Tilburg University

Strategic claim games corresponding to an NTU-game

Borm, P.E.M.; Tijs, S.H.

Publication date:
1992

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Borm, P. E. M., & Tijs, S. H. (1992). *Strategic claim games corresponding to an NTU-game*. (Reprint series / CentER for Economic Research; Vol. 98). Unknown Publisher.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM
R

8823

1992

NR.98

98

ntER

for
nic Research

REPRINT

* C I N O 1 0 6 3 *

Strategic Claim Games Corresponding to an NTU-Game

by

P.E.M. Borm

and

S.H. Tijs

Reprinted from Games and Economic Behavior,
Vol. 4, No. 1, 1992

Reprint Series

no. 98

CENTER FOR ECONOMIC RESEARCH

Research Staff

Helmut Bester
Eric van Damme

Board

Helmut Bester
Eric van Damme, director
Arie Kapteyn

Scientific Council

Eduard Bomhoff	Erasmus University Rotterdam
Willem Buiters	Yale University
Jacques Drèze	Université Catholique de Louvain
Theo van de Klundert	Tilburg University
Simon Kuipers	Groningen University
Jean-Jacques Laffont	Université des Sciences Sociales de Toulouse
Merton Miller	University of Chicago
Stephen Nickell	University of Oxford
Pieter Ruys	Tilburg University
Jacques Sijben	Tilburg University

Residential Fellows

Lans Bovenberg	Center
Anna Lusardi	Princeton University
Jan Magnus	Center
Eduardo Siandra	UCLA
Hideo Suehiro	Kobe University

Doctoral Students

Roel Beetsma
Paul de Bijl
Hans Bloemen
Sjaak Hurkens
Frank de Jong
Pieter Kop Jansen

Address: Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
Phone : +31 13 663050
Telex : 52426 kub nl
Telefax: +31 13 663066
E-mail : "center@htikub5.bitnet"

ISSN 0924-7874

1992

**Strategic Claim Games
Corresponding to an NTU-Game**

by
P.E.M. Borm
and
S.H. Tijs

Reprinted from Games and Economic Behavior,
Vol. 4, No. 1, 1992

**Reprint Series
no. 98**

Strategic Claim Games Corresponding to an NTU-Game

P. E. M. BORM AND S. H. TIJS

Department of Econometrics, University of Tilburg, The Netherlands

Received March 24, 1989

This paper introduces a way to construct a game $\Gamma(V)$ in strategic form to a standard NTU-game V . In the strategic claim game $\Gamma(V)$ the players are allowed to make claims on the coalition they want to participate in, and on the payoff they want to attain. Conversely, to construct an NTU-game to each game in strategic form the paper follows R. J. Aumann (1961, in *Contributions to the Theory of Games* (A. W. Tucker and R. D. Luce, Eds.), Vol. IV, pp. 287-324. Princeton, NJ: Princeton Univ. Press), but instead of correlation only coordination of the players' actions is allowed. Applying this procedure to the claim game $\Gamma(V)$, an NTU-game V is associated with an NTU-game $\bar{V}$. It is found that $\bar{V}$ is the superadditive hull of V . Further, it is shown that if V is superadditive, strong core elements of V exactly correspond to the payoff vectors of strong Nash equilibria for $\Gamma(V)$. Finally, some applications of the Aumann procedure are discussed. *Journal of Economic Literature* Classification Numbers: C71, C72. © 1992 Academic Press, Inc.

1. INTRODUCTION

In 1944 von Neumann and Morgenstern indicated a way to construct a superadditive game in *coalitional form* to each game in *strategic form*. It was proved that in this way all superadditive games in coalitional form could be obtained. More generally, Aumann (1961, 1967) constructed a superadditive *NTU-game* (NTU = nontransferable utility) to each game in strategic form. In both of these procedures the players are allowed to act highly cooperative in the sense that they can *correlate* their actions. In particular, this implies that Aumann's procedure always leads to NTU-games in which all sets of attainable payoff vectors for the various coalitions are convex. In this paper we adapt Aumann's procedure by only allowing the players to *coordinate* their actions. Again, the resulting NTU-game is superadditive. The starting point for our research was the following question. If one does not require convexity in the definition of

an NTU-game, can all superadditive NTU-games be obtained from games in strategic form by using the "coordinated" Aumann procedure?

The "almost" affirmative answer to this question is found by defining a strategic *claim game* $\Gamma(V)$ to each *standard* NTU-game V . The construction of a claim game bears some resemblance to the way Nash (1950, 1953) constructed a game in strategic form to each two-person bargaining game, and also to the way von Neumann and Morgenstern (1944) showed that each superadditive game in coalitional form arises from a game in strategic form. According to Nash both players can make claims on their own payoffs. If this pair of claim fits (i.e., is a feasible pair), the players get their claims. If not, the disagreement point is reached. Von Neumann and Morgenstern allowed the players to make claims on the coalition they want to participate. A coalition S forms if all the players in S claim S . If S is formed, each of the players in S obtains an equal share of the worth of S . If a player's claim is not consistent in the above sense, he obtains the payoff he can make on his own. In a claim game $\Gamma(V)$ of Section 2 the players are allowed to make claims on both a coalition and a payoff.

Applying the coordinated Aumann procedure to the claim game $\Gamma(V)$ the *claim associate* $\bar{V}$ of V is obtained. In Section 2 it is shown that the claim associate $\bar{V}$ is the *superadditive hull* of V , i.e., the smallest superadditive game containing V . This especially implies that the class of standard superadditive NTU-games can be seen as arising from games in strategic form.

Subsequently, Section 3 concentrates on the implementation of cooperative solution concepts (cf. Ichiishi, 1987). It is shown that if V is superadditive, imputations and strong core elements of V exactly correspond to the payoff vectors that are obtained by payoff undominated equilibria and strong equilibria of $\Gamma(V)$, respectively.

Finally, in Section 4, it is seen that the NTU-game $\bar{V}$, which is obtained by applying the correlated Aumann procedure to the claim game $\Gamma(V)$, is the convex hull of the claim associate $\bar{V}$. Further, we discuss the possibility of defining *finite* claim games for a special class of NTU-games.

Notation. Let $x, y \in \mathbf{R}^t$. Then $x \geq y$ ($x > y$) if $x_i \geq y_i$ ($x_i > y_i$) for all $i \in \{1, \dots, t\}$. For a finite set A , let $|A|$ denote the number of elements of A and let $\langle A \rangle$ denote the set of all possible partitions of A , i.e.,

$$\langle A \rangle := \left\{ \{A_1, \dots, A_r\} \mid r \in \mathbf{N}, \bigcup_{k=1}^r A_k = A, A_k \cap A_l = \emptyset \right. \\ \left. \text{for all } k, l \in \{1, \dots, r\} \text{ with } k \neq l \right\}.$$

Finally, for a convex set $C \subset \mathbf{R}^t$, $\text{Ext}(C)$ denotes the set of extreme points of C . Here, $c \in C$ is called *extreme* if for all $c_1, c_2 \in C$ with $c = \frac{1}{2}(c_1 + c_2)$ we have that $c_1 = c_2$.

