	151 220	348 271	378 652	120 838
1979	504 280	132 813	371 467	379 458	124 823
1980	508 369	115 358	393 011	380 086	128 284
1981	512 686	99 235	413 451	381 425	131 261
1982	516 938	84 658	432 280	383 138	133 800
1983	520 971	71 712	449 259	385 019	135 952
1984	524 779	60 381	464 398	387 016	137 763
1985	527 976	50 584	477 392	388 695	139 281
1986	530 490	42 197	488 293	389 943	140 547
1987	532 293	35 076	497 217	390 694	141 599
1988	533 476	29 070	504 406	391 006	142 471
1989	534 223	24 035	510 188	391 031	143 192
1990	534 670	19 831	514 839	390 883	143 787
1991	534 985	16 335	518 650	390 708	144 277
1992	535 370	13 438	521 832	390 590	144 680
1993	535 593	11 041	524 552	390 581	145 012
1994	535 973	9 064	526 909	390 689	145 283

Table 71: subdivision of the total demand based on the time series 1950-1969.

Year	Total demand	Replacement demand	Expansion demand	Demand for new cars	Demand for used cars
1972	487 462	292 772	194 690	394 628	92 835
1973	515 581	291 495	224 086	414 001	101 580
1974	540 234	284 676	255 558	430 113	110 120
1975	560 313	272 825	287 488	442 008	118 305
1976	576 011	256 783	319 228	450 002	126 008
1977	588 703	237 606	351 097	455 566	133 137
1978	598 468	216 424	382 044	458 838	139 629
1979	606 720	194 318	412 402	461 261	145 459
1980	613 989	172 233	441 756	463 363	150 626
1981	620 563	150 920	469 643	465 409	155 154
1982	626 812	130 924	495 888	467 730	159 081
1983	632 669	112 592	520 077	470 210	162 459
1984	638 043	96 103	541 940	472 701	165 342
1985	642 740	81 506	561 234	474 952	167 787
1986	646 627	68 750	577 877	476 778	169 850
1987	649 697	57 726	591 971	478 115	171 582
1988	651 993	48 283	603 710	478 964	173 030
1989	653 653	40 256	613 397	479 415	174 238
1990	654 827	33 472	621 355	479 585	175 242
1991	655 685	27 769	627 916	479 611	176 075
1992	656 372	22 996	633 376	479 607	176 765
1993	656 984	19 013	637 971	479 649	177 335
1994	656 581	15 701	641 880	479 774	177 806

Table 72: *subdivision of the total demand based on the time series 1950-1970.*

Year	Total demand	Expansion demand	Replacement demand	Demand for new cars	Demand for used cars
1972	529 384	331 239	198 145	431 420	97 964
1973	565 426	335 427	229 999	457 399	108 027
1974	596 833	333 247	263 586	478 809	118 024
1975	623 508	324 863	298 645	495 738	127 770
1976	644 974	310 890	334 084	507 877	137 097
1977	663 089	292 298	370 791	517 223	145 866
1978	677 352	270 281	407 071	523 377	153 974
1979	688 943	246 101	442 842	527 585	161 357
1980	699 139	220 961	478 178	531 153	167 986
1981	707 770	195 903	511 867	533 907	173 863
1982	715 766	171 755	544 011	536 749	179 016
1983	723 180	149 111	574 069	539 691	183 489
1984	729 901	128 354	601 547	542 561	187 340
1985	735 910	109 678	626 232	545 281	190 630
1986	741 039	93 132	647 907	547 615	193 424
1987	745 291	78 663	666 628	549 506	195 784
1988	748 653	66 141	682 512	550 884	197 768
1989	751 213	55 403	695 810	551 782	199 431
1990	753 118	46 261	706 857	552 300	200 818
1991	754 530	38 525	716 005	552 555	201 974
1992	755 621	32 012	723 609	552 686	202 934
1993	756 517	26 551	729 966	552 787	203 731
1994	757 315	21 988	735 327	552 924	204 391

Table 73: subdivision of the total demand based on the time series 1950-1971.

Year	Total demand	Expansion demand	Replacement demand	Demand for new cars	Demand for used cars
1972	575 985	374 583	201 402	472 828	103 157
1973	622 353	386 725	235 628	507 594	114 759
1974	663 436	391 975	271 460	536 917	126 518
1975	699 763	389 950	309 812	561 545	138 217
1976	730 074	380 798	349 276	580 433	149 641
1977	756 443	365 177	391 265	595 846	160 596
1978	777 747	344 155	433 591	606 826	170 921
1979	794 905	319 067	475 837	614 412	180 493
1980	809 830	291 339	518 491	620 597	189 233
1981	822 025	262 347	559 678	624 921	197 103
1982	832 986	233 299	599 687	628 883	204 102
1983	842 971	205 166	637 805	632 713	210 257
1984	851 932	178 664	673 268	636 314	215 617
1985	860 053	154 260	705 793	639 808	220 245
1986	867 165	132 209	734 956	642 954	224 211
1987	873 286	112 596	760 690	645 696	227 589
1988	878 348	95 375	782 973	647 897	230 450
1989	882 377	80 420	801 957	649 513	232 863
1990	885 504	67 548	817 955	650 614	234 890
1991	887 876	56 553	831 323	651 290	236 586
1992	889 700	47 220	842 479	651 697	238 003
1993	891 132	39 337	851 795	651 949	239 183
1994	892 324	32 709	859 615	652 159	240 164

Graph 17: income per capita in Holland and the United States.

Graph 18: mortality rates of the cars.

Graph 19: subdivision of total demand, based on the time series 1950–1971.
demand $\times 10^5$

Appendices

APPENDIX A

Cars in thousands.

$$\begin{aligned} z' &= -1,93 - 0,57 t - 0,076 i \\ (t) & \quad (-1,71) \quad (-5,98) \quad (-0,52) \quad (q = 13) \\ (e) & \quad \quad \quad 92\% \quad \quad 8\% \\ F(R^2_{x_i, \hat{x}}) & \quad \quad \quad 0,37 \quad \quad 0,37 \quad \quad (q_1 = 1, q_2 = 14) \end{aligned}$$

$$\begin{aligned} s^2_{\epsilon} &= 2,96 \\ R^2 &= 0,73 \\ \overline{R}^2 &= 0,69 \\ F &= 17,98 \quad (q_1 = 2, q_2 = 13) \\ \hat{d} &= 0,716 \quad (N=16, K=2) \\ \hat{\rho} &= 0,64 \\ |r_{ij}| &= 0,974 \\ \chi^2 &= 0,152 \quad (q = 1) \end{aligned}$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i
t	1,00	-0,16
i	-0,61	1,00

