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Equilibria in Repeated
Prisoner's Dilemma

by
Eric van Damme

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Reprinted from Journal of Economic Theory,
Vol. 47, No. 1, February 1989

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Renegotiation-Proof Equilibria in Repeated Prisoners' Dilemma

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In this note it is shown that the requirement of weak renegotiation proofness does not eliminate any subgame perfect equilibrium outcome of the repeated prisoner's dilemma. Specifically it is demonstrated that, when players are sufficiently patient, any feasible and individually rational outcome can be sustained by means of an equilibrium in which the punishing player always profits by carrying out the punishment. *Journal of Economic Literature* Classification Number: 026. © 1989 Academic Press, Inc.

1. INTRODUCTION

It is well known that repeating a game may lead to the emergence of new equilibria. Recently, Fudenberg and Maskin [9] have shown that, when either there are only 2 players or when a full dimensionality condition holds, any feasible and individually rational outcome of the one-shot game can be obtained by a subgame perfect equilibrium of the infinitely repeated (discounted) supergame provided that the players are sufficiently patient. Benoit and Krishna [4] have shown that this "Perfect Folk Theorem" also holds for finite repetitions provided that the underlying game has at least 2 Nash equilibrium payoffs for each player.

When one reviews the proof of the Perfect Folk Theorem one sees that this result depends crucially on the fact that according to the subgame perfectness concept all kinds of self-lacerating threats are credible: Many equilibrium paths can be sustained only by threats that hurt the punished as well as those who punish. Such equilibria are unstable whenever players cannot suppress the temptation to renegotiate. In this case, a player will deviate on the expectation that she can convince her opponents to

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"forgive" her since it actually (after the deviation has occurred) is in their best interest to do so. The game of Fig. 1 may illustrate this phenomenon.

$$\begin{array}{cc} 1, 1 & 0, 0 \\ 0, 0 & -1, -1 \end{array}$$

FIGURE 1

If the interests of the players coincide in this game, there is a unique Nash equilibrium and this results in the Pareto efficient outcome (1, 1). However, each player can only guarantee a payoff of 0, hence, the Perfect Folk Theorem [9, Theorem 1] implies that any payoff (x, x) with $x \in [0, 1]$ can be approximately obtained by a subgame perfect equilibrium of the discounted supergame if the discount factor is sufficiently close to 1. Specifically, for an integer T let x be given by

$$x = -1(1 - \delta^T) + 1\delta^T, \quad (1)$$

then the following strategy pair is a subgame perfect equilibrium of the δ -discounted supergame as long as $x \geq 0$.

- (a) First play T times $(-1, -1)$; thereafter, always play $(1, 1)$;
- (b) If there are any deviations restart (a).

Equation (1) implies that the strategy from (2) results in the (normalized) discounted payoff of x . Furthermore, as δ tends to 1, the set of x that can be written as in (1) becomes dense in $[0, 1]$. Take an equilibrium as in (2) with $T > 0$. If one is playing against a machine that is programmed to play according to this equilibrium, there is nothing one can do but go along and play the dominated action for T periods. However, an agreement to play the strategies from (2) does not seem to be self-enforcing: Both players have an incentive to propose¹ to skip the initial punishment phase and when one player makes such a proposal the opponent should agree since it is in his best interest to do so. Hence, only repeated play of (1, 1) seems self-enforcing.

Joe Farrell [6] has proposed the concept of renegotiation-proofness to exclude the equilibria in which planned punishments hurt the punishers themselves. This concept requires that, in equilibrium, no continuation payoff be Pareto dominated by another continuation payoff. For the

¹ This terminology seems to presume that communication between the players is possible and the reader may argue that in this case the relevant solution concept is Aumann's correlated equilibrium [3] rather than Nash equilibrium. However, this is irrelevant: Even if no player can articulate the proposal, the logic underlying the argument should convince both players not to punish each other (and themselves).