2. NTU-GAMES, CLAIM GAMES, AND CLAIM ASSOCIATES

An NTU-game is an ordered pair (N, V) , where $N := \{1, \dots, n\}$ is the set of players and, with $2^N := \{S \mid S \subset N\}$ representing the set of possible coalitions, V is a mapping which assigns to each coalition $S \in 2^N \setminus \{\emptyset\}$ a subset $V(S)$ of $\mathbf{R}^S$ such that $V(S)$ is *comprehensive*, i.e., if $a \in V(S)$ and $b \in \mathbf{R}^S$ is such that $b \leq a$, then $b \in V(S)$.

This can be interpreted as follows. If a coalition S forms, each of the payoff vectors $a \in V(S)$ is attainable by S , giving a payoff (utility) a_i to player $i \in S$. We often identify (N, V) with V . The set of all n -person NTU-games is denoted by NTU^n . A game $V \in \text{NTU}^n$ is called *superadditive* if

$$V(S) \times V(T) \subset V(S \cup T) \quad (1)$$

for all $S, T \in 2^N \setminus \{\emptyset\}$ with $S \cap T = \emptyset$

Note that each n -person game (N, v) in *coalitional form* with $v: 2^N \rightarrow \mathbf{R}$ and $v(\emptyset) = 0$ gives rise to an NTU-game (N, V) , where

$$V(S) := \left\{ a \in \mathbf{R}^S \mid \sum_{i \in S} a_i \leq v(S) \right\} \quad (S \in 2^N \setminus \{\emptyset\}).$$

Another example is provided by two-person *bargaining games* (C, d) , where C is a compact subset of $\mathbf{R}^2$ representing the feasible utility pairs and $d \in \mathbf{R}^2$ is the disagreement point. Such a bargaining game can be viewed as an NTU-game $(\{1, 2\}, V)$ by defining

$$V(\{i\}) = \{a \in \mathbf{R} \mid a \leq d_i\} \text{ for } i \in \{1, 2\} \quad \text{and} \quad V(\{1, 2\}) = \text{Compr}(C),$$

where $\text{Compr}(A) := \{b \in \mathbf{R}^I \mid \text{there is an } a \in A \text{ such that } b \leq a\}$ denotes the *comprehensive hull* of a subset A of $\mathbf{R}^I$.

For an n -person game $\Gamma = (X_1, \dots, X_n, K_1, \dots, K_n)$ in strategic form, X_i and $K_i: X_1 \times \dots \times X_n \rightarrow \mathbf{R}$ represent the strategy space and the payoff function of player $i \in \{1, \dots, n\}$, respectively. The set of all n -person games in strategic form is denoted by SG^n . Furthermore, for $S \in 2^N \setminus \{\emptyset\}$ and $x = (x_1, \dots, x_n) \in \prod_{i \in N} X_i$,

$$X_S := \prod_{i \in S} X_i, \quad x_S := (x_i)_{i \in S} \in X_S, \quad K_S(x) := (K_i(x))_{i \in S} \in \mathbf{R}^S$$

and x is identified with $(x_S, x_{N \setminus S})$.

Following Aumann (1961, 1967), but allowing the players only to coordinate their actions, we construct NTU-games A_Γ and B_Γ to each game Γ in strategic form in the following way.

DEFINITION. Let $\Gamma = (X_1, \dots, X_n, K_1, \dots, K_n) \in SG^n$. Then $A_\Gamma \in NTU^n$ and $B_\Gamma \in NTU^n$ are defined by

$$A_\Gamma(S) := \{a \in \mathbf{R}^S \mid \exists_{\hat{x}_S \in X_S} \forall_{x_{N \setminus S} \in X_{N \setminus S}} : K_S(\hat{x}_S, x_{N \setminus S}) \geq a\} \quad (2)$$

and

$$B_\Gamma(S) := \{a \in \mathbf{R}^S \mid \forall_{x_{N \setminus S} \in X_{N \setminus S}} \exists_{\hat{x}_S \in X_S} : K_S(\hat{x}_S, x_{N \setminus S}) \geq a\} \quad (3)$$

for all $S \in 2^N \setminus \{\emptyset\}$.

Note that $A_\Gamma(S)$ corresponds to a pessimistic view on the cooperative behavior of the coalition S , while $B_\Gamma(S)$ corresponds to a more optimistic point of view.

EXAMPLE 1. Consider the two-person game Γ in strategic form with strategy spaces $\{T, B\}$ for player 1 and $\{L, R\}$ for player 2, and where the payoffs are determined by

$$\begin{array}{cc} & \begin{array}{cc} \text{player 2} \\ L & R \end{array} \\ \begin{array}{c} \text{player 1} \\ T \\ B \end{array} & \begin{pmatrix} (1, -1) & (-1, 1) \\ (-1, 1) & (1, -1) \end{pmatrix} \end{array}$$

Then

$$\begin{aligned} A_\Gamma(\{1\}) &= A_\Gamma(\{2\}) = \{a \in \mathbf{R} \mid a \leq -1\}, \\ B_\Gamma(\{1\}) &= B_\Gamma(\{2\}) = \{a \in \mathbf{R} \mid a \leq 1\} \end{aligned}$$

and

$$A_\Gamma(\{1, 2\}) = B_\Gamma(\{1, 2\}) = \text{Compr}(\{(-1, 1), (1, -1)\}).$$

Note that B_Γ is not superadditive.

It is straightforward to show

LEMMA 2.1. A_Γ is superadditive for all $\Gamma \in SG^n$.

Conversely, for a large class of NTU-games, we can define a corresponding game in strategic form in the following way.

DEFINITION. Let $V \in NTU^n$ be called *standard* if $V(\{i\})$ is nonempty, upper-bounded, and closed for all $i \in N$.

For a standard NTU-game V , the corresponding *claim game* $\Gamma(V) \in SG^n$ is given by $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$, where for each $i \in N$

$$X_i := \{S \in 2^N \mid i \in S\} \times \mathbf{R} \quad (4)$$

and for $x = (S_j, t_j)_{j \in N} \in \prod_{j \in N} X_j$

$$K_i(x) = \begin{cases} t_i & \text{if } S_j = S_i \text{ for all } j \in S_i \text{ and } (t_j)_{j \in S_i} \in V(S_i) \\ \min\{t_i, v(i)\} & \text{otherwise} \end{cases} \quad (5)$$

with $v(i) := \max(V(\{i\}))$.

In a claim game $\Gamma(V)$ the strategy (S_i, t_i) of player $i \in N$ can be interpreted as follows. Player i wants the coalition $S_i \ni i$ to form and he claims a payoff equal to t_i . His demands are met if the wishes with respect to the coalition formation are consistent for all players in S_i (i.e., $S_j = S_i$ for all $j \in S_i$) and the payoff (demand) vector for S_i is attainable in V (i.e., $(t_j)_{j \in S_i} \in V(S_i)$). If one of these two conditions is violated, player i in principle gets his own "individual rational" payoff $v(i)$. However, if his claim t_i is less than $v(i)$, he obtains a payoff equal to t_i .