The comparison of the critical t-values with the calculated values reveals that t is significant and i is not. There is no multicollinearity between t and i. The tvalue of F is 4,60; the calculated values – only 0,37 – are considerably lower. The value of the determinant, 0,974, leads to the same conclusions, as do the elements of the multicollinearity pattern. The ρ -value indicates that autocorrelation exists, as the calculated value exceeds the critical alue of 0,50.

$$\begin{aligned}
 z' &= -2,26 - 0,56t + 0,078 p_2 \\
 (t) & \quad (-2,53) \quad (-6,01) \quad (0,71) \quad (q = 13) \\
 (e) & \quad \quad \quad 89\% \quad 11\% \\
 F(R^2_{x_i, \hat{x}}) & \quad \quad \quad 0,01 \quad 0,01 \quad (q_1 = 1, q_2 = 14)
 \end{aligned}$$

$$\begin{aligned}
 s_e^2 &= 2,91 \\
 R^2 &= 0,77 \\
 \bar{R}^2 &= 0,70 \\
 \bar{F} &= 18,41 (q_1 = 2, q_2 = 13) \\
 \hat{d} &= 0,76 (N = 16, K = 2) \\
 \hat{\rho} &= 0,62 \\
 |r_{ij}| &= 0,999 \\
 \chi^2 &= 0,003 (q = 1)
 \end{aligned}$$

r_{ij} t_{ij}	t	p ₂
t	1,00	0,02
p ₂	0,09	1,00

In this case as well the comparison of the calculated t-coefficient of t and p₂ with the critical value of t shows that t is significant, while p₂ is not. There is no multicollinearity, which is proved by the value of the determinant and the multicollinearity pattern. The ρ -value, which is 0,62, indicates the existence of autocorrelation between the residues.

$$\begin{aligned}
 z' &= -1,82 - 0,54t + 0,025i + 0,21p_1 \\
 (t) & \quad (-1,65) \quad (-5,71) \quad (0,15) \quad (1,32) \quad (q=12) \\
 (e) & \quad \quad \quad 78\% \quad 2\% \quad 20\% \\
 F(R^2_{x_i, \hat{x}}) & \quad \quad \quad 0,51 \quad 2,08 \quad 1,99 \quad (q_1=2, q_2=13)
 \end{aligned}$$

$$\begin{aligned}
 s^2_e &= 2,80 \\
 R^2 &= 0,768 \\
 \bar{R}^2 &= 0,710 \\
 F &= 13,26 \quad (q_1=3, \quad q_2=12) \\
 \hat{d} &= 0,94 \quad (N=16, \quad K=3) \\
 \hat{\rho} &= 0,52 \\
 |r_{ij}| &= 0,746 \\
 \chi^2 &= 1,673 \quad (q=3)
 \end{aligned}$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i	p ₁
t	1,00	-0,24	-0,22
i	-0,90	1,00	-0,47
p ₁	-0,81	-1,93	1,00

The comparison of the calculated t-values and the critical value which at $95t^{12}$ is equal to 1,782 shows that both price and income are insignificant. In other words: the explaining variable time – which in fact explains 78% of the variance – is the only significant variable. The multicollinearity pattern moreover shows that multicollinearity exists between the variable p_1 and i : the calculated t-value (-1,93) is greater than $95t^{13}$ which is equal to 1,771. In view of the fact that a critical limit of 0,50 was assumed, it must be stated that autocorrelation is present, as exceeds this critical value.

$$z' = -2,17 - 0,56t - 0,020i + 0,069p_2$$

(t) (-1,70) (-5,66) (-0,10) (0,47) (q = 12)

(e) 89% 2% 9%

$F(R^2_{x_i}, \hat{x})$ 0,33 4,38 4,10 (q₁ = 2, q₂ = 13)

$$s_e^2 = 3,15$$

$$R^2 = 0,74$$

$$\bar{R}^2 = 0,67$$

$$F = 11,34 (q_1 = 3, q_2 = 12)$$

$$\hat{d} = 0,762 (N = 16, K = 3)$$

$$\hat{\rho} = 0,61$$

$$|r_{ij}| = 0,592$$

$$\chi^2 = 2,946 (q = 3)$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i	p ₂
t	1,00	-0,22	-0,15
i	-0,81	1,00	-0,62
p ₂	-0,56	-2,86	1,00

Here, too, the calculation shows that the variable t is significant, while the variables i and p₂ are not; furthermore the multicollinearity pattern shows that multicollinearity exists between p₂ and i. The calculated t-value is -2,86 and the critical value is 1,771. Another conclusion is that autocorrelation exists. The value of ρ (0,61) is greater than 0,50.

Cars per 100 families

$z' = -1,00 - 0,91 t - 0,096 i$
(t) (-0,55) (-5,98) (-0,41) (q = 13)
(e) 94% 6%
 $F(R^2_{x_i, \hat{x}})$ 0,37 0,37 (q₁ = 1, q₂ = 14)

$s^2_{\epsilon} = 7,66$
 $R^2 = 0,73$
 $\overline{R^2} = 0,69$
 $F = 18,03$ (q₁ = 2, q₂ = 13)
 $\hat{d} = 0,530$ (N = 16, K = 2)
 $\hat{\rho} = 0,72$
 $|r_{ij}| = 0,974$
 $\chi^2 = 0,152$ (q = 1)

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i
t	1,00	-0,16
i	-0,61	1,00

It appears from the calculated t-values that t is significant, but that i is not. The time is practically the only explaining variable. The multicollinearity pattern shows that no multicollinearity exists between t and i, which can also be derived from the value of the determinant. The autocorrelation coefficient ρ is, however, considerably too high.

$$\begin{aligned}
 z' &= -0,78 - 0,88t + 0,24p_1 \\
 (t) &(-0,51) \quad (-6,04) \quad (1,09) \quad (q = 13) \\
 (e) &85\% \quad 15\% \\
 F(R^2_{x_i, \hat{x}}) &0,22 \quad 0,22 \quad (q_1 = 1, q_2 = 14)
 \end{aligned}$$

$$\begin{aligned}
 s^2_{\epsilon} &= 7,11 \\
 R^2 &= 0,75 \\
 \bar{R}^2 &= 0,72 \\
 F &= 19,94 \quad (q_1 = 2, q_2 = 13) \\
 \hat{d} &= 0,64 \quad (N = 16, K = 2) \\
 \hat{\rho} &= 0,66 \\
 |r_{ij}| &= 0,984 \\
 \chi^2 &= 0,091 \quad (q = 1)
 \end{aligned}$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	p ₁
t	1,00	-0,12
p ₁	0,47	1,00

The application of the t-test shows that t can be distinguished significantly from zero, while p₁ cannot. In other words: p₁ is unacceptable as explaining variable. As is shown by the multicollinearity pattern and the value of the determinant, no multicollinearity exists. The value of ρ indicates, however, that autocorrelation exists between the residuals.

