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A. van den Nouweland

P. Borm and S. Tijs

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Allocation Rules for Hypergraph Communication Situations

By A. van den Nouweland, P. Borm and S. Tijs¹

Abstract: This paper considers hypergraph communication situations, where for a group of agents the economic possibilities are described by a coalitional game and the communication possibilities are described by a hypergraph in which the nodes are the agents and the edges are the subgroups of agents who can effect communication. Axiomatic characterizations are provided for two allocation rules, the Myerson value and the position value.

1 Introduction

For a group $N := \{1, \dots, n\}$ of agents the economic possibilities of each subgroup of players who decide to cooperate are described by a *coalitional game* (N, v) with player set N and characteristic function $v: 2^N \rightarrow \mathbb{R}$ with $v(\emptyset) = 0$. The communication possibilities are described by a (*communication*) *hypergraph* $(N, \mathcal{H})$, where $\mathcal{H} \subseteq 2^N$. The interpretation is that communication is only possible within a *conference* $H \in \mathcal{H}$ and all players in H have to be present before communication can take place. We will implicitly assume that each player can communicate with himself and therefore we restrict our attention to hypergraphs $(N, \mathcal{H})$ with $\mathcal{H} \subseteq \{H \in 2^N \mid |H| \geq 2\}$. For convenience we assume that the game v is *zero-normalized*, i.e. $v(\{i\}) = 0$ for all $i \in N$. The triple $(N, v, \mathcal{H})$ is called a (*hypergraph*) *communication situation*.

An example of a hypergraph communication structure appears in sports, for example football. Football teams have committees and some members of those committees are also in the board of the national football association of their country, which in turn has representatives in international football associations. Basic decisions can be made within each of these three types of organizations and interaction between two organizations is possible if they have at least one member in common.

Communication situations where the underlying communication structure is a graph were first studied in Myerson (1977). He introduced corresponding *graph-restricted games* and characterized the *Shapley value* (Shapley (1953)) of these games in terms of efficiency and fairness. Owen (1986) also studied graph-restricted games and their Shapley value, the *Myerson value* (cf. Aumann and Myerson (1988)), for situations where the underlying graph is a tree. Meessen (1988) introduced a second value for graph communication situations, the *position value*, and this value was

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characterized axiomatically in Borm, Owen and Tijs (1991) in case the communication graphs contain no cycles. They also gave a new axiomatic characterization of the Myerson value for the same class of communication situations. The idea of direct communication between two players was generalized towards communication within conferences by Myerson (1980).

The present paper extends the Myerson value and the position value towards hypergraph communication situations. Section 2 provides three axiomatic characterizations of the Myerson value, inspired by Myerson ((1977) and (1980)). In section 3 the position value is characterized axiomatically in case the communication hypergraphs contain no cycles. Further, a characterization of the Myerson value for the same class is provided. Finally, section 4 addresses the irredundancy of the axioms in the characterization of the position value given in section 3.

Preliminaries: Let S be a finite set. Then we denote the number of elements of S by $|S|$ and the set of subsets of S by 2^S .

For a coalitional game (N, v) the *Shapley value* $\Phi(N, v)$ (cf. Shapley (1953)) is defined by

$$\Phi_i(N, v) = \sum_{S \in 2^{N \setminus \{i\}}} \frac{(|S| - 1)!(n - |S|)!}{n!} (v(S) - v(S \setminus \{i\}))$$

for all $i \in N$.

If $S \subseteq N$, then the *unanimity game* (N, u_S) on S is defined by

$$u_S(T) = \begin{cases} 1 & \text{if } S \subseteq T \\ 0 & \text{else.} \end{cases}$$

The $2^n - 1$ unanimity games with player set N form a basis of the space of all coalitional games with player set N . Finally, we define the empty sum to be zero.

2 The Myerson Value

Let HCS^N denote the class of all (hypergraph) communication situations with fixed player set N as described in section 1. If $(N, v, \mathcal{H}) \in HCS^N$, then a *coalition* $S \subseteq N$ can effect communication in the conferences in $\mathcal{H}(S) := \{H \in \mathcal{H} \mid H \subseteq S\}$. We define *interaction sets* of $(S, \mathcal{H}(S))$ as follows:

- 1) $\{i\}$ is an interaction set for all $i \in S$
- 2) H is an interaction set for all $H \in \mathcal{H}(S)$
- 3) if T^1 and T^2 are interaction sets such that $T^1 \cap T^2 \neq \emptyset$, then $T^1 \cup T^2$ is an interaction set
- 4) all interaction sets are obtained by 1), 2) and 3).

The interaction sets of $(S, \mathcal{H}(S))$ are the subgroups of players in S who can interact with respect to communication in $(S, \mathcal{H}(S))$. The hypergraph $(S, \mathcal{H}(S))$ determines a partition of S into *interaction components*: a coalition $T \subseteq S$ is a component within S if and only if T is an interaction set of $(S, \mathcal{H}(S))$ and there is no interaction set $\bar{T}$ of $(S, \mathcal{H}(S))$ such that $T \subseteq \bar{T} \subseteq S$. We denote the resulting partition of S by $S/\mathcal{H}$. Now we are ready to introduce a *reward function* $r^v: 2^N \times 2^{\mathcal{H}} \rightarrow \mathbb{R}$, which assigns to a coalition $S \subseteq N$ and a set of conferences $\mathcal{A} \subseteq \mathcal{H}$ the reward that the coalition can achieve when effecting communication in all conferences of $\mathcal{A}(S)$. For all $S \subseteq N$ and all $\mathcal{A} \subseteq \mathcal{H}$ we define

$$r^v(S, \mathcal{A}) := \sum_{C \in S/\mathcal{A}} v(C).$$

An *allocation rule* on HCS^N is a map $F: HCS^N \rightarrow \mathbb{R}^n$.

