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Dynamic Policy in Linear Models with Rational Expectations of Future Events: A Computer Package

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Abstract. A computer package is presented, called POREM, for policy optimisation of linear dynamic, continuous-time models with constant coefficients and rational expectations of future events, based on infinite horizons and quadratic preferences. It is possible to calculate cooperative, decentralised Nash and decentralised Stackelberg outcomes and for each outcome it is possible to allow for pre-commitment and for lack of pre-commitment vis-à-vis private sector agents. It is possible to allow for hierarchical games, that is to allow for a group of Stackelberg leaders and a group of Stackelberg followers. The input of the model is very user-friendly and can be done with the aid of mnemonics. The package is programmed in FORTRAN77 and a single-precision version is available for personal computers

Key words. Rational expectations, optimal control; differential games, Nash equilibrium, Stackelberg equilibrium, time inconsistency

1. Introduction

There has been an explosion in the use of macroeconomic and macroeconometric models that incorporate rational expectations of future events, which has been coined the 'Rational Expectations Revolution' (Begg, 1982). Mostly these arise from the presence of financial asset prices and forward-looking, optimising agents. For example, share prices may reflect the expected discounted value of the stream of future dividends. An algorithm and computer package for the simulation of linear dynamic models, in continuous or discrete time, with rational expectations of future events, called PSREM, has been presented in Markink and van der Ploeg (1989). In this companion paper our main interest is the optimal formulation of economic policy from such models. These policies are, in the absence of pre-commitment, typically time inconsistent (Kydland and Prescott, 1977; Calvo, 1978). For example, a government (monopoly trade union) may announce that it will levy most taxes on labour rather than on capital (that it will not ask for high wages) in the future in order to encourage firms to invest a lot in capital, but once the capital has been installed it is a quasi-fixed factor whose rent can be reaped by renegeing through higher capital taxes (wages) (Fischer, 1980; van der Ploeg, 1987a). Alternatively, a central bank may announce low monetary growth to

induce workers to settle for low growth in nominal wages, and then to renege with a surprise increase in monetary growth (Calvo, 1978). Since pre-commitment is not always feasible, attention has been given to time-consistent solutions. One possibility is the 'loss-of-leadership' solution (Buiter, 1986), but this has been criticised for its lack of credibility (Oudiz and Sachs, 1985). An alternative is a dynamic programming solution, specially developed for governments facing atomistic agents (Cohen and Michel, 1988), which has the advantage that it is a credible solution. Both of these solutions have been frequently applied in the literature on international policy coordination (see, for example, the studies in Buiter and Marston, 1985).

The main problem with the theory of optimal formulation of economic policy from ad-hoc macroeconomic models with rational expectations described so far is that it does not really deal explicitly with the Lucas critique of econometric policy evaluation. The problem of time inconsistency is best viewed as a dynamic game, say a government or monopoly trade union, and a number of follower-players, such as (atomistic) private sector agents or firms. Game theory cannot be properly applied unless the preferences of all players are explicitly specified, but the literature based on ad-hoc macroeconomic models does not permit one to write down the preferences of the private sector. This lack of micro foundations prevents a proper analysis of the problem of time inconsistency. In addition, it is difficult to formulate a social welfare function for a dominant player such as the government without knowledge of the utility functions of private sector agents. This explains the enormous development in the applications of dynamic game theory to economics (e.g., van der Ploeg and de Zeeuw, 1989). A large part of the methodology is summarised in Başar and Olsder (1982), which discusses open-loop and feedback Nash equilibrium solutions (Starr and Ho, 1969a) as well as open-loop and feedback Stackelberg equilibrium solutions (Simaan and Cruz, 1973a,b). A confusion that persists in part of the literature is that time consistency of the open-loop Nash equilibrium solution implies subgame perfection. This is nonsense, because the feedback (or subgame-perfect) Nash equilibrium solution produces very different outcomes. Subgame perfection does always imply time consistency. The open-loop Stackelberg equilibrium solution is, typically, time inconsistent and is analogous to the optimal control of a model with rational expectations of future events. The feedback Stackelberg equilibrium solution imposes subgame perfection and tends to the Cohen and Michel solution as the number of follower-players tends to infinity. The purpose of this paper is to present a computer package for the policy optimisation of rational-expectations models, called POREM, which can also be used to calculate the differential-game outcomes mentioned above.

Section 2 discusses the (open-loop) simulation of linear continuous-time models with constant coefficients and rational expectations of future events. Section 3 considers the open-loop optimal (cooperative) control of such models under the

presence of pre-commitment and discusses the problem of time inconsistency. Sections 4 and 5 analyse decentralised, non-cooperative Nash and Stackelberg outcomes with pre-commitment, respectively. Section 6 discusses the problem of time inconsistency and the loss-of-leadership solution. Section 7 discusses sub-game perfection, credibility and the Cohen and Michel solution to the problem of time inconsistency. Section 8 gives some numerical examples and Section 9 concludes the paper. The Appendix provides a user's guide to the package.

2. Simulation of Linear Models with Rational Expectations of Future Events

All linear dynamic continuous-time models with constant coefficients can be written as a simultaneous system of state equations

$$E_1 \dot{\tilde{x}}(t) + E_2 \Delta \tilde{x}(t) + E_3 \tilde{y}(t) + E_4 \tilde{u}(t) = 0 \quad (1)$$

and of output equations

$$E_5 \dot{\tilde{x}}(t) + E_6 \Delta \tilde{x}(t) + E_7 \tilde{y}(t) + E_8 \tilde{u}(t) = 0 \quad (2)$$

where t denotes time, Δ denotes the time derivative, $\tilde{x}(t)$ denotes the vector of state variables at time t , $\tilde{y}(t)$ denotes the vector of output variables at time t and $\tilde{u}(t)$ denotes the vector of exogenous variables at time t .

It is assumed that the state vector consists of a sub-vector of predetermined state variables, $\tilde{x}_s(t)$, and a sub-vector of non-predetermined state variables, $\tilde{x}_u(t)$, so that $\tilde{x} \equiv (\tilde{x}_s', \tilde{x}_u')'$. The non-predetermined variables are, typically, asset prices. For continuous-time models $\Delta \tilde{x}_u(t) = \lim_{s \rightarrow t} [\partial \tilde{x}_u^e(s, t) / \partial s]$, where $\tilde{x}_u^e(s, t)$ denotes the expectation of $\tilde{x}_u(s)$, $s > t$, formed at time t . Weak consistency of expectations requires that $\tilde{x}_u^e(t, t) = \tilde{x}_u(t)$, perfect hindsight requires that $\tilde{x}_u^e(s, t) = \tilde{x}_u(s, t)$, $s < t$, and perfect foresight requires that $\tilde{x}_u^e(s, t) = \tilde{x}_u(s)$, $s > t$.

It is assumed that the matrices E_7 and $E_2 - E_3 E_7^{-1} E_6$ are non-singular, so that (1)–(2) can be solved to give the reduced-form state-space model:

$$\Delta \tilde{x}(t) = \tilde{A} \tilde{x}(t) + \tilde{B} \tilde{u}(t), \quad \tilde{x}_s(0) = \tilde{x}_s^0, \quad (3)$$

$$\tilde{y}(t) = \tilde{C} \tilde{x}(t) + \tilde{D} \tilde{u}(t), \quad (4)$$

where

$$[\tilde{A}, \tilde{B}] \equiv -(E_2 - E_3 E_7^{-1} E_6)^{-1} [E_1 - E_3 E_7^{-1} E_5, E_4 - E_3 E_7^{-1} E_8] \quad (5)$$

$$[\tilde{C}, \tilde{D}] \equiv -E_7^{-1} [E_5 + E_6 A, E_8 + E_6 B]. \quad (6)$$

It is easy to allow for higher-order derivatives and integrals in (1) and (2) by appropriately augmenting the state vector. The solution to (3) is well known (Buiter, 1984):

$$\begin{aligned}\tilde{x}_s(t) = & M_{ss} \exp(\Lambda_s t) M_{ss}^{-1} \tilde{x}_s^0 + \int_0^t M_{ss} \exp[\Lambda_s(t-s)] M_{ss}^{-1} \tilde{B}_s \tilde{u}(s) ds \\ & - \int_0^t M_{ss} \exp[\Lambda_s(t-s)] M_{ss}^{-1} A_{su} N_{uu}^{-1} \int_s^\infty \exp[\Lambda_u(s-\tau)] \\ & \times [N_{us}, N_{uu}] \tilde{B} \tilde{u}^e(\tau, s) d\tau ds\end{aligned}\quad (7)$$

$$\begin{aligned}\tilde{x}_u(t) = & -N_{uu}^{-1} N_{us} \tilde{x}_s(t) - N_{uu}^{-1} \int_t^\infty \exp[\Lambda_u(t-s)] \\ & \times [N_{us}, N_{uu}] \tilde{B} \tilde{u}^e(s, t) ds\end{aligned}\quad (8)$$

where the spectral decomposition of $\tilde{A}$ is given by $\tilde{A} = M \Lambda M^{-1} = N^{-1} \Lambda N$, the diagonal matrix Λ contains first the eigenvalues with negative real parts and then the eigenvalues with positive real parts, the columns of the matrix M contain the eigenvectors of $\tilde{A}$, and $\tilde{A}_{ss} - \tilde{A}_{su} N_{uu}^{-1} N_{us} = M_{ss} \Lambda_s M_{ss}^{-1}$. It is assumed that the saddlepoint property is satisfied, that is, the number of eigenvalues with negative real parts must equal $\dim(\tilde{x}_s)$. It is also assumed that all eigenvalues are distinct. The solution (7)–(8) assumes that all explosive trajectories are ruled out by a kind of ‘transversality’ condition, so that there is a unique, convergent, perfect-foresight solution. It follows that the state variables are a decaying function of the initial values of the predetermined state variables and of past and current values of the exogenous variables and a function of past and current expectations of all future exogenous variables. It is possible to extend this methodology to allow for finite-horizon two-point-boundary-value problems, which have an initial condition for the predetermined variables and a terminal condition, for the non-predetermined variables at some finite date in the future (Markink and van der Ploeg, 1989).