$$z' = -1,43 - 0,90t + 0,072p_2$$

(t) (-0,98) (-5,98) (0,40) (q = 13)

(e) 94% 6%

$F(R^2_{x_i, \hat{x}})$ 0,01 0,01 ($q_1 = 1, q_2 = 14$)

$$s^2_{\epsilon} = 7,66$$

$$R^2 = 0,73$$

$$\bar{R}^2 = 0,69$$

$$F = 18,02 \quad (q_1 = 2, q_2 = 13)$$

$$\hat{d} = 0,529 \quad (N = 16, K = 2)$$

$$\hat{\rho} = 0,73$$

$$|r_{ij}| = 0,999$$

$$\chi^2 = 0,003 \quad (q = 1)$$

r_{ij} t_{ij}	t	p ₂
t	1,00	-0,02
p ₂	-0,09	1,00

Here again it is found that only the time is a significant variable: it contributes 94% to the variance. There is no multicollinearity, as is revealed by the multicollinearity pattern and the value of the determinant. Autocorrelation does exist; $\rho = 0,73$ is considerably higher than the acceptable critical limit of 0,50.

$$z' = -0,87 - 0,88t + 0,027i + 0,26p_1$$

(t) (-0,48) (-5,61) (0,10) (0,97) (q = 12)

(e) 82% 2% 16%

$F(R^2_{x_i, \hat{x}})$ 0,51 2,08 1,99 (q₁ = 2, q₂ = 13)

$$s^2_{\epsilon} = 7,69$$

$$R^2 = 0,75$$

$$\bar{R}^2 = 0,69$$

$$F = 12,29 \quad (q_1 = 3, q_2 = 12)$$

$$\hat{d} = 0,636 \quad (N = 16, K = 3)$$

$$\hat{\rho} = 0,66$$

$$|r_{ij}| = 0,746$$

$$\chi^2 = 1,673 \quad (q = 3)$$

$r_{ij} \backslash t_{ij}$	t	i	p ₁
t	1,00	-0,24	-0,22
i	-0,90	1,00	-0,47
p ₁	-0,81	-1,93	1,00

The calculated t-value shows that the coefficients of i and p₁ cannot be distinguished significantly from zero, in other words: i and p₁ are not acceptable as explaining variables. The multicollinearity pattern furthermore shows that multicollinearity exists between p₁ and i. The value of ρ indicates that autocorrelation also exists.

$$\begin{aligned}
 z' &= -1,15 - 0,90t - 0,061i + 0,043p_2 \\
 (t) &(-0,56) \quad (-5,65) \quad (-0,19) \quad (0,18) \quad (q = 12) \\
 (e) &92\% \quad 4\% \quad 4\% \\
 F(R^2_{x_i, \hat{x}}) &0,33 \quad 4,38 \quad 4,10 \quad (q_1 = 2, \quad q_2 = 13)
 \end{aligned}$$

$$\begin{aligned}
 s_e^2 &= 8,28 \\
 R^2 &= 0,74 \\
 \bar{R}^2 &= 0,67 \\
 F &= 11,14 \quad (q_1 = 3, \quad q_2 = 12) \\
 \hat{d} &= 0,538 \quad (N = 16, \quad K = 3) \\
 \hat{\rho} &= 0,72 \\
 |r_{ij}| &= 0,597 \\
 \chi^2 &= 2,946 \quad (q = 3)
 \end{aligned}$$

$r_{ij} \backslash t_{ij}$	t	i	p ₂
t	1,00	-0,22	-0,15
i	-0,81	1,00	-0,62
p ₂	0,56	-2,86	1,00

The variables i and p₂ are not significant, as the calculated t-coefficients are insignificant. Multicollinearity exists between the variables p₂ and i, as is shown by the multicollinearity pattern, for $95t^{13} = 1,771 > -2,86$. The value of ρ indicates that autocorrelation is present.

Cars per 100 inhabitants.

$$z' = 3,77 - 2,08t + 0,077i$$

$$(t) \quad (0,80) \quad (-5,31) \quad (0,13) \quad (q = 13)$$

$$(e) \quad 98\% \quad 2\%$$

$$F(R^2_{x_i, \hat{x}}) \quad 0,37 \quad 0,37 \quad (q_1 = 1, q_2 = 14)$$

$$s^2_{\epsilon} = 51,12$$

$$R^2 = 0,69$$

$$\bar{R}^2 = 0,64$$

$$F = 14,58 \quad (q_1 = 2, q_2 = 13)$$

$$\hat{d} = 0,32 \quad (N = 16, K = 2)$$

$$\hat{\rho} = 0,81$$

$$|r_{ij}| = 0,974$$

$$\chi^2 = 0,152 \quad (q = 11)$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i
t	1,00	-0,16
i	-0,60	1,00

In this relationship the time is the only significant variable, in other words: the income is insignificant. The calculated t-value is only 0,13. There is no multicollinearity, as appears from the multicollinearity pattern and the value of the determinant. The value of ρ indicates that autocorrelation is present.

$$z' = 4,17 - 2,09t + 0,016p_1$$

(t) (1,02) (-5,35) (0,026) (q=13)

(e) 99,5% 0,5%

$F(R^2_{x_i, \hat{x}})$ 0,22 0,22 (q₁=1, q₂=14)

$$s^2_{\epsilon} = 51,18$$

$$R^2 = 0,69$$

$$\bar{R}^2 = 0,64$$

$$F = 14,55 \quad (q_1 = 2, q_2 = 13)$$

$$\hat{d} = 0,323 \quad (N = 16, K = 2)$$

$$\hat{\rho} = 0,81$$

$$|r_{ij}| = 0,984$$

$$\chi^2 = 0,091 \quad (q = 1)$$

r_{ij} t_{ij}	t	p ₁
t	1,00	-0,12
p ₁	-0,47	1,00

The calculation shows that the income is insignificant. The multicollinearity pattern reveals that no multicollinearity exists between p_1 and t . The value of ρ is unacceptable; consequently autocorrelation is present between the variables.

$$z' = 4,09 - 2,10t - 0,13p_2$$

$$(t) \quad (1,09) \quad (-5,42) \quad (-0,28) \quad (q = 13)$$

$$(e) \quad 95\% \quad 5\%$$

$$F(R^2_{x_i, \hat{x}}) \quad 0,01 \quad 0,01 \quad (q_1 = 1, q_2 = 14)$$

$$s^2_{\epsilon} = 50,87$$

$$R^2 = 0,69$$

$$\bar{R}^2 = 0,65$$

$$F = 14,68 \quad (q_1 = 2, q_2 = 13)$$

$$\hat{d} = 0,334 \quad (N = 16, K = 2)$$

$$\hat{\rho} = 0,81$$

$$|r_{ij}| = 0,999$$

$$\chi^2 = 0,003 \quad (q = 1)$$

r_{ij} t_{ij}	t	p ₂
t	1,00	-0,02
p ₂	-0,09	1,00

The calculated t-values show that the explaining variable time is significant, while the income is not. The variables are not multicollinear, as appears from the value of the determinant and the multicollinearity pattern. The value of ρ is much too high, however. Consequently autocorrelation will certainly be present.