The first allocation rule we consider is an extension of the Myerson value (Myerson (1977) and Aumann and Myerson (1988)) towards hypergraph communication situations. The (extended) *Myerson value* μ assigns to a communication situation the Shapley value of the so-called *point game*. In formula, for all $(N, v, \mathcal{H}) \in HCS^N$

$$\mu(N, v, \mathcal{H}) := \Phi(N, r^v_{\mathcal{H}}),$$

where the point game $(N, r^v_{\mathcal{H}})$ is defined as follows: for all $S \subseteq N$

$$r^v_{\mathcal{H}}(S) := r^v(S, \mathcal{H}).$$

Example 2.1: Consider the communication situation $(N, v, \mathcal{H})$, where $N = \{1, \dots, 6\}$, v is the unanimity game $u_{\{1,2,3\}}$ and $\mathcal{H} = \{\{1, 4\}, \{2, 5\}, \{3, 6\}, \{4, 5, 6\}\}$. We calculate the point game:

$$r^v_{\mathcal{H}}(S) = \begin{cases} 1 & \text{if } S = N \\ 0 & \text{else.} \end{cases}$$

Hence, the Myerson value for this situation is

$$\mu(N, v, \mathcal{H}) = \Phi(N, r^v_{\mathcal{H}}) = \left(\frac{1}{6}, \dots, \frac{1}{6}\right).$$

Myerson (1977) proved that the Myerson value is the unique allocation rule on the set of graph communication situations that is fair and component-efficient. We will prove a similar result for hypergraph communication situations, inspired by Myerson (1980), where non-transferable utility games are considered. First we introduce some properties.

- An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ is called *component-efficient* if for all $(N, v, \mathcal{H}) \in HCS^N$ and all components $C \in N/\mathcal{H}$

$$\sum_{i \in C} F_i(N, v, \mathcal{H}) = v(C).$$

- An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ is called *fair* if for all $(N, v, \mathcal{H}) \in HCS^N$, all $H \in \mathcal{H}$ and all $i, j \in H$

$$F_i(N, v, \mathcal{H}) - F_i(N, v, \mathcal{H} \setminus \{H\}) = F_j(N, v, \mathcal{H}) - F_j(N, v, \mathcal{H} \setminus \{H\}).$$

So, according to a fair allocation rule all players in a conference H lose the same amount if conference H is deleted.

- An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ has *balanced contributions* if for all $(N, v, \mathcal{H}) \in HCS^N$ and all $i, j \in N$

$$F_i(N, v, \mathcal{H}) - F_i(N, v, \mathcal{H}_{-i}) = F_j(N, v, \mathcal{H}) - F_j(N, v, \mathcal{H}_{-j}),$$

where

$$\mathcal{H}_{-i} := \{H \in \mathcal{H} \mid i \notin H\}.$$

- An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ satisfies the *Shapley-formula* if for all $(N, v, \mathcal{H}) \in HCS^N$ and all $i \in N$

$$F_i(N, v, \mathcal{H}) = \sum_{S \subseteq N} \frac{(|S| - 1)! (n - |S|)!}{n!} \left(\sum_{j \in N} F_j(N, v, \mathcal{H}(S)) - \sum_{j \in N} F_j(N, v, \mathcal{H}(S)_{-i}) \right).$$

The relations between the last three properties are given in

Lemma 2.1:

- An allocation rule on HCS^N that satisfies the Shapley formula has balanced contributions.
- An allocation rule on HCS^N that has balanced contributions is fair.

For the proof of this lemma we refer to Myerson (1980).

The following lemma shows that the Myerson value satisfies all four properties introduced above.

Lemma 2.2: The Myerson value μ is component-efficient and satisfies the Shapley formula.

Proof: To prove component-efficiency let $(N, v, \mathcal{H}) \in HCS^N$ be fixed and let $C \in N/\mathcal{H}$. We split up $(N, r_{\mathcal{H}}^v)$ into two games, (N, r^C) and $(N, r^{N \setminus C})$, where for all $S \subseteq N$

$$r^C(S) := r_{\mathcal{H}}^v(S \cap C) \quad \text{and} \quad r^{N \setminus C}(S) := r_{\mathcal{H}}^v(S \setminus C).$$

Since C is a component of $(N, \mathcal{H})$ we know that

$$r_{\mathcal{H}}^v = r^C + r^{N \setminus C}.$$

It is straightforward to verify that $\Phi_i(N, r^{N \setminus C}) = 0$ for all $i \in C$. Hence,

$$\begin{aligned} \sum_{i \in C} \mu_i(N, v, \mathcal{H}) &= \sum_{i \in C} \Phi_i(N, r^C) + \sum_{i \in C} \Phi_i(N, r^{N \setminus C}) \\ &= \sum_{i \in C} \Phi_i(N, r^C) = r^C(N) = r_{\mathcal{H}}^v(C) = v(C), \end{aligned}$$

where the first and the third equality follow from the definition of the Shapley value.

Now let $(N, v, \mathcal{H}) \in HCS^N$ and $i \in N$ be fixed. Note that

$$\mu_i(N, v, \mathcal{H}) = \Phi_i(N, r_{\mathcal{H}}^v) = \sum_{S \subseteq N} \frac{(|S| - 1)!(n - |S|)!}{n!} (r_{\mathcal{H}}^v(S) - r_{\mathcal{H}}^v(S \setminus \{i\})).$$

Hence, to prove that μ satisfies the Shapley formula it suffices to show that for all $S \subseteq N$

$$r_{\mathcal{H}}^v(S) = \sum_{j \in N} \mu_j(N, v, \mathcal{H}(S)) \quad \text{and} \quad r_{\mathcal{H}}^v(S \setminus \{i\}) = \sum_{j \in N} \mu_j(N, v, \mathcal{H}(S)_{-i})$$

and this is a direct consequence of the component-efficiency for μ , the fact that v is zero-normalized and the fact that $\mathcal{H}(S \setminus \{i\}) = \mathcal{H}(S)_{-i}$. $\square$

Three axiomatic characterizations of the Myerson value are given in

Theorem 2.3: The Myerson value is component-efficient and fair, has balanced contributions and satisfies the Shapley formula. Furthermore, the Myerson value is the unique component-efficient allocation rule on HCS^N that satisfies either of the other three properties.