In most applications the above methodology is used to investigate the effects of anticipated and unanticipated shocks in policy instruments on the economy. A user-friendly package PSREM is available for this purpose, which also allows for discrete-time and sampled-data systems (Markink and van der Ploeg, 1989). However, in this paper we are mainly concerned with optimal policy formulation. Hence, the vector $\hat{y}$ will include the policy instruments as they are now endogenous whilst the vector $\tilde{u}$ will include the desired values for the output variables as they are exogenous. The system (1)–(2) then gives the first-order conditions of an optimal control problem. Section 3–5 show how this can be done for cooperative, Nash–Cournot and Stackelberg outcomes, respectively.

3. Cooperative Optimal Control with Pre-commitment

It is assumed that the objective functions of each controller are quadratic and that the state-space model is linear with constant coefficients. A disadvantage of quadratic objective functions is that preferences need not be symmetric. For example, unemployment may be more undesirable than over-employment. Never-

theless, the assumption of quadratic objective functions is retained for analytical convenience. Alternatively, quadratic objective functions can be viewed as a second-order Taylor series approximation of the true objective function. Similarly, the linear state-space model can be viewed as a first-order Taylor series approximation of the true non-linear state-space model*. The controllers have an infinite planning horizon and may discount the future. The problem is thus for each controller to choose its policy instruments to minimise at time t an objective or loss function of the form

$$L_i(t) = \int_t^\infty \frac{1}{2} \left[\|y(s) - y_i^d(s)\|_{Q_i}^2 + \sum_{j=1}^N \|u_j(s) - u_{ij}^d(s)\|_{R_{ij}}^2 \right] \times \exp[-\rho(s-t)] ds \quad (9)$$

subject to the linear state-space model

$$\Delta x = Ax + \sum_{i=1}^N B_i u_i + B_{N+1} u_{N+1} = Ax + \bar{B} \bar{u} + b = Ax + Bu \quad (10)$$

and the output equations

$$y = Cx + \sum_{i=1}^N D_i u_i + D_{N+1} u_{N+1} = Cx + \bar{D} \bar{u} + d = Cx + Du, \quad (11)$$

where $\|y\|_Q^2 \equiv y' Q y$ denotes the weighted Euclidean norm, y , y^d , x , u_i , $i = 1, \dots, N$, u_{N+1} , Q_i , R_{ij} and ρ denotes the vector of output (or target) variables, the vector of desired values for the output variables, the vector of state variables, the vector of controllable exogenous variables or policy instruments of controller i for $i = 1, \dots, N$, the vector of uncontrollable exogenous variables, the penalty matrices and the rate of discount, respectively. The policy instruments of the various controllers can be aggregated in one vector, $\bar{u} \equiv (u_1', \dots, u_N')'$, so that $\bar{B} \equiv (B_1, \dots, B_N)$, $\bar{D} \equiv (D_1, \dots, D_N)$, $B \equiv (\bar{B}, B_{N+1})$, $D \equiv (\bar{D}, D_{N+1})$. The penalty matrices, Q_i and R_{ij} , are symmetric and positive semi-definite. The loss function includes the policy instruments of all the controllers in order to avoid instrument instability and to allow for a direct impact upon welfare. The present formulation allows for non-zero desired values of the target variables, so that allowance can be made for long-run and persistent policy trade offs. The reduced-form, state-space model (10)–(11) can be derived from a general structural-form model with higher-order derivatives of the state variables and policy instruments (see Section 2 and Markink and van der Ploeg, 1989). The vector of state variables, x , consists of a sub-vector of predetermined state variables, x_s , and a sub-vector of non-predetermined state variables, x_u , so that $x \equiv (x_s', x_u')'$.

Cooperation among the controllers can lead to Pareto-efficient strategies. They

* One needs to be careful here, because it may be more efficient to have a quadratic approximation to the Hamiltonian system (Mayne, 1966).

are derived from minimising a weighted combination of the objective functions of each of the controllers:

$$\text{Min}_{\{u(s), s \geq t\}} \sum_{i=1}^N \alpha_i L_i(t), \quad \alpha_i \geq 0, \quad \sum_{i=1}^N \alpha_i = 1 \quad (12)$$

subject to (9), (10) and (11). There are two possible outcomes under cooperative control depending on whether each controller can convince private sector agents that it can pre-commit itself to announced policies or not. Sections 6 and 7 consider situations where each controller has no credibility and thus cannot manipulate the expectations of private sector agents. Here it is assumed that individual controllers can pre-commit and stick to announced policies. This situation will be referred to as *cooperation with pre-commitment* (CP) and is relevant for both open-loop and closed-loop information patterns.

The CP-outcome can be derived with the aid of Pontryagin's Minimum Principle, hence define the Hamiltonian

$$H \equiv \sum_{i=1}^N \frac{1}{2} \alpha_i \left[\|Cx + \bar{D}\bar{u} + d - y_i^d\|_{Q_i}^2 + \sum_{j=1}^N \|u_j - u_{ij}^d\|_{R_{ij}}^2 \right] + \psi'(Ax + \bar{B}\bar{u} + b) \quad (13)$$

where ψ denotes the vector of (undiscounted) discounted shadowprices (or co-states) associated with x . The first-order conditions are (10), (11),

$$H_{\bar{u}} = \bar{D}'(Qy - q) + \hat{R}_i \bar{u} - r + \bar{B}'\psi = 0 \quad (14)$$

and

$$\rho\psi - \Delta\psi = H_x = A'\psi + C'(Qy - q), \quad (15)$$

where $Q \equiv \sum_{j=1}^N \alpha_j Q_j$ is the average penalty matrix for the target variables of the various controllers, $q \equiv \sum_{j=1}^N \alpha_j Q_j y_j^d$, $R_i \equiv \sum_{j=1}^N \alpha_j R_{ji}$ is the average penalty matrix for the policy instruments of controller i ,

$$r_i \equiv \sum_{j=1}^N \alpha_j R_{ji} u_{ji}^d, \quad \hat{R}_i \equiv \begin{bmatrix} R_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_N \end{bmatrix} \quad \text{and} \quad r \equiv (r'_1, \dots, r'_N)'$$

The predetermined state variables are historically fixed at the beginning of the planning period, $x_s(0) = x_s^0$, and the corresponding shadowprices must satisfy the appropriate transversality conditions. The non-predetermined state variables, x_u , are unconstrained by their past history and free to take on any value at the beginning of the planning period. Hence, their marginal contribution to the optimal value of the cooperative loss should be zero. It follows that the corresponding shadowprices at the beginning of the planning period should equal zero, $\psi_u(0) = 0$ (cf., Calvo, 1978).

The easiest way to solve the outcome under cooperation with pre-commitment is to transform the first-order conditions into the general structural-form, state-space model (1)–(2), because then the standard techniques for the simulation of linear dynamic models with rational expectations of future events (Buiter, 1984; Markink and van der Ploeg, 1989) can be used. Since the predetermined state variables and the shadowprices associated with the non-predetermined state variables behave in a similar fashion, they are collected in $\tilde{x}_s \equiv (x'_s, -\psi'_u)'$. Similarly, the non-predetermined state variables and the (non-predetermined) shadowprices associated with the predetermined state variables are collected in $\tilde{x}_u \equiv (\psi'_s, x'_u)'$. The policy instruments have now become endogenous, so that $\tilde{y} \equiv (y', \tilde{u}')'$. Finally, the vector of exogenous variables now includes the desired values for the targets and instruments in the loss functions so that $\tilde{u} \equiv (q', r', u'_{N+1})'$. It follows that the structural-form matrices for the outcome under cooperation with pre-commitment are given by:

$$\begin{aligned} E_1 &\equiv \begin{bmatrix} A_{ss} & 0 & 0 & A_{su} \\ A_{us} & 0 & 0 & A_{uu} \\ 0 & A'_{us} & \rho I - A'_{ss} & 0 \\ 0 & A'_{uu} - \rho I & -A'_{su} & 0 \end{bmatrix}, & E_2 &\equiv \begin{bmatrix} -I & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \\ 0 & I & 0 & 0 \end{bmatrix}, \\ E_3 &\equiv \begin{bmatrix} 0 & \bar{B} \\ -C'Q & 0 \end{bmatrix}, & E_4 &\equiv \begin{bmatrix} 0 & 0 & B_{N+1} \\ C' & 0 & 0 \end{bmatrix}, \\ E_5 &\equiv \begin{bmatrix} C_s & 0 & 0 & C_u \\ 0 & -\bar{B}'_u & \bar{B}'_s & 0 \end{bmatrix}, & E_6 &\equiv 0, & E_7 &\equiv \begin{bmatrix} -I & \bar{D} \\ \bar{D}'Q & \hat{R}_i \end{bmatrix}, \\ E_8 &\equiv \begin{bmatrix} 0 & 0 & D_{N+1} \\ -\bar{D}' & -I & 0 \end{bmatrix}, & \tilde{x}_s(0) &\equiv (x_s^0, 0')'. \end{aligned} \quad (16)$$