formal definition see Section 2. It is easily seen that in the discounted supgame of Fig. 1 only playing (1, 1) at every stage is renegotiation-proof. Farrell conjectured that the renegotiation-proofness requirement generally would greatly reduce the set of subgame perfect equilibrium outcomes. Specifically he conjectured that in repeated prisoners' dilemma only the repetition of the one-shot Nash outcome can be renegotiation-proof. In Section 3 we will show that the latter conjecture is wrong: Requiring renegotiation-proofness does not reduce the set at all.² The key point is that one need not revert to grim punishments to sustain cooperation in repeated prisoners' dilemma, rather one can punish by using some kind of Tit-For-Tat strategy: If you deviate from the cooperative path (i.e., if (C, D) occurs), I demand that you allow me to defect once (i.e., to play (D, C)) in order to restore the balance, and only after you have granted me my most preferred payoff will I indeed return to the cooperative path. For low enough discount rates this is indeed an equilibrium and since the punisher profits by punishing, he clearly has an incentive to carry it out, hence, the equilibrium is renegotiation-proof.

2. RENEGOTIATION-PROOF EQUILIBRIA

Let $\Gamma = (S_1, \dots, S_n, U_1, \dots, U_n)$ be an n -person game where S_i denotes the set of mixed³ strategies of player i and U_i is this player's payoff function, $U_i: S \rightarrow \mathbb{R}$ where $S = \prod_{i=1}^n S_i$. For $\delta \in (0, 1)$ denote by $\Gamma(\delta)$ the infinite repetition of Γ that starts at time 0 and in which all players discount future payoffs with δ . Since mixed actions are assumed to be observable ex post, a history (subgame) at time $t \geq 1$ can be identified with an element $(s^0, s^1, \dots, s^{t-1})$ of the t -fold Cartesian product of S . For convenience, we set $S^0 = \{0\}$ and $H = \bigcup_t S^t$ denotes the set of all histories. Σ_i is the set of strategies of player i in $\Gamma(\delta)$, hence, $\Sigma_i = S_i^H$ (the set of mappings from H to S_i) and $\Sigma = \prod_i \Sigma_i$ is the set of strategy profiles. A path π is an element of S^∞ . If $\pi = \{s^t\}_{t=0}^\infty$, then we write $\pi^T = \{s^t\}_{t=0}^T$ and $\pi^{-T} = \{s^t\}_{t=0}^{T-1}$. The path $\pi(\sigma)$ induced by the strategy profile σ is inductively defined by

$$\pi(\sigma) = \{s^t(\sigma)\}_{t=0}^\infty \text{ with } s^0(\sigma) = \sigma(0) \quad \text{and} \quad s^t(\sigma) = \sigma(\pi^{-t}(\sigma)) \quad (t \geq 1).$$

² In their recent paper [7], Farrell and Maskin introduce several notions of renegotiation-proofness and this sentence is correct for their weakest concept, i.e., the one introduced is Farrell [6]. If $\delta \geq \frac{1}{2}$, then only subsets of the Pareto efficient frontier of prisoners' dilemma (Fig. 3) satisfy the strongest renegotiation-proofness requirement of [7].

³ As in Fudenberg and Maskin [9] we do not distinguish between mixed and pure strategies, hence we assume that the private randomizations from period t are publicly observable before the beginning of period $t+1$. This assumption is important; i.e., the main result no longer holds if this assumption is dropped (see the discussion at the end of Section 3).