$$\begin{aligned}
 z' &= 3,80 - 2,08t + 0,11i + 0,066p_1 \\
 (t) &\quad (0,78) \quad (-4,96) \quad (0,15) \quad (0,093) \quad (q = 12) \\
 (e) &\quad \quad \quad 95\% \quad \quad 3\% \quad \quad 2\% \\
 F(R^2_{x_i, \hat{x}}) &\quad \quad \quad 0,51 \quad \quad 2,08 \quad \quad 1,99 \quad (q_1 = 2, q_2 = 13)
 \end{aligned}$$

$$\begin{aligned}
 s^2_{\hat{\epsilon}} &= 55,34 \\
 R^2 &= 0,77 \\
 \bar{R}^2 &= 0,61 \\
 F &= 13,26 \quad (q_1 = 13, q_2 = 12) \\
 \hat{d} &= 0,321 \quad (N = 16, K = 3) \\
 \hat{\rho} &= 0,81 \\
 |r_{ij}| &= 0,746 \\
 \chi^2 &= 1,673 \quad (q = 3)
 \end{aligned}$$

$\begin{matrix} r_{ij} \\ t_{ij} \end{matrix}$	t	i	p ₁
t	1,00	-0,24	-0,22
i	-0,90	1,00	-0,47
p ₁	-0,81	-1,93	1,00

The values of the t-coefficients of t, i and p₁ reveal that only the explaining variable time is significant, as it has the only t-value greater than 1,782. This means that both price and income cannot be considered as explaining variables. Moreover multicollinearity exists between p₁ and i, as appears from the multicollinearity pattern. The value of $\rho = 0,81$ shows that autocorrelation is also present.

$$z' = 4,31 - 2,10t - 0,049i - 0,15p_2$$

(t) (0,81) (-5,09) (-0,060) (-0,25) (q = 12)

(e) 93% 1% 6%

$F(R^2_{x_i, \hat{x}})$ 0,33 4,38 4,10 (q₁ = 2, q₂ = 13)

$$s^2_{\epsilon} = 55,09$$

$$R^2 = 0,69$$

$$\bar{R}^2 = 0,62$$

$$F = 9,04 \quad (q_1 = 3, q_2 = 12)$$

$$\hat{d} = 0,337 \quad (N = 16, K = 3)$$

$$\hat{\rho} = 0,81$$

$$|r_{ij}| = 0,597$$

$$\chi^2 = 2,946$$

r_{ij} t_{ij}	t	i	p ₂
t	1,00	-0,22	-0,15
i	-0,81	1,00	-0,62
p ₂	-0,56	-2,86	1,00

Here, too, only the time is significant, while the price and income are not. Multicollinearity exists between the variables p_2 and i , as appears from the coefficient $-2,86$. The value of ρ is much too high again, so that it must be concluded that autocorrelation is also present.

APPENDIX B

	on 1-8-'55	on 31-12-'55	on 1-8-'56
SURVEY OF 1955			
New production	11029	18249	
Exported	<u>2664</u>	<u>3503</u>	
Available remainder	8365	14746	
Imported as new cars	<u>29611</u>	<u>49243</u>	
Available for the home market	37976	63989	
Issue of part I accelerated procedure	37976	63989	
Additions, Part II	35717	63500	
Tax cards according to the Central Bureau of Statistics	35360		60143
Corrected for import (used)			59961

SITUATION ON 31-12-1955

(1) Issue of part I accelerated procedure	63989		
(2) Issue part I after receipt of application form	2242		
(3) Issue of part I, Total		66231	
(4) New cars available for the home market	63989		
(5) Used, registered, imported	2242		
(6) Available for the home market		66231	
(7) (1) - (4)	0		
(8) (2) - (5)	0		
(9) Part I new = (1) + (2) - (5)	63989		
(10) Mutations in trade part I = (3) - (6)		0	
(11) Cum mutations in trade part I			0
(12) Additions part II	63500		
(13).Mutations in trade new = (4)-(12)		489	
(14) Cum mutations in trade new			489

SITUATION ON 1-8-1956

(5) Used, registered, imported	173		
(6) Available for the home market		63673	
(15) Registered tax cards	60134		
(16) Shortage = (6)-(15)		3539	5,55%

SITUATION ON 1-8-1957

(5) Used, registered, imported	295		
(6) Available for the home market		60429	
(15) Registered tax cards	59657		
(16) Shortage = (6)-(15)		772	1,27%

SITUATION ON 1-8-1958

(5) Used, registered, imported	356		
(6) Available for the home market		60013	

(15) Registered tax cards	58000		
(16) Shortage = (6)-(15)		2013	3,35%
SITUATION ON 1-8-1959			
(5) Used, registered, imported	449		
(6) Available for the home market		58449	
(15) Registered tax cards	55659		
(16) Shortage = (6)-(15)		2790	4,77%
SITUATION ON 1-8-1960			
(5) Used, registered, imported	1081		
(6) Available for the home market		56740	
(15) Registered tax cards	54189		
(16) Shortage = (6)-(15)		2551	4,49%
SITUATION ON 1-8-1961			
(5) Used, registered, imported	1235		
(6) Available for the home market		55424	
(15) Registered tax cards	54155		
(16) Shortage = (6)-(15)		1269	2,28%
SITUATION ON 1-8-1962			
(5) Used, registered, imported	2163		
(6) Available for the home market		56318	
(15) Registered tax cards	54368		
(16) Shortage = (6)-(15)		1950	3,46%
SITUATION ON 1-8-1963			
(5) Used, registered, imported	2991		
(6) Available for the home market		57359	
(15) Registered tax cards	53708		
(16) Shortage = (6)-(15)		3651	6,36%
SITUATION ON 1-8-1964			
(5) Used, registered, imported	2880		
(6) Available for the home market		56588	
(15) Registered tax cards	45699		
(16) Shortage = (6)-(15)		10889	19,24%
SITUATION ON 1-8-1965			
(5) Used, registered, imported	1255		
(6) Available for the home market		46954	
(15) Registered tax cards	37070		
(16) Shortage = (6)-(15)		9884	21,05%