The CP-outcome can thus be solved with a standard simulation package for models with rational expectations of future events, such as PSREM (see Section 2). The set of Pareto-efficient solutions is not unique, because they depend on the weights (α_i) given to the objectives of the various controllers. This set can be restricted somewhat, because each of the controllers has to do at least as well under cooperation as under non-cooperation (say, the NP-outcome discussed in Section 4). Hence, Pareto-efficiency requires also that $L_i^{CP} \leq L_i^{NP}$, $i = 1, N$ and $L_i^{CP} < L_i^{NP}$ for at least one i . Particular Pareto-efficient outcomes are the *Nash Bargaining Solution with pre-commitment* (NBP) and the *Kalai–Smorodinski Solution with pre-commitment* (KP). The NBP-outcome minimises the 'Nash product', $\prod_{i=1}^N (L_i^{CP} - L_i^{NP})$. The appropriate outcome values of the weights given to the various controllers can easily be found by trial and error and must satisfy

$$\begin{aligned} 0 \leq \alpha_i &\equiv \prod_{j=1, j \neq i}^N (L_j^{CP} - L_j^{NP}) \\ \cdot \left[\sum_{k=1}^N \prod_{j=1, j \neq k}^N (L_j^{CP} - L_j^{NP}) \right] &\leq 1. \end{aligned} \quad (17)$$

The papers collected in Binmore and Dasgupta (1987) have recently provided a non-cooperative or behavioural foundation, rather than an axiomatic foundation, of the Nash Bargaining Solution, but it is not clear yet how this would fit in within a differential-game framework. The main lesson is that the threat point need not necessarily be the non-cooperative Nash–Cournot outcome, but should reflect the discounted costs of not coming to an agreement.

The KP-outcome replaces the axiom of independence of irrelevant alternatives with the axiom of individual monotonicity. The KP-outcome makes use of the concept of 'ideal' points, which are the minimum losses each controller can obtain when the other controllers fully cooperate (i.e. the cooperative outcome corresponding to $\alpha_i = 1$ for controller i). In the space of losses for the various controllers, the KP-outcome corresponds to the intersection of the set of Pareto-efficient outcomes and the line which connects the 'threat' points, say the L_i^{NC} , and the 'ideal' points.

4. Non-cooperative Nash Outcome with Pre-commitment

This section considers the *non-cooperative Nash outcome with pre-commitment* (NP), which is relevant for open-loop information patterns. None of the controllers cooperate. Instead, each controller minimises its loss given the past and future values of the policy instruments of the other controllers.

The Hamiltonian of controller i is defined as

$$H^i \equiv \frac{1}{2} \left\| Cx + \sum_{j=1}^N (D_j u_j) + D_{N+1} u_{N+1} - y^d \right\|_{Q_i}^2 + \frac{1}{2} \sum_{j=1}^N \|u_j - u_{ij}^d\|_{R_{ij}}^2 + \psi_i' \left[Ax + \sum_{j=1}^N (B_j u_j) + B_{N+1} u_{N+1} \right], \quad (18)$$

where ψ_i denotes the vector of (undiscounted) shadowprices (or co-states) associated with x for controller i . The first-order conditions are (10), (11),

$$H_{u_i}^i = D_i'(Q_i y - q_i) + R_{ii} u_i - r_{ii} + B_i' \psi_i = 0, \quad i = 1, \dots, N \quad (19)$$

and

$$\rho \psi_i - \Delta \psi_i = H_x^i = C'(Q_i y - q_i) + A' \psi_i, \quad i = 1, \dots, N, \quad (20)$$

where $q_i \equiv Q_i y^d$ and $r_{ij} \equiv R_{ij} u_{ij}^d$. As before, the predetermined state variables are historically given at the beginning of the planning period, $x_s(0) = x_s^0$. The marginal contribution of the non-predetermined state variables at time zero to the loss of each controller must be zero, so that $\psi_u^i(0) = 0$, $i = 1, \dots, N$.

In order to fit the first-order conditions associated with the NP-outcome into the general structural form, state-space model (1)–(2), it is convenient to define $\tilde{x}_s \equiv (x_s', -\psi_{1u}', \dots, -\psi_{Nu}')'$, $\tilde{x}_u \equiv (\psi_{1s}', \dots, \psi_{Ns}', x_u')'$, $\tilde{y} \equiv (y', \bar{u}')'$ and $\tilde{u} \equiv (q_1', \dots, q_N', r_{11}', \dots, r_{NN}', u_{N+1}')'$. It follows that the structural form matrices for the NP-outcome are given by:

$$\begin{aligned}
E_1 &\equiv \begin{bmatrix} A_{ss} & 0 & 0 & A_{su} \\ A_{us} & 0 & 0 & A_{uu} \\ 0 & \hat{A}'_{us} & \rho \hat{I} - \hat{A}'_{ss} & 0 \\ 0 & \hat{A}'_{uu} - \rho \hat{I} & -\hat{A}'_{su} & 0 \end{bmatrix}, \quad E_2 \equiv \begin{bmatrix} -I & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & -\hat{I} & 0 \\ 0 & \hat{I} & 0 & 0 \end{bmatrix}, \\
E_3 &\equiv \begin{bmatrix} 0 & \bar{B} \\ -C'_s Q_1 & 0 \\ \vdots & \vdots \\ -C'_s Q_N & 0 \\ -C'_u Q_1 & 0 \\ \vdots & \vdots \\ -C'_u Q_N & 0 \end{bmatrix}, \quad E_4 \equiv \begin{bmatrix} 0 & 0 & B_{N+1} \\ \hat{C}'_s & 0 & 0 \\ \hat{C}'_u & 0 & 0 \end{bmatrix}, \\
E_5 &\equiv \begin{bmatrix} C_s & 0 & 0 & C_u \\ 0 & -\hat{B}'_{iu} & \hat{B}'_{is} & 0 \end{bmatrix}, \quad E_6 \equiv 0, \quad E_7 \equiv \begin{bmatrix} -I & \bar{D} \\ D'_1 Q_1 & \\ \vdots & \hat{R}_{ii} \\ D'_N Q_N & \end{bmatrix}, \\
E_8 &\equiv \begin{bmatrix} 0 & 0 & D_{N+1} \\ -\hat{D}'_i & -\hat{I} & 0 \end{bmatrix}, \quad \tilde{x}'_s(0) = (x_s^{0'}, 0')', \quad (21)
\end{aligned}$$

where the notation $\hat{\cdot}$ denotes repetition in a diagonal matrix for the N controllers (as for $\hat{R}_i$ in Section 3). The NP-outcome can thus also easily be cast into the formulation discussed in Section 2 and thus be solved in the usual manner with PSREM.

The non-cooperative Nash outcome, typically, yields higher losses for all controllers than the cooperative outcome due to a variety of externalities. The main externalities are: (i) the policy instruments of each controller affect (through the B_j) the state-space dynamics of all other controllers; (ii) the policy instruments of each controller affect (through the D_j) the target variables of all other controllers; (iii) the policy instruments of each controller affect (through the R_{ij}) the loss of all other controllers. The third form of externality does not affect the NP-outcome at all, since each controller does not take account of the effects of the changes in policy of other controllers. The CP-outcome internalises the external effects on its loss mentioned under (i) and (ii).

The special case of the non-cooperative Nash outcome with pre-commitment when there are no non-predetermined state variables, such as asset prices, present ($\dim(x_u) = 0$) has been originally worked out by Starr and Ho (1969a, b) and is discussed at length in Başar and Olsder (1982). For this special case, the NP-outcome is time consistent as there is no incentive to reoptimise at a later stage. However, this does not mean that the NP-outcome is subgame perfect or credible and, in general, it differs from the feedback Nash outcome. For example, monitoring of weapon stocks in a dynamic armaments game implies that the feedback Nash outcome is relevant and leads to lower weapon stocks and higher

welfare than the NP-outcome (van der Ploeg and de Zeeuw, 1990). Hence, time consistency does not imply subgame perfection but subgame perfection always implies time consistency. The computer package developed in de Zeeuw (1984) can be used to calculate the feedback Nash equilibrium when there are no non-predetermined state variables present. When non-predetermined state variables are present, the NP-outcome is typically time inconsistent (as $\psi_u(t) \neq 0$, $t > 0$) in addition to be not subgame perfect.