Players i 's (normalized) discounted payoff in $\Gamma(\delta)$ associated with path $\pi = \{s^t\}_t$ is defined by

$$U_i^\delta(\pi) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t U_i(s^t) \quad (3)$$

and this player's payoff associated to σ is $U_i^\delta(\sigma) \equiv U_i^\delta(\pi(\sigma))$. If $\sigma_i \in \Sigma_i$ and h is a history, then σ_{ih} , the continuation of σ_i after h , is defined by

$$\sigma_{ih}(h') = \sigma_i(h, h') \quad \text{for } h' \in H,$$

with the understanding that $(h, 0) = (0, h) = h$. The profile σ is a subgame perfect equilibrium of $\Gamma(\delta)$ if σ_h is a Nash equilibrium for every h , i.e.,

$$U_i^\delta(\sigma_h) \geq U_i^\delta(\sigma_h \setminus \sigma'_i) \quad \text{for all } h, \sigma'_i, \quad (4)$$

where $\sigma_h \setminus \sigma'_i$ denotes the profile in which all players j use σ_{jh} except player i who plays σ'_i . In Section 3 attention will be restricted to simple strategy profiles (Abreu [1]): Let $\Pi = \{\pi^k\}_{k=0}^n$ be a collection of $n+1$ paths with $\pi^k = \{s^{kt}\}_{t=0}^{\infty}$. Then the simple strategy profile induced by Π (to be denoted by $\sigma(\Pi)$) is described by

start with π^0 ,

if player i is the unique deviator restart π^i .

Equation (4) implies that $\sigma(\Pi)$ is a subgame perfect equilibrium if and only if

$$U_i^\delta(\pi^{kt}) \geq (1 - \delta) \max_{s_i} U_i^\delta(s^{kt} \setminus s_i) + \delta U_i^\delta(\pi^i) \quad \text{all } i, k, t. \quad (5)$$

The payoff vector $U(\sigma_h)$ Pareto dominates $U(\sigma_{h'})$ if

$$U_i^\delta(\sigma_h) > U_i^\delta(\sigma_{h'}) \quad \text{for all } i.$$

σ is said to be a renegotiation-proof equilibrium (or RPE) Farrell [6] (if σ is subgame perfect and no continuation payoff $U^\delta(\sigma_h)$ is dominated by another continuation payoff $U^\delta(\sigma_{h'})$). If an equilibrium σ is not renegotiation-proof, $\sigma_{h'}$, then there exist subgames h and h' such that when h' is reached the players would strictly prefer to continue with (and to renegotiate to) σ_h . Hence, it seems that σ_h and $\sigma_{h'}$ cannot be considered self-enforcing at the same time and equilibria that are not RPE seem inherently unstable. Note that the RPE concept requires only internal consistency of the equilibrium: No payoff vector that the players consider as feasible (i.e., that can occur in equilibrium) can be dominated by another payoff vector that is considered feasible. External dominance is not excluded and it is perfectly well possible that one RPE is dominated by

another. Namely, not that any profile σ that requires that always the same equilibrium of Γ be played is an RPE of $\Gamma(\delta)$ for every δ , so that any subgame consistent equilibrium (Selten [15]) is an RPE, hence, existence of RPE poses no problems. Consequently, if the static game has equilibria (1, 1) and (2, 2) then always playing (2, 2) will be an RPE as well as always playing (1, 1). This already shows that the RPE concept of this paper is not a final answer to the question of what are "reasonable" equilibria in repeated games; i.e., the current concept may be too weak to capture the idea of renegotiation-proofness fully. It is important to complement the RPE idea with a concept of external dominance and the reader is urged to read recent papers by Farrell and Maskin [7] and Bernheim and Ray [5] in which several such concepts are proposed. (The reader should also be aware of work of Pearce [12] who does not view the RPE condition as being necessary for renegotiation-proofness).

The RPE concept is weaker than the concept of strong perfect equilibrium that has been introduced in Rubinstein [14]. The latter requires that there not exist a subgame and a coalition who can improve its payoffs by switching to a different continuation strategy in the subgame. It is easy to see that every strong perfect equilibrium is an RPE, but that the converse is not true. In fact, strong perfect equilibria need not exist in discounted supergames, not even in the 2-person case (Rubinstein [14]). Finally, it should be noted that not every subgame perfect equilibrium is an RPE. The repetition of the game of Fig. 1 has always playing (1, 1) as the unique RPE of $\Gamma(\delta)$ for every value of δ : Since players' payoffs are identical in Γ , the second condition of an RPE forces all continuation payoffs to be identical, hence, current actions cannot influence future payoffs and an equilibrium of Γ (therefore (1, 1)) must be played at every stage.