SITUATION ON 1-8-1966

(5) Used, registered, imported	484		
(6) Available for the home market		37554	
(15) Registered tax cards	29259		
(16) Shortage = (6)–(15)		8295	22,08%

	on	on	on
	1-8-'56	31-12-'56	1-8-'57

SURVEY OF 1956

New production	10603	21361	
Exported	<u>2620</u>	<u>4249</u>	
Available remainder	7983	17112	
Imported as new cars	<u>36982</u>	<u>56474</u>	
Available for the home market	44965	73586	
Issue of part I accelerated procedure	44965	73586	
Additions, part II	43961	68896	
Tax cards according to the Central Bureau of Statistics	41299		61207
Corrected for import (used)			61061

SITUATION ON 31-12-1956

(1) Issue of part I accelerated procedure	73586		
(2) Issue part I after receipt of application form	3344		
(3) Issue of part I, Total		76930	
(4) New cars available for the home market	73586		
(5) Used, registered, imported	3344		
(6) Available for the home market		76930	
(7) (1)–(4)	0		
(8) (2)–(5)	0		
(9) Part I new = (1) + (2)–(5)	73586		
(10) Mutations in trade part I = (3)–(6)	0	0	
(11) Cum mutations in trade part I	0		0
(12) Additions part II	68896		
(13) Mutations in trade new = (4)–(12)		4690	
(14) Cum mutations in trade new			5179

SITUATION ON 1-8-1957

(5) Used, registered, imported	146		
(6) Available for the home market		69042	
(15) Registered tax cards	61207		
(16) Shortage = (6)–(15)		7835	11,34%

SITUATION ON 1-8-1958

(5) Used, registered, imported	324		
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(6) Available for the home market		61531	
(15) Registered tax cards	59500		
(16) Shortage = (6)–(15)		2031	3,30%
SITUATION ON 1-8-1959			
(5) Used, registered, imported	462		
(6) Available for the home market		59962	
(15) Registered tax cards	57994		
(16) Shortage = (6)–(15)		1968	3,28%
SITUATION ON 1-8-1960			
(5) Used, registered, imported	1063		
(6) Available for the home market		59057	
(15) Registered tax cards	56807		
(16) Shortage = (6)–(15)		2250	3,80%
SITUATION ON 1-8-1961			
(5) Used, registered, imported	1191		
(6) Available for the home market		57998	
(15) Registered tax cards	56869		
(16) Shortage = (6)–(15)		1129	1,94%
SITUATION ON 1-8-1962			
(5) Used, registered, imported	2117		
(6) Available for the home market		58984	
(15) Registered tax cards	57519		
(16) Shortage = (6)–(15)		1467	2,48%
SITUATION ON 1-8-1963			
(5) Used, registered, imported	3692		
(6) Available for the home market		61211	
(15) Registered tax cards	58179		
(16) Shortage = (6)–(15)		3032	4,95%
SITUATION ON 1-8-1964			
(5) Used, registered, imported	4082		
(6) Available for the home market		62261	
(15) Registered tax cards	51915		
(16) Shortage = (6)–(15)		10346	16,61%
SITUATION ON 1-8-1965			
(5) Used, registered, imported	2281		
(6) Available for the home market		54196	
(15) Registered tax cards	45743		
(16) Shortage = (6)–(15)		8453	15,59%

SITUATION ON 1-8-1966

(5) Used, registered, imported	890		
(6) Available for the home market		46633	
(15) Registered tax cards	38848		
(16) Shortage = (6)-(15)		7785	16,69%

on	on	on
1-8-'57	31-12-'57	1-8-'58

SURVEY OF 1957

New production	10531	15814	
Exported	<u>2471</u>	<u>3485</u>	
Available remainder	8060	12329	
Imported as new cars	<u>32035</u>	<u>52538</u>	
Available for the home market	40095	64867	
Issue of part I accelerated procedure	37200	61926	
Additions, part II	41299	65243	
Tax cards according to the Central Bureau of Statistics	40157		62500
Corrected for import (used)			62340

SITUATION ON 31-12-1957

(1) Issue of part I accelerated procedure	61926		
(2) Issue part I after receipt of application form	15793		
(3) Issue of part I, Total		77719	
(4) New cars available for the home market	64867		
(5) Used, registered, imported	3139		
(6) Available for the home market		68006	
(7) (1)-(4)	-2941		
(8) (2)-(5)	12654		
(9) Part I new = (1)+(2)-(5)	74580		
(10) Mutations in trade part I = (3)-(6)		9713	
(11) Cum mutations in trade part I			9713
(12) Additions part II	65243		
(13) Mutations in trade new = (4)-(12)		-376	
(14) Cum mutations in trade new			4803

SITUATION ON 1-8-1958

(5) Used, registered, imported	160		
(6) Available for the home market		65403	
(15) Registered tax cards	62500		
(16) Shortage = (6)-(15)		2903	4,43%

SITUATION ON 1-8-1959

(5) Used, registered, imported	440		
(6) Available for the home market		62940	
(15) Registered tax cards	61098		
(16) Shortage = (6)-(15)		1842	2,92%

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SITUATION ON 1-8-1960				
(5) Used, registered, imported	1001			
(6) Available for the home market		62099		
(15) Registered tax cards	58863			
(16) Shortage = (6)-(15)		3236		5,21%
SITUATION ON 1-8-1961				
(5) Used, registered, imported	957			
(6) Available for the home market		59820		
(15) Registered tax cards	59077			
(16) Shortage = (6)-(15)		743		1,24%
SITUATION ON 1-8-1962				
(5) Used, registered, imported	1768			
(6) Available for the home market		60845		
(15) Registered tax cards	59715			
(16) Shortage = (6)-(15)		1130		1,85%
SITUATION ON 1-8-1963				
(5) Used, registered, imported	3301			
(6) Available for the home market		63016		
(15) Registered tax cards	60687			
(16) Shortage = (6)-(15)		2329		3,69%
SITUATION ON 1-8-1964				
(5) Used, registered, imported	5022			
(6) Available for the home market		65709		
(15) Registered tax cards	59517			
(16) Shortage = (6)-(15)		6192		9,42%
SITUATION ON 1-8-1965				
(5) Used, registered, imported	3672			
(6) Available for the home market		63189		
(15) Registered tax cards	57161			
(16) Shortage = (6)-(15)		6028		9,53%
SITUATION ON 1-8-1966				
(5) Used, registered, imported	1820			
(6) Available for the home market		58981		
(15) Registered tax cards	50352			
(16) Shortage = (6)-(15)		8629		14,63%
SURVEY OF 1958				
		on	on	on
		1-8-'58	31-12-'58	1-8-'59
New production		10640	15811	

Exported	5289	8030	
Available remainder	5351	7781	
Imported as new cars	26502	43703	
Available for the home market	31853	51484	
Issue of part I accelerated procedure	32017	51516	
Additions, part II	35276	56188	
Tax cards according to the Central Bureau of Statistics	33918		51432
Corrected for import (used)			51183