5. Non-cooperative Stackelberg Outcome with Pre-commitment

This section considers the *non-cooperative Stackelberg outcome with pre-commitment* (SP), which again is relevant for open-loop information patterns. Hence, as in Sections 3 and 4, it is assumed that all controllers can pre-commit themselves to their announced sequences of future policies. This can be achieved via constitutional law or via reputational forces. The first M controllers are Stackelberg followers and display non-cooperative Nash behaviour among each other. The remaining $N - M$ controllers are Stackelberg leaders vis-à-vis the first M controllers and also display non-cooperative Nash behaviour among themselves. This is a form of hierarchical control often found in economics. For example, in the field of international policy coordination, the countries of the Group of Three (US, Japan and Germany) could adopt the role of Stackelberg leaders whilst the remaining countries (Canada, UK, France, Italy among others) adopt the role of follower. Of course, it is possible to think of a hierarchical model where the latter $N - M$ controllers cooperate among themselves and at the same time adopt a leadership role vis-à-vis the other M controllers. This situation is best handled by treating the latter $N - M$ controllers as one single controller, who is the Stackelberg leader.

The first M controllers adopt a following role and thus their behaviour is described by

$$D'_j(Q_j y - q_j) + R_{jj} u_j - r_{jj} + B'_j \psi_j = 0, \quad j = 1, \dots, M \quad (22)$$

$$\rho \psi_j - \Delta \psi_j = C'(Q_j y - q_j) + A' \psi_j, \quad j = 1, \dots, M \quad (23)$$

(cf. Equations ((19)–(20)). The remaining $N - M$ controllers adopt a leadership role and minimise their loss subject to (10), (11), (22) and (23) given expectations about the exogenous variables and the values of the policy instruments adopted by the other Stackelberg leaders. It follows that the Hamiltonians for the $N - M$ Stackelberg leaders are defined as

$$H^i \equiv \frac{1}{2} \left\| Cx + \sum_{j=1}^N (D_j u_j) + D_{N+1} u_{N+1} - y^d \right\|_{Q_i}^2 + \frac{1}{2} \sum_{j=1}^N \|u_j - u_{ij}^d\|_{R_{ij}}^2 \\ + \psi_i' \left[Ax + \sum_{j=1}^N (B_j u_j) + B_{N+1} u_{N+1} \right]$$

$$\begin{aligned}
& + \sum_{j=1}^M \mu'_{ij} \left\{ (\rho I - A') \psi_j - C' Q_j \left[Cx + \sum_{k=1}^N (D_k u_k) + D_{N+1} u_{N+1} \right] + C' q_j \right\} \\
& + \sum_{j=1}^M v'_{ij} \left\{ R_{ij} u_j - r_{ij} + D'_j Q_j \left[Cx + \sum_{k=1}^N (D_k u_k) + D_{N+1} u_{N+1} \right] \right. \\
& \quad \left. - D'_j q_j + B'_j \psi_j \right\}, \quad i = M+1, \dots, N,
\end{aligned} \quad (24)$$

where μ_{ij} denotes the vector of (undiscounted) shadowprices associated with ψ_j for controller i and v_{ij} denotes the Lagrange multipliers associated with equation (22) for controller i . The first-order conditions follow from Pontryagin's Minimum Principle and are given by (10), (11), (22), (23),

$$\begin{aligned}
H^i_{u_i} &= D'_i (Q_i y - q_i) + R_{ii} u_i - r_{ii} + B'_i \psi_i - \sum_{k=1}^M (D'_i Q_k C \mu_{ik}) \\
&+ \sum_{k=1}^M (D'_i Q_k D_k v_{ik}) = 0, \quad i = M+1, \dots, N,
\end{aligned} \quad (25)$$

$$\begin{aligned}
H^i_{u_j} &= D'_j (Q_i y - q_i) + R_{ij} u_j - r_{ij} + B'_j \psi_i - \sum_{k=1}^M (D'_j Q_k C \mu_{ik}) \\
&+ \sum_{k=1}^M (D'_j Q_k D_k v_{ik}) + R_{ij} v_{ij} = 0, \quad j = 1, \dots, M, \\
i &= M+1, \dots, N,
\end{aligned} \quad (26)$$

$$\begin{aligned}
\rho \psi_i - \Delta \psi_i &= H^i_x = C' (Q_i y - q_i) + A' \psi_i - \sum_{k=1}^M (C' Q_k C \mu_{ik}) \\
&+ \sum_{k=1}^M (C' Q_k D_k v_{ik}), \quad i = M+1, \dots, N
\end{aligned} \quad (27)$$

and

$$\begin{aligned}
\rho \mu_{ij} - \Delta \mu_{ij} &= H^i_{\psi_j} = (\rho I - A) \mu_{ij} + B_j v_{ij}, \quad j = 1, \dots, M, \\
i &= M+1, \dots, N.
\end{aligned} \quad (28)$$

Note that the Stackelberg leaders take account of the third type of externality associated with the effect of changes in policies of the followers on their losses through the R_{ij} , which is not taken account of in the NP-outcome. The predetermined variables now consist of the predetermined state variables, x_s , the shadowprices of the non-predetermined state variables, ψ_{iu} , $i = 1, \dots, N$, and the shadowprices of the followers' shadowprices associated with the predetermined state variables, μ_{ijs} , $j = 1, \dots, M$, $i = M+1, \dots, N$. The remaining variables are non-predetermined. Hence, it is sensible to define

$$\tilde{x}'_s \equiv (x'_s, -\psi'_{1u}, \dots, -\psi'_{Nu}, \mu'_{M+1,1,s}, \dots, \mu'_{M+1,M,s}, \dots, \mu'_{N,1,s}, \dots, \mu'_{N,M,s})',$$

$$\begin{aligned}\tilde{x}_u &\equiv (\psi'_{1s}, \dots, \psi'_{Ns}, \mu'_{M+1,1,u}, \dots, \mu'_{M+1,M,u}, \dots, \mu'_{N,1,u}, \dots, \mu'_{N,M,u}, x'_u)', \\ \tilde{y} &\equiv (y', \tilde{u}, v'_{M+1,1}, \dots, v'_{M+1,M}, \dots, v'_{N,1}, \dots, v'_{N,M}),\end{aligned}$$

and

$$\tilde{u} \equiv (q'_1, \dots, q'_N, r'_{11}, \dots, r'_{NN}, r'_{M+1,1}, \dots, r'_{M+1,M}, \dots, r'_{N,1}, \dots, r'_{NM}, u'_{N+1})'$$

in order to be able to cast the first-order conditions into the form (1)–(2) discussed in Section 2. The structural-form matrices for the Stackelberg outcome with pre-commitment are then given by:

$$\begin{aligned}E_1 &\equiv \begin{bmatrix} A_{ss} & 0 & 0 & 0 & 0 & A_{su} \\ A_{us} & 0 & 0 & 0 & 0 & A_{uu} \\ 0 & \hat{A}'_{us} & \hat{V}_{ss} & \rho \hat{I} - A'_{ss} & \hat{V}_{su} & 0 \\ 0 & \hat{A}'_{uu} - \rho I & \hat{V}_{us} & -\hat{A}'_{su} & \hat{V}_{uu} & 0 \\ 0 & 0 & -\hat{A}_{ss} & 0 & -\hat{A}_{su} & 0 \\ 0 & 0 & -\hat{A}_{us} & 0 & -\hat{A}_{uu} & 0 \end{bmatrix}, \\ E_2 &\equiv \begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \\ 0 & 0 & 0 & -\hat{I} & 0 & 0 \\ 0 & \hat{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{I} & 0 \end{bmatrix}, \\ E_3 &\equiv \begin{bmatrix} 0 & \bar{B} & 0 \\ -C'_s Q_1 & & \hat{W}_s \\ \vdots & 0 & \hat{W}_s \\ -C'_s Q_N & & \\ -C'_u Q_1 & & \hat{W}_u \\ \vdots & 0 & \hat{W}_u \\ -C'_u Q_N & & \\ 0 & 0 & \hat{B}^* \end{bmatrix}, \quad E_4 \equiv \begin{bmatrix} 0 & 0 & B_{N+1} \\ \hat{C}'_s & 0 & 0 \\ \hat{C}'_u & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (29)\end{aligned}$$

and similarly for $E_5, \dots, E_8$, where

$$\begin{aligned}V &\equiv C'[Q_1, \dots, Q_N]C, \quad W \equiv -C'[Q_1 D_1, \dots, Q_M D_M], \\ B_s^* &\equiv \begin{bmatrix} B_{1s} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_{Ms} \end{bmatrix}, \quad B_u^* \equiv \begin{bmatrix} B_{1u} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_{Mu} \end{bmatrix},\end{aligned}$$

and the notation $\hat{V}_{su}$ defines a block-diagonal matrix of N block-rows and N block-columns where the first M diagonal blocks are zero matrices and the remaining $N - M$ diagonal blocks are given by V_{su} . Since $\psi_{iu}(0) = 0$, $i = 1, \dots, N$

and $\mu_{ij_s}(0) = 0$, $i = M+1, \dots, N$, $j = 1, \dots, M$, the initial conditions are $\hat{x}_s(0) = (x_s^{0'}, 0')'$. The solution can now easily be found with the aid of the method discussed in Section 2.