3. PRISONERS' DILEMMA

The version of prisoners' dilemma that we will consider is given in Fig. 2:

	C	D
C	4, 4	0, 5
D	5, 0	1, 1

FIGURE 2

For this prisoners' dilemma it can be shown that always playing (D, D) is the unique equilibrium of $\Gamma(\delta)$ for every $\delta < \frac{1}{4}$ (Sorin [16], van Damme

[17]). When $\delta \geq \frac{1}{4}$ new subgame perfect equilibria emerge. For example, the trigger strategy pair

$$\sigma_i(h) = \begin{cases} C & \text{if } h=0 \text{ or } h=(C, C)^t \text{ for some } t \\ D & \text{otherwise} \end{cases}$$

(start with C and continue with C as long as (C, C) was played at all previous stages, otherwise switch to D forever) is subgame perfect in this case (Friedman [8]). However, this is not an RPE since the continuation payoff after (D, D) (viz. $(1, 1)$) is dominated by the continuation payoff after (C, C) (viz. $(4, 4)$). The following strategy pair describes an RPE that results in the cooperative outcome at every stage: Each player starts with C and continues to play cooperatively as long as the other does so; If player i defects in period t , (and player j does not), then player j will defect until player i chooses C . As soon as player i has shown repentance by playing C , player j forgives the initial defection and also returns to the cooperative mode of play. Formally this behavior is described by the simple strategy profile induced by $\Pi = (\pi^0, \pi^1, \pi^2)$ with

π^0 : always play (C, C)

π^1 : play (C, D) at $t=0$, otherwise play (C, C)

π^2 : play (D, C) at $t=0$, otherwise play (C, C) .

To verify that $\sigma(\Pi)$ is an RPE when $\delta \geq \frac{1}{4}$, note that the continuation payoffs are given by

$$U^\delta(\pi^{kt}) = \begin{cases} (4, 4) & \text{if } k=0 \text{ or } t>0, \\ (4\delta, 5-\delta) & \text{if } k=1 \text{ and } t=0, \\ (5-\delta, 4\delta) & \text{if } k=2 \text{ and } t=0, \end{cases}$$

so that no continuation payoff dominates another. By symmetry, only 3 constraints from (2.3) are really relevant: It should not be profitable for player 1 to deviate from π^0 , π^1 , or π^2 at $t=0$. These constraints are given by, respectively,

$$4 \geq (1-\delta) \cdot 5 + \delta \cdot 4\delta$$

$$4\delta \geq (1-\delta) \cdot 1 + \delta \cdot 4\delta$$

$$5-\delta \geq (1-\delta) \cdot 5 + \delta \cdot 4\delta$$

and all 3 constraints are satisfied for $\delta \geq \frac{1}{4}$.

The following proposition generalizes the above observation: If mixed actions can be observed ex post, then any feasible and individually rational payoff of Γ can be obtained by means of an RPE of $\Gamma(\delta)$ for every $\delta \geq \frac{1}{4}$.

PROPOSITION. Let Γ be the prisoners' dilemma of Fig. 2, let X be the convex hull of $\{(1, 1), (0, 5), (5, 0), (4, 4)\}$ and let X^+ be the points $x \in X$ with $x_1 \geq 1$ and $x_2 \geq 1$. If $x \in X^+$ and $\delta \geq \frac{1}{4}$, there exists an RPE σ of $\Gamma(\delta)$ with $U^\delta(\sigma) = x$.