DEFINITION. Let $V \in \text{NTU}^n$ be standard. Let $x = (S_i, t_i)_{i \in N}$ be a strategy vector in the claim game $\Gamma(V)$. Then the *fitting* $F_x \subset 2^N \setminus \{\emptyset\}$ is the partition of N , defined as follows. For $S \subset N$ with $|S| \geq 2$ we have that $S \in F_x$ if

$$S_i = S \text{ for all } i \in S \quad \text{and} \quad (t_i)_{i \in S} \in V(S).$$

Further, with $i \in N$, we have that $\{i\} \in F_x$ if and only if

$$i \notin \bigcup \{S \in F_x \mid |S| \geq 2\}.$$

Consequently,

$$K_T(x) \in V(T) \quad \text{for all } T \in F_x. \quad (6)$$

Claim games are *tight* (cf. Ichiishi, 1986, p. 281) in the sense that the associated NTU-games of (2) and (3) are identical. This is seen in

LEMMA 2.2. *Let $V \in \text{NTU}^n$ be standard. Then $A_{\Gamma(V)} = B_{\Gamma(V)}$.*

Proof. Let $S \in 2^N \setminus \{\emptyset\}$ and $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$. Since it is clear that $A_{\Gamma(V)}(S) \subset B_{\Gamma(V)}(S)$, it suffices to show the converse.

Let $b \in B_{\Gamma(V)}(S)$. Let $\hat{x}_S = (S_i, t_i)_{i \in S} \in X_S$ be such that

$$K_S(\hat{x}_S, (\{j\}, v(j))_{j \in N \setminus S}) \geq b.$$

Then, without loss of generality, we may assume that $S_i \subset S$ for all $i \in S$. Consequently, for all $x_{N \setminus S} \in X_{N \setminus S}$,

$$T \cap S \neq \emptyset, T \in F_{(\hat{x}_S, x_{N \setminus S})} \Rightarrow T \in F_{(\hat{x}_S, (\{j\}, v(j))_{j \in N \setminus S})}$$

and so

$$K_S(\hat{x}_S, x_{N \setminus S}) = K_S(\hat{x}_S, (\{j\}, v(j))_{j \in N \setminus S}) \geq b.$$

Hence, $b \in A_{\Gamma(V)}(S)$. ■

DEFINITION. Let $V \in \text{NTU}^n$ be standard. Then the *claim associate* $\bar{V} \in \text{NTU}^n$ is defined by $\bar{V} := A_{\Gamma(V)} (= B_{\Gamma(V)})$.

It is easily verified that a claim associate $\bar{V}$ is standard because $\bar{V}(\{i\}) = V(\{i\})$ for all $i \in N$. Theorem 2.1 below shows that the claim associate $\bar{V}$ is the superadditive hull of V . In particular this implies that each standard superadditive NTU-game can be seen as arising from a (claim) game in strategic form.

THEOREM 2.1. Let $V \in \text{NTU}^n$ be standard. Then the claim associate $\bar{V}$ is the smallest superadditive game containing V .

Proof. Let $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$ and $S \in 2^N \setminus \{\emptyset\}$. Then, clearly, $V(S) \subset \bar{V}(S)$ because, with $a \in V(S)$ and $\hat{x}_S := (S, a_i)_{i \in S} \in X_S$, it is found that $K_S(\hat{x}_S, x_{N \setminus S}) = a$ for all $x_{N \setminus S} \in X_{N \setminus S}$, which implies that $a \in \bar{V}(S)$.

Let $W \in \text{NTU}^n$ be superadditive and assume that $V \subset W$. For proving the theorem, it suffices to show that $\bar{V} \subset W$. First we make the following observations. Let $V^1 \in \text{NTU}^n$ be standard and let $\Gamma(V^1) = (X_1, \dots, X_n, L_1, \dots, L_n)$.

(a) If $V \subset V^1$, then $\bar{V} \subset \bar{V}^1$. This immediately follows from the fact that $K_N(x) \leq L_N(x)$ for all $x \in X_N$ (cf. (5)).

(b) If V^1 is superadditive then $\bar{V}^1 = V^1$. For this, it suffices to show that $\bar{V}^1(S) \subset V^1(S)$. Let $a \in \bar{V}^1(S)$ and let $\bar{x}_S \in X_S$ be such that

$$L_S(\bar{x}_S, (\{j\}, v(j))_{j \in N \setminus S}) \geq a.$$

It is clear that the fitting $F_{(\bar{x}_S, (\{j\}, v(j))_{j \in N \setminus S})}$ induces a partition $\{S(1), \dots, S(r)\}$ of S such that, according to (6), $a_{S(k)} = (a_i)_{i \in S(k)} \in V^1(S(k))$ for all $k \in \{1, \dots, r\}$. Hence,

$$a = (a_{S(k)})_{k \in \{1, \dots, r\}} \in \prod_{k=1}^r V^1(S(k)) \subset V^1(S)$$

by superadditivity of V^1 . Consequently, $a \in V^1(S)$.

Choosing $V^1 \in \text{NTU}^n$ such that $V^1(\{i\}) = V(\{i\})$ for all $i \in N$ and $V^1(S) = W(S)$ for all $S \in 2^N$ with $|S| \geq 2$, (a) and (b) imply that

$$\bar{V} \subset \bar{V}^1 = V^1 \subset W.$$

This finishes the proof. ■

Theorem 2.1 straightforwardly implies the following “inside” description for a claim associate.

COROLLARY 2.2. *Let $V \in \text{NTU}^n$ and $S \in 2^N \setminus \{\emptyset\}$. Then*

$$\bar{V}(S) = \bigcup_{P \in (S)} \prod_{T \in P} V(T).$$

In a strategic claim game $\Gamma(V)$ it may seem unnatural for player i to ever claim a payoff that is less than his individual rational level $v(i)$. By shrinking the strategy spaces in order to rule these claims out in advance, one obtains a *restricted claim game* $\Gamma'(V)$ and a corresponding *restricted claim associate* $\bar{V}'$. It is not difficult to verify the following relation between the claim associates $\bar{V}$ and $\bar{V}'$. For all (nonempty) coalitions S , it holds that

$$\bar{V}'(S) = \text{Compr}(\{a \in \bar{V}(S) \mid a \geq (v(i))_{i \in S}\}). \quad (7)$$

3. IMPLEMENTATION OF COOPERATIVE SOLUTION CONCEPTS

First we recall the definitions of some well-known solution concepts for NTU-games.

DEFINITION. Let $V \in \text{NTU}^n$ be standard. The *imputation set* $I(V)$ is defined by

$$I(V) := \{a \in \mathbf{R}^n \mid a \in U(V(N)), a \geq (v(i))_{i \in N}\},$$

where, for $A \subset \mathbf{R}^I$, $U(A)$ denotes the set of *undominated elements* of A , i.e.,

$$U(A) := \{a \in A \mid \neg \exists b \in A : b \geq a \text{ and } b \neq a\}.$$

The *strong core* $SC(V)$ consists of those payoff vectors in $V(N)$ which are “stable” with respect to domination for each subcoalition S . More specifically,

$$SC(V) := \{a \in V(N) \mid \neg \exists_{S \in 2^{N \setminus \{0\}}} : a_S \in V(S) \setminus U(V(S))\}.$$

The *core* $C(V)$ consists of those payoff vectors in $V(N)$ which are "stable" with respect to *strict* domination, i.e.,

$$C(V) := \{a \in V(N) \mid \neg \exists_{S \in 2^{N \setminus \{0\}}} \exists_{b \in V(S)} : b > a_S\}.$$

From these definitions it is clear that the strong core is a subset of both the core and the imputation set. Furthermore, for a two-person (standard) NTU-game the imputation set and the strong core coincide.