Proof: It remains to prove that there is at most one allocation rule on HCS^N that is fair and component-efficient. Suppose F and G are two fair and component-efficient allocation rules on HCS^N . Let $(N, v, \mathcal{H}) \in HCS^N$ be such that $F(N, v, \mathcal{H}) \neq G(N, v, \mathcal{H})$ and $F(N, v, \mathcal{A}) = G(N, v, \mathcal{A})$ for all $\mathcal{A} \subseteq \mathcal{H}$. Without

loss of generality, let $k \in N$ be such that $F_k(N, v, \mathcal{H}) > G_k(N, v, \mathcal{H})$ and suppose $\{i, k\} \subseteq H \in \mathcal{H}$. Then

$$\begin{aligned} F_i(N, v, \mathcal{H}) - F_k(N, v, \mathcal{H}) &= F_i(N, v, \mathcal{H} \setminus \{H\}) - F_k(N, v, \mathcal{H} \setminus \{H\}) \\ &= G_i(N, v, \mathcal{H} \setminus \{H\}) - G_k(N, v, \mathcal{H} \setminus \{H\}) \\ &= G_i(N, v, \mathcal{H}) - G_k(N, v, \mathcal{H}). \end{aligned}$$

By repeatedly applying this argument we obtain

$$\sum_{i \in C_k} F_i(N, v, \mathcal{H}) > \sum_{i \in C_k} G_i(N, v, \mathcal{H}),$$

where $C_k \in N/\mathcal{H}$ is the component containing k , and this is in contradiction with component-efficiency for F and G . $\square$

3 The Position Value

In Meessen (1988) the position value was introduced for graph communication situations. The (extended) *position value* π for hypergraph communication situations is defined with the aid of the *conference game* $(\mathcal{H}, r_N^v)$, where for all $(N, v, \mathcal{H}) \in HCS^N$ and all $\mathcal{A} \subseteq \mathcal{H}$

$$r_N^v(\mathcal{A}) := r^v(N, \mathcal{A}).$$

The position value is obtained by first assigning to each conference its Shapley value in the conference game and then dividing the value of each conference equally among its participants. In formula: for all $(N, v, \mathcal{H}) \in HCS^N$ and all $i \in N$

$$\pi_i(N, v, \mathcal{H}) := \sum_{H \in \mathcal{H}_i} |H|^{-1} \cdot \Phi_H(\mathcal{H}, r_N^v),$$

where $\mathcal{H}_i := \{H \in \mathcal{H} \mid i \in H\}$ denotes the set of all conferences of which i is a member.

Example 3.1: Consider the communication situation as described in example 2.1. We calculate the conference game:

$$r_N^v(\mathcal{A}) = \begin{cases} 1 & \text{if } \mathcal{A} = \mathcal{H} \\ 0 & \text{else.} \end{cases}$$

Hence, $\Phi_H(\mathcal{H}, r_N^v) = \frac{1}{4}$ for all $H \in \mathcal{H}$ and

$$\begin{aligned} \pi_1(N, v, \mathcal{H}) &= \pi_2(N, v, \mathcal{H}) = \pi_3(N, v, \mathcal{H}) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}, \\ \pi_4(N, v, \mathcal{H}) &= \pi_5(N, v, \mathcal{H}) = \pi_6(N, v, \mathcal{H}) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{5}{24}. \end{aligned}$$

Note that $\pi(N, v, \mathcal{H}) \neq \mu(N, v, \mathcal{H})$.

Example 3.2: Consider the communication situation $(N, v, \mathcal{H})$, where $N = \{1, 2, 3\}$, v is the unanimity game $u_{\{1,2\}}$ and $\mathcal{H} = \{\{1, 2\}, \{2, 3\}, \{1, 3\}\}$. Then some calculation shows that $\pi(N, v, \mathcal{H}) = (\frac{5}{12}, \frac{5}{12}, \frac{2}{12})$. Note that although player 3 is a zero-player in the point game, he obtains a positive payoff according to the position value because player 3 can act as an intermediate between players 1 and 2.

In order to characterize the position value axiomatically we first introduce three more properties of allocation rules.

- An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ is called *additive* if for all $(N, v, \mathcal{H})$ and $(N, w, \mathcal{H}) \in HCS^N$

$$F(N, v + w, \mathcal{H}) = F(N, v, \mathcal{H}) + F(N, w, \mathcal{H}).$$

- If $(N, v, \mathcal{H}) \in HCS^N$, then a conference $H \in \mathcal{H}$ is called *superfluous* (for $(N, v, \mathcal{H})$) if for all $\mathcal{A} \subseteq \mathcal{H}$

$$r^v(N, \mathcal{A} \setminus \{H\}) = r^v(N, \mathcal{A}).$$

This means that the deletion of a superfluous conference from a subsystem of conferences does not affect the reward of the grand coalition.

An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ has the *superfluous conference property* if for all $(N, v, \mathcal{H}) \in HCS^N$ and all conferences $H \in \mathcal{H}$ that are superfluous for $(N, v, \mathcal{H})$

$$F(N, v, \mathcal{H} \setminus \{H\}) = F(N, v, \mathcal{H}).$$

- The *influence* of a player in a communication situation is defined as follows:

$$I_i(N, v, \mathcal{H}) := \sum_{H \in \mathcal{H}_i} |H|^{-1}.$$

One could think of $I_i(N, v, \mathcal{H})$ as a natural measure of the importance of player i in the hypergraph $(N, \mathcal{H})$.