Proof. Let $\delta \geq \frac{1}{4}$ be fixed. For $s_i \in S_i$, write $p_i = s_i(C)$ for the probability that s_i assigns to C . Then the expected payoff $U(s)$ can be written as

$$U(s) = (1 + 4p_2 - p_1, 1 + 4p_1 - p_2) \quad (6)$$

and it follows that any $x \in X$ can be obtained as the expected payoff of some mixed strategy pair. For $p \in [0, 1]$, define

$$S(p) = \{s \in S; s_i(C) = p \text{ and } s_j(C) \leq p \text{ (} i \neq j \in \{1, 2\} \text{)}\},$$

and

$$U(p) = \{U(s); s \in S(p)\}.$$

Note that $U(p)$ consists of 2 negatively sloped line segments that have exactly one (diagonal) point in common, and that $\{U(p); p \in [0, 1]\}$ is a partition of X . ($U(0) = \{(1, 1)\}$, $p = 1$ yields the Pareto efficient frontier of X , see Fig. 3).

Take $p \in [0, 1]$ fixed. We will first construct paths π^1 and π^2 with respective payoffs

$$\left(1, 1 + \frac{15}{4}p\right) \quad \text{and} \quad \left(1 + \frac{15}{4}p, 1\right). \quad (7)$$

FIGURE 3

(These payoffs are the endpoints of $U(p) \cap X^+$.) Choose T such that

$$\delta^{T+1} < \frac{1}{4} \leq \delta^T$$

and define $q \in [0, p]$ by

$$q = \frac{p(1 - 4\delta^{T+1})}{4(1 - \delta)\delta^T}.$$

Then the path $\pi^i = \{s^{it}\}_{t=0}^\infty$ is defined by

$$s_i^{it} = p \quad \text{for all } t$$

$$s_j^{it} = \begin{cases} 0 & \text{for } t = 0, \dots, T-1 \\ q & \text{for } t = T \\ p & \text{for } t > T \end{cases} \quad (j \neq i).$$

(Hence, player i cooperates with probability p at all times, player j first defects for T periods, then cooperates once with probability q and thereafter always cooperates with probability p .) It is easily verified that the paths π^1 and π^2 result in the payoffs from (7). We claim that the simple strategy profiles induced by (π^i, π^j) ($i, j = 1, 2$) both are subgame perfect equilibria. The relevant constraints from (5) are

$$U_i^\delta(\pi^{ki}) \geq (1 - \delta) \max_{s_i} U_i^\delta(s^{ki} \setminus s_i) + \delta \quad \text{all } i, k, t,$$

which, after substituting (6), is seen to be equivalent to

$$\delta U_i^\delta(\pi^{k,t+1}) \geq (1 - \delta) s_i^{kt}(C) + \delta \quad \text{all } i, k, t. \quad (8)$$

By symmetry, it suffices to verify (8) for $k = 1$. First take $i = 1$. Then the left hand side of (8) is nondecreasing in t while the right hand side is constant, so that it suffices to check $t = 0$. By substituting

$$1 = U_1^\delta(\pi^{1,0}) = (1 - \delta)(1 - p) + \delta U_1^\delta(\pi^{1,1})$$

into (8) one sees that (8) is satisfied with equality for $i = k = 1$ and $t = 0$, hence, player 1 cannot profitably deviate from π^1 . A similar argument establishes that player 2 cannot profitably deviate from π^1 : This player's continuation payoffs are nonincreasing in t , while his gain from deviation is nondecreasing, so that it suffices to show that he cannot profitably deviate at $T + 1$, i.e.,

$$\delta(1 + 3p) \geq (1 - \delta)p + \delta,$$

and this condition is satisfied for $\delta \geq \frac{1}{4}$.

Hence, if $\Pi^i = (\pi^i, \pi^1, \pi^2)$, then $\sigma(\Pi^i)$ is a subgame perfect equilibrium. In fact, this is an RPE since all stage payoffs associated to $\sigma(\Pi^i)$ belong to $U(p)$ and no payoff in this set dominates another.