The Stackelberg outcome presented above is typically time inconsistent, even when there are no forward-looking state variables present ($\dim(x_u) = 0$). The reason is that the dominant players can manipulate the future actions of the followers, through the forward-looking co-states (ψ_j , $j = 1, \dots, M$), and thus the future state of the economy ($x_s = x$), but renege at a later date (by resetting the μ_{ij} , $j = 1, \dots, M$, $i = M+1, \dots, N$ to zero). Hence, even when there are forward-looking state variables, the SP-outcome is time inconsistent and thus not subgame perfect. A good example is the time inconsistency of optimal taxation of capital; here the leader is the government, the follower is the representative producer-consumer, the state variable is the capital stock, the policy instruments of the producer-consumer are consumption, labour supply and investment and the policy instruments of the government are the tax rates on labour and on capital income (Fischer, 1980). The government then has an incentive to announce a low tax on capital and a high tax on labour income, but once the capital is in place to renege and levy a high tax on capital and a low tax on labour income.

6. Time Consistency and 'Loss of Leadership'

The optimal strategies discussed in Sections 3–5 yield rational expectations equilibria with open-loop information patterns and binding contracts or pre-commitment. The presence of pre-commitment ensures that controllers cannot renege on their announced policies as time proceeds. A potential time inconsistency arises from the fact that by merely announcing changes in *future* policy one can achieve improvement in the *current* state of the economy. Once the future becomes the present, it may no longer be optimal to stick to the announced change in policy and, in the absence of pre-commitment, controllers may renege (cf., Kydland and Prescott, 1977; Calvo, 1978).

This and the following section are concerned with appropriate solutions when binding contracts are not available or pre-commitment is not feasible. In such situations the open-loop equilibria discussed so far are not credible, because of the problem of time inconsistency discussed above. An early proposal for resolving the problem of time inconsistency when pre-commitment is not feasible is the 'loss of leadership' solution (Buiter, 1986). This solution builds on the static solutions of Kydland and Prescott (1977) and Barro and Gordon (1983), which assume that controllers give up any attempt to manipulate the expectations of the private sector and to manipulate the outcomes for the non-predetermined state variables. Such a solution is analogous to each controller re-optimising at each instant of time, so that the marginal contribution of the non-predetermined variables to their welfare is always equal to zero. For Section 3 this implies that

$\psi_u(t) = 0$, for all $t \geq 0$, for Section 4 this implies that $\psi_{iu}(t) = 0$, $i = 1, \dots, N$, for all $t \geq 0$, and for Section 5 this implies that $\psi_{iu}(t) = 0$, $i = 1, \dots, N$ and $\mu_{ijs}(t) = 0$, $i = M + 1, \dots, N$, $j = 1, \dots, M$ for all $t \geq 0$. The costate equations associated with these variables can thus simply be dropped from the solutions. This leads to the *cooperative outcome with loss of leadership* (CL), the *non-cooperative Nash outcome with loss of leadership* (NL), and the *non-cooperative Stackelberg outcome with loss of leadership* (SL), respectively. The loss of leadership outcomes are time consistent, but as continuous cheating means that expectations are never fulfilled they are not credible (see the critique in Oudiz and Sachs, 1985).

This can easily be seen when one considers the special case where there are no non-predetermined state variables present and where $\sum_{k=1}^M D'_i Q_k D_k = 0$, $i = M + 1, \dots, N$ and $\sum_{k=1}^M C' Q_k D_k = 0$ hold. This will be the case when $\dim(x_u) = 0$ and $\bar{D} = 0$. This case rules out rational expectations of future events, unless they arise from the optimising behaviour of the followers, and rules out some of the externalities between leaders and followers. The main form of externalities left are through the B_j (and R_{ij}). The non-cooperative Stackelberg outcome with pre-commitment is then time inconsistent as soon as pre-commitment is no longer feasible, because typically the $\mu_{ijs}(t)$, $i = M + 1, \dots, N$, $j = 1, \dots, M$ will differ from zero for $t > 0$. This source of time inconsistency arises from the forward-looking nature of the followers' reaction functions. The loss of leadership solution simply set $\mu_{ijs}(t) = 0$, $i = M + 1, \dots, N$, $j = 1, \dots, M$ for all $t \geq 0$, but then it is easy to show that the SP-outcome reduces to the NP-outcome. Hence, for this special case the loss of leadership outcome corresponds to the open-loop Nash outcome and is therefore not credible or subgame perfect. It thus follows that the feedback Nash or feedback Stackelberg outcomes (Başar and Olsder, 1982) are much more satisfactory solutions to the problem of time inconsistency in models with rational expectations than the loss of leadership solution. When private sector agents are atomistic, one can develop special iterative solutions (Cohen and Michel, 1988). Such solutions are discussed in the following section.

7. Credibility and Atomistic Behaviour of Private Sector Agents

Here the situation of one Stackelberg leader ($N - M = 1$) and an infinite number of atomistic Stackelberg followers and no non-predetermined state variables is considered. For simplicity, it is assumed that $\rho = 0$, $y_1^d = y_2^d = 0$, $b = 0$, and $R_{ij} = 0$, $i = 1, 2$, $j = 1, 2$. If all followers are identical, they can be replaced by an atomistic representative follower ($M = 1$, $N = 2$). The optimal reaction of the policy instruments of the atomistic followers are given by

$$u_1 = -(D'_1 Q_1 D_1)^{-1} D'_1 [Q_1 (Cx + D_2 u_2 + d) + \psi_1] \quad (30)$$

and the associated co-states of the followers satisfy

$$-\dot{\psi}_1 = C' [Q_1 (Cx + D_1 u_1 + D_2 u_2 + d) + \psi_1]. \quad (31)$$

Hence, the leader faces the following system:

$$\begin{pmatrix} \dot{x} \\ \dot{\psi}_1 \end{pmatrix} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{pmatrix} \begin{pmatrix} x \\ \psi_1 \end{pmatrix} + \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix} u_2 + \begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \end{pmatrix} \quad (32)$$

$$y = (\bar{C}_1 \bar{C}_2) \begin{pmatrix} x \\ \psi \end{pmatrix} + \bar{D} u_2 + \bar{d} \quad (33)$$

where

$$\bar{A}_{11} = A - B_1(D_1' Q_1 D_1)^{-1} D_1' Q_1 C, \quad \bar{A}_{12} = -B_1(D_1' Q_1 D_1)^{-1} D_1',$$

$$\bar{A}_{21} = -C' Q_1 [I - D_1(D_1' Q_1 D_1)^{-1} D_1' Q_1] C,$$

$$\bar{A}_{22} = -C' [I - Q_1 D_1(D_1' Q_1 D_1)^{-1} D_1'],$$

$$\bar{B}_1 = B_2 - B_1(D_1' Q_1 D_1)^{-1} D_1' Q_1 D_2,$$

$$\bar{B}_2 = -C' Q_1 [I - D_1(D_1' Q_1 D_1)^{-1} D_1' Q_1] D_2, \text{ etc.}$$

The following iterative algorithm yields the feedback Stackelberg equilibrium outcome (Cohen and Michel, 1984): (i) calculate the 'loss-of-leadership' solution for the leader (by assuming $\psi = 0$) and obtain the optimal feedback rule for the leader, say $u_2 = Gx + g$; (ii) substitute the rule into (32) and obtain the saddle-path of this system, say $\psi_1 = Hx + h$; (iii) calculate the optimal rule for the leader given the system $\dot{x} = (\bar{A}_{11} + \bar{A}_{12}H)x + \bar{B}_1 u_2 + (\bar{b}_1 + \bar{A}_{12}h)$, say $u_2 = Gx + g$; (iv) go back to (ii) and continue, until convergence of the H and h has been achieved. It is straightforward to extend this algorithm to allow for non-zero values of ρ , y_1^d , y_2^d , b and R_{ij} , $i = 1, 2$, $j = 1, 2$. However, it is conceptually rather more difficult to allow for more than one leader or more than one representative follower as one would then not be able to avoid dynamic programming.

Many studies have used the above method to find time-consistent solutions for optimal government policy of models with rational expectations of future events (e.g., Buiter, 1983, 1986; Oudiz and Sachs, 1985). The idea is to think of the co-state of the representative follower as non-predetermined state variables often found in *ad hoc* macroeconomic models (e.g. asset prices such as exchange rates and share prices). It is not clear that this is very satisfactory, because the arbitrage equations (such as the uncovered interest parity condition) are postulated and the preferences of the atomistic private sector agents are never defined.