A communication situation $(N, v, \mathcal{H})$ is called *conference anonymous* if there exists a function $f: \{0, 1, \dots, |\mathcal{H}|\} \rightarrow \mathbb{R}$ such that for all $\mathcal{A} \subseteq \mathcal{H}$

$$r^v(N, \mathcal{A}) = f(|\mathcal{A}|). \quad (3.1)$$

An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ has the *influence property* if for each conference anonymous communication situation $(N, v, \mathcal{H}) \in HCS^N$ there exists an $\alpha \in \mathbb{R}$ such that for all $i \in N$

$$F_i(N, v, \mathcal{H}) = \alpha \cdot I_i(N, v, \mathcal{H}).$$

Lemma 3.1: The position value $\pi: HCS^N \rightarrow \mathbb{R}$ satisfies additivity, component-efficiency, the superfluous conference property and the influence property.

Proof: Additivity follows straightforwardly. To prove component-efficiency, let $(N, v, \mathcal{H}) \in HCS^N$ be fixed and $C \in N/\mathcal{H}$. Then

$$\begin{aligned} \sum_{i \in C} \pi_i(N, v, \mathcal{H}) &= \sum_{i \in C} \sum_{H \in \mathcal{H}_i} |H|^{-1} \Phi_H(\mathcal{H}, r_N^v) = \sum_{H \in \mathcal{H}(C)} \Phi_H(\mathcal{H}, r_N^v) \\ &\stackrel{(*)}{=} \sum_{H \in \mathcal{H}(C)} \Phi_H(\mathcal{H}(C), r_N^v) = r_N^v(\mathcal{H}(C)) = v(C), \end{aligned}$$

where equality (*) follows from the definition of the Shapley value and the fact that

$$r_N^v(\mathcal{A} \cup \{H\}) - r_N^v(\mathcal{A}) = r_N^v((\mathcal{A} \cap \mathcal{H}(C)) \cup \{H\}) - r_N^v(\mathcal{A} \cap \mathcal{H}(C))$$

for all $\mathcal{A} \subseteq \mathcal{H}$ and all $H \in \mathcal{H}(C)$.

Now let H be a superfluous conference for $(N, v, \mathcal{H}) \in HCS^N$. This conference is a zero-player in the game $(\mathcal{H}, r_N^v)$ and hence $\Phi_H(\mathcal{H}, r_N^v) = 0$. From this we see that $\pi(N, v, \mathcal{H} \setminus \{H\}) = \pi(N, v, \mathcal{H})$. Suppose $(N, v, \mathcal{H})$ is conference anonymous and let f be as in (3.1). Because all conferences are symmetric in the game $(\mathcal{H}, r_N^v)$

$$\Phi_H(\mathcal{H}, r_N^v) = r_N^v(\mathcal{H}) \cdot |\mathcal{H}|^{-1}$$

for all $H \in \mathcal{H}$. Hence,

$$\pi_i(N, v, \mathcal{H}) = \sum_{H \in \mathcal{H}_i} |H|^{-1} \cdot \Phi_H(\mathcal{H}, r_N^v) = r_N^v(\mathcal{H}) \cdot |\mathcal{H}|^{-1} \cdot I_i(N, v, \mathcal{H})$$

and we can choose $\alpha = r_N^v(\mathcal{H}) \cdot |\mathcal{H}|^{-1}$. □

Theorem 3.2 below shows that the four properties of lemma 3.1 characterize the position value if the hypergraphs contain no cycles. We first investigate cycle-free hypergraphs. Let $(N, \mathcal{H})$ be a hypergraph. A *chain* (of length k) in $(N, \mathcal{H})$ is a sequence $(x_1, H_1, x_2, \dots, H_k, x_{k+1})$ where $x_1, \dots, x_k$ are all distinct players and $H_1, \dots, H_k$ are all distinct conferences such that $\{x_\ell, x_{\ell+1}\} \subseteq H_\ell$ for all $\ell \in \{1, \dots, k\}$. A *cycle* is a chain $(x_1, H_1, x_2, \dots, H_k, x_{k+1})$ of length $k \geq 2$ with $x_1 = x_{k+1}$. If $(N, \mathcal{H})$ is a cycle-free hypergraph and $T \subseteq N$ is contained in an interaction component of $(N, \mathcal{H})$, then $\bar{T} := \bigcap \{T^0 \mid T \subseteq T^0 \subseteq N, T^0/\mathcal{H} = \{T^0\}\}$ is the unique set such that $T \subseteq \bar{T} \subseteq N$ and $\bar{T}$ is an interaction component of $(\bar{T}, \mathcal{H}(\bar{T}))$ and there is no S with $S \subseteq \bar{T}$ that has the same properties. This is a straightforward generalization of the results in Owen (1986).

Defining $HCS^N := \{(N, v, \mathcal{H}) \in HCS^N \mid (N, \mathcal{H}) \text{ contains no cycles}\}$, we have

Theorem 3.2: The position value π is the unique allocation rule on HCS_N^π that satisfies additivity, component-efficiency, the superfluous conference property and the influence property.

Proof: Lemma 3.1 shows that the four properties are satisfied by the position value. Let $F: HCS_N^\pi \rightarrow \mathbb{R}^n$ be an allocation rule that satisfies the four properties. Let $(N, \mathcal{H})$ be a cycle-free hypergraph, let $\beta \in \mathbb{R}$, $T \in 2^N$ with $|T| \geq 2$ and $v := \beta u_T$. Since π and F are additive it suffices to show that $F(N, v, \mathcal{H}) = \pi(N, v, \mathcal{H})$.