SITUATION ON 31-12-1958

(1) Issue of part I accelerated procedure	51516		
(2) Issue part I after receipt of application form	8149		
(3) Issue of part I, Total		59665	
(4) New cars available for the home market	51484		
(5) Used, registered, imported	3328		
(6) Available for the home market		54812	
(7) (1)-(4)	32		
(8) (2)-(5)	4821		
(9) Part I new = (1)+(2)-(5)	56337		
(10) Mutations in trade part I = (3)-(6)		4853	
(11) Cum mutations in trade part I			14566
(12) Additions part II	56188		
(13) Mutations in trade new = (4)-(12)		-4704	
(14) Cum mutations in trade new			99

SITUATION ON 1-8-1959

(5) Used, registered, imported	249		
(6) Available for the home market		56437	
(15) Registered tax cards	51432		
(16) Shortage = (6)-(15)		5005	8,86%

SITUATION ON 1-8-1960

(5) Used, registered, imported	929		
(6) Available for the home market		52361	
(15) Registered tax cards	49634		
(16) Shortage = (6)-(15)		2727	5,20%

SITUATION ON 1-8-1961

(5) Used, registered, imported	885		
(6) Available for the home market		50519	
(15) Registered tax cards	49763		
(16) Shortage = (6)-(15)		756	1,49%

SITUATION ON 1-8-1962

(5) Used, registered, imported	1509		
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(6) Available for the home market		51272	
(15) Registered tax cards	50156		
(16) Shortage = (6)-(15)		1116	2,17%
SITUATION ON 1-8-1963			
(5) Used, registered, imported	2997		
(6) Available for the home market		53153	
(15) Registered tax cards	51483		
(16) Shortage = (6)-(15)		1670	3,14%
SITUATION ON 1-8-1964			
(5) Used, registered, imported	5684		
(6) Available for the home market		57167	
(15) Registered tax cards	53145		
(16) Shortage = (6)-(15)		4022	7,03%
SITUATION ON 1-8-1965			
(5) Used, registered, imported	5714		
(6) Available for the home market		58859	
(15) Registered tax cards	55133		
(16) Shortage = (6)-(15)		3726	6,33%
SITUATION ON 1-8-1966			
(5) Used, registered, imported	3709		
(6) Available for the home market		58842	
(15) Registered tax cards	53359		
(16) Shortage = (6)-(15)		5483	9,31%

	on 1-8-'59	on 31-12-'59	on 1-8-'60
SURVEY OF 1959			
New production	15117	28339	
Exported	5348	8894	
Available remainder	9769	19445	
Imported as new cars	37969	59074	
Available for the home market	47738	78519	
Issue of part I accelerated procedure	47803	78590	
Additions, part II	47019	76847	
Tax cards according to the Central Bureau of Statistics	46644		73666
Corrected for import (used)			73545

SITUATION ON 31-12-1959			
(1) Issue of part I accelerated procedure	78590		
(2) Issue part I after receipt of application form	10886		
(3) Issue of part I, Total		89476	

(4) New cars available for the home market	78519		
(5) Used, registered, imported	5469		
(6) Available for the home market		83988	
(7) (1)–(4)	71		
(8) (2)–(5)	5417		
(9) Part I new = (1)+(2)–(5)	84007		
(10) Mutations in trade part I = (3)–(6)		5488	
(11) Cum mutations in trade part I			20054
(12) Additions part II	76847		
(13) Mutations in trade new = (4)–(12)		1682	
(14) Cum mutations in trade new			1771

SITUATION ON 1-8-1960

(5) Used, registered, imported	121		
(6) Available for the home market		76968	
(15) Registered tax cards	73666		
(16) Shortage = (6)–(15)		3302	4,29%

SITUATION ON 1-8-1961

(5) Used, registered, imported	1031		
(6) Available for the home market		74697	
(15) Registered tax cards	72765		
(16) Shortage = (6)–(15)		1932	2,58%

SITUATION ON 1-8-1962

(5) Used, registered, imported	1323		
(6) Available for the home market		74088	
(15) Registered tax cards	71682		
(16) Shortage = (6)–(15)		2406	3,24%

SITUATION ON 1-8-1963

(5) Used, registered, imported	2286		
(6) Available for the home market		73968	
(15) Registered tax cards	71445		
(16) Shortage = (6)–(15)		2523	3,41%

SITUATION ON 1-8-1964

(5) Used, registered, imported	4727		
(6) Available for the home market		76172	
(15) Registered tax cards	74581		
(16) Shortage = (6)–(15)		1591	2,08%

SITUATION ON 1-8-1965

(5) Used, registered, imported	5656		
(6) Available for the home market		80237	

(15) Registered tax cards	78429		
(16) Shortage = (6)–(15)		1808	2,25%

SITUATION ON 1-8-1966

(5) Used, registered, imported	4769		
(6) Available for the home market		83198	
(15) Registered tax cards	77127		
(16) Shortage = (6)–(15)		6071	7,29%

	on 1-8-'60	on 31-12-'60	on 1-8-'61
SURVEY OF 1960			
New production	30612	46565	
Exported	10935	16264	
Available remainder	19677	30301	
Imported as new cars	44530	70816	
Available for the home market	64207	101117	
Issue of part I accelerated procedure	63515	98862	
Additions, part II	63015	100154	
Tax cards according to the Central Bureau of Statistics	63162		95312
Corrected for import (used)			94241

SITUATION ON 31-12-1960

(1) Issue of part I accelerated procedure	98862		
(2) Issue part I after receipt of application form	13068		
(3) Issue of part I, Total		111930	
(4) New cars available for the home market	101117		
(5) Used, registered, imported	8676		
(6) Available for the home market		109793	
(7) (1)–(4)	–2255		
(8) (2)–(5)	4392		
(9) Part I new = (1)+(2)–(5)	103254		
(10) Mutations in trade part I = (3)–(6)		2137	
(11) Cum mutations in trade part I			22191
(12) Additions part II	100154		
(13) Mutations in trade new = (4)–(12)		963	
(14) Cum mutations in trade new			2734

SITUATION ON 1-8-1961

(5) Used, registered, imported	1071		
(6) Available for the home market		101225	
(15) Registered tax cards	95312		
(16) Shortage = (6)–(15)		5913	5,84%

SITUATION ON 1-8-1962			
(5) Used, registered, imported	1625		
(6) Available for the home market		96937	
(15) Registered tax cards	93056		
(16) Shortage = (6)–(15)		3881	4,00%

SITUATION ON 1-8-1963			
(5) Used, registered, imported	2166		
(6) Available for the home market		95222	
(15) Registered tax cards	91631		
(16) Shortage = (6)–(15)		3591	3,77%

SITUATION ON 1-8-1964			
(5) Used, registered, imported	4119		
(6) Available for the home market		95750	
(15) Registered tax cards	94811		
(16) Shortage = (6)–(15)		939	0,98%