Now we recall the definitions of the various types of equilibria for games in strategic form that play a role in implementing imputations and strong core elements.

DEFINITION. Let $\Gamma = (X_1, \dots, X_n, K_1, \dots, K_n) \in SG^n$. The set $E(\Gamma)$ of *Nash equilibria* for Γ is defined by

$$E(\Gamma) := \{x \in X_N \mid \neg \exists_{i \in N} \exists_{y_i \in X_i} : K_i(y_i, x_{N \setminus \{i\}}) > K_i(x)\},$$

the set $SE(\Gamma)$ of *strong Nash equilibria* by

$$SE(\Gamma) := \{x \in X_N \mid \neg \exists_{S \in 2^{N \setminus \{0\}}} \exists_{y_S \in X_S} : K_S(y_S, x_{N \setminus S}) \geq K_S(x) \text{ and } K_S(y_S, x_{N \setminus S}) \neq K_S(x)\},$$

and the set $PUE(\Gamma)$ of *payoff undominated equilibria* by

$$PUE(\Gamma) := \{x \in E(\Gamma) \mid \neg \exists_{y \in X_N} : K_N(y) \geq K_N(x) \text{ and } K_N(y) \neq K_N(x)\}.$$

In Theorem 3.1 below it is seen that each imputation for a (standard) NTU-game V comes forward as a payoff vector attained by a Nash equilibrium for the claim game $\Gamma(V)$. Moreover, if V is superadditive, then the imputation set $I(V)$ is equal to the set of payoff vectors attained by payoff undominated equilibria for $\Gamma(V)$.

THEOREM 3.1. Let $V \in NTU^n$ be standard and $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$. Then the following two assertions holds.

- (i) For all $a \in I(V)$ there is an $x \in E(\Gamma(V))$ such that $K_N(x) = a$.
- (ii) If V is superadditive, then $a \in I(V)$ if and only if there exists an $x \in PUE(\Gamma(V))$ such that $K_N(x) = a$.

Proof. (i) Let $a \in I(V)$. Define $x := (N, a_j)_{j \in N} \in X_N$. It is clear that $K_N(x) = a$. Suppose $x \notin E(\Gamma(V))$. Then there is a player $i \in N$ and a strategy $y_i = (S_i, t_i) \in X_i$ such that

$$t_i = K_i(y_i, x_{N \setminus \{i\}}) > a_i \geq v(i).$$

Hence, $S_i = N$ and $b := (t_i, (a_j)_{j \in N \setminus \{i\}}) \in V(N)$. However, this contradicts the fact that $a \in U(V(N))$ since $b \geq a$ and $b \neq a$.

(ii) Let V be superadditive. First assume $a \in I(V)$. By defining $x := (N, a_i)_{i \in N}$ we have (cf. (i)) $K_N(x) = a$ and $x \in E(\Gamma(V))$. Suppose $x \notin \text{PUE}(\Gamma(V))$. Then there exists a strategy vector $y \in X_N$ such that $K_N(y) \geq a$ and $K_N(y) \neq a$. Using (6) we can find a partition $\{N(1), \dots, N(r)\}$ of N such that

$$K_N(y) \in \bigcap_{k=1}^r V(N(k)) \subset V(N),$$

by superadditivity of V . However, this should imply that $a \notin U(V(N))$. Conversely, let $x \in \text{PUE}(\Gamma(V))$ and define $a := K_N(x)$. Trivially, $a \geq (v(i))_{i \in N}$. Superadditivity and (6) imply that $a \in V(N)$. It remains to prove that a is undominated in $V(N)$. Suppose we can find a vector $b \in V(N)$ such that $b \geq a$, $b \neq a$. Defining $y := (N, b_i)_{i \in N} \in X_N$ we find that $K_N(y) = b \geq a = K_N(x)$. Since $b \neq a$, this contradicts the fact that x is payoff undominated. ■

Now we show that each strong core element for a (standard) NTU-game V comes forward as a payoff vector attained by a strong Nash equilibrium for the claim game $\Gamma(V)$. Moreover, if V is superadditive, then each payoff vector attained by a strong equilibrium corresponds to a strong core element.

THEOREM 3.2. *Let $V \in \text{NTU}^n$ be standard and $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$. Then the following two assertions hold.*

- (i) *For all $a \in \text{SC}(V)$ there is an $x \in \text{SE}(\Gamma(V))$ such that $K_N(x) = a$.*
- (ii) *If V is superadditive, then $a \in \text{SC}(V)$ if and only if there exists an $x \in \text{SE}(\Gamma(V))$ such that $K_N(x) = a$.*

Proof. We only prove (i). For the proof of (ii) one can use a similar line of argument as in the corresponding part of the proof of Theorem 3.1(ii).

Let $a \in \text{SC}(V)$ and define $x := (N, a_j)_{j \in N}$. Suppose $x \notin \text{SE}(\Gamma(V))$. Then there exists a coalition $S \in 2^N \setminus \{\emptyset\}$, a player $i \in S$, and a strategy vector $y_S = (S_j, t_j)_{j \in S} \in X_S$ such that

$$t_S = K_S(y_S, x_{N \setminus S}) \geq a_S \geq (v(j))_{j \in S} \quad \text{and} \quad t_i > a_i$$

Hence, $S_i \in F_{(y_S, x_{N \setminus S})}$ and $|S_i| \geq 2$. If $S_i \subset S$, then (6) implies that

$$V(S_i) \ni K_{S_i}(y_S, x_{N \setminus S}) = t_{S_i} \geq a_{S_i}.$$

However, since $t_i > a_i$, this would imply that $a_{S_i} \notin U(V(S_i))$ and contradict the fact that $a \in SC(V)$. So we may assume that $(N \setminus S) \cap S_i \neq \emptyset$. Consequently, $S_i = N$ and

$$v(N) \ni K_N(y_S, x_{N \setminus S}) = (t_S, a_{N \setminus S}) \geq a.$$

This, however, would imply that $a \notin U(V(N))$. ■

In particular, Theorem 3.2 implies that, for two-person bargaining games, the Nash bargaining solution (which by definition is an element of the strong core) corresponds to a payoff vector attained by a (strong) Nash equilibrium of a (claim) game in a strategic form.

Now we illustrate that core elements cannot be implemented in the same way as imputations and strong core elements. More specifically, the superadditive game V of Example 2 has core elements which do not correspond to any payoff vector attained by a Nash equilibrium for the claim game $\Gamma(V)$.