- If T is not contained in an interaction component of $(N, \mathcal{H})$, then $\pi_i(N, v, \mathcal{H}) = 0$ for all $i \in N$. Furthermore, the influence property and component-efficiency imply $F_i(N, v, \mathcal{H}) = 0$ for all $i \in N$.
- If there is an interaction component C of $(N, \mathcal{H})$ such that $T \subseteq C$, then consider $\bar{T} := \bigcap \{T^0 \mid T \subseteq T^0 \subseteq N, T^0 / \mathcal{H} = \{T^0\}\}$. Obviously,

$$r_N^v(\mathcal{H}) = \begin{cases} \beta & \text{if } \mathcal{H}(\bar{T}) \subseteq \mathcal{H} \\ 0 & \text{else} \end{cases}$$

and

$$\Phi_H(\mathcal{H}, r_N^v) = \begin{cases} \beta \cdot |\mathcal{H}(\bar{T})|^{-1} & \text{if } H \in \mathcal{H}(\bar{T}) \\ 0 & \text{else.} \end{cases}$$

Hence, for all $i \in N$

$$\pi_i(N, v, \mathcal{H}) = \sum_{H \in \mathcal{H}_i} |H|^{-1} \Phi_H(\mathcal{H}, r_N^v) = \beta \cdot |\mathcal{H}(\bar{T})|^{-1} \sum_{H \in \mathcal{H}_i \cap \mathcal{H}(\bar{T})} |H|^{-1}.$$

The superfluous conference property implies $F(N, v, \mathcal{H}) = F(N, v, \mathcal{H}(\bar{T}))$. Note that the communication situation $(N, v, \mathcal{H}(\bar{T}))$ is conference anonymous with $f(0) = \dots = f(|\mathcal{H}(\bar{T})| - 1) = 0$, $f(|\mathcal{H}(\bar{T})|) = \beta$. Consequently, the influence property and component-efficiency for F imply $F(N, v, \mathcal{H}) = \pi(N, v, \mathcal{H})$. $\square$

We can also provide an axiomatic characterization of the Myerson value on the class HCS_N^π using the following property.

- A communication situation $(N, v, \mathcal{H}) \in HCS_N^\pi$ is called *point anonymous* if there exists a function $f: \{0, 1, \dots, |P|\} \rightarrow \mathbb{R}$ such that for all $S \subseteq N$

$$r^v(S, \mathcal{H}) = f(|S \cap P|), \quad (3.2)$$

where $P := \{i \in N \mid \exists H \in \mathcal{H}: i \in H\}$ is the set of players who belong to at least one conference.

An allocation rule $F: HCS^N \rightarrow \mathbb{R}^n$ has the *communication ability property* if for each point anonymous communication situation $(N, v, \mathcal{H})$ there exists an $\alpha \in \mathbb{R}$ such that for all $i \in N$

$$F_i(N, v, \mathcal{H}) = \begin{cases} \alpha & \text{if } i \in P \\ 0 & \text{else.} \end{cases}$$

Similarly to theorem 3.2 one can prove

Theorem 3.3: The Myerson value μ is the unique allocation rule on HCS^N that satisfies additivity, component-efficiency, the superfluous conference property and the communication ability property.

4 Irredundancy of the Axioms

In section 3 we characterized the position value $\pi: HCS^N \rightarrow \mathbb{R}^n$ axiomatically by additivity, component-efficiency, the superfluous conference property and the influence property. In this section we show that none of these four properties is redundant: for each of the four properties we provide an allocation rule on HCS^N that does not satisfy that specific property but that does satisfy the remaining three properties.

The irredundancy of component-efficiency follows immediately by considering the allocation rule that assigns zero to each player in every communication situation. The irredundancy of the influence property follows by considering the Myerson value (see theorem 3.3 and examples 2.1 and 3.1). To show the irredundancy of the superfluous conference property we first prove the following

Lemma 4.1: Let $(N, v, \mathcal{H}) \in HCS^N$ be a conference anonymous communication situation. Then there exists an $\alpha \in \mathbb{R}$ such that for all $C \in N/\mathcal{H}$

$$r^v(N, \mathcal{H}(C)) = \alpha \cdot |\mathcal{H}(C)|. \quad (4.1)$$

Proof: Let f be as in (3.1). If $C \in N/\mathcal{H}$ is such that $|C| = 1$, then $r^v(N, \mathcal{H}(C)) = 0$ and $|\mathcal{H}(C)| = 0$, so $r^v(N, \mathcal{H}(C)) = \alpha \cdot |\mathcal{H}(C)|$ for every $\alpha \in \mathbb{R}$. If there is exactly one component C such that $|C| > 1$, then we simply define $\alpha = v(C) \cdot |\mathcal{H}(C)|^{-1}$. Now suppose C_1 and C_2 are two different components of $(N, \mathcal{H})$ such that $|C_1| > 1$ and $|C_2| > 1$ and let $c := |\mathcal{H}(C_1)| \geq 1$. Let $k \in \{1, \dots, c\}$ and $\mathcal{A} \subseteq \mathcal{H}(C_1)$ such that $|\mathcal{A}| = k - 1$. Let $H \in \mathcal{H}(C_2)$, then

$$\begin{aligned} f(k-1) + f(1) &= f(|\mathcal{A}|) + f(|\{H\}|) = r^v(N, \mathcal{A}) + r^v(N, \{H\}) \\ &= r^v(N, \mathcal{A} \cup \{H\}) = f(|\mathcal{A} \cup \{H\}|) = f(k). \end{aligned}$$

Since $k \in \{1, \dots, c\}$ was arbitrary, we see that $f(k) = f(k-1) + f(1)$ for all $k \in \{1, \dots, c\}$ and it follows with induction that $f(c) = c \cdot f(1)$. Because C_1 and C_2 are arbitrary we obtain

$$r''(N, \mathcal{H}(C)) = f(1) \cdot |\mathcal{H}(C)|$$

for all $C \in N/\mathcal{H}'$ with $|C| > 1$. □

We define the *influence rule* ρ that for each communication situation divides the worth of each communication component equally among all conferences of that component and, subsequently, divides the so obtained share of each conference equally among its members. In formula, for all $(N, v, \mathcal{H}) \in HCS^N$ and $i \in N$

$$\rho_i(N, v, \mathcal{H}) := \begin{cases} I_i(N, v, \mathcal{H}) \cdot |\mathcal{H}(C_i)|^{-1} \cdot v(C_i) & \text{if } i \in P \\ 0 & \text{else,} \end{cases}$$

where P is as in (3.2) and C_i is the unique component of $(N, \mathcal{H})$ that contains player i .