SITUATION ON 1-8-1965			
(5) Used, registered, imported	5853		
(6) Available for the home market		100664	
(15) Registered tax cards	99453		
(16) Shortage = (6)–(15)		1211	1,20%

SITUATION ON 1-8-1966			
(5) Used, registered, imported	6644		
(6) Available for the home market		106097	
(15) Registered tax cards	99915		
(16) Shortage = (6)–(15)		6182	5,82%

	on 1-8-'61	on 31-12-'61	on 1-8-'62
SURVEY OF 1961			
New production	25543	40337	
Exported	<u>8435</u>	<u>12785</u>	
Available remainder	17108	27552	
Imported as new cars	<u>61630</u>	<u>91775</u>	
Available for the home market	78738	119327	
Issue of part I accelerated procedure	81204	120790	
Additions, part II	79074	120195	
Tax cards according to the Central Bureau of Statistics	77307		114991
Corrected for import (used)			114075

SITUATION ON 31-12-1961	
(1) Issue of part I accelerated procedure	120790

(2) Issue part I after receipt of application form	15580		
(3) Issue of part I, Total		136370	
(4) New cars available for the home market	119327		
(5) Used, registered, imported	13374		
(6) Available for the home market		132701	
(7) (1)–(4)	1463		
(8) (2)–(5)	2206		
(9) Part I new = (1)+(2)–(5)	122996		
(10) Mutations in trade part I = (3)–(6)		3669	
(11) Cum mutations in trade part I			25860
(12) Additions part II	120195		
(13) Mutations in trade new = (4)–(12)		–868	
(14) Cum mutations in trade new			1866

SITUATION ON 1-8-1962

(5) Used, registered, imported	916		
(6) Available for the home market		121111	
(15) Registered tax cards	114991		
(16) Shortage = (6)–(15)		6120	5,05%

SITUATION ON 1-8-1963

(5) Used, registered, imported	1894		
(6) Available for the home market		116885	
(15) Registered tax cards	112025		
(16) Shortage = (6)–(15)		4860	4,15%

SITUATION ON 1-8-1964

(5) Used, registered, imported	2913		
(6) Available for the home market		114938	
(15) Registered tax cards	114565		
(16) Shortage = (6)–(15)		373	0,32%

SITUATION ON 1-8-1965

(5) Used, registered, imported	4311		
(6) Available for the home market		118876	
(15) Registered tax cards	118129		
(16) Shortage = (6)–(15)		747	0,62%

SITUATION ON 1-8-1966

(5) Used, registered, imported	6320		
(6) Available for the home market		124449	
(15) Registered tax cards	118733		
(16) Shortage = (6)–(15)		5716	4,59%

	on 1-8-'62	on 31-12-'62	on 1-8-'63
SURVEY OF 1962			
New production	25661	43111	
Exported	<u>9027</u>	<u>13423</u>	
Available remainder	16634	29688	
Imported as new cars	79795	122594	
Available for the home market	<u>96429</u>	<u>152282</u>	
Issue of part I accelerated procedure	96258	148008	
Additions, part II	95697	146031	
Tax cards according to the Central Bureau of Statistics	92718		136808
Corrected for import (used)			136807
SITUATION ON 31-12-1962			
(1) Issue of part I accelerated procedure	148008		
(2) Issue part I after receipt of application form	17871		
(3) Issue of part I, Total		165879	
(4) New cars available for the home market	152282		
(5) Used, registered, imported	19962		
(6) Available for the home market		172244	
(7) (1)-(4)	-4274		
(8) (2)-(5)	-2091		
(9) Part I new = (1)+(2)-(5)	145917		
(10) Mutations in trade part I = (3)-(6)		-6365	
(11) Cum mutations in trade part I			19495
(12) Additions part II	146031		
(13) Mutations in trade new = (4)-(12)		6251	
(14) Cum mutations in trade new			8117
SITUATION ON 1-8-1963			
(5) Used, registered, imported	1		
(6) Available for the home market		146032	
(15) Registered tax cards	136808		
(16) Shortage = (6)-(15)		9224	6,31%
SITUATION ON 1-8-1964			
(5) Used, registered, imported	2199		
(6) Available for the home market		139007	
(15) Registered tax cards	139513		
(16) Shortage = (6)-(15)		-506	-0,36%
SITUATION ON 1-8-1965			
(5) Used, registered, imported	3846		
(6) Available for the home market		143359	
(15) Registered tax cards	141172		
(16) Shortage = (6)-(15)		2187	1,52%
			149

SITUATION ON 1-8-1966

(5) Used, registered, imported	5754		
(6) Available for the home market		146926	
(15) Registered tax cards	143039		
(16) Shortage = (6)-(15)		3887	2,64%

on	on	on
1-8-'63	31-12-'63	1-8-'64

SURVEY OF 1963

New production	24197	42226	
Exported	<u>7131</u>	<u>13012</u>	
Available remainder	17066	29214	
Imported as new cars	<u>95500</u>	<u>152174</u>	
Available for the home market	112566	181388	
Issue of part I accelerated procedure	120897	187911	
Additions, part II	116719	181853	
Tax cards according to the Central Bureau of Statistics	111053		182497
Corrected for import (used)			180707

SITUATION ON 31-12-1963

(1) Issue of part I accelerated procedure	187911		
(2) Issue part I after receipt of application form	33085		
(3) Issue of part I, Total		220996	
(4) New cars available for the home market	181388		
(5) Used, registered, imported	28310		
(6) Available for the home market		209698	
(7) (1)-(4)	6523		
(8) (2)-(5)	4775		
(9) Part I new = (1)+(2)-(5)	192686		
(10) Mutations in trade part I = (3)-(6)		11298	
(11) Cum mutations in trade part I			30793
(12) Additions part II	181853		
(13) Mutations in trade new = (4)-(12)		-465	
(14) Cum mutations in trade new			7652

SITUATION ON 1-8-1964

(5) Used, registered, imported	1790		
(6) Available for the home market		183643	
(15) Registered tax cards	182497		
(16) Shortage = (6)-(15)		1146	0,62%

SITUATION ON 1-8-1965

(5) Used, registered, imported	4033		
(6) Available for the home market		186530	
(15) Registered tax cards	185721		
(16) Shortage = (6)-(15)		809	0,43%

SITUATION ON 1-8-1966

(5) Used, registered, imported	6023		
(6) Available for the home market		191744	
(15) Registered tax cards	189171		
(16) Shortage = (6)-(15)		2573	1,34%

	on 1-8-'64	on 31-12-'64	on 1-8-'65
SURVEY OF 1964			
New production	35346	60925	
Exported	15399	29409	
Available remainder	19947	31516	
Imported as new cars	136508	194716	
Available for the home market	156455	226232	
Issue of part I accelerated procedure	159535	229817	
Additions, part II	154811	226090	
Tax cards according to the Central Bureau of Statistics	152706		224418
Corrected for import (used)			222204