EXAMPLE 2. Let the two-person NTU-game V be given by

$$V(\{1\}) = V(\{2\}) = \{a \in \mathbf{R} \mid a \leq 0\}$$

and

$$V(\{1, 2\}) = \{(a, b) \in \mathbf{R}^2 \mid a \leq 5, b \leq 5, a + b \leq 6\}.$$

Then $(5, 0) \in C(V)$, while the set of all payoff vectors attained by Nash equilibria for $\Gamma(V)$ equals

$$\{(0, 0)\} \cup \{(a, b) \in \mathbf{R}^2 \mid a + b = 6, a \geq 1, b \geq 1\}.$$

Remark. It may be noted that in all results of this section one can replace $\Gamma(V)$ by the restricted claim game $\Gamma'(V)$ because the various types of equilibria for $\Gamma(V)$ and $\Gamma'(V)$ coincide. Further, using appropriate definitions, it is possible to extend all results toward ε -(strong) cores, ε -(strong) Nash equilibria, etc.

4. CORRELATION AND FINITE CLAIM GAMES

In this section we follow Aumann (1961, 1967) and allow correlation of the players' actions in the construction of an NTU-game to a game in strategic form.

DEFINITION. Let $\Gamma = (X_1, \dots, X_n, K_1, \dots, K_n) \in SG^n$. For $S \subset N$, the set C_S of *correlated strategies* for S in Γ is defined by

$$C_S := \left\{ \sum_{k=1}^r \lambda_k \delta(x_S^k) \mid r \in \mathbf{N}, \sum_{k=1}^r \lambda_k = 1, \lambda_k \geq 0 \right. \\ \left. \text{and } x_S^k \in X_S \text{ for all } k \in \{1, \dots, r\} \right\},$$

where, for each $x_S \in X_S$, $\delta(x_S)$ represents the probability measure on X_S which assigns probability one to x_S . So correlated strategies for S in Γ correspond to probability measures on X_S with finite support. Allowing correlated strategies, the payoff functions K_i , $i \in N$, are extended in the following natural way. For $S \subset N$, $c_S = \sum_{k=1}^r \lambda_k \delta(x_S^k) \in C_S$, and $c_{N \setminus S} = \sum_{l=1}^s \mu_l \delta(x_{N \setminus S}^l) \in C_{N \setminus S}$, we define

$$K_S(c_S, c_{N \setminus S}) := \sum_{k=1}^r \sum_{l=1}^s \lambda_k \mu_l K_S(x_S^k, x_{N \setminus S}^l).$$

Further, the NTU-game $\tilde{A}_\Gamma$ corresponding to Γ is defined by

$$\tilde{A}_\Gamma(S) := \{a \in \mathbf{R}^S \mid \exists \hat{c}_S \in C_S \forall c_{N \setminus S} \in C_{N \setminus S} : K_S(\hat{c}_S, c_{N \setminus S}) \geq a\} \quad (8)$$

for all $S \in 2^N \setminus \{\emptyset\}$.

It is not difficult to verify that $A_\Gamma \subset \tilde{A}_\Gamma$, $\tilde{A}_\Gamma$ is superadditive and $\tilde{A}_\Gamma(S)$ is convex for all coalitions S . Further, if one defines $\tilde{B}_\Gamma$ is the obvious way (cf. (3)), then the same arguments as in the proof of Lemma 2.2 imply that $\tilde{A}_{\Gamma(V)} = \tilde{B}_{\Gamma(V)}$ for all standard NTU-games V .

DEFINITION. Let $V \in \text{NTU}^n$ be standard. Then the *correlated claim associate* $\tilde{V} \in \text{NTU}^n$ is defined by $\tilde{V} = \tilde{A}_{\Gamma(V)} (= \tilde{B}_{\Gamma(V)})$.

THEOREM 4.1. Let $V \in \text{NTU}^n$ be standard. Then $\tilde{V}$ is the convex hull of $\bar{V}$.

Proof. Let $S \in 2^N \setminus \{\emptyset\}$. Clearly, $\bar{V}(S) \subset \tilde{V}(S)$. So, by convexity of $\tilde{V}(S)$, $\text{Conv}(\bar{V}(S)) \subset \tilde{V}(S)$. To prove the converse, let $a \in \tilde{V}(S)$. Define $W \in \text{NTU}^n$ by $W := \text{Conv}(\bar{V})$. Then W is superadditive because $\bar{V}$ is, and $W(\{i\}) = \bar{V}(\{i\}) = V(\{i\})$ for all $i \in N$. Further, since $V \subset W$, we have that $\tilde{V} \subset \tilde{W}$. Hence, $a \in \tilde{W}(S)$ and therefore, with $\Gamma(W) = (X_1, \dots, X_n, K_1, \dots, K_n)$, there exists a correlated strategy vector $c_S = \sum_{k=1}^r \lambda_k \delta(x_S^k) \in C_S$ such that

$$a \leq K_S(c_S, (\{j\}, v(j))_{j \in N \setminus S}) \\ = \sum_{k=1}^r \lambda_k K_S(x_S^k, (\{j\}, v(j))_{j \in N \setminus S}) \in \text{Conv}(W(S)) = W(S),$$

because the superadditivity of W and (6) imply that $K_S(x_S^k, (\{j\}, v(j))_{j \in N \setminus S}) \in W(S)$ for all $k \in \{1, \dots, r\}$. Consequently, $a \in W(S)$. ■

In particular, Theorem 4.1 implies that all (standard) superadditive NTU-games with convex sets of attainable payoff vectors can be obtained from games in strategic form by means of the correlated Aumann procedure.

Now we introduce a special class of NTU-games V for which it is possible to define a *finite claim game* $\Gamma^f(V)$ in such a way that by applying the (correlated) Aumann procedure to $\Gamma^f(V)$, we again obtain the correlated claim associate $\bar{V}$ that was originally derived from the infinite claim game $\Gamma(V)$.

DEFINITION. A game $V \in \text{NTU}^n$ is called *finitely generated* if, for all $S \in 2^N \setminus \{\emptyset\}$, $V(S)$ is convex, $1 \leq |\text{Ext}(V(S))| < \infty$, and

$$V(S) = \text{Comp}(\text{Conv}(\text{Ext}(V(S)))). \quad (9)$$

Note that each finitely generated game is standard. Let $V \in \text{NTU}^n$ be finitely generated and $\Gamma(V) = (X_1, \dots, X_n, K_1, \dots, K_n)$.

The *finite claim game* $\Gamma^f(V)$ corresponding to V is given by $\Gamma^f(V) = (X_1^f, \dots, X_n^f, K_1, \dots, K_n)$, where, for each $i \in N$,

FIG. 1. (—) Boundary of $V(\{1, 2\})$; (---) boundary of $\bar{V}(\{1, 2\})$; (···) boundary of $\tilde{V}(\{1, 2\})$.

$$X_i^f := \bigcup_{S \in 2^N \setminus \{\emptyset\}: S \ni i} \{(S, t) \mid t = a_i \text{ for some } a \in \text{Ext}(V(S))\} \quad (10)$$

and the payoff function K_i is restricted to $X_N^f \subset X_N$.