Theorem 4.2: The influence rule satisfies additivity, component-efficiency and the influence property.

Proof: Additivity and component-efficiency follow immediately from the definition. Now let $(N, v, \mathcal{H}) \in HCS^N$ be conference anonymous and let $\alpha \in \mathbb{R}$ be as in (4.1). For $i \in P$ we have

$$\begin{aligned} \rho_i(N, v, \mathcal{H}) &= I_i(N, v, \mathcal{H}) \cdot |\mathcal{H}(C_i)|^{-1} \cdot v(C_i) \\ &= I_i(N, v, \mathcal{H}) \cdot |\mathcal{H}(C_i)|^{-1} \cdot r''(N, \mathcal{H}(C_i)) \\ &= \alpha \cdot I_i(N, v, \mathcal{H}) \end{aligned}$$

and for $i \in N \setminus P$ we have

$$\rho_i(N, v, \mathcal{H}) = 0 = \alpha \cdot I_i(N, v, \mathcal{H}).$$

Hence, ρ satisfies the influence property. □

The allocation rule ρ does not satisfy the superfluous conference property, as is seen in

Example 4.1: Consider the communication situation $(N, v, \mathcal{H})$, where $N = \{1, 2, 3\}$, $v = u_{\{1,2\}}$ and $\mathcal{H} = \{\{1, 2\}, \{1, 3\}\}$. It is easily checked that conference $\{1, 3\}$ is superfluous, but

$$\begin{aligned} \rho(N, v, \mathcal{H}) &= \frac{1}{2} \left(1, \frac{1}{2}, \frac{1}{2}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right) \quad \text{and} \\ \rho(N, v, \mathcal{H} \setminus \{\{1, 3\}\}) &= \left(\frac{1}{2}, \frac{1}{2}, 0\right). \end{aligned}$$

To prove the irredundancy of the additivity property we first explore superfluous conferences. The following lemma states that in a conference anonymous communication situation either all or none of the conferences are superfluous.

Lemma 4.3: Let $(N, v, \mathcal{H}) \in HCS^N$ be conference anonymous and have a superfluous conference, then $r^v(N, \mathcal{A}) = 0$ for all $\mathcal{A} \subseteq \mathcal{H}$.

Proof: Let f be as in (3.1) and let $H \in \mathcal{H}$ be superfluous. It suffices to show that $f(k) = 0$ for all $k \in \{0, 1, \dots, |\mathcal{H}|\}$. Since v is zero-normalized we know $f(0) = r^v(N, \emptyset) = 0$. Now let $k \in \{1, \dots, |\mathcal{H}|\}$ and $\mathcal{A} \subseteq \mathcal{H} \setminus \{H\}$ with $|\mathcal{A}| = k - 1$, then

$$f(k) = r^v(N, \mathcal{A} \cup \{H\}) = r^v(N, \mathcal{A}) = f(k - 1).$$

It now follows with induction that $f(k) = 0$ for all $k \in \{0, \dots, |\mathcal{H}|\}$. $\square$

For all $(N, v, \mathcal{H}) \in HCS^N$ we define

$$\mathcal{H}^* := \{H \in \mathcal{H} \mid H \text{ is not superfluous for } (N, v, \mathcal{H})\}.$$

Lemma 4.4: Let $(N, v, \mathcal{H}) \in HCS^N$ have a superfluous conference H , then $\mathcal{H}^* = (\mathcal{H} \setminus \{H\})^*$.

Proof: Suppose $H' \in \mathcal{H} \setminus \{H\}$ is superfluous for $(N, v, \mathcal{H} \setminus \{H\})$. Then, for all $\mathcal{A} \subseteq \mathcal{H} \setminus \{H\}$

$$r^v(N, \mathcal{A} \cup \{H\}) = r^v(N, \mathcal{A}) = r^v(N, \mathcal{A} \setminus \{H'\}) = r^v(N, (\mathcal{A} \setminus \{H'\}) \cup \{H\}),$$

which implies that H' is superfluous for $(N, v, \mathcal{H})$. Hence, $\mathcal{H}^* \subseteq (\mathcal{H} \setminus \{H\})^*$. It is straightforward to verify that every conference ($\neq H$) that is superfluous for $(N, v, \mathcal{H})$ is also superfluous for $(N, v, \mathcal{H} \setminus \{H\})$. Thus, $(\mathcal{H} \setminus \{H\})^* = \mathcal{H}^*$. $\square$

To show the irredundancy of the additivity property we define the allocation rule γ as follows: for all $(N, v, \mathcal{H}) \in HCS^N$

$$\gamma(N, v, \mathcal{H}) := \begin{cases} \rho(N, v, \mathcal{H}) & \text{if } (N, v, \mathcal{H}) \text{ is conference anonymous} \\ \rho(N, v, \mathcal{H}^*) & \text{else.} \end{cases}$$

Theorem 4.5: The allocation rule γ satisfies component-efficiency, the superfluous conference property and the influence property.