8. Numerical Examples in International Monetary Economics

Consider the following Mundell-Fleming two-country model (van der Ploeg, 1989):

$$y = -r + 0.375c + 0.75y^*; \quad y^* = -r - 0.375c + 0.75y \quad (34)$$

$$l \equiv m - p = y - 2i; \quad l^* \equiv m^* - p^* = y^* - 2i^* \quad (35)$$

$$p = w + s; \quad p^* = w^* + s^* \quad (36)$$

$$\dot{w} = 0.25(y - \bar{y}), \quad \bar{y} \equiv \bar{n} - s; \quad \dot{w}^* = 0.25(y^* - \bar{y}^*), \quad \bar{y}^* \equiv \bar{n}^* - s^* \quad (37)$$

$$\dot{i} \equiv r + \dot{p} + 0.25E\dot{c} = \dot{i}^* + E\dot{e} = r^* + \dot{p}^* - 0.25E\dot{c}, \quad c \equiv p^* + e - p, \quad (38)$$

All variables are in logarithmic deviations from their steady-state values. Foreign variables are denoted with an asterisk. Equation (34) gives the IS-curves: aggregate demand, y , is a negative function of the real interest rate, r , and a

Table I. Policy responses to a common adverse supply shock ($s = s^* = 1$) under alternative exchange-rate regimes

Variable	Regime No Policy	FLOAT: No pre- commitm.	FLOAT: pre- commitm.	EMS Germany	EMS rest of Europe	EMU
Output:						
0	-0.500	-0.501	-0.557	-0.636	-0.719	-0.636
5	-0.857	-0.726	-0.846	-0.774	-0.829	-0.774
∞	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
loss	36.840	34.041	35.972	34.565	35.824	34.565
Real income:						
0	-1.0	-1.0	-1.0	-1.045	-0.955	-1.0
5	-1.0	-1.0	-1.0	-1.029	-0.971	-1.0
∞	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
loss	40.0	40.0	40.0	40.833	39.188	40.0
Nominal wage rate:						
0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.357	0.470	0.329	0.362	0.277	0.362
∞	0.5	1.043	0.682	0.955	0.708	0.955
Nominal exchange rate:						
0	0.0	0.0	0.0	0.180	-0.180	0.0
5	0.0	0.0	0.0	0.203	-0.203	0.0
∞	0.0	0.0	0.246	-0.246	0.0	0.0
Nominal money supply:						
0	0.0	-0.001	-0.057	-0.136	-0.219	-0.136
5	0.0	0.244	-0.017	0.088	-0.052	0.088
∞	0.0	0.543	0.182	0.455	0.208	0.455
loss	0.0	8.832	0.771	5.418	1.126	6.418
Real interest rate:						
0	0.125	0.125	0.139	0.164	0.175	0.159
5	0.214	0.181	0.211	0.197	0.204	0.194
∞	0.25	0.25	0.25	0.25	0.25	0.25
Welfare loss:	76.840	75.807	76.126	76.482	75.237	75.649

Notes: (i) International policy coordination under a FLOAT (with or without credibility) and under the EMS yield the same outcomes as under EMU; the table presents the non-cooperative (Nash-Cournot) outcomes under FLOAT and EMS.

(ii) The losses for output, real income and the money supply refer to the discounted squared deviations from desired values.

Table II. Policy responses to an idiosyncratic adverse supply shock under alternative exchange-rate regimes

Variable	Regime	FLOAT Nash ($s = 1$)	FLOAT Pareto ($s = 1$)	EMS Nash ($s = 1$)	EMS Nash ($s^* = 1$)	EMU fixed e ($s = 1$)	EMU flexible e ($s = 1$)
y :	0	-0.705	-0.765	-0.786	0.067	-0.510	-0.749
	5	-0.873	-0.838	-0.863	0.034	-0.714	-0.837
	∞	-1.0	-10.0	-1.0	0.0	-1.0	-1.0
	loss	36.505	35.704	36.470	0.015	33.591	35.749
y^*	0	0.148	0.129	0.095	-0.732	-0.126	0.113
	5	0.027	0.064	0.059	-0.833	-0.060	0.063
	∞	0.0	0.0	0.0	-1.0	0.0	0.0
	loss	0.031	0.053	0.042	35.870	0.049	0.048
ω :	0	-0.498	-0.408	-0.481	-0.474	-0.75	-0.492
	5	-0.472	-0.472	-0.459	-0.512	-0.604	-0.471
	∞	-0.417	-0.417	-0.417	-0.582	-0.417	-0.419
	loss	7.685	7.729	7.456	12.507	9.358	7.680
ω^* :	0	-0.502	-0.520	-0.519	-0.526	-0.25	-0.508
	5	-0.528	-0.528	-0.514	-0.488	-0.396	-0.529
	∞	-0.583	-0.583	-0.583	-0.418	-0.583	-0.582
	loss	12.667	12.603	12.946	7.839	11.275	12.670
w :	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.240	0.248	0.216	0.061	0.473	0.255
	∞	0.599	0.766	0.602	0.106	1.144	0.746
w^* :	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.089	0.114	0.095	0.267	-0.111	0.107
	∞	0.083	0.189	0.25	0.705	-0.189	0.207
e :	0	-1.009	-1.078	-1.076	0.897	0.0	-1.033
	5	-0.959	-0.978	-1.044	0.841	0.0	-0.968
	∞	-0.818	-0.757	-0.981	0.735	0.0	-0.797
m :	0	0.035	-0.053	-0.036	-0.183	0.240	0.001
	5	0.108	0.147	0.103	-0.154	0.509	0.168
	∞	0.349	0.516	0.352	-0.144	0.894	0.496
	loss	3.109	6.649	3.292	0.828	24.811	6.487
m^* :	0	-0.093	-0.084	-0.155	0.019	-0.376	-0.138
	5	-0.125	-0.059	-0.096	0.184	-0.421	-0.081
	∞	-0.167	-0.061	0.0	0.455	-0.439	-0.041
	loss	0.882	0.096	0.111	5.851	7.643	0.113
welfare		44.811	44.763	44.584	12.687	47.910	44.726
welfare*		12.874	12.676	13.011	44.879	12.853	12.740
Av. welfare		57.685	57.439	57.595	57.566	60.763	57.466

Notes: (i) The EMS outcomes under international policy coordination correspond to the EMU outcomes under international policy coordination and under floating exchange rates. The FLOAT outcomes assume that central banks can pre-commit to their announced monetary policies.

(ii) The losses for output, real income and the money supply refer to the discounted squared deviations from desired values.

positive function of competitiveness, c , and foreign income, y^* . Equation (35) gives the LM-curves: money demand, l , is a positive function of income, y , and a negative function of the nominal interest rate, i . Equation (36) shows that producers' prices, p , are a constant mark-up on the unit cost of labour, $w + s$, where s denotes an adverse productivity shock. Equation (37) gives the Phillips curve: nominal wage inflation, $\dot{w}$, is an increasing function of the excess of output over full-employment output, $\bar{y} \equiv \bar{n} - s$ where $\bar{n}$ denotes the supply of labour. Equation (38) defines the real interest rates and gives the uncovered interest parity condition. The share of imports in total expenditures is 25%. Policy follows from minimising a intertemporal welfare loss function

$$W \equiv \int_0^{\infty} \exp(-0.025t) [y^2 + (w - p - 0.25c)^2 + 0.2m^2] dt \quad (39)$$

and similarly for abroad. Hence, governments try to achieve full employment and maintain real income whilst not manipulating the money supply too much.

Three exchange-rate regimes are possible. The first is floating exchange rates (FLOAT) in which both countries can adjust their money supplies and the exchange rate adjusts to ensure equilibrium in the balance of payments. The second is managed exchange rates (EMS) in which the foreign country sets the money supply (m^*) and the home country chooses the value of its exchange rate vis-à-vis the foreign currency (e). The third is monetary union (EMU) in which the nominal exchange rate is irrevocably fixed and both countries jointly determine the global money supply ($m_E \equiv \frac{1}{2}(m + m^*)$). Table I presents the non-cooperative Nash-Cournot responses to a common adverse supply shock under the three regimes. The cooperative outcomes under a FLOAT and the EMS corresponds to the outcome under EMU and yield the lowest (average) welfare loss whilst the no-policy outcome yields the highest welfare loss. Coordination under a FLOAT leads to higher welfare losses, irrespective of whether there is pre-commitment or not. Lack of pre-commitment under a FLOAT leads to higher money supplies, higher prices and lower real incomes and to smaller output losses. Under a non-cooperative FLOAT with pre-commitment each central bank tries to export inflation with competitive, futile attempts to appreciate the currency, which leads to too tight money supplies and to too high welfare losses. Table II shows the effects of a country-specific, adverse supply shock. Now a FLOAT typically yields lower welfare losses than maintaining a fixed exchange rate. Further details on the economic interpretation of these results can be found in van der Ploeg (1989).

9. Concluding Remarks

Various solutions to differential games with rational expectations of future events have been discussed. There are three cooperative (or single-controller) outcomes: (i) pre-commitment to announced policies; (ii) lack of pre-commitment/loss-of-

leadership outcome; and (iii) lack of pre-commitment/dynamic-programming outcome. The first one corresponds to *rules* and the latter two require atomistic agents and correspond to *discretion*. The package POREM allows for outcomes (i) and (ii), but unfortunately not for (iii). The paper also discusses and the package POREM also allows for (i) and (ii) for the non-cooperative Nash and non-cooperative Stackelberg outcomes. The computer package developed in de Zeeuw (1984) can be used to calculate feedback Nash and feedback Stackelberg outcomes when there are no expectations of future events. The algorithm of Cohen and Michel (1988) applies to very special cases (one Stackelberg leader, atomistic followers and no non-predetermined state variables) only and the translation from atomistic followers to *ad hoc* macroeconomic models with rational expectations of future agents is somewhat questionable.