Without proof we state

THEOREM 4.2. *Let $V \in \text{NTU}^n$ be finitely generated. Then $\bar{V} = \bar{A}_{\Gamma(V)} = \bar{A}_{\Gamma^f(V)}$.*

EXAMPLE 3. Let the finitely generated two-person NTU-game V be defined by

$$V(\{1\}) = V(\{2\}) = \{a \in \mathbf{R} \mid a \leq 3\}$$

and

$$V(\{1, 2\}) = \{(a, b) \in \mathbf{R}^2 \mid a \leq 4, b \leq 4, a + b \leq 5\}.$$

Then $V(\{i\}) = \bar{V}(\{i\}) = \hat{V}(\{i\})$ for all $i \in \{1, 2\}$ and

$$\bar{V}(\{1, 2\}) = V(\{1, 2\}) \cup \{(a, b) \in \mathbf{R}^2 \mid a \leq 3, b \leq 3\}$$

$$\hat{V}(\{1, 2\}) = \{(a, b) \in \mathbf{R}^2 \mid a \leq 4, b \leq 4, a + 2b \leq 9, 2a + b \leq 9\}$$

as represented in Fig. 1.

ACKNOWLEDGMENTS

The authors thank Jos Potters and Bezalel Peleg for their stimulating and helpful remarks.

REFERENCES

- AUMANN, R. J. (1961). "The Core of a Cooperative Game without Side Payments," in *Contributions to the Theory of Games* (A. W. Tucker and R. D. Luce, Eds.), Vol. IV, pp. 287–324. Princeton, NJ: Princeton Univ. Press.
- AUMANN, R. J. (1967). "A Survey of Cooperative Games without Side Payments," in *Essays in Mathematical Economics* (M. Shubik, Ed.), pp. 3–27. Princeton, NJ: Princeton Univ. Press.
- ICHISHI, T. (1986). "The Effectivity Function Approach to the Core," in *Contributions to Mathematical Economics* (W. Hildenbrand and A. Mas-Colell, Eds.), pp. 269–293. New York/Amsterdam: North-Holland.

- ICHIISHI, T. (1987). "Strong Equilibria," in *Nonlinear and Convex Analysis* (B.-L. Lin and S. Simons, Eds.), pp. 107-125, Proceedings in Honor of Ky Fan. New York: Dekker.
- NASH, J. F. (1950). "The Bargaining Problem," *Econometrica* **18**, 155-162.
- NASH, J. F. (1953). "Two-Person Cooperative Games," *Econometrica* **21**, 128-140.
- VON NEUMANN, J., AND MORGENSTERN, O. (1944). *Theory of Games and Economic Behavior*, 3rd ed. (1953), pp. 238-245. Princeton, NJ: Princeton Univ. Press.

Reprint Series, CentER, Tilburg University, The Netherlands:

- No. 1 G. Marini and F. van der Ploeg, Monetary and fiscal policy in an optimising model with capital accumulation and finite lives, *The Economic Journal*, vol. 98, no. 392, 1988, pp. 772 - 786.
- No. 2 F. van der Ploeg, International policy coordination in interdependent monetary economies, *Journal of International Economics*, vol. 25, 1988, pp. 1 - 23.
- No. 3 A.P. Barten, The history of Dutch macroeconomic modelling (1936-1986), in W. Driehuis, M.M.G. Fase and H. den Hartog (eds.), *Challenges for Macroeconomic Modelling*, Contributions to Economic Analysis 178, Amsterdam: North-Holland, 1988, pp. 39 - 88.
- No. 4 F. van der Ploeg, Disposable income, unemployment, inflation and state spending in a dynamic political-economic model, *Public Choice*, vol. 60, 1989, pp. 211 - 239.
- No. 5 Th. ten Raa and F. van der Ploeg, A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, vol. 6, no. 1, 1989, pp. 2 - 19.
- No. 6 E. van Damme, Renegotiation-proof equilibria in repeated prisoners' dilemma, *Journal of Economic Theory*, vol. 47, no. 1, 1989, pp. 206 - 217.
- No. 7 C. Mulder and F. van der Ploeg, Trade unions, investment and employment in a small open economy: a Dutch perspective, in J. Muysken and C. de Neubourg (eds.), *Unemployment in Europe*, London: The Macmillan Press Ltd, 1989, pp. 200 - 229.
- No. 8 Th. van de Klundert and F. van der Ploeg, Wage rigidity and capital mobility in an optimizing model of a small open economy, *De Economist*, vol. 137, nr. 1, 1989, pp. 47 - 75.
- No. 9 G. Dhaene and A.P. Barten, When it all began: the 1936 Tinbergen model revisited, *Economic Modelling*, vol. 6, no. 2, 1989, pp. 203 - 219.
- No. 10 F. van der Ploeg and A.J. de Zeeuw, Conflict over arms accumulation in market and command economies, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 91 - 119.
- No. 11 J. Driffill, Macroeconomic policy games with incomplete information: some extensions, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 289 - 322.
- No. 12 F. van der Ploeg, Towards monetary integration in Europe, in P. De Grauwe et al., *De Europese Monetaire Integratie: vier visies*, Wetenschappelijke Raad voor het Regeringsbeleid V 66, 's-Gravenhage: SDU uitgeverij, 1989, pp. 81 - 106.

- No. 13 R.J.M. Alessie and A. Kapteyn, Consumption, savings and demography, in A. Wenig, K.F. Zimmermann (eds.), *Demographic Change and Economic Development*, Berlin/Heidelberg: Springer-Verlag, 1989, pp. 272 - 305.
- No. 14 A. Hoque, J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model, *Journal of Econometrics*, vol. 39, no. 3, 1988, pp. 327 - 346.
- No. 15 R. Alessie, A. Kapteyn and B. Melenberg, The effects of liquidity constraints on consumption: estimation from household panel data, *European Economic Review*, vol. 33, no. 2/3, 1989, pp. 547 - 555.
- No. 16 A. Holly and J.R. Magnus, A note on instrumental variables and maximum likelihood estimation procedures, *Annales d'Économie et de Statistique*, no. 10, April-June, 1988, pp. 121 - 138.
- No. 17 P. ten Hacken, A. Kapteyn and I. Woittiez, Unemployment benefits and the labor market, a micro/macro approach, in B.A. Gustafsson and N. Anders Klevmarken (eds.), *The Political Economy of Social Security*, Contributions to Economic Analysis 179, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 143 - 164.
- No. 18 T. Wansbeek and A. Kapteyn, Estimation of the error-components model with incomplete panels, *Journal of Econometrics*, vol. 41, no. 3, 1989, pp. 341 - 361.
- No. 19 A. Kapteyn, P. Kooreman and R. Willemse, Some methodological issues in the implementation of subjective poverty definitions, *The Journal of Human Resources*, vol. 23, no. 2, 1988, pp. 222 - 242.
- No. 20 Th. van de Klundert and F. van der Ploeg, Fiscal policy and finite lives in interdependent economies with real and nominal wage rigidity, *Oxford Economic Papers*, vol. 41, no. 3, 1989, pp. 459 - 489.
- No. 21 J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model with an intercept, *Journal of Econometrics*, vol. 42, no. 2, 1989, pp. 157 - 179.
- No. 22 F. van der Ploeg, Two essays on political economy: (i) The political economy of overvaluation, *The Economic Journal*, vol. 99, no. 397, 1989, pp. 850 - 855; (ii) Election outcomes and the stockmarket, *European Journal of Political Economy*, vol. 5, no. 1, 1989, pp. 21 - 30.
- No. 23 J.R. Magnus and A.D. Woodland, On the maximum likelihood estimation of multivariate regression models containing serially correlated error components, *International Economic Review*, vol. 29, no. 4, 1988, pp. 707 - 725.
- No. 24 A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for stationary point problems on polytopes, *Mathematics of Operations Research*, vol. 14, no. 3, 1989, pp. 383 - 399.
- No. 25 E. van Damme, Stable equilibria and forward induction, *Journal of Economic Theory*, vol. 48, no. 2, 1989, pp. 476 - 496.