Proof: To prove component-efficiency let $(N, v, \mathcal{H}) \in HCS^N$ and $C \in N / \mathcal{H}$ be fixed. If $(N, v, \mathcal{H})$ is conference anonymous then component-efficiency for ρ gives us

$$\sum_{i \in C} \gamma_i(N, v, \mathcal{H}) = v(C).$$

If $(N, v, \mathcal{H})$ is not conference anonymous, we have to consider $(N, v, \mathcal{H}^*)$. Note that $N/\mathcal{H}^*$ is a finer partition than $N/\mathcal{H}$. Hence, there are $k \in \mathbb{N}$ and $C_1, \dots, C_k \in N/\mathcal{H}^*$ such that $\{C_1, \dots, C_k\}$ is a partition of C . Now

$$\begin{aligned} \sum_{i \in C} \gamma_i(N, v, \mathcal{H}) &= \sum_{j=1}^k \sum_{i \in C_j} \rho_i(N, v, \mathcal{H}^*) = \sum_{j=1}^k v(C_j) = \sum_{T \in N/\mathcal{H}^*: T \subseteq C} v(T) \\ &= \sum_{T \in N/\mathcal{H}^*(C)} v(T) = r^v(N, \mathcal{H}^*(C)) = r^v(N, \mathcal{H}(C)) = v(C). \end{aligned}$$

To prove the influence property for γ , we first note that it follows from the definition of ρ that $\pi(N, v, \mathcal{H}) = \rho(N, v, \mathcal{H}) = \gamma(N, v, \mathcal{H})$ for each conference anonymous communication situation $(N, v, \mathcal{H})$. From the influence property for π it now follows that γ satisfies the influence property. To prove the superfluous conference property let $H \in \mathcal{H}$ be superfluous for $(N, v, \mathcal{H}) \in HCS^N$. If $(N, v, \mathcal{H})$ is conference anonymous then $r^v(N, \mathcal{A}) = 0$ for all $\mathcal{A} \subseteq \mathcal{H}$ (lemma 4.3) and $\gamma(N, v, \mathcal{H} \setminus \{H\}) = 0 = \gamma(N, v, \mathcal{H})$. So suppose $(N, v, \mathcal{H})$ is not conference anonymous. We distinguish between two cases. If $(N, v, \mathcal{H} \setminus \{H\})$ is not conference anonymous, then lemma 4.4 implies $\gamma(N, v, \mathcal{H} \setminus \{H\}) = \rho(N, v, (\mathcal{H} \setminus \{H\})^*) = \rho(N, v, \mathcal{H}^*) = \gamma(N, v, \mathcal{H})$. If $(N, v, \mathcal{H} \setminus \{H\})$ is conference anonymous, then trivially $(N, v, \mathcal{H}^*)$ is conference anonymous too and

$$\begin{aligned} \gamma(N, v, \mathcal{H} \setminus \{H\}) &= \rho(N, v, \mathcal{H} \setminus \{H\}) = \pi(N, v, \mathcal{H} \setminus \{H\}) \stackrel{(1)}{=} \pi(N, v, \mathcal{H}^*) \\ &= \rho(N, v, \mathcal{H}^*) = \gamma(N, v, \mathcal{H}) \end{aligned}$$

where equality (1) follows from the fact that π satisfies the superfluous conference property. $\square$

The allocation rule γ does not satisfy the additivity property, as is seen in

Example 4.2: Consider the communication situation $(N, v, \mathcal{H})$, where $N = \{1, 2, 3, 4\}$, $v = u_{\{1,2\}} + u_{\{2,3\}}$ and $\mathcal{H} = \{\{1, 4\}, \{2, 4\}, \{3, 4\}\}$. $(N, v, \mathcal{H})$ is not conference anonymous and has no superfluous conferences, hence,

$$\gamma(N, v, \mathcal{H}) = \rho(N, v, \mathcal{H}^*) = \rho(N, v, \mathcal{H}) = \frac{2}{3}(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}).$$

We calculate $\pi(N, v, \mathcal{H}) = (\frac{1}{4}, \frac{1}{4}, 0, \frac{1}{2}) + (0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}) = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{2})$.

Since $(N, v, \mathcal{H}) \in HCS^N$ and $\gamma(N, v, \mathcal{H}) \neq \pi(N, v, \mathcal{H})$, we see that γ cannot satisfy additivity.

Remark: Borm, Owen and Tijs (1991) gave an axiomatic characterization of the position value on the class of communication situations where the underlying hypergraph is a cycle-free graph. The axioms they used are restrictions of the axioms we use to characterize the position value on the class HCS^N . Since we chose examples 4.1 and 4.2 such that they belong to the class Borm et al. considered, the results in this section immediately imply that their axioms are irredundant too.

References

- Aumann RJ, Myerson RB (1988) Endogenous formation of links between players and of coalitions: an application of the Shapley value. In: *The Shapley Value*, Roth AE (ed.), Cambridge University Press, Cambridge, 175-191.
- Berge C (1989) Hypergraphs. North-Holland, Mathematical library, pp. 1-3, 155-162.
- Borm PEM, Owen G, Tijs SH (1991) On the position value for communication situations. To appear in *SIAM J. on Disc. Math.*
- Meessen R (1988) Communication games. Master's Thesis (in Dutch), Department of Mathematics, University of Nijmegen, The Netherlands.
- Myerson RB (1977) Graphs and cooperation in games. *Math. Oper. Res.* 2, 225-229.
- Myerson RB (1980) Conference structures and fair allocation rules. *Intern. J. of Game Theory* 9, 169-182.
- Owen G (1986) Values of graph-restricted games. *SIAM J. Alg. Disc. Meth.* 7, 210-220.
- Shapley LS (1953) A value for n -person games. *Annals of Math. Studies* 28, 307-317.

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- No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', *European Economic Review*, vol. 34, no. 5, 1990, pp. 1011 - 1022.
- No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.
- No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, *Oxford Economic Papers*, vol. 42, no. 3, 1990, pp. 501 - 525.
- No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, *Journal of Time Series Analysis*, vol. 11, no. 3, 1990, pp. 239 - 248.
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- No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, *Journal of Econometrics*, vol. 46, no. 3, 1990, pp. 333 - 346.
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- No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, *Sankhya: The Indian Journal of Statistics*, vol. 52, series B, part. 1, 1990, pp. 1 - 13.
- No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, *Journal of Econometrics*, vol. 48, no. 1/2, 1991, pp. 83 - 117.
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- No. 64 Y. Dai, G. van der Laan, A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for the nonlinear stationary point problem on an unbounded polyhedron, *Siam Journal of Optimization*, vol. 1, no. 2, 1991, pp. 151 - 165.
- No. 65 M. McAleer and C.R. McKenzie, Keynesian and new classical models of unemployment revisited, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 359 - 381.
- No. 66 A.J.J. Talman, General equilibrium programming, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 387 - 397.