Appendix: User's Guide

It is strongly recommended to also consult the User's Guide of the companion package PSREM (Markink and van der Ploeg, 1989), because the same input that is used for this simulation package is also used for the optimisation package POREM. The package POREM is developed for use on IBM-compatible personal computers, but a mainframe version is also available. The package is started by typing the command POREM. The package assumes that the model is formulated in continuous time and that the horizon is infinite. (PSREM also allows for sampled-data and discrete-time models and for finite horizons.) POREM then asks what kind of game you want to play:

- (1) Simulation, running PSREM with the POREM-package
- (2) Cooperation, Pareto
- (3) Non-cooperation, Nash
- (4) Non-cooperation, Stackelberg.

Subsequently, POREM asks you whether you want pre-commitment or 'loss of leadership'. If you answer with a 0, POREM assumes pre-commitment; a 1 means 'loss of leadership'.

The package then asks what print level is required. The printlevels are:

- ≥ 5 - the matrices $E_1, E_2, \dots, E_8$;
- ≥ 4 - eigenvectors of the matrix A ;
- ≥ 3 - the matrices A, B, C and D ;
- ≥ 2 - the steady-state values of x and y ;
- ≥ 1 - eigenvalues, settling times, moduli and periods associated with the matrix A and the state and output equations given in mnemonics;
- ≥ 0 - time-trajectories for the state, output and exogenous variables.

When the level is negative, the printing is done in a more compact format. For the case of Pareto, the package asks you to give the weights for the cooperative outcome (α_i). For the Stackelberg case, you are asked to give the number of followers (M). At the end of the computations, POREM asks whether the user wants to plot the time-trajectories for the variables and, if so, what the name of the graphics file is.

The input file should contain the following information:

- The title of the exercise (on one line with a maximum of 80 characters).
- For the mainframe version, the number of characters that fit on one line of output (default is 80), the number of characters used to print a number (default is 10), the number of decimals (default is 4) to be printed. If you give a zero, then the default value will be taken.
- The number of predetermined and the number of non-predetermined state variables followed by the names (mnemonics) of the state variables (on separate lines and each name must be no longer than 10 characters and start with a letter).
- The number of output variables followed by the mnemonics of these variables (on separate lines and each name must be no longer than 10 characters and start with a letter).
- The number of exogenous variables followed by the mnemonics of these variables (on separate lines and each name must be no longer than 10 characters and start with a letter).
- An integer, which is zero if the matrices $E_1, E_2, \dots, E_8$ are given directly and which is unity if the model is given in terms of the mnemonics.
- (i) If the matrices $E_1, E_2, \dots, E_8$ are read, there must be 8 integers to indicate the way the corresponding matrix is read:
 - 1 - the matrix need not be given, but is set to minus the identity matrix (only for E_1, E_2 , and E_7);
 - 0 - the matrix need not be given, but is set to the zero matrix;
 - 1 - all elements of the matrix will be read row by row;
 - 2 - the elements of the matrix are given in a sparse format, that is for each non-zero element there is a line of input with the row number, column number and value of the element and the list of non-zero elements is concluded with the line 000.

Hence, to give the model directly in the state-space format (10)–(11) one could enter the line 1-10110-1. This line of input is followed by the elements of the matrices $E_1, E_2, \dots, E_8$ (unless the above integer is -1 or 0).

- (ii) If the model is given in terms of mnemonics, a listing of the state equations and the output equations (cf., the package PSREM). If 'GDP' is a mnemonic,

then 'dGDP' denotes $dGDP/dt$. One cannot use 'GDP' and 'dGDP' both as variable names, because otherwise the package cannot distinguish between the operator 'd' and the variable 'dGDP'. The syntax of each equation is: $[sign][value^*] mnemonic\{sign[value^*] mnemonic\} = 0$ where [...] means optional and {..} means repetition (0 or more times). A value is read in free-field format, but may not contain the exponentiation character (E). The right-hand side of the equation can also have the same syntax as the left-hand side of the equation. Spaces or linefeeds have no meaning; however, if a mnemonic is discovered at the end of a line, POREM skips to the next equation.

- The number of players (N) and the discount factor (ρ).
- The names of the players, in the same way as the names of the variables (maximal 10 characters).
- The number of instruments that each player is able to control. The instruments themselves are not mentioned, they follow from the order in which the players and the instruments are given above.
- Penalty weights for the output variables, given as a matrix with $\dim(y)$ rows and columns for each player. Thus the i th column indicates the diagonal matrix Q_i .
- A 0 (zero) or 1 (one) indicating whether you want to input penalty matrices R_{ij} .
- If you answered with a 1, the matrices R_{ij} as one matrix with a column for each player. The i th column contains the diagonal matrices R_{ii} , $i = 1, 2, \dots, N$.
- The number of different values taken on by the vector of exogenous variables.
- The transition times (in units of the sampling interval) at which the vector of exogeneous variables changes.
- The values taken on by the i th exogeneous variable at each of the transition times. The order is:

$$- y_i^d \quad i = 1, \dots, N$$

$$- y_{ij}^d \text{ (if } R_{ij} \text{ exists)}, \quad i = 1, \dots, N, \quad j = 1, \dots, N$$

$$- u_{N+1}$$

- The number of intervals in time and the duration of each interval over which the model needs to be solved.
- An integer which says how the initial values of the predetermined state variables are given:
 - 1 - values are read,
 - 0 - zero values,
 - 1 - initial steady-state values,
 - 2 - difference between initial and final steady-state values.
- If the above integer is -1, the initial values of the predetermined state variables x_i^0 .