- No. 26 A.P. Barten and L.J. Bettendorf, Price formation of fish: An application of an inverse demand system, *European Economic Review*, vol. 33, no. 8, 1989, pp. 1509 - 1525.
- No. 27 G. Noldeke and E. van Damme, Signalling in a dynamic labour market, *Review of Economic Studies*, vol. 57 (1), no. 189, 1990, pp. 1 - 23.
- No. 28 P. Kop Jansen and Th. ten Raa, The choice of model in the construction of input-output coefficients matrices, *International Economic Review*, vol. 31, no. 1, 1990, pp. 213 - 227.
- No. 29 F. van der Ploeg and A.J. de Zeeuw, Perfect equilibrium in a model of competitive arms accumulation, *International Economic Review*, vol. 31, no. 1, 1990, pp. 131 - 146.
- No. 30 J.R. Magnus and A.D. Woodland, Separability and aggregation, *Economica*, vol. 57, no. 226, 1990, pp. 239 - 247.
- No. 31 F. van der Ploeg, International interdependence and policy coordination in economies with real and nominal wage rigidity, *Greek Economic Review*, vol. 10, no. 1, June 1988, pp. 1 - 48.
- No. 32 E. van Damme, Signaling and forward induction in a market entry context, *Operations Research Proceedings 1989*, Berlin-Heidelberg: Springer-Verlag, 1990, pp. 45 - 59.
- No. 33 A.P. Barten, Toward a levels version of the Rotterdam and related demand systems, *Contributions to Operations Research and Economics*, Cambridge: MIT Press, 1989, pp. 441 - 465.
- No. 34 F. van der Ploeg, International coordination of monetary policies under alternative exchange-rate regimes, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 91 - 121.
- No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', *European Economic Review*, vol. 34, no. 5, 1990, pp. 1011 - 1022.
- No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.
- No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, *Oxford Economic Papers*, vol. 42, no. 3, 1990, pp. 501 - 525.
- No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, *Journal of Time Series Analysis*, vol. 11, no. 3, 1990, pp. 239 - 248.
- No. 39 Th. van de Klundert, Wage differentials and employment in a two-sector model with a dual labour market, *Metroeconomica*, vol. 40, no. 3, 1989, pp. 235 - 256.

- No. 40 Th. Nijman and M.F.J. Steel, Exclusion restrictions in instrumental variables equations, *Econometric Reviews*, vol. 9, no. 1, 1990, pp. 37 - 55.
- No. 41 A. van Soest, I. Woittiez and A. Kapteyn, Labor supply, income taxes, and hours restrictions in the Netherlands, *Journal of Human Resources*, vol. 25, no. 3, 1990, pp. 517 - 558.
- No. 42 Th.C.M.J. van de Klundert and A.B.T.M. van Schaik, Unemployment persistence and loss of productive capacity: a Keynesian approach, *Journal of Macroeconomics*, vol. 12, no. 3, 1990, pp. 363 - 380.
- No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, *Journal of Econometrics*, vol. 46, no. 3, 1990, pp. 333 - 346.
- No. 44 E. van Damme, R. Selten and E. Winter, Alternating bid bargaining with a smallest money unit, *Games and Economic Behavior*, vol. 2, no. 2, 1990, pp. 188 - 201.
- No. 45 C. Dang, The D_1 -triangulation of $\mathbb{R}^n$ for simplicial algorithms for computing solutions of nonlinear equations, *Mathematics of Operations Research*, vol. 16, no. 1, 1991, pp. 148 - 161.
- No. 46 Th. Nijman and F. Palm, Predictive accuracy gain from disaggregate sampling in ARIMA models, *Journal of Business & Economic Statistics*, vol. 8, no. 4, 1990, pp. 405 - 415.
- No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, *Sankhya: The Indian Journal of Statistics*, vol. 52, series B, part. 1, 1990, pp. 1 - 13.
- No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, *Journal of Econometrics*, vol. 48, no. 1/2, 1991, pp. 83 - 117.
- No. 49 F. van der Ploeg and C. Withagen, Pollution control and the ramsey problem, *Environmental and Resource Economics*, vol. 1, no. 2, 1991, pp. 215 - 236.
- No. 50 F. van der Ploeg, Money and capital in interdependent economies with overlapping generations, *Economica*, vol. 58, no. 230, 1991, pp. 233 - 256.
- No. 51 A. Kapteyn and A. de Zeeuw, Changing incentives for economic research in the Netherlands, *European Economic Review*, vol. 35, no. 2/3, 1991, pp. 603 - 611.
- No. 52 C.G. de Vries, On the relation between GARCH and stable processes, *Journal of Econometrics*, vol. 48, no. 3, 1991, pp. 313 - 324.
- No. 53 R. Alessie and A. Kapteyn, Habit formation, interdependent preferences and demographic effects in the almost ideal demand system, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 404 - 419.

- No. 54 W. van Groenendaal and A. de Zeeuw, Control, coordination and conflict on international commodity markets, *Economic Modelling*, vol. 8, no. 1, 1991, pp. 90 - 101.
- No. 55 F. van der Ploeg and A.J. Markink, Dynamic policy in linear models with rational expectations of future events: A computer package, *Computer Science in Economics and Management*, vol. 4, no. 3, 1991, pp. 175 - 199.
- No. 56 H.A. Keuzenkamp and F. van der Ploeg, Savings, investment, government finance, and the current account: The Dutch experience, in G. Alogoskoufis, L. Papademos and R. Portes (eds.), *External Constraints on Macroeconomic Policy: The European Experience*, Cambridge: Cambridge University Press, 1991, pp. 219 - 263.
- No. 57 Th. Nijman, M. Verbeek and A. van Soest, The efficiency of rotating-panel designs in an analysis-of-variance model, *Journal of Econometrics*, vol. 49, no. 3, 1991, pp. 373 - 399.
- No. 58 M.F.J. Steel and J.-F. Richard, Bayesian multivariate exogeneity analysis - an application to a UK money demand equation, *Journal of Econometrics*, vol. 49, no. 1/2, 1991, pp. 239 - 274.
- No. 59 Th. Nijman and F. Palm, Generalized least squares estimation of linear models containing rational future expectations, *International Economic Review*, vol. 32, no. 2, 1991, pp. 383 - 389.
- No. 60 E. van Damme, Equilibrium selection in 2×2 games, *Revista Espanola de Economia*, vol. 8, no. 1, 1991, pp. 37 - 52.
- No. 61 E. Bennett and E. van Damme, Demand commitment bargaining: the case of apex games, in R. Selten (ed.), *Game Equilibrium Models III - Strategic Bargaining*, Berlin: Springer-Verlag, 1991, pp. 118 - 140.
- No. 62 W. Güth and E. van Damme, Gorbys games - a game theoretic analysis of disarmament campaigns and the defense efficiency - hypothesis -, in R. Avenhaus, H. Karkar and M. Rudnianski (eds.), *Defense Decision Making - Analytical Support and Crisis Management*, Berlin: Springer-Verlag, 1991, pp. 215 - 240.
- No. 63 A. Roell, Dual-capacity trading and the quality of the market, *Journal of Financial Intermediation*, vol. 1, no. 2, 1990, pp. 105 - 124.
- No. 64 Y. Dai, G. van der Laan, A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for the nonlinear stationary point problem on an unbounded polyhedron, *Siam Journal of Optimization*, vol. 1, no. 2, 1991, pp. 151 - 165.
- No. 65 M. McAleer and C.R. McKenzie, Keynesian and new classical models of unemployment revisited, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 359 - 381.
- No. 66 A.J.J. Talman, General equilibrium programming, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 387 - 397.