Next, let $x \in U(p) \cap X^+$ and assume $x_1 \leq x_2$. Then $x = R(s)$ where $s_1(C) = p$ and $p_2 = s_2(C) \leq p$. Let $\tau \in \mathbb{N}$ be such that

$$\delta^{\tau+1} < \frac{p_2}{p} \leq \delta^\tau$$

and define $r \in [0, p]$ by means of

$$r = \frac{p_2 - \delta^{\tau+1}p}{(1-\delta)\delta^\tau}.$$

Then x results when the following path $\pi^0 = \{s^{0t}\}_t$ is played

$$\begin{aligned} s_1^{0t}(C) &= p && \text{for all } t \\ s_2^{0t}(C) &= \begin{cases} 0 & \text{for } t=0, \dots, \tau-1 \\ r & \text{for } t=\tau \\ p & \text{for } t>\tau, \end{cases} \end{aligned}$$

and similarly as above it can be verified that the simple strategy profile induced by $\Pi = (\pi^0, \pi^1, \pi^2)$ is a subgame perfect equilibrium. When π^0, π^1 , and π^2 have the same value of p , then all continuation payoffs belong to $U(p)$, hence, there exist no dominance relationships and $\sigma(\Pi)$ is an RPE. Since $\bigcup_p (U(p) \cap X^+) = X^+$, this shows that any $x \in X^+$ can be obtained by means of an RPE. ■

Note that, for $p=1$, the above equilibrium is a strong perfect equilibrium, hence, we have shown that any point on the Pareto efficient frontier of X^+ can be attained by means of a strong perfect equilibrium of $\Gamma(\delta)$ for any $\delta \geq \frac{1}{4}$.

To conclude this section we show that the proposition is not correct if mixed strategies cannot be observed. Let Γ be the prisoners' dilemma from Fig. 2 and let $\Gamma^*(\delta)$ be the infinite repetition (with discount factor δ) in which players only get to hear the outcomes of the private lotteries. We claim that the payoff $(\frac{3}{2}, 2)$, resulting from, e.g., $(\frac{1}{2}(C+D), C)$, is not even a Nash equilibrium payoff of $\Gamma^*(\frac{1}{4})$ (also see Sorin [16] in which the set of all Nash payoffs is computed). Namely, assume Π is a path in $\Gamma^*(\frac{1}{4})$ resulting in $(\frac{3}{2}, 2)$. Then player 2 has to choose C in every round along Π since the payoff is efficient. Furthermore, player 1 cannot start with D , as in this case player 2's payoff is bounded by $\frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 4 = 1$. Similarly, player 1 cannot start with C as then player 2 would get at least 3. Hence, player 1 should randomize. Let (v_i, w_i) be the average continuation payoff along

Π at $t = 1$ after (i, C) ($i = C, D$). Then (v_i, w_i) is a convex combination of $(4, 4)$ and $(5, 0)$ and, if Π is a Nash equilibrium path, player 1 should be indifferent between C and D at $t = 0$. Hence,

$$4 \leq v_C, v_D \leq 5 \quad \text{and} \quad \frac{3}{4} \cdot 4 + \frac{1}{4} \cdot v_C = \frac{3}{4} \cdot 5 + 1 \cdot v_D$$

but these conditions clearly are inconsistent. Fudenberg and Maskin [9, Theorem 5] show that, for subgame perfect equilibria, in the limit as δ tends to 1 it is immaterial whether mixed strategies or only the outcomes can be observed; i.e., the sets of subgame perfect equilibrium payoffs of $\Gamma(\delta)$ and $\Gamma^*(\delta)$ have the same limit as δ tends to 1. It is not clear whether this result also holds for RPE's. The proof of the proposition however shows that, as δ tends to 1, the efficient points can be sustained as an RPE also in the case in which only outcomes of lotteries are observable.

4. CONCLUDING REMARKS

The preceding sections may have given the impression that the renegotiation-proofness requirement only bites for games in which the set of feasible and individually rational payoff vectors is less than full dimensional. The game of Fig. 4 may show that this impression is misleading.