SITUATION ON 31-12-1964

(1) Issue of part I accelerated procedure	229817		
(2) Issue part I after receipt of application form	42293		
(3) Issue of part I, Total		272110	
(4) New cars available for the home market	226232		
(5) Used, registered, imported	39437		
(6) Available for the home market		265669	
(7) (1)-(4)	3585		
(8) (2)-(5)	2856		
(9) Part I new = (1)+(2)-(5)	232673		
(10) Mutations in trade part I = (3)-(6)		6441	
(11) Cum mutations in trade part I			37234
(12) Additions part II	226090		
(13) Mutations in trade new = (4)-(12)		142	
(14) Cum mutations in trade new			7794

SITUATION ON 1-8-1965

(5) Used, registered, imported	2214		
(6) Available for the home market		228304	
(15) Registered tax cards	224418		
(16) Shortage = (6)-(15)		3886	1,70%

SITUATION ON 1-8-1966

(5) Used, registered, imported	5484		
(6) Available for the home market		229902	

(15) Registered tax cards	228429		
(16) Shortage = (6)-(15)		1473	0,64%

	on 1-8-'65	on 31-12-'65	on 1-8-'66
SURVEY OF 1965			
New production	35954	59801	
Exported	16565	27304	
Available remainder	19389	32497	
Imported as new cars	150421	264030	
Available for the home market	169810	296527	
Issue of part I accelerated procedure	177226	307062	
Additions, part II	173381	303789	
Tax cards according to the Central Bureau of Statistics	167509		301027
Corrected for import (used)			297550

SITUATION ON 31-12-1965			
(1) Issue of part I accelerated procedure	307062		
(2) Issue part I after receipt of application form	43287		
(3) Issue of part I, Total		350349	
(4) New cars available for the home market	296527		
(5) Used, registered, imported	41781		
(6) Available for the home market		338308	
(7) (1)-(4)	10535		
(8) (2)-(5)	1506		
(9) Part I new = (1)+(2)-(5)	308568		
(10) Mutations in trade part I = (3)-(6)		12041	
(11) Cum mutations in trade part I			49275
(12) Additions part II	303789		
(13) Mutations in trade new = (4)-(12)		-7262	
(14) Cum mutations in trade new			532

SITUATION ON 1-8-1966			
(5) Used, registered, imported	3477		
(6) Available for the home market		307266	
(15) Registered tax cards	301027		
(16) Shortage = (6)-(15)		6239	2,03%

	on 1-8-'66	on 31-12-'66	on 1-8-'67
SURVEY OF 1966			
New production	31733	54499	
Exported	15302	26474	

Available remainder	16431	28025
Imported as new cars	<u>120289</u>	<u>197275</u>
Available for the home market	136720	225300
Issue of part I accelerated procedure	135973	223063
Additions, part II	130002	213389
Tax cards according to the Central Bureau of Statistics	126432	
Corrected for import (used)		

SITUATION ON 31-12-1966

(1) Issue of part I accelerated procedure	223063		
(2) Issue part I after receipt of application form	52030		
(3) Issue of part I, Total		275093	
(4) New cars available for the home market	225300		
(5) Used, registered, imported	48603		
(6) Available for the home market		273903	
(7) (1)-(4)	-2237		
(8) (2)-(5)	3427		
(9) Part I new = (1)+(2)-(5)	226490		
(10) Mutations in trade part I = (3)-(6)		1190	
(11) Cum mutations in trade part I			50465
(12) Additions part II	213389		
(13) Mutations in trade new = (4)-(12)		11911	
(14) Cum mutations in trade new			12443

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STELLINGEN

I

Indien de fabrikant van bepaalde duurzame produktiemiddelen deze apparatuur verhuurt, ontstaat hieruit vaak een zodanige binding tussen leverancier en afnemer dat deze financieringsvorm het karakter verkrijgt van een pseudo-deelneming, zodat de afnemer de uit het huurcontract voortvloeiende vaste verplichtingen niet meer behoort te beschouwen als zijnde analoog aan die van financiering van vreemd vermogen.

II

In Europa wordt in verhouding tot de Verenigde Staten de financiële structuur nog teveel geanalyseerd op basis van de balansverhoudingen in plaats van op basis van de grootte en stabiliteit van de cash flow.

III

In een planningproces zijn beslissingen van strategische en taktische aard verschillende categorieën van besluitvorming. Zij kunnen niet alternerend als basis voor het bedrijfsbeleid worden gebezigd. De taktische beslissingen dienen daarom een duidelijke afbakening te vinden in een zogenaamd overall plan en niet een opportunistisch alternatief te zijn.

IV

De toepassingsmogelijkheid van de lineaire beslissingsregel van Holt, Modigliani, Muth en Simon voor bedrijfs- en algemeen economische problemen is door de eis van de kwadratuur van de kostenfunctie veel realiteitswaarde ontnomen, daar alternatieve beslissingen geen waarheidsgetrouwe uitkomsten meer geven.

V

Employees c.q. hoger personeel, dat op grond van hun specialistische kennis en ervaring op latere leeftijd door een onderneming wordt aangetrokken, behoort een zodanige pensioensvoorziening te verkrijgen, dat — bij het bereiken van de pensioengerechtigde leeftijd — het totale pensioen niet lager is dan het pensioen dat ook verkregen zou zijn als het totaal aantal werkzame jaren bij een — i.c. de laatste — onderneming zou zijn doorgebracht.

VI

Het verdient aanbeveling meer tijd te besteden aan het onderricht in de economische wetenschappen bij de hogere maatschappelijke beroepsopleidingen in het algemeen en die van de beroepskeuze adviseurs in het bijzonder.

VII

Het voorstel van de A.N.W.B. om een kentekenbewijs voor automobielen in te voeren, dat uit drie van elkaar gescheiden delen bestaat, verdient alle steun met name van die instanties welke op enigerlei wijze zijn belast met de statistische registratie en bewerking van de groeidata van het wagenpark.

VIII

Gezien de nog bestaande verschillen tussen de economische structuren van de lidstaten van de E.E.G. dient men bij het streven naar een stelsel van vaste wisselkoersen tussen de munteenheden van de lidstaten — in de overgangsfase — rekening te houden met de noodzaak van een beperkt flexibel wisselkoerssysteem.

IX

In een op consumptie gerichte gemeenschap is het gebruik van een losbladig felicitatieregister aan te bevelen bij die recepties waarbij het aanbieden van gelukwensen in de tijd vooraf gaat aan het tekenen van het betreffende register.

Stellingen behorende bij het proefschrift van G.G.J. Bos: A logistic approach to the demand for private cars.

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