EXAMPLE OF SECTION 8

Monetary interdependence under a clean float

2 1

 w ws e

13

 y ys $infpc$ $infpcs$ c ri ris r rs i is $m1$ $ms1$

4

 m ms s ss

1

$$dw = 0.25*y + 0.25*s$$

$$dws = 0.25*ys + 0.25*ss$$

$$de = i - is$$

$$y = -r + 0.375*c + 0.75*ys$$

$$ys = -rs - 0.375*c + 0.75*y$$

$$m - w - s = y - 2.0*i$$

$$ms - ws - ss = ys - 2.0*is$$

$$r = i - infpc$$

$$rs = is - infpcs$$

$$infpc = 0.75*dw + 0.25*de + 0.25*dws$$

$$infpcs = -0.25*de + 0.75*dws + 0.25*dw$$

$$c = ws + e - w + ss - s$$

$$ri = -s - 0.25*c$$

$$ris = -ss + 0.25*c$$

$$m1 = m$$

$$ms1 = ms$$

2 0.025 npl discount rate

France

Germany

1 1

1.0 0.0 *y*

0.0 1.0

0.0 0.0 *infpc*

0.0 0.0

0.0 0.0 *c*

1.0 0.0 *ri*

0.0 1.0

0.0 0.0 *r*

0.0 0.0

0.0 0.0 *i*

0.0 0.0

0.2 0.0 *m1*

0.0 0.2 *m1s*

0 no weights to policies

2 no. of shocks

0.0 0.0 target values

0.0 0.0 *y*

0.0 0.0 *ys*

0.0 0.0 *infpc*

0.0 0.0

0.0 0.0 *c*

0.0 0.0 *ri*

0.0 0.0

0.0 0.0 *r*

0.0 0.0

0.0 0.0 *i*

0.0 0.0

0.0 0.0 *m1*

0.0 0.0 *m1s*

0.0 0.0 *y*

0.0 0.0

0.0 0.0 *infpc*

0.0 0.0

0.0 0.0 *c*

0.0 0.0 *ri*

0.0 0.0

0.0 0.0 *r*

0.0 0.0

0.0 0.0 *i*

0.0 0.0
 0.0 0.0 *m1*
 0.0 0.0 *m1s*
 0.0 1.0 *s*
 0.0 1.0 *ss*
 17 5.0
 1

References

- Barro, R.J. and Gordon, D.B. (1983) Rules, discretion and reputation in a model of monetary policy, *Journal of Monetary Economics* 12, 101–121.
- Başar, T. and Olsder, G.J. (1982) *Dynamic Non-Cooperative Game Theory*, Academic Press, New York.
- Begg, D.K.H. (1982) *The Rational Expectations Revolution in Macroeconomics: Theory and Evidence*, Philip Allan, Oxford.
- Binmore, K. and Dasgupta, P. (eds.) (1987) *The Economics of Bargaining*, Basil Blackwell, Oxford.
- Buiter, W.H. (1983) Optimal and time-consistent policies in continuous time rational expectations models, *Discussion Paper No. A39*, LSE Econometrics Programme, London School of Economics.
- Buiter, W.H. (1984) Saddlepoint problems in continuous time rational expectations models: A general method and some macroeconomic examples, *Econometrica* 52(3), 665–680.
- Buiter, W.H. (1986) Policy evaluation and design for continuous time linear rational expectations models: Some recent developments, in M.H. Peston and R.E. Quandt (eds.), *Prices, Competition and Equilibrium*, Philip Allan, Oxford.
- Buiter, W.H. and Marston, R.C. (eds.) (1985) *International Economic Policy Coordination*, Cambridge University Press, Cambridge.
- Calvo, G.A. (1978) On the time consistency of optimal policy in a monetary economy, *Econometrica* 46, 1411–1428.
- Cohen, D. and Michel, P. (1988) How should control theory be used to calculate a time-consistent government policy? *Review of Economic Studies* LV(2), 263–274.
- Fischer, S. (1980) Dynamic inconsistency, cooperation and the benevolent dissembling government, *Journal of Economic Dynamics and Control* 2(1), 93–108.
- Kalai, E. and Smorodinski, M. (1975) Other solutions to Nash's bargaining problem, *Econometrica* 43(3), 513–518.
- Kydland, F.E. and Prescott, E.C. (1977) Rules rather than discretion: The time-inconsistency of optimal plans, *Journal of Political Economy* 85, 473–491.
- Lucas, R.E. Jr. (1976) Econometric policy evaluation: A critique, *Carnegie-Rochester Conference Series on Public Policy* 6, 19–46.
- Markink, A.J. and van der Ploeg, F. (1989) Dynamic policy simulation of linear models with rational expectations of future events: A computer package, *Discussion Paper No. 8906*, Centre for Economic Research, Tilburg University.
- Mayne, D. (1966) A second-order gradient method for determining optimal trajectories of nonlinear discrete-time systems, *International Journal of Control* 3, 1.
- Nash, J.F. (1950) The bargaining problem, *Econometrica* 18, 155–162.
- Nash, J.F. (1953) Two-person cooperative games, *Econometrica* 21, 128–140.
- Oudiz, G. and J. Sachs (1985) International policy coordination in dynamic macroeconomic models, in W.H. Buiter and R.C. Marston (eds.), *International Economic Policy Coordination*, Cambridge University Press, Cambridge.
- Ploeg, F. van der (1987a) Trade unions, investment and employment. A non-cooperative approach, *European Economic Review* 31, 1465–1492.
- Ploeg, F. van der (1987b) Optimal government policy in a small open economy with rational expectations and uncertain election outcomes, *International Economic Review* 28(2), 469–491.

- Ploeg, F. van der (1989) Monetary interdependence under alternative exchange-rate regimes: A European perspective, *Discussion Paper* No. 358, Centre for Economic Policy Research, London.
- Ploeg, F. van der and de Zeeuw, A.J. (eds.) (1989) *Dynamic Policy Games in Economics*, North-Holland, Amsterdam.
- Ploeg, F. van der and de Zeeuw, A.J. (1990) Perfect-equilibrium in a competitive model of arms accumulation, *International Economic Review* 31(1), 131-146.
- Roberts, S.M. and Shipman, J.S. (1972) *Two-point Boundary Value Problems: Shooting Methods*, Elsevier, New York.
- Simaan, M. and Cruz, J.B. Jr. (1973a) On the Stackelberg strategy in nonzero-sum games, *Journal of Optimization Theory and Applications* 11(3/4), 184-206.
- Simaan, M. and Cruz, J.B. Jr. (1973b) Additional aspects of the Stackelberg strategy in nonzero-sum games, *Journal of Optimization Theory and Applications* 11(6), 613-626.
- Starr, A.W. and Ho, Y.C. (1969a) Further properties of non-zero sum differential games, *Journal of Optimization Theory and Applications* 3(3), 184-206.
- Starr, A.W. and Ho, Y.C. (1969b) Further properties of nonzero-sum differential games, *Journal of Optimization Theory and Applications* 3(4), 207-219.
- Zeeuw, A.J. de (1984) *Difference Games and Linked Econometric Policy Model*, Ph.D. thesis, Tilburg University, The Netherlands.

Reprint Series, Center, Tilburg University, The Netherlands:

- No. 1 G. Marini and F. van der Ploeg, Monetary and fiscal policy in an optimising model with capital accumulation and finite lives, The Economic Journal, Vol. 98, No. 392, 1988, pp. 772 - 786.
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- No. 10 F. van der Ploeg and A.J. de Zeeuw, Conflict over arms accumulation in market and command economies, in F. van der Ploeg and A.J. de Zeeuw (eds.), Dynamic Policy Games in Economics, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 91 - 119.
- No. 11 J. Driffill, Macroeconomic policy games with incomplete information: some extensions, in F. van der Ploeg and A.J. de Zeeuw (eds.), Dynamic Policy Games in Economics, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 289 - 322.
- No. 12 F. van der Ploeg, Towards monetary integration in Europe, in P. De Grauwe e.a., De Europese Monetair Integratie: vier visies, Wetenschappelijke Raad voor het Regeringsbeleid V 66, 's-Gravenhage: SDU uitgeverij, 1989, pp. 81 - 106.

- No. 13 R.J.M. Alessie and A. Kapteyn, Consumption, savings and demography, in A. Wenig, K.F. Zimmermann (eds.), Demographic Change and Economic Development, Berlin/Heidelberg: Springer-Verlag, 1989, pp. 272 - 305.
- No. 14 A. Hoque, J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model, Journal of Econometrics, Vol. 39, No. 3, 1988, pp. 327 - 346.
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- No. 23 J.R. Magnus and A.D. Woodland, On the maximum likelihood estimation of multivariate regression models containing serially correlated error components, International Economic Review, Vol. 29, No. 4, 1988, pp. 707 - 725.
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- No. 30 J.R. Magnus and A.D. Woodland, Separability and Aggregation, Economica, vol. 57, no. 226, 1990, pp. 239 - 247.
- No. 31 F. van der Ploeg, International interdependence and policy coordination in economies with real and nominal wage rigidity, Greek Economic Review, vol. 10, no. 1, June 1988, pp. 1 - 48.
- No. 32 E. van Damme, Signaling and forward induction in a market entry context, Operations Research Proceedings 1989, Berlin-Heidelberg: Springer-Verlag, 1990, pp. 45 - 59.
- No. 33 A.P. Barten, Toward a levels version of the Rotterdam and related demand systems, Contributions to Operations Research and Economics, Cambridge: MIT Press, 1989, pp. 441 - 465.
- No. 34 F. van der Ploeg, International coordination of monetary policies under alternative exchange-rate regimes, Advanced Lectures in Quantitative Economics, London-Orlando: Academic Press Ltd., 1990, pp. 91 - 121.
- No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', European Economic Review, vol. 34, no. 5, 1990, pp. 1011 - 1022.
- No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), Panel Data and Labor Market Studies, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.
- No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, Oxford Economic Papers, vol. 42, no. 3, 1990, pp. 501 - 525.
- No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, Journal of Time Series Analysis, vol. 11, no. 3, 1990, pp. 239 - 248.

- No. 39 Th. van de Klundert, Wage differentials and employment in a two-sector model with a dual labour market, Metroeconomica, vol. 40, no. 3, 1989, pp. 235 - 256.
- No. 40 Th. Nijman and M.F.J. Steel, Exclusion restrictions in instrumental variables equations, Econometric Reviews, vol. 9, no. 1, 1990, pp. 37 - 55.
- No. 41 A. van Soest, I. Woittiez and A. Kapteyn, Labor supply, income taxes, and hours restrictions in the Netherlands, Journal of Human Resources, vol. 25, no. 3, 1990, pp. 517 - 558.
- No. 42 Th.C.M.J. van de Klundert and A.B.T.M. van Schaik, Unemployment persistence and loss of productive capacity: a Keynesian approach, Journal of Macroeconomics, vol. 12, no. 3, 1990, pp. 363 - 380.
- No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, Journal of Econometrics, vol. 46, no. 3, 1990, pp. 333 - 346.
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- No. 46 Th. Nijman and F. Palm, Predictive accuracy gain from disaggregate sampling in ARIMA models, Journal of Business & Economic Statistics, vol. 8, no. 4, 1990, pp. 405 - 415.
- No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, Sankhya: The Indian Journal of Statistics, vol. 52, series B, part. 1, 1990, pp. 1 - 13.
- No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, Journal of Econometrics, vol. 48, no. 1/2, 1991, pp. 83 - 117.
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- No. 50 F. van der Ploeg, Money and capital in interdependent economies with overlapping generations, Economica, vol. 58, no. 230, 1991, pp. 233 - 256.
- No. 51 A. Kapteyn and A. de Zeeuw, Changing incentives for economic research in the Netherlands, European Economic Review, vol. 35, no. 2/3, 1991, pp. 603 - 611.
- No. 52 C.G. de Vries, On the relation between GARCH and stable processes, Journal of Econometrics, vol. 48, no. 3, 1991, pp. 313 - 324.

- No. 53 R. Alessie and A. Kapteyn, Habit formation, interdependent preferences and demographic effects in the almost ideal demand system, The Economic Journal, vol. 101, no.406, 1991, pp. 404 - 419.
- No. 54 W. van Groenendaal and A. de Zeeuw, Control, coordination and conflict on international commodity markets, Economic Modelling, vol. 8 no. 1, 1991, pp. 90 - 101.
- No. 55 F. van der Ploeg and A.J. Markink, Dynamic policy in linear models with rational expectations of future events: A computer package, Computer Science in Economics and Management, vol. 4, no. 3, 1991, pp. 175 - 199.

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