2, 2	1, 0
0, 0	0, 0

FIGURE 4

In this game, player 1's minmax level is 1 while player 2 can only guarantee a payoff of 0, hence, the set of feasible and individually rational payoff vectors is the convex hull of $\{(1, 0), (1, 1), (2, 2)\}$. We claim that, for any value of δ , any RPE of $\Gamma(\delta)$ yields player 2 at least a normalized discounted payoff of 1. To verify this claim, we will use the fact that the set of payoffs that can be obtained by means of RPEs is closed, a fact that can be demonstrated by adapting the proof of Harris [11] that the set of subgame perfect equilibrium payoffs is closed. (also see Theorem 8.5.6 in van Damme [17; cf. Lemma 1 in Farrell and Maskin [7]). Let σ be an RPE that is worst for player 2, hence,

$$U_2^\delta(\sigma) = \min\{U_2^\delta(\sigma'); \sigma' \text{ is RPE of } \Gamma(\delta)\}$$

and let $\pi = \pi(\sigma)$ be the path induced by σ . Since π^1 (the subpath of π starting at $t = 1$) is also sustainable by an RPE we must have

$$U_2^\delta(\pi^1) \geq U_2^\delta(\pi).$$

Since σ is an RPE, the continuation payoff $U^\delta(\pi^1)$ cannot be dominated by $U^\delta(\pi)$, hence,

$$U_1^\delta(\pi^1) \leq U_1^\delta(\pi). \quad (9)$$

Let $s = \sigma(0)$, then

$$U_1^\delta(\pi) = (1 - \delta)U_1(s) + \delta U_1^\delta(\pi^1),$$

and substituting (9) yields

$$U_1^\delta(\pi) \leq U_1(s). \quad (10)$$

Since σ is a subgame perfect equilibrium, the left hand side of (10) is at least player 1's minmax level of 1; consequently player 1 must choose his first action with probability at least $\frac{1}{2}$ at $t = 0$. The constraint that it not be profitable for player 2 to deviate at $t = 0$ implies

$$U_2^\delta(\pi) \geq (1 - \delta) \max_{s_2'} U_2(s \setminus s_2') + \delta U_2^\delta(\pi)$$

or, equivalently,

$$U_2^\delta(\pi) \geq \max_{s_2'} U_2(s \setminus s_2'). \quad (11)$$

Given that player 1 chooses his first action with probability at least $\frac{1}{2}$, it follows that the right hand side of (11) is at least 1 and this establishes the claim.

It remains to be investigated how powerful the RPE concept is in general. For advances in this direction we refer to Farrell and Maskin [7]. Let us just remark that the concept can be quite powerful in specific contexts. For example, consider the Green/Porter oligopoly model [10, 13] in which players cannot observe opponents' outputs and in which there is demand uncertainty. In Abreu *et al.* [2], the optimal symmetric subgame perfect equilibrium has been characterized by means of 2 price regions: If price in the previous period was low the players should produce high outputs, whereas output should be low if price was high. This equilibrium, however, is not an RPE⁴ since the (expected) continuation payoff in a period requiring high outputs is dominated by the continuation payoff of a period with low outputs. If price in the previous period was unexpectedly low, then all players have an incentive to argue that this low price was due only to weak demand and that they should continue producing low outputs. Hence, in this model, the unique RPE requires producing the Cournot output at every stage.

⁴ In this stochastic context, "continuation payoff" should be replaced by "expected continuation payoff" in the definition of RPE.

At the other extreme there is the repeated Cournot oligopoly model in which there is no uncertainty and in which output levels can be observed. In this model one has the unsatisfactory result that the monopoly price can be sustained by a subgame perfect equilibrium for any number of oligopolists, provided that they are sufficiently patient. For this model, the RPE concept leads to the conclusion that (even for δ arbitrarily close to 1) the monopoly price can be sustained only when there are at most 3 players and that, as the number of players tends to infinity, only prices close to the Cournot price can be sustained. These issues are more thoroughly discussed in my paper [18].

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