- No. 67 J.R. Magnus and B. Pesaran, The bias of forecasts from a first-order autoregression, *Econometric Theory*, vol. 7, no. 2, 1991, pp. 222 - 235.
- No. 68 F. van der Ploeg, Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, *European Economy - The Economics of EMU*, Commission of the European Communities, special edition no. 1, 1991, pp. 136 - 164.
- No. 69 H. Keuzenkamp, A precursor to Muth: Tinbergen's 1932 model of rational expectations, *The Economic Journal*, vol. 101, no. 408, 1991, pp. 1245 - 1253.
- No. 70 L. Zou, The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect, *Journal of Public Economics*, vol. 46, no. 1, 1991, pp. 51 - 89.
- No. 71 E. Bomhoff, Between price reform and privatization: Eastern Europe in transition, *Finanzmarkt und Portfolio Management*, vol. 5, no. 3, 1991, pp. 241 - 251.
- No. 72 E. Bomhoff, Stability of velocity in the major industrial countries: a Kalman filter approach, *International Monetary Fund Staff Papers*, vol. 38, no. 3, 1991, pp. 626 - 642.
- No. 73 E. Bomhoff, Currency convertibility: when and how? A contribution to the Bulgarian debate, *Kredit und Kapital*, vol. 24, no. 3, 1991, pp. 412 - 431.
- No. 74 H. Keuzenkamp and F. van der Ploeg, Perceived constraints for Dutch unemployment policy, in C. de Neubourg (ed.), *The Art of Full Employment - Unemployment Policy in Open Economies*, Contributions to Economic Analysis 203, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1991, pp. 7 - 37.
- No. 75 H. Peters and E. van Damme, Characterizing the Nash and Raiffa bargaining solutions by disagreement point axioms, *Mathematics of Operations Research*, vol. 16, no. 3, 1991, pp. 447 - 461.
- No. 76 P.J. Deschamps, On the estimated variances of regression coefficients in misspecified error components models, *Econometric Theory*, vol. 7, no. 3, 1991, pp. 369 - 384.
- No. 77 A. de Zeeuw, Note on 'Nash and Stackelberg solutions in a differential game model of capitalism', *Journal of Economic Dynamics and Control*, vol. 16, no. 1, 1992, pp. 139 - 145.
- No. 78 J.R. Magnus, On the fundamental bordered matrix of linear estimation, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 583 - 604.
- No. 79 F. van der Ploeg and A. de Zeeuw, A differential game of international pollution control, *Systems and Control Letters*, vol. 17, no. 6, 1991, pp. 409 - 414.
- No. 80 Th. Nijman and M. Verbeek, The optimal choice of controls and pre-experimental observations, *Journal of Econometrics*, vol. 51, no. 1/2, 1992, pp. 183 - 189.

- No. 81 M. Verbeek and Th. Nijman, Can cohort data be treated as genuine panel data?, *Empirical Economics*, vol. 17, no. 1, 1992, pp. 9 - 23.
- No. 82 E. van Damme and W. Güth, Equilibrium selection in the Spence signaling game, in R. Selten (ed.), *Game Equilibrium Models II - Methods, Morals, and Markets*, Berlin: Springer-Verlag, 1991, pp. 263 - 288.
- No. 83 R.P. Gilles and P.H.M. Ruys, Characterization of economic agents in arbitrary communication structures, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 325 - 345.
- No. 84 A. de Zeeuw and F. van der Ploeg, Difference games and policy evaluation: a conceptual framework, *Oxford Economic Papers*, vol. 43, no. 4, 1991, pp. 612 - 636.
- No. 85 E. van Damme, Fair division under asymmetric information, in R. Selten (ed.), *Rational Interaction - Essays in Honor of John C. Harsanyi*, Berlin/Heidelberg: Springer-Verlag, 1992, pp. 121 - 144.
- No. 86 F. de Jong, A. Kemna and T. Kloek, A contribution to event study methodology with an application to the Dutch stock market, *Journal of Banking and Finance*, vol. 16, no. 1, 1992, pp. 11 - 36.
- No. 87 A.P. Barten, The estimation of mixed demand systems, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 31 - 57.
- No. 88 T. Wansbeek and A. Kapteyn, Simple estimators for dynamic panel data models with errors in variables, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 238 - 251.
- No. 89 S. Chib, J. Osiewalski and M. Steel, Posterior inference on the degrees of freedom parameter in multivariate-t regression models, *Economics Letters*, vol. 37, no. 4, 1991, pp. 391 - 397.
- No. 90 H. Peters and P. Wakker, Independence of irrelevant alternatives and revealed group preferences, *Econometrica*, vol. 59, no. 6, 1991, pp. 1787 - 1801.
- No. 91 G. Alogoskoufis and F. van der Ploeg, On budgetary policies, growth, and external deficits in an interdependent world, *Journal of the Japanese and International Economies*, vol. 5, no. 4, 1991, pp. 305 - 324.
- No. 92 R.P. Gilles, G. Owen and R. van den Brink, Games with permission structures: The conjunctive approach, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 277 - 293.
- No. 93 J.A.M. Potters, I.J. Curiel and S.H. Tijs, Traveling salesman games, *Mathematical Programming*, vol. 53, no. 2, 1992, pp. 199 - 211.
- No. 94 A.P. Jurg, M.J.M. Jansen, J.A.M. Potters and S.H. Tijs, A symmetrization for finite two-person games, *Zeitschrift für Operations Research - Methods and Models of Operations Research*, vol. 36, no. 2, 1992, pp. 111 - 123.

No. 95

A. van den Nouweland, P. Borm and S. Tijs, Allocation rules for hypergraph communication situations, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 255 - 268.

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