- No. 67 J.R. Magnus and B. Pesaran, The bias of forecasts from a first-order autoregression, *Econometric Theory*, vol. 7, no. 2, 1991, pp. 222 - 235.
- No. 68 F. van der Ploeg, Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, *European Economy - The Economics of EMU*, Commission of the European Communities, special edition no. 1, 1991, pp. 136 - 164.
- No. 69 H. Keuzenkamp, A precursor to Muth: Tinbergen's 1932 model of rational expectations, *The Economic Journal*, vol. 101, no. 408, 1991, pp. 1245 - 1253.
- No. 70 L. Zou, The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect, *Journal of Public Economics*, vol. 46, no. 1, 1991, pp. 51 - 89.
- No. 71 E. Bomhoff, Between price reform and privatization: Eastern Europe in transition, *Finanzmarkt und Portfolio Management*, vol. 5, no. 3, 1991, pp. 241 - 251.
- No. 72 E. Bomhoff, Stability of velocity in the major industrial countries: a Kalman filter approach, *International Monetary Fund Staff Papers*, vol. 38, no. 3, 1991, pp. 626 - 642.
- No. 73 E. Bomhoff, Currency convertibility: when and how? A contribution to the Bulgarian debate, *Kredit und Kapital*, vol. 24, no. 3, 1991, pp. 412 - 431.
- No. 74 H. Keuzenkamp and F. van der Ploeg, Perceived constraints for Dutch unemployment policy, in C. de Neubourg (ed.), *The Art of Full Employment - Unemployment Policy in Open Economies*, Contributions to Economic Analysis 203, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1991, pp. 7 - 37.
- No. 75 H. Peters and E. van Damme, Characterizing the Nash and Raiffa bargaining solutions by disagreement point axioms, *Mathematics of Operations Research*, vol. 16, no. 3, 1991, pp. 447 - 461.
- No. 76 P.J. Deschamps, On the estimated variances of regression coefficients in misspecified error components models, *Econometric Theory*, vol. 7, no. 3, 1991, pp. 369 - 384.
- No. 77 A. de Zeeuw, Note on 'Nash and Stackelberg solutions in a differential game model of capitalism', *Journal of Economic Dynamics and Control*, vol. 16, no. 1, 1992, pp. 139 - 145.
- No. 78 J.R. Magnus, On the fundamental bordered matrix of linear estimation, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 583 - 604.
- No. 79 F. van der Ploeg and A. de Zeeuw, A differential game of international pollution control, *Systems and Control Letters*, vol. 17, no. 6, 1991, pp. 409 - 414.
- No. 80 Th. Nijman and M. Verbeek, The optimal choice of controls and pre-experimental observations, *Journal of Econometrics*, vol. 51, no. 1/2, 1992, pp. 183 - 189.

- No. 81 M. Verbeek and Th. Nijman, Can cohort data be treated as genuine panel data?, *Empirical Economics*, vol. 17, no. 1, 1992, pp. 9 - 23.
- No. 82 E. van Damme and W. Güth, Equilibrium selection in the Spence signaling game, in R. Selten (ed.), *Game Equilibrium Models II - Methods, Morals, and Markets*, Berlin: Springer-Verlag, 1991, pp. 263 - 288.
- No. 83 R.P. Gilles and P.H.M. Ruys, Characterization of economic agents in arbitrary communication structures, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 325 - 345.
- No. 84 A. de Zeeuw and F. van der Ploeg, Difference games and policy evaluation: a conceptual framework, *Oxford Economic Papers*, vol. 43, no. 4, 1991, pp. 612 - 636.
- No. 85 E. van Damme, Fair division under asymmetric information, in R. Selten (ed.), *Rational Interaction - Essays in Honor of John C. Harsanyi*, Berlin/Heidelberg: Springer-Verlag, 1992, pp. 121 - 144.
- No. 86 F. de Jong, A. Kemna and T. Kloek, A contribution to event study methodology with an application to the Dutch stock market, *Journal of Banking and Finance*, vol. 16, no. 1, 1992, pp. 11 - 36.
- No. 87 A.P. Barten, The estimation of mixed demand systems, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 31 - 57.
- No. 88 T. Wansbeek and A. Kapteyn, Simple estimators for dynamic panel data models with errors in variables, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 238 - 251.
- No. 89 S. Chib, J. Osiewalski and M. Steel, Posterior inference on the degrees of freedom parameter in multivariate t regression models, *Economics Letters*, vol. 37, no. 4, 1991, pp. 391 - 397.
- No. 90 H. Peters and P. Wakker, Independence of irrelevant alternatives and revealed group preferences, *Econometrica*, vol. 59, no. 6, 1991, pp. 1787 - 1801.
- No. 91 G. Alogoskoufis and F. van der Ploeg, On budgetary policies, growth, and external deficits in an interdependent world, *Journal of the Japanese and International Economies*, vol. 5, no. 4, 1991, pp. 305 - 324.
- No. 92 R.P. Gilles, G. Owen and R. van den Brink, Games with permission structures: The conjunctive approach, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 277 - 293.
- No. 93 J.A.M. Potters, I.J. Curiel and S.H. Tijs, Traveling salesman games, *Mathematical Programming*, vol. 53, no. 2, 1992, pp. 199 - 211.
- No. 94 A.P. Jurg, M.J.M. Jansen, J.A.M. Potters and S.H. Tijs, A symmetrization for finite two-person games, *Zeitschrift für Operations Research - Methods and Models of Operations Research*, vol. 36, no. 2, 1992, pp. 111 - 123.

- No. 95 A. van den Nouweland, P. Borm and S. Tijs, Allocation rules for hypergraph communication situations, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 255 - 268.
- No. 96 E.J. Bomhoff, Monetary reform in Eastern Europe, *European Economic Review*, vol. 36, no. 2/3, 1992, pp. 454 - 458.
- No. 97 F. van der Ploeg and A. de Zeeuw, International aspects of pollution control, *Environmental and Resource Economics*, vol. 2, no. 2, 1992, pp. 117 - 139.
- No. 98 P.E.M. Borm and S.H. Tijs, Strategic claim games corresponding to an NTU-game, *Games and Economic Behavior*, vol. 4, no. 1, 1992, pp. 58 - 71.

P.O. BOX 90153 5000 LE TILBURG THE NETHERLANDS

Bibliotheek K. U. Brabant

17 000 01